Geometric Constructions through Python

G V V Sharma*

Contents

***Abstract*—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.**

1 Right Triangle

1.1 Draw $\triangle ABC$ right angled at $\mathbf{B}$ such that $AB = c = 6, BC = a = 8$.
Solution: The coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{1}$$

1.2 Let $\mathbf{D}, \mathbf{F}, \mathbf{F}$ be the mid points of BC, CA and AB respectively in $\triangle ABC$. Draw AD, BE and CF.
Solution:

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = \frac{\mathbf{C} + \mathbf{A}}{2} = \frac{1}{2}\begin{pmatrix} a \\ c \end{pmatrix} \tag{3}$$

$$\mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix} 0 \\ c \end{pmatrix} \tag{4}$$

1.3 Draw AD, BE and CF.

1.4 Draw $\triangle DEF$ in the previous problem.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

2 Cirumcircle of Right Triangle

2.1 Show that

$$\mathbf{A} - \mathbf{E} = \begin{pmatrix} -ka \\ c - kc \end{pmatrix} \tag{5}$$

$$\mathbf{B} - \mathbf{E} = -k\begin{pmatrix} a \\ c \end{pmatrix} \tag{6}$$

$$\mathbf{C} - \mathbf{E} = \begin{pmatrix} a - ak \\ -kc \end{pmatrix} \tag{7}$$

where

$$k = \frac{1}{2} \tag{8}$$

Solution: Substituting $\mathbf{A}$ from (1) and $\mathbf{E}$ from (2)

$$\mathbf{A} - \mathbf{E} = \begin{pmatrix} 0 \\ c \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} - k\begin{pmatrix} a \\ c \end{pmatrix} \tag{9}$$

Thus,

$$\mathbf{A} - \mathbf{E} = \begin{pmatrix} 0 \\ c \end{pmatrix} - k\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} - \begin{pmatrix} ka \\ kc \end{pmatrix} = \begin{pmatrix} 0 - (ka) \\ c - (kc) \end{pmatrix}$$
$$= \begin{pmatrix} 0 - ka \\ c - kc \end{pmatrix} = \begin{pmatrix} -ka \\ c - kc \end{pmatrix} \tag{10}$$

Similarly, $\mathbf{B} - \mathbf{E}, \mathbf{C} - \mathbf{E}$ can be obtained.

2.2 Find EA^2.

Solution:

$$\begin{aligned}
EA^2 &= \|\mathbf{A}-\mathbf{E}\|^2 = \left\|\begin{pmatrix}0\\0\end{pmatrix}-\begin{pmatrix}-ka\\c-kc\end{pmatrix}\right\|^2\\
&= \left\|\begin{pmatrix}0-(-ka)\\0-(c-kc)\end{pmatrix}\right\|^2 = \left\|\begin{pmatrix}ka\\-c+kc\end{pmatrix}\right\|^2\\
&= (ka)^2+(-c+kc)^2\\
&= k^2a^2+(-c+kc)(-c+kc)\\
&= k^2a^2+(-c)(-c+kc)+(kc)(-c+kc)\\
&= k^2a^2+(-c)(-c)+(-c)(kc)+(kc)(-c)\\
&\quad +(kc)(kc)\\
&= k^2a^2+c^2-kc^2-kc^2+k^2c^2\\
&= k^2a^2+c^2+(-1-1)kc^2+k^2c^2\\
&= k^2a^2+c^2-2kc^2+k^2c^2
\end{aligned}$$

$\because 2k=1$,

$$\begin{aligned}
k^2a^2+c^2-2kc^2+k^2c^2 &= k^2a^2+c^2-c^2+k^2c^2\\
&= k^2a^2+k^2c^2\\
&= k^2\left(a^2+c^2\right)
\end{aligned}$$

