Vector Algebra and Three Dimensional Geometry: JEE Maths

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1. Let $\overrightarrow{A}$, $\overrightarrow{B}$, $\overrightarrow{C}$ be vectors of length 3, 4, 5 respectively. Let $\overrightarrow{A}$ be perpendicular to $\overrightarrow{B}+\overrightarrow{C}$, $\overrightarrow{B}$ to $\overrightarrow{C}+\overrightarrow{A}$ and $\overrightarrow{C}$ to $\overrightarrow{A}+\overrightarrow{B}$. Then the length of vector $\overrightarrow{A}+\overrightarrow{B}+\overrightarrow{C}$ is..............
2. The unit vector perpendicular to the plane determined by P(1, -1, 2), Q(2, 0, -1) and R(0, 2, 1) is.........
3. The area of the triangle whose vertices are A(1, -1, 2), B(2, 1, -1) and C(3, -1, 2) is........
4. A, B, C, and D are four points in a plane with position vectors a, b, c, and d respectively such that
$$(\overrightarrow{a}-\overrightarrow{d})(\overrightarrow{b}-\overrightarrow{c})=(\overrightarrow{b}-\overrightarrow{d})(\overrightarrow{c}-\overrightarrow{a})=0$$
The Point D, then is the.................of the triangle ABC.
5. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and the vectors $\overrightarrow{A}=(1,a,a^2)$, $\overrightarrow{B}=(1,b,b^2)$, $\overrightarrow{C}=(1,c,c^2)$ are non-olar, then the product abc = ...
6. If $\overrightarrow{A}\overrightarrow{B}\overrightarrow{C}$ are three non-polar vectors, then $\frac{\overrightarrow{A}.\overrightarrow{B}\times\overrightarrow{C}}{\overrightarrow{C}\times\overrightarrow{A}.\overrightarrow{B}}+\frac{\overrightarrow{B}.\overrightarrow{A}\times\overrightarrow{C}}{\overrightarrow{C}.\overrightarrow{A}\times\overrightarrow{B}}=$ ....
7. If $\overrightarrow{A}$ = (1, 1, 1), $\overrightarrow{C}$ = (0, 1, -1) are given vectors, then a vector B satisfying the equations $\overrightarrow{A}\times\overrightarrow{B}=\overrightarrow{C}$ and $\overrightarrow{A}.\overrightarrow{B}=3$.............
8. If the vectors a$\hat{i}+\hat{j}+\hat{k}$, $\hat{i}$ + b$\hat{j}+\hat{j}$, $\hat{i}+\hat{j}$ + c$\hat{k}$ $(a\neq b\neq c\neq 1)$ are co-planar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$ ....
9. Let b = $4\hat{i}+3\hat{j}$ and $\overrightarrow{c}$ be two vectors perpendicular to each other in the xy - plane. All vectors in the same plane having projections 1 and 2 along $\overrightarrow{b}$ and $\overrightarrow{c}$, respectively are given by..........
10. The components of a vector $\overrightarrow{a}$ along and perpendicular to a non-zero vector $\overrightarrow{b}$ are............. and ....respectively.
11. Given that $\overrightarrow{a}$ = (1, 1, 1), $\overrightarrow{c}$ = (0, 1, -1), $\overrightarrow{a}\overrightarrow{b}=3$ and $\overrightarrow{a}\times\overrightarrow{b}=\overrightarrow{c}$, then $\overrightarrow{b}$ = ...
12. A unit vector co-planr with $\overrightarrow{i}+\overrightarrow{j}+2\overrightarrow{k}$ and $\overrightarrow{i}+2\overrightarrow{j}+\overrightarrow{k}$ and perpendicular to $\overrightarrow{i}+\overrightarrow{j}+\overrightarrow{k}$ is............
13. A unit vector perpendicular to the plane determined by the points P(1, -1, 2), Q(2, 0, -1) and R(0, 2, 1) is..........
14. A non-zero vector $\overrightarrow{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and plane determined by the vectors $\hat{i}$ - $\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\overrightarrow{a}$ and the vector $\hat{i}$ - $2\hat{j}$ + $2\hat{k}$ is...........
15. If $\overrightarrow{b}$ and $\overrightarrow{c}$ are any two non-collinear unit vectors and $\overrightarrow{a}$ is any vector $(\overrightarrow{a}.\overrightarrow{b})\overrightarrow{b}+(\overrightarrow{a}.\overrightarrow{c})\overrightarrow{c}+\frac{\overrightarrow{a}.(\overrightarrow{b}\times\overrightarrow{c})}{\left|\overrightarrow{b}\times\overrightarrow{c}\right|}$ $(\overrightarrow{b}\times\overrightarrow{c})=$ ....
16. Let OA = a, OB = 10a + 2b and OC = b where O, A, and C are non-collinear points. Let p denote thea area of the quadrilateral OABC, and let q denote the area of the parallelgram with OA and OC as adjacent sides. If p = kq, then k = ...

(B). True/False

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17. Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ be unit vectors suppose that $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$, and the angle between $\vec{B}$ and $\vec{C}$ is $\frac{\pi}{6}$. Then $\vec{A} = \pm 2(\vec{B} \times \vec{C})$.
18. If X.A = 0, X.B = 0, X.C = 0 for some non-zero vector X, then [A B C] = 0.
19. The points with position vectors a + b, a - b, and a + kb are collinear for all real values of k.
20. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $(\vec{a}\,\vec{b}).(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a}) = 2\vec{a}.(\vec{b} \times \vec{c})$.

(C). MCQs with One Correct Answer

21. The scalar $\vec{A}.(\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})$ equals:
    a) 0
    b) $[\vec{A}\vec{B}\vec{C}] + [\vec{B}\vec{C}\vec{A}]$
    c) $[\vec{A}\vec{B}\vec{C}]$
    d) None of these
22. For non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $|(\vec{a} \times \vec{b}).\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds if and only if
    a) $\vec{a}.\vec{b} = 0$, $\vec{b}.\vec{c} = 0$
    b) $\vec{b}.\vec{c} = 0$, $\vec{c}.\vec{a} = 0$
    c) $\vec{c}.\vec{a} = 0$, $\vec{a}.\vec{b} = 0$
    d) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$
23. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA}$ = 2i - 2j, $\overrightarrow{OB}$ = i + j - k, $\overrightarrow{OC}$ = 3i - k, is
    a) $\frac{4}{13}$
    b) 4
    c) $\frac{2}{7}$
    d) None of these
24. The points with position vectors 60i + 3j, 40i - 8j, ai - 52j are collinear if
    a) a = -40
    b) a = 40
    c) a = 20
    d) None of these
25. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors and $\vec{p}$, $\vec{q}$, $\vec{r}$, are vectors defined by the relations

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$$

    then the value of the expression $(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$ is eqaul to
    a) 0
    b) 1
    c) 2
    d) 3
26. Let a, b, c be distinct non-negative numbers. If the vectors a$\hat{i}$ + a$\hat{j}$ + c$\hat{k}$, $\hat{i}$ + $\hat{k}$ and c$\hat{i}$ + c$\hat{j}$ + b$\hat{k}$ lie in a plane, then c is
    a) the arithmetic mean of a and b
    b) the geomemetic mean of a and b
    c) the harmonic mean of a and b
    d) equal to zero
27. Let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectively, with respect to O $|\vec{p}| = p$, $|\vec{q}| = q$. The points R and S divide PQ internally and externally in the ratio 2:3 respectively. If OR and OS are perpendicular then

a) $9q^2 = 4q^2$
b) $4p^2 = 9q^2$
c) 9p = 4q
d) 4p = 9q

28. Let α, β, γ be distinct real numbers. The points with postion vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$
a) are collinear
b) form an equilateral triangle
c) form an scalene triangle
d) form a right angled triangle

29. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$, $\vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a} \cdot \vec{d} = 0 = [\vec{b}\vec{c}\vec{d}]$, then $\vec{d}$ equals
a) $\pm \frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$
b) $\pm \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$
c) $\pm \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{3}}$
d) $\pm\hat{k}$

30. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{(\vec{b}+\vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is
a) $\frac{3\pi}{4}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) π

31. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$, $|\vec{w}| = 5$, then $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is
a) 47
b) -25
c) 0
d) 25

32. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non polar vectors, then $\vec{a} + \vec{b} + \vec{c}$. $[(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})]$ equals
a) 0
b) $1[\vec{a}\ \vec{b}\ \vec{c}]$
c) $2[\vec{a}\ \vec{b}\ \vec{c}]$
d) $-[\vec{a}\ \vec{b}\ \vec{c}]$

