Computational Approach to School Mathematics

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Contents

***Abstract*—This book provides a computational approach to school mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/computation/codes
```

1 Trigonometry

1. Suppose

$$\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx \tag{1.0.1.1}$$

is an identity in x, where $C_0, C_1, ...C_n$ are constants, and $C_n \neq 0$ then find the value of n.

2. Find the solution set of the system of equations

$$x + y = \frac{2\pi}{3} \tag{1.0.2.1}$$

$$\cos x + \cos y = \frac{3}{2}, \tag{1.0.2.2}$$

where x and y are real.

3. Find the set of all x in the interval[0,π] for which

$$2\sin^2 x - 3\sin x + 1 \geq 0 \tag{1.0.3.1}$$

4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. Find the minimum value of the arithmetic mean of $\cos(\alpha + \frac{\pi}{2}), \cos(\beta + \frac{\pi}{2})$ and $\cos(\gamma + \frac{\pi}{2})$.

5. Find the value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$.

6. If

$$K = \sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18}), \tag{1.0.6.1}$$

then find the numerical value of K ?

7. If A>0,B>0 and

$$A + B = \frac{\pi}{3}, \tag{1.0.7.1}$$

then find the maximum value of tan A tan B.

8. Find the general value of θ satisfying the equation

$$\tan^2\theta + \sec 2\theta = 1. \tag{1.0.8.1}$$

9. Find the real roots of the equation

$$\cos^7 x + \sin^4 x = 1 \tag{1.0.9.1}$$

in the interval $(-\pi, \pi)$.

10. If $\tan\theta = -\frac{4}{3}$, then find $\sin\theta$.

11. If $\alpha + \beta + \gamma = 2\pi$ then
    a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
    c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    d) None of these

12. Given

$$A = \sin^2\theta + \cos^4\theta \tag{1.0.12.1}$$

then for all real values of θ
    a) $1 \leq A \leq 2$
    b) $\frac{3}{4} \leq A \leq 1$
    c) $\frac{13}{16} \leq A \leq 1$
    d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

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13. The equation
$$2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}; 0 < x < \frac{\pi}{2} \quad (1.0.13.1)$$
has
    a) no real solution
    b) One real solution
    c) more than the one solution
    d) none of these
14. The general solution of the trigonometric equation
$$\sin x + \cos x = 1 \quad (1.0.14.1)$$
is given by :
    a) x $= 2\mathrm{n}\pi; n = 0, \pm 1, \pm 2...$
    b) x $= 2\mathrm{n}\pi + \frac{\pi}{2}; n = 0, \pm 1, \pm 2...$
    c) x $= \mathrm{n}\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}; n = 0, \pm 1, \pm 2...$
    d) none of these
15. The value of expression $\sqrt{3} cosec 20° - \sec 20°$ is equal to
    a) 2
    b) $\frac{2\sin 20°}{\sin 40°}$
    c) 4
    d) $\frac{4\sin 20°}{\sin 40°}$
16. The general solution of
$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x \quad (1.0.16.1)$$
is
    a) $n\pi + \frac{\pi}{8}$
    b) $\frac{n\pi}{2} + \frac{\pi}{8}$
    c) $(-1)^n\frac{n\pi}{2} + \frac{\pi}{8}$
    d) $2n\pi + \cos^{-1}\frac{3}{2}$
17. The equation
$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0 \quad (1.0.17.1)$$
In the variable x, has real roots. Then p can take any value in the interval
    a) $(0, 2\pi)$
    b) $(-\pi, 0)$
    c) $(-\frac{\pi}{2}, \frac{\pi}{2})$
    d) $(0, \pi)$
18. Number of solutions of the equation
$$\tan x + \sec x = 2\cos x \quad (1.0.18.1)$$
lying in the interval $[0, 2\pi]$ is :
    a) 0
    b) 1
    c) 2
    d) 3
19. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals
    a) $\tan(x - \frac{\pi}{4})$
    b) $\tan(\frac{\pi}{4} - x)$
    c) $\tan(x + \frac{\pi}{4})$
    d) $\tan^2(x + \frac{\pi}{4})$
20. Let n be a positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$.Then
    a) $6 \le n \le 8$
    b) $4 < n \le 8$
    c) $4 \le n \le 8$
    d) $4 < n < 8$
