

Optimization through School Geometry

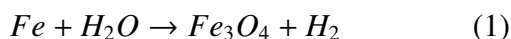
G V V Sharma*

Abstract—This manual shows how to balance chemical equations using matrices.

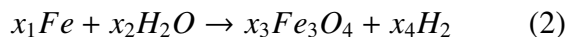
Download python codes using

svn co <https://github.com/gadepall/school/trunk/training>

Balance the following chemical equation



Solution: Let the balanced version of (1) be



which results in the following equations

$$\begin{aligned} (x_1 - 3x_3) Fe &= 0 \\ (2x_2 - 2x_4) H &= 0 \end{aligned} \quad (3)$$

$$\begin{aligned} (x_2 - 4x_3) H &= 0 \\ x_1 + 0.x_2 - 3x_3 + 0.x_4 &= 0 \\ \Rightarrow 0.x_1 + 2x_2 + 0.x_3 - 2x_4 &= 0 \\ 0.x_1 + x_2 - 4x_3 + 0.x_4 &= 0 \end{aligned} \quad (4)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \quad (5)$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad (6)$$

The matrix in (5) can be row reduced as follows

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \quad (7)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow 4R_1 - 3R_3} \begin{pmatrix} 4 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \quad (8)$$

$$\xrightarrow{\begin{matrix} R_1 \leftarrow \frac{1}{4} \\ R_3 \leftarrow -\frac{1}{4}R_3 \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{4} \end{pmatrix} \quad (9)$$

Thus,

$$\mathbf{x} = x_4 \begin{pmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{4} \\ 1 \end{pmatrix} \quad (10)$$

Substituting $x_4 = 4$ in (10),

$$\mathbf{x} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \quad (11)$$

and (2) becomes



*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.