# Sequences and series

G V V Sharma*

***Abstract*—Solved problems from JEE mains papers related to Sequences and series in coordinate geometry are available in this document. These problems are solved using linear algebra/matrix analysis.**

1 The sum of integers from 1 to 100 that are divisible by 2 or 5 is.......................
2 The solution of the equation $log_7 log_5(\sqrt{x+5}+\sqrt{x}) = 0$ is ....................
3 The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 +$ ................is $\frac{n(n+1)^2}{2}$, when n is even.When n is odd,the sum is ...............
4 Let the harmonic mean and geometric mean of two positive numbers be the ratio 4:5.Then the two numbers are in the ratio..................
5 For any odd integer n≥ 1,$n^3 - (n-1)^3 + ... + (-1)^{(n-1)}1^3$=............
6 Let p and q be roots of the equation

$$x^2 - 2x + A = 0 \tag{6.1}$$

and let r and s be the roots of the equation

$$x^2 - 18x + B = 0 \tag{6.2}$$

. If $p < q < r < s$ are in arithmetic progression,then A =.......... and B=........

**MCQs with One Correct Answer**

7 If x,y and z are $p^th$,$q^th$ and $r^th$ terms respectively of an A.P.and also of a G.P.,then $x^y - zy^z - xz^x - y$ is equal to:
 a) xyz
 b) 0
 c) 1
 d) None of these
8 The third term of a geometric progression is 4.The product of the first five terms is
 a) $4^3$
 b) $4^5$
 c) $4^4$
 d) None of these
9 The rational number,which equals the number $2.\overline{357}$ with recurring decimal is
 a) $\frac{2355}{1001}$
 b) $\frac{2379}{997}$
 c) $\frac{2355}{999}$
 d) none of these
10 If a,b,c are in G.P.,then the equations

$$ax^2 + 2bx + c = 0 \tag{10.1}$$

and

$$dx^2 + 2ex + f = 0 \tag{10.2}$$

have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in......
 a) A.P.
 b) G.P.
 c) H.P.
 d) None of these
11 Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$.....is equal to
 a) $2^n - n - 1$
 b) $1 - 2^-n$
 c) $n + 2^-n - 1$
 d) $2^n + 1$
12 The number $log_2 7$ is
 a) an integer
 b) a rational number
 c) an irrational number
 d) a prime number
13 If $ln(a+c)$, $ln(a-c)$, $ln(a-2b+c)$ are in A.P.,then
 a) a,b,c are in A.P.
 b) $a^2$, $b^2$, $c^2$ are in A.P.
 c) a,b,c are in G.P.
 d) a,b,c are in H.P.
14 Let $a_1, a_2, .....a_10$ be in A.P.and $h_1, h_2, .....h_{10}$ be in H.P.If $a_1 = h_1 = 2$ and $a_{10} = h_{10}$=3, then $a_4 h_7$ is
 a) 2
 b) 3
 c) 5

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. 

d) 6

15 The harmonic mean of the roots of the equation

$$(5+\sqrt{2})x^2-(4+\sqrt{5})x+8+2\sqrt{5}=0 \tag{15.1}$$

is

a) 2
b) 4
c) 6
d) 8

16 Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then

a) a=$\frac{4}{7}$,r=$\frac{3}{7}$
b) a=2,r=$\frac{3}{8}$
c) a=$\frac{3}{2}$,r=$\frac{1}{2}$
d) a=3,r=$\frac{1}{4}$

17 Let $\alpha$, $\beta$ be the roots of

$$x^2-x+p=0 \tag{17.1}$$

and $\gamma$, $\delta$ be the roots of

$$x^2-4x+q=0 \tag{17.2}$$

. If $\alpha$, $\beta$, $\gamma$, $\delta$ are in G.P., then the integral values of p and q respectively, are

a) -2,-32
b) -2,3
c) -6,3
d) -6,-32

18 Let the positive numbers a,b,c,d be in A.P.Then abc,abd,acd,bcd are

a) NOT in A.P./G.P./H.P.
b) in A.P.
c) in G.P.
d) in H.P.

