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### 1 Miscellaneous Examples 1

**Abstract**—This book provides a computational approach to school algebra and discrete mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/ncert/codes>

#### 1 MISCELLANEOUS EXAMPLES

1. Divide  $p(x)$  by  $g(x)$ , where  $p(x) = x + 3x^2 - 1$  and  $g(x) = 1 + x$ .
2. Divide the polynomial  $p(x) = 3x^4 - 4x^3 - 3x - 1$  by  $x - 1$ .
3. Find the value of  $k$ , if  $x - 1$  is a factor of  $p(x) = 4x^3 + 3x^2 - 4x + k$ .
4. Divide  $2x^2 + 3x + 1$  by  $x + 2$ .
5. Divide  $3x^3 + x^2 + 2x + 5$  by  $1 + 2x + x^2$ .
6. Find all the zeroes of  $2x^4 - 3x^3 - 3x^2 + 6x - 2$ , if you know that two of its zeroes are  $\sqrt{2}$  and  $-\sqrt{2}$ .
7. Find the remainder when  $x^3 - ax^2 + 6x - a$  is divided by  $x - a$ .
8. Find the value of  $k$ , if  $x - 1$  is a factor of  $p(x)$  in each of the following cases:
  - a)  $p(x) = x^2 + x + k$
  - b)  $p(x) = kx^2 - \sqrt{2}x + 1$
  - c)  $p(x) = 2x^2 + kx + \sqrt{2}$
  - d)  $p(x) = kx^2 - 3x + k$
9. Divide the polynomial  $p(x)$  by the polynomial  $g(x)$  and find the quotient and remainder in each of the following:
  - a)  $p(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$ .
  - b)  $p(x) = x^4 - 3x^2 + 4x + 5, g(x) = x^2 + 1 - x$ .
  - c)  $p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$ .
10. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:
  - a)  $t^2 - x, 2t^4 + 3t^3 - 2t^2 - 9t - 12$ .
  - b)  $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$ .
  - c)  $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$ .
11. Obtain all the other zeroes of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ , if two of its zeroes are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$ .
12. On dividing  $x^3 - 3x^2 + x + 2$  by a polynomial  $g(x)$ , the quotient and remainder were  $x - 2$  and  $-2x + 4$  respectively. Find  $g(x)$ .
13. Verify that the numbers given alongside the cubic polynomials below are their zeroes. Also verify if the relationship between the zeroes and the coefficients in each case:
  - a)  $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$
  - b)  $x^3 - 4x^2 + 5x - 2; 2, 1, 1$
14. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -4 respectively.
15. If two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find the other zeroes.
16. If the polynomial  $x^4 - 6x^3 + 16x^2 - 25x + 10$  is divided by another polynomial  $x^2 - 2x + k$ , the remainder comes out to be  $x + a$ , find  $k$  and  $a$ .
17. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
18. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a

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particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

19. The product of Sunita's age (in years) two years ago and her age four years from now is one more than twice her present age. What is her present age?
20. Find two consecutive odd positive integers, sum of whose squares is 290.
21. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
22. The product of two consecutive positive integers is 306. We need to find the integers.
23. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
24. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.
25. Find two numbers whose sum is 27 and product is 182.
26. Find two consecutive positive integers, sum of whose squares is 365.
27. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.
28. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is  $\frac{1}{3}$ . Find his present age.
29. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.
30. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
31. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the

speed of the train.

32. Two water taps together can fill a tank in  $9\frac{3}{8}$  hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
33. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.
34. Sum of the areas of two squares is  $468 m^2$  find the sides of the two squares.
35. Find the values of k for each of the following quadratic equations, so that they have two equal roots:
  - a)  $2x^2 + kx + 3 = 0$
  - b)  $kx(x - 2) + 6 = 0$
36. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.
37. If

$$\mathbf{x} = \sqrt{\begin{pmatrix} a \\ -b \\ c \\ -d \end{pmatrix}} \quad (1.0.37.1)$$

prove that

$$\|\mathbf{x}\|^2 = \frac{\left\| \begin{pmatrix} a \\ b \end{pmatrix} \right\|^2}{\left\| \begin{pmatrix} c \\ d \end{pmatrix} \right\|^2} \quad (1.0.37.2)$$

38. For any two complex numbers  $\mathbf{z}_1, \mathbf{z}_2$ , prove that

$$\Re \mathbf{z}_1 \mathbf{z}_2 = \Re \mathbf{z}_1 \Re \mathbf{z}_2 - \Im \mathbf{z}_1 \Im \mathbf{z}_2 \quad (1.0.38.1)$$

39. If  $\mathbf{x} = \begin{pmatrix} a \\ b \\ a \\ -b \end{pmatrix}$ , show that  $\|\mathbf{x}\| = 1$
40. If  $\mathbf{x} = \frac{\begin{pmatrix} x \\ 1 \end{pmatrix}}{2x^2+1}$ , prove that  $\|\mathbf{x}\|^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$ .
41. If  $\begin{pmatrix} x \\ y \end{pmatrix}^3 = \mathbf{uv}$ , then show that  $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$ .