21. If ω is an imaginary cube root of unity then the value of $\sin\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\}$ is
    a) $-\frac{\sqrt{3}}{2}$
    b) $-\frac{1}{\sqrt{2}}$
    c) $\frac{1}{\sqrt{2}}$
    d) $\frac{\sqrt{3}}{2}$
22. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$
    a) 11
    b) 12
    c) 13
    d) 14
23. The general values of θ satisfying equation
$$2\sin^2\theta - 3\sin\theta - 2 = 0 \quad (1.0.23.1)$$
is
    a) $n\pi + (-1)^n\frac{\pi}{6}$
    b) $n\pi + (-1)^n\frac{\pi}{2}$
    c) $n\pi + (-1)^n\frac{5\pi}{6}$
    d) $n\pi + (-1)^n\frac{7\pi}{6}$
24. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if
    a) $x + y \ne 0$
    b) $x = y, x \ne 0$
    c) $x = y$
    d) $x \ne 0, y \ne 0$
25. In a triangle PQR, $\angle R = \pi/2$.If $\tan(\frac{P}{2})$ and $\tan(\frac{Q}{2})$ are the roots of the equation
$$ax^2 + bx + c = 0 (a \ne 0) \quad (1.0.25.1)$$
then
    a) a+b=c

b) b+c=a
c) a+c=b
d) b=c

26. Let f(θ) =$\sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is
a) ≥ 0 only when $\theta \geq 0$
b) ≤ 0 for all real θ
c) ≥ 0 for all real θ
d) ≤ 0 only when $\theta \leq 0$

27. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$=0
in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is
a) 0
b) 2
c) 1
d) 3

28. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)...(\cos\alpha_n)$, under the restrictions, $0 \leq \alpha_1, \alpha_2, ...\alpha_n \leq \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)...(\cot\alpha_n) = 1$ is
a) $\frac{1}{2^{\frac{n}{2}}}$
b) $\frac{1}{2^n}$
c) $\frac{1}{2n}$
d) 1

29. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$,then $\tan\alpha$ equals
a) $2(\tan\beta + \tan\gamma)$
b) $\tan\beta + \tan\gamma$
c) $\tan\beta + 2\tan\gamma$
d) $2\tan\beta + \tan\gamma$

30. The number of integral values of k for which the equation

$$7\cos x + 5\sin x = 2k + 1 \tag{1.0.30.1}$$

has a solution is
a) 4
b) 8
c) 10
d) 12

31. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}, \cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to
a) $(\frac{\pi}{3}, \frac{\pi}{2}]$
b) $(\frac{\pi}{2}, \frac{2\pi}{3})$
c) $(\frac{2\pi}{3}, \frac{5\pi}{6}]$
d) $(\frac{5\pi}{6}, \pi]$

32. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ where $\alpha, \beta\epsilon[-\pi, \pi]$. Pairs of α, βwhich satisfy both the equations is/are
a) 0
b) 1
c) 2
d) 4

33. The values of $\theta\epsilon(0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are
a) $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$
b) $(\frac{\pi}{8}, \frac{5\pi}{6})$
c) $(0, \frac{\pi}{8}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$
d) $(\frac{41\pi}{48}, \pi)$

34. Let $\theta\epsilon(0, \frac{\pi}{4})$ and $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then
a) $t_1 > t_2 > t_3 > t_4$
b) $t_4 > t_3 > t_1 > t_2$
c) $t_3 > t_1 > t_2 > t_4$
d) $t_2 > t_3 > t_1 > t_4$

35. The number of solutions of the pair of equations

$$2\sin^2\theta - \cos 2\theta = 0 \tag{1.0.35.1}$$
$$2\cos^2\theta - 3\sin\theta = 0 \tag{1.0.35.2}$$

in the interval [0,2π] is
a) zero
b) one
c) two
d) four

36. For $x\epsilon(0, \pi)$, the equation

$$\sin x + 2\sin 2x - \sin 3x = 3 \tag{1.0.36.1}$$

has
a) infinitely many solutions
b) three solutions
c) one solution
d) no solution

37. Let $S = \{x\epsilon(-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\}$. The sum of all distinct solutions of the equation

$$\sqrt{3}\sec x + \text{cosec} x + 2(\tan x - \cot x) = 0 \tag{1.0.37.1}$$

in the set S is equal to
a) $-\frac{7\pi}{9}$
b) $-\frac{2\pi}{9}$
c) 0
d) $\frac{5\pi}{9}$

38. The value of

$\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to

a) $3-\sqrt{3}$
b) $2(3-\sqrt{3})$
c) $2(\sqrt{3}-1)$
d) $2(2-\sqrt{3})$

39. $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$ is equal to
a) $\frac{1}{2}$
b) $\cos(\frac{\pi}{8})$
c) $\frac{1}{8}$
