Indefinite Integrals: JEE Maths

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1. If $\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax + Blog(9e^{2x} - 4) + C$, then A =....., B =....., and C =.....

MCQs with One Correct Answer:

2. The value of the integral $\int\frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$ is
   a) $\sin x - 6\tan^{-1}(\sin x) + C$
   b) $\sin x - 2(\sin x)^{-1} + C$
   c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$
   d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$
3. If $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x$, then $f(\frac{1}{\sqrt{3}})$ is
   a) $\frac{1}{3}$
   b) $\frac{1}{\sqrt{3}}$
   c) 3
   d) $\sqrt{3}$
4. $\int\frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx =$
   a) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + C$
   b) $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + C$
   c) $\frac{\sqrt{2x^4-2x^2+1}}{x} + C$
   d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + C$
5. Let $I = \int\frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J = \int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. Then for an arbitary constant C, then the value of J - I equals
   a) $\frac{1}{2}log(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}) + C$
   b) $\frac{1}{2}log(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}) + C$
   c) $\frac{1}{2}log(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}) + C$
   d) $\frac{1}{2}log(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}) + C$
6. The integral $\int\frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals(for some arbitary constant K)
   a) $-\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\} + K$
   b) $\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\} + K$
   c) $-\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\} + K$
   d) $\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\} + K$

Subjective Problems:

7. Evaluate
$$\int\frac{\sin x}{\sin x - \cos x}dx$$
8. Evaluate
$$\int\frac{x^2}{(a+bx)^2}$$
9. Evaluate
$$\int(e^{logx} + \sin x)\cos x dx$$
10. Evaluate:
$$\int\frac{(x-1)e^x}{(x+1)^3}dx$$
11. Evaluate the following
$$\int\frac{dx}{x^2(x^4+1)^3/4}$$
12. Evaluate the following
$$\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$$
13. Evaluate:
$$\int[\frac{(\cos 2x)^1/2}{\sin x}]dx$$
14. Evaluate
$$\int(\sqrt{\tan x} + \sqrt{\cot x})dx$$
15. Find the indefinite integral
$$\int(\frac{1}{x^{1/3}+4^{1/4}} + \frac{ln(1+x^{1/6})}{x^{1/3}+x^{1/2}})dx$$
16. Find the indefinite integral
$$\int\cos 2\theta ln(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta})d\theta$$
17. Evaluate
$$\int\frac{(x+1)}{x(1+xe^x)^2}dx$$

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18. Integrate

$$\frac{x^3+3x+2}{(x^2+1)^2(x+1)}dx$$

19. Evaluate

$$\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx.$$

20. For any natural number m, evaluate

$$\int (x^{3m}+x^{2m}+x^m)(2x^{2m}+3x^m+6)^{l/m}dx, x>0$$

Assertion and Reason Type Questions:

21. Let F(x) be an indefinite integral of $\sin^2 x$.
STATEMENT-1: The function F(x) satisfies $F(x+\pi)=F(x)$ for all real x.
STATEMENT-2: $\sin^2(x+\pi)=\sin^2 x$ for all real x.
a) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1.
b) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1.
c) Statement-1 is True, Statement-2 is False.
d) Statement-1 is False, Statement-2 is True.

Section - B:

22. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + Blog\sin(x-alpha)+C$, then the value of (A, B) is
a) $(-\cos\alpha, \sin\alpha)$
b) $(\cos\alpha, \sin\alpha)$
c) $(-\sin\alpha, \cos\alpha)$
d) $(\sin\alpha, -\cos\alpha)$

23. $\int \frac{dx}{\cos x-\sin x}$ is equal to
a) $\frac{1}{\sqrt{2}}log|\tan(\frac{x}{2})+\frac{3\pi}{8}|+C$
b) $\frac{1}{\sqrt{2}}log|\cot(\frac{x}{2})|+C$
c) $\frac{1}{\sqrt{2}}log|\tan(\frac{x}{2})-\frac{3\pi}{8}|+C$
d) $\frac{1}{\sqrt{2}}log|\tan(\frac{x}{2})-\frac{\pi}{8}|+C$

