Problems in Linear Algebra

Contents

1 Introduction

1.1 Definitions

1. The *inner product* of $\mathbf{P}$ and $\mathbf{Q}$ is defined as
$$\mathbf{P}^T\mathbf{Q} = p_1q_1 + p_2q_2 \tag{1.1.1}$$
2. The *norm* of a vector
$$\mathbf{P} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \tag{1.1.2}$$
is defined as
$$\|\mathbf{P}\| = \sqrt{p_1^2 + p_2^2} \tag{1.1.2}$$
3. The *length* of PQ is defined as
$$\|\mathbf{P} - \mathbf{Q}\| \tag{1.1.3}$$
4. The *direction vector* of the line PQ is defined as
$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} p_1 - q_1 \\ p_2 - q_2 \end{pmatrix} \tag{1.1.4}$$
5. *Orthogonality*
$$PQ \perp RS \iff (\mathbf{P} - \mathbf{Q})^T (\mathbf{R} - \mathbf{S}) = 0 \tag{1.1.5}$$
6. The point dividing PQ in the ratio $k : 1$ is
$$\mathbf{R} = \frac{k\mathbf{P} + \mathbf{Q}}{k + 1} \tag{1.1.6}$$
7. The *area* of $\triangle PQR$ is the *determinant*
$$\begin{vmatrix} 1 & 1 & 1 \\ \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{vmatrix} \tag{1.1.7}$$

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1.2 Points

1. Find the distance between
$$\mathbf{P} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \tag{1.2.1}$$
2. Find the length of PQ for
   a) $\mathbf{P} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$;
   b) $\mathbf{P} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$;
   c) $\mathbf{P} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -b \\ a \end{pmatrix}$.
3. Using direction vectors, show that $\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ are the vertices of a parallelogram.
4. Using Baudhayana's theorem, show that the points $\begin{pmatrix} -3 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 10 \end{pmatrix}$ are the vertices of a right-angled traingle. Repeat using orthogonality.
5. Plot the points $\begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and prove that they are the vertices of a rectangle.
6. Show that $\mathbf{B} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ are the vertices of an isosceles triangle.
7. In the last question, find the distance of the vertex $\mathbf{A}$ of the triangle from the middle point of the base BC.
8. Prove that the points $\begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ are the vertices of a square.
9. Prove that the points $\mathbf{A} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are the vertices of a parallelogram. Find $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$, the mid points of AB, BC, CD, AD respectively. Show that EG and FH bisect each other.

10. Prove that the points $\begin{pmatrix}21\\-2\end{pmatrix}$, $\begin{pmatrix}15\\10\end{pmatrix}$, $\begin{pmatrix}-5\\0\end{pmatrix}$ and $\begin{pmatrix}1\\-12\end{pmatrix}$ are the vertices of a rectangle, and find the coordinates of its centre.
11. Find the lengths of the medians of the triangle whose vertices are at the points $\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}0\\3\end{pmatrix}$ and $\begin{pmatrix}-1\\-2\end{pmatrix}$.
12. Find the coordinates of the points that divide the line joining the points $\begin{pmatrix}-35\\-20\end{pmatrix}$ and $\begin{pmatrix}5\\-10\end{pmatrix}$ into four equal parts.
13. Find the coordinates of the points of trisection of the line joining the points $\begin{pmatrix}-5\\5\end{pmatrix}$ and $\begin{pmatrix}25\\10\end{pmatrix}$.
14. Prove that the middle point of the line joining the points $\begin{pmatrix}-5\\12\end{pmatrix}$ and $\begin{pmatrix}9\\-2\end{pmatrix}$ is a point of trisection of the line joining the points $\begin{pmatrix}-8\\-5\end{pmatrix}$ and $\begin{pmatrix}7\\10\end{pmatrix}$.
15. The points $\begin{pmatrix}8\\5\end{pmatrix}$, $\begin{pmatrix}-7\\-5\end{pmatrix}$ and $\begin{pmatrix}-5\\5\end{pmatrix}$ are three of the vertices of a parallelogram. Find the coordinates of the remaining vertex which is to be taken as opposite to $\begin{pmatrix}-7\\-5\end{pmatrix}$.
16. The point $\begin{pmatrix}2\\6\end{pmatrix}$ is the intesection of the diagonals of a parallelogram two of whose vertices are at the points $\begin{pmatrix}7\\16\end{pmatrix}$ and $\begin{pmatrix}10\\2\end{pmatrix}$. Find the coordinates of the remaining vertices.
17. Find the area of the triangle whose vertices are the points $\begin{pmatrix}2\\3\end{pmatrix}$, $\begin{pmatrix}-4\\7\end{pmatrix}$ and $\begin{pmatrix}5\\-2\end{pmatrix}$.
18. Find the coordinates of points which divide the join of $\begin{pmatrix}2\\3\end{pmatrix}$, $\begin{pmatrix}-4\\5\end{pmatrix}$ externally in the ratio 2 : 3, and also externally in the ratio 3 : 2.
19. Prove the centroid of $\triangle ABC$ is

