Discrete Mathematics

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Contents

***Abstract*—This book provides a computational approach to discrete mathematics by solving problems in related areas from IIT-JEE. Links to sample C/Python codes are available in the text. The book provides sufficient math basics for Machine Learning and is also recommended for high school students who wish to explore topics in Artificial Intelligence.**

Download python codes using

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svn co https://github.com/gadepall/school/trunk/jee/
    discrete/codes
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1 Signal Processing: Z Transform

1. Let

$$a(n) = \frac{\alpha^n - \beta^n}{\alpha - \beta} u(n) \tag{1.0.1.1}$$

$$b(n) = a(n-1) + a(n+1) - \delta(n) \tag{1.0.1.2}$$

where α, β are the roots of the equation

$$z^2 - z - 1 = 0 \tag{1.0.1.3}$$

and

$$u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \ge 0 \end{cases} \tag{1.0.1.4}$$

$$\delta(n) = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \tag{1.0.1.5}$$

2. Verify your results through a C program.

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3. Show that the Z transform of $u(n)$

$$U(z) \triangleq \sum_{n=-\infty}^{\infty} u(n) z^{-n} \tag{1.0.3.1}$$

$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{1.0.3.2}$$

4. Show that

$$A(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}} \tag{1.0.4.1}$$

5. Let

$$y(n) = a(n) * u(n) \triangleq \sum_{k=-\infty}^{\infty} a(k) u(n-k) \tag{1.0.5.1}$$

Show that

$$y(n) = \sum_{k=0}^{n} a(k) \tag{1.0.5.2}$$

6. Show that

$$Y(z) = A(z) U(z) \tag{1.0.6.1}$$

$$= \frac{z^{-1}}{\left(1 - z^{-1} - z^{-2}\right)\left(1 - z^{-1}\right)} \tag{1.0.6.2}$$

7. Show that

$$w(n) = [a(n+2) - 1]\, u(n-1) \tag{1.0.7.1}$$

$$= a(n+2) - u(n+1) + 2\delta(n) \tag{1.0.7.2}$$

8. Is $W(z) = Y(z)$?
9. Verify if

$$\sum_{n=1}^{\infty} \frac{a(n)}{10^n} = \frac{10}{89} \tag{1.0.9.1}$$

10. Verify if

$$\sum_{n=1}^{\infty} \frac{b(n)}{10^n} = \frac{8}{89} \tag{1.0.10.1}$$

2 Algebra: Modular Arithmetic

Let $AP(a;d)$ denote an A.P. with $d > 0$

1. Express $AP(a;d)$ in modulo arithmetic.
Solution:
$$A \equiv a \pmod{d} \tag{2.0.10.1}$$
2. Express the intersection of $AP(1;3), AP(2;5)$ and $AP(3;7)$ using modulo arithmetic.
Solution: The desired AP can be expressed as
$$A \equiv 1 \pmod{3} \tag{2.0.10.2}$$
$$\equiv 2 \pmod{5} \tag{2.0.10.3}$$
$$\equiv 3 \pmod{7} \tag{2.0.10.4}$$
3. Two numbers are said to be coprime if their greatest common divisor (gcd) is 1. Verify if (3,5), (5,7) and (3,7) are pairwise coprime.
4. Does a solution for (2.0.10.2) exist?
Solution: The Chinese remainder theorem guarantees that the system in (2.0.10.2) has a solution since 3,5,7 are pairwise coprime.
5. Simplify
$$(7 \times 5) \pmod{3} \tag{2.0.10.5}$$
Solution: (2.0.10.5) can be expressed as
$$(7 \times 5) \pmod{3} = 35 \pmod{3}$$
$$= 2 \pmod{3} \tag{2.0.10.6}$$
6. Find x in
$$2x = 1 \pmod{3} \tag{2.0.10.7}$$
Solution: By inspection, for $x = 2$,
$$2x = 2 \times 2 = 4 = 3 + 1 = 1 \pmod{3} \tag{2.0.10.8}$$
Thus $x = 2$ is a solution of (2.0.10.7).
7. In general, x in
$$ax = 1 \pmod{d} \tag{2.0.10.9}$$
is defined to be the modular multiplicative inverse of (2.0.10.1).
8. Show that the multiplicative inverse of
$$(3 \times 5) \pmod{7} = y = 1 \tag{2.0.10.10}$$
9. Show that the multiplicative inverse of
$$(3 \times 7) \pmod{5} = z = 1 \tag{2.0.10.11}$$
10. Find $a + d$.
Solution:
$$(5 \times 7 \times 1 \times x) + (3 \times 5 \times 3 \times y) + (3 \times 7 \times 2 \times z) = 157 \tag{2.0.10.12}$$
11. Find a and d.
Solution:
$$d = LCM(3,5,7) = 105 \tag{2.0.10.13}$$
$$A = 157 \pmod{105}$$
$$= 52 \pmod{105}$$
$$\implies a = 52 \tag{2.0.10.14}$$
12. Given the APs
$$a_1 \pmod{d_1} \tag{2.0.10.15}$$
$$a_2 \pmod{d_2} \tag{2.0.10.16}$$
$$a_3 \pmod{d_3}, \tag{2.0.10.17}$$
such that
$$gcd(d_1, d_2) = gcd(d_2, d_3) = gcd(d_3, d_1) = 1, \tag{2.0.10.18}$$
show that their intersection
$$a \pmod{d} \tag{2.0.10.19}$$
is obtained through
$$a + d = (d_1 \times d_2 \times a_3 \times x) + (d_2 \times d_3 \times a_1 \times y) + (d_3 \times d_1 \times a_2 \times z) \tag{2.0.10.20}$$
$$d = LCM(d_1, d_2, d_3), \tag{2.0.10.21}$$
where x, y, z are the modular multiplicative inverses given by
$$x = [(d_1 \times d_2) \pmod{d_3}]^{-1} \tag{2.0.10.22}$$
$$y = [(d_2 \times d_3) \pmod{d_1}]^{-1} \tag{2.0.10.23}$$
$$z = [(d_3 \times d_1) \pmod{d_2}]^{-1} \tag{2.0.10.24}$$
respectively.
13. Write a C program to find x, y and z.

3 Discrete Fourier Transform

1. Show that
$$\sum_{k=0}^{n-1} e^{\jmath\frac{2\pi k}{n}} = \begin{cases} 1 & n = 1, \\ 0 & n > 1 \end{cases} \tag{3.0.1.1}$$
2. Show that
$$\sum_{k=0}^{n} \cos\left(\frac{2k+r}{n+2}\pi\right) = -\cos\left(\frac{r-2}{n+2}\pi\right) \tag{3.0.2.1}$$
Solution: From (3.0.1.1),
$$\sum_{k=0}^{n+1} e^{\jmath\frac{2k+r}{n+2}\pi} = 0$$
$$\implies \sum_{k=0}^{n} e^{\jmath\frac{2k+r}{n+2}\pi} + e^{\jmath\frac{2(n+1)+r}{n+2}\pi} = 0$$
$$\implies \sum_{k=0}^{n} e^{\jmath\frac{2k+r}{n+2}\pi} = -e^{\jmath\frac{2(n+2)+r-2}{n+2}\pi}$$
$$= -e^{\jmath\frac{r-2}{n+2}\pi} \tag{3.0.2.2}$$
Taking the real part on both sides yields (3.0.2.1).
3. Show that
$$f(n) = \frac{\sum_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} \sin^2\left(\frac{k+2}{n+2}\pi\right)} \tag{3.0.3.1}$$
$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)}{n + \cos\left(\frac{2\pi}{n+2}\right)} \tag{3.0.3.2}$$
Solution: Let
$$\theta_n = \frac{\pi}{n+2} \tag{3.0.3.3}$$
$$\because \sin\{(k+1)\theta_n\}\sin\{(k+2)\theta_n\},$$
$$= \frac{1}{2}\left[\cos\theta_n - \cos\{(2k+3)\theta_n\}\right] \tag{3.0.3.4}$$
from (3.0.3.1) and (3.0.2.1),
$$f(n) = \frac{n\cos\theta_n - \sum_{k=0}^{n}\cos\{(2k+3)\theta_n\}}{n - \sum_{k=0}^{n}\cos\{(2k+4)\theta_n\}}$$
$$= \frac{n\cos\left(\frac{\pi}{n+2}\right) + \cos\left(\frac{\pi}{n+2}\right)}{n + \cos\left(\frac{2\pi}{n+2}\right)} \tag{3.0.3.5}$$
resulting in (3.0.3.2). Verify if
4.
$$f(4) = \frac{\sqrt{3}}{2} \tag{3.0.4.1}$$
5.
$$\lim_{n\to\infty} f(n) = \frac{1}{2} \tag{3.0.5.1}$$
6.
$$\sin\left(7\cos^{-1} f(5)\right) = 0 \tag{3.0.6.1}$$
7. If
$$\alpha = \tan\left(\cos^{-1} f(6)\right) \tag{3.0.7.1}$$
verify if
$$\alpha^2 + 2\alpha - 1 = 0 \tag{3.0.7.2}$$

