JEE Problems in Linear Algebra: 3D

Abstract—**A collection of problems from JEE mains papers related to 3D geometry are available in this document. Students are expected to solve these using linear algebra.**

1. $\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ is a solution of
$$\begin{pmatrix} 1 & -8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{1}$$
such that $\mathbf{A}$ lies on the plane
$$\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \mathbf{x} = 6. \tag{2}$$
Find $2a_1 + a_2 + a_3$.
2. For any two 3×3 matrices A and B, let $A + B = 2B^T$ and $3A + 2B = I_3$. Which of the following is true?
   a) $5A + 10B = 2I_3$.
   b) $10A + 5B = 3I_3$.
   c) $2A + B = 3I_3$.
   d) $3A + 6B = 2I_3$.
3. If the line,
$$L_1 : \frac{x_1 - 3}{1} = \frac{x_2 + 2}{-1} = \frac{x_3 + \lambda}{-2} \tag{3}$$
lies in the plane
$$\begin{pmatrix} 2 & -4 & 3 \end{pmatrix} \mathbf{x} = 2, \tag{4}$$
find the shortest distance between L_1 and
$$L_2 : \frac{x_1 - 1}{12} = \frac{x_2}{9} = \frac{x_3}{4} \tag{5}$$
4. Given
$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^T \tag{6}$$
$$\mathbf{B} = \begin{pmatrix} 0 & 3 & 4 \end{pmatrix}^T \tag{7}$$
and $\mathbf{B}_2$ such that
$$BB_2 \parallel OA \tag{8}$$
$$\mathbf{B}_2^T \mathbf{A} = 0 \tag{9}$$
where $\mathbf{O}$ is the origin, find $(\mathbf{B} - \mathbf{B}_2) \times \mathbf{B}_2$.
5. Find the distance between the point $\begin{pmatrix} 1 & -5 & 9 \end{pmatrix}^T$ from the plane
$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \mathbf{x} = 5, \tag{10}$$
along the line $x_1 = x_2 = x_3$.
6. The line
$$L : \frac{x_1 - 3}{2} = \frac{x_2 + 2}{-1} = \frac{x_3 + 4}{3} \tag{11}$$
lies in the plane
$$\begin{pmatrix} l & m & -1 \end{pmatrix} \mathbf{x} = 9, \tag{12}$$
Find $l^2 + m^2$.
7. Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be three unit vectors such that
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \frac{\sqrt{3}}{2} (\mathbf{B} + \mathbf{C}). \tag{13}$$
If $\mathbf{B}$ is not parallel to $\mathbf{C}$, then find the angle between and $\mathbf{A}$ and $\mathbf{B}$.
8. Find the range of the shortest distance between the lines
$$L_1 : \frac{x_1}{2} = \frac{x_2}{2} = \frac{x_3}{1} \tag{14}$$
$$L_2 : \frac{x_1 + 2}{-1} = \frac{x_2 - 4}{8} = \frac{x_3 - 5}{4} \tag{15}$$
9. Find the distance of the point $\begin{pmatrix} 1 & -2 & 4 \end{pmatrix}^T$ from the plane passing through the point $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}^T$ and perpendicular to the planes
$$\begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \mathbf{x} = 3 \tag{16}$$
$$\text{and } \begin{pmatrix} 2 & -2 & 1 \end{pmatrix} \mathbf{x} = -12. \tag{17}$$
10. In $\triangle ABC$, right angled at $\mathbf{A}$,
$$\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ p \end{pmatrix} \mathbf{C} = \begin{pmatrix} 5 \\ q \\ -4 \end{pmatrix} \tag{18}$$
sketch the point $\begin{pmatrix} p \\ q \end{pmatrix}$

11. Find the distance of the point $\begin{pmatrix}1 & 3 & -7\end{pmatrix}$ from the plane passing through the point $\begin{pmatrix}1 & -1 & -1\end{pmatrix}$, having normal perpendicular to both the lines

$$L_1 : \frac{x_1 - 1}{1} = \frac{x_2 + 2}{-2} = \frac{x_3 - 4}{3} \tag{19}$$

$$L_2 : \frac{x_1 - 2}{2} = \frac{x_2 + 1}{-1} = \frac{x_3 + 7}{-1} \tag{20}$$

12. If the image of the point

$$\mathbf{P} = \begin{pmatrix}1 \\ -2 \\ 3\end{pmatrix} \tag{21}$$

in the plane

$$\begin{pmatrix}2 & 3 & -4\end{pmatrix}\mathbf{x} = -22. \tag{22}$$

measured parallel to the line

$$L : \frac{x_1}{1} = \frac{x_2}{4} = \frac{x_3}{5} \tag{23}$$

is Q, find PQ.