$$\mathbf{O} = \frac{\mathbf{A}+\mathbf{B}+\mathbf{C}}{3} \tag{1.2.19}$$

1.3 *Loci*

1. A point moves so that its distance from the point $\begin{pmatrix}2\\1\end{pmatrix}$ is double its distance from the point $\begin{pmatrix}1\\2\end{pmatrix}$. Find the equation of its locus.

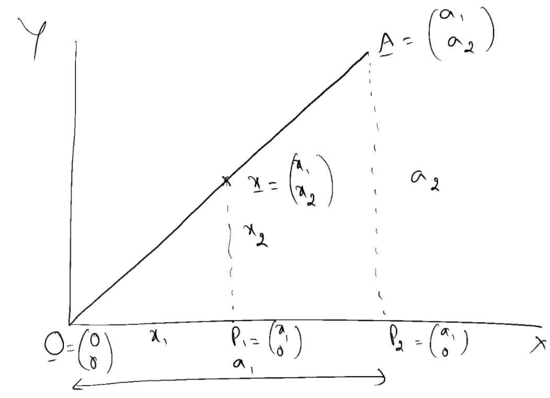

Fig. 2.1.1

2. Find the equation of the perpendicular bisector of the line joining the points $\begin{pmatrix}3\\-4\end{pmatrix}$ and $\begin{pmatrix}-2\\3\end{pmatrix}$.
3. Find the equation of the circle of radius 5 with centre at $\begin{pmatrix}3\\-4\end{pmatrix}$.
4. A point moves so that its distance from the y-axis is equal to the distance from the point $\begin{pmatrix}2\\1\end{pmatrix}$. Find the equation of its locus.
5. A point moves so that the sum of the squares of its distance from the points $\begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}4\\3\end{pmatrix}$ is constant. Find the equation of the locus.
6. A point moves so that its distance from the axis of x is twice its distance from the point $\begin{pmatrix}0\\1\end{pmatrix}$. Find the equation of the locus.
7. A point moves in such a way that with the points $\begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}-3\\4\end{pmatrix}$ it forms a triangle of area 8.5. Show that its locus has an equation

$$\left\{\begin{pmatrix}1 & 5\end{pmatrix}\mathbf{x}\right\}\left\{\begin{pmatrix}1 & 5\end{pmatrix}\mathbf{x} - 34\right\} = 0 \tag{1.3.7}$$

2 The Straight Line

2.1 *Definitions*

1. The points $\mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{A} = \begin{pmatrix}a_1\\a_2\end{pmatrix}$ are as shown in Fig. 2.1.1. Find the equation of OA.

