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Abstract—Solved problems from JEE mains papers related to conic sections in coordinate geometry are available in this document. These problems are solved using linear algebra/matrix analysis.

- Two parabolas with a common vertex and with axes along x -axis and y -axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, find the equation of the common tangent to the two parabolas.
- A hyperbola passes through the point

$$\mathbf{P} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \quad (2.1)$$

and has foci at $\begin{pmatrix} \pm 2 \\ 0 \end{pmatrix}$. Find the equation of the tangent to this hyperbola at \mathbf{P} .

- Find the product of the perpendiculars drawn from the foci of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 25 & 0 \\ 0 & 9 \end{pmatrix} \mathbf{x} = 225 \quad (3.1)$$

upon the tangent to it at the point

$$\frac{1}{2} \begin{pmatrix} 3 \\ 5\sqrt{3} \end{pmatrix} \quad (3.2)$$

- Let P be the point on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - (8 \ 0) \mathbf{x} = 0 \quad (4.1)$$

which is at a minimum distance from the centre C of the circle

$$\mathbf{x}^T \mathbf{x} + (0 \ 12) \mathbf{x} = 1 \quad (4.2)$$

Find the equation of the circle passing through

C and having its centre at (P) .

- Consider an ellipse, whose centre is at the origin and its major axis is along the x -axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then find the area of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse.
- Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation

$$9e^2 - 18e + 5 = 0 \quad (6.1)$$

If

$$\mathbf{S} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (6.2)$$

is a focus and

$$\begin{pmatrix} 5 & 0 \end{pmatrix} \mathbf{x} = 9 \quad (6.3)$$

is the corresponding directrix of this hyperbola, then find $a^2 - b^2$.

- A variable line drawn through the intersection of the lines

$$\begin{pmatrix} 4 & 3 \end{pmatrix} \mathbf{x} = 12 \quad (7.1)$$

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 12 \quad (7.2)$$

meets the coordinate axes at \mathbf{A} and \mathbf{B} , then find the locus of the midpoint of AB .

Solution: The intersection of the lines in (7.1) is obtained through the matrix equation

$$\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 \\ 12 \end{pmatrix} \quad (7.3)$$

by forming the augmented matrix and row

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reduction as

$$\begin{pmatrix} 4 & 3 & 12 \\ 3 & 4 & 12 \end{pmatrix} \leftrightarrow \begin{pmatrix} 4 & 3 & 12 \\ 0 & 7 & 12 \end{pmatrix} \leftrightarrow \begin{pmatrix} 28 & 0 & 48 \\ 0 & 7 & 12 \end{pmatrix} \leftrightarrow \begin{pmatrix} 7 & 0 & 12 \\ 0 & 7 & 12 \end{pmatrix} \quad (7.4)$$

resulting in

$$\mathbf{C} = \frac{1}{7} \begin{pmatrix} 12 \\ 12 \end{pmatrix} \quad (7.5)$$

Let the \mathbf{R} be the mid point of AB . Then,

$$\mathbf{A} = 2 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{R} \quad (7.6)$$

$$\mathbf{B} = 2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{R} \quad (7.7)$$

Let the equation of AB be

$$\mathbf{n}^T (\mathbf{x} - \mathbf{C}) = 0 \quad (7.8)$$

Since \mathbf{R} lies on AB ,

$$\mathbf{n}^T (\mathbf{R} - \mathbf{C}) = 0 \quad (7.9)$$

Also,

$$\mathbf{n}^T (\mathbf{A} - \mathbf{B}) = 0 \quad (7.10)$$

Substituting from (7.6) in (7.10),

$$\mathbf{n}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{R} = 0 \quad (7.11)$$

From (7.9) and (7.11),

$$(\mathbf{R} - \mathbf{C}) = k \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{R} \quad (7.12)$$

for some constant k . Multiplying both sides of (7.12) by

$$\mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (7.13)$$

$$\begin{aligned} \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbf{R} - \mathbf{C}) &= k \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{R} \\ &= k \mathbf{R}^T \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{R} = 0 \end{aligned} \quad (7.14)$$

$$\therefore \mathbf{R}^T \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{R} = 0 \quad (7.15)$$

which can be easily verified for any \mathbf{R} . from

(7.14),

$$\begin{aligned} \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbf{R} - \mathbf{C}) &= 0 \\ \Rightarrow \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{R} - \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{C} &= 0 \\ \Rightarrow \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{R} - \mathbf{C}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{R} &= 0 \end{aligned} \quad (7.16)$$

which is the desired locus.