4 Combinatorics

1. Find
$$\sum_{k=0}^{n} k \tag{4.0.1.1}$$
Solution: (4.0.1.1) can be expressed as
$$\frac{n(n+1)}{2} \tag{4.0.1.2}$$
2. Find
$$\sum_{k=0}^{n} {}^{n}C_k k^2 \tag{4.0.2.1}$$
Solution:
$$(1+x)^n = \sum_{k=0}^{n} {}^{n}C_k x^k \tag{4.0.2.2}$$
$$\implies n(1+x)^{n-1} = \sum_{k=0}^{n} k\,{}^{n}C_k x^{k-1} \tag{4.0.2.3}$$
upon differentiation. Multiplying (4.0.2.3) by x and differentiating,
$$\frac{d}{dx}\left[nx(1+x)^{n-1}\right] = \sum_{k=0}^{n} k^2\,{}^{n}C_k x^{k-1} \tag{4.0.2.4}$$
$$\implies n(n-1)x(1+x)^{n-2} + n(1+x)^{n-1}$$
$$= \sum_{k=0}^{n} k^2\,{}^{n}C_k x^{k-1} \tag{4.0.2.5}$$

Substituting $x = 1$ in (4.0.2.5),

$$\sum_{k=0}^{n} {}^nC_k k^2 = n(n-1)2^{n-2} + n2^{n-1}$$
$$= n(n+1)2^{n-2} \quad (4.0.2.6)$$

3. Find
$$\sum_{k=0}^{n} {}^nC_k k \quad (4.0.3.1)$$
Solution: Substituting $x = 1$ in (4.0.2.3),
$$\sum_{k=0}^{n} {}^nC_k k = n2^{n-1} \quad (4.0.3.2)$$

4. Find
$$\sum_{k=0}^{n} {}^nC_k 3^k \quad (4.0.4.1)$$
Solution: Substituting $x = 2$ in (4.0.2.2),
$$\sum_{k=0}^{n} {}^nC_k 3^k = 4^n \quad (4.0.4.2)$$

5. If
$$\begin{vmatrix} \frac{n(n+1)}{2} & n(n+1)2^{n-2} \\ n2^{n-1} & 4^n \end{vmatrix} = 0 \quad (4.0.5.1)$$
for some n, find
$$\sum_{k=0}^{n} \frac{{}^nC_k}{k+1} \quad (4.0.5.2)$$
Solution: (4.0.5.1) can be expressed as
$$n(n+1)2^{2n-3} \begin{vmatrix} 1 & 1 \\ n & 4 \end{vmatrix} = 0 \quad (4.0.5.3)$$
$$\implies n = 4 \quad (4.0.5.4)$$
Integrating (4.0.2.2) from 0 to 1,
$$\frac{2^{n+1}}{n+1} = \sum_{k=0}^{n} \frac{{}^nC_k}{k+1} \quad (4.0.5.5)$$
Substituting $n = 4$ in the above,
$$\sum_{k=0}^{n} \frac{{}^nC_k}{k+1} = \frac{2^5-1}{5} = \frac{31}{5} \quad (4.0.5.6)$$

5 JEE Exercises:Sequences and Series

1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is......................
2. The solution of the equation $log_7 log_5(\sqrt{x+5} + \sqrt{x}) = 0$ is ....
3. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 +$ ....is $\frac{n(n+1)^2}{2}$, when n is even.When n is odd,the sum is ....
4. Let the harmonic mean and geometric mean of two positive numbers be the ratio 4:5.Then the two numbers are in the ratio..................
5. For any odd integer n≥ 1,$n^3 - (n-1)^3 + ... + (-1)^{(n-1)}1^3$=............
6. Let p and q be roots of the equation
$$x^2 - 2x + A = 0 \quad (5.0.6.1)$$
and let r and s be the roots of the equation
$$x^2 - 18x + B = 0 \quad (5.0.6.2)$$
. If $p < q < r < s$ are in arithmetic progression,then A =.......... and B=........

MCQs with One Correct Answer

7. If x,y and z are p^th,q^th and r^th terms respectively of an A.P.and also of a G.P.,then $x^y - zy^z - xz^x - y$ is equal to:
   a) xyz
   b) 0
   c) 1
   d) None of these
8. The third term of a geometric progression is 4.The product of the first five terms is
   a) 4^3
   b) 4^5
   c) 4^4
   d) None of these
9. The rational number,which equals the number $2.\overline{357}$ with recurring decimal is
   a) $\frac{2355}{1001}$
   b) $\frac{2379}{997}$
   c) $\frac{2355}{999}$
   d) none of these
10. If a,b,c are in G.P.,then the equations
$$ax^2 + 2bx + c = 0 \quad (5.0.10.1)$$
and
$$dx^2 + 2ex + f = 0 \quad (5.0.10.2)$$
have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in......
   a) A.P.
   b) G.P.

c) H.P.
d) None of these

11. Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$.....is equal to
a) $2^n - n - 1$
b) $1 - 2^- n$
c) $n + 2^- n - 1$
d) $2^n + 1$

12. The number $log_2 7$ is
a) an integer
b) a rational number
c) an irrational number
d) a prime number

13. If $ln(a+c)$, $ln(a-c)$, $ln(a-2b+c)$ are in A.P.,then
a) a,b,c are in A.P.
b) a^2, b^2, c^2 are in A.P.
c) a,b,c are in G.P.
d) a,b,c are in H.P.

14. Let $a_1, a_2, ...a_1 0$ be in A.P.and $h_1, h_2, ...h_{10}$ be in H.P.If $a_1 = h_1 = 2$ and $a_{10} = h_{10}$=3, then $a_4 h_7$ is
a) 2
b) 3
c) 5
d) 6

15. The harmonic mean of the roots of the equation

$$(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0 \tag{5.0.15.1}$$

is
a) 2
b) 4
c) 6
d) 8

16. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then
a) a=$\frac{4}{7}$,r=$\frac{3}{7}$
b) a=2,r=$\frac{3}{8}$
c) a=$\frac{3}{2}$,r=$\frac{1}{2}$
d) a=3,r=$\frac{1}{4}$

17. Let α, β be the roots of

$$x^2 - x + p = 0 \tag{5.0.17.1}$$

and γ, δ be the roots of

$$x^2 - 4x + q = 0 \tag{5.0.17.2}$$

. If α, β, γ, δ are in G.P., then the integral values of p and q respectively, are
a) -2,-32
b) -2,3
c) -6,3
d) -6,-32

18. Let the positive numbers a,b,c,d be in A.P.Then abc,abd,acd,bcd are
a) NOT in A.P./G.P./H.P.
b) in A.P.
c) in G.P.
d) in H.P.

