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CONTENTS

0.1 Trigonometry 1

Abstract—This book provides a computational approach to school mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/ncert/computation/codes>

0.1 Trigonometry

1. Suppose

$$\sin^3 x \sin 3x = \sum_{m=0}^n C_m \cos mx \quad (0.1.1.1)$$

is an identity in x , where C_0, C_1, \dots, C_n are constants, and $C_n \neq 0$ then find the value of n .

2. Find the solution set of the system of equations

$$x + y = \frac{2\pi}{3} \quad (0.1.2.1)$$

$$\cos x + \cos y = \frac{3}{2}, \quad (0.1.2.2)$$

where x and y are real.

3. Find the set of all x in the interval $[0, \pi]$ for which

$$2 \sin^2 x - 3 \sin x + 1 \geq 0 \quad (0.1.3.1)$$

4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. Find the minimum value of the arithmetic mean of $\cos(\alpha + \frac{\pi}{2}), \cos(\beta + \frac{\pi}{2})$ and $\cos(\gamma + \frac{\pi}{2})$.

5. Find the value of

$$\sin \frac{\pi}{14} \sin \frac{3\pi}{14} \sin \frac{5\pi}{14} \sin \frac{7\pi}{14} \sin \frac{9\pi}{14} \sin \frac{11\pi}{14} \sin \frac{13\pi}{14}.$$

6. If

$$K = \sin\left(\frac{\pi}{18}\right) \sin\left(\frac{5\pi}{18}\right) \sin\left(\frac{7\pi}{18}\right), \quad (0.1.6.1)$$

then find the numerical value of K ?

7. If $A > 0, B > 0$ and

$$A + B = \frac{\pi}{3}, \quad (0.1.7.1)$$

then find the maximum value of $\tan A \tan B$.

8. Find the general value of θ satisfying the equation

$$\tan^2 \theta + \sec 2\theta = 1. \quad (0.1.8.1)$$

9. Find the real roots of the equation

$$\cos^7 x + \sin^4 x = 1 \quad (0.1.9.1)$$

in the interval $(-\pi, \pi)$.

10. If $\tan \theta = -\frac{4}{3}$, then find $\sin \theta$.

11. If $\alpha + \beta + \gamma = 2\pi$ then

$$a) \tan \frac{\alpha}{2} + \tan \frac{\beta}{2} + \tan \frac{\gamma}{2} = \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}$$

$$b) \tan \frac{\alpha}{2} \tan \frac{\beta}{2} + \tan \frac{\beta}{2} \tan \frac{\gamma}{2} + \tan \frac{\gamma}{2} \tan \frac{\alpha}{2} = 1$$

$$c) \tan \frac{\alpha}{2} + \tan \frac{\beta}{2} + \tan \frac{\gamma}{2} = -\tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}$$

d) None of these

12. Given

$$A = \sin^2 \theta + \cos^4 \theta \quad (0.1.12.1)$$

then for all real values of θ

$$a) 1 \leq A \leq 2$$

$$b) \frac{3}{4} \leq A \leq 1$$

$$c) \frac{13}{16} \leq A \leq 1$$

$$d) \frac{3}{4} \leq A \leq \frac{13}{16}$$

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13. The equation

$$2 \cos^2 \frac{x}{2} \sin^2 x = x^2 + x^{-2}; 0 < x < \frac{\pi}{2} \quad (0.1.13.1)$$

has

- a) no real solution
- b) One real solution
- c) more than the one solution
- d) none of these

14. The general solution of the trigonometric equation

$$\sin x + \cos x = 1 \quad (0.1.14.1)$$

is given by :

- a) $x = 2n\pi; n = 0, \pm 1, \pm 2 \dots$
- b) $x = 2n\pi + \frac{\pi}{2}; n = 0, \pm 1, \pm 2 \dots$
- c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}; n = 0, \pm 1, \pm 2 \dots$
- d) none of these

15. The value of expression $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$ is equal to

- a) 2
- b) $\frac{2 \sin 20^\circ}{\sin 40^\circ}$
- c) 4
- d) $\frac{4 \sin 20^\circ}{\sin 40^\circ}$

16. The general solution of

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x \quad (0.1.16.1)$$

is

- a) $n\pi + \frac{\pi}{8}$
- b) $\frac{n\pi}{2} + \frac{\pi}{8}$
- c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$
- d) $2n\pi + \cos^{-1} \frac{3}{2}$

