Geometric Constructions through Python

G V V Sharma*

Contents

Abstract—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.
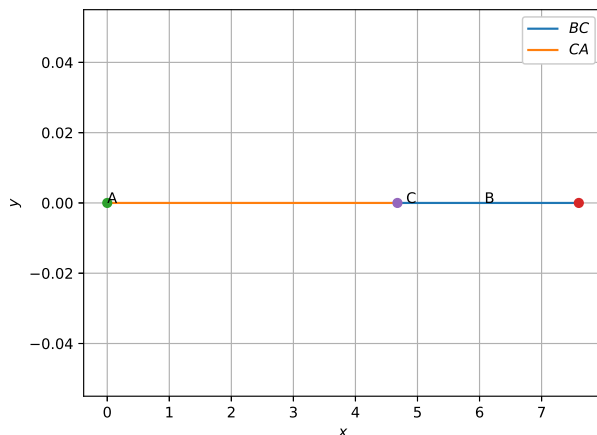
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Fig. 1.1

1 Triangle

1.1 Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8.
Solution: Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \qquad (1)$$

Then the point **C**

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k + 1} \qquad (2)$$

divides AB in the ration k : 1. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1

codes/draw_section.py

1.2 Draw $\triangle ABC$ where $\angle B = 90°, a = 4$ and $b = 3$.
Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \qquad (3)$$
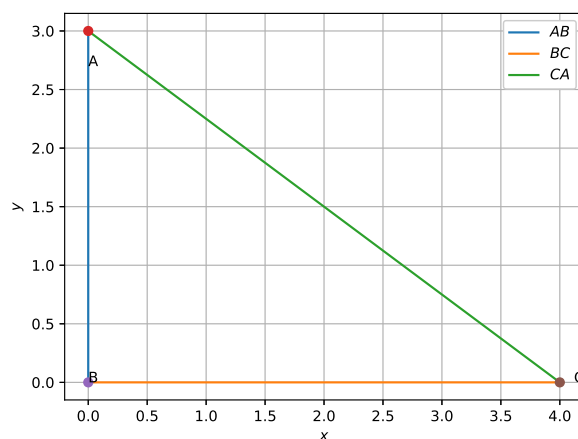
The following code plots Fig. 1.2

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All solutions in this manual is released under GNU GPL. Free and open source.

codes/rt_triangle.py

Fig. 1.2

1.3 Construct a triangle of sides $a = 4$, $b = 5$ and $c = 6$.

Solution: Let the vertices of $\triangle ABC$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \qquad (4)$$

Then

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \qquad (5)$$
$$\|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \qquad (6)$$
$$\|\mathbf{A} - \mathbf{C}\|^2 = b^2 \qquad (7)$$

From (7), yielding

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \qquad (8)$$
$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \qquad (9)$$
$$= a^2 + c^2 - 2ap \qquad (10)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \qquad (11)$$

From (6),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \qquad (12)$$
$$\implies q = \sqrt{c^2 - p^2} \qquad (13)$$
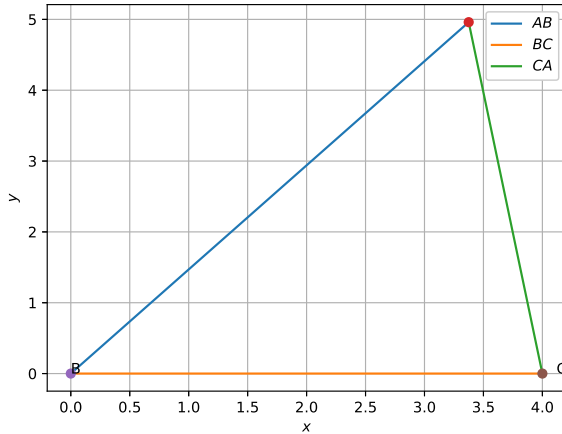
The following code plots Fig. 1.3

codes/draw_triangle.py

Fig. 1.3

1.4 Construct a triangle of sides $a = 5$, $b = 6$ and $c = 7$. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.
Solution: The sides of the similar triangle are $\frac{7}{5}a, \frac{7}{5}b$ and $\frac{7}{5}c$.

