

Math Document Template

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Abstract—This is a document explaining for a question on the concept of linear algebra.

Download all python codes from

```
svn co https://github.com/Ashuwin/summer_20/
trunk/linear_algebra/codes
```

and latex-tikz codes from

```
svn co https://github.com/Ashuwin/summer_20/
trunk/linear_algebra/figs
```

1 TRIANGLE

1.1 Problem

- Find the area of triangle whose vertices are

- $\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \end{pmatrix}$
- $\begin{pmatrix} -5 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

1.2 Solution

- The area of triangle ABC :

Solution: The area of triangle ABC using cross product is obtained as:

$$\frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| \quad (1.2.1.1)$$

$$\frac{1}{2} \left\| \begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \end{pmatrix} \right\| \quad (1.2.1.2)$$

$$\frac{1}{2} \left\| \begin{pmatrix} -3 \\ -3 \end{pmatrix} \times \begin{pmatrix} 0 \\ -7 \end{pmatrix} \right\| = \frac{21}{2} \quad (1.2.1.3)$$

Area of $\triangle ABC = 10.5 \text{ units}^2$ and it is found in the following python code:

```
codes/triangle/tri_area_ABC.py
```

$\triangle ABC$ in Fig.1.2.1 is generated using the following python code

```
codes/triangle/triangle1.py
```

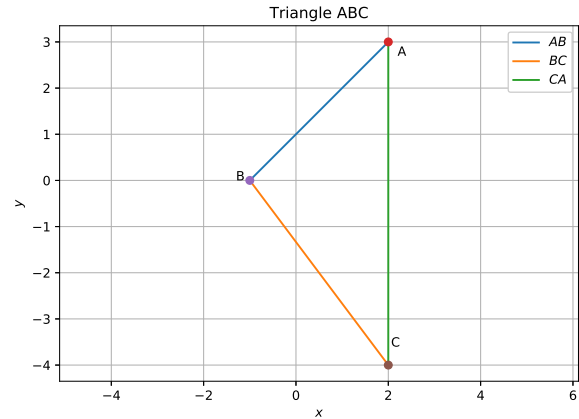


Fig. 1.2.1: Triangle ABC using python

- The area of triangle PQR :

Solution: The area of triangle PQR using Heron's formula is obtained as:

$$\frac{1}{2} \|(\mathbf{Q} - \mathbf{P}) \times (\mathbf{R} - \mathbf{P})\| \quad (1.2.2.1)$$

$$\frac{1}{2} \left\| \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -5 \\ -1 \end{pmatrix} \times \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ -1 \end{pmatrix} \right\| \quad (1.2.2.2)$$

$$\frac{1}{2} \left\| \begin{pmatrix} 8 \\ -4 \end{pmatrix} \times \begin{pmatrix} 10 \\ 3 \end{pmatrix} \right\| = \frac{64}{2} \quad (1.2.2.3)$$

Area of $\triangle PQR = 32 \text{ units}^2$ and it is found in the following python code:

```
codes/triangle/tri_area_PQR.py
```

$\triangle PQR$ in Fig.1.2.2 is generated using the following python code

```
codes/triangle/triangle2.py
```

2 QUADRILATERAL

2.1 Problem

- Find the area of the quadrilateral whose vertices are, taken in order, are $\begin{pmatrix} -4 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

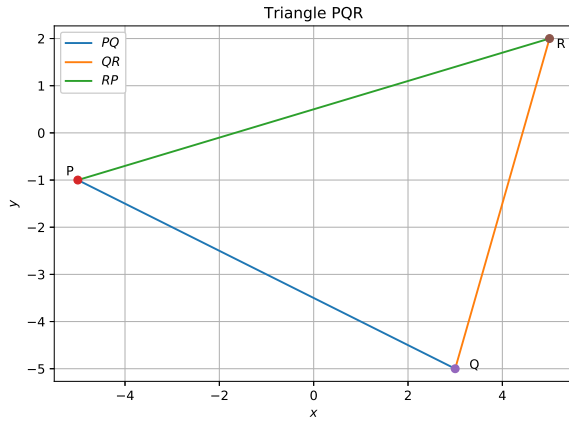


Fig. 1.2.2: Triangle PQR using python

2.2 Solution

1. The area of triangle ABC :

Solution: The area of triangle ABC using cross product is obtained as:

$$\frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| \quad (2.2.1.1)$$

$$\frac{1}{2} \left\| \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \end{pmatrix} \right\| \quad (2.2.1.2)$$

$$\frac{1}{2} \left\| \begin{pmatrix} 1 \\ -7 \end{pmatrix} \times \begin{pmatrix} 7 \\ -4 \end{pmatrix} \right\| = \frac{45}{2} \quad (2.2.1.3)$$