33. Let a = 2i + j - 2k and b = i + j. If c is a vector such that a.c = $|c|$, $|c - a| = 2\sqrt{2}$ and the angle between $(a \times b)$ and c is 30°, then $|(a \times b) \times c| =$
a) $\frac{2}{3}$
b) $\frac{3}{2}$
c) 2
d) 3

34. a = 2i + j + k, b = i + 2j - k and a unit vector c be coplanar. If c is perpendicular to a, then c =
a) $\frac{1}{\sqrt{2}}$(-j + k)
b) $\frac{1}{\sqrt{3}}$(-i - j - k)
c) $\frac{1}{\sqrt{5}}$(i - 2j)
d) $\frac{1}{\sqrt{3}}$(i - j - k)

35. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form the sides BC, CA, AB respectively of a triangle ABC, then
   a) $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$
   b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
   c) $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$
   d) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$
36. Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be such that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0.$$

   Let P_1 and P_2 be planes determined by the pairs of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{d}$ respectively. Then the angle between P_1 and P_2 is
   a) 0
   b) $\frac{\pi}{4}$
   c) $\frac{\pi}{3}$
   d) $\frac{\pi}{2}$
37. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit co-planar vectors, then the scalar triple product $[2\vec{a} - \vec{b}, 2\vec{b} - \vec{c}, 2\vec{c} - \vec{a}]$ =
   a) 0
   b) 1
   c) $-\sqrt{3}$
   d) $\sqrt{3}$
38. Let

$$\vec{a} = \vec{i} - \vec{k}$$

$$\vec{b} = x\vec{i} + \vec{j} + (1-x)\vec{k}$$

$$\vec{c} = y\vec{i} + x\vec{j} + (1+x-y)\vec{k}$$

   Then $[\vec{a}\,\vec{b}\,\vec{c}]$ depends on
   a) only x
   b) only y
   c) Neither x nor y
   d) both x and y
39. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

   does not exceed.
   a) 4
   b) 9
   c) 8
   d) 6
40. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is
   a) 45°
   b) 60°

c) $\cos^{-1}(\frac{1}{3})$
d) $\cos^{-1}(\frac{2}{7})$

41. Let $\vec{V} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{W} = \vec{i} + 3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $|\vec{U}\vec{V}\vec{W}|$ is
a) -1
b) $\sqrt{10} + \sqrt{6}$
c) $\sqrt{59}$
d) $\sqrt{60}$

42. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane 2x - 4y + z = 7, is
a) 7
b) -7
c) no real value
d) 4

43. The value of $'a'$ so that the volume of parallelopiped formed by $\vec{i}$ + a$\vec{j}$ + $\vec{k}$, $\vec{j}$ + a$\vec{k}$ and a$\vec{i}$ + $\vec{k}$ becomes minimum is
a) -3
b) 3
c) $\frac{1}{\sqrt{3}}$
d) $\sqrt{3}$

44. If $\vec{a} = (\vec{i} + \vec{j} + \vec{k})$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \vec{j} - \vec{k}$, then $\vec{b}$ is
a) $\vec{i} - \vec{j} + \vec{k}$
b) $2\vec{j} - \vec{k}$
c) $\vec{i}$
d) $2\vec{i}$

45. If the lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} and \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

intersect, then the value of k is
a) 3/2
b) 9/2
c) -2/9
d) -3/2

46. The unit vector which is orthogonal to the vector $3\vec{i} + 2\vec{j} + \vec{k}$ and is co-planar with the vectors $2\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} - \vec{j} + \vec{k}$ is
a) $\frac{2\vec{i}-6\vec{j}+\vec{k}}{\sqrt{41}}$
b) $\frac{2\vec{i}-3\vec{j}}{\sqrt{13}}$
c) $\frac{3\vec{i}-\vec{k}}{10}$
d) $\frac{4\vec{i}+3\vec{j}-3\vec{k}}{34}$

47. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B, and C. If the centroid D(x, y, z) of triangle ABC satisfies the relation

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$$

then the value of k is

a) 3
b) 1
c) $\frac{1}{3}$
d) 9

48. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero, non-polar vectors and

$$\vec{b_1} = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a},$$

$$\vec{b_2} = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$$

$$\vec{c_1} = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b_1}$$

$$\vec{c_2} = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b_1}.\vec{c}}{|\vec{b_1}|^2}\vec{b_1}$$

$$\vec{c_3} = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b_1}$$

$$\vec{c_4} = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} = \frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b_1}$$

then the set of orthogonal vectors ts

a) $(\vec{a}, \vec{b_1}, \vec{c_3})$
b) $(\vec{a}, \vec{b_1}, \vec{c_2})$
c) $(\vec{a}, \vec{b_1}, \vec{c_1})$
d) $(\vec{a}, \vec{b_2}, \vec{c_2})$

49. A plane which is perpendicular to two planes

$$2x - 2y + z = 0 \tag{49.1}$$

$$x - y + 2z = 4 \tag{49.2}$$

passes through (1, -2 ,1). The distance of the plane from the point (1, 2, 2) is

a) 0
b) 1
c) $\sqrt{2}$
d) $2\sqrt{2}$

50. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is

a) $4\hat{i} - \hat{j} + 4\hat{k}$
b) $3\hat{i} + \hat{j} - 3\hat{k}$
c) $2\hat{i} + \hat{j} - 2\hat{k}$

d) $4\hat{i} + \hat{j} - 4\hat{k}$

51. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar is
   a) 0
   b) 1
   c) 2
   d) 3
52. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct?
   a) $(\vec{a} \times \vec{b}) = (b \times \vec{c}) = (\vec{c} \times \vec{a}) = \vec{0}$
   b) $(\vec{a} \times \vec{b}) = (b \times \vec{c}) = (\vec{c} \times \vec{a}) \neq \vec{0}$
   c) $(\vec{a} \times \vec{b}) = (b \times \vec{c}) = (\vec{a} \times \vec{c}) \neq \vec{0}$
   d) $(\vec{a} \times \vec{b}), (b \times \vec{c}), (\vec{c} \times \vec{a})$ are mutually perpendicular.
53. The edges of a parallelopiped are of unit length and are to parallel to non-coplanar unit vectors $\hat{a}$, $\hat{b}$, $\hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = \frac{1}{2}$. Then, the volume of the parallelopiped is
   a) $\frac{1}{\sqrt{2}}$
   b) $\frac{1}{2\sqrt{2}}$
   c) $\frac{\sqrt{3}}{2}$
   d) $\frac{1}{\sqrt{3}}$
54. Let two non-collinear vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$(where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,
   a) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and M $= (1 + \hat{a}.\hat{b})^{1/2}$
   b) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and M $= (1 + \hat{a}.\hat{b})^{1/2}$
   c) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and M $= (1 + 2\hat{a}.\hat{b})^{1/2}$
   d) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and M $= (1 + 2\hat{a}.\hat{b})^{1/2}$
55. Let P(3, 2, 6) be a point in a space and Q be a point on the line

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

   Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x - 4y + 3z = 1 is
   a) $\frac{1}{4}$
   b) $\frac{-1}{4}$
   c) $\frac{1}{8}$
   d) $\frac{-1}{8}$
56. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$, then
   a) $\vec{a}, \vec{b}, \vec{c}$ are non-polar
   b) $\vec{b}, \vec{c}, \vec{d}$ are non-polar
   c) $\vec{b}, \vec{d}$ are non-parallel
   d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel
57. A line with positive direction cosines passes through the point P(2, -1, 2) and make equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at a point Q. The length of the line segment PQ equals
   a) 1
   b) $\sqrt{2}$

c) $\sqrt{3}$
d) 2

58. Let P, Q, R and S be the points on the plane with position vectors -2$\hat{i}$ - $\hat{j}$, 4$\hat{i}$, 3$\hat{i}$ + 3$\hat{j}$ and -3$\hat{i}$ + 2$\hat{j}$ respectively. The qudrilateral PQRS must be a
a) parallelgram, which is neither a rhombus nor a rectangle
b) square
c) rectangle, but not a square
d) rhombus, but a square

59. Equation of the plane containing the straight line
$$\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$$
and perpendicular to the plane containing the straight lines
$$\frac{x}{3} = \frac{y}{4} = \frac{z}{2} and \frac{x}{4} = \frac{y}{2} = \frac{z}{3}$$
is
a) x + 2y - 2z = 0
b) 3x + 2y - 2z = 0
c) x - 2y + z = 0
d) 5x + 2y - 4z = 0

60. If the distance of the point P(1, -2, 1) from the plane x + 2y - 2z = α, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is
a) $(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3})$
b) $(\frac{4}{3}, \frac{-4}{3}, \frac{1}{3})$
c) $(\frac{1}{3}, \frac{2}{3}, \frac{10}{3})$
d) $(\frac{2}{3}, \frac{-1}{3}, \frac{5}{2})$