19. If the sum of the first 2n terms of the A.P.2,5,8,.....,is equal to the sum of the first n terms of the A.P.57,59,61,.....,then n equals
a) 10
b) 12
c) 11
d) 13

20. Suppose a,b,c are in A.P. and a^2, b^2, c^2 are in G.P.if $a < b < c$ and a+b+c=$\frac{3}{2}$, then the value of a is
a) $\frac{1}{2\sqrt{2}}$
b) $\frac{1}{2\sqrt{3}}$
c) $\frac{1}{2}$-$\frac{1}{\sqrt{3}}$
d) $\frac{1}{2}$-$\frac{1}{\sqrt{2}}$

21. An infinite G.P. has first term 'x' and sum '5', then x belongs to
a) $x < -10$
b) $-10 < x < 0$
c) $0 < x < 10$
d) $x > 10$

22. In the quadratic equation

$$ax^2 + bx + c = 0, \tag{5.0.22.1}$$

$\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$, are in G.P.where α, β are the root of $ax^2 + bx + c = 0$, then
a) $\Delta \neq 0$
b) $b\Delta = 0$
c) $c\Delta 0$
d) $\Delta = 0$

23. In the sum of first n terms of an A.P.is cn^2, then the sum of squares of these n terms is
a) $\frac{n(4n^2-1)c^2}{6}$
b) $\frac{n(4n^2+1)c^2}{3}$

c) $\frac{n(4n^2-1)c^2}{3}$
d) $\frac{n(4n^2+1)c^2}{6}$

24. Let a_1, a_2, a_3,........be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is
a) 22
b) 23
c) 24
d) 25

25. Let $b_1 > 1$ for $i = 1, 2, ..., 101$.Suppose $\log_e b_1, \log_e b_2, ... \log_e b_{101}$ are in Arithmetic progression(A.P) with the common difference $\log_e 2$. Suppose $a_1, ...a_{101}$ are in A.P such that $a_1 = b_1$ and $a_{51} = b_{51}$.If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ...a_{53}$, then
a) $s > t$ and $a_{101} > b_{101}$
b) $s > t$ and $a_{101} < b_{101}$
c) $s < t$ and $a_{101} > b_{101}$
d) $s < t$ and $a_{101} < b_{101}$

MCQs with One or More than One Correct

26. If the first and the $(2n-1)$st terms of an A.P., a G.P. and an H.P. are equal and their n-th terms are a,b and c respectively , then
a) a=b=c
b) $a \geq b \geq c$
c) a+c=b
d) $ac - b^2 = 0$.

27. For $0 < \phi < \frac{\Pi}{2}$, if $x = \sum_{n=0}^{\infty}(\cos^{2n})\phi$, $y = \sum_{n=0}^{\infty}(\sin^{2n})\phi$, $z = \sum_{n=0}^{\infty}(\cos^{2n})\phi(\sin^{2n})\phi$ then:
a) xyz = xz+y
b) xyz = xy+z
c) xyz = x+y+z
d) xyz = yz+x

28. Let n be an odd integer.If $\sin n\theta = \sum_{r=0}^{n}(b)_r \sin^r\theta$, for every value of θ,then
a) $b_0 = 1, b_1 = 3$
b) $b_0 = 0, b_1 = n$
c) $b_0 = -1, b_1 = n$
d) $b_0 = 0, b_1 = n^2 - 3n + 3$

29. Let T_r be the r^{th} term of an A.P.,for r=1,2,3,..... If for some positive integers m,n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals
a) $\frac{1}{mn}$
b) $\frac{1}{m}+\frac{1}{n}$
c) 1
d) 0

30. If $x > 1, y > 1, z > 1$ are in G.P.,then $\frac{1}{1+lnx}, \frac{1}{1+lny}, \frac{1}{1+lnz}$ are in
a) A.P.
b) H.P.
c) G.P.
d) None of these

31. For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...\frac{1}{(2^n)-1}$. Then
a) $a(100) \leq 100$
b) $a(100) > 100$
c) $a(200) \leq 100$
d) $a(200) > 100$

32. A straight line through the vertex P of a triangle PQR intersects the side QR at the points **S** and the circumcircle of the triangle PQR at the point **T**.If S is not the centre of the circumcircle, then
a) $\frac{1}{(PS}+\frac{1}{(ST} < \frac{2}{\sqrt{QSXSR}}$
b) $\frac{1}{(PS}+\frac{1}{(ST} > \frac{2}{\sqrt{QSXSR}}$
c) $\frac{1}{(PS}+\frac{1}{(ST} < \frac{4}{QR}$
d) $\frac{1}{(PS}+\frac{1}{(ST} > \frac{2}{QR}$

33. Let $S_n = \sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ for n=1,2,3......Then

34. Let $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(S)
a) 1056
b) 1088
c) 1120
d) 1332

35. Let α and β be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$.For all positive integers n,define $a_n = \frac{\alpha^n-\beta^n}{\alpha-\beta}, n \geq 1 b_1$ and $b_n = a_{n-1} + a_{n+1}, n \geq 2$ Then which of the following options is /are correct?
a) $\sum_{n=1}^{\infty}\frac{\alpha_n}{10^n} = \frac{10}{89}$
b) $b_n = \alpha^n + \beta^n$ for all $n \geq 1$
c) $a_1 + a_2 + a_3 + ...a_n = a_{n+2} - 1$ for all $n \geq 1$
d) $\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \frac{8}{89}$

Subjective Problems

36. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation, $2A + G^2 = 27$. Find the two numbers.

37. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°, and the common difference is 5°. Find the number of sides of the polygon.

38. Does there exists a geometric progression con-

taining 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible ?

39. Find three numbers a,b,c between 2 and 18 such that
   a) their sum is 25
   b) the numbers 2,a,b are consecutive terms of an A.P. and
   c) the numbers b,c,18 are consecutive terms of a G.P.
40. If $a > 0, b > 0$ and $c > 0$, prove that $(a + b + c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \geq 9$
41. If n is a natural number such that $n = p_1^{\alpha}1.p_2^{\alpha}2.p_3^{\alpha}3....p_k^{\alpha}k$ and $p_1, p_2, ...p_k$ are distinct primes, then show that $lnn \geq kln2$
42. Find the sum of the series: $\sum_{r=0}^{n}(-1)^r nC_r[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}}$......upto m terms]
43. Solve for x the following equation: $log_{2x+3}(6x^2+23x+21) = 4-log_{3x+7}(4x^2+12x+9)$
44. If $log_3 2$, $log_3(2^x - 5)$, $log_3(2^x - \frac{7}{2})$ are in arithmetic progression, determine the value of x.
45. Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $[\frac{n+1}{n-1}]^2$p.
46. If S_1, S_2, S_3,......., S_n are the sums of infinite geometric series whose first terms are 1, 2, 3,......, n and whose common ratios are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$,..........$\frac{1}{n+1}$ respectively, then find the values of S_1^2+ S_2^2+ S_3^2,......., S_{2n-1}^2
47. The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie.
48. Let a,b,c,d are the real numbers in G.P. If u,v,w, satisfy the system of equations $u+2v+3w = 6$ $4u + 5v + 6w = 12$ $6u + 9v = 4$ then show that the root of the equation $(\frac{1}{u} + \frac{1}{v} + \frac{1}{w})x^2 + [(b - c)^2 + (c - a)^2 + (d - b)^2]x + u + v + w = 0$ and $20x^2 + 10(a - d)^2 x - 9 = 0$ are reciprocals of the each other.
49. The fouth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer.
50. Let a_1, a_2,.......,a_n be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of a_1, a_2,.......,a_n. Find an expression for the geometric mean of G_1, G_2,......, G_n, in terms of A_1, A_2,.......,A_n, H_1,H_2,......,H_n,
51. Let a, b be positive real numbers.If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that $\frac{G_1 G_2}{H_1 H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$.
52. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either a = b = c or a, b, $-\frac{c}{2}$ form a G.P.
53. If $a_n = \frac{3}{4} - [\frac{3}{4}]^2 + [\frac{3}{4}]^3 + ...(-1)^{n-1}[\frac{3}{4}]^n$ and $b_n = 1 - a_n$, then find the least natural number n_0 such that $b_n > a_n \forall n \geq n_0$.

Comprehension Based Questions

PASSAGE - 1

Let V_r denote the sum of first r terms of an arithmetic progression(A.P.) whose first term is r and the common difference is (2r-1). Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r=1,2,...

54. The sum $V_1 + V_2 + ... + V_n$ is
   a) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$
   b) $\frac{1}{12}n(n + 1)(3n^2 + n + 2)$
   c) $\frac{1}{2}n(2n^2 - n + 1)$
   d) $\frac{1}{3}(2n^3 - 2n + 3)$
55. T_r is always
   a) an odd number
   b) an even number
   c) a prime number
   d) a composite number
56. Which one of the following is a correct statement ?
   a) Q_1, Q_2, Q_3,.... are in A.P. with common difference 5
   b) Q_1, Q_2, Q_3,.... are in A.P. with common difference 6
   c) Q_1, Q_2, Q_3,.... are in A.P. with common difference 11
   d) $Q_1 = Q_2 = Q_3$=........

