

Presentation Template

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January 21, 2020

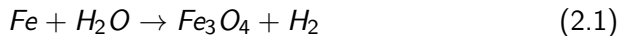
1 Problem

2 Solution

- Linear Equation
- Matrix Equation
- Row Reduction
- Balanced Equation

Problem Statement

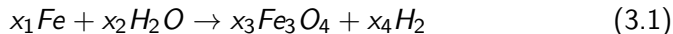
Balance the following chemical equation



using a matrix equation.

Linear Equation

Let the balanced version of (2.1) be



which results in the following equations

$$\begin{aligned}(x_1 - 3x_3) Fe &= 0 \\ (2x_2 - 2x_4) H &= 0 \\ (x_2 - 4x_3) H &= 0\end{aligned} \quad (3.2)$$

Matrix Equation

The linear equations can be expressed as

$$\begin{aligned}x_1 + 0.x_2 - 3x_3 + 0.x_4 &= 0 \\0.x_1 + 2x_2 + 0.x_3 - 2x_4 &= 0 \\0.x_1 + x_2 - 4x_3 + 0.x_4 &= 0\end{aligned}\tag{3.3}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0}\tag{3.4}$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}\tag{3.5}$$

Row Reduction

(3.4) can be row reduced as follows

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \quad (3.6)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \quad (3.7)$$

$$\xleftrightarrow{R_1 \leftarrow 4R_1 - 3R_3} \begin{pmatrix} 4 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \quad (3.8)$$

$$\xleftrightarrow{\begin{matrix} R_1 \leftarrow \frac{1}{4}R_1 \\ R_3 \leftarrow -\frac{1}{4}R_3 \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{4} \end{pmatrix} \quad (3.9)$$

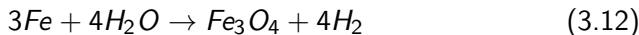
Balanced Equation

Thus,

$$x_1 = \frac{3}{4}x_4, x_2 = x_4, x_3 = \frac{1}{4}x_4 \quad (3.10)$$

$$\Rightarrow \mathbf{x} = x_4 \begin{pmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \quad (3.11)$$

upon substituting $x_4 = 4$. (3.1) then becomes



The code in

<https://github.com/gadepall/school/raw/master/training/chemistry/codes/chembal.py>

verifies (3.10).