61. Two adjacent sides of a parallelgram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelgram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α os given by
a) $\frac{8}{9}$
b) $\frac{\sqrt{17}}{9}$
c) $\frac{1}{9}$
d) $\frac{4\sqrt{5}}{9}$

62. Let $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\overrightarrow{v}$ in the plane of $\overrightarrow{a}$ and $\overrightarrow{b}$, whose projection on $\overrightarrow{c}$ is $\frac{1}{\sqrt{3}}$, is given by
a) $\hat{i} - 3\hat{j} + 3\hat{k}$
b) $-3\hat{i} - 3\hat{j} - \hat{k}$
c) $3\hat{i} - \hat{j} + 3\hat{k}$
d) $\hat{i} + 3\hat{j} - 3\hat{k}$

63. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, -1, 4) with the plane 5x - 4y - z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is
a) $\frac{1}{\sqrt{2}}$
b) $\sqrt{2}$
c) 2
d) $2\sqrt{2}$

64. The equation of the plane passing through the line of intersection of the planes

$$x + 2y + 3z = 2$$

$$x - y + z = 3$$

and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, -1) is
a) $5x - 11y + z = 17$
b) $\sqrt{3}x + y = 3\sqrt{2} - 1$
c) $x + y + z = \sqrt{3}$
d) $x - \sqrt{2}y = 1 - \sqrt{2}$

65. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is
a) 0
b) 3
c) 4
d) 8

66. Let P be the image of the point(3, 1, 7) with respect to the plane x - y + z = 3. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{z} = \frac{z}{1}$ is
a) $x + y - 3z = 0$
b) $3x + z = 0$
c) $x - 4y + 7z = 0$
d) $2x - y = 0$

67. The equation of the plane passing through the point(1, 1, 1) and perpendicular to the planes 2x + y - 2z = 0 and 3x - 6y - 2z = 7, is
a) $14x + 2y - 15z = 1$
b) $14x - 2y + 15z = 27$
c) $14x + 2y + 15z = 31$
d) $-14x + 2y + 15z = 3$

68. Let O be the origin and let PQR be an arbitrary triangle. The points S is such that

$$\begin{aligned}\overrightarrow{OP}.\overrightarrow{OQ} + \overrightarrow{OR}.\overrightarrow{OS} &= \overrightarrow{OR}.\overrightarrow{OP} + \overrightarrow{OQ}.\overrightarrow{OS} \\ &= \overrightarrow{OQ}.\overrightarrow{OR} + \overrightarrow{OP}.\overrightarrow{OS}\end{aligned}$$

Then the triangle PQR has S as its
a) Centroid
b) Circumcentre
c) Incentre
d) Orthocentre

(D). MCQs with One or More than One Correct

69. Let $\vec{a} = a_1 i + a_2 j + a_3 k$, $\vec{b} = b_1 i + b_2 j + b_3 k$, $\vec{c} = c_1 i + c_2 j + c_3 k$, be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$

is equal to

a) 0
b) 1
c) $\frac{1}{4}(a_1^2 + a_2^2 + a_2^3)(b_1^2 + b_2^2 + b_3^2)$
d) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^3)$

70. The number of vectors of unit length perpendicular to vectors $\overrightarrow{a}$ = (1, 1, 0) and $\overrightarrow{b}$ = (0, 1, 1) is
a) 0
b) 1
c) 2
d) 3
e) ∞
f) None of these

71. Let $\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\overrightarrow{b}$ and $\overrightarrow{c}$, whose projection on $\overrightarrow{a}$ is magnitudu of $\sqrt{2/3}$, is
a) $2\hat{i} + 3\hat{j} - 3\hat{k}$
b) $2\hat{i} + 3\hat{j} + 3\hat{k}$
c) $-2\hat{i} - \hat{j} + 5\hat{k}$
d) $2\hat{i} + \hat{j} + 5\hat{k}$

72. The vector $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$ is
a) a unit vector
b) makes an angle $\frac{\pi}{3}$ with the vector $(2\hat{i} - 4\hat{j} + 3\hat{k})$
c) parallel to the vector $(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k})$
d) perpendicular to the vector $(3\hat{i} + 2\hat{j} - 2\hat{k})$

73. If a = i + j + k, $\overrightarrow{b}$ = 4i + 3j + 4k and c = i + αj + βk are linearly dependent vectors and $|c| = \sqrt{3}$, then
a) $\alpha = 1, \beta = -1$
b) $\alpha = 1, \beta = \pm 1$
c) $\alpha = -1, \beta = \pm 1$
d) $\alpha = \pm 1, \beta = 1$

74. For three vectors u, v, w which of the following expression in not equal to any one of the remaining three?
a) u.(v $\times$ w)
b) (v $\times$ w).u
c) v.(u $\times$ w)
d) (u $\times$ v).w

75. Which of the following expressions are meaningful?
a) u(v $\times$ w)
b) (u . v).w
c) (u . v)w
d) u $\times$(v . w)

76. Let a and b two non-collinear unit vectors. If u = a - (a.b)b and v = a $\times$ b, then $|v|$ is
a) $|u|$
b) $|u| + |u.a|$
c) $|u| + |u.b|$
d) $|u| + u.(a + b)$

77. Let $\overrightarrow{A}$ be a parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and that P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\overrightarrow{A}$ and

a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is

a) $\frac{\pi}{2}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{6}$
d) $\frac{3\pi}{4}$

78. The vectors which are coplanr with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ are

a) $\hat{j} - \hat{k}$
b) $-\hat{i} - \hat{j}$
c) $\hat{i} - \hat{j}$
d) $-\hat{j} + \hat{k}$

79. If the straight lines

$$\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2} and \frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$$

are co-planar, then the plane(s) containing these two lines is(are)

a) y + 2z = -1
b) y + z = -1
c) y - z = -1
d) y - 2z = -1

80. A line l passing through the origin is perpendicular to the lines

$$l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, -\infty < t < \infty$$

$$l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$$

Then, the coordinates of the points on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is(are)

a) $(\frac{7}{5}, \frac{7}{3}, \frac{5}{3})$
b) (-1, -1, 0)
c) (1, 1, 1)
d) $(\frac{7}{9}, \frac{7}{9}, \frac{8}{9})$

81. Two lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$$

are coplanar. Then α can take value(s)

a) 1
b) 2
c) 3
d) 4

82. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

a) $\vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$
b) $\vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$

c) $\overrightarrow{a}.\overrightarrow{b}$ = -$(\overrightarrow{a}.\overrightarrow{y})(\overrightarrow{b}.\overrightarrow{z})$
d) $\overrightarrow{a}$ = $(\overrightarrow{a}.\overrightarrow{y})(\overrightarrow{z} - \overrightarrow{y})$

83. From a point P$(\lambda, \lambda, \lambda)$ perpendicular to PQ and PR are drawn respectively on the lines y = x, z = 1. If P is such that $\angle$ QPR is a right angle, then the possible value(s) of λ is(are)
a) $\sqrt{2}$
b) 1
c) -1
d) - $\sqrt{2}$

84. In R^3, consider the planes P_1: y = 0 and P_2: X + Z = 1. Let P_3 be the plane, different from P_1 and P_2 , which passes through the intersection of P_1 and P_2. If the distance the point (0, 1, 0) from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relation is(are) true?
a) $2\alpha + \beta + 2\gamma + 2 = 0$
b) $2\alpha - \beta + 2\gamma + 4 = 0$
c) $2\alpha + \beta - 2\gamma - 10 = 0$
d) $2\alpha - \beta + 2\gamma - 8 = 0$

85. In R^3,let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes

$$P_1 : x + 2y - z + 1 = 0$$

$$P_2 : 2x - y + z - 1 = 0$$

Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?
a) $(0, \frac{-5}{6}, \frac{-2}{3})$
b) $(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6})$
c) $(\frac{-5}{6}, 0, \frac{1}{3})$
d) $\frac{-1}{3}, 0, \frac{2}{3})$

86. Let Δ PQR be a triangle. Let $\overrightarrow{a} = \overrightarrow{QR}$, $\overrightarrow{b} = \overrightarrow{RP}$ and $\overrightarrow{c} = \overrightarrow{PQ}$. If $|\overrightarrow{a}| = 12$, $|\overrightarrow{b}| = 4\sqrt{3}$, $\overrightarrow{b}.\overrightarrow{c} = 24$, then the which of the following is(are) true?
a) $\frac{|c|^2}{2} - |\overrightarrow{a}| = 12$
b) $\frac{|c|^2}{2} + |\overrightarrow{a}| = 30$
c) $|\overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{c} \times \overrightarrow{a}| = 48\sqrt{3}$
d) $\overrightarrow{a}.\overrightarrow{b}$ = -72

87. Consider a pyramid OPQRS located in the first octant(x $\geq$ 0, y $\geq$ 0, z $\geq$ 0) with O as origin, and OP and OR along x-axis and along y-axis respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point, T of diagonal OQ such that TS = 3. Then
a) the acute angle between OQ and OS is $\frac{\pi}{3}$
b) the equation of the plane containing the triangle OQS is x-y = 0
c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
d) The perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$.

