Math Document Template

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***Abstract*—This is a document explaining for a question on the concept of triangles.**

Download all python codes from

```
svn co https://github.com/Ashuwin/Summer_2020/
    trunk/triangle/codes
```

and latex-tikz codes from

```
svn co https://github.com/Ashuwin/Summer_2020/
    trunk/triangle/figs
```

1 Problem

In $\triangle PQR$, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$

2 Construction

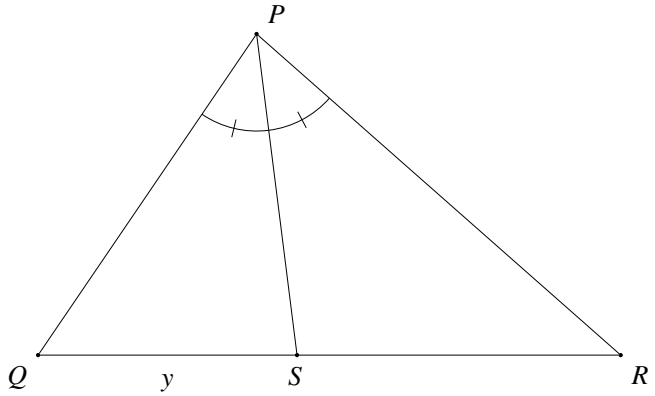

Fig. 2.0: Triangle by Latex-Tikz

2.1. The figure obtained looks like Fig. 2.0. $PR > PQ$, $\angle QPS = \angle SPR = x$

2.2. The design parameters used for construction

Solution: See Table. 2.2.

2.3. Find the coordinates of various points:

Design Parameters	
Parameters	Value
PQ	4
PR	5
QR	6

TABLE 2.2: Triangle PQR

Solution: From the given information,

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{2.3.2}$$

$$\mathbf{R} = \begin{pmatrix} p \\ 0 \end{pmatrix} \tag{2.3.3}$$

$$\mathbf{S} = \begin{pmatrix} y \\ 0 \end{pmatrix} \tag{2.3.4}$$

where

$$a = (p^2 + r^2 - q^2)/2p$$

$$b = \sqrt{r^2 - a^2}$$

2.4. Point S can be found by Triangle angle bisector theorem.

$$\frac{\|\mathbf{QS}\|}{\|\mathbf{PQ}\|} = \frac{\|\mathbf{SR}\|}{\|\mathbf{PR}\|}$$

$$\frac{y}{4} = \frac{6-y}{5}$$

$$5y = 24 - 4y$$

$$9y = 24$$

$$y = \frac{8}{3}$$

2.5. The derived parameters used for construction

Solution: From the given information, The values are listed in 2.5

2.6. Draw fig. 2.6.

Solution: The following Python code generates

Derived values	
Parameter	values
P	$\begin{pmatrix} 2.25 \\ 3.3072 \end{pmatrix}$
S	$\begin{pmatrix} 8/3 \\ 0 \end{pmatrix}$

TABLE 2.5: *TrianglePQR*

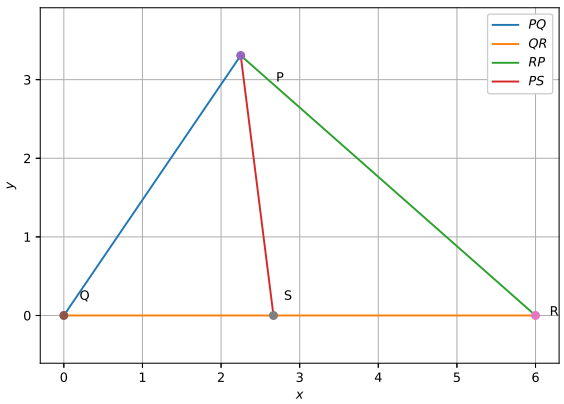

Fig. 2.6: Triangle generated using python

Fig. 2.6

codes/tri.py

and the equivalent latex-tikz code generating Fig. 2.6 is

figs/triangle.tex

3 Solution

Solution by linear algebra:
$\angle PSQ$ is the angle between **SP** and **SQ**
$\angle PSR$ is the angle between **SP** and **SR**
Equation of line

$$\mathbf{SQ} = \mathbf{Q} - \mathbf{S}$$

$$\mathbf{SQ} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix}$$

$$\mathbf{SQ} = \begin{pmatrix} \frac{-8}{3} \\ 0 \end{pmatrix}$$

Equation of line

$$\mathbf{SP} = \mathbf{P} - \mathbf{S}$$

$$\mathbf{SP} = \begin{pmatrix} 2.25 \\ 3.3072 \end{pmatrix} - \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix}$$

$$\mathbf{SP} = \begin{pmatrix} -0.416 \\ 3.3072 \end{pmatrix}$$

Equation of line

$$\mathbf{SR} = \mathbf{R} - \mathbf{S}$$

$$\mathbf{SR} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{8}{3} \\ 0 \end{pmatrix}$$

$$\mathbf{SR} = \begin{pmatrix} 3.33 \\ 0 \end{pmatrix}$$

Angle between two lines $\mathbf{u}$ and $\mathbf{v}$ $= \cos\theta = \frac{\mathbf{u}.\mathbf{v}}{\|\mathbf{u}\|.\|\mathbf{v}\|}$

$$\cos\angle PSR = \frac{\mathbf{SP}.\mathbf{SR}}{\|\mathbf{SP}\|\,.\,\|\mathbf{SR}\|}$$

$$\cos\angle PSR = \frac{\begin{pmatrix} -0.416 \\ 3.3072 \end{pmatrix}.\begin{pmatrix} 3.33 \\ 0 \end{pmatrix}}{(3.33).\frac{10}{3}}$$

$$\cos\angle PSR = -0.1248$$

$$\angle PSR = \cos^{-1} -0.1248 = 97.17^\circ$$

$$\cos\angle PSQ = \frac{\mathbf{SP}.\mathbf{SQ}}{\|\mathbf{SP}\|\,.\,\|\mathbf{SQ}\|}$$

$$\cos\angle PSQ = \frac{\begin{pmatrix} -0.416 \\ 3.3072 \end{pmatrix}.\begin{pmatrix} \frac{-8}{3} \\ 0 \end{pmatrix}}{(3.33).\frac{8}{3}}$$

$$\cos\angle PSQ = 0.1249$$

$$\angle PSQ = \cos^{-1} 0.1249 = 82.82^\circ$$

Therefore, $\angle PSR > \angle PSQ$