d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

40. The expression $3[\sin^4(\frac{3\pi}{2}-\alpha)+\sin^4(3\pi+\alpha]-2[\sin^6(\frac{\pi}{2}+\alpha)+\sin^6(5\pi-\alpha)]$ is equal to
a) 0
b) 1
c) 3
d) $\sin 4\alpha+\cos 6\alpha$
e) none of these

41. The number of all possible triplets (a_1, a_2, a_3) such that

$$a_1+a_2\cos(2x)+a_3\sin^2(x)=0 \quad (1.0.41.1)$$

for all x is
a) zero
b) one
c) three
d) infinite
e) none

42. The values of θ lying between $\theta=0$ and $\theta=\pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix}=0 \quad (1.0.42.1)$$

are
a) $\frac{7\pi}{24}$
b) $\frac{5\pi}{24}$
c) $\frac{11\pi}{24}$
d) $\frac{\pi}{24}$

43. Let

$$2\sin^2 x+3\sin x-2>0 \quad (1.0.43.1)$$

$$x^2-x-2<0 \quad (1.0.43.2)$$

(x is measured in radians). Then x lies in the interval
a) $(\frac{\pi}{6},\frac{5\pi}{6})$
b) $(-1,\frac{5\pi}{6})$
c) (-1 , 2)
d) $(\frac{\pi}{6},2)$

44. The minimum value of the expression $\sin\alpha+\sin\beta+\sin\gamma$, where$\alpha,\beta,\gamma$are real numbers satisfying $\alpha+\beta+\gamma=\pi$ is
a) Positive
b) zero
c) negative
d) -3

45. The number of values of x in the interval $[0,\pi]$ satisfying the equation

$$3\sin^2 x-7\sin x+2=0 \quad (1.0.45.1)$$

is
a) 0
b) 5
c) 6
d) 10

46. Which of the following number(s) is/are/rational?
a) $\sin 15^\circ$
b) $\cos 15^\circ$
c) $\sin 15^\circ\cos 15^\circ$
d) $\sin 15^\circ\cos 75^\circ$

47. For a positive integer n, let $f_n(\theta)=\tan(\frac{\theta}{2})(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)......(1+\sec 2^n\theta)$.Then
a) $f_2(\frac{\pi}{16})=1$
b) $f_3(\frac{\pi}{32})=1$
c) $f_4(\frac{\pi}{64})=1$
d) $f_5(\frac{\pi}{128})=1$

48. If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then
a) $\tan^2 x=\frac{2}{3}$
b) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$
c) $\tan^2 x=\frac{1}{3}$
d) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

49. For $0<\theta<\frac{\pi}{2}$. the solution(s) of $\sum_{m=1}^{6}\text{cosec}(\theta+\frac{(m-1)\pi}{4})\text{cosec}(\theta+\frac{m\pi}{4})=4\sqrt{2}$ is(are)
a) $\frac{\pi}{4}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{12}$
d) $\frac{5\pi}{12}$

50. Let $\theta,\varphi\epsilon[0,2\pi]$ be such that $2\cos\theta(1-\sin\varphi)=\sin^2\theta(\tan\frac{\theta}{2}+\cot\frac{\theta}{2})\cos\varphi-1, \tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$, then φ can not satisfy

a) $0 < \varphi < \frac{\pi}{2}$
b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$
d) $\frac{3\pi}{2} < \varphi < 2\pi$

51. The number of points in$(-\infty, \infty)$, for which

$$x^2 - x\sin x - \cos x = 0 \quad (1.0.51.1)$$

is
a) 6
b) 4
c) 2
d) 0

52. Let

$$f(x) = x\sin \pi x, x > 0 \quad (1.0.52.1)$$

Then for all natural numbers n, $f'(x)$ vanishes at
a) A unique point in the interval (n,n+$\frac{1}{2}$)
b) A unique point in the interval (n+$\frac{1}{2}$, n+1)
c) A unique point in the interval (n,n+1)
d) Two points in the interval (n,n+1)

53. Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$.Then which of the following is/are true?
a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
c) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

54. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha+\beta)$.

55. (a) Draw the graph of

$$y = \frac{1}{\sqrt{2}}(\sin x + \cos x) \quad (1.0.55.1)$$

from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
(b) If $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and α,β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$

56. Given $\alpha+\beta-\gamma = \pi$, prove that

$$\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma \quad (1.0.56.1)$$

57. Given A= { x: $\frac{\pi}{6} \le x \le \frac{\pi}{3}$} and

$$f(x) = \cos x - x(1+x); \quad (1.0.57.1)$$

find f(A)

58. For all θ in $[0, \pi/2]$ show that,

$$\cos(\sin\theta) \ge \sin(\cos\theta) \quad (1.0.58.1)$$

.