PASSAGE - 2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

57. Which one of the following statements is correct ?
   a) $G_1 > G_2 > G_3 >$ ....

b) $G_1 < G_2 < G_3 <$ ....
c) $G_1 = G_2 = G_3 =$ ....
d) $G_1 < G_3 < G_5$ and $G_2 > G_4 > G_6 >$ ....

58. Which one of the following statements is correct ?
a) $A_1 > A_2 > A_3 >$ ....
b) $A_1 < A_2 < A_3 <$ ....
c) $A_1 > A_3 > A_5 >$ .... and $A_2 < A_4 < A_6 <$ ....
d) $A_1 < A_3 < A_5 <$ .... and $A_2 > A_4 > A_6 >$ ....

59. Which one of the following statements is correct ?
a) $H_1 > H_2 > H_3 >$ ....
b) $H_1 < H_2 < H_3 <$ ....
c) $H_1 > H_3 > H_5 >$ .... and $H_2 < H_4 < H_6 <$ ....
d) $H_1 < H_3 < H_6 <$ .... and $H_2 > H_4 > H_6 >$ ....

Assertion Reson type quations

60. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.
STATEMENT - 1: The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. and
STATEMENT - 2: The numbers b_1, b_2, b_3, b_4 are in H.P.
a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a NOT a correct explanation for STATEMENT - 1
c) STATEMENT - 1 is True, STATEMENT - 2 is False
d) STATEMENT - 1 is False, STATEMENT - 2 is True

Integer Value Correct Type

61. Let $S_k, k = 1, 2, ..., 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2 - 3k + 1)S_k\right|$ is

62. $a_1, a_2, a_3, ..., a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k= 3,4,.....,11, if $\frac{a_1^2+a_2^2+.....+a_{11}^2}{11} = 90$,then the value of $\frac{a_1+a_2+.....+a_{11}}{11}$ is equal to

63. Let a_1, a_2, a_3,.......,a_{100} be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100$.For any integer n with $1 \leq n \leq 20$, let m=5n .If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is......

64. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k-20=..........

65. Let a ,b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b+2, then the value of $\frac{a^2+a-14}{a+1}$ is

66. Suppose that all the terms of an arithmetic progression(A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6:11 and the seventh term is lies in between 130 and 140, then the common difference of this A.P. is

67. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3)......(1 + x^{100})$ is

68. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

69. Let X be the set consisting of the first 2018 terms of the arithmetic progression $1, 6, 11, ...,$ and Y be the set consisting of the first 2018 terms of the arithmetic progression $9, 16, 23, ...$ Then, the number of elements in the set $X \cup Y$ is ....

70. Let AP(a;d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If AP(1;3) AP(2;5) AP(3;7)= AP(a;d) then a+d equals.....

Section-B JEE Main/AIEEE

71. If 1, $log_9(3^{1-x} + 2), log_3(4.3^x - 1)$ are in A.P. then x equals
a) $log_3 4$
b) $1 - log_3 4$
c) $1 - log_4 3$
d) $log_4 3$

72. l, m, n are the p^{th}, q^{th} , and r^{th} term of a G.P. all positive, then $\begin{pmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log nn & r & 1 \end{pmatrix}$ equals
a) -1
b) 2
c) 1

d) 0

73. The value of $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.........\infty$ is

a) 1
b) 2
c) $\frac{3}{2}$
d) 4

74. Fifth term of a G.P. is 2, then the product of its 9 terms is

a) 256
b) 512
c) 1024
d) none of these

75. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is

a) 5
b) $\frac{3}{5}$
c) $\frac{8}{5}$
d) $\frac{1}{5}$

76. $1^3 - 2^3 + 3^3 - 4^3 + ... + 9^3$=

a) 425
b) -425
c) 475
d) -475

77. The sum of the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4}$....... upto ∞ is equal to

a) $log_e \frac{4}{e}$
b) $2log_e 2$
c) $log_e 2 - 1$
d) $log_e 2$

78. If $S_n = \sum_{r=0}^{n} \frac{1}{nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{nC_r}$, then $\frac{t_n}{S_n}$ is equal to

a) $\frac{2n-1}{2}$
b) $\frac{1}{2}$n-1
c) n-1
d) $\frac{1}{2}$n

79. Let T_r be the r^{th} term of an A.P.whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a-d equals

a) $\frac{1}{m} + \frac{1}{n}$
b) 1
c) $\frac{1}{mn}$
d) 0

80. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

a) $[\frac{n(n+1)}{2}]^2$
b) $\frac{n^2(n+1)}{2}$
c) $\frac{n(n+1)^2}{4}$
d) $\frac{3n(n+1)}{2}$

81. The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!}$..... is

a) $\frac{(e^2-2)}{e}$
b) $\frac{(e-1)^2}{2e}$
c) $\frac{(e^2-1)}{2e}$
d) $\frac{(e^2-1)}{2}$

82. If the coefficient of r^{th},$(r+1)^th$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation

a) $m^2 - m(4r-1) + 4r^2 - 2 = 0$
b) $m^2 - m(4r+1) + 4r^2 + 2 = 0$
c) $m^2 - m(4r+1) + 4r^2 - 2 = 0$
d) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

83. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1$ then x, y, z are in

a) G.P.
b) A.P.
c) Arithmetic - Geometric Progression
d) H.P.

84. The sum of series $1+\frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.6!}.....\infty$ is

a) $\frac{(e-1)}{\sqrt{e}}$
b) $\frac{(e+1)}{\sqrt{e}}$
c) $\frac{(e-1)}{2\sqrt{e}}$
d) $\frac{(e+1)}{2\sqrt{e}}$

85. Let a_1, a_2, a_3.....be terms in A.P. If $\frac{a_1+a_2+......+a_p}{a_1+a_2+......+a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

a) $\frac{41}{11}$
b) $\frac{7}{2}$.
c) $\frac{2}{7}$
d) $\frac{11}{41}$

86. If a_1, a_2, a_3.....a_n are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to

a) $n(a_1 - a_n)$
b) $(n-1)(a_1 - a_n)$
c) $n(a_1a_n)$
d) $(n-1)(a_1a_n)$

87. The sum of series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}$..... upto ∞ is

a) $e^{-\frac{1}{2}}$
b) $e^{+\frac{1}{2}}$
c) e^{-2}
d) e^{-1}

88. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals
   a) $\sqrt{5}$
   b) $\frac{1}{2}(\sqrt{5}-1)$
   c) $\frac{1}{2}(1-\sqrt{5})$
   d) $\frac{1}{2}\sqrt{5}$
89. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
   a) -4
   b) -12
   c) 12
   d) 4
90. The sum to infinite term of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\ldots$ is
   a) 3
   b) 4
   c) 6
   d) 2
91. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots\ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots\ldots$ are in A.P. with common difference -2, then the time taken by him to count all notes is
   a) 34 minutes
   b) 125 minutes
   c) 135 minutes
   d) 24 minutes
92. A man saves 200 in each of the first three months of his service. In each of the subsequent months his saving increases by 40 more than the saving of immediately previous month. His total savings from the start of service will be 11040 after
   a) 19 months
   b) 20 months
   c) 21 months
   d) 18 months
93. **Statement - 1:** The sum of the series 1+(1+2+4)+(4+6+9)+(9+12+16)+.....+(361+380+400) is 8000.
   Statement - 2: $\sum_{k=1}^{n}(k^3-(k-1)^3) = n^3$, for any natural number n.
   a) Statement - 1 is false, Statement - 2 is true.
   b) Statement - 1 is true, Statement - 2 is true, Statement - 2 is a correct explanation for Statement - 1
   c) Statement - 1 is true, Statement - 2 is true, Statement - 2 is a not a correct explanation for Statement - 1
   d) Statement - 1 is true, Statement - 2 is false.
94. The sum of the first 20 terms of sequence 0.7,0.77,0.777,....,is
   a) $\frac{7}{81}(179-10^{-20})$
   b) $\frac{7}{9}(99-10^{-20})$
   c) $\frac{7}{81}(179+10^{-20})$
   d) $\frac{7}{9}(99+10^{-20})$
95. If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + \ldots\ldots\ldots + 10(11)^9 = k(10)^9$, then k is equal to :
   a) 100
   b) 110
   c) $\frac{122}{10}$
   d) $\frac{441}{100}$
96. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ration of the G.P. is:
   a) $2-\sqrt{3}$
   b) $2+\sqrt{3}$
   c) $\sqrt{2}+\sqrt{3}$
   d) $3+\sqrt{2}$
97. The sum of the first 9 terms of the series.$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots$
   a) 142
   b) 192
   c) 71
   d) 96
98. If m is the A.M. of two distinct real numbers l and $n (l, n > 1)$ and G_1, G_2 and G_3 are the three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals:
   a) $4lmn^2$
   b) $4l^2m^2n^2$
   c) $4l^2mn$
   d) $4lm^2n$
99. If the $2^{nd}, 5^{th}$ and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:
   a) 1
   b) $\frac{7}{4}$

