

	<p>Pimpri Chinchwad Education Trust's Pimpri Chinchwad College of Engineering An Autonomous Institute (Permanently affiliated to Savitribai Phule Pune University)</p>	<p>SET – I</p>
<p align="center">End Term Examination</p>		

First Year B. Tech. (Mech / Civil / E & TC / Comp / Comp-Regional / IT / AI-ML)

Multivariate Calculus [BSC]

[BFE2206/BFE2209]

Even Semester (2023-24)

Total No. of Questions-04

Total No. of Printed Pages-02

[Time: 2 Hr. 30 min.]

[Max. Marks: 80]

PRN									
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Instructions:

IMP: Verify that you have received a question paper with correct course, code, branch etc.

- All questions are compulsory.
- Assume suitable data wherever necessary.
- Neat labelled diagrams must be drawn wherever necessary.
- Figure to right indicates full marks.
- Use of a non-programmable calculator is allowed.

		Marks
Q.1	Attempt the following	[15M]
A	If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, Prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u)$	[5M]
B	If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	[5M]
C	Find the Fourier series expansion of the function, $f(x) = \frac{x^2+1}{2}, -\pi \leq x \leq \pi$ and $f(x+2\pi) = f(x)$	[5M]
Q.2	Attempt the following	
A)	Attempt any <u>two</u> of the following	[8M]
	i) Evaluate $\int_0^{\infty} \sqrt[4]{x} e^{-\sqrt{x}} dx$	[4M]
	ii) Evaluate $\int_0^1 x^3 (1-\sqrt{x})^5 dx$	[4M]
	iii) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$	[4M]

B)	Attempt any <u>two</u> of the following	[12M]
	i) Prove that $\int_0^1 \frac{x^a - x^b}{\log x} dx = \log \left(\frac{a+1}{b+1} \right); a > 0, b > 0$	[6M]
	ii) Show that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} [\text{erf}(b) - \text{erf}(a)]$	[6M]
	iii) If $f(x) = \int_0^x (x-t)^2 G(t) dt$ then show that $\frac{d^3 f}{dx^3} - 2G(x) = 0$	[6M]
Q.3	Attempt the following	
A)	Attempt any <u>two</u> of the following	[8M]
	i) Evaluate $\int_0^1 \int_{x^2}^x xy(x+y) dx dy$	[4M]
	ii) Evaluate $\int_{-1}^1 dz \int_0^z dx \int_{x-z}^{x+z} (x+y+z) dy$	[4M]
	iii) Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.	[4M]
B)	Attempt any <u>two</u> of the following	[12M]
	i) Evaluate $\iiint x^2 yz dx dy dz$ throughout the volume bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	[6M]
	ii) Evaluate $\iint y dx dy$ over the area bounded by $y = x^2$ and $x + y = 2$	[6M]
	iii) Evaluate $\iiint_V z^2 dx dy dz$ over the volume bounded by surfaces $x^2 + y^2 = a^2, x^2 + y^2 = z$ and $z = 0$.	[6M]
Q.4	Attempt the following	
A)	Attempt any <u>three</u> of the following	[18M]
	i) Find the volume bounded by $x^2 + y^2 = 4z$ cut off by the plane $z = 4$	[6M]
	ii) Find area included between $y^2 = 4x$ and $2x - y - 4 = 0$.	[6M]
	iii) Find the area in XY plane bounded by the Lemniscate $r^2 = a^2 \cos 2\theta$	[6M]
	iv) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 4$ and $z = 0$	[6M]
B)	Attempt any <u>one</u> of the following	[7M]
	i) Find the centre of gravity (C.G.) of the area of the cardioid $r = a(1 + \cos \theta)$	[7M]
	ii) Find moment of inertia of the area included between the curves $y^2 = x$ and $x^2 = y$.	[7M]

**** End of Question Paper****

	<p>Pimpri Chinchwad Education Trust's Pimpri Chinchwad College of Engineering An Autonomous Institute (Affiliated to Savitribai Phule Pune University) पिंपरी चिंचवड एज्युकेशन ट्रस्ट पिंपरी चिंचवड कॉलेज ऑफ इंजिनीअरिंग (सावित्रीबाई फुले पुणे विद्यापीठाशी संलग्नित स्वायत्त संस्था)</p>	SET – I संच - १
		SEMESTER- II सत्र - २
End Term Examination [अंतिम सत्र परीक्षा]		

**First Year B. Tech. [प्रथम वर्ष बी. टेक.] (Mech / Civil / E & TC / Comp /
 Comp-Regional / IT / AI-ML)
 Multivariate Calculus [BSC]**

[BFE2206/BFE2209]

Even Semester [सम सत्र] (2023-24)

Total No. of Questions: 04

एकूण प्रश्न संख्या: ४

Time: 2 Hr. 30 min.

वेळ : २ तास ३० मिनिटे

Total No. of Printed Pages: 04

एकूण छापील पाने: ०४

Max. Marks: 80

कमाल गुण : ८०

PRN									
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Instructions:

IMP: Verify that you have received a question paper with correct course, code, branch etc.

- All questions are compulsory.
- Assume suitable data wherever necessary.
- Neat labelled diagrams must be drawn wherever necessary.
- Figure to right indicates full marks.
- Use of a non-programmable calculator is allowed.
- You are allowed to write the answers in Marathi or English or Both.

सूचना:

महत्वाचे: आपल्याला योग्य ती प्रश्नपत्रिका मिळाली आहे का ते तपासून घ्या.