Area of $\triangle ABC = 22.5 \text{ units}^2$ and it is found in the following python code:

```
codes/tri_area_ABC.py
```

2. The area of triangle ACD :

Solution: The area of triangle ACD using Heron's formula is obtained as:

$$\frac{1}{2} \|(\mathbf{C} - \mathbf{A}) \times (\mathbf{D} - \mathbf{A})\| \quad (2.2.2.1)$$

$$\frac{1}{2} \left\| \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 2 \end{pmatrix} \right\| \quad (2.2.2.2)$$

$$\frac{1}{2} \left\| \begin{pmatrix} 7 \\ -4 \end{pmatrix} \times \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right\| = \frac{31}{2} \quad (2.2.2.3)$$

Area of $\triangle ACD = 15.5 \text{ units}^2$ and it is found in the following python code:

```
codes/tri_area_ACD.py
```

3. The area of quadrilateral $ABCD$:

Solution: Area of Quadrilateral $ABCD = \text{Area of } \triangle ABC + \text{Area of } \triangle ACD = 38 \text{ units}^2$

4. Quadrilateral $ABCD$ in Fig.2.2.4 is generated using the following python code

```
codes/quadrilateral/quad.py
```

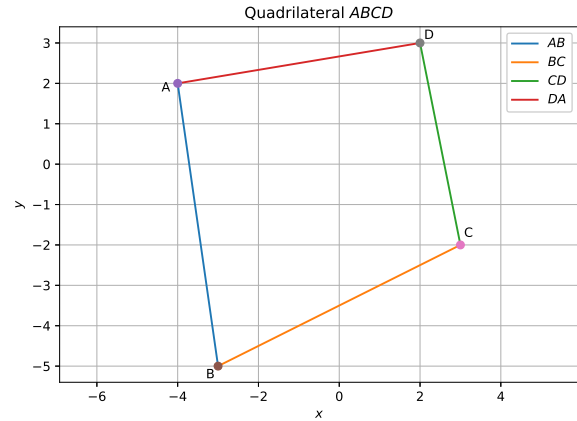


Fig. 2.2.4: Quadrilateral $ABCD$ using python

3 LINE EXERCISES

3.1 Complex numbers

3.1.1 Problem:

1. Find the conjugate of $\begin{pmatrix} 3 \\ -2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

3.1.2 Solution:

1. A complex number $\begin{pmatrix} a \\ b \end{pmatrix}$ can be represented as 2×2 matrix:

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

Multiplying the given complex numbers after converting them to a 2×2 matrix,

$$\begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix} \quad (3.1.2.1.1)$$

$$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \quad (3.1.2.1.2)$$

$$\begin{pmatrix} 12 & -5 \\ 5 & 12 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}$$

Converting the matrices back to complex numbers,

$$\frac{\begin{pmatrix} 12 \\ 5 \end{pmatrix}}{\begin{pmatrix} 4 \\ 3 \end{pmatrix}} \quad (3.1.2.1.3)$$

Multiplying the conjugate of denominator to both numerator and denominator,

$$\frac{\begin{pmatrix} 12 \\ 5 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \end{pmatrix}}{\begin{pmatrix} 4 \\ 3 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \end{pmatrix}} \quad (3.1.2.1.4)$$

Multiplying the complex numbers after converting them to a 2 x 2 matrix,

$$\frac{\begin{pmatrix} 12 & -5 \\ 5 & 12 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix}}{\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix}} \quad (3.1.2.1.5)$$

$$\frac{\begin{pmatrix} 63 & 16 \\ -16 & 63 \end{pmatrix}}{\begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}} \quad (3.1.2.1.6)$$

$$\frac{1}{25} \begin{pmatrix} 63 \\ -16 \end{pmatrix} \quad (3.1.2.1.7)$$

$$\text{Conjugate of the complex number} = \frac{1}{25} \begin{pmatrix} 63 \\ 16 \end{pmatrix}$$

which can be simplified to obtain

$$\begin{pmatrix} 8 & -28 \end{pmatrix} \mathbf{x} = -56 \quad (3.2.2.1.2)$$

Choose $\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ as the point lies on the x-axis

$$\begin{pmatrix} 8 & -28 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = -56 \quad (3.2.2.1.3)$$