59. Without using tables, Prove that $(\sin 12°)(\sin 48°)(\sin 54°) = \frac{1}{8}$

60. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right) = 1$

61. Find all the solution of $4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$

62. Find the values of x$\epsilon(-\pi, \pi)$ which satisfy the equation

$$8^{(1+|\cos x|+|cos^2 x|+|cos^3 x|+.....)} = 4^3 \quad (1.0.62.1)$$

63. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

64. ABC is a triangle such that $\sin(2A+B) = \sin(C-A) = -\sin(B+2c) = \frac{1}{2}$ If A,B and C are in arithmetic progression, determine the values of A, B and C.

65. if exp{$(\sin^2 x + \sin^4 x + \sin^6 x + ...\infty)$ In 2 } satisifies the equation

$$x^2 - 9x + 8 = 0 \quad (1.0.65.1)$$

, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$.

66. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3.

67. Determine the smallest positive value of x(in degrees) for which $\tan(x+100°) = \tan(x+50°)\tan(x)\tan(x-50°)$.

68. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x\epsilon[0, 2\pi]$

69. Find all values of θ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ satisfying the equation

$$(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0 \quad (1.0.69.1)$$

70. Prove that the values of the function $\frac{\sin x\cos 3x}{\sin 3x\cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x.

71. Prove that $\sum_{k=1}^{n-1}(n-k)cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \ge 3$ is an integer

72. If any triangle ABC, Prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{c}{2}$

73. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$.

This section contains 1 paragraph, Based on each paragraph,there are 2 questions. Each question has four options (A),(B),(C) and (D) ONLY ONE of these four options is correct.

PARAGRAPH 1

Let O be the origin, and **OX**, **OY**, **OZ** be three unit vectors in the directions of the sides **QR**, **RP**, **PQ** respectively, of a triangle PQR

74. $|\mathbf{OX} \times \mathbf{OY}| =$
 a) $\sin(P+Q)$
 b) $\sin 2R$
 c) $\sin(P+R)$
 d) $\sin(Q+R)$
75. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is
 a) $-\frac{5}{3}$
 b) $-\frac{3}{2}$
 c) $\frac{3}{2}$
 d) $\frac{5}{3}$
76. The number of all possible values of θ where $0 < \theta < \pi$,for which the system of equations

 $(y+z)\cos 3\theta = (xyz)\sin 3\theta$

 $x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

 $(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$
 have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$,is
77. The number of values of θ in the interval, $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0 , ±1,±2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is
78. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is
79. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of [k] is
 Note: [k] denotes the largest integer less than or equal to k.
80. The positive integer value of $n > 3$ satisfying the equation
 $\frac{1}{\sin(\frac{\pi}{n})} = \frac{1}{\sin(\frac{2\pi}{n})} + \frac{1}{\sin(\frac{3\pi}{n})}$ is
81. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is
82. Let a,b,c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$ has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is
83. The period of $\sin^2\theta$ is
 a) π^2
 b) π
 c) 2π
 d) $\pi/2$
84. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
 a) 2
 b) 3
 c) 0
 d) 1
85. Which one is not periodic
 a) $|\sin 3x| + \sin^2 x$
 b) $\cos\sqrt{x} + \cos^2 x$
 c) $\cos 4x + \tan^2 x$
 d) $\cos 2x + \sin x$
86. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$
 a) $-\frac{6}{65}$
 b) $\frac{3}{\sqrt{130}}$
 c) $\frac{6}{65}$
 d) $-\frac{3}{\sqrt{130}}$
87. If u = $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$then the difference between the maximum and minimum values of u^2 is given by
 a) $(a-b)^2$
 b) $2\sqrt{a^2+b^2}$
 c) $(a+b)^2$
 d) $2(a^2+b^2)$
88. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
 a) $\frac{2}{5}$
 b) $\frac{1}{5}$
 c) $\frac{3}{5}$
 d) $\frac{2}{3}$
89. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

 $$2\sin^2 x + 5\sin x - 3 = 0 \qquad (1.0.89.1)$$

 is
 a) 4
 b) 6
 c) 1
 d) 2
90. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is

a) $\frac{(1-\sqrt{7})}{4}$
b) $\frac{(4-\sqrt{7})}{3}$
c) $-\frac{(4+\sqrt{7})}{3}$
d) $\frac{(1+\sqrt{7})}{4}$

91. Let A and B denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta) = -\frac{3}{2}$,then :
a) A is false and B is true
b) Both A and B are true
c) both A and B are false
d) A is true and B is false

92. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha,\beta \le \frac{\pi}{4}$, Then $\tan 2\alpha =$
