

Geometric Constructions through Python

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CONTENTS

1	Triangle	1
2	Circle	4
3	Quadrilaterals	4

Abstract—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.

Download all codes for this manual from

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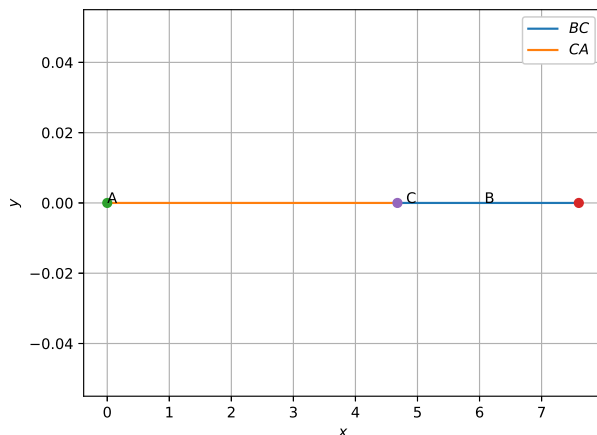


Fig. 1.1

1 TRIANGLE

1.1 Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8.

Solution: Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \quad (1)$$

Then the point C

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k + 1} \quad (2)$$

divides AB in the ratio $k : 1$. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1

codes/draw_section.py

1.2 Draw $\triangle ABC$ where $\angle B = 90^\circ$, $a = 4$ and $b = 3$.

Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (3)$$

The following code plots Fig. 1.2

codes/rt_triangle.py

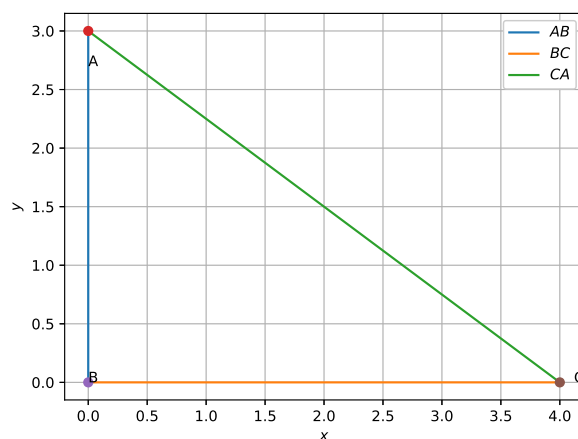


Fig. 1.2

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1.3 Construct a triangle of sides $a = 4\text{cm}$, $b = 5\text{cm}$ and $c = 6\text{cm}$.

Solution: Let the vertices of $\triangle ABC$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (4)$$

Then

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = (p - a)^2 + q^2 \quad (5)$$

$$c^2 = \|\mathbf{A} - \mathbf{B}\|^2 = p^2 + q^2 \quad (6)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (7)$$

$$q = \sqrt{c^2 - p^2} \quad (8)$$

The following code plots Fig. 1.3

```
codes/draw_triangle.py
```

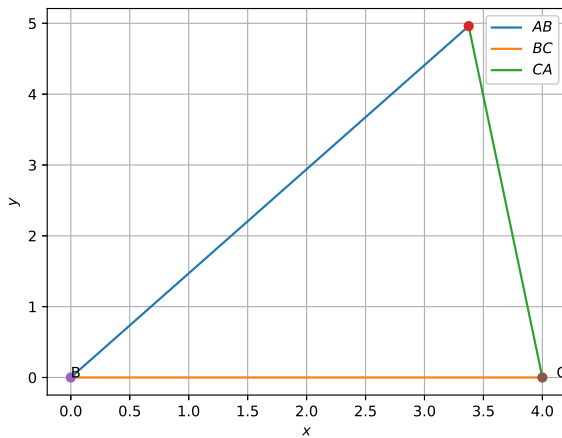


Fig. 1.3

- 1.4 Construct a triangle of sides $a = 5\text{cm}$, $b = 6\text{cm}$ and $c = 7\text{cm}$. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.