$$\Rightarrow x = -7 \quad (3.2.2.1.4)$$

The point is $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$

The Lines in Fig.3.2.2.1 is generated using the following python code

codes/line/point_vector/point_vector.py

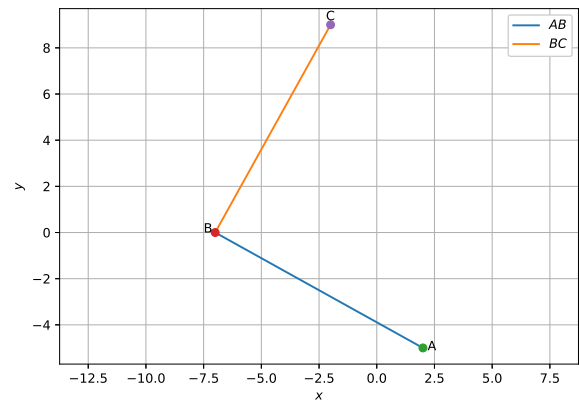


Fig. 3.2.2.1: Lines generated using python

3.2 Points and Vectors

3.2.1 Problem: Find the point on the x-axis which is equidistant from

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}, \begin{pmatrix} -2 \\ 9 \end{pmatrix}$$

3.2.2 Solution:

1. From the given information,

$$\left\| \mathbf{x} - \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right\|^2 = \left\| \mathbf{x} - \begin{pmatrix} -2 \\ 9 \end{pmatrix} \right\|^2 \quad (3.2.2.1.1)$$

$$\|\mathbf{x}\|^2 + \left\| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right\|^2 - 2 \begin{pmatrix} 2 & -5 \end{pmatrix} \mathbf{x} = \|\mathbf{x}\|^2 + \left\| \begin{pmatrix} -2 \\ 9 \end{pmatrix} \right\|^2 - 2 \begin{pmatrix} -2 & 9 \end{pmatrix} \mathbf{x}$$

3.3 Points on a line

3.3.1 Problem: If $\mathbf{A} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ respectively, find the coordinates of \mathbf{P} such that $AP = \frac{3}{7}AB$ and \mathbf{P} lies on the line segment AB

3.3.2 Solution:

$$1. \mathbf{A} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Then \mathbf{P} that divides \mathbf{A}, \mathbf{B} in the ratio $k:1$ is

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (3.3.2.1.1)$$

For the given problem, $k = \frac{3}{4}$

Using the equation 3.3.2.1.1, the desired point

is

$$\mathbf{P} = \frac{\frac{3}{4} \begin{pmatrix} 2 \\ -4 \end{pmatrix} + 1 \begin{pmatrix} -2 \\ -2 \end{pmatrix}}{\frac{3}{4} + 1} \quad (3.3.2.1.2)$$

$$\mathbf{P} = \begin{pmatrix} -2/7 \\ -20/7 \end{pmatrix} \quad (3.3.2.1.3)$$

The following python code plots the Fig.??

codes/point_line/int_sec.py

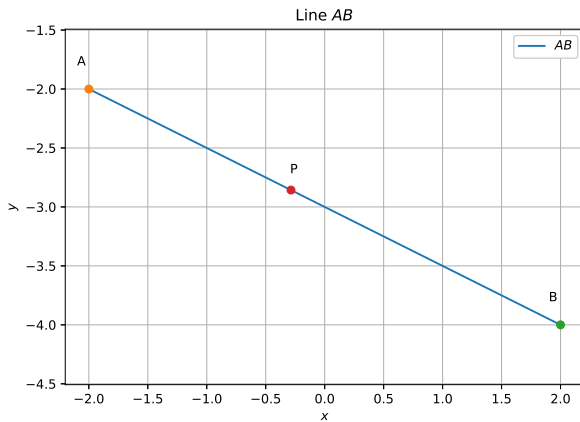


Fig. 3.3.2.1: Line AB using python

3.4 Lines and Planes

3.4.1 Problem: Write four solutions for each of the following equations

a) $\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = 7$

b) $\begin{pmatrix} \pi & 1 \end{pmatrix} \mathbf{x} = 9$

c) $\begin{pmatrix} 1 & -4 \end{pmatrix} \mathbf{x} = 0$

3.4.2 Solution:

1. \mathbf{x} are randomly chosen and substituted in the equation and solutions are found.

a)