Solution: Let $\mathbf{x} = \begin{pmatrix}x_1\\x_2\end{pmatrix}$ be any point on OA. Then, using similar triangles,

$$\frac{x_2}{x_1} = \frac{a_2}{a_1} = m \tag{2.1.1.1}$$

$$\implies x_2 = mx_1 \tag{2.1.1.2}$$

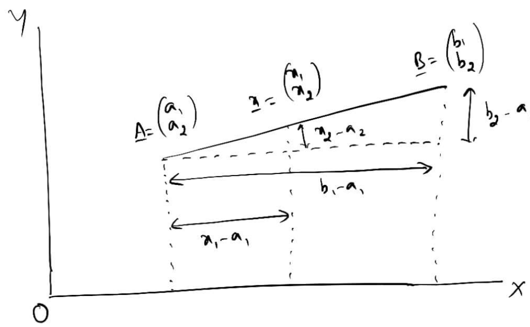

Fig. 2.1.2

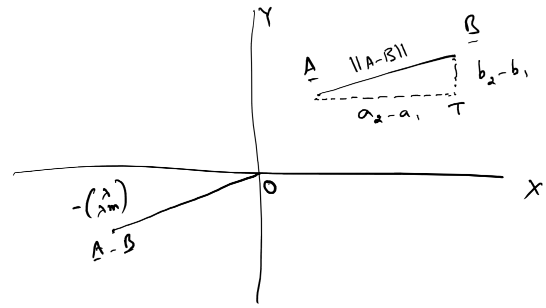

Fig. 2.1.5

where m is known as the slope of the line. Thus, the equation of the line is

$$\mathbf{x} = \begin{pmatrix} x_1 \\ mx_1 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ m \end{pmatrix} = x_1 \mathbf{m} \tag{2.1.1.3}$$

In general, the above equation is written as

$$\mathbf{x} = \lambda \mathbf{m}, \tag{2.1.1.4}$$

where $\mathbf{m}$ is the direction vector of the line.

2. Find the equation of AB in Fig. 2.1.2
Solution: From Fig. 2.1.2,

$$\frac{x_2 - a_2}{x_1 - a_1} = \frac{b_2 - a_2}{b_1 - a_1} = m \tag{2.1.2.1}$$

$$\implies x_2 = mx_1 + a_2 - ma_1 \tag{2.1.2.2}$$

From (2.1.2.2),

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ mx_1 + a_2 - ma_1 \end{pmatrix} \tag{2.1.2.3}$$

$$= \mathbf{A} + (x_1 - a_1) \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2.1.2.4}$$

$$= \mathbf{A} + \lambda \mathbf{m} \tag{2.1.2.5}$$

3. *Translation:* If the line shifts from the origin by $\mathbf{A}$, (2.1.2.5) is obtained from (2.1.1.4) by adding $\mathbf{A}$.
4. Find the length of $\mathbf{A}$ in Fig. 2.1.1
Solution: Using Baudhayana's theorem, the length of the vector $\mathbf{A}$ is defined as

$$\|\mathbf{A}\| = OA = \sqrt{a_1^2 + a_2^2} = \sqrt{\mathbf{A}^T \mathbf{A}}. \tag{2.1.4.1}$$

Also, from (2.1.1.4),

$$\|\mathbf{A}\| = \lambda \sqrt{1 + m^2} \tag{2.1.4.2}$$

Note that λ is the variable that determines the length of $\mathbf{A}$, since m is constant for all points on the line.

5. Find $\mathbf{A} - \mathbf{B}$.
Solution: See Fig. 2.1.5. From (2.1.2.5), for some λ,

$$\mathbf{B} = \mathbf{A} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2.1.5.1}$$

$$\implies \mathbf{A} - \mathbf{B} = -\lambda \begin{pmatrix} 1 \\ m \end{pmatrix}, \tag{2.1.5.2}$$

$\mathbf{A} - \mathbf{B}$ is marked in Fig. 2.1.5.