19 If the sum of the first 2n terms of the A.P.2,5,8,.....,is equal to the sum of the first n terms of the A.P.57,59,61,.....,then n equals

a) 10
b) 12
c) 11
d) 13

20 Suppose a,b,c are in A.P. and $a^2,b^2,c^2$ are in G.P.if $a<b<c$ and a+b+c=$\frac{3}{2}$, then the value of a is

a) $\frac{1}{2\sqrt{2}}$
b) $\frac{1}{2\sqrt{3}}$
c) $\frac{1}{2}$-$\frac{1}{\sqrt{3}}$
d) $\frac{1}{2}$-$\frac{1}{\sqrt{2}}$

21 An infinite G.P. has first term 'x' and sum '5', then x belongs to

a) $x<-10$
b) $-10<x<0$
c) $0<x<10$
d) $x>10$

22 In the quadratic equation

$$ax^2+bx+c=0, \tag{22.1}$$

$\Delta=b^2-4ac$ and $\alpha+\beta$, $\alpha^2+\beta^2$, $\alpha^3+\beta^3$, are in G.P.where $\alpha,\beta$ are the root of $ax^2+bx+c=0$, then

a) $\Delta\neq 0$
b) $b\Delta=0$
c) $c\Delta 0$
d) $\Delta=0$

23 In the sum of first n terms of an A.P.is $cn^2$, then the sum of squares of these n terms is

a) $\frac{n(4n^2-1)c^2}{6}$
b) $\frac{n(4n^2+1)c^2}{3}$
c) $\frac{n(4n^2-1)c^2}{3}$
d) $\frac{n(4n^2+1)c^2}{6}$

24 Let $a_1,a_2,a_3$,........be in harmonic progression with $a_1=5$ and $a_{20}=25$. The least positive integer n for which $a_n<0$ is

a) 22
b) 23
c) 24
d) 25

25 Let $b_1>1$ for $i=1,2,....,101$.Suppose $\log_e b_1,\log_e b_2,.....\log_e b_{101}$ are in Arithmetic progression(A.P) with the common difference $\log_e 2$. Suppose $a_1,......a_{101}$ are in A.P such that $a_1=b_1$ and $a_{51}=b_{51}$.If $t=b_1+b_2+......+b_{51}$ and $s=a_1+a_2+...a_{53}$, then

a) $s>t$ and $a_{101}>b_{101}$
b) $s>t$ and $a_{101}<b_{101}$
c) $s<t$ and $a_{101}>b_{101}$
d) $s<t$ and $a_{101}<b_{101}$

**MCQs with One or More than One Correct**

26 If the first and the $(2n-1)$st terms of an A.P., a G.P. and an H.P. are equal and their n-th terms are a,b and c respectively , then

a) a=b=c
b) $a\geq b\geq c$

c) a+c=b
d) $ac - b^2 = 0$.
27 For $0 < \phi < \frac{\Pi}{2}$, if $x = \sum_{n=0}^{\infty}(\cos^{2n})\phi$, $y = \sum_{n=0}^{\infty}(\sin^{2n})\phi$, $z = \sum_{n=0}^{\infty}(\cos^{2n})\phi(\sin^{2n})\phi$ then:
a) xyz = xz+y
b) xyz = xy+z
c) xyz = x+y+z
d) xyz = yz+x
28 Let n be an odd integer.If $\sin n\theta = \sum_{r=0}^{n}(b)_r \sin^r \theta$, for every value of $\theta$,then
a) $b_0 = 1, b_1 = 3$
b) $b_0 = 0, b_1 = n$
c) $b_0 = -1, b_1 = n$
d) $b_0 = 0, b_1 = n^2 - 3n + 3$
29 Let $T_r$ be the $r^{th}$ term of an A.P.,for r=1,2,3,..... If for some positive integers m,n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $T_{mn}$ equals
a) $\frac{1}{mn}$
b) $\frac{1}{m}+\frac{1}{n}$
c) 1
d) 0
30 If $x > 1, y > 1, z > 1$ are in G.P.,then $\frac{1}{1+lnx}, \frac{1}{1+lny}, \frac{1}{1+lnz}$ are in
a) A.P.
b) H.P.
c) G.P.
d) None of these
31 For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + .....\frac{1}{(2^n)-1}$. Then
a) $a(100) \leq 100$
b) $a(100) > 100$
c) $a(200) \leq 100$
d) $a(200) > 100$
32 A straight line through the vertex P of a triangle PQR intersects the side QR at the points **S** and the circumcircle of the triangle PQR at the point **T**.If S is not the centre of the circumcircle, then
a) $\frac{1}{(PS}+ \frac{1}{(ST} < \frac{2}{\sqrt{QSXSR}}$
b) $\frac{1}{(PS}+ \frac{1}{(ST} > \frac{2}{\sqrt{QSXSR}}$
c) $\frac{1}{(PS}+ \frac{1}{(ST} < \frac{4}{QR}$
d) $\frac{1}{(PS}+ \frac{1}{(ST} > \frac{2}{QR}$
33 Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$ for n=1,2,3......Then
34 Let $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}} k^2$. Then $S_n$ can take value(S)
a) 1056
b) 1088
c) 1120
d) 1332
35 Let $\alpha$ and $\beta$ be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$.For all positive integers n,define $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1 b_1$ and $b_n = a_{n-1} + a_{n+1}, n \geq 2$ Then which of the following options is /are correct?
a) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$
b) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$
c) $a_1 + a_2 + a_3 + ....a_n = a_{n+2} - 1$ for all $n \geq 1$
d) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