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = 7 \quad (3.4.2.1.1)$$

Solution: Let $\mathbf{x} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ Substituting in equation 3.4.2.1.1, $\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = 7$
 $\Rightarrow a = \frac{7}{2}$

Let $\mathbf{x} = \begin{pmatrix} 0 \\ b \end{pmatrix}$ Substituting in equation

$$3.4.2.1.1, \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} = 7$$

$$\Rightarrow b = 7$$

Let $\mathbf{x} = \begin{pmatrix} c \\ 1 \end{pmatrix}$ Substituting in equation

$$3.4.2.1.1, \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} c \\ 1 \end{pmatrix} = 7$$

$$\Rightarrow c = 3$$

Let $\mathbf{x} = \begin{pmatrix} 1 \\ d \end{pmatrix}$ Substituting in equation

$$3.4.2.1.1, \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ d \end{pmatrix} = 7$$

$$\Rightarrow d = 5$$

b)

$$\begin{pmatrix} \pi & 1 \end{pmatrix} \mathbf{x} = 9 \quad (3.4.2.1.2)$$

Solution: Let $\mathbf{x} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ Substituting in equation

$$3.4.2.1.2, \begin{pmatrix} \pi & 1 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = 9$$

$$\Rightarrow a = \frac{9}{\pi}$$

Let $\mathbf{x} = \begin{pmatrix} 0 \\ b \end{pmatrix}$ Substituting in equation

$$3.4.2.1.2, \begin{pmatrix} \pi & 1 \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} = 9$$

$$\Rightarrow b = 9$$

Let $\mathbf{x} = \begin{pmatrix} c \\ 1 \end{pmatrix}$ Substituting in equation

$$3.4.2.1.2, \begin{pmatrix} \pi & 1 \end{pmatrix} \begin{pmatrix} c \\ 1 \end{pmatrix} = 9$$

$$\Rightarrow c = \frac{8}{\pi}$$

Let $\mathbf{x} = \begin{pmatrix} 1 \\ d \end{pmatrix}$ Substituting in equation

$$3.4.2.1.2, \begin{pmatrix} \pi & 1 \end{pmatrix} \begin{pmatrix} 1 \\ d \end{pmatrix} = 9$$

$$\Rightarrow d = 9 - \pi$$

c)

$$\begin{pmatrix} 1 & -4 \end{pmatrix} \mathbf{x} = 0 \quad (3.4.2.1.3)$$

Solution: Let $\mathbf{x} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ Substituting in equation

$$3.4.2.1.3, \begin{pmatrix} 1 & -4 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = 0$$

$$\Rightarrow a = 0$$

Let $\mathbf{x} = \begin{pmatrix} 0 \\ b \end{pmatrix}$ Substituting in equation

$$3.4.2.1.3, \begin{pmatrix} 1 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} = 0$$

$$\Rightarrow b = 0$$

Let $\mathbf{x} = \begin{pmatrix} c \\ 1 \end{pmatrix}$ Substituting in equation

$$3.4.2.1.3, (1 \quad -4) \begin{pmatrix} c \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow c = 4$$

Let $\mathbf{x} = \begin{pmatrix} 1 \\ d \end{pmatrix}$ Substituting in equation

$$3.4.2.1.3, (1 \quad -4) \begin{pmatrix} 1 \\ d \end{pmatrix} = 0$$

$$\Rightarrow d = \frac{1}{4}$$

3.5 Motion in a plane

3.5.1 Problem:

1. A man can swim with a speed of 4.0km/h in still water. How long does he take to cross a river 1.0km wide if the river flows steadily at 3.0km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

3.5.2 Solution:

1. Let the man be at point $\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

The speed of the man is $\mathbf{u} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

The speed of the river is $\mathbf{v} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Since the swimmer dive the river normal to the flow of river, therefore time taken by swimmer to cross the river,

$$t = \frac{d}{\|\mathbf{u}\|} = \frac{1\text{km}}{4\text{km/h}} = 15\text{mins}$$