1.5 Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = p = 4$cm
Solution: Using Baudhayana's theorem,

$$b = c = \sqrt{p^2 + \left(\frac{a}{2}\right)^2} \qquad (14)$$

1.6 Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60°$.
Solution: In Fig. (1.6), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \qquad (15)$$
$$\cos B = \frac{x}{b}, \qquad (16)$$

Thus,

$$a = x + y = b \cos C + c \cos B, \qquad (17)$$
$$b = c \cos A + a \cos C \qquad (18)$$
$$c = b \cos A + a \cos B \qquad (19)$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (20)$$

Using the properties of determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \qquad (21)$$

$$= \frac{b^2 + c^2 - a^2}{2bc} \qquad (22)$$

From (22)

$$b^2 = c^2 + a^2 - 2ca \cos B \qquad (23)$$

which is computed by the following code

codes/cos_form.py

1.7 Draw $\triangle ABC$ with $a = 7, \angle B = 45°$ and $\angle A = 105°$.
Solution: In Fig. (1.6),

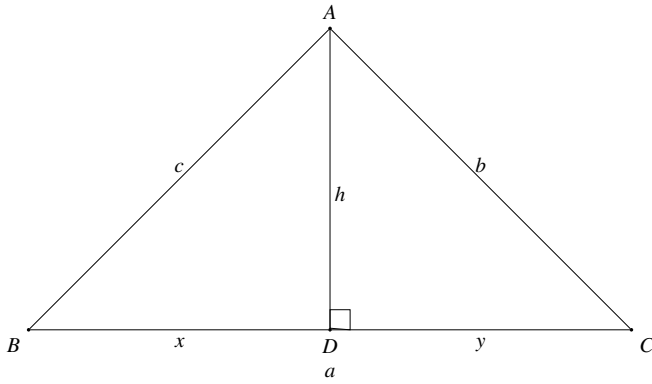$$\sin B = \frac{h}{c} \qquad (24)$$
$$\sin C = \frac{h}{b} \qquad (25)$$

Fig. 1.6: The cosine formula

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \tag{26}$$

Thus,

$$c = \frac{a \sin C}{\sin A} \tag{27}$$

where

$$C = 180 - A - B \tag{28}$$

1.8 $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.
Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \tag{29}$$
$$\implies (18 - c)^2 = 12^2 + c^2 \tag{30}$$

which can be simplified to obtain

$$c^2 + 36c^2 - 180 = 0 \tag{31}$$
$$\implies (c + 18)^2 - 18^2 - 180 = 0 \tag{32}$$
$$\tag{33}$$

which can be simplified as

$$\implies (c + 18)^2 = \left(18^2 + 180\right) \tag{34}$$
$$\implies c = -18 \pm \sqrt{18^2 + 180} \tag{35}$$

1.9 In $\triangle ABC$, $a = 7$, $\angle B = 75°$ and $b+c = 13$. Find b and c and sketch $\triangle ABC$.

1.10 In $\triangle ABC$, $a = 8$, $\angle B = 45°$ and $c - b = 3.5$. Sketch $\triangle ABC$.
Solution: The general solution of a quadratic equation

$$\alpha x^2 + \beta x + \gamma = 0 \tag{36}$$

is

$$x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \tag{37}$$

Using this and (23), b and c can be obtained.

1.11 In $\triangle ABC$, $a = 6$, $\angle B = 60°$ and $b-c = 2$. Sketch $\triangle ABC$.

1.12 In $\triangle ABC$, given that $a + b + c = 11$, $\angle B = 45°$ and $\angle C = 45°$, find a, b, c.
Solution: We have

$$a = b \cos C + c \cos B \tag{38}$$
$$b \sin C = c \sin B \tag{39}$$
$$a + b + c = 11 \tag{40}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \tag{41}$$

Solving the equivalent matrix equation gives the desired answer.

1.13 Draw $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 30°$ and $\angle C = 90°$.

1.14 Construct $\triangle xyz$ where $xy = 4.5, yz = 5$ and $zx = 6$.

1.15 Draw an equilateral triangle of side 5.5.

1.16 Draw $\triangle PQR$ with $PQ = 4, QR = 3.5$ and $PR = 4$. What type of triangle is this?

1.17 Construct $\triangle ABC$ such that $AB = 2.5, BC = 6$ and $AC = 6.5$. Find $\angle B$.

1.18 Construct $\triangle PQR$, given that $PQ = 3, QR = 5.5$ and $\angle PQR = 60°$.

1.19 Draw $\triangle ABC$ if $AB = 3, AC = 5$ and $\angle C = 30°$.

1.20 Construct $\triangle DEF$ such that $DE = 5, DF = 3$ and $\angle D = 90°$.

1.21 Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°.

1.22 Construct $\triangle ABC$ with $BC = 7.5, AC = 5$ and $\angle C = 60°$.

1.23 Construct $\triangle XYZ$ if $XY = 6, \angle X = 30°$ and $\angle Y = 100°$.

1.24 If $AC = 7, \angle A = 60°$ and $\angle B = 50°$, can you draw the triangle?

1.25 Construct $\triangle ABC$ given that $\angle A = 60°, \angle B = 30°$ and $AB = 5.8$.

1.26 Construct $\triangle PQR$ if $PQ = 5, \angle Q = 105°$ and $\angle R = 40°$.

1.27 Can you construct $\triangle DEF$ such that $EF =$

2.11 Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90°$. *BD* is the perpendicular from **B** on *AC*. The circle through **B, C, D** is drawn. Construct the tangents from **A** to this circle.