2.3 Show that

$$EB^2 = k^2\left(a^2+c^2\right) \tag{11}$$

2.4 Show that

$$EC^2 = k^2\left(a^2+c^2\right) \tag{12}$$

2.5 Draw the circumcircle of $\triangle ABC$ with centre $\mathbf{E}$ and radius

$$R = EA = EB = EC = k\sqrt{(a^2+c^2)} \tag{13}$$

3 Tangent

3.1 In the right $\triangle ABC$, right angled at $\mathbf{A}$, $AC = b = 8, AB = c = 6$ and

$$\mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}a\\0\end{pmatrix}, \tag{14}$$

find

$$\mathbf{A} = \begin{pmatrix}p\\q\end{pmatrix} \tag{15}$$

Solution: $\because$

$$AB^2 = \|\mathbf{A}-\mathbf{B}\|^2, \tag{16}$$

$$\implies AB^2 = \left\|\begin{pmatrix}p\\q\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right\|^2 \tag{17}$$

$$= \left\|\begin{pmatrix}p-0\\q-0\end{pmatrix}\right\|^2 = \left\|\begin{pmatrix}p\\q\end{pmatrix}\right\|^2 \tag{18}$$

$$\implies c^2 = p^2+q^2 \tag{19}$$

$$\text{or, } c^2-q^2 = p^2 \tag{20}$$

Similarly,

$$AC^2 = \|\mathbf{A}-\mathbf{C}\|^2 = \left\|\begin{pmatrix}p\\q\end{pmatrix}-\begin{pmatrix}a\\0\end{pmatrix}\right\|^2 \tag{21}$$

$$= \left\|\begin{pmatrix}p-a\\q-0\end{pmatrix}\right\|^2 = \left\|\begin{pmatrix}p-a\\q\end{pmatrix}\right\|^2 \tag{22}$$

Thus,

$$\begin{aligned}
AC^2 &= b^2 = (p-a)^2+q^2\\
&= (p-a)(p-a)+q^2\\
&= (p)(p-a)+(-a)(p-a)+q^2\\
&= p^2+p(-a)+(-a)(p) \qquad (23)\\
&\quad +(-a)(-a)+q^2\\
&= p^2-pa-pa+a^2\\
&= p^2+pa(-1-1)+a^2+q^2\\
\implies b^2 &= p^2-2pa+a^2+q^2\\
\implies b^2 &- \left(-2pa+a^2+q^2\right) = p^2\\
\implies b^2 &+ 2pa-a^2-q^2 = p^2 \qquad (24)
\end{aligned}$$

From (20) and (24), equating the L.H.S,

$$\begin{aligned}
c^2-q^2 &= b^2+2pa-a^2-q^2\\
\implies c^2-q^2-\left(b^2+2pa-a^2-q^2\right) &= 0\\
\implies c^2-q^2-b^2-2pa+a^2+q^2 &= 0\\
\implies c^2-q^2+q^2-b^2-2pa+a^2 &= 0\\
\implies c^2+q^2(-1+1)-b^2-2pa+a^2 &= 0\\
\implies c^2+q^2(0)-b^2-2pa+a^2 &= 0\\
\implies c^2-b^2-2pa+a^2 &= 0\\
\implies c^2-b^2+a^2 &= 2pa\\
\implies \frac{c^2-b^2+a^2}{2a} &= p
\end{aligned} \tag{25}$$

From (25), p is obtained. q can be obtained from (20) as

$$
\begin{aligned}
c^2 &= p^2 + q^2 \\
\implies c^2 - p^2 &= q^2 \\
\implies \sqrt{c^2 - p^2} &= q
\end{aligned}
\tag{26}
$$

3.2 Draw a circle with centre $\mathbf{B}$ and radius $c = 6$.

3.3 Let

$$
D = \begin{pmatrix} p \\ -q \end{pmatrix} \tag{27}
$$

Draw $\triangle ABC$ and $\triangle ADC$.