Distance covered down the river = $t \times \|\mathbf{v}\|$

$$x = \frac{1}{4}\text{hr} \times 3\text{km/h} = 750\text{m}$$

The code for diagrammatic representation(3.5.2.1) of the solution is

codes/line/motion_plane/man_river.py

3.6 Matrix

3.6.1 Problem:

1. Find the values of x, y and z from the following equations:

$$\text{a) } \begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix}$$

$$\text{b) } \begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$$

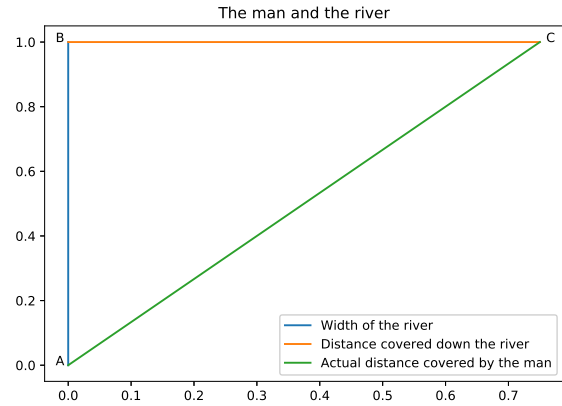


Fig. 3.5.2.1

$$\text{c) } \begin{pmatrix} x+y+z \\ x+y \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

3.6.2 Solution:

1. This problem is solved by comparing the respective elements in both the matrices

a)

$$\begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix} \quad (3.6.2.1.1)$$

$$x = 1, y = 4, z = 3 \quad (3.6.2.1.2)$$

b)

$$\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix} \quad (3.6.2.1.3)$$

$$5 + z = 5 \quad (3.6.2.1.4)$$

$$\Rightarrow z = 0 \quad (3.6.2.1.5)$$

$$x + y = 6 \quad (3.6.2.1.6)$$

$$xy = 8 \quad (3.6.2.1.7)$$

$$x = 4, y = 2 \quad (3.6.2.1.8)$$

$$x = 2, y = 4 \quad (3.6.2.1.9)$$

$$x = 4, y = 2, z = 0 \text{ or } x = 2, y = 4, z = 0$$

$$\text{c) } \begin{pmatrix} x+y+z \\ x+y \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

Expressing it as $Ax = b$ and $x = A^{-1}b$,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix} \quad (3.6.2.1.10)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix} \quad (3.6.2.1.11)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad (3.6.2.1.12)$$

$$x = 2, y = 3, z = 4 \quad (3.6.2.1.13)$$

3.7 Determinants

3.7.1 Problem:

1. If $A = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$, find $|A|$.

3.7.2 Solution:

1. To find the value of the determinant,

$$\begin{pmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{pmatrix} \quad (3.7.2.1.1)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 5 & 4 & -9 \end{pmatrix} \quad (3.7.2.1.2)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 5R_1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \quad (3.7.2.1.3)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.7.2.1.4)$$

$$\text{Det } |A| = 1 \times -1 \times 0 = 0$$

The value of the determinant is found in the following python code

```
codes/line/determinants/det.py
```

3.8.2 Solution:

1. Let $3x + 2y = 6$ intersects the x-axis and y-axis at **A** and **B** respectively.

a) Let $\mathbf{A} = \begin{pmatrix} x \\ 0 \end{pmatrix}$

$$3x = 6 \quad (3.8.2.1.1)$$

$$\Rightarrow x = 2 \quad (3.8.2.1.2)$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (3.8.2.1.3)$$

b) Let $\mathbf{B} = \begin{pmatrix} 0 \\ y \end{pmatrix}$

$$2y = 6 \quad (3.8.2.1.4)$$

$$\Rightarrow y = 3 \quad (3.8.2.1.5)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (3.8.2.1.6)$$

- c) Origin $= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ does not satisfy the equation $3x + 2y < 6$.

\Rightarrow The solution is the right side of the line $3x + 2y = 6$

2. The following python code is the diagrammatic representation of the solution in Fig.3.8.2.2

```
codes/linear_inequalities/linear_inequalities.py
```

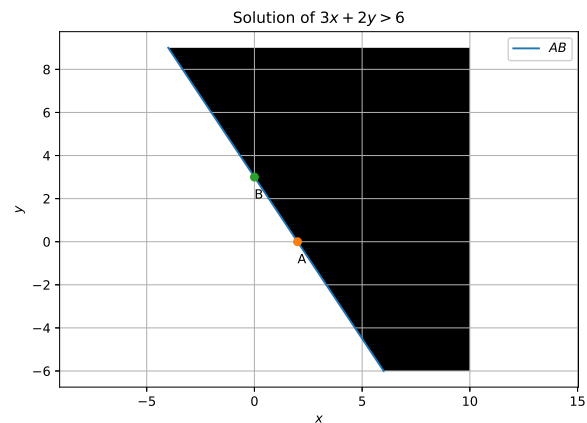


Fig. 3.8.2.2

3.8 Linear Inequalities

3.8.1 Problem:

1. Solve $3x + 2y > 6$ graphically

3.9 Miscellaneous

3.9.1 Problem:

1. The base of an equilateral triangle with side $2a$ lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

3.9.2 Solution:

1. Let $\triangle ABC$ be an equilateral triangle. Let AB be the base and M be the midpoint.

a) $A = \begin{pmatrix} 0 \\ m \end{pmatrix}$ (as point A lies on the y-axis)