**Subjective Problems**

36 The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation, $2A + G^2 = 27$. Find the two numbers.
37 The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°, and the common difference is 5°. Find the number of sides of the polygon.
38 Does there exists a geometric progression containing 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible ?
39 Find three numbers a,b,c between 2 and 18 such that
a) their sum is 25
b) the numbers 2,a,b are consecutive terms of an A.P. and
c) the numbers b,c,18 are consecutive terms of a G.P.
40 If $a > 0, b > 0$ and $c > 0$, prove that $(a + b + c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \geq 9$
41 If n is a natural number such that $n = p_1^{\alpha}1.p_2^{\alpha}2.p_3^{\alpha}3....p_k^{\alpha}k$ and $p_1, p_2, ......p_k$ are distinct primes, then show that $lnn \geq kln2$
42 Find the sum of the series: $\sum_{r=0}^{n}(-1)^r nC_r[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}}$......upto m terms]
43 Solve for x the following equation: $log_{2x+3}(6x^2+23x+21) = 4-log_{3x+7}(4x^2+12x+9)$
44 If $log_3 2$, $log_3(2^x - 5)$, $log_3(2^x - \frac{7}{2})$ are in arithmetic progression, determine the value of x.
45 Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $[\frac{n+1}{n-1}]^2$p.
46 If $S_1$, $S_2$, $S_3$,......., $S_n$ are the sums of infinite geometric series whose first terms are 1, 2,

3,......, n and whose common ratios are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$,..........$\frac{1}{n+1}$ respectively, then find the values of $S_1^2$+ $S_2^2$+ $S_3^2$,......., $S_{2n-1}^2$

47 The real numbers $x_1, x_2, x_3$ satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P. Find the intervals in which $\beta$ and $\gamma$ lie.

48 Let a,b,c,d are the real numbers in G.P. If u,v,w, satisfy the system of equations $u + 2v + 3w = 6$ $4u + 5v + 6w = 12$ $6u + 9v = 4$ then show that the root of the equation $(\frac{1}{u} + \frac{1}{v} + \frac{1}{w})x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]x + u + v + w = 0$ and $20x^2 + 10(a-d)^2x - 9 = 0$ are reciprocals of the each other.

49 The fouth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer.

50 Let $a_1$, $a_2$,.......,$a_n$ be positive real numbers in geometric progression. For each n, let $A_n$, $G_n$, $H_n$ be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1$, $a_2$,.......,$a_n$. Find an expression for the geometric mean of $G_1$, $G_2$,......, $G_n$, in terms of $A_1$, $A_2$,.......,$A_n$, $H_1$,$H_2$,......,$H_n$,

51 Let a, b be positive real numbers.If a, $A_1$, $A_2$, b are in arithmetic progression, a, $G_1$, $G_2$, b are in geometric progression and a, $H_1$, $H_2$, b are in harmonic progression, show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$.

52 If a, b, c are in A.P., $a^2$, $b^2$, $c^2$ are in H.P., then prove that either a = b = c or a, b, $-\frac{c}{2}$ form a G.P.

53 If $a_n = \frac{3}{4} - [\frac{3}{4}]^2 + [\frac{3}{4}]^3 + ......(-1)^{n-1}[\frac{3}{4}]^n$ and $b_n = 1 - a_n$, then find the least natural number $n_0$ such that $b_n > a_n \forall n \geq n_0$.