Solution: The following code draws the circle and tangents in Fig. 3.3

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
  len =10
  x_AB = np.zeros((2,len))
  lam_1 = np.linspace(0,1,len)
  for i in range(len):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB


#Triangle sides
a = 10
c = 6
b = np.sqrt(a**2-c**2)

p = (a**2 + c**2-b**2 )/(2*a)
q = np.sqrt(c**2-p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])
D = np.array([p,-q])

#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)
x_CD = line_gen(C,D)

#Plotting all lines
plt.plot(x_AB[0,:],x_AB[1,:],label='$AB$')
plt.plot(x_BC[0,:],x_BC[1,:],label='$BC$')
plt.plot(x_CA[0,:],x_CA[1,:],label='$CA$')
plt.plot(x_CD[0,:],x_CD[1,:],label='$CD$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) , '
    A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) ,
    'C')
plt.plot(D[0], D[1], 'o')
plt.text(D[0] * (1 - 0.2), D[1] * (1) , 'D')

#Plotting the circle

theta = np.linspace(0,2*np.pi,50)
x = c*np.cos(theta)
y = c*np.sin(theta)

plt.plot(x,y)

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')
#if using termux
plt.savefig('../figs/circle.pdf')
plt.savefig('../figs/circle.eps')
subprocess.run(shlex.split("termux-open ../
    figs/circle.pdf"))
#else
#plt.show()
```

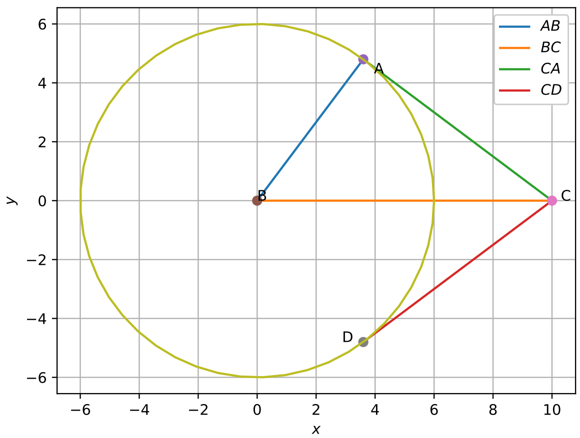

Fig. 3.3

4 Incircle

4.1 Consider the right angled $\triangle ABC$, right angled at $\mathbf{B}$ with $a = 8, b = 10, c = 6$. Let

$$\begin{aligned} x + y &= a \\ y + z &= b \\ z + x &= c \end{aligned} \tag{28}$$

Show that

$$\begin{aligned} x &= \frac{a + c - b}{2} \\ y &= \frac{b + a - c}{2} \\ z &= \frac{c + b - a}{2} \end{aligned} \tag{29}$$

4.2 Find $\mathbf{D}, \mathbf{E}, \mathbf{F}$ such that

$$\begin{aligned} \mathbf{D} &= \frac{x\mathbf{C} + y\mathbf{B}}{x + y} \\ \mathbf{E} &= \frac{y\mathbf{A} + z\mathbf{C}}{y + z} \\ \mathbf{F} &= \frac{z\mathbf{B} + x\mathbf{A}}{z + x} \end{aligned} \tag{30}$$

4.3 Let

$$\mathbf{I} = \begin{pmatrix} p \\ q \end{pmatrix} \tag{31}$$

If

$$\mathbf{D} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}, \tag{32}$$

and

$$ID = IE \tag{33}$$

show that

$$p(d_1 - e_1) + q(d_2 - e_2) = \frac{e_1^2 + e_2^2 - d_1^2 - d_2^2}{2} \tag{34}$$

4.4 If

$$IE = IF, \tag{35}$$

show that

$$p(e_1 - f_1) + q(e_2 - f_2) = \frac{f_1^2 + f_2^2 - e_1^2 - e_2^2}{2} \tag{36}$$

4.5 Find $\mathbf{I}$ and r if

$$ID = IE = IF = r \tag{37}$$

5 Exercises

5.1 Plot $\triangle ABC$ for $a = 8, b = 11$ and $c = 13$.
Solution: The following program plots $\triangle ABC$ in Fig. 5.1

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
  len =10
  x_AB = np.zeros((2,len))
  lam_1 = np.linspace(0,1,len)
  for i in range(len):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB


#Triangle sides
a = 8
b = 11
c = 13
```

```
p = (a**2 + c**2-b**2 )/(2*a)
q = np.sqrt(c**2-p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])


#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)

#Plotting all lines
plt.plot(x_AB[0,:],x_AB[1,:],label='$AB$')
plt.plot(x_BC[0,:],x_BC[1,:],label='$BC$')
plt.plot(x_CA[0,:],x_CA[1,:],label='$CA$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) , '
    A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) ,
    'C')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor

#if using termux
plt.savefig('../figs/triangle.pdf')
plt.savefig('../figs/triangle.eps')
subprocess.run(shlex.split("termux-open ../
    figs/triangle.pdf"))
#else
#plt.show()
```