17. The equation

$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0 \quad (0.1.17.1)$$

In the variable x , has real roots. Then p can take any value in the interval

- a) $(0, 2\pi)$
- b) $(-\pi, 0)$
- c) $(-\frac{\pi}{2}, \frac{\pi}{2})$
- d) $(0, \pi)$

18. Number of solutions of the equation

$$\tan x + \sec x = 2 \cos x \quad (0.1.18.1)$$

lying in the interval $[0, 2\pi]$ is :

- a) 0
- b) 1
- c) 2
- d) 3

19. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals

- a) $\tan(x - \frac{\pi}{4})$
- b) $\tan(\frac{\pi}{4} - x)$
- c) $\tan(x + \frac{\pi}{4})$
- d) $\tan^2(x + \frac{\pi}{4})$

20. Let n be a positive integer such that $\sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then

- a) $6 \leq n \leq 8$
- b) $4 < n \leq 8$
- c) $4 \leq n \leq 8$
- d) $4 < n < 8$

21. If ω is an imaginary cube root of unity then the value of $\sin \{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\}$ is

- a) $-\frac{\sqrt{3}}{2}$
- b) $-\frac{1}{\sqrt{2}}$
- c) $\frac{1}{\sqrt{2}}$
- d) $\frac{\sqrt{3}}{2}$

22. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$

- a) 11
- b) 12
- c) 13
- d) 14

23. The general values of θ satisfying equation

$$2 \sin^2 \theta - 3 \sin \theta - 2 = 0 \quad (0.1.23.1)$$

is

- a) $n\pi + (-1)^n \frac{\pi}{6}$
- b) $n\pi + (-1)^n \frac{\pi}{2}$
- c) $n\pi + (-1)^n \frac{5\pi}{6}$
- d) $n\pi + (-1)^n \frac{7\pi}{6}$

24. $\sec^2 \theta = \frac{4xy}{(x+y)^2}$ is true if and only if

- a) $x + y \neq 0$
- b) $x = y, x \neq 0$
- c) $x = y$
- d) $x \neq 0, y \neq 0$

25. In a triangle PQR, $\angle R = \pi/2$. If $\tan(\frac{P}{2})$ and $\tan(\frac{Q}{2})$ are the roots of the equation

$$ax^2 + bx + c = 0 (a \neq 0) \quad (0.1.25.1)$$

then

- a) $a+b=c$

- b) $b+c=a$
 c) $a+c=b$
 d) $b=c$

26. Let $f(\theta) = \sin \theta (\sin \theta + \sin 3\theta)$. Then $f(\theta)$ is

- a) ≥ 0 only when $\theta \geq 0$
 b) ≤ 0 for all real θ
 c) ≥ 0 for all real θ
 d) ≤ 0 only when $\theta \leq 0$

27. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is

- a) 0
 b) 2
 c) 1
 d) 3

28. The maximum value of

$(\cos \alpha_1)(\cos \alpha_2) \dots (\cos \alpha_n)$, under the restrictions, $0 \leq \alpha_1, \alpha_2, \dots, \alpha_n \leq \frac{\pi}{2}$ and $(\cot \alpha_1)(\cot \alpha_2) \dots (\cot \alpha_n) = 1$ is

- a) $\frac{1}{2^{\frac{n}{2}}}$
 b) $\frac{1}{2^n}$
 c) $\frac{1}{2^n}$
 d) 1

29. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan \alpha$ equals

- a) $2(\tan \beta + \tan \gamma)$
 b) $\tan \beta + \tan \gamma$
 c) $\tan \beta + 2 \tan \gamma$
 d) $2 \tan \beta + \tan \gamma$

30. The number of integral values of k for which the equation

$$7 \cos x + 5 \sin x = 2k + 1 \quad (0.1.30.1)$$

has a solution is

- a) 4
 b) 8
 c) 10
 d) 12

31. Given both θ and ϕ are acute angles and $\sin \theta = \frac{1}{2}$, $\cos \phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to

- a) $(\frac{\pi}{3}, \frac{\pi}{2}]$
 b) $(\frac{\pi}{2}, \frac{2\pi}{3})$
 c) $(\frac{2\pi}{3}, \frac{5\pi}{6}]$
 d) $(\frac{5\pi}{6}, \pi]$

32. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the

equations is/are

- a) 0
 b) 1
 c) 2
 d) 4

33. The values of $\theta \in (0, 2\pi)$ for which $2 \sin^2 \theta - 5 \sin \theta + 2 > 0$, are

- a) $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$
 b) $(\frac{\pi}{8}, \frac{5\pi}{6})$
 c) $(0, \frac{\pi}{8}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$
 d) $(\frac{41\pi}{48}, \pi)$

34. Let $\theta \in (0, \frac{\pi}{4})$ and $t_1 = (\tan \theta)^{\tan \theta}$, $t_2 = (\tan \theta)^{\cot \theta}$, $t_3 = (\cot \theta)^{\tan \theta}$ and $t_4 = (\cot \theta)^{\cot \theta}$, then

- a) $t_1 > t_2 > t_3 > t_4$
 b) $t_4 > t_3 > t_1 > t_2$
 c) $t_3 > t_1 > t_2 > t_4$
 d) $t_2 > t_3 > t_1 > t_4$

35. The number of solutions of the pair of equations

$$2 \sin^2 \theta - \cos 2\theta = 0 \quad (0.1.35.1)$$

$$2 \cos^2 \theta - 3 \sin \theta = 0 \quad (0.1.35.2)$$

in the interval $[0, 2\pi]$ is

- a) zero
 b) one
 c) two
 d) four

36. For $x \in (0, \pi)$, the equation

$$\sin x + 2 \sin 2x - \sin 3x = 3 \quad (0.1.36.1)$$

has

- a) infinitely many solutions
 b) three solutions
 c) one solution
 d) no solution

37. Let $S = \{x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2}\}$. The sum of all distinct solutions of the equation

$$\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0 \quad (0.1.37.1)$$

in the set S is equal to

- a) $-\frac{7\pi}{9}$
 b) $-\frac{2\pi}{9}$
 c) 0
 d) $\frac{5\pi}{9}$

38. The value of

$\sum_{k=1}^{13} \frac{1}{\sin(\frac{\pi}{4} + \frac{(k-1)\pi}{6}) \sin(\frac{\pi}{4} + \frac{k\pi}{6})}$ is equal to

- a) $3 - \sqrt{3}$
- b) $2(3 - \sqrt{3})$
- c) $2(\sqrt{3} - 1)$
- d) $2(2 - \sqrt{3})$

39. $(1 + \cos \frac{\pi}{8})(1 + \cos \frac{3\pi}{8})(1 + \cos \frac{5\pi}{8})(1 + \cos \frac{7\pi}{8})$ is equal to

- a) $\frac{1}{2}$
- b) $\cos(\frac{\pi}{8})$
- c) $\frac{1}{8}$
- d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

40. The expression $3[\sin^4(\frac{3\pi}{2} - \alpha) + \sin^4(3\pi + \alpha)] - 2[\sin^6(\frac{\pi}{2} + \alpha) + \sin^6(5\pi - \alpha)]$ is equal to

- a) 0
- b) 1
- c) 3
- d) $\sin 4\alpha + \cos 6\alpha$
- e) none of these

41. The number of all possible triplets (a_1, a_2, a_3) such that

$$a_1 + a_2 \cos(2x) + a_3 \sin^2(x) = 0 \quad (0.1.41.1)$$

for all x is

- a) zero
- b) one
- c) three
- d) infinite
- e) none

42. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0 \quad (0.1.42.1)$$

are

- a) $\frac{7\pi}{24}$
- b) $\frac{5\pi}{24}$
- c) $\frac{11\pi}{24}$
- d) $\frac{\pi}{24}$

43. Let

$$2 \sin^2 x + 3 \sin x - 2 > 0 \quad (0.1.43.1)$$

$$x^2 - x - 2 < 0 \quad (0.1.43.2)$$

(x is measured in radians). Then x lies in the interval

- a) $(\frac{\pi}{6}, \frac{5\pi}{6})$

- b) $(-1, \frac{5\pi}{6})$
- c) $(-1, 2)$
- d) $(\frac{\pi}{6}, 2)$

44. The minimum value of the expression $\sin \alpha + \sin \beta + \sin \gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$ is

- a) Positive
- b) zero
- c) negative
- d) -3

45. The number of values of x in the interval $[0, \pi]$ satisfying the equation

$$3 \sin^2 x - 7 \sin x + 2 = 0 \quad (0.1.45.1)$$

is

- a) 0
- b) 5
- c) 6
- d) 10

46. Which of the following number(s) is/are/rational?

- a) $\sin 15^\circ$
- b) $\cos 15^\circ$
- c) $\sin 15^\circ \cos 15^\circ$
- d) $\sin 15^\circ \cos 75^\circ$

