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The Straight Line

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***Abstract*—Solved problems from JEE mains papers related to 2D lines in coordinate geometry are available in this document. These problems are solved using linear algebra/matrix analysis.**

1. A straight line through the origin **O** meets the lines

$$\begin{pmatrix}4 & 3\end{pmatrix}\mathbf{x} = 10 \tag{1}$$

$$\begin{pmatrix}8 & 6\end{pmatrix}\mathbf{x} + 5 = 0 \tag{2}$$

at **A** and **B** respectively. Find the ratio in which **O** divides AB.
Solution: Let

$$\mathbf{n} = \begin{pmatrix}4 & 3\end{pmatrix} \tag{3}$$

Then (1) can be expressed as

$$\mathbf{n}^T\mathbf{x} = 10 \tag{4}$$

$$2\mathbf{n}^T\mathbf{x} = -5 \tag{5}$$

and since $\mathbf{A}, \mathbf{B}$ satisfy (4) respectively,

$$\mathbf{n}^T\mathbf{A} = 10 \tag{6}$$

$$2\mathbf{n}^T\mathbf{B} = -5 \tag{7}$$

Let **O** divide the segment AB in the ratio $k : 1$. Then

$$\mathbf{O} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{8}$$

$$\because \mathbf{O} = \mathbf{0}, \tag{9}$$

$$\mathbf{A} = -k\mathbf{B} \tag{10}$$

Substituting in (6), and simplifying,

$$\mathbf{n}^T\mathbf{B} = \frac{10}{-k} \tag{11}$$

$$\mathbf{n}^T\mathbf{B} = \frac{-5}{2} \tag{12}$$

resulting in

$$\frac{10}{-k} = \frac{-5}{2} \implies k = 4 \tag{13}$$

2. The point

$$\begin{pmatrix}2\\1\end{pmatrix} \tag{14}$$

is translated parallel to the line

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 4 \tag{15}$$

by $2\sqrt{3}$ units. If the new point **Q** lies in the third quadrant, then find the equation of the line passing through **Q** and perpendicular to L.
3. A variable line drawn through the intersection of the lines

$$\begin{pmatrix}4 & 3\end{pmatrix}\mathbf{x} = 12 \tag{16}$$

$$\begin{pmatrix}3 & 4\end{pmatrix}\mathbf{x} = 12 \tag{17}$$

meets the coordinate axes at **A** and **B**, then find the locus of the midpoint of AB.
4. Two sides of a rhombus are along the lines

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 1 = 0 \tag{18}$$

$$\begin{pmatrix}7 & -1\end{pmatrix}\mathbf{x} - 5 = 0. \tag{19}$$

If its diagonals intersect at

$$\begin{pmatrix}-1\\-2\end{pmatrix}, \tag{20}$$

find its vertices.
5. Let k be an integer such that the triangle with

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vertices

$$\begin{pmatrix} k \\ -3k \end{pmatrix}, \begin{pmatrix} 5 \\ k \end{pmatrix}, \begin{pmatrix} -k \\ 2 \end{pmatrix} \tag{21}$$

has area 28. Find the orthocentre of this triangle.

6. If an equlateral triangle, having centroid at the origin, has a side along the line

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2, \tag{22}$$

then find the area of this triangle.

7. Find the locus of the point of intersection of the straight lines

$$\begin{pmatrix} t & -2 \end{pmatrix} \mathbf{x} - 3t = 0 \tag{23}$$

$$\begin{pmatrix} 1 & -2t \end{pmatrix} \mathbf{x} + 3 = 0 \tag{24}$$

8. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then find the sum of the x-coordinates of the vertices of the square.
9. Find the locus of the point of intersection of the lines

$$\begin{pmatrix} \sqrt{2} & -1 \end{pmatrix} \mathbf{x} + 4\sqrt{2}k = 0 \tag{25}$$

$$\begin{pmatrix} \sqrt{2}k & k \end{pmatrix} \mathbf{x} - 4\sqrt{2} = 0 \tag{26}$$

10. The sides of a rhombus ABC are parallel to the lines

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} + 2 = 0 \tag{27}$$

$$\begin{pmatrix} 7 & -1 \end{pmatrix} \mathbf{x} + 3 = 0. \tag{28}$$

If the diagonals of the rhombus intersect at

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{29}$$

and the vertex $\mathbf{A}$ (different) from the origin is on the y-axis, then find the ordinate of A.