

Geometry through Python



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Abstract—This manual shows how to construct geomet-		

Abstract—This manual shows how to construct geometric figures using Python.

1 Triangle

1.1 Consider $\triangle ABC$ with BC = a, CA = b and AB = c. Find the coordinates of **A**, **B** and **C**. **Solution:** Let

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{3}$$

1.2 Let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \tag{4}$$

Find det(A).

Solution: The *determinant* is obtained as

$$\det(\mathbf{A}) = 1 \times -1 - 3 \times 1 = -4. \tag{5}$$

1.3 Write a program for finding det (A).Solution: The following program finds the determinant.

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#Code by GVV Sharma

#March 14, 2019

#released under GNU GPL

import numpy as np

a1 = 1

a2 = 1

b1 = 3

b2 = -1

c1 = 8

c2 = 12

A = np.array(([a1,a2],[b1,b2]))

x = np.linalg.det(A)

print(x)

1.4 Write your own function for calculating det A Solution: The following routine finds the determinant.

#Code by GVV Sharma #March 14, 2019 #released under GNU GPL import numpy as np def det(A): a1 = A[0][0]a2 = A[0][1]b1 = A[1][0]b2 = A[1][1]y = a1*b2 - a2*b1return y a1 = 1a2 = 1b1 = 3b2 = -1A = np.array(([a1,a2],[b1,b2]))

```
x = det(A)
print(x)
```

1.5 Write a program to check if two lines intersect. **Solution:** Two lines intersect if $det(\mathbf{A}) \neq 0$. The following code checks for this condtion.

```
#Code by GVV Sharma
#March 16, 2019
#released under GNU GPL
import numpy as np

a1 = 1
a2 = 1
b1 = 3
b2 = -1

A = np.array(([a1,a2],[b1,b2]))
x = np.linalg.det(A)
if x != 0:
    print('The_lines_intersect')
else:
    print('The_lines_do_not_intersect')
```

1.6 Let

$$\mathbf{A}_1 = \begin{pmatrix} 8 & 1 \\ 12 & -1 \end{pmatrix} \tag{6}$$

$$\mathbf{A}_2 = \begin{pmatrix} 1 & 8 \\ 3 & 12 \end{pmatrix} \tag{7}$$

Verify that

$$x_1 = \frac{\det \mathbf{A}_1}{\det \mathbf{A}} \text{ and } x_2 = \frac{\det \mathbf{A}_2}{\det \mathbf{A}}$$
 (8)

satisfy (??).

Solution:

$$\frac{\det \mathbf{A}_1}{\det \mathbf{A}} = \frac{-20}{-4} = 5 \tag{9}$$

Similarly,

$$\frac{\det \mathbf{A}_2}{\det \mathbf{A}} = \frac{-12}{-4} = 3 \tag{10}$$

2 Graphical Solution

2.1 Find a graphical solution for (??).

Solution: The following code plots Fig. 2.1. It is obvious that the two equations in (??) represent the lines y_1 and y_2 in Fig. 2.1 and intersect at $\binom{5}{3}$

```
#Code by GVV Sharma
#March 14, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt
#if using termux
import subprocess
import shlex
#end if
x = \text{np.linspace}(-2,8,20)
y1 = 8-x
y2 = 3*x-12
fig = plt.figure()
ax = fig.add subplot(1, 1, 1)
# Major ticks every 2, minor ticks every 1
major ticks = np.arange(-10, 10, 2)
minor ticks = np.arange(-10, 10, 1)
ax.set xticks(major ticks)
ax.set xticks(minor ticks, minor=True)
ax.set yticks(major ticks)
ax.set yticks(minor ticks, minor=True)
# If you want different settings for the grids:
ax.grid(which='minor', alpha=0.2)
ax.grid(which='major', alpha=0.5)
#Plotting all lines
ax.plot(x,y1,label='\$y 1\$')
ax.plot(x,y2,label='\$y 2\$')
plt.xlabel('$x$')
plt.ylabel('$y$')
ax.legend(loc='best')
#if using termux
plt.savefig('../figs/draw line.pdf')
plt.savefig('../figs/draw line.eps')
subprocess.run(shlex.split("termux-open ../
    figs/draw line.pdf"))
#else
#plt.show()
```

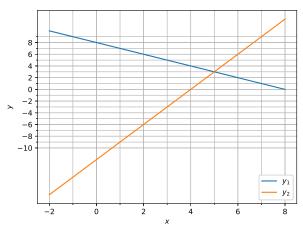


Fig. 2.1

2.2 The **np.linspace** function above generates an arithmetic sequence with first term -2, last term 8 and number of terms 20. Write your own linspace function and verify.