88. Let $\hat{u} = u_1 i + u_2 j + u_3 k$ be a unit vector in R^3 and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector $\overrightarrow{v}$ in following R^3 such that $|\overrightarrow{u} \times \overrightarrow{v}| = 1$ and $\overrightarrow{w}(\overrightarrow{u} \times \overrightarrow{v}) = 1$. Which of the following statement(s) is(are) correct?
a) There is exactly one choice for such $\overrightarrow{v}$

b) There are infinitely many choices for such $\overrightarrow{v}$
c) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$
d) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

89. Let

$$P_1 : 2x + y - z = 3$$

$$P_2 : x + 2y + z = 2$$

be two planes. Then, which of the following statement(s) is(are) correct?
a) The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1
b) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2.
c) The acute angle between P_1 and P_2 is 60°
d) If P_3 is the plane passing through the point(4, 2, -2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$.

90. Let L_1 and L_2 denote the lines

$$\overrightarrow{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$$

$$\overrightarrow{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?
a) $\overrightarrow{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, t ∈ R
b) $\overrightarrow{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, t ∈ R
c) $\overrightarrow{r} = t(2\hat{i} + 2\hat{j} - \hat{k})$, t ∈ R
d) $\overrightarrow{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, t ∈ R

91. Three lines

$$L_1 : \overrightarrow{r} = \lambda\hat{i}, \lambda \in R$$

$$L_2 : \overrightarrow{r} = \hat{k} + \mu\hat{j}, \mu \in R$$

$$L_3 : \overrightarrow{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P,Q and R are collinear?
a) $\hat{k}$ - $\frac{1}{2}\hat{j}$
b) $\hat{k}$
c) $\hat{k} + \hat{j}$
d) $\hat{k} + \frac{1}{2}\hat{j}$

(E). Subjective Problems

92. From a point O inside a triangle ABC, perpendiculars OD, OE, OF are drawn to the sides BC, CA, AB respectively. Prove that the perpendiculars from A, B, C to the sides EF, FD, DE are concurrent.

93. A_1, A_2,............A_n are the vertices of a regular plane polygon with n sides and O is its centre. Show that

$$\sum_{i=1}^{n-1}(\overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}}) = (1 - n)(\overrightarrow{OA_2} \times \overrightarrow{OA_1})$$

94. Find all values of λ such that x, y, z $\neq$ (0, 0, 0) and

$$(\vec{i} + \vec{j} + 3\vec{k})x + (3\vec{i} - 3\vec{j} + \vec{k})y + (-4\vec{i} + 5\vec{j})z$$
$$= \lambda(x\vec{i} \times \vec{j}y + \vec{k}z)$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along the coordinate axes.

95. A vector $\vec{A}$ has components $\vec{A_1}$, $\vec{A_2}$, $\vec{A_3}$ ina right-handed rectangular Cartesian coordinate system oxyz. The coordinate system is rotated about the x-axis through an angle of $\frac{\pi}{2}$. Find the components of A in the new coordinate system, interms of $\vec{A_1}$, $\vec{A_2}$, $\vec{A_3}$.

96. The position vectors of the points A, B, C and D are $(3\hat{i} - 2\hat{j} - \hat{k})$, $(2\hat{i} + 3\hat{j} - 4\hat{k})$, $(-\hat{i} + \hat{j} + 2\hat{k})$ and $(4\hat{i} + 5\hat{j} + \lambda\hat{k})$ respectively. If the points A, B, C and D lie on a plane, find the value of λ?

97. If A, B, C, D are any four points in space, Prove that

$$|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}| = 4$$

(area of triangle ABC)

98. Let OACB be a parallelgram with O at the origin and OC a diagonal. Let D be the mid-point of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio.

99. If vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$$

100. In a triangle OAB, E is the midpoint of BO and D is a point on AB such that AD : DB = 2 : 1. If OD and AE intersect at P, Determine the ratio OP : PD using vectors methods?

101. Let $\vec{A} = 2\hat{i} + \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$. Determine a vector $\vec{R}$. Satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}.\vec{A} = 0$

102. Determine the value of 'c' so that for all real x, the vector $cx\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

103. In a triangle ABC, D and E are points on BC and AC respectively, such that BD = 2DC and AE = 3EC. Let P be the point of intersection of AD and BE. Find BP/PE using vector methods.

104. If the vectors $\vec{b}$, $\vec{c}$, $\vec{d}$ are not coplanar, then prove that the vector

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b})$$
$$+(\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$$

is parallel to $\vec{a}$.

105. The position vectors of the vertices A, B, C of a tetrahedron ABCD are $\hat{i} + \hat{j} + \hat{k}$, $\hat{i}$ and $3\hat{i}$ respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of tetrahedron is $\frac{2\sqrt{2}}{3}$. find the position vector of the point E for all its possible positions.

106. If A, B and C vectors such that $|B| = |C|$, Prove that

$$[(A + B) \times (A + C)] \times (B \times C)(B + C) = 0.$$

107. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid-points of the parallel sides. (You may assume that the trapezium is not a parallelgram.)

108. For any two vectors u and v, prove that

a) $(u.v)^2 + |u \times v|^2 = |u|^2|v|^2$
b) $(1 + |u|^2)(1 + |v|^2) = (1 - u.v)^2 + |u + v + (u \times v)|^2$

109. Let u and v be unit vectors. If w is a vector such that w + (w × u) = v, then prove that $|u \times v|.w \leq 1/2$ and that the equality holds if and onlt if u is perpendicular to v.

110. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point concurrency in terms of the position vectors of the vertices.

111. Find 3-dimensional vectors $\overrightarrow{v_1}$, $\overrightarrow{v_2}$, $\overrightarrow{v_3}$ satisfying

$$\overrightarrow{v_1}.\overrightarrow{v_1} = 4, \overrightarrow{v_1}.\overrightarrow{v_2} = -2, \overrightarrow{v_1}.\overrightarrow{v_3} = 6,$$

$$\overrightarrow{v_2}.\overrightarrow{v_2} = 2, \overrightarrow{v_2}.\overrightarrow{v_3} = -5, \overrightarrow{v_3}.\overrightarrow{v_3} = 29$$

112. Let

$$\overrightarrow{A(t)} = f_1(t)\hat{i} + f_2(t)\hat{j}$$

$$\overrightarrow{B(t)} = g_1(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1]$$

where f_1, f_2, g_1, g_2 are continuous functions. If $\overrightarrow{A(t)}$ and $\overrightarrow{B(t)}$ are non-zero vectors for all t and $\overrightarrow{A(0)} = 2\hat{i} + 3\hat{j}$, $\overrightarrow{A(1)} = 6\hat{i} + 2\hat{j}$, $\overrightarrow{B(0)} = 3\hat{i} + 2\hat{j}$, $\overrightarrow{B(1)} = 2\hat{i} + 6\hat{j}$. Then show that $\overrightarrow{A(t)}$ and $\overrightarrow{B(t)}$ are parallel for some t.

113. Let V be the volume of the parallelopiped formed by the vectors

$$\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\overrightarrow{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\overrightarrow{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

If a_r, b_r, c_r where r = 1, 2, 3, are non-negative real numbers and

$$\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$$

Show that $V \leq L^3$.

114. a) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).
b) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the midpoint of PQ lies on it.