47. For a positive integer n, let $f_n(\theta) = \tan(\frac{\theta}{2})(1 + \sec \theta)(1 + \sec 2\theta)(1 + \sec 4\theta) \dots (1 + \sec 2^{n-1}\theta)$. Then

- a) $f_2(\frac{\pi}{16}) = 1$
- b) $f_3(\frac{\pi}{32}) = 1$
- c) $f_4(\frac{\pi}{64}) = 1$
- d) $f_5(\frac{\pi}{128}) = 1$

48. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then

- a) $\tan^2 x = \frac{2}{3}$
- b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
- c) $\tan^2 x = \frac{1}{3}$
- d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

49. For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^6 \operatorname{cosec}(\theta + \frac{(m-1)\pi}{4}) \operatorname{cosec}(\theta + \frac{m\pi}{4}) = 4\sqrt{2}$ is(are)

- a) $\frac{\pi}{4}$
- b) $\frac{\pi}{6}$
- c) $\frac{\pi}{12}$
- d) $\frac{5\pi}{12}$

50. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2 \cos \theta (1 - \sin \varphi) = \sin^2 \theta (\tan \frac{\theta}{2} + \cot \frac{\theta}{2}) \cos \varphi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin \theta < -\frac{\sqrt{3}}{2}$, then φ can not satisfy

- a) $0 < \varphi < \frac{\pi}{2}$
 b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
 c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$
 d) $\frac{3\pi}{2} < \varphi < 2\pi$

51. The number of points in $(-\infty, \infty)$, for which

$$x^2 - x \sin x - \cos x = 0 \quad (0.1.51.1)$$

is

- a) 6
 b) 4
 c) 2
 d) 0

52. Let

$$f(x) = x \sin \pi x, x > 0 \quad (0.1.52.1)$$

Then for all natural numbers n , $f'(x)$ vanishes at

- a) A unique point in the interval $(n, n+\frac{1}{2})$
 b) A unique point in the interval $(n+\frac{1}{2}, n+1)$
 c) A unique point in the interval $(n, n+1)$
 d) Two points in the interval $(n, n+1)$

53. Let α and β be non-zero real numbers such that $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true?

- a) $\tan(\frac{\alpha}{2}) + \sqrt{3} \tan(\frac{\beta}{2}) = 0$
 b) $\sqrt{3} \tan(\frac{\alpha}{2}) + \tan(\frac{\beta}{2}) = 0$
 c) $\tan(\frac{\alpha}{2}) - \sqrt{3} \tan(\frac{\beta}{2}) = 0$
 d) $\sqrt{3} \tan(\frac{\alpha}{2}) - \tan(\frac{\beta}{2}) = 0$

54. If $\tan \alpha = \frac{m}{m+1}$ and $\tan \beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$.

55. (a) Draw the graph of

$$y = \frac{1}{\sqrt{2}}(\sin x + \cos x) \quad (0.1.55.1)$$

from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$

(b) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$

56. Given $\alpha + \beta - \gamma = \pi$, prove that

$$\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma = 2 \sin \alpha \sin \beta \cos \gamma \quad (0.1.56.1)$$

57. Given $A = \{x: \frac{\pi}{6} \leq x \leq \frac{\pi}{3}\}$ and

$$f(x) = \cos x - x(1+x); \quad (0.1.57.1)$$

find $f(A)$

58. For all θ in $[0, \pi/2]$ show that,

$$\cos(\sin \theta) \geq \sin(\cos \theta) \quad (0.1.58.1)$$

59. Without using tables, Prove that $(\sin 12^\circ)(\sin 48^\circ)(\sin 54^\circ) = \frac{1}{8}$

60. Show that $16 \cos(\frac{2\pi}{15}) \cos(\frac{4\pi}{15}) \cos(\frac{8\pi}{15}) \cos(\frac{16\pi}{15}) = 1$

61. Find all the solution of $4 \cos^2 x \sin x - 2 \sin^2 x = 3 \sin x$

62. Find the values of $x \in (-\pi, \pi)$ which satisfy the equation

$$8(1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + \dots) = 4^3 \quad (0.1.62.1)$$

63. Prove that $\tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha = \cot \alpha$

64. ABC is a triangle such that $\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}$. If A, B and C are in arithmetic progression, determine the values of A, B and C.