2.12 Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through **C**

3 QUADRILATERALS

3.1 Draw *ABCD* with $AB = a = 4.5, BC == b = 5.5, CD = c = 4, AD = d = 6$ and $AC = e = 7$.
Solution: Fig. 3.1 shows a rough sketch of *ABCD*. Letting

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (45)$$

it is trivial to sketch $\triangle ABC$ from Problem 1.3. $\triangle ACD$ is can be obtained by rotating an equivalent triangle with *AC* on the *x*-axis by an angle θ with

$$\mathbf{D} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} e \\ 0 \end{pmatrix} \quad (46)$$

and

$$\cos \theta = \frac{a^2 + e^2 - b^2}{2ae} \quad (47)$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (48)$$

The coordinates of the rotated triangle *ACD* are

$$\mathbf{D} = \mathbf{P} \begin{pmatrix} p \\ q \end{pmatrix} \quad (49)$$

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (50)$$

$$\mathbf{C} = \mathbf{P} \begin{pmatrix} e \\ 0 \end{pmatrix} \quad (51)$$

where

$$\mathbf{P} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (52)$$

3.2 Construct a quadrilateral *ABCD* such that $AB = 5, \angle A = 50°, AC = 4, BD = 5$ and $AD = 6$.

3.3 Construct *PQRS* where $PQ = 4, QR = 6, RS = 5, PS = 5.5$ and $PR = 7$.

3.4 Draw *JUMP* with $JU = 3.5, UM = 4, MP = 5, PJ = 4.5$ and $PU = 6.5$

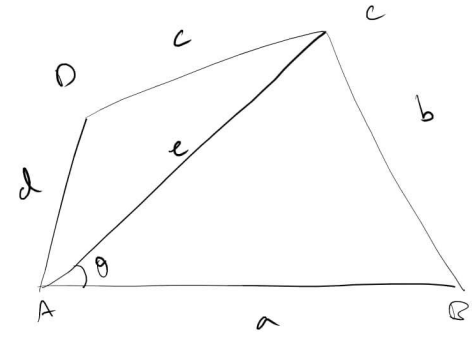Fig. 3.1

3.5 Draw the parallelogram *MORE* with $OR = 6, RE = 4.5$ and $EO = 7.5$.
Solution: Diagonals of a parallelogram bisect each other. Opposite sides of a parallelogram are equal and parallel .

3.6 Draw the rhombus *BEST* with $BE = 4.5$ and $ET = 6$.
Solution: Diagonals of a rhombus bisect each other at right angles.

3.7 Construct a quadrilateral *ABCD* such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7$ and $AD = 5.5$.

3.8 Can you construct a quadrilateral *PQRS* with $PQ = 3, RS = 3, PS = 7.5, PR = 8$ and $SQ = 4$?

3.9 Construct *LIFT* such that $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.

3.10 Draw *GOLD* such that $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$.

3.11 DRAW rhombus *BEND* such that $BN = 5.6, DE = 6.5$.

3.12 construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

3.13 Can you construct the above quadrilateral MIST if $\angle M = 100°$ instead of 75°.

3.14 Can you constrcut the quadrilateral PLAN if $PL = 6, LA = 9.5, \angle P = 75°, \angle L = 150°$ and $\angle A = 140°$?

3.15 Construct *MORE* where $MO = 6, OR = 4.5, \angle M = 60°, \angle O = 105°, \angle R = 105°$.

3.16 Construct *PLAN* where $PL = 4, LA = 6.5, \angle P = 90°, \angle A = 110°$ and $\angle N = 85°$.

3.17 Constrcut parallelogram *HEAR* where $HE = 5, EA = 6, \angle R = 85°$.

3.18 Draw rectangle *OKAY* with $OK = 7$ and $KA = $

    5.

3.19 Construct $ABCd$, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105°$ and $\angle C = 80°$.

3.20 Construct $DEAR$ with $DE = 4, EA = 5, AR = 4.5, \angle E = 60°$ and $\angle A = 90°$.

3.21 Construct $TRUE$ with $TR = 3.5, RU = 3, UE = 4 \angle R = 75°$ and $\angle U = 120°$.

3.22 Draw a square of side 4.5.

3.23 Can you construct a rhombus $ABCD$ with $AC = 6$ and $BD = 7$?

3.24 Construct a kite $EASY$ if $AY = 8, EY = 4$ and $SY = 6$.

    Solution: The diagonals of a kite are perpendicular to each other.

3.25 Draw a square $READ$ with $RE = 5.1$.

3.26 Draw a rhombus who diagonals are 5.2 and 6.4.

3.27 Draw a rectangle with adjacent sides 5 and 4.

3.28 Draw a parallelogram $OKAY$ with $OK = 5.5$ and $KA = 4.2$.