6. Show that $AB = \|\mathbf{A} - \mathbf{B}\|$
7. Show that the equation of AB is

$$\mathbf{x} = \mathbf{A} + \lambda (\mathbf{B} - \mathbf{A}) \tag{2.1.7.1}$$

8. The *normal* to the vector $\mathbf{m}$ is defined as

$$\mathbf{n}^T \mathbf{m} = 0 \tag{2.1.8.1}$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{2.1.8.2}$$

9. From (2.1.7.1), the equation of a line can also be expressed as

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} + \lambda \mathbf{n}^T (\mathbf{B} - \mathbf{A}) \tag{2.1.9.1}$$

$$\implies \mathbf{n}^T \mathbf{x} = c \tag{2.1.9.2}$$

10. The unit vectors on the x and y axis are defined as

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{2.1.10.1}$$

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.1.10.2}$$

11. If a be the *intercept* of the line

$$\mathbf{n}^T \mathbf{x} = c \tag{2.1.11.1}$$

on the $x-$axis, then $\begin{pmatrix} a \\ 0 \end{pmatrix}$ is a point on the line. Thus,

$$\mathbf{n}^T \begin{pmatrix} a \\ 0 \end{pmatrix} = c \tag{2.1.11.2}$$

$$\implies a = \frac{c}{\mathbf{n}^T \mathbf{e}_1} \tag{2.1.11.3}$$

12. The *rotation matrix* is defined as

$$\mathbf{Q} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{2.1.12}$$

where θ is anti-clockwise.

13.

$$\mathbf{Q}^T\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I} \tag{2.1.13}$$

where $\mathbf{I}$ is the *identity matrix*. The rotation matrix $\mathbf{Q}$ is also an *orthogonal matrix*.

14. Find the equation of line L in Fig. 2.1.14.
Solution: The equation of the $x-$axis is

$$\mathbf{x} = \lambda\mathbf{e}_1 \tag{2.1.14.1}$$

Translation by p units along the $y-$axis results in

$$L_0 : \quad \mathbf{x} = \lambda\mathbf{e}_1 + p\mathbf{e}_2 \tag{2.1.14.2}$$

Rotation by $90° - \alpha$ in the anti-clockwise direction yields

$$\begin{aligned} L : \quad \mathbf{x} &= \mathbf{Q}\left\{\lambda\mathbf{e}_1 + p\mathbf{e}_2\right\} && (2.1.14.3) \\ &= \lambda\mathbf{Q}\mathbf{e}_1 + p\mathbf{Q}\mathbf{e}_2 && (2.1.14.4) \end{aligned}$$

where

$$\begin{aligned} \mathbf{Q} &= \begin{pmatrix} \cos(\alpha - 90) & -\sin(\alpha - 90) \\ \sin(\alpha - 90) & \cos(\alpha - 90) \end{pmatrix} && (2.1.14.5) \\ &= \begin{pmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{pmatrix} && (2.1.14.6) \end{aligned}$$

From (2.1.14.4),

$$\begin{aligned} L : \quad \mathbf{e}_2^T\mathbf{Q}^T\mathbf{x} &= \lambda\mathbf{e}_2^T\mathbf{Q}^T\mathbf{Q}\mathbf{e}_1 + p\mathbf{e}_2^T\mathbf{Q}^T\mathbf{Q}\mathbf{e}_2 \\ &= \lambda\mathbf{e}_2^T\mathbf{e}_1 + p\mathbf{e}_2^T\mathbf{e}_2 && (2.1.14.7) \end{aligned}$$

resulting in

$$L : \quad \begin{pmatrix} \cos\alpha & \sin\alpha \end{pmatrix}\mathbf{x} = p \tag{2.1.14.8}$$

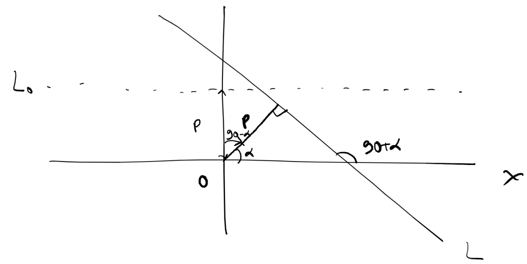

Fig. 2.1.14

15. Show that the distance from the orgin to the line

$$\mathbf{n}^T\mathbf{x} = c \tag{2.1.15.1}$$

is

$$p = \frac{c}{\|n\|} \tag{2.1.15.2}$$

16. Show that the point of intersection of two lines

$$\mathbf{n}_1^T\mathbf{x} = c_1 \tag{2.1.16.1}$$

$$\mathbf{n}_2^T\mathbf{x} = c_2 \tag{2.1.16.2}$$

is given by

$$\mathbf{x} = \left(\mathbf{N}^T\right)^{-1}\mathbf{c} \tag{2.1.16.3}$$

where

$$\mathbf{N} = \begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix} \tag{2.1.16.4}$$