$B = \begin{pmatrix} 0 \\ n \end{pmatrix}$ (as point B lies on the y-axis)

$M = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (as point M lies on the origin)

$C = \begin{pmatrix} p \\ 0 \end{pmatrix}$ (as point C lies on the x-axis)

$$\|A - B\| = \|B - C\| = \|C - A\| = 2a \quad (3.9.2.1.1)$$

- b) M is the mid-point of A and B

$$M = \frac{A + B}{2} \quad (3.9.2.1.2)$$

$$\Rightarrow m = -n \quad (3.9.2.1.3)$$

$$\|A - B\| = 2a \quad (3.9.2.1.4)$$

$$\Rightarrow m = -n = a \quad (3.9.2.1.5)$$

- c)

$$\|C - A\| = 2a \quad (3.9.2.1.6)$$

$$\sqrt{p^2 + a^2} = 2a \quad (3.9.2.1.7)$$

$$\Rightarrow p = \pm \sqrt{3}a \quad (3.9.2.1.8)$$

- d) The vertices are,

$$A = \begin{pmatrix} 0 \\ a \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ -a \end{pmatrix}$$

$$C = \begin{pmatrix} \pm \sqrt{3}a \\ 0 \end{pmatrix}$$

$\triangle ABC$ in Fig.3.9.2.1 is generated using the following python code

```
codes/line/miscellaneous/tri_equi.py
```

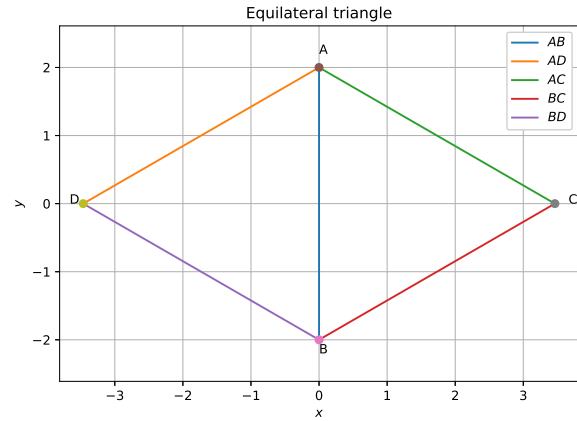


Fig. 3.9.2.1: Triangles ABC and ABD using python

the line $\begin{pmatrix} 4 & 1 \end{pmatrix} \mathbf{x} = 16$.

4.2 Solution

1. The vector form of general equation of circle,

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{O}^T \mathbf{x} + F = 0 \quad (4.2.1.1)$$

whose centre is \mathbf{O} .

2. Point $A = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ lies on the circle. So, point A satisfies the equation 4.2.1.1

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix}^T \begin{pmatrix} 4 \\ 1 \end{pmatrix} - 2\mathbf{O}^T \begin{pmatrix} 4 \\ 1 \end{pmatrix} + F = 0$$

$$2\mathbf{O}^T \begin{pmatrix} 4 \\ 1 \end{pmatrix} - F = 17 \quad (4.2.2.1)$$

3. Point $B = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ lies on the circle. So, point B satisfies the equation 4.2.1.1

$$\begin{pmatrix} 6 \\ 5 \end{pmatrix}^T \begin{pmatrix} 6 \\ 5 \end{pmatrix} - 2\mathbf{O}^T \begin{pmatrix} 6 \\ 5 \end{pmatrix} + F = 0 \quad (4.2.3.1)$$

$$2\mathbf{O}^T \begin{pmatrix} 6 \\ 5 \end{pmatrix} - F = 61 \quad (4.2.3.2)$$

4. Centre \mathbf{O} lies on the line $\begin{pmatrix} 4 & 1 \end{pmatrix} \mathbf{x} = 16$

$$\begin{pmatrix} 4 & 1 \end{pmatrix} \mathbf{O} = 16 \quad (4.2.4.1)$$

5. Solving equations 4.2.2.1, 4.2.3.2 and 4.2.4.1

$$\mathbf{O} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, F = 15$$

4 CIRCLE

4.1 Problem

1. Find the equation of the circle passing through the points $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and whose centre is on

