Math Document Template

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Abstract—This is a document explaining for a question on the concept of concentric circles.

Download all python codes from

svn co https://github.com/Ashuwin/Summer_2020/
    trunk/circle/codes

and latex-tikz codes from

svn co https://github.com/Ashuwin/Summer_2020/
    trunk/circle/figs

1 PROBLEM

If a line intersects two concentric circles(circles with same centre) with centre O at A, B, C and D. Prove that $AB = CD$.

2 CONSTRUCTION

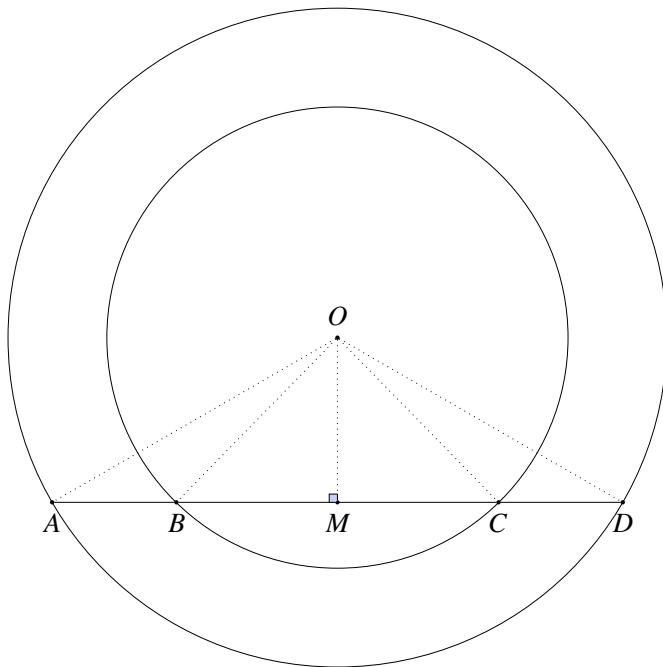Fig. 2.0: Concentric circles by Latex-Tikz

2.1. The design parameters used for construction
    Solution: See Table. 2.1.

Design Parameters	
Parameters	Values
Radius 1	7
Radius 2	10
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
A	$\begin{pmatrix} -\sqrt{75} \\ -5 \end{pmatrix}$
B	$\begin{pmatrix} -\sqrt{24} \\ -5 \end{pmatrix}$
C	$\begin{pmatrix} \sqrt{24} \\ -5 \end{pmatrix}$
D	$\begin{pmatrix} \sqrt{75} \\ -5 \end{pmatrix}$

TABLE 2.1: Concentric circles and line $ABCD$

2.2. Finding Coordinates of Point **M** Let **M** be the mid-point of AD(or BC)

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$$

2.3. The derived parameters used for construction

Solution: From the given information, The values are listed in 2.3

Derived values	
Parameter	values
S	$\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

TABLE 2.3: Concentric circles and line $ABCD$

2.4. Draw fig. 2.4.
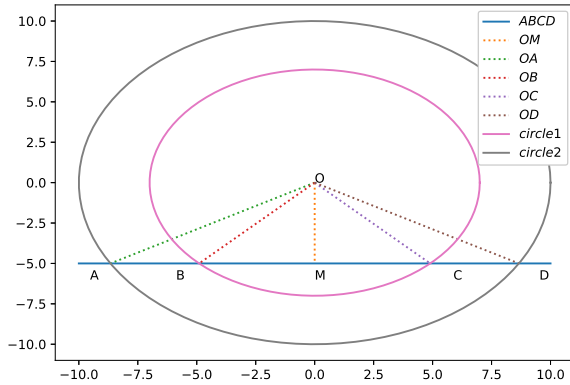   **Solution:** The following Python code generates Fig. 2.4

codes/circle.py

Fig. 2.4: Concentric circles generated using python

and the equivalent latex-tikz code generating Fig. 2.4 is

figs/circle.tex

3 Solution

Finding scalar product between line OM and AD

$$(\mathbf{M} - \mathbf{O})^T(\mathbf{D} - \mathbf{A})$$

$$\left(\begin{pmatrix} 0 \\ -5 \end{pmatrix}\right)^T\left(\begin{pmatrix} 2\sqrt{74} \\ 0 \end{pmatrix}\right) = 0$$

$$\implies OM \perp AD$$

$\triangle OMB \cong \triangle OMC$ by RHS congruency because:
1) $\angle OMB = \angle OMC = 90°$
2) $OB = OC = 7$ (Radius of circle 1)
3) $OM = OM$ (Common side)

$$\implies BM = CM$$

$\triangle OMA \cong \triangle OMD$ by RHS congruency because:
1) $\angle OMA = \angle OMD = 90°$
2) $OA = OD = 10$ (Radius of circle 2)
3) $OM = OM$ (Common side)

$$\implies AM = DM$$

$\because BM = CM$ and $AM = DM$,

$$\implies AB = CD$$

Hence proved.