115. If $\overrightarrow{u}$, $\overrightarrow{v}$, $\overrightarrow{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\overrightarrow{u}$ and $\overrightarrow{v}$ and $\overrightarrow{w}$, $\overrightarrow{w}$ and $\overrightarrow{u}$ respectively and $\overrightarrow{x}$, $\overrightarrow{y}$, $\overrightarrow{z}$ are unit vectors along the bisection of the angles α, β, γ respectively. Prove that

$$[(\overrightarrow{x} \times \overrightarrow{y})(\overrightarrow{y} \times \overrightarrow{z})(\overrightarrow{z} \times \overrightarrow{x})] = \frac{1}{16}[\overrightarrow{u}\,\overrightarrow{v}\,\overrightarrow{w}]^2 \sec^2\frac{\alpha}{2} \sec^2\frac{\beta}{2} \sec^2\frac{\gamma}{2}.$$

116. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ and $\overrightarrow{d}$ are distinct vectors such that $\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \times \overrightarrow{d}$ and $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$. Prove that

$$(\overrightarrow{a} - \overrightarrow{d}).(\overrightarrow{b} - \overrightarrow{c}) \neq 0$$

$$(i.e, \overrightarrow{a}.\overrightarrow{b} + \overrightarrow{d}.\overrightarrow{c} \neq \overrightarrow{d}.\overrightarrow{b} + \overrightarrow{a}.\overrightarrow{c})$$

117. Find the equation of the plane passing through (1, 1, 1) and parallel to the lines L_1, L_2 having direction ratios (1, 0, -1), (1, -1, 0). Find the volume of tetrahedron formed by origin and the points where these planes intersect the coordinate axes.
118. A parallelopiped $'S'$ has base points A, B, C and D and upper face points A', B', C', D'. This parallelopiped is compressed by upper face $A'B'C'D'$ to form a new parallelopiped $'T'$ having upper face points A'',B'',C'',D''. Volume of parallelopiped T is 90 perecnt of the volume of the parallelopiped S. Prove that the locus of a A'', is a plane.
119. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two lines on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C whose permutation A', B', C' can be chosen such that
    a) A is on L_1, B on P_1 but not on L_1 and C not on P_1
    b) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2
120. Find the equation of the plane containing the line

$$2x - y + z - 3 = 0$$

$$3x + y + z = 5$$

and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, -1).
121. If the incident ray on a surface is along the unit vector $\hat{v}$ the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.

(F). Match the following

122. Match the following:

Column I	**Column II**
(A) Two rays x+y=$\lvert a\rvert$ and ax-y=1 intersects each other in the first quadrant in the interval a $\in (a_0, \infty)$, the value of a_0 is	(p) 2
(B) Point (α, β, γ) lies on the plane x+y+z=2. Let $\overrightarrow{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k} \times (\hat{k} \times \hat{a})$=0, then γ=	(q) $\frac{4}{3}$
(C) $\lvert \int_0^1 (1 - y^2)dy\rvert + \lvert \int_1^0 (y^2 - 1)dy\rvert$	(r) $\lvert \int_0^1 \sqrt{1 - x}dx\rvert + \lvert \int_{-1}^0 \sqrt{1 + x}dx\rvert$
(D) If $\sin A \sin B \sin C + \cos A \cos B = 1$, then value of $\sin C$=	(s) 1

123. Match the statements/expressions in **Column-I** with the values given in **Column-II**.

Column I	**Column II**
(A) Roots of the equation $2\sin^2\theta + \sin^2 2\theta$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the unction f(x)=$[\frac{6x}{\pi}]\cos[\frac{3x}{\pi}]$	(q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vector $\overrightarrow{a}$ and $\overrightarrow{b}$ where $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ are unit vectors satisfying $\overrightarrow{a} + \overrightarrow{b} + \sqrt{3}\overrightarrow{c} = 0$	(s) $\frac{\pi}{2}$

124. Consider the following linear equations

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

$$cx + ay + bz = 0$$

Match the conditions/expressions in **Column-I** with statements in **Column-II** and indicate your answer by darkening the bubbles in the 4×4 matrix given in the *ORS*.

Column I	**Column II**
(A) a+b+c $\neq$ 0 and $a^2+b^2+c^2$=ab+bc+ca	(p) the equation represents planes meeting only at single point
(B) a+b+c=0 and $a^2+b^2+c^2 \neq$ ab+bc+ca	(q) the equation represents the line x=y=z
(C) a+b+c $\neq$ 0 and $a^2+b^2+c^2 \neq$ ab+bc+ca	(r) the equation represents identical planes
(D) a+b+c=0 and $a^2+b^2+c^2$=ab+bc+ca	(s) the equation represents the whole of the 3 dimensional space

125. Mtach the statements/expressions given in **Column-I** with the values in given in **Column-II**.

Column I	**Column II**
(A) The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $(0, \frac{\pi}{2})$	(p) 1
(B) Values of k for which the planes kx+4y+z=0, 4x+ky+2z=0 and 2x+2y+z=0 intersect in a straight line	(q) 2
(C) Values of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert\lvert x+2\rvert = 4k$ has integer solutions	(r) 3
(D) If $y' = y + 1$ and y(0)=1, then values of y(ln2)	(s) 4

126. Mtach the statement in **Column-I** with the values in given in **Column-II**.

Column I	**Column II**
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-8/3}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length PQ=d, then d^2	(p) -4
(B) The values of x for $\tan^{-1}(x+3) - \tan^{-1}(x-3)$ $=\sin^{-1}(\frac{3}{5})$ are	(q) 0
(C) Non-zero vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ satisfy $\overrightarrow{a}.\overrightarrow{b} = 0$ $(\overrightarrow{b} - \overrightarrow{a}).(\overrightarrow{b} + \overrightarrow{c})$ and $2\lvert\overrightarrow{b} + \overrightarrow{c}\rvert = \lvert\overrightarrow{b} - \overrightarrow{a}\rvert$. If $\overrightarrow{a} = \mu\overrightarrow{b} + 4\overrightarrow{c}$, then the possible values of μ are	(r) 4
(D) Let f be the function in $[-\pi, \pi]$ given by f(0)=9 and $f(x) = \sin(\frac{9x}{2})/\sin(\frac{x}{2})$ for $x \neq 0$ The value of $\frac{2}{\pi}\int_{\pi}^{\pi} f(x)dx$ is	(s) 5

127. Mtach the statement in **Column-I** with the values in given in **Column-II**.

Column I	Column II
(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f(\frac{\pi}{6})$	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{ln3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(r) $\frac{\pi}{3}$
(D) The maximum value of $\|Arg(\frac{1}{1-z})\|$ for $\|z\|=1$, $z \neq 1$ is given by	(s) π

128. Mtach the List-I with List-II and select the correct answer using the code given the below lists..

Column I	Column II
(A) Volume of parallelopiped determined by the vectors $\vec{a}$ and $\vec{b}$ and $\vec{c}$ is 2. Then the volume the parallelopiped determined by the vectors $2(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(p) 100
(B) Volume of parallelopiped determined by the vectors $\vec{a}$ and $\vec{b}$ and $\vec{c}$ is 5. Then the volume the parallelopiped determined by the vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(p) 30
(C) Area of the triangle with adjacent sides determined by the vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $\vec{a} - \vec{b}$ is	(r) 24
(D) Area of the parallelgram with adjacent sides determined by the vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelgram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(s) 60

codes:

	P	Q	R	S
(a)	4	2	3	1
(b)	2	3	1	4
(c)	3	4	1	2
(d)	1	4	3	2

129. Mtach the statements/expressions given in **Column-I** with the values in given in **Column-II**.

Column I	Column II
(A) In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i} + \beta\hat{j}$ on $\sqrt{3}\hat{i} + \hat{j}$ is $\sqrt{3}$ and if $\alpha = 2 + \sqrt{3}\beta$, then possible value of $\|\alpha\|$ is	(p) 1
(B) Let a and b real numbers such that the function $f(x) = -3ax^2 - 2, x < 1$ and $f(x) = bx + a^2, x \geq 1$ if differentiable for all $x \in R$. Then possible value of a is(are)	(q) 2
(C) Let $\omega \neq 1$ be a complex cube root of unity. If $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$ then possible value(s) of n is(are)	(r) 3
(D) Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that a,5,q,b is an arithmetic progression, then the value(s) of $\|q - a\|$ is(are)	(s) 4

130. Consider the lines

$$L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$$

$$L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$$

and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let ax+by+cz=d be the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to the planes P_1 and P_2.