Solution: The sides of the similar triangle are $\frac{7}{5}a$, $\frac{7}{5}b$ and $\frac{7}{5}c$.

- 1.5 Construct an isosceles triangle whose base is $a = 8\text{cm}$ and altitude $AD = p = 4\text{cm}$

Solution: Using Baudhayana's theorem,

$$b = c = \sqrt{p^2 + \left(\frac{a}{2}\right)^2} \quad (9)$$

- 1.6 Draw $\triangle ABC$ with $a = 6$, $c = 5$ and $\angle B = 60^\circ$.

Solution: In Fig. (1.6), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \quad (10)$$

$$\cos B = \frac{x}{a}, \quad (11)$$

Thus,

$$a = x + y = b \cos C + c \cos B, \quad (12)$$

$$b = c \cos A + a \cos C \quad (13)$$

$$c = b \cos A + a \cos B \quad (14)$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (15)$$

Using the properties of determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \quad (16)$$

$$= \frac{b^2 + c^2 - a^2}{2bc} \quad (17)$$

From (17)

$$b^2 = c^2 + a^2 - 2ca \cos B \quad (18)$$

which is computed by the following code

```
codes/cos_form.py
```

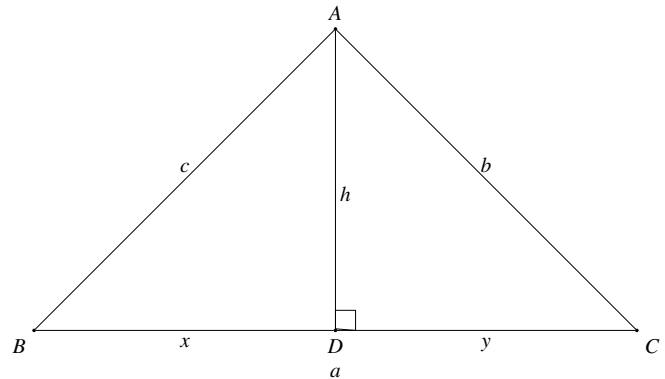


Fig. 1.6: The cosine formula

- 1.7 Draw $\triangle ABC$ with $a = 7$, $\angle B = 45^\circ$ and $\angle A = 105^\circ$.

Solution: In Fig. (1.6),

$$\sin B = \frac{h}{c} \quad (19)$$

$$\sin C = \frac{h}{b} \quad (20)$$

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (21)$$

Thus,

$$c = \frac{a \sin C}{\sin A} \quad (22)$$

where

$$C = 180 - A - B \quad (23)$$

- 1.8 $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.

Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (24)$$

$$\Rightarrow (18 - c)^2 = 12^2 + c^2 \quad (25)$$

which can be simplified to obtain

$$c^2 + 36c^2 - 180 = 0 \quad (26)$$

$$\Rightarrow (c + 18)^2 - 18^2 - 180 = 0 \quad (27)$$

which can be simplified as

$$\Rightarrow (c + 18)^2 = (18^2 + 180) \quad (29)$$

$$\Rightarrow c = -18 \pm \sqrt{18^2 + 180} \quad (30)$$

- 1.9 In $\triangle ABC$, $a = 7$, $\angle B = 75^\circ$ and $b+c = 13$. Find b and c and sketch $\triangle ABC$.
1.10 In $\triangle ABC$, $a = 8$, $\angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.

Solution: The general solution of a quadratic equation

$$\alpha x^2 + \beta x + \gamma = 0 \quad (31)$$

is

$$x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad (32)$$

Using this and (18), b and c can be obtained.

- 1.11 In $\triangle ABC$, $a = 6$, $\angle B = 60^\circ$ and $b-c = 2$. Sketch $\triangle ABC$.
1.12 In $\triangle ABC$, given that $a + b + c = 11$, $\angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c .

Solution: We have

$$a = b \cos C + c \cos B \quad (33)$$

$$b \sin C = c \sin B \quad (34)$$

$$a + b + c = 11 \quad (35)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (36)$$

Solving the equivalent matrix equation gives the desired answer.