5.2 Find $\mathbf{O}$ and R such that

$$R = OA = OB = OC \tag{38}$$

5.3 Draw a circle with centre $\mathbf{B}$ and radius 6. If $\mathbf{C}$ be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.
Solution: From the given information, in

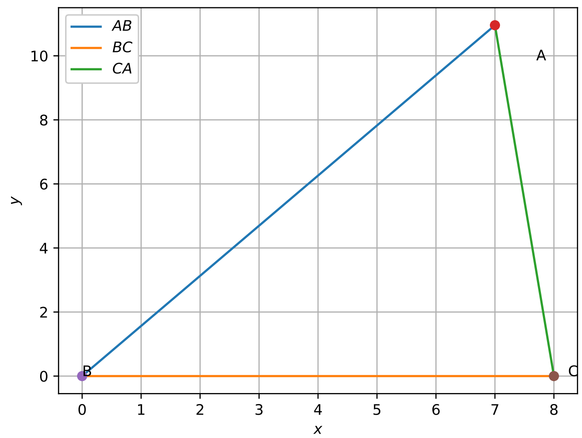

Fig. 5.1

$\triangle ABC, AC \perp AB, a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \tag{39}$$

5.4 Write a program to compute p and q when $a = 8, b = 11$ and $c = 13$.

5.5 In $\triangle ABC$, a and $\angle B$ are known and $b + c = k$. If

$$b^2 = a^2 + c^2 - 2ac\cos B \tag{40}$$

show that

$$c = \frac{a^2 - k^2}{2(a\cos B - k)} \tag{41}$$

5.6 In $\triangle ABC$, $a = 7, \angle B = 75^\circ$ and $b + c = 13$. Find b and c and sketch $\triangle ABC$.

5.7 In $\triangle ABC$, $a = 8, \angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.

5.8 In $\triangle ABC$, $a = 6, \angle B = 60^\circ$ and $b - c = 2$. Sketch $\triangle ABC$.

5.9 $\triangle ABC$ is right angled at $\mathbf{B}$. If $a = 12$ and $b + c = 18$, find a, b, c and draw the triangle.
Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \tag{42}$$

5.10 In $\triangle ABC$, given that $a + b + c = 11, \angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c.
Solution: We have

$$a = b\cos C + c\cos B \tag{43}$$
$$b\sin C = c\sin B \tag{44}$$
$$a + b + c = 11 \tag{45}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \tag{46}$$

Solving the equivalent matrix equation gives the desired answer.

5.11 Draw $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 30^{\circ}$ and $\angle C = 90^{\circ}$, find a, b, c.

5.12 Draw a square of side 3.

5.13 Draw a parallelogram with sides 12 and 5.

5.14 Draw a circle with centre **O** and diameter $AC = 6$. Choose any point B on the circle and draw $\triangle ABC$.

5.15 In $\triangle ABC$, $a = 8, b = 11, c = 13$. Find

$$R = \frac{a}{2\sin A}. \tag{47}$$

Let **D** be the mid point of BC. Find the point **O** such that $\triangle ODB$ is right angled at **D** and $OD = R$. Draw the circle with centre **O** and radius R.

5.16 Let

$$r = \frac{abc}{2(a+b+c)}. \tag{48}$$

and

$$IB = r\sqrt{\frac{2}{1-\cos B}}. \tag{49}$$

Draw a circle with centre **I** and radius r.

5.17 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

5.18 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

5.19 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°.

5.20 Draw a line segment AB of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

5.21 Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^{\circ}$. BD is the perpendicular from **B** on AC. The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.