Column I	**Column II**
(P) a=	(1) 13
(Q) b=	(2) -3
(R) c=	(3) 1
(S) d=	(4) -2

codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

131. Mtach the statements/expressions given in **Column-I** with the values in given in **Column-II**.

Column I	**Column II**
(A) In a triangle ΔXYZ, let a,b,c be the lengths of the sides opposite to the angles X,Y,Z respectively. If $2(a^2 - b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$ then possible values of n for which $\cos(n\pi\lambda) = 0$ is(are)	(p) 1
(B) In a triangle ΔXYZ, let a,b,c be the lengths of the sides opposite to the angles X,Y,Z respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X$ then possible values of $\frac{a}{b}$ is(are)	(q) 2
(C) In a R^2 let $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors of X,Y and Z w.r.t. to the origin O respectively. If the distance Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible values of $\lvert\beta\rvert$ is(are)	(r) 3
(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by x=0, x=2, $y^2 = 4x$ and $y = \lvert\alpha x - 1\rvert + \lvert\alpha x - 2\rvert + \alpha x$, where $\alpha \in (0, 1)$ Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0, 1$ is(are)	(s) 5

(G). Comprehension Based Questions:

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

132. The unit vector perpendicular to both L_1 and L_2 is
 a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$
 b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$
 c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$
 d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$
133. The shortest distance between L_1 and L_2 is
 a) 0
 b) $\frac{17}{\sqrt{3}}$
 c) $\frac{41}{5\sqrt{3}}$
 d) $\frac{17}{5\sqrt{3}}$
134. The distance of the point (1, 1, 1) from the plane passing through the point (-1, -2, -1) and whose normal is perpendicular to both the lines L_1 and L_2 is
 a) $\frac{2}{\sqrt{75}}$
 b) $\frac{7}{\sqrt{75}}$
 c) $\frac{13}{\sqrt{75}}$
 d) $\frac{23}{\sqrt{75}}$

(H). Assertion and Reason Type Questions

135. Consider the planes 3x - 6y - 2z = 15 and 2x + y - 2z = 5.
 STATEMENT-1: The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.
 STATEMENT-2: The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of given planes.
 a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
 b) Statement-1 is true, Statement-2 is false
 c) Statement-1 is false, Statement-2 is true
 d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
136. Let the vectors $\overrightarrow{PQ}$, $\overrightarrow{QR}$, $\overrightarrow{RS}$,$\overrightarrow{ST}$, $\overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.
 STATEMENT-1: $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST} \neq \overrightarrow{0}$
 STATEMENT-2: $\overrightarrow{PQ} \times \overrightarrow{RS} = \overrightarrow{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \overrightarrow{0}$.
 a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
 b) Statement-1 is true, Statement-2 is false
 c) Statement-1 is false, Statement-2 is true
 d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
137. Consider three planes

$$P_1 : x - y + z = 1$$

$$P_2 : x + y - z = 1$$

$$P_3 : x - 3y + 3z = 2$$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2 respectively.
STATEMENT-1: At least two of the lines L_1, L_2, L_3 are non-parallel
STATEMENT-2: The three planes does not have a common point.

a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
b) Statement-1 is true, Statement-2 is false
c) Statement-1 is false, Statement-2 is true
d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(I). Integer Value Correct Type:

138. If $\vec{a}$ and $\vec{b}$ are vectors in space given by

$$\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$$

$$\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$$

then find the value of $(2\vec{a} + \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$.

139. If the distance between the plane Ax - 2y + z = d and the plane containing the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

is $\sqrt{6}$, then find $|d|$.

140. Let $\vec{a} = -\vec{i} - \vec{k}$, $\vec{b} = -\vec{i} + \vec{j}$ and $\vec{c} = \vec{i} + 2\vec{j} + 3\vec{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

141. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9,$$

then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

142. Consider the set of eight vectors

$$V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$$

Three non-copolanar vectors can be chosen from V in 2^p ways. Then p is

143. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k - 20 =

144. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-copolar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If

$$(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) = p\vec{a} + q\vec{b} + r\vec{c},$$

where p, q, r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

145. Suppose that $\vec{p}$, $\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R^3. Let the components of a vector $\vec{s}$ along $\vec{p}$, $\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p} + \vec{q}$

$+\vec{r}$), $(\vec{p} - \vec{q} + \vec{r})$ and $(-\vec{p} - \vec{q} + \vec{r})$ are x, y and z, respectively. then the value of 2x+y+z is

146. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b}=0$. For some x, y $\in$ R, let

$$\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$$

If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is

147. Let P be a point in the first octant, whose image Q in the plane x + y = 3(that is the line segment PQ is perpendicular to the plane x + y = 3 and mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is

148. Consider the cube in the first octant with sides OP, OQ, OR of length 1, along the x-axis, y-axis, z-axis respectively, where O(0, 0, 0) is the origin. Let $S(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p} = \vec{SP}$, $\vec{q} = \vec{SQ}$, $\vec{r} = \vec{SR}$ and $\vec{t} = \vec{ST}$, then the value of $|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})|$ is

149. Three lines are given by $\vec{r} = \lambda\hat{i}$, $\lambda \in$ R; $\vec{r} = \mu(\hat{i} + \hat{j})$, $\mu \in R$ and $\vec{r} = v(\hat{i} + \hat{j} + \hat{k})$, v $\in$ R. Let the lines cut the plane x+y+z=1 at the points A, B, C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals

150. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\hat{a} + \beta\hat{b}$, $\alpha, \beta \in R$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})).\vec{c}$ equals

Section-B

151. A plane which passes through the point (3, 2, 0) and the line

$$\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$$

is

a) x - y + z = 1
b) x + y + z = 5
c) x + 2y -z = 1
d) 2x - y + z = 5

152. If $|\vec{a}| = 4$, $|\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $(\vec{a} \times \vec{b})^2$ is equal to

a) 48
b) 16
c) $\vec{a}$
d) none of these

153. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors show that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 7$, $|\vec{b}| = 5$, $|\vec{c}| = 3$ then angle between vector $\vec{b}$ and $\vec{c}$ is

a) 60^o
b) 30^o
c) 45^o
d) 90^o

154. If $|\vec{a}| = 5$, $|\vec{b}| = 4$, $|\vec{c}| = 3$ thus what will be the value of $|\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}|$, given that $\vec{a} + \vec{b} + \vec{c} = 0$

a) 25
b) 50
c) -25
d) -50

155. If the vectors $\overrightarrow{c}$, $\overrightarrow{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\overrightarrow{b} = \hat{j}$ are such that $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ form a right handed system then $\overrightarrow{c}$ is
   a) $z\hat{i} - x\hat{k}$
   b) 0
   c) y$\hat{j}$
   d) $-z\hat{i} + x\hat{k}$
156. $\overrightarrow{a} = 3\hat{i}-5\hat{j}$ and $\overrightarrow{b} = 6\hat{i}+3\hat{j}$ are two vectors and $\overrightarrow{c}$ is a vector such that $\overrightarrow{c} = \overrightarrow{a}\times\overrightarrow{b}$ then $|\overrightarrow{a}| : |\overrightarrow{b}| : |\overrightarrow{c}|$
   a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$
   b) $\sqrt{34} : \sqrt{45} : 39$
   c) 34 : 39 : 45
   d) 39 : 35 : 34
157. If $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{c} = \overrightarrow{c} \times \overrightarrow{a}$ then $\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} =$
   a) abc
   b) -1
   c) 0
   d) 2
158. The d.r. of a normal to the plane through (1,0,0), (0,1,0) which makes an angle $\pi/4$ with plane x+y=3 are
   a) 1, $\sqrt{2}$, 1
   b) 1, 1, $\sqrt{2}$
   c) 1, 1, 2
   d) $\sqrt{2}$, 1, 1
159. Let $\overrightarrow{u} = \hat{i} + \hat{j}$, $\overrightarrow{v} = \hat{i} - \hat{j}$ and $\overrightarrow{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\overrightarrow{u}.\hat{n} = 0$ and $\overrightarrow{v},\hat{n} = 0$, then $|\overrightarrow{w}.\hat{n}|$ is equal to
   a) 3
   b) 0
   c) 1
   d) 2
160. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}-3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done the by the forces is
   a) 50 units
   b) 20 units
   c) 30 units
   d) 40 units
161. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is
   a) $\sqrt{288}$
   b) $\sqrt{18}$
   c) $\sqrt{72}$
   d) $\sqrt{33}$
162. The shortest distance from the plane

$$12x + 4y + 3z = 327 \tag{162.1}$$

to the sphere

$$x^2 + y^2 + z^2 + 4x - 2y - 6z = 155 \tag{162.2}$$

is

a) 39
b) 26
c) $11\frac{4}{13}$
d) 13

163. The two lines

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$$

and

$$\frac{x-1}{k} = \frac{y-4}{1} = \frac{z-5}{1}$$

are coplanar if

a) k = 3 or -2
b) k = 0 or -1
c) k = 1 or -1
d) k = 0 or -3

164. $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors such that $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $\vec{c} = 3$, then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = 0$ is equal to

a) 1
b) 0
c) -7
d) 7

165. The radius of the circle in which the sphere

$$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0 \tag{165.1}$$

is cut by the plane

$$x + 2y + 2z + 7 = 0 \tag{165.2}$$

is

a) 4
b) 1
c) 2
d) 3

166. A tetrahedron has vertices O(0,0,0), A(1,2,1), B(2,1,3) and C(-1,1,2). Then the angle between the faces OAB and ABC will be

a) 90^o
b) $\cos^{-1}(\frac{19}{35})$
c) $\cos^{-1}(\frac{17}{31})$
d) 30^o

167. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$=0 and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals

a) 0
b) 2
c) -1
d) 1

168. Consider a points A, B, C, D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$

respectively. Then ABCD is a
a) parallelogram but not a rhombus
b) square
c) rhombus
d) rectangle