- 1.13 Draw $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 30^\circ$ and $\angle C = 90^\circ$.
1.14 Construct $\triangle xyz$ where $xy = 4.5$, $yz = 5$ and $zx = 6$.
1.15 Draw an equilateral triangle of side 5.5.
1.16 Draw $\triangle PQR$ with $PQ = 4$, $QR = 3.5$ and $PR = 4$. What type of triangle is this?
1.17 Construct $\triangle ABC$ such that $AB = 2.5$, $BC = 6$ and $AC = 6.5$. Find $\angle B$.
1.18 Construct $\triangle PQR$, given that $PQ = 3$, $QR = 5.5$ and $\angle PQR = 60^\circ$.
1.19 Draw $\triangle ABC$ if $AB = 3$, $AC = 5$ and $\angle C = 30^\circ$.
1.20 Construct $\triangle DEF$ such that $DE = 5$, $DF = 3$ and $\angle D = 90^\circ$.
1.21 Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110° .
1.22 Construct $\triangle ABC$ with $BC = 7.5$, $AC = 5$ and $\angle C = 60^\circ$.
1.23 Construct $\triangle XYZ$ if $XY = 6$, $\angle X = 30^\circ$ and $\angle Y = 100^\circ$.
1.24 If $AC = 7$, $\angle A = 60^\circ$ and $\angle B = 50^\circ$, can you draw the triangle?
1.25 Construct $\triangle ABC$ given that $\angle A = 60^\circ$, $\angle B = 30^\circ$ and $AB = 5.8$.
1.26 Construct $\triangle PQR$ if $PQ = 5$, $\angle Q = 105^\circ$ and $\angle R = 40^\circ$.
1.27 Can you construct $\triangle DEF$ such that $EF = 7.2$, $\angle E = 110^\circ$ and $\angle F = 180^\circ$?
1.28 Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.
1.29 Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.
1.30 Construct right angled \triangle whose hypotenuse is 6 and one of the legs is 4.
1.31 Construct an isosceles right angled $\triangle ABC$ right angled at C such $AC = 6$.

1.32 Construct the following triangles.

2 CIRCLE

2.1 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents **AC** and **CD** to the circle.

Solution: The tangent is perpendicular to the radius. From the given information, in $\triangle ABC$, $AC \perp AB$, $a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \quad (37)$$

The following code plots Fig. 2.1

```
codes/draw_circle_eg.py
```

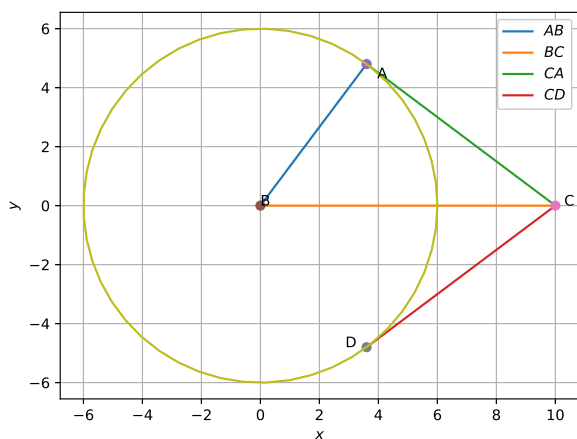


Fig. 2.1

2.2 Draw a circle of diameter 6.1

2.3 Draw a circle of radius 3. Mark any point **A** on the circle, point **B** inside the circle and point **C** outside the circle.

For any angle θ , a point on the circle with radius 3 has coordinates

$$3 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (38)$$

2.4 With the same centre **O**, draw two circles of radii 4 and 2.5

2.5 Draw a circle of radius 3 and any two of its diameters. draw the ends of these diameters. What figure do you get?

2.6 Let **A** and **B** be two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is $AB \perp CD$?

2.7 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

Solution: Take the centre of both circles to be at the origin.

