

Math Document Template

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Abstract—This is a document explaining for a question on the concept of triangles.

Download all python codes from

```
svn co https://github.com/Ashuwin/Summer_2020/trunk/triangle/codes
```

and latex-tikz codes from

```
svn co https://github.com/Ashuwin/Summer_2020/trunk/triangle/figs
```

1 PROBLEM

In $\triangle PQR$, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$

2 CONSTRUCTION

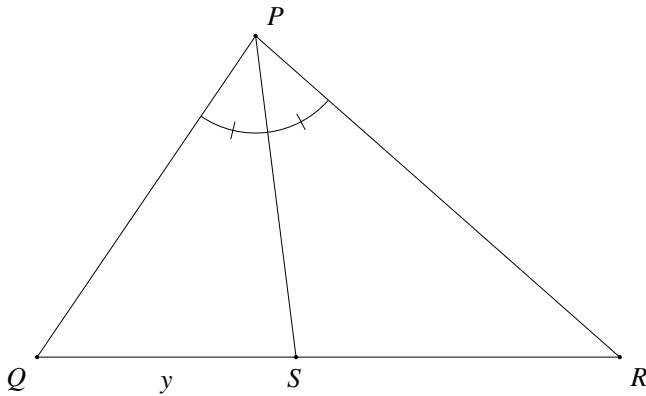


Fig. 2.0: Triangle by Latex-Tikz

2.1. The figure obtained looks like Fig. 2.0. $PR > PQ$, $\angle QPS = \angle SPR = x$

2.2. The design parameters used for construction

Solution: See Table. 2.2.

2.3. Find the coordinates of various points:

Design Parameters	
Parameters	Value
PQ	4
PR	5
QR	6

TABLE 2.2: Triangle PQR

Solution: From the given information,

$$\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.3.1)$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (2.3.2)$$

$$\mathbf{R} = \begin{pmatrix} p \\ 0 \end{pmatrix} \quad (2.3.3)$$

$$\mathbf{S} = \begin{pmatrix} y \\ 0 \end{pmatrix} \quad (2.3.4)$$

where

$$a = (p^2 + r^2 - q^2)/2p$$

$$b = \sqrt{r^2 - a^2}$$

2.4. Point S can be found by Triangle angle bisector theorem.

$$\frac{\|QS\|}{\|PQ\|} = \frac{\|SR\|}{\|PR\|}$$

$$\frac{y}{4} = \frac{6-y}{5}$$

$$5y = 24 - 4y$$

$$9y = 24$$

$$y = \frac{8}{3}$$

2.5. The derived parameters used for construction

Solution: From the given information, The values are listed in 2.5

2.6. Draw fig. 2.6.

Solution: The following Python code generates

Derived values	
Parameter	values
P	$\begin{pmatrix} 2.25 \\ 3.3072 \end{pmatrix}$
S	$\begin{pmatrix} 8/3 \\ 0 \end{pmatrix}$

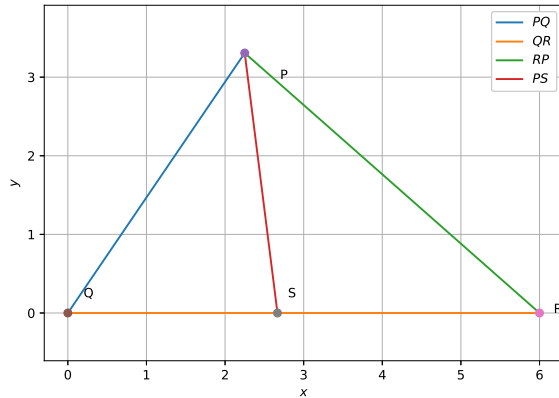
TABLE 2.5: *TrianglePQR*

Fig. 2.6: Triangle generated using python

Fig. 2.6

```
codes/tri.py
```

and the equivalent latex-tikz code generating Fig. 2.6 is

```
figs/triangle.tex
```

3 SOLUTION

Solution by linear algebra:

$\angle PSQ$ is the angle between **SP** and **SQ**

$\angle PSR$ is the angle between **SP** and **SR**

Equation of line

$$\mathbf{SQ} = \mathbf{Q} - \mathbf{S}$$

$$\mathbf{SQ} = (0\mathbf{i} + 0\mathbf{j}) - \left(\frac{8}{3}\mathbf{i} + 0\mathbf{j}\right)$$

$$\mathbf{SQ} = -\frac{8}{3}\mathbf{i} + 0\mathbf{j}$$

Equation of line

$$\mathbf{SP} = \mathbf{P} - \mathbf{S}$$

$$\mathbf{SP} = (2.25\mathbf{i} + 3.3072\mathbf{j}) - \left(\frac{8}{3}\mathbf{i} + 0\mathbf{j}\right)$$

$$\mathbf{SP} = -0.416\mathbf{i} + 3.3072\mathbf{j}$$

Equation of line

$$\mathbf{SR} = \mathbf{R} - \mathbf{S}$$

$$\mathbf{SR} = (6\mathbf{i} + 0\mathbf{j}) - \left(\frac{8}{3}\mathbf{i} + 0\mathbf{j}\right)$$

$$\mathbf{SR} = 3.33\mathbf{i} + 0\mathbf{j}$$

Angle between two lines **u** and **v** $= \cos \theta \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|}$

$$\cos \angle PSR = \frac{\mathbf{SP} \cdot \mathbf{SR}}{\|\mathbf{SP}\| \cdot \|\mathbf{SR}\|}$$

$$\cos \angle PSR = \frac{(-0.416\mathbf{i} + 3.3072\mathbf{j}) \cdot (3.33\mathbf{i} + 0\mathbf{j})}{(3.33) \cdot \frac{10}{3}}$$

$$\cos \angle PSR = -0.1248$$

$$\angle PSR = \cos^{-1} -0.1248 = 97.17^\circ$$

$$\cos \angle PSQ = \frac{\mathbf{SP} \cdot \mathbf{SQ}}{\|\mathbf{SP}\| \cdot \|\mathbf{SQ}\|}$$

$$\cos \angle PSQ = \frac{(-0.416\mathbf{i} + 3.3072\mathbf{j}) \cdot \left(-\frac{8}{3}\mathbf{i} + 0\mathbf{j}\right)}{(3.33) \cdot \frac{8}{3}}$$

$$\cos \angle PSQ = 0.1249$$

$$\angle PSQ = \cos^{-1} 0.1249 = 82.82^\circ$$

Therefore, $\angle PSR > \angle PSQ$