**Comprehension Based Questions**

**PASSAGE - 1**

Let $V_r$ denote the sum of first r terms of an arithmetic progression(A.P.) whose first term is r and the common difference is (2r-1). Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r=1,2,...

54 The sum $V_1 + V_2 + .... + V_n$ is

a) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$
b) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$
c) $\frac{1}{2}n(2n^2 - n + 1)$
d) $\frac{1}{3}(2n^3 - 2n + 3)$

55 $T_r$ is always

a) an odd number
b) an even number
c) a prime number
d) a composite number

56 Which one of the following is a correct statement ?

a) $Q_1, Q_2, Q_3$,.... are in A.P. with common difference 5
b) $Q_1, Q_2, Q_3$,.... are in A.P. with common difference 6
c) $Q_1, Q_2, Q_3$,.... are in A.P. with common difference 11
d) $Q_1 = Q_2 = Q_3$=........

**PASSAGE - 2**

Let $A_1, G_1, H_1$ denote the arithmetic, geometric and harmonic means respectively, of two distinct positive numbers. For $n \geq 2$, let $A_{n-1}$ and $H_{n-1}$ have arithmetic, geometric and harmonic means as $A_n, G_n, H_n$ respectively.

57 Which one of the following statements is correct ?

a) $G_1 > G_2 > G_3 >$ ..........
b) $G_1 < G_2 < G_3 <$ ..........
c) $G_1 = G_2 = G_3 =$ ..........
d) $G_1 < G_3 < G_5$ and $G_2 > G_4 > G_6 >$ ..........

58 Which one of the following statements is correct ?

a) $A_1 > A_2 > A_3 >$ ..........
b) $A_1 < A_2 < A_3 <$ ..........
c) $A_1 > A_3 > A_5 >$ .... and $A_2 < A_4 < A_6 <$ ..........
d) $A_1 < A_3 < A_5 <$ .... and $A_2 > A_4 > A_6 >$ ..........

59 Which one of the following statements is correct ?

a) $H_1 > H_2 > H_3 >$ ..........
b) $H_1 < H_2 < H_3 <$ ..........
c) $H_1 > H_3 > H_5 >$ .... and $H_2 < H_4 < H_6 <$ ..........
d) $H_1 < H_3 < H_6 <$ .... and $H_2 > H_4 > H_6 >$ ..........

**Assertion Reson type quations**

60 Suppose four distinct positive numbers $a_1$, $a_2$, $a_3$, $a_4$ are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT - 1: The numbers $b_1$, $b_2$, $b_3$, $b_4$ are neither in A.P. nor in G.P. and

STATEMENT - 2: The numbers $b_1$, $b_2$, $b_3$, $b_4$ are in H.P.

a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a NOT a correct explanation for STATEMENT - 1
c) STATEMENT - 1 is True, STATEMENT - 2 is False
d) STATEMENT - 1 is False, STATEMENT - 2 is True

**Integer Value Correct Type**

61 Let $S_k, k = 1, 2, ......, 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2 - 3k + 1)S_k\right|$ is

62 $a_1, a_2, a_3, ......., a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k= 3,4,.....,11, if $\frac{a_1^2+a_2^2+.....+a_{11}^2}{11} = 90$,then the value of $\frac{a_1+a_2+.....+a_{11}}{11}$ is equal to

63 Let $a_1$, $a_2$, $a_3$,.......,$a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100$.For any integer n with $1 \leq n \leq 20$, let m=5n .If $\frac{S_m}{S_n}$ does not depend on n, then $a_2$ is......

64 A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k-20=..........

65 Let a ,b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b+2, then the value of $\frac{a^2+a-14}{a+1}$ is

66 Suppose that all the terms of an arithmetic progression(A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6:11 and the seventh term is lies in between 130 and 140, then the common difference of this A.P. is

67 The coefficient of $x^9$ in the expansion of $(1 + x)(1 + x^2)(1 + x^3)......(1 + x^{100})$ is

68 The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

69 Let X be the set consisting of the first 2018 terms of the arithmetic progression $1, 6, 11, ....,$ and Y be the set consisting of the first 2018 terms of the arithmetic progression $9, 16, 23, .....$ Then, the number of elements in the set $X \cup Y$ is ....

70 Let AP(a;d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If AP(1;3) AP(2;5) AP(3;7)= AP(a;d) then a+d equals.....