65. if $\exp\{(\sin^2 x + \sin^4 x + \sin^6 x + \dots \infty) \ln 2\}$ satisfies the equation

$$x^2 - 9x + 8 = 0 \quad (0.1.65.1)$$

, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$.

66. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3.

67. Determine the smallest positive value of x (in degrees) for which $\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan(x) \tan(x - 50^\circ)$.

68. Find the smallest positive number p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution $x \in [0, 2\pi]$

69. Find all values of θ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ satisfying the equation

$$(1 - \tan \theta)(1 + \tan \theta) \sec^2 \theta + 2^{\tan^2 \theta} = 0 \quad (0.1.69.1)$$

70. Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x .

71. Prove that $\sum_{k=1}^{n-1} (n-k) \cos \frac{2k\pi}{n} = -\frac{n}{2}$, where $n \geq 3$ is an integer

72. If any triangle ABC, Prove that $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$

73. Find the range of values of t for which $2 \sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

This section contains 1 paragraph, Based on each paragraph, there are 2 questions. Each question has four options (A), (B), (C) and (D) ONLY ONE of these four options is correct.

PARAGRAPH 1

Let O be the origin, and **OX, OY, OZ** be three unit vectors in the directions of the sides **QR, RP, PQ** respectively, of a triangle PQR

74. $|\mathbf{OX} \times \mathbf{OY}| =$

- a) $\sin(P + Q)$
- b) $\sin 2R$
- c) $\sin(P + R)$
- d) $\sin(Q + R)$

75. If the triangle PQR varies, then the minimum value of $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$ is

- a) $-\frac{5}{3}$
- b) $-\frac{3}{2}$
- c) $\frac{3}{2}$
- d) $\frac{5}{3}$

76. The number of all possible values of θ where $0 < \theta < \pi$, for which the system of equations

$$(y + z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$$

$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$
have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

77. The number of values of θ in the interval, $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan \theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

78. The maximum value of the expression $\frac{1}{\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta}$ is

79. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is

Note: $[k]$ denotes the largest integer less than or equal to k .

80. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin(\frac{\pi}{n})} = \frac{1}{\sin(\frac{2\pi}{n})} + \frac{1}{\sin(\frac{3\pi}{n})} \text{ is}$$

81. The number of distinct solutions of the equation $\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

82. Let a, b, c be three non-zero real numbers such that the equation : $\sqrt{3}a \cos x + 2b \sin x = c, x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is

83. The period of $\sin^2 \theta$ is

- a) π^2

- b) π
- c) 2π
- d) $\pi/2$

84. The number of solution of $\tan x + \sec x = 2 \cos x$ in $[0, 2\pi)$ is

- a) 2
- b) 3
- c) 0
- d) 1

85. Which one is not periodic

- a) $|\sin 3x| + \sin^2 x$
- b) $\cos \sqrt{x} + \cos^2 x$
- c) $\cos 4x + \tan^2 x$
- d) $\cos 2x + \sin x$

86. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the value of $\cos \frac{\alpha - \beta}{2}$

- a) $-\frac{6}{65}$
- b) $\frac{3}{\sqrt{130}}$
- c) $\frac{6}{65}$
- d) $-\frac{3}{\sqrt{130}}$

87. If $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$ then the difference between the maximum and minimum values of u^2 is given by

- a) $(a - b)^2$
- b) $2\sqrt{a^2 + b^2}$
- c) $(a + b)^2$
- d) $2(a^2 + b^2)$

88. A line makes the same angle θ , with each of the x and z axis. If the angle β , which it makes with y -axis, is such that $\sin^2 \beta = 3 \sin^2 \theta$, then $\cos^2 \theta$ equals

- a) $\frac{2}{5}$
- b) $\frac{1}{5}$
- c) $\frac{3}{5}$
- d) $\frac{4}{5}$

89. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

$$2 \sin^2 x + 5 \sin x - 3 = 0 \quad (0.1.89.1)$$

is

- a) 4
- b) 6
- c) 1
- d) 2

90. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is

- a) $\frac{(1-\sqrt{7})}{4}$
 b) $\frac{(4-\sqrt{7})}{3}$
 c) $-\frac{(4+\sqrt{7})}{3}$
 d) $\frac{(1+\sqrt{7})}{4}$