\Rightarrow Equation of the circle is $x^2 + y^2 - 6x - 8y + 15 = 0$

The vector form is

$$\mathbf{x}^T \mathbf{x} - 2 \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} + 15 = 0$$

6. The circle in Fig.4.2.6 is generated using the following python code

```
codes/circle/circle.py
```

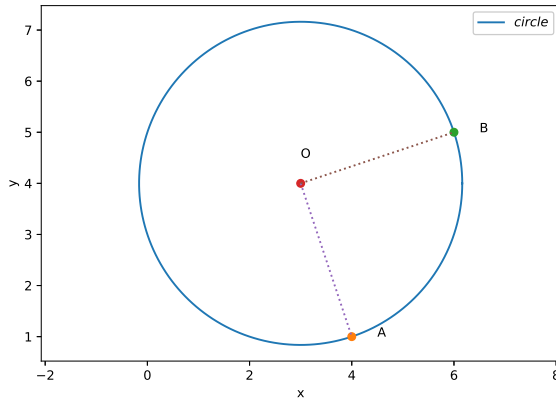


Fig. 4.2.6: Circle generated using python

5 CONICS

5.1 Problem

1. Find the roots of the quadratic equations:

- $x^2 - 3x - 10 = 0$
- $2x^2 + x - 6 = 0$
- $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
- $2x^2 - x + \frac{1}{8} = 0$
- $100x^2 - 20x + 1 = 0$

5.2 Solution

1. For a general polynomial equation of degree 2

$$p(x, y) \Rightarrow Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

The vector form is

$$\mathbf{x}^T \begin{pmatrix} A & B/2 \\ B/2 & C \end{pmatrix} \mathbf{x} + \begin{pmatrix} D & E \end{pmatrix} \mathbf{x} + F = 0 \quad (5.2.1.1)$$

- a) $x^2 - 3x - 10 = 0$

The vector form from the equation is 5.2.1.1 is

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -3 & 0 \end{pmatrix} \mathbf{x} - 10 = 0 \quad (5.2.1.2)$$

The values of \mathbf{x} are found in the following python code

```
codes/conics/conics_1.py
```

$$\mathbf{x} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

which can be verified from the Fig.5.2.1. The following python code generates the fig.5.2.1

```
codes/conics/conics_1.py
```

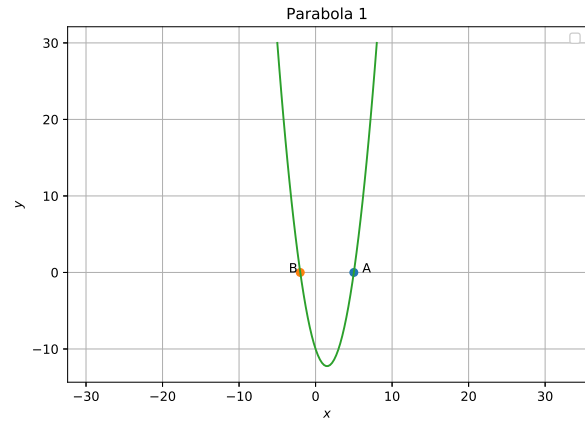


Fig. 5.2.1: Parabola 1

- b) $2x^2 + x - 6 = 0$

The vector form from the equation is 5.2.1.1 is

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 6 = 0 \quad (5.2.1.3)$$

The values of \mathbf{x} are found in the following python code

```
codes/conics/conics_2.py
```

$$\mathbf{x} = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

which can be verified from the Fig.5.2.1. The following python code generates the fig.5.2.1

```
codes/conics/conics_2.py
```

- c) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

The vector form from the equation is 5.2.1.1 is

$$\mathbf{x}^T \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 7 & 0 \end{pmatrix} \mathbf{x} + 5\sqrt{2} = 0 \quad (5.2.1.4)$$