**Section-B JEE Main/AIEEE**

71 If 1, $log_9(3^{1-x} + 2), log_3(4.3^x - 1)$ are in A.P. then x equals
a) $log_3 4$
b) $1 - log_3 4$
c) $1 - log_4 3$
d) $log_4 3$

72 l, m, n are the $p^{th}, q^{th}$ , and $r^{th}$ term of a G.P. all positive, then $\begin{pmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log nn & r & 1 \end{pmatrix}$ equals
a) -1
b) 2
c) 1
d) 0

73 The value of $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.........\infty$ is
a) 1
b) 2
c) $\frac{3}{2}$
d) 4

74 Fifth term of a G.P. is 2, then the product of its 9 terms is
a) 256
b) 512
c) 1024
d) none of these

75 Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is
a) 5
b) $\frac{3}{5}$
c) $\frac{8}{5}$
d) $\frac{1}{5}$

76 $1^3 - 2^3 + 3^3 - 4^3 + .... + 9^3 =$
a) 425
b) -425
c) 475
d) -475

77 The sum of the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4}.......$ upto $\infty$ is equal to
a) $log_e \frac{4}{e}$

b) $2log_e2$
c) $log_e2 - 1$
d) $log_e2$

78 If $S_n = \sum_{r=0}^{n} \frac{1}{nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{nC_r}$, then $\frac{t_n}{S_n}$ is equal to
a) $\frac{2n-1}{2}$
b) $\frac{1}{2}$n-1
c) n-1
d) $\frac{1}{2}$n

79 Let $T_r$ be the $r^{th}$ term of an A.P.whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a-d equals
a) $\frac{1}{m} + \frac{1}{n}$
b) 1
c) $\frac{1}{mn}$
d) 0

80 The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + .....$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is
a) $[\frac{n(n+1)}{2}]^2$
b) $\frac{n^2(n+1)}{2}$
c) $\frac{n(n+1)^2}{4}$
d) $\frac{3n(n+1)}{2}$

81 The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!}.....$ is
a) $\frac{(e^2-2)}{e}$
b) $\frac{(e-1)^2}{2e}$
c) $\frac{(e^2-1)}{2e}$
d) $\frac{(e^2-1)}{2}$

82 If the coefficient of $r^{th}$,$(r+1)^th$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation
a) $m^2 - m(4r-1) + 4r^2 - 2 = 0$
b) $m^2 - m(4r+1) + 4r^2 + 2 = 0$
c) $m^2 - m(4r+1) + 4r^2 - 2 = 0$
d) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

83 If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1$ then x, y, z are in
a) G.P.
b) A.P.
c) Arithmetic - Geometric Progression
d) H.P.

84 The sum of series $1+\frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.6!}.....\infty$ is
a) $\frac{(e-1)}{\sqrt{e}}$
b) $\frac{(e+1)}{\sqrt{e}}$
c) $\frac{(e-1)}{2\sqrt{e}}$
d) $\frac{(e+1)}{2\sqrt{e}}$

85 Let $a_1$, $a_2$, $a_3$.....be terms in A.P. If $\frac{a_1+a_2+......+a_p}{a_1+a_2+......+a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
a) $\frac{41}{11}$
b) $\frac{7}{2}$.
c) $\frac{2}{7}$
d) $\frac{11}{41}$

86 If $a_1$, $a_2$, $a_3$.....$a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ..... + a_{n-1}a_n$ is equal to
a) $n(a_1 - a_n)$
b) $(n-1)(a_1 - a_n)$
c) $n(a_1a_n)$
d) $(n-1)(a_1a_n)$

87 The sum of series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}.....$ upto $\infty$ is
a) $e^{-\frac{1}{2}}$
b) $e^{+\frac{1}{2}}$
c) $e^{-2}$
d) $e^{-1}$

88 In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals
a) $\sqrt{5}$
b) $\frac{1}{2}(\sqrt{5} - 1)$
c) $\frac{1}{2}(1 - \sqrt{5})$
d) $\frac{1}{2}\sqrt{5}$

89 The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
a) -4
b) -12
c) 12
d) 4

90 The sum to infinite term of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + .....$ is
a) 3
b) 4
c) 6
d) 2

91 A person is to count 4500 currency notes. Let $a_n$ denote the number of notes he counts in the $n^{th}$ minute. If $a_1 = a_2 = ..... = a_{10} = 150$ and $a_{10}, a_{11}, .....$ are in A.P. with common difference -2, then the time taken by him to count all notes

is

a) 34 minutes
b) 125 minutes
c) 135 minutes
d) 24 minutes

92 A man saves 200 in each of the first three months of his service. In each of the subsequent months his saving increases by 40 more than the saving of immediately previous month. His total savings from the start of service will be 11040 after

a) 19 months
b) 20 months
c) 21 months
d) 18 months

93 **Statement - 1:** The sum of the series 1+(1+2+4)+(4+6+9)+(9+12+16)+.....+(361+380+400) is 8000.