c) $\frac{8}{5}$
d) $\frac{4}{3}$

100. If the sum of the first ten terms of the series $(1\frac{3}{5})^2 + (2\frac{2}{5})^2 + (3\frac{1}{5})^2 + 4^2 + (4\frac{4}{5})^2 + ...$, is $\frac{16}{5}$m then m is equal to :
a) 100
b) 99
c) 102
d) 101

101. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + ... + (x+n-1)(x+n) = 10n$ has two consecutive integral solutions, then n is equal to :
a) 11
b) 12
c) 9
d) 10

102. For any three positive real numbers a, b and c,
$$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c) \tag{5.0.102.1}$$
Then:
a) a, b and c are in G.P.
b) b, c and a are in G.P.
c) b, c and a are in A.P.
d) a, b and c are in A.P.

103. Let a, b, c $\in$ R. If $f(x) = ax^2+bx+c$ is such that $a+b+c = 3$ $f(x+y) = f(x)+f(y)+xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to :
a) 255
b) 330
c) 165
d) 190

104. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to
a) 68
b) 34
c) 33
d) 66

105. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ If $B - 2A = 100\Lambda$, then Λ is equal to :
a) 248
b) 464
c) 496
d) 232

106. If a, b, and c be three distinct real numbers in G.P. and a+b+c=xb, then x cannot be :
a) -2
b) -3
c) 4
d) 2

107. Let $a_1, a_2, a_3, ..., a_{30}$ be in A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{2i-1}$. If $a_5 = 27$ and S-2T=75, Then a_{10} is equal to :
a) 52
b) 57
c) 47
d) 42

108. Three circles of radii a, b, c $(a < b < c)$ touch each other externally. If they have X-axis as a common tangent, then :
a) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$
b) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$
c) a, b, c are in A.P.
d) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ are in A.P.

109. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, ...$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P.,then the ordered pair (d, a_{10}) is equal to:
a) (50, 50+46A)
b) (50, 50+45A)
c) (A, 50+45A)
d) (A, 50+46A)

6 JEE Exercises:Mathematical Induction and Binomial Theorem

1. The large of $99^{50} + 100^{50}$ and 101^{50} is........
2. The sum of the coefficients of the polynomial$(1 + x - 3x^2)^{2163}$ is....
3. If
$$(1 + ax)^n = 1 + 8x + 24x^2 + ... \tag{6.0.3.1}$$
then a=....and n=.....
4. let n be positive integer. If the coefficients of 2nd,3rd and 4th terms in the expansion of $(1+x)^n$ are in A.P.,then value of n is.....
5. the sum of the rational terms in the expansion of $(\sqrt{2} + 3^{\frac{1}{5}})^{10}$ is ....
6. Given Positive integers r> 1,n> 1 and that the coefficient of (3r)th and (r+2)th terms in the binomial expression of $(1+x)^{2n}$ are equal.Then

a) n=2r
b) n=3r
c) n=2r+1
d) none of these

7. The Coefficient of x^4 in$(\frac{x}{2} - \frac{3}{x^2})^{10}$ is
a) $\frac{405}{256}$
b) $\frac{504}{259}$
c) $\frac{450}{263}$
d) none of these

8. The expression $(x+(x^3-1)^{\frac{1}{2}})^5+(x-(x^3-1)^{\frac{1}{2}})^5$ is a polynomial of degree
a) 5
b) 6
c) 7
d) 8

9. If in the expression of $(1+x)^m(1-x)^n$,the coefficient of x and x^2 are 3 and -6 respectively,then m is
a) 6
b) 9
c) 12
d) 24

10. For $2 \leq r \leq n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$
a) $\binom{n+1}{r-1}$
b) $2\binom{n+1}{r+1}$
c) $2\binom{n+2}{r}$
d) $\binom{n+2}{r}$

11. In the binomial expression of $(a-b)^n$,n≥5,the sum of the 5th and 6th terms is Zero.Then $\frac{a}{b}$ equals
a) $\frac{n-5}{6}$
b) $\frac{n-4}{5}$
c) $\frac{5}{n-4}$
d) $\frac{6}{n-5}$

12. The sum
$$\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i},$$
(where $\binom{p}{q}$=0 if p¡q) is maximum when m is
a) 5
b) 10
c) 15
d) 20

13. Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is
a) ${}^{12}C_6 + 3$
b) ${}^{12}C_6 + 1$
c) ${}^{12}C_6$
d) ${}^{12}C_6 + 2$

14. If ${}^{n-1}C_r = (k^2-3)^nC_{r+1}$,Then k∈
a) $(-\infty, -2]$
b) $(2, -\infty, -2]$
c) $[-\sqrt{3}, \sqrt{3}]$
d) $(\sqrt{3}, -2]$

15. The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} .... + \binom{30}{20}\binom{30}{30}$ is where $\binom{n}{r} =^n C_r$
a) $\binom{30}{10}$
b) $\binom{30}{15}$
c) $\binom{60}{30}$
d) $\binom{31}{10}$

16. For r=0,1,....10,let $A_r, B_r and C_r$ denote respectively,the coefficient of x^r in the expansions of $(1+x)^{10}, (1+x)^{20} and (1+x)^{30}$.Then
$$\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$
is equal to
a) $(B_{10} - C_{10})$
b) $A_{10}(B_{10}^2 C_{10} A_{10})$
c) 0
d) $(C_{10} - B_{10})$

17. Coefficient of x^{11} in the expansion of $(1+x^2)^4$ $(1+x^3)^7$ $(1+x^4)^{12}$ is
a) 1051
b) 1106
c) 1113
d) 1120

18. If C_r stands for ${}^nC_r$,then the sum of the series $\frac{2(\frac{n}{2})!(\frac{n}{2})!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - ..... + (-1)^n(n+1)C_n^2]$,where n is an even positive integer,is equal to
a) 0
b) $(-1)^{\frac{n}{2}}(n+1)$
c) $(-1)^{\frac{n}{2}}(n+2)$

d) $(-1)^n n$
e) none of these

19. if $a_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$,then $\sum_{r=0}^{n} \frac{r}{^nC_r}$ equals
a) $(n-1)a_n$
b) na_n
c) $\frac{1}{2}na_n$
d) none of these

20. Given that

$$C_1 + 2C_2x + 3C_3x^2 + ... + 2nC_{2n}x^{2n-1} = 2n(1+x)^{2n-1} \quad (6.0.20.1)$$

where $C_r = \frac{(2n!)}{r!(2n-r)!}$ r=0,1,2,....,2n Prove that $C_1^2 - 2C_2^2 + 3C_3^2 - ...... - 2nC_2^2n = (-1)^n nC_n$.

21. Prove that $7^{2n} + (2^{3n-3})(3^{n-1})$ is divisible by 25 for any natural numbers n

22. If

$$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n \quad (6.0.22.1)$$

then show that the sum of the products of the C_i's taken at a time,represented by $\sum_{0\le i<j\le n}\sum C_iC_j$ is equal to $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$

23. Use mathematical induction to prove : If n is any odd positive integer,then $n(n^2-1)$ is divisible by 24.

24. If p be a natural number then prove that $p^{n+1} + (p+1)^{2n-1}$ is divisible by p^2+p+1 for every position integer n.