The values of \mathbf{x} are found in the following python code

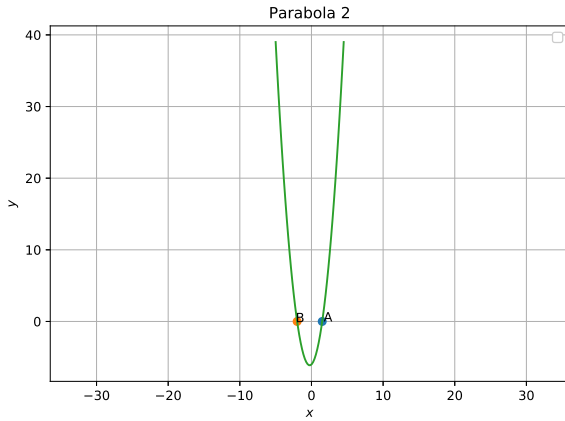


Fig. 5.2.1: Parabola 2

```
codes/conics/conics_3.py
```

$\mathbf{x} = \begin{pmatrix} -1.414 \\ 0 \end{pmatrix}, \begin{pmatrix} -3.535 \\ 0 \end{pmatrix}$ which can be verified from the Fig.5.2.1. The following python code generates the fig.5.2.1

```
codes/conics/conics_3.py
```

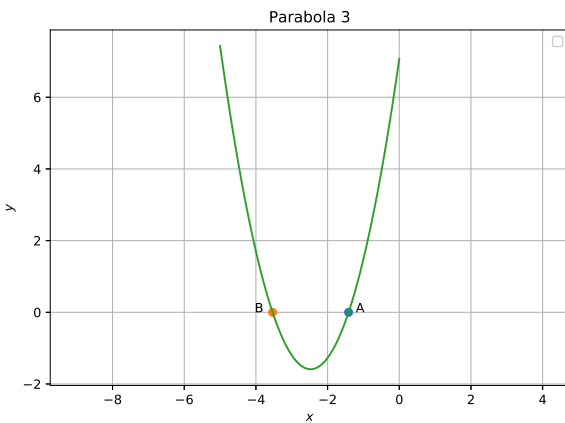


Fig. 5.2.1: Parabola 3

d) $2x^2 - x + \frac{1}{8} = 0$

The vector form from the equation is 5.2.1.1 is

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} + \frac{1}{8} = 0 \quad (5.2.1.5)$$

The values of \mathbf{x} are found in the following python code

```
codes/conics/conics_4.py
```

$$\mathbf{x} = \begin{pmatrix} 0.25 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.25 \\ 0 \end{pmatrix}$$

which can be verified from the Fig.5.2.1. The following python code generates the fig.5.2.1

```
codes/conics/conics_4.py
```

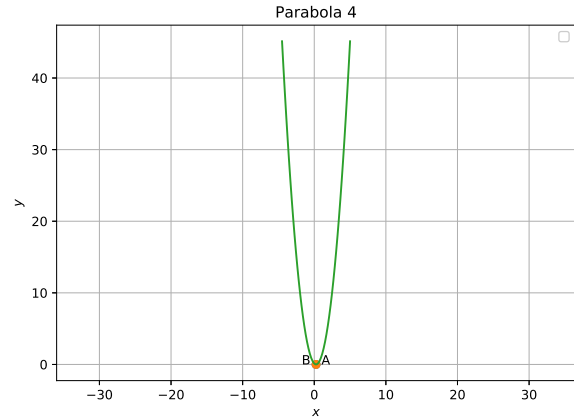


Fig. 5.2.1: Parabola 4

e) $100x^2 - 20x + 1 = 0$

The vector form from the equation is 5.2.1.1 is

$$\mathbf{x}^T \begin{pmatrix} 100 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -20 & 0 \end{pmatrix} \mathbf{x} + 1 = 0 \quad (5.2.1.6)$$

The values of \mathbf{x} are found in the following python code

```
codes/conics/conics_5.py
```

$\mathbf{x} = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.1 \\ 0 \end{pmatrix}$ which can be verified from the Fig.5.2.1. The following python code generates the fig.5.2.1

```
codes/conics/conics_5.py
```

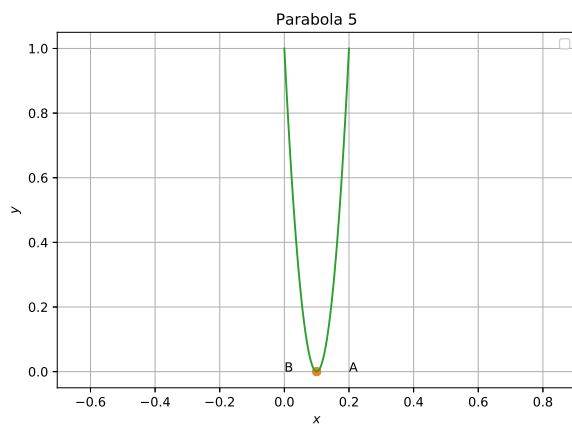


Fig. 5.2.1: Parabola 5