169. If $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ are three non-coplanar vectors, then

$$(\overrightarrow{u}+\overrightarrow{v}-\overrightarrow{w}).(\overrightarrow{u}-\overrightarrow{v})\times(\overrightarrow{v}-\overrightarrow{w})$$

equals
a) $3\overrightarrow{u}.\overrightarrow{v}\times\overrightarrow{w}$
b) 0
c) $\overrightarrow{u}.\overrightarrow{v}\times\overrightarrow{w}$
d) $\overrightarrow{u}.\overrightarrow{w}\times\overrightarrow{v}$

170. Two system of rectangle axes have the same origin. If a plane cuts them at distances a,b,c and a', b', c' from the origin then
a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}=0$
b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=0$
c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}=0$
d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}=0$

171. Distance between two parallel planes

$$2x+y+2z=8 \tag{171.1}$$

$$4x+2y+4z+5=0 \tag{171.2}$$

is
a) $\frac{9}{2}$
b) $\frac{5}{2}$
c) $\frac{7}{2}$
d) $\frac{3}{2}$

172. A line with direction cosines proportional to 2, 1, 2 meets each of the lines

$$x=y+a=z \tag{172.1}$$

$$x+a=2y=2z \tag{172.2}$$

The coordinates os each of the points of intersection are given by
a) (2a,3a,3a), (2a,a,a)
b) (3a,2a,3a), (a,a,a)
c) (3a,2a,3a), (a,a,2a)
d) (3a,3a,3a), (a,a,a)

173. If the straight lines

$$x=1+s, y=-3-\lambda s, z=1+\lambda s \tag{173.1}$$

$$x=\frac{t}{2}, y=1+t, z=2-t \tag{173.2}$$

a) 0
b) -1

c) $\frac{-1}{2}$
d) -2

174. The intersection of the spheres

$$x^2 + y^2 + z^2 + 7x - 2y - z = 13 \tag{174.1}$$

$$x^2 + y^2 + z^2 - 3x + 3y + 4z = 8 \tag{174.2}$$

is the same as the intersection of one of the sphere and the plane

a) 2x-y-z=1
b) x-2y-z=1
c) x-y-2z=1
d) x-y-z=1

175. Let $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ be three non zero vectors such that no two of these are collinear. If the vector $\overrightarrow{a} + 2\overrightarrow{b}$ is collinear with $\overrightarrow{c}$ and $\overrightarrow{b} + 3\overrightarrow{c}$ is collinear with $\overrightarrow{a}$(λ being some non-zero scalar) then $\overrightarrow{a} + 2\overrightarrow{b} + 6\overrightarrow{c}$ equals

a) 0
b) $\lambda\overrightarrow{b}$
c) $\lambda\overrightarrow{c}$
d) $\lambda\overrightarrow{a}$

176. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The work done in standard units by the forces is given by

a) 15
b) 30
c) 25
d) 40

177. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are non-coplanar vectors and λ is a real number, then the vctors $\overrightarrow{a} + 2\overrightarrow{b} + 3\overrightarrow{c}$, $\lambda\overrightarrow{b} + 4\overrightarrow{c}$ and $(2\lambda - 1)\overrightarrow{c}$ are non coplanar for

a) no values of λ
b) all except one value of λ
c) all except two values of λ
d) all value of λ

178. Let $\overrightarrow{u}$, $\overrightarrow{v}$, $\overrightarrow{w}$ be such that $\lceil\overrightarrow{u}|$=1, $\lceil\overrightarrow{v}|$=2, $\lceil\overrightarrow{w}|$=3. If the projection $\overrightarrow{v}$ along $\lceil\overrightarrow{u}|$ is equal to that of $\overrightarrow{w}$ along $\overrightarrow{u}$ and $\overrightarrow{v}$, $\overrightarrow{w}$ are perpendicular to each other then $\lceil\overrightarrow{u} - \overrightarrow{v} + \overrightarrow{w}|$ equals

a) 14
b) $\sqrt{7}$
c) $\sqrt{14}$
d) 2

179. Let $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ be non-zero vectors such that $(\overrightarrow{a} \times \overrightarrow{b}) \times \overrightarrow{c} = \frac{1}{3}|\overrightarrow{b}|\lceil\overrightarrow{c}\lceil\overrightarrow{a}$. If θ is the acute angle between the vectors $\overrightarrow{b}$ and $\overrightarrow{c}$, then $\sin\theta$ equals

a) $\frac{2\sqrt{2}}{3}$
b) $\frac{\sqrt{2}}{3}$
c) $\frac{2}{3}$
d) $\frac{1}{3}$

180. If C is the mid-point of AB and P is any point outside AB, then

a) $\overrightarrow{AB} + \overrightarrow{PB} = 2\overrightarrow{PC}$

b) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$
c) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \overrightarrow{0}$
d) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{0}$

181. If the angle θ between the line

$$\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$$

and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ then the value of λ is

a) $\frac{5}{3}$
b) $\frac{-3}{5}$
c) $\frac{3}{4}$
d) $\frac{-4}{3}$

182. The angle between the lines

$$2x = 3y = -z$$

$$6x = -y = -4z$$

is

a) 0^o
b) 90^o
c) 45^o
d) 30^o

183. If the plane

$$2ax - 3ay + 4az + 6 = 0$$

passes through the midpoint of the line joining the centres of the spheres

$$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13 \tag{183.1}$$

$$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8 \tag{183.2}$$

the a equals

a) -1
b) 1
c) -2
d) 2

184. The distance between the lines

$$\overrightarrow{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$$

and the plane

$$\overrightarrow{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$$

is

a) $\frac{10}{9}$
b) $\frac{10}{3\sqrt{3}}$
c) $\frac{3}{10}$
d) $\frac{10}{3}$

185. For any vector $\overrightarrow{a}$, the value of

$$(\overrightarrow{a} \times \hat{i})^2 + (\overrightarrow{a} \times \hat{j})^2 + (\overrightarrow{a} \times \hat{k})^2$$

is equal to

a) $3\overrightarrow{a}^2$
b) $\overrightarrow{a}^2$
c) $2\overrightarrow{a}^2$
d) $4\overrightarrow{a}^2$

186. If non-zero numbers a,b,c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. The point is

a) (-1, 2)
b) (-1 -2)
c) (1, -2)
d) $(1, \frac{-1}{2}$

187. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

a) the Geometric Mean of a and b
b) the Arithmetic Mean of a and b
c) equal to zero
d) the Harmonic Mean of a and b

188. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are non-coplanar vectors and λ is a real number the

$$\lambda(\overrightarrow{a} + \overrightarrow{b})\lambda^2\overrightarrow{b}\lambda\overrightarrow{c} = [\overrightarrow{a}(\overrightarrow{b} + \overrightarrow{c})\overrightarrow{b}]$$

for

a) exactly one value of λ
b) no value of λ
c) exactly three values of λ
d) exactly two values of λ

189. Let $\overrightarrow{a} = \hat{i} - \hat{k}$, $\overrightarrow{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\overrightarrow{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then $[\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}]$ depends on

a) only y
b) only x
c) both x and y
d) neither x nor y

190. The plane

$$x + 2y - z = 4$$

cuts the sphere

$$x^2 + y^2 + z^2 - x + z - 2 = 0 \tag{190.1}$$

in a circle of radius

a) 3
b) 1
c) 2
d) $\sqrt{2}$

191. If $(\overrightarrow{a} \times \overrightarrow{b}) \times \overrightarrow{c} = \overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c})$ where $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ are any three vectors such that $\overrightarrow{a}.\overrightarrow{b} \neq 0$, $\overrightarrow{b}.\overrightarrow{c} \neq 0$ then $\overrightarrow{a}$ and $\overrightarrow{c}$ are

a) inclined at an angle of $\frac{\pi}{3}$ between them
b) inclined at an angle of $\frac{\pi}{6}$ between them
c) perpendicular
d) parallel

192. The values of a , for which points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right angled triangle with C=$\frac{\pi}{2}$ are
a) 2 and 1
b) -2 and -1
c) -2 and 1
d) 2 and -1

193. The two lines x=ay+b, z=cy+d; and x=$a'y+b'$, z=$c'y+d'$ are perpendicular to each other if
a) $aa'+cc'=-1$
b) $aa'+cc'=1$
c) $\frac{a}{a'}+\frac{c}{c'}$=-1
d) $\frac{a}{a'}+\frac{c}{c'}$=1