25. Given $s_n = 1 + q + q^2 + ... + q^n$; $s_n = 1 + \frac{q+1}{2} + (\frac{q+1}{2})^2 + ... + (\frac{q+1}{2})^n$, $q \neq 1$ prove that $^{n+1}C_1 +^{n+1} C_2 +^{n+1} C_3 + ... +^{n+1} C_n s_n = 2^n S_n$

26. Use method of mathematical induction that $2.7^n + 3.5^n - 5$ is divisible by 24 for all n>0

27. Prove by mathematical induction that $\frac{(2n)!}{2^{2n}(n!)^2} \le \frac{1}{(3n+1)^{\frac{1}{2}}}$ for all positive integers n.

28. Let R=$(5\sqrt{5}+11)^{2n+1}$ and f=R-[R],where [] denotes the greater integer function.Prove that $Rf = 4^{2n+4}$

29. Using mathematical induction prove that $^mC_0^nC_k +^m C_1^nC_{k-1} + ... +^m C_k^nC_0 =^{m+n} C_k$,where m,n,k are positive integers,and $^pC_q = 0$ for p<q.

30. Prove that $C_0 - 2^2C_1 + 3^2C_2 - ...... + (-1)^n(n+1)^2C_n = 0$,n>2,where $C_r =^n C_r$.

31. Prove that $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ is a integer for every positive integer n.

32. using induction or otherwise ,Prove that for any non-negative integers m,n,r and k, $\sum_{m=0}^{k}(n-m)\frac{(r+m)!}{m!} = \frac{(r+k+1)!}{k!}[\frac{n}{r+1} - \frac{k}{r+2}]$

33. If

$$\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r \quad (6.0.33.1)$$

and $a_k = 1$ for all k ≥ n,then show that $b_n =^{2n+1} C_{n+1}$

34. Let p≥ 3 be an Integer and α,β be the roots of

$$x^2 - (p+1)x + 1 = 0 \quad (6.0.34.1)$$

Using mathematical induction show that $\alpha^n+\beta^n$ (i) is an integer and (ii) is not divisible by p

35. Using mathematical induction Prove that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + ... \tan^{-1} 1/(n^2+n+1) = \tan^{-1} n/(n+2)$

36. Prove that $\sum_{r=1}^{k}(-3)^{r-1}\ ^{3n}C_{2r-1} = 0$, where $k = \frac{3n}{2}$ and n is an even positive integer.

37. If x is not an integral multiple of 2π use mathematical induction to prove that: $\cos x + \cos 2x + ... + \cos nx = \cos\frac{n+1}{2}x \sin\frac{nx}{2}\ \mathrm{cosec}\frac{x}{2}$

38. let n be positive integer and

$$(1+x+x^2) = a_0 + a_1x + ... + a_{2n}x^{2n} \quad (6.0.38.1)$$

show that $a_0^2 - a_2^1 + a_2^2..... + a_{2n}^2 = a_n$

39. Using mathematical induction prove that for every integer n≥ 1,$(3^{2n}-1)$ is divisible by 2^{n+2} but not by 2^{n+3}

40. Let $0 < A_i < \pi$ for i=1,2,....,n.Use mathematical induction to prove that $\sin A_1 + \sin A_2... + \sin A_n \le n\sin(\frac{A_1+A_2+...+A_n}{n})$ where ≥ 1 is natural number.
(You may use the fact that $p\sin x + (1-p)\sin y \le \sin[px + (1-p)y]$,where 0≤p≤1 and 0≤x,y≤ π)

41. Let p be prime and m a positive integer.By mathematical induction on m or otherwise prove that whenever r is an integer such that p does not divide r,p divides $^{mp}C_r$, [Hint: you may use the fact that $(1+x)^{(m+1)p} = (1+x)^p(1+x)^{mp}$]

42. Let n be any positive integer.prove that $\sum_{k=0}^{m}\frac{\binom{2n-k}{k}}{\binom{2n-k}{n}}\cdot\frac{(2n-4k+1)}{(2n-2k+1)}2^{n-2k} = \frac{\binom{n}{m}}{\binom{2n-2m}{n-m}}2^{n-2m}$ for each non-be gatuve integer m ≤ n.
(Here $\binom{p}{q} =^p C_q$)

43. For any Positive integer m, n(with n $\geq$ m),let $\binom{n}{m} = ^n C_m$. Prove that $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \dots + \binom{m}{m} = \binom{n+1}{m+2}$ Hence or otherwise, prove that $\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \dots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}$
44. for every positive integer n,Prove that $\sqrt{(4n+1)} < \sqrt{n} + \sqrt{n+1} < \sqrt{4n+2}$. Hence or otherwise, prove that $[\sqrt{n} + \sqrt{(n+1)}] = [\sqrt{4n+1}]$ where [x] denotes the greater integers not exceeding x.
45. Let a,b,c be the positive real numbers such that $b^2 - 4ac > 0$ and let α_1=c prove by induction that$\alpha n + 1 = \frac{a\alpha_n^2}{(b^2 - 2a(\alpha_1 + \alpha_2 + \dots + \alpha_n))}$is well defined and $\alpha n + 1 < \frac{\alpha_n}{2}$ for all n = 1,2,.....(Here, 'well-defined' means that the denominator in the expresion for α_{n+1} is not zero.)
46. Use the mathematical induction to show that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all n=1,2,......
47. prove that $2^k \binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{2}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n-2}{k-2} - \dots(-1)^k\binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$
48. A coin has probability p of showing head when tossed.It is tossed n times.Let p_n denote the probability that no two (or more) consecutive heads occur Prove that $p_1 = 1, p_2 = 1 - p^2$ and $p_n = (1-p).p_(n-1) + p(1-p)p_{n-2}$ for all n $\geq$ 3.
   Prove by induction on n, that $p_n = A\alpha^n + B\beta^n$ for all n $\geq$ 1,
   where α and β are the roots of quadratic equation
   $$x^2 - (1-p)x - p(1-p) = 0 \qquad (6.0.48.1)$$
   and $A = \frac{p^2+\beta-1}{\alpha\beta-\alpha^2}, B = \frac{p^2+\alpha-1}{\alpha\beta-\beta^2}$.
49. The coefficient of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14.Then n=
50. Let m be the smallest positive integer such that the Coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + \dots + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1)^{51}C_3$ for some positive integers n.Then value of n is
51. Let X=$(^{10}C_1)^2 + 2(^{10}C_2)^2 + 3(^{10}C_3)^2 + \dots + (^{10}C_{10})^2$ where $^{10}C_r$,r$\in$ 1,2,3,.....,10 denote binomial coefficient then ,the value of $\frac{1}{1430}$X is.......
52. suppose det$\begin{pmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} nC_k k^2 \\ \sum_{k=0}^{n} nC_k k & \sum_{k=0}^{n} nC_k 3^k \end{pmatrix} = 0$ holds for some positive integer n.The $\sum_{k=0}^{n} \frac{^nC_k}{k+1}$ equals
53. the coefficient of x^p and x^q in the expansion of $(1+x)^{p+q}$ are
   a) equal
   b) equal with opposite signs
   c) reciprocals of each other
   d) none of these
54. If sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is
   a) 1594
   b) 792
   c) 924
   d) 2924
55. the positive integer just greater than $(1+0.0001)^{10000}$is
   a) 4
   b) 5
   c) 2
   d) 3
56. r and n are the positive integers r>1,n>2 and coefficient of $(r+2)^{th}$ term and $3r^{th}$ tern in the expansion of $(1+x)^{2n}$ are equal,then n equals
   a) 3r
   b) 3r+1
   c) 2r
   d) 2r+1
57. If $\sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}$ having n radical sign then by methods of mathematical induction which is true
   a) $a_n > 7 \forall n \geq 1$
   b) $a_n < 7 \forall n \geq 1$
   c) $a_n < 4 \forall n \geq 1$
   d) $a_n < 3 \forall n \geq 1$
58. If x is positive,the first negative term in the expansion of $(1+x)^{\frac{27}{5}}$ is
   a) 6th term
   b) 7th term
   c) 5th term
   d) 8th term
59. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is

a) 35
b) 32
c) 33
d) 34

60. Let S(K)=1+3+5+....+(2K-1)=3+K^2 Then which of the following is true
a) principle of mathematical induction can be used to prove the formula
b) S(K)→S(K+1)
c) S(K)→S(K+1)
d) S(1) is correct