17. The *angle between two lines* is given by

$$\cos^{-1}\frac{\mathbf{n}_1^T\mathbf{n}_2}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} \tag{2.1.17.1}$$

2.2 *Intercepts*

1. Find the intercepts made on the axes by the straight lines whose equations are

a) $\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 2$
b) $\begin{pmatrix} 1 & -3 \end{pmatrix}\mathbf{x} = -5$
c) $\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = 0$
d) $\begin{pmatrix} \frac{1}{a+b} & \frac{1}{a-b} \end{pmatrix}\mathbf{x} = \frac{1}{a^2-b^2}$
e) $\begin{pmatrix} 1 & -m \end{pmatrix}\mathbf{x} = -c$

2. Write down the equations of straight lines which make the following pairs of intercepts on the axes:

a) 3,-4
b) -5,6
c) $\frac{1}{a}, \frac{1}{b}$
d) $2a, -2a$

3. A straight line passes through a fixed point $\begin{pmatrix} h \\ k \end{pmatrix}$ and cuts the axes in $\mathbf{A}, \mathbf{B}$. Parallels to the axes through $\mathbf{A}$ and $\mathbf{B}$ intersect in $\mathbf{P}$. Find the equation of the locus of $\mathbf{P}$.

2.3 Line Equation

1. Find the equations of two straight lines at a distance 3 from the origin and making an angle of 120° with OX.
2. Find the equation of a straight line making an angle of 60° with OX and passing through the point $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$. Transform the equation to the form
$$\begin{pmatrix} \cos\alpha & \sin\alpha \end{pmatrix} \mathbf{x} = p \tag{2.3.2}$$
3. Find the equation of the straight line that passes through the points $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. What is its inclination to OX?
4. Find the equation of the straight line through the point $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ that makes equal intercepts on the axes.
5. Find the equations of the sides of a triangle whose vertices are $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.
6. For the same triangle find the equations of the medians
7. Find the equation of a straight line passing through the point $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ parallel to the line $\begin{pmatrix} 4 & -1 \end{pmatrix} \mathbf{x} + 7 = 0$.
8. Find the intercepts on the axes made by a straight line which passes through the point $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and makes an angle of 30° with OX.
9. Find the equation of the straight line through the points $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and of the parallel line through $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
10. What is the distance from the origin of the line $\begin{pmatrix} 4 & -1 \end{pmatrix} \mathbf{x} = 7$? Write down the equation of a parallel line at double the distance.
11. Find the equation of the straight line through the point $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ parallel to the line joining the origin to the point $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
12. Write down the equation of the straight line which makes intercepts 2 and -7 on the axes, and of the parallel line through the point $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.
13. Find the equations of the straight line joining the points $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and of the parallel line through the origin.
14. ABC is a triangle and $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are the points $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$. Find the equation of the straight line through $\mathbf{A}$ parallel to BC.
15. Find the equation of a line parallel to $\begin{pmatrix} 2 & 5 \end{pmatrix} \mathbf{x} = 11$ passing through the middle point of the join of the points $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -11 \end{pmatrix}$.
16. The base of a triangle passes through a fixed point $\begin{pmatrix} f \\ g \end{pmatrix}$ and the sides are bisected at right angles by the axes. Prove that the locus of the vertex is the line
$$\begin{pmatrix} g & f \end{pmatrix} \mathbf{x} = 0 \tag{2.3.16}$$