2.8 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

Solution: Take the diameter to be on the x -axis.

2.9 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60° .

Solution: The tangent is perpendicular to the radius.

2.10 Draw a line segment **AB** of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \quad (39)$$

2.11 Let **ABC** be a right triangle in which $a = 8$, $c = 6$ and $\angle B = 90^\circ$. **BD** is the perpendicular from **B** on **AC**. The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.

2.12 Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through **C**

3 QUADRILATERALS

3.1 Construct a quadrilateral **ABCD** such that $AB = 5$, $\angle A = 50^\circ$, $AC = 4$, $BD = 5$ and $AD = 6$.

3.2 Construct **PQRS** where $PQ = 4$, $QR = 6$, $RS = 5$, $PS = 5.5$ and $PR = 7$.

3.3 Draw **ABCD** with $AB = 4.5$, $BC = 5.5$, $CD = 4$, $AD = 6$ and $AC = 7$.

3.4 Draw **JUMP** with $JU = 3.5$, $UM = 4$, $MP = 5$, $PJ = 4.5$ and $PU = 6.5$

3.5 Draw the parallelogram **MORE** with $OR = 6$, $RE = 4.5$ and $EO = 7.5$.

Solution: Diagonals of a parallelogram bisect each other. Opposite sides of a parallelogram are equal and parallel .

- 3.6 Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.
Solution: Diagonals of a rhombus bisect each other at right angles.
- 3.7 Construct a quadrilateral $ABCD$ such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7$ and $AD = 5.5$.
- 3.8 Can you construct a quadrilateral $PQRS$ with $PQ = 3, RS = 3, PS = 7.5, PR = 8$ and $SQ = 4$?
- 3.9 Construct $LIFT$ such that $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.
- 3.10 Draw $GOLD$ such that $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$.
- 3.11 DRAW rhombus $BEND$ such that $BN = 5.6, DE = 6.5$.
- 3.12 construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75^\circ, \angle I = 105^\circ$ and $\angle S = 120^\circ$.
- 3.13 Can you construct the above quadrilateral MIST if $\angle M = 100^\circ$ instead of 75° .
- 3.14 Can you construct the quadrilateral PLAN if $PL = 6, LA = 9.5, \angle P = 75^\circ, \angle L = 150^\circ$ and $\angle A = 140^\circ$?
- 3.15 Construct $MORE$ where $MO = 6, OR = 4.5, \angle M = 60^\circ, \angle O = 105^\circ, \angle R = 105^\circ$.
- 3.16 Construct $PLAN$ where $PL = 4, LA = 6.5, \angle P = 90^\circ, \angle A = 110^\circ$ and $\angle N = 85^\circ$.
- 3.17 Construct parallelogram $HEAR$ where $HE = 5, EA = 6, \angle R = 85^\circ$.
- 3.18 Draw rectangle $OKAY$ with $OK = 7$ and $KA = 5$.
- 3.19 Construct $ABCd$, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105^\circ$ and $\angle C = 80^\circ$.
- 3.20 Construct $DEAR$ with $DE = 4, EA = 5, AR = 4.5, \angle E = 60^\circ$ and $\angle A = 90^\circ$.
- 3.21 Construct $TRUE$ with $TR = 3.5, RU = 3, UE = 4, \angle R = 75^\circ$ and $\angle U = 120^\circ$.
- 3.22 Draw a square of side 4.5.
- 3.23 Can you construct a rhombus $ABCD$ with $AC = 6$ and $BD = 7$?
- 3.24 Construct a kite $EASY$ if $AY = 8, EY = 4$ and $SY = 6$.
Solution: The diagonals of a kite are perpendicular to each other.
- 3.25 Draw a square $READ$ with $RE = 5.1$.
- 3.26 Draw a rhombus whose diagonals are 5.2 and 6.4.
- 3.27 Draw a rectangle with adjacent sides 5 and 4.
- 3.28 Draw a parallelogram $OKAY$ with $OK = 5.5$ and $KA = 4.2$.