13. Let

$$A = \begin{pmatrix}2 \\ 1 \\ -2\end{pmatrix} \text{ and } B = \begin{pmatrix}1 \\ 1 \\ 0\end{pmatrix}. \tag{24}$$

If

$$|\mathbf{C} - \mathbf{A}| = 3, \tag{25}$$

$$|(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}| = 3, \tag{26}$$

$$\frac{\mathbf{C}^T (\mathbf{A} \times \mathbf{B})}{|\mathbf{C}|\, |\mathbf{A} \times \mathbf{B}|} = \frac{\sqrt{3}}{2}, \tag{27}$$

then find $\mathbf{A}^T\mathbf{C}$.

14. Find b such that the planes

$$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 1 \tag{28}$$

$$\begin{pmatrix}1 & a & 1\end{pmatrix}\mathbf{x} = 1 \tag{29}$$

$$\begin{pmatrix}a & b & 1\end{pmatrix}\mathbf{x} = 0 \tag{30}$$

do not intersect.

15. If the shortest distance between the lines

$$x_1 + 2\lambda = 2x_2 = -12x_3 \tag{31}$$

$$x_1 = x_2 + 4\lambda = 6x_3 - 12\lambda \tag{32}$$

is $4\sqrt{2}$, find λ.

16. Find the perpendicular distance from the point

$$\mathbf{A} = \begin{pmatrix}3 \\ 1 \\ 1\end{pmatrix} \tag{33}$$

on the plane passing through the point

$$\mathbf{B} = \begin{pmatrix}1 \\ 2 \\ 3\end{pmatrix} \tag{34}$$

and containing the line

$$\mathbf{x} = \begin{pmatrix}1 \\ 1 \\ 0\end{pmatrix} + \lambda \begin{pmatrix}2 \\ 1 \\ 4\end{pmatrix} \tag{35}$$

17. If

$$|\mathbf{a}| = 1 \tag{36}$$

$$|\mathbf{b}| = 2 \tag{37}$$

$$|\mathbf{c}| = 4 \tag{38}$$

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0. \tag{39}$$

Find

$$4\mathbf{a}^T\mathbf{b} + 3\mathbf{b}^T\mathbf{c} + 3\mathbf{c}^T\mathbf{a} \tag{40}$$

18. Find the coordinates of the foot of the perpendicular from the point

$$\mathbf{B} = \begin{pmatrix}1 \\ -2 \\ 1\end{pmatrix} \tag{41}$$

on the plane containing the lines

$$L_1 : \frac{x_1 + 1}{6} = \frac{x_2 - 1}{7} = \frac{x_3 - 3}{8} \tag{42}$$

$$L_2 : \frac{x_1 - 1}{3} = \frac{x_2 - 2}{5} = \frac{x_3 - 3}{7} \tag{43}$$

19. Find the intersection of the planes

$$\begin{pmatrix}3 & -1 & 1\end{pmatrix}\mathbf{x} = 1 \tag{44}$$

$$\begin{pmatrix}1 & 4 & -2\end{pmatrix}\mathbf{x} = 2 \tag{45}$$

20. If $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are the vertices of a parallelogram such that

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix}8 \\ -6 \\ 0\end{pmatrix} \tag{46}$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix}3 \\ 4 \\ -12\end{pmatrix} \tag{47}$$

Find its area.

21. Find λ for which the planes

$$\begin{pmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{pmatrix} \mathbf{x} = 0 \tag{48}$$

do not intersect at the origin.

22. In $\triangle ABC$,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \mathbf{C} = \begin{pmatrix} \lambda \\ 5 \\ \mu \end{pmatrix} \tag{49}$$

The median through $\mathbf{A}$ is equally inclined to the coordinate axes. Find $\left(\lambda^3 + \mu^3 + 5\right)$

23. If

$$L_1 : \frac{x_1 - 1}{1} = \frac{x_2 - 2}{2} = \frac{x_3 + 3}{\lambda^2} \tag{50}$$

$$L_2 : \frac{x_1 - 3}{1} = \frac{x_2 - 2}{\lambda^2} = \frac{x_3 - 1}{2} \tag{51}$$

are coplanar, find the number of distinct real values of λ.

24. The circumcentre of $\triangle ABC$ is

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B} + \mathbf{C}}{4} \tag{52}$$

Find its orthocentre.