61. The coefficient of middle term in the binomial expansion in powers of x of$(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals to
a) $\frac{3}{5}$
b) $\frac{10}{3}$
c) $\frac{-3}{10}$
d) $\frac{-5}{3}$

62. The coefficient of x^n in the expansion of (1+x) $(1-x)^n$ is
a) $(-1)^{n-1}n$
b) $(-1)^n(1-n)$
c) $(-1)^{n-1}(n-1)^2$
d) n-1

63. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is
a) ${}^{55}C_4$
b) ${}^{55}C_3$
c) ${}^{56}C_3$
d) ${}^{56}C_4$

64. If A=$\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$ and I=$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, then which one of the following holds for all n ≥ 1, by the principle of mathematical induction
a) $A^n = nA-(n-1)I$
b) $A^n = 2^{n-1}A-(n-1)I$
c) $A^n = nA+(n-1)I$
d) $A^n = 2^{n-1}A+(n-1)I$

65. If the coefficient of x^7 in $[ax^2+\frac{1}{bx}]^{11}$ equals the coefficient of x^{-7} in $[ax-\frac{1}{bx^2}]^{11}$, then a and b satisfy the relation
a) a-b=1
b) a+b=1
c) $\frac{a}{b}=1$
d) ab=1

66. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}}-(1+\frac{1}{2}x)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as
a) $1-\frac{3}{8}x^2$
b) $3x+\frac{3}{8}x^2$
c) $-\frac{3}{8}x^2$
d) $\frac{x}{2}-\frac{3}{8}x^2$

67. If expansion in power of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+.....$ then a_n is
a) $\frac{b^n-a^n}{b-a}$
b) $\frac{a^n-b^n}{b-a}$
c) $\frac{a^{n+1}-b^{n+1}}{b-a}$
d) $\frac{b^{n+1}-a^{n+1}}{b-a}$

68. For natural numbers m,n if
$$(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+....... \tag{6.0.68.1}$$
and $a_1=a_2=10$, then $\begin{pmatrix}m\\n\end{pmatrix}$ is
a) $\begin{pmatrix}20\\45\end{pmatrix}$
b) $\begin{pmatrix}35\\20\end{pmatrix}$
c) $\begin{pmatrix}45\\35\end{pmatrix}$
d) $\begin{pmatrix}35\\45\end{pmatrix}$

69. In the binomial expansion of $(a-b)^n$, $n\geq 5$, the sum of 5th and 6th terms is zero then a/b equals
a) $\frac{n-5}{6}$
b) $\frac{n-4}{5}$
c) $\frac{5}{n-4}$
d) $\frac{6}{n-5}$

70. The sum of the series ${}^{20}C_0 -{}^{20}C_1 +{}^{20}C_2 -{}^{20}C_3 + ... - ..... +{}^{20}C_{10}$ is
a) 0
b) ${}^{20}C_{10}$
c) $-{}^{20}C_{10}$
d) $\frac{1}{2}{}^{20}C_{10}$

71. **statement-1**:
$\sum_{r=0}^{n}(r+1)^nC_r = (n+2)2^{n-1}$
statement-2:
$\sum_{r=0}^{n}(r+1)^nC_rx^r = (1+x)^n + nx(1+x)^{n-1}$
a) statement-1 is true,statement-2 is true
b) statement-1 is true,statement-2 is true;statement-2 is correct explanation for statement-1
c) statement-1 is true,statement-2 is true;statement-2 is not correct explanation

for statement-1

d) statement-1 is true,statement-2 is false

72. The reminder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is:

a) 2
b) 7
c) 8
d) 0

73. $S_1 = \sum_{j=1}^{10} j(j-1)^{10}C_J$,
$S_2 = \sum_{j=1}^{10} {}^{10}C_j$ and
$S_3 = \sum_{j=1}^{10} j^2 \; {}^{10}C_j$
statement-1:$S_3 = 55 \times 2^9$
statement-2:$S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

a) statement-1 is true,statement-2 is true;statement-2 is not correct explanation for statement-1
b) statement-1 is true,statement-2 is false
c) statement-1 is false,statement-2 is true
d) statement-1 is true,statement-2 is true;statement-2 is correct explanation for statement-1

74. The Coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

a) -132
b) -144
c) 132
d) 144

75. If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is:

a) an irrational number
b) an odd positive integer
c) an even positive integer
d) a rational number other than positive integer

76. The term independent of x in the expansion $(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}})^{10}$ is

a) 4
b) 120
c) 210
d) 310

77. If the coefficient of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x both zero,then $\begin{pmatrix} a \\ b \end{pmatrix}$ is equal to:

a) $\begin{pmatrix} 14 \\ \frac{272}{3} \end{pmatrix}$
b) $\begin{pmatrix} 16 \\ \frac{272}{3} \end{pmatrix}$
c) $\begin{pmatrix} 16 \\ \frac{251}{3} \end{pmatrix}$
d) $\begin{pmatrix} 14 \\ \frac{251}{3} \end{pmatrix}$

78. The sum of the coefficients of integral power of x in the binomial expansion $(1 - 2\sqrt{x})^{50}$ is:

a) $\frac{1}{2}(3^{50} - 1)$
b) $\frac{1}{2}(2^{50} + 1)$
c) $\frac{1}{2}(3^{50} + 1)$
d) $\frac{1}{2}(3^{50})$

79. The number of terms in the expansion of $(1 - \frac{2}{x} + \frac{4}{x^2})^n$, $x \neq 0$, is 28, Then the sum of the all the terms in the expansion is:

a) 243
b) 729
c) 64
d) 2187

80. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is

a) $2^{20} - 2^{10}$
b) $2^{21} - 2^{11}$
c) $2^{21} - 2^{10}$
d) $2^{20} - 2^9$

81. The sum of the all coefficients of all odd degree terms in the expansion of $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$, (x>1) is:

a) 0
b) 1
c) 2
d) -1

82. If the forth term in the Binomial expansion of $(\frac{2}{x} + x^{log 8x})^6$ (x>0) is 20×8^7, then a value of x is:

a) 8^3
b) 8^2
c) 8
d) 8^{-2}

7 JEE Exercises:Permutation and Combination

1. In a certain test, a_i students gave wrong answers to atleast i questions, where $i = 1, 2, ..., k$. No student gave more than k wrong answers. The total number of wrong answers given is.....

2. The side AB, BC and CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is.....

3. Total number of ways in which six $'+'$ and four $'-'$ signs can be arranged in a line such that no two $'-'$ signs occur together is......

4. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is.....

5. The product of any r consecutive natural numbers is always divisible by r!

6. ${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$ and ${}^{n}C_{r+1} = 126$, then r is :
   a) 1
   b) 2
   c) 3
   d) None of these.

7. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated are
   a) 69760
   b) 30240
   c) 99748
   d) none of these

8. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to
   a) ${}^{47}C_5$
   b) ${}^{52}C_5$
   c) ${}^{52}C_4$
   d) none of these

9. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4; and then the men select the chairs from amongst the remaining. The number of possible arrangments is
   a) ${}^{6}C_3 \times {}^{4}C_2$
   b) $nPr42 \times nPr43$
   c) ${}^{4}C_2 + nPr43$
   d) none of these

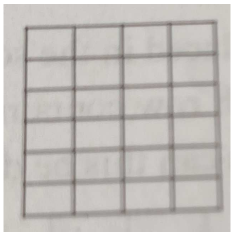
10. A five-digit numbers divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is
    a) 216
    b) 240
    c) 600
    d) 3125

11. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?
    a) 16
    b) 36
    c) 60
    d) 180

12. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals
    a) 5
    b) 7
    c) 6
    d) 4

13. The number of arrangements of the letters of the word BANANA in which the two N's do not apppear adjacently is
    a) 40
    b) 60
    c) 80
    d) 100

14. A rectangle with sides of length (2m-1) and (2n-1) units is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



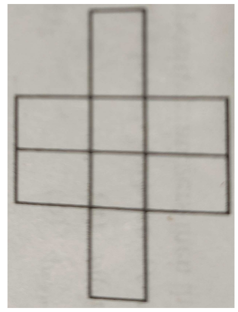
a) $(m + n - 1)^2$
b) 4^{m+n-1}
c) m^2n^2
d) $m(m + 1)n(n + 1)$

15. If the LCM of p, q is $r^2t^4s^2$, where r, s, t are prime numbers and p,q are the positive integers then the number of ordered pair (p, q) is
a) 252
b) 254
c) 225
d) 224

16. The letters of the word COCHIN are computed and all the computations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is
a) 360
b) 192
c) 96
d) 48

17. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only, is
a) 55
b) 66
c) 77
d) 88

18. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is
a) 75
b) 150
c) 210
d) 243



19. Six cards and six envelopes are numbered 1,2,3,4,5,6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is
a) 264
b) 265
c) 53
d) 67

20. A Debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is
a) 380
b) 320
c) 260
d) 95

21. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only three digits 2,5 and 7. The smallest value of n for which this is possible is
a) 6
b) 7
c) 8
d) 9

22. Six X's have to be placed in the sqaures of figure given below in such a way that each row contains atleast one X. In how many different ways can this be done

23. Five balls of different colours are to be placed in the boxes of different size. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty?

24. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is $\frac{m!(m+1)!}{(m-n+1)!}$

25. 7 relatives of a man comprises 4 ladies and 3 gentlemen; his wife also has 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways they can invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives?

26. A box contains two white balls, three black balls, four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw?

27. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the others side. Determine the number of ways in which the sitting arragements can be made.

28. A committee of 12 is to be formed from 9 women and 8 men.In how many ways this can be done if at least five women have to be included in a committee? In how many of these committees
    a) The women are in majority?
    b) The men are in majority?

29. Prove by computation or otherwise $\frac{(n^2)!}{(n!)^n}$ is an integer ($n \epsilon I^+$).

30. If total number of runs scored in n matches is $(\frac{n+1}{4})$ $(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the kth match are given by K. 2^{n+1-k}, where $1 \leq k \leq n$. Find n.

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

31. The value of b_6 is
    a) 7
    b) 8
    c) 9
    d) 11

32. which of the following is correct?
    a) $a_{17} = a_{16} + a_{15}$
    b) $c_{17} \neq c_{16} + c_{15}$
    c) $b_{17} \neq b_{16} + c_{16}$
    d) $a_{17} = c_{17} + b_{16}$

33. Consider the set of eight vectors V={a$\hat{i}$+b$\hat{j}$+c$\hat{k}$:a,b,c$\epsilon\{-1, 1\}$ }. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is

34. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

35. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment by joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

36. Let n be number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

37. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}$ =

38. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is

39. Let $|X|$ denote the number of elements in a set X. Let S= $\{1, 2, 3, 4, 5, 6\}$ be a sample space, where each element is likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \leq |B| < |A|$, equals.

40. Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is...

41. Total no of four digit odd numbers that can be formed using 0,1,2,3,5,7 (using the repetition allowed) are
    a) 216
    b) 375
    c) 400
    d) 720

42. Number greater than 1000 but less than 4000 is formed using the digits 0,1,2,3,4 (repetition allowed). Their number is
    a) 125
    b) 105
    c) 375
    d) 625

43. Five digit number divisible by 3 is formed using 0,1,2,3,4 and 5 without repetition. Total number of such numbers are
    a) 312
    b) 3125
    c) 120
    d) 216

44. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
    a) 3000
    b) 3050
    c) 3600
    d) 3250

45. If ${}^{n}C_{r}$ denotes the number of combination of n things taken r at a time, then the expression ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2 \times {}^{n}C_{r}$ equals
    a) ${}^{n+1}C_{r+1}$
    b) ${}^{n+2}C_{r}$
    c) ${}^{n+2}C_{r+1}$
    d) ${}^{n+1}C_{r}$

46. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
    a) 346
    b) 140
    c) 196
    d) 280

47. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by
    a) $7! \times 5!$
    b) $6! \times 5!$
    c) $30!$
    d) $5! \times 4!$

48. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order
    a) 480
    b) 240
    c) 360
    d) 120

49. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is
    a) ${}^{8}C_{3}$
    b) 21
    c) 3^{8}
    d) 5

50. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number
    a) 601

b) 600
c) 603
d) 602

51. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is
a) 5040
b) 6210
c) 385
d) 1110

52. The set S = $\{1, 2, 3.....12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is
a) $\frac{12!}{(4!)^3}$
b) $\frac{12!}{(4!)^4}$
c) $\frac{12!}{3!(4!)^3}$
d) $\frac{12!}{3!(4!)^4}$

53. In a Shop there are five types of ice-creams available. A child buys six ice-creams. Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$. Statement-2 : The number of different ways the child can but the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.
a) Statement-1 is False, Statement-2 is True
b) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
c) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
d) Statement-1 is True, Statement-2 is False

54. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
a) $8.{}^6C_4.{}^7C_4$
b) $6.7.{}^8C_4$
c) $6.8.{}^7C_4$
d) $7.{}^6C_4.{}^8C_4$

55. From 6 different novels and 3 different ditionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is :
a) at least 500 but less than 750
b) at least 750 but less than 1000
c) at least 1000
d) at least 500

56. There are two urns. Urn A has 3 distinct red balls and Urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is
a) 36
b) 66
c) 108
d) 3

57. Statement-1 : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$. Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.
a) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
b) Statement-1 is True, Statement-2 is False
c) Statement-1 is False, Statement-2 is True
d) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

58. These are 10 points in a plane, out of these 6 are collinear, if N is the number of triangles formed by joining these points. then:
a) $n \leq 100$
b) $100 < n \leq 140$
c) $140 < n \leq 190$
d) $n > 190$

59. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green, and 7 black balls is :
a) 880

b) 629
c) 630
d) 879

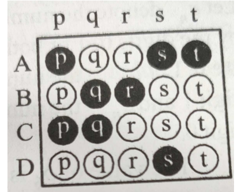
60. Let T_n be the number of all possible traingles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is:
a) 7
b) 5
c) 10
d) 8

61. The number of integers greater than 6000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :
a) 120
b) 72
c) 216
d) 192

62. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :
a) 52^{nd}
b) 58^{th}
c) 46^{th}
d) 59^{th}

63. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :
a) 484
b) 485
c) 468
d) 469

64. From 6 different novels and 3 different dictionaries; 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :



a) less than 500
b) at least 500 but less than 750
c) at least 750 but less than 1000
d) at least 1000

65. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:
a) 500
b) 200
c) 300
d) 350

66. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then:
a) m + n = 68
b) m = n = 78
c) n = m - 8
d) m = n = 68

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C, D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statements in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example : If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s, then the correct darkening of bubbles will look like

67. Consider all possible computations of the letters of the word ENDEANOEL. Match the Statements/Expressions in Column-I with

the Statements/Expressions in column-II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column-I	Column-II
(A) The number of computations containing the word ENDEA is	(p)5!
(B) The number of computations in which the letter E occurs in the first and the last positions is	(q)$2 \times 5!$
(C) The number of computations in which none of the letters D, L, N occurs in the last five positions is	(r)$7 \times 5!$
(D) The number of computations in which the letters A, E, O occur only in odd positions is	(s)$21 \times 5!$

68. In a high school, a committe has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.
   a) Let α_1 be the total number of ways in which the committe can be formed such that the committe has 5 members, having exactly 3 boys and 2 girls.
   b) Let α_2 be the total number of ways in which the committe can be formed such that the committe has at least 2 members and having an equal number of boys and girls.
   c) Let α_3 be the total number of ways in which the committe can be formed such that the committe has 5 members at least 2 of them being girls.
   d) Let α_4 be the total number of ways in which the committe can be formed such that the committe has 4 members, having at least 2 girls and such that both M_1 and G_1 are NOT in the committe together.

LIST-I	LIST-II
P. The value of α_1 is	1. 136
Q. The value of α_2 is	2. 189
R. The value of α_3 is	3. 192
S. The value of α_4 is	4. 200
	5. 381
	6. 461

The correct option is:

a) P→4; Q→6; R→2; S→1
b) P→1; Q→4; R→2; S→3
c) P→4; Q→6; R→5; S→2
d) P→4; Q→2; R→3; S→1