**Statement - 2:** $\sum_{k=1}^{n}(k^3-(k-1)^3) = n^3$, for any natural number n.

a) Statement - 1 is false, Statement - 2 is true.
b) Statement - 1 is true, Statement - 2 is true, Statement - 2 is a correct explanation for Statement - 1
c) Statement - 1 is true, Statement - 2 is true, Statement - 2 is a not a correct explanation for Statement - 1
d) Statement - 1 is true, Statement - 2 is false.

94 The sum of the first 20 terms of sequence 0.7,0.77,0.777,....,is

a) $\frac{7}{81}(179 - 10^{-20})$
b) $\frac{7}{9}(99 - 10^{-20})$
c) $\frac{7}{81}(179 + 10^{-20})$
d) $\frac{7}{9}(99 + 10^{-20})$

95 If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + ........ + 10(11)^9 = k(10)^9$, then k is equal to :

a) 100
b) 110
c) $\frac{122}{10}$
d) $\frac{441}{100}$

96 Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ration of the G.P. is:

a) $2-\sqrt{3}$
b) $2+\sqrt{3}$
c) $\sqrt{2}+\sqrt{3}$
d) $3+\sqrt{2}$

97 The sum of the first 9 terms of the series. $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} +.......$

a) 142
b) 192
c) 71
d) 96

98 If m is the A.M. of two distinct real numbers l and $n(l, n > 1)$ and $G_1, G_2$ and $G_3$ are the three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals:

a) $4lmn^2$
b) $4l^2m^2n^2$
c) $4l^2mn$
d) $4lm^2n$

99 If the $2^{nd}, 5^{th}$ and $9^{th}$ terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:

a) 1
b) $\frac{7}{4}$
c) $\frac{8}{5}$
d) $\frac{4}{3}$

100 If the sum of the first ten terms of the series $(1\frac{3}{5})^2 + (2\frac{2}{5})^2 + (3\frac{1}{5})^2 + 4^2 + (4\frac{4}{5})^2+......$, is $\frac{16}{5}$m then m is equal to :

a) 100
b) 99
c) 102
d) 101

101 If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + ..... + (x+n-1)(x+n) = 10n$ has two consecutive integral solutions, then n is equal to :

a) 11
b) 12
c) 9
d) 10

102 For any three positive real numbers a, b and c,

$$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c) \tag{102.1}$$

Then:

a) a, b and c are in G.P.
b) b, c and a are in G.P.
c) b, c and a are in A.P.
d) a, b and c are in A.P.

103 Let a, b, c ∈ R. If $f(x) = ax^2+bx+c$ is such that $a+b+c = 3$ $f(x+y) = f(x)+f(y)+xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to :

a) 255
b) 330
c) 165
d) 190

104 Let $a_1, a_2, a_3, ......, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.If $a_1^2 + a_2^2 + ..... + a_{17}^2 = 140m$, then m is equal to

a) 68
b) 34
c) 33
d) 66

105 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+.....$ If $B-2A = 100\Lambda$, then $\Lambda$ is equal to :

a) 248
b) 464
c) 496
d) 232

106 If a, b, and c be three distinct real numbers in G.P. and a+b+c=xb, then x cannot be :

a) -2
b) -3
c) 4
d) 2

107 Let $a_1, a_2, a_3, ......, a_{30}$ be in A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{2i-1}$. If $a_5 = 27$ and S-2T=75, Then $a_{10}$ is equal to :

a) 52
b) 57
c) 47
d) 42

108 Three circles of radii a, b, c ($a < b < c$) touch each other externally. If they have X-axis as a common tangent, then :

a) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$
b) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$
c) a, b, c are in A.P.
d) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

109 Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, ..........$ be $50n+\frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P.,then the ordered pair $(d, a_{10})$ is equal to:

a) (50, 50+46A)
b) (50, 50+45A)
c) (A, 50+45A)
d) (A, 50+46A)