- सर्व प्रश्न अनिवार्य आहेत.
- आवश्यक तेथे योग्य डेटा गृहीत धरा.
- योग्य जागी व्यवस्थित नामनिर्देशित केलेली आकृती काढणे आवश्यक आहे.
- व्यवस्थित नामनिर्देशित केलेल्या आकृतीला पूर्ण गुण मिळतील.
- नॉन-प्रोग्रामेबल कॅल्क्युलेटर वापरण्यास परवानगी आहे.
- प्रश्नांची उत्तरे मराठी वा इंग्रजी वा दोन्ही भाषेत लिहण्याची मुभा आहे.

BFE2206/BFE2209: प्रश्नपत्रिका [मराठी भाषा]

प्र. 1	खालील प्रश्न सोडवा	[15M]
अ	जर $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, तर सिद्ध करा $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u)$	[5M]
ब	जर $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ तर शोधा $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	[5M]
क	Fourier Series विस्तार शोधा, $f(x) = \frac{x^2+1}{2}, -\pi \leq x \leq \pi$ आणि $f(x+2\pi) = f(x)$	[5M]
प्र. 2	खालील प्रश्न सोडवा	
अ	खालीलपैकी कोणतेही <u>दोन</u> प्रश्न सोडावा	[8M]
	i) सोडावा $\int_0^{\infty} \sqrt[4]{x} e^{-\sqrt{x}} dx$	[4M]
	ii) सोडावा $\int_0^1 x^3 (1-\sqrt{x})^5 dx$	[4M]
	iii) सोडावा $\int_0^{\infty} \frac{x^4}{4^x} dx$	[4M]
ब	खालीलपैकी कोणतेही <u>दोन</u> प्रश्न सोडावा	[12M]
	i) सिद्ध करा $\int_0^1 \frac{x^a - x^b}{\log x} dx = \log\left(\frac{a+1}{b+1}\right); a > 0, b > 0$	[6M]
	ii) दाखवा की $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} [\text{erf}(b) - \text{erf}(a)]$	[6M]
	iii) जर $f(x) = \int_0^x (x-t)^2 G(t) dt$ तर दाखवा की $\frac{d^3 f}{dx^3} - 2G(x) = 0$	[6M]
प्र. 3	खालील प्रश्न सोडवा	
अ	खालीलपैकी कोणतेही <u>दोन</u> प्रश्न सोडावा	[8M]
	i) सोडावा $\int_0^1 \int_{x^2}^x xy(x+y) dx dy$	[4M]
	ii) सोडावा $\int_{-1}^1 dz \int_0^z dx \int_{x-z}^{x+z} (x+y+z) dy$	[4M]
	iii) सोडावा $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ वर्तुळांमधील कंकणाकृती प्रदेशावर $x^2 + y^2 = 4$ आणि $x^2 + y^2 = 9$.	[4M]
ब	खालीलपैकी कोणतेही <u>दोन</u> प्रश्न सोडावा	[12M]
	i) सोडावा $\iiint x^2 yz dx dy dz$ अंतर्गत Volume = 0, $y = 0, z = 0$ आणि $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ बांधलेले.	[6M]
	ii) सोडावा $\iint y dx dy$ सीमा असलेल्या क्षेत्रावर $y = x^2$ आणि $x + y = 2$	[6M]
	iii) सोडावा $\iiint_V z^2 dx dy dz$ ज्यांच्या सतहे $x^2 + y^2 = a^2, x^2 + y^2 = z$ आणि $z = 0$ ने मर्यादा ठरविली आहे.	[6M]
प्र. ४	खालील प्रश्न सोडवा	[xM]

अ	खालीलपैकी कोणतेही <u>तीन</u> प्रश्न सोडावा	[18M]
	i) $x^2 + y^2 = 4z$ आणि plane $z = 4$ द्वारे कट केलेले area शोधा	[6M]
	ii) $y^2 = 4x$ आणि $2x - y - 4 = 0$ मधील क्षेत्र शोधा.	[6M]
	iii) XY plane आणि Leminiscate $r^2 = a^2 \cos 2\theta$ ने बांधलेले क्षेत्र शोधा	[6M]
	iv) Cylinder $x^2 + y^2 = 4$ आणि plane $y + z = 4, z = 0$ ने बांधलेला volume शोधा	[6M]
ब	खालीलपैकी कोणताही <u>एक</u> प्रश्न सोडावा	[7M]
	i) Cardioide $r = a(1 + \cos \theta)$ क्षेत्राच्या केंद्रबिंदू (C.G.) शोधा	[7M]
	ii) Moment of inertia शोधा ज्यामध्ये $y^2 = x$ आणि $x^2 = y$ या वर्तुळांमध्ये आलेले क्षेत्र समाविष्ट आहे.	[7M]

BFE2206/BFE2209: Question Paper [English]

		Marks
Q.1	Attempt the following	[15M]
A	If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, Prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u)$	[5M]
B	If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	[5M]
C	Find the Fourier series expansion of the function, $f(x) = \frac{x^2+1}{2}, -\pi \leq x \leq \pi$ and $f(x + 2\pi) = f(x)$	[5M]
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	ii) Evaluate $\iint y dx dy$ over the area bounded by $y = x^2$ and $x + y = 2$	[6M]
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Q.4	Attempt the following	
A)	Attempt any three of the following	[18M]
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**** End of Question Paper****