Solution: The code is available below.

```
#Code by GVV Sharma
#March 14, 2019
#released under GNU GPL
import numpy as np

def linspace(first,last,k):
    t= np.zeros((k,1))
    t[0]=first
    d = (last-first)/(k-1)
    for n in range(2,k):
        t[n-1] = t[0]+(n-1)*d
    return t

x = np.linspace(-2,8,20)
print(x)
```

3 C PROGRAMMING

3.1 Write a C program to generate an arithmetic sequence with $t_0 = -2$, $t_{n-1} = 8$, n = 20 and print it to the file **ap.dat**.

Solution:

```
#include <stdio.h>
int main(void)
{
FILE *fp;
```

```
float t_0 = -2.0, t_k = 8.0, d,t_n;
int k = 20, n;

//Common difference
d = (t_k-t_0)/(k-1);
fp = fopen("ap.dat","w");
for(n = 0; n < k; n++)
{
t_n = t_0+n*d;
printf("%f\n",t_n);
fprintf(fp,"%f\n",t_n);
}
fclose(fp);
return 0;
}</pre>
```

3.2 Now execute the following code.

```
#Code by GVV Sharma
#March 15, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt
#if using termux
import subprocess
import shlex
#end if
x = np.loadtxt('ap.dat',dtype='float')
y1 = 8-x
y2 = 3*x-12
fig = plt.figure()
ax = fig.add \quad subplot(1, 1, 1)
# Major ticks every 2, minor ticks every 1
major ticks = np.arange(-10, 10, 2)
minor ticks = np.arange(-10, 10, 1)
ax.set xticks(major ticks)
ax.set xticks(minor ticks, minor=True)
ax.set yticks(major ticks)
ax.set yticks(minor ticks, minor=True)
# If you want different settings for the grids:
ax.grid(which='minor', alpha=0.2)
ax.grid(which='major', alpha=0.5)
#Plotting all lines
```

```
ax.plot(x,y1,label='$y_1$')
ax.plot(x,y2,label='$y_2$')

plt.xlabel('$x$')
plt.ylabel('$y$')
ax.legend(loc='best')

#if using termux
plt.savefig('../figs/draw_line.pdf')
plt.savefig('../figs/draw_line.eps')
subprocess.run(shlex.split("termux-open ../
figs/draw_line.pdf"))

#else
#plt.show()
```

- 3.3 Do all computations in Problem 2.1 using C and store the data into files. Import this data so that Python is used only for plotting.
- 3.4 Write a function for computing the common difference d given t_0 , t_{n-1} and n.

Solution:

```
#include <stdio.h>
float comm diff(float,float,int);
int main(void)
FILE *fp;
float t 0 = -2.0, t k = 8.0, d,t n;
int k = 20, n;
//Common difference
d = comm \ diff(t \ 0,t \ k,k);
fp = fopen("ap.dat","w");
for(n = 0; n < k; n++)
t_n = t_0 + n*d;
printf("%f \setminus n",t n);
fprintf(fp,"%f\n",t n);
fclose(fp);
return 0;
float comm diff(float first,float last,int n)
float d;
d = (last-first)/(n-1);
return d;
```

4 Python programming exercises

4.1 Find A^{-1} .

Solution: The *inverse* of **A** is obtained as

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} -1 & -1 \\ -3 & 1 \end{pmatrix} \tag{11}$$

$$= \frac{1}{-4} \begin{pmatrix} -1 & -1 \\ -3 & 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$
 (12)

- 4.2 Write your own function for calculating A^{-1}
- 4.3 Let

$$\mathbf{c} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} \tag{13}$$

Find $\mathbf{A}^{-1}\mathbf{b}$

Solution: From (11) and (13),

$$\mathbf{A}^{-1}\mathbf{b} = \frac{1}{4} \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 8 \\ 12 \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} 1 \times 8 + 1 \times 12 \\ 3 \times 8 - 1 \times 12 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 20 \\ 12 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$
(15)

- 4.4 Verify that (15) is a solution of (??).
- 4.5 Write a program to find the solution of (??). **Solution:** The following program finds the solution

```
#Code by GVV Sharma

#March 14, 2019

#released under GNU GPL

import numpy as np

a1 = 1
a2 = 1
b1 = 3
b2 = -1
c1 = 8
c2 = 12

A = np.array(([a1,a2],[b1,b2]))
c = np.array([c1,c2])
Ainv = np.linalg.inv(A)
x = np.matmul(Ainv,c)

print(x)
```

4.6 Write your own program for **np.matmul**.

5 C PROGRAMMING EXERCISES

5.1 A geometric sequence is defined as

$$t_{n-1} = t_0 r^{n-1} (16)$$

Write a function for generating the nth term of a geometric sequence from t_0 , r and n.

