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**Abstract**—This book provides a collection of the international maths olympiad problems in algebra.

1. For what real values of  $x$  is

$$\sqrt{(x + \sqrt{2x - 1})} + \sqrt{(x - \sqrt{2x - 1})} = A \quad (1.1)$$

given

- a)  $A = \sqrt{2}$ ,
- b)  $A = 1$ ,
- c)  $A = 2$

where only non-negative real numbers are admitted for square roots?

2. Let  $a, b, c$  be real numbers. Consider the quadratic equation in  $\cos x$  :

$$a \cos^2 x + b \cos x + c = 0. \quad (2.1)$$

Using the numbers  $a, b, c$ , form a quadratic equation in  $\cos 2x$ , whose roots are the same as those of the original equation. Compare the equations in  $\cos x$  and  $\cos 2x$  for  $a = 4, b = 2, c = -1$ .

3. Find all real roots of the equation

$$\sqrt{x^2 - p} + 2\sqrt{x^2 - 1} = x \quad (3.1)$$

where  $p$  is a real parameter.

4. Find all solutions  $x_1, x_2, x_3, x_4, x_5$  of the system

$$x_5 + x_2 = yx_1 \quad (4.1)$$

$$x_1 + x_3 = yx_2 \quad (4.2)$$

$$x_2 + x_4 = yx_3 \quad (4.3)$$

$$x_3 + x_5 = yx_4 \quad (4.4)$$

$$x_4 + x_1 = yx_5 \quad (4.5)$$

where  $y$  is a parameter.

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