2.4 Point of Intersection

1. Find the vertices of the triangle whose sides are
$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} + 6 = 0, \tag{2.4.1.1}$$
$$\begin{pmatrix} 2 & -5 \end{pmatrix} \mathbf{x} + 4 = 0, \tag{2.4.1.2}$$
$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} - 6 = 0 \tag{2.4.1.3}$$
2. Prove that the lines
$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} + 25 = 0, \tag{2.4.2.1}$$
$$\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} + 7 = 0 \tag{2.4.2.2}$$
$$\begin{pmatrix} 3 & 5 \end{pmatrix} \mathbf{x} = 11 \tag{2.4.2.3}$$
are concurrent, and find the coordinates of their common point.
3. Find the equation of a line parallel to the line
$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{x} = 3 \tag{2.4.3.1}$$
and passing through the intersection of the lines
$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 7 \tag{2.4.3.2}$$
$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 5 \tag{2.4.3.3}$$

4. Find the equation of the line joining the origin to the point of intersection of the lines

$$\begin{pmatrix}3 & -5\end{pmatrix}\mathbf{x} = 11 \tag{2.4.4.1}$$

$$\begin{pmatrix}2 & 7\end{pmatrix}\mathbf{x} + 4 = 0 \tag{2.4.4.2}$$

5. Find the acute angle between the lines

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -7 \tag{2.4.5.1}$$

$$\begin{pmatrix}2+\sqrt{3} & 1\end{pmatrix}\mathbf{x} = 11 \tag{2.4.5.2}$$

6. Find the angle between the lines

$$\begin{pmatrix}-2 & 1\end{pmatrix}\mathbf{x} = 5 \tag{2.4.6.1}$$

$$\begin{pmatrix}2 & 4\end{pmatrix}\mathbf{x} + 11 = 0 \tag{2.4.6.2}$$

7. Find the equation of a straight line through the point $\begin{pmatrix}2\\-4\end{pmatrix}$ at right angles to the line

$$\begin{pmatrix}5 & 7\end{pmatrix}\mathbf{x} + 12 = 0 \tag{2.4.7}$$

and find the point in which the lines intersect.

8. Find the equation of a straight line through the origin and at right angles to the line

$$\begin{pmatrix}a & b\end{pmatrix}\mathbf{x} + c = 0 \tag{2.4.8}$$

9. Find the equation of a straight line at right angles to the line

$$\begin{pmatrix}5 & -2\end{pmatrix}\mathbf{x} + 11 = 0 \tag{2.4.9.1}$$

and passing through the intersecton of the lines

$$\begin{pmatrix}1 & 2\end{pmatrix}\mathbf{x} + 1 = 0, \tag{2.4.9.2}$$

$$\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} = 7. \tag{2.4.9.3}$$

10. The origin is a corner of a square and two of its sides have equations

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 0 \tag{2.4.10.1}$$

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 3. \tag{2.4.10.2}$$

Find the equations of the other two sides.

11. Write down the equations of the perpendiculars from the origin to the lines

$$\begin{pmatrix}1 & 5\end{pmatrix}\mathbf{x} = 13, \tag{2.4.11.1}$$

$$\begin{pmatrix}5 & 1\end{pmatrix}\mathbf{x} = 13 \tag{2.4.11.2}$$

and find the equation of the line joining the feet of the perpendiculars.

12. Prove that the line

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 11 \tag{2.4.12.1}$$

makes equal angles with the lines

$$\begin{pmatrix}1 & -\left(2-\sqrt{3}\right)\end{pmatrix}\mathbf{x} + 2 = 0, \tag{2.4.12.2}$$

$$\begin{pmatrix}\left(2-\sqrt{3}\right) & -1\end{pmatrix}\mathbf{x} + 5 = 0 \tag{2.4.12.3}$$

13. **A** is the point $\begin{pmatrix}-4\\0\end{pmatrix}$ and **B** is the point $\begin{pmatrix}3\\0\end{pmatrix}$. Find the locus of a point **P** such that the angles APO, OPB are equal, where **O** is the origin.