Be Like Bernoulli

Step into the shoes of a researcher and write your very own treatise on a specific topic pertaining to mathematics and related fields.

Problems (Choose any one):

1. Use Einstein's theory of special relativity to explain why time can be considered a dimension. Explore the mathematics of Minkowski Space-Time to do so. Explain why the Lorentz transformation is a linear transformation. Show how time travel is physically possible by proving it mathematically using the Lorentz transformation (Brownie points for deriving the infamous $E = mc^2$).

2. Clearly explain the Josephus problem and, given that the number of soldiers is n, derive a function $J(n)$ for computing the number associated with the survivor.

3. Derive the equation for the line of best fit using Least squares interpolation, which requires knowledge of partial derivatives. Explain partial derivatives both mathematically and visually. Use basic linear algebra to determine the conditions for which the line of best fit cannot be found.

4. Consider the generalized Tower of Hanoi problem:
    a. Derive an algorithm to determine the minimum number of moves required to move n disks across p pegs.
    b. Prove the algorithm's correctness and establish an upper bound on the time complexity of the algorithm with respect to n and p by denoting the same with use of a recurrence relation.

Let

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}\ ,\ s \in C$$

Using the Dirichlet series representation of the same and the assumption that the real part of s is greater than 1 i.e. R(s) > 1, prove that

$$\zeta(s) = \prod_{p\ prime} \frac{1}{(1-p^{-s})}$$

5. Explain the challenges in computing
   $\int_{-\infty}^{\infty} e^{-x^2} dx$
   Evaluate the solution in two ways, namely:
   a. By considering the double integral of
      $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} dx\, dy$
      by means of multivariable calculus.
   b. By considering the same double integral as mentioned in the previous part but solving with a change of variables to polar coordinates.

      In obtaining the final answer, demonstrate visually, the process of computing the solution.

Scoring rubric:

- Clarity and precision of explanation
- Level of mathematical detail
- Originality and creativity

General Guidelines:

- The event is house-wise and the research paper must be written by a team of 3 11th grade students from the same house.
- Representation through diagrams and figures in the research paper is encouraged.
- The maximum word count is 1000 words - excluding variables, notations, equations and figures.
- The research paper must be submitted to Utkarsh Bajaj in a PDF format. Mode of submission is via email. (Email ID - utkarshbajaj03@gmail.com)
- The final date and time of submission is 11:59 pm IST, 13th August 2020.