a) $\frac{56}{33}$
b) $\frac{19}{12}$
c) $\frac{20}{7}$
d) $\frac{25}{16}$

93. If A = $\sin^2 x + \cos^4 x$, then for all real x:
a) $\frac{13}{16} \le A \le 1$
b) $1 \le A \le 2$
c) $\frac{3}{4} \le A \le \frac{13}{16}$
d) $\frac{3}{4} \le A \le 1$

94. In a $\triangle PQR$, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :
a) $\frac{5\pi}{6}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{4}$
d) $\frac{3\pi}{4}$

95. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC=p and CD=q, then AB is equal to :
a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$
b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$
c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$
d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

96. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as:
a) $\sin A\cos A + 1$
b) $\sec A cosec A + 1$
c) $\tan A + \cot A$
d) $\sec A + cosec A$

97. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x\epsilon R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals
a) $\frac{1}{4}$
b) $\frac{1}{12}$
c) $\frac{1}{6}$
d) $\frac{1}{3}$

98. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x + \cos 4x = 0$ is:
a) 7
b) 9
c) 3
d) 5

99. If $5(\tan^2 x-\cos^2 x) = 2\cos 2x+9$,then the value of cos4x is :
a) $-\frac{7}{9}$
b) $-\frac{3}{5}$
c) $\frac{1}{3}$
d) $\frac{2}{9}$

100. If sum of all the solutions of the equation 8 $\cos x.(\cos(\frac{\pi}{6}+x)(\cos(\frac{\pi}{6}-x)-\frac{1}{2})-1$ in $[0,\pi]$ is $k\pi$. then k is equal to :
a) $\frac{13}{9}$
b) $\frac{8}{9}$
c) $\frac{20}{9}$
d) $\frac{2}{3}$

101. For any $\theta\epsilon(\frac{\pi}{4},\frac{\pi}{2})$ the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta+\cos\theta)^2 + 4\sin^2\theta$ equals:
a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
b) $13 - 4\cos^6\theta$
c) $13 - 4\cos^2\theta + 6\cos^4\theta$
d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

102. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is:
a) $\frac{3}{4} + \cos 20°$
b) $\frac{3}{4}$
c) $\frac{3}{2}(1+\cos 20°)$
d) $\frac{3}{2}$

103. Let S=$\{\theta\epsilon[-2\pi,2\pi] : 2\cos^\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is
a) $\frac{13\pi}{6}$
b) $\frac{5\pi}{3}$
c) 2
d) 1

Match the Following

DIRECTIONS (Q.1): *Each question contains*

statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B , C and D, while the statements in Column-II are labelled p,q,r,s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to theses questions have to be darkened as illustrated in the following example:
If the correct matches are A-p, s and t; B-q and r; C-p and q; D -s then the correct darkening of bubbles will look like the given

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a) In this question there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.
$\frac{\sin 3\alpha}{\cos 2\alpha}$ is

Column-I	**Column-II**
(A) Positive	(p)$(\frac{13\pi}{48}, \frac{14\pi}{48})$
(B) Negative	(q)$(\frac{14\pi}{48}, \frac{18\pi}{48})$
	(r)$(\frac{18\pi}{48}, \frac{23\pi}{48})$
	(s)$(0, \frac{\pi}{2})$

b) Let
f(x)=sin$(\pi\cos x)$ and g(x)=cos$(2\pi\sin x)$be two functions defined for $x > 0$. Define the following sets whose elements are written i n the increasing order.

X $=\{x : f(x) = 0\}$,Y $=\{x : f'(x) = 0\}$

Z $=\{x : g(x) = 0\}$,W $=\{x : g'(x) = 0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

Column-I	**Column-II**
(A)X	(p)$\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$
(B)Y	(q)an arithmetic progression
(C)Z	(r)NOT an arithmetic progression
(D)W	(s)$\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$
	(t)$\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$
	(u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

i) (IV),(P),(R),(S)
ii) (III),(P),(Q),(U)
iii) (III),(R),(U)
iv) (IV),(Q),(T)

c) Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\pi\sin x)$ be two functions defined for $x > 0$.Define the following sets whose elements are written in the increasing order

X $=\{x : f(x) = 0\}$,Y $=\{x : f'(x) = 0\}$

Z $=\{x : g(x) = 0\}$,W $=\{x : g'(x) = 0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

Column-I	**Column-II**
(A)X	(p)$\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$
(B)Y	(q)an arithmetic progression
(C)Z	(r)NOT an arithmetic progression
(D)W	(s)$\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$
	(t)$\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$
	(u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

i) (I),(Q),(U)
ii) (I),(P),(R)
iii) (II),(R),(S)
iv) (II),(Q),(T)

2 Quadratic Equations and Inequations