91. Let A and B denote the statements

A : $\cos \alpha + \cos \beta + \cos \gamma = 0$

B : $\sin \alpha + \sin \beta + \sin \gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :

- a) A is false and B is true
 b) Both A and B are true
 c) both A and B are false
 d) A is true and B is false

92. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$, Then $\tan 2\alpha =$

- a) $\frac{56}{33}$
 b) $\frac{19}{12}$
 c) $\frac{20}{7}$
 d) $\frac{25}{16}$

93. If $A = \sin^2 x + \cos^4 x$, then for all real x:

- a) $\frac{13}{16} \leq A \leq 1$
 b) $1 \leq A \leq 2$
 c) $\frac{3}{4} \leq A \leq \frac{13}{16}$
 d) $\frac{1}{4} \leq A \leq 1$

94. In a $\triangle PQR$, If $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to :

- a) $\frac{5\pi}{6}$
 b) $\frac{\pi}{6}$
 c) $\frac{\pi}{4}$
 d) $\frac{3\pi}{4}$

95. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to :

- a) $\frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$
 b) $\frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$
 c) $\frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$
 d) $\frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$

96. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as:

- a) $\sin A \cos A + 1$
 b) $\sec A \operatorname{cosec} A + 1$
 c) $\tan A + \cot A$
 d) $\sec A + \operatorname{cosec} A$

97. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in \mathbb{R}$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

- a) $\frac{1}{4}$
 b) $\frac{1}{12}$
 c) $\frac{1}{6}$
 d) $\frac{1}{3}$

98. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:

- a) 7
 b) 9
 c) 3
 d) 5

99. If $5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$, then the value of $\cos 4x$ is :

- a) $-\frac{7}{9}$
 b) $-\frac{3}{5}$
 c) $\frac{1}{3}$
 d) $\frac{2}{9}$

100. If sum of all the solutions of the equation $8 \cos x (\cos(\frac{\pi}{6} + x)(\cos(\frac{\pi}{6} - x) - \frac{1}{2}) - 1)$ in $[0, \pi]$ is $k\pi$. then k is equal to :

- a) $\frac{13}{9}$
 b) $\frac{8}{9}$
 c) $\frac{20}{9}$
 d) $\frac{2}{3}$

101. For any $\theta \in (\frac{\pi}{4}, \frac{\pi}{2})$ the expression $3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4 \sin^2 \theta$ equals:

- a) $13 - 4 \cos^2 \theta + 6 \sin^2 \theta \cos^2 \theta$
 b) $13 - 4 \cos^6 \theta$
 c) $13 - 4 \cos^2 \theta + 6 \cos^4 \theta$
 d) $13 - 4 \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta$

102. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is:

- a) $\frac{3}{4} + \cos 20^\circ$
 b) $\frac{1}{4}$
 c) $\frac{1}{2}(1 + \cos 20^\circ)$
 d) $\frac{1}{2}$

103. Let $S = \{\theta \in [-2\pi, 2\pi] : 2 \cos^2 \theta + 3 \sin \theta = 0\}$. Then the sum of the elements of S is

- a) $\frac{13\pi}{6}$
 b) $\frac{5\pi}{3}$
 c) 2
 d) 1

Match the Following

DIRECTIONS (Q.1): Each question contains

statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A-p, s and t; B-q and r; C-p and q; D -s then the correct darkening of bubbles will look like the given

- a) In this question there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.
- $\frac{\sin 3\alpha}{\cos 2\alpha}$ is

Column-I	Column-II
(A) Positive	(p) $(\frac{13\pi}{48}, \frac{14\pi}{48})$
(B) Negative	(q) $(\frac{14\pi}{48}, \frac{18\pi}{48})$
	(r) $(\frac{18\pi}{48}, \frac{23\pi}{48})$
	(s) $(0, \frac{\pi}{2})$

- b) Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

List-I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

Column-I	Column-II
(A) X	(p) $\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$
(B) Y	(q) an arithmetic progression
(C) Z	(r) NOT an arithmetic progression
(D) W	(s) $\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$
	(t) $\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$
	(u) $\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

- i) (IV), (P), (R), (S)
 ii) (III), (P), (Q), (U)
 iii) (III), (R), (U)
 iv) (IV), (Q), (T)
- c) Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

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	(u) $\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

- i) (I), (Q), (U)
 ii) (I), (P), (R)
 iii) (II), (R), (S)
 iv) (II), (Q), (T)