42. If  $\alpha, \beta$  are different complex numbers with  $\|\beta\| = 1$ , then find  $\left\| \frac{\beta - \alpha}{1 - \alpha^* \beta} \right\|$ .
43. Find the number of non-zero integral solutions of the equation  $\|1 - 1\|^x = 2^x$ .
44. If  $\begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix} \begin{pmatrix} g \\ h \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix}$ , then show that  $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$ .
45. If  $\left( \frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \right) = 1$ , then find the least positive integral value of  $m$ .
46. The length  $L$  (in cm) of a copper rod is a linear function of its Celsius temperature  $C$ . In an experiment, if  $L = 124.942$  when  $C = 20$  and  $L = 125.134$  when  $C = 110$ , express  $L$  in terms of  $C$ .
47. The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?
48. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.
49. The taxi fare in a city is as follows: For the first kilometre, the fare is ₹8 and for the subsequent distance it is ₹5 per km. Taking the distance covered as  $x$  km and total fare as ₹ $y$ , write a linear equation for this information, and draw its graph.
50. Yamini and Fatima, two students of Class IX of a school, together contributed ₹100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹ $x$  and ₹ $y$ .) Draw the graph of the same.
51. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:
- $$F = \frac{9}{5}C + 32 \quad (1.0.51.1)$$
- a) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.
- b) If the temperature is  $30^\circ\text{C}$ , what is the temperature in Fahrenheit?
- c) If the temperature is  $95^\circ\text{F}$ , what is the temperature in Celsius?
- d) If the temperature is  $0^\circ\text{C}$ , what is the temperature in Fahrenheit and if the temperature is  $0^\circ\text{F}$ , what is the temperature in Celsius?
- e) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.
52. Romila went to a stationery shop and purchased 2 pencils and 3 erasers for ₹9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for ₹18. Represent this situation algebraically and graphically. Find the cost of each pencil and eraser.
53. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically. Find their respective ages.
54. The coach of a cricket team buys 3 bats and 6 balls for ₹3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and geometrically. Find the cost of each bat and ball.
55. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹300. Represent the situation algebraically and geometrically. Find the cost of apples and grape.
56. Form the pair of linear equations in the following problems, and find their solutions.
57. 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
58. 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and that of one pen.
59. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

60. The difference between two numbers is 26 and one number is three times the other. Find them.
61. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
62. The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.
63. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?
64. A fraction becomes  $\frac{9}{11}$ , if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes  $\frac{5}{6}$ . Find the fraction.
65. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages
66. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹2000 per month, find their monthly incomes.
67. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?
68. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes  $\frac{1}{2}$ , if we only add 1 to the denominator. What is the fraction?
69. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
70. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
71. Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.
72. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.
73. The cost of 5 oranges and 3 apples is ₹35 and the cost of 2 oranges and 4 apples is ₹28. Let us find the cost of an orange and an apple.
74. From a bus stand in Bangalore, if we buy 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, the total cost is ₹46; but if we buy 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur the total cost is ₹74. Find the fares from the bus stand to Malleswaram, and to Yeshwanthpur.
75. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day.
76. A fraction becomes  $\frac{1}{3}$  when 1 is subtracted from the numerator and it becomes when 8 is added to its denominator. Find the fraction.
77. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
78. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?
79. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

80. Solve the pair of equations:

$$\begin{aligned} \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= 13 \\ \begin{pmatrix} 5 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= -2 \end{aligned} \quad (1.0.80.1)$$

81. Solve the pair of equations by reducing them to a pair of linear equations

$$\begin{aligned} \begin{pmatrix} 5 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{x-1} \\ \frac{1}{y-2} \end{pmatrix} &= 2 \\ \begin{pmatrix} 6 & -3 \end{pmatrix} \begin{pmatrix} \frac{1}{x-1} \\ \frac{1}{y-2} \end{pmatrix} &= 1 \end{aligned} \quad (1.0.81.1)$$

82. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

83. Solve the following pairs of equations

a)

$$\begin{aligned} \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= 2 \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= \frac{13}{6} \end{aligned} \quad (1.0.83.1)$$

b)

$$\begin{aligned} \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{x}} \\ \frac{1}{\sqrt{y}} \end{pmatrix} &= 2 \\ \begin{pmatrix} 4 & -9 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{x}} \\ \frac{1}{\sqrt{y}} \end{pmatrix} &= -1 \end{aligned} \quad (1.0.83.2)$$

c)

$$\begin{aligned} \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= 14 \\ \begin{pmatrix} 3 & -4 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= 23 \end{aligned} \quad (1.0.83.3)$$

d)

$$\begin{aligned} \begin{pmatrix} 10 & 2 \end{pmatrix} \begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} &= 4 \\ \begin{pmatrix} 15 & -5 \end{pmatrix} \begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} &= -2 \end{aligned} \quad (1.0.83.4)$$

e)

$$\begin{aligned} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{3x+y} \\ \frac{1}{3x-y} \end{pmatrix} &= \frac{3}{4} \\ \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{3x+y} \\ \frac{1}{3x-y} \end{pmatrix} &= -\frac{1}{8} \end{aligned} \quad (1.0.83.5)$$

84. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

85. 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

86. Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

87. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

88. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II].

89. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

90. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.