24. $\int\{\frac{(logx-1)}{1+(logx)^2}\}^2dx$ is equal to
a) $\frac{logx}{(logx)^2+1}+C$
b) $\frac{x}{x^2+1}+C$
c) $\frac{xe^x}{1+x^2}+C$
d) $\frac{x}{(logx)^2+1}+C$

25. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals
a) $log\tan(\frac{x}{2}+\frac{\pi}{12})+C$
b) $log\tan(\frac{x}{2}-\frac{\pi}{12})+C$
c) $\frac{1}{2}log\tan(\frac{x}{2}+\frac{\pi}{12})+C$
d) $\frac{1}{2}log\tan(\frac{x}{2}-\frac{\pi}{12})+C$

26. The value of $\sqrt{2}\int\frac{\sin x dx}{\sin(x-\frac{\pi}{4})}$ is
a) $x+log|\cos(x-\frac{\pi}{4})|+C$
b) $x-log|\sin(x-\frac{\pi}{4})|+C$
c) $x+log|\sin(x-\frac{\pi}{4})|+C$
d) $x-log|\cos(x-\frac{\pi}{4})|+C$

27. If the $\int\frac{5\tan x}{\tan x-2}dx = x+aln|\sin x-2\cos x|+k$, then a is equal to:
a) -1
b) -2
c) 1
d) 2

28. If $\int f(x)dx=\psi(x)$, then $\int x^5 f(x^3)dx$ is equal to
a) $\frac{1}{3}[x^3\psi(x^3)-\int x^2\psi(x^3)dx]+C$
b) $\frac{1}{3}[x^3\psi(x^3)-3\int x^3\psi(x^3)dx]+C$
c) $\frac{1}{3}[x^3\psi(x^3)-\int x^2\psi(x^3)dx]+C$
d) $\frac{1}{3}[x^3\psi(x^3)-\int x^3\psi(x^3)dx]+C$

29. The integral $\int(1+x-\frac{1}{x})e^{x+\frac{1}{x}}dx$ is equal to
a) $(x+1)e^{x+\frac{1}{x}}+C$
b) $(-x)e^{x+\frac{1}{x}}+C$
c) $(x-1)e^{x+\frac{1}{x}}+C$
d) $(x)e^{x+\frac{1}{x}}+C$

30. The integral $\int\frac{dx}{x^2(x^4+1)^{3}/4}$ equals
a) $-(x^4+1)^{1/4}+C$
b) $-(\frac{x^4+1}{x^4})^{1/4}+C$
c) $(\frac{x^4+1}{x^4})+C$
d) $(x^4+1)^{1/4}+C$

31. The integral $\int\frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to:
a) $\frac{x^5}{2(x^5+x^3+1)^2}+C$
b) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$
c) $\frac{-x^5}{(x^5+x^3+1)^2}+C$
d) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$

32. Let $I_n=\int\tan x dx, (n>1)$. $I_4+I_6=a\tan^5 x+bx^5+C$, where C is constant of integration, then the ordered pair (a, b) is equal to:
a) $(-\frac{1}{5},0)$
b) $(-\frac{1}{5},1)$
c) $(\frac{1}{5},0)$
d) $(\frac{1}{5},-1)$

33. The integral

$$\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^3 x\cos^2+\cos^5 x)^2}dx$$

is equal to

a) $\frac{-1}{3(1+\tan^3 x)} + C$
b) $\frac{1}{1+\cot^3 x} + C$
c) $\frac{-1}{1+\cot^3 x} + C$
d) $\frac{1}{3(1+\tan^3 x)} + C$

34. For $x^2 \neq n\pi + 1, n \in N$(the set of natural numbers), the integral

$$\int x \sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}} dx$$

is equal to

a) $log_e|\frac{1}{2}\sec^2(x^2-1)| + C$
b) $\frac{1}{2}log_e|\sec(x^2-1)| + C$
c) $\frac{1}{2}log_e|\sec^2(\frac{x^2-1}{2})| + C$
d) $log_e|\sec(\frac{x^2-1}{2})| + C$

35. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to

a) $-3\tan^{-1/3} x + C$
b) $-\frac{3}{4}\tan^{-4/3} x + C$
c) $-3\cot^{-1/3} x + C$
d) $3\tan^{-1/3} x + C$