Solution:

```
#include <stdio.h>
#include <math.h>
//nth term of a GP
//t_0 = first term
//t_n = nth term
//r = common ratio
//k = number of terms

int main(void)
{
float t_0 = 1, r = 2, t_n;
int n = 3;

t_n = t_0*pow(r,n);
printf("%f\n",t_n);
return 0;
}
```

- 5.2 Write a function to calculate simple interest and amount.
- 5.3 Write a function to calculate compound interest and amount.
- 5.4 Write a function to calculate the circumference of a circle.

Solution:

```
#include <stdio.h>
#include <math.h>

float circumference(float);
int main(void)
{
  printf("%f\n",circumference(0.5));
  return 0;
}
float circumference(float r)
{
  return 2*M_PI*r;
}
```

5.5 Write a function to calculate the area of a circle.

5.6 Write a program to find the sum of the first n terms of an arithmetic sequence. Verify by finding the sum of the numbers $1, \ldots, 10$.

Solution:

```
#include <stdio.h>
//sum to n terms of an AP
//t 0 = first term
//t n = nth term
//d = commo \ difference
//k = number \ of \ terms
int main(void)
float t 0 = 1, d = 1,s n, t n;
int k = 10, n;
s n = 0:
//For Loop
for(n = 0; n < k; n++)
t \quad n = t \quad 0 + n*d;
s \quad n = s \quad n+t \quad n;
printf("%f \ n",s \ n);
return 0;
```

- 5.7 Write a program to find the sum of the first n terms of a geometric sequence.
- 5.8 Write a program to check if given lengths can form the sides of a triangle.

Solution:

```
#include <stdio.h>
#include <math.h>
//verify if given lengths
//are sides of a triangle.
int main(void)
{
float a,b,c;

printf("Enter_the_sides_of_the_triangle\n");
 scanf("%f_%f_%f", &a,&b,&c);

if(a+b < c)
{
 printf("No_triangle\n");
}
else if(b+c < a)
{</pre>
```

```
printf("No_triangle\n");
}
else if(c+a < b)
{
printf("No_triangle\n");
}
else
{
printf("Yes_Triangle\n");
}
return 0;
}</pre>
```

5.9 Write a program to find A^{-1} and print it. Solution:

```
#include <stdio.h>
#include <stdlib.h>
//This program shows how to use pointers as
    2-D arrays
//Function declaration
double **createMat(int m,int n);
void readMat(int m,int n,double **p);
void print(int m,int n,double **p);
double detMat(double **p);
double **invMat(double **p);
//End function declaration
int main() //main function begins
//Defining the variables
int m,n://integers
double **A,**A inv,det;
m = 2;
n = 2;
printf("Enter_the_values_of_the_matrix\n");
A = createMat(m,n);//creating the matrix
det = detMat(A);
readMat(m,n,A)://reading values into the
    matrix a
A inv = invMat(A);
print(m,n,A inv);//printing the matrix a
return 0;
```

```
double **invMat(double **p)
double **q, det;
det = detMat(p);
q = createMat(2,2);
q[0][0] = p[1][1]/det;
q[0][1] = -p[0][1]/det;
q[1][0] = -p[1][0]/det;
q[1][1] = p[0][0]/det;
return q;
double detMat(double **A)
double det;
\det = A[0][0]*A[1][1]-A[0][1]*A[1][0];
return det;
//Defining the function for matrix creation
double **createMat(int m,int n)
 int i:
 double **a;
//Allocate memory to the pointer
a = (double **)malloc(m * sizeof( *a));
    for (i=0; i<m; i++)
         a[i] = (double *)malloc(n * sizeof(
              *a[i]));
 return a;
//End function for matrix creation
//Defining the function for reading matrix
void readMat(int m,int n,double **p)
 int i,j;
 for(i=0;i<m;i++)
  for(j=0;j< n;j++)
   scanf("%lf",&p[i][j]);
//End function for reading matrix
```

```
//Defining the function for printing
void print(int m,int n,double **p)
int i,j;
 for(i=0;i<m;i++)
  for(j=0;j<n;j++)
  printf("%lf_",p[i][j]);
 printf("\n");
```

- 5.10 Write a program to find the intersection of two lines using Cramer's rule.
- 5.11 Write a program to verify if two lines intersect. 5.12 Write a program to find $A^{-1}c$.