194. The image of the point (-1,3,4) in the plane x-2y=0 is
a) $(\frac{-17}{3},\frac{-19}{3}),4$
b) (15,11,4)
c) $(\frac{-17}{3},\frac{-19}{3}),1$
d) None of these

195. If a line makes an angle of $\pi/4$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is
a) $\frac{\pi}{4}$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{3}$

196. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for
a) no value of θ
b) exactly one value of θ
c) exactly two values of θ
d) more than two values of θ

197. If (2,3,5) is one end of a diameter of the sphere

$$x^2+y^2+z^2-6x-12y-2z+20=0 \tag{197.1}$$

then the coordinates of the other end of the diameter are
a) (4,3,5)
b) (4,3,-3)
c) (4,9,-3)
d) (4,-3,3)

198. Let $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}$, $\overrightarrow{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\overrightarrow{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vectors $\overrightarrow{c}$ lies in the plane of $\overrightarrow{a}$ and $\overrightarrow{b}$, then x equals
a) -4
b) -2
c) 0
d) 1

199. If L be the line of intersection of the planes

$$2x + 3y + z = 1$$

$$x + 3y + 2z = 2$$

If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals

a) 1
b) $\frac{1}{\sqrt{2}}$
c) $\frac{1}{\sqrt{3}}$
d) $\frac{1}{2}$

200. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then whoch one of the following fives possible values of α and β?

a) $\alpha = 2, \beta = 2$
b) $\alpha = 1, \beta = 2$
c) $\alpha = 2, \beta = 1$
d) $\alpha = 1, \beta = 1$

201. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = 7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

a) 0
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) π

202. The line passing through the points (5,1,a) and (3,b,1) crosses the yz-plane at the point $(0, \frac{17}{2}, \frac{-13}{2})$. Then

a) a=2, b=8
b) a=4, b=6
c) a=6, b=4
d) a=8, b=2

203. If the straight lines

$$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$$

and

$$\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$$

intersect at a point, then the integer k is equal to

a) -5
b) 5
c) 2
d) -2

204. Let the line

$$\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$$

lie in the plane x+3y-αz+β=0. Then (α, β) equals

a) (-6,7)
b) (5,-15)

c) (-5,5)
d) (6,-17)

205. The projections of a vector om the three coordinates axis are 6, -3, 2 respectively. The direction cosnies of the vector are:
a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$
b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$
d) 6, -3, 2

206. If $\vec{u}$, $\vec{v}$, $\vec{w}$ are non=coplanar vectors and p,q are real numbers then the equality

$$[3\vec{u}\, p\vec{v}\, p\vec{w}] - [p\vec{v}\, \vec{w} q\vec{u}] - [2\vec{w} q\vec{v} q\vec{u}] = 0$$

holds for:
a) exactly two values of (p,q)
b) more than two but not all values of (p,q)
c) all values of (p,q)
d) exactly one value of (p,q)

207. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$
a) $2\hat{i} - \hat{j} + 2\hat{k}$
b) $\hat{i} - \hat{j} - 2\hat{k}$
c) $\hat{i} + \hat{j} - 2\hat{k}$
d) $-\hat{i} + \hat{j} - 2\hat{k}$

208. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{2}k$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ)=
a) 2,-3
b) -2,3
c) 3,-2
d) -3,2

209. **Statement-1**: The point A(3,1,6) is the mirror image of the point B(1,3,4) in the plane x-y+z=5
Statement-2: The plane x-y+z=5 bisects the line segment joining A(3,1,6) and B(1,3,4).
a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
b) Statement-1 is true, Statement-2 is false
c) Statement-1 is false, Statement-2 is true
d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

210. A line AB in three-dimensional space makes angle 45^o and 120^o with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals
a) 45^o
b) 60^o
c) 75^o
d) 30^o

211. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane x+2y+3z=4 is $\cos^{-1}(\sqrt{\frac{5}{14}})$, then λ equals
a) $\frac{3}{2}$
b) $\frac{2}{5}$
c) $\frac{5}{3}$
d) $\frac{2}{3}$

212. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b})[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is
a) -3

b) 5
c) -3
d) -5

213. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to

a) $\vec{c} + (\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}})\vec{b}$
b) $\vec{b} + (\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}})\vec{c}$
c) $\vec{c} - (\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}})\vec{b}$
d) $\vec{b} - (\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}})\vec{c}$

214. **Statement-1**: The point A(1,0,7) is the mirror image of the point B(1,6,3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
Statement-2: The plane $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1,0,7) and B(1,6,3).

a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
b) Statement-1 is true, Statement-2 is false
c) Statement-1 is false, Statement-2 is true
d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

215. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c} = \vec{a} + 2\vec{b}$ and $\vec{d} = 5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is:

a) $\frac{\pi}{6}$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{4}$

216. A equation of a plane parallel to the plane

$$x - 2y + 2z - 5 = 0 \tag{216.1}$$

and at a unit distance from the origin is:

a) x-2y+2z-3=0
b) x-2y+2z+1=0
c) x-2y+2z-1=0
d) x-2y+2z+5=0

217. If the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$

and

$$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

intersect, then k is equal to

a) -1
b) $\frac{2}{9}$
c) $\frac{9}{2}$
d) 0

218. Let ABCD be a parallelogram such that $\vec{AB} = \vec{q}$, $\vec{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coinside with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by:

a) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

b) $\overrightarrow{r} = -\overrightarrow{q} + \frac{(\overrightarrow{p}.\overrightarrow{q})}{(\overrightarrow{p}.\overrightarrow{p})}\overrightarrow{p}$
c) $\overrightarrow{r} = \overrightarrow{q} - \frac{(\overrightarrow{p}.\overrightarrow{q})}{(\overrightarrow{p}.\overrightarrow{p})}\overrightarrow{p}$
d) $\overrightarrow{r} = -3\overrightarrow{q} - \frac{3(\overrightarrow{p}.\overrightarrow{q})}{(\overrightarrow{p}.\overrightarrow{p})}\overrightarrow{p}$

219. Distance between the two parallel planes

$$2x + y + 2z = 8$$

$$4x + 2y + 4z + 5 = 0$$

is
a) $\frac{3}{2}$
b) $\frac{5}{2}$
c) $\frac{7}{2}$
d) $\frac{9}{2}$

220. If the lines

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$$

and

$$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$$

are coplanr, then k can have
a) any value
b) exactly one value
c) exactly two values
d) exactly three values

221. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is
a) $\sqrt{18}$
b) $\sqrt{72}$
c) $\sqrt{33}$
d) $\sqrt{45}$

222. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane 2x-y+z+3=0 is the line:
a) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$
b) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$
c) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$
d) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

223. The angle between the lines whose direction cosines satisfy the equations

$$l + m + n = 0$$

$$l^2 = m^2 + n^2$$

is
a) $\frac{\pi}{6}$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{4}$

224. Let $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ be three non-zero vectors such that no two of them are collinear and

$$(\overrightarrow{a} \times \overrightarrow{b}) \times \overrightarrow{c} = \frac{1}{3}|\overrightarrow{b}||\overrightarrow{c}|\overrightarrow{a}$$

If θ is the angle between vectors $\overrightarrow{b}$ and $\overrightarrow{c}$, then a value of $\sin\theta$ is:

a) $\frac{2}{3}$
b) $\frac{-2\sqrt{3}}{3}$
c) $\frac{2\sqrt{2}}{3}$
d) $\frac{-\sqrt{2}}{3}$

225. The equation of the plane containing the line

$$2x - 5y + z = 3$$

$$x + y + 4z = 5$$

and parallel to the plane x+3y+6z=1 is:

a) x+3y+6z=7
b) 2x+6y+12z=-13
c) 2x+6y+12z=13
d) x+3y+6z=-7

226. The distance of the point (1,0,2) from the point of intersection of the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$$

and the plane x-y+z=16, is

a) $3\sqrt{21}$
b) 13
c) $2\sqrt{14}$
d) 8

227. If the line

$$\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$$

lies in the plane, lx+my-z=9, then $l^2 + m^2$ is equal to:

a) 5
b) 2
c) 26
d) 18

228. Let $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$ be three unit vectors such that

$$\overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c}) = \frac{\sqrt{3}}{2}(\overrightarrow{b} + \overrightarrow{c})$$

If $\overrightarrow{b}$ is not parallel to $\overrightarrow{c}$, then the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is:

a) $\frac{2\pi}{3}$
b) $\frac{5\pi}{6}$
c) $\frac{3\pi}{4}$
d) $\frac{\pi}{2}$

229. The distance of the point (1,-5,9) from the plane x-y+z=5 measured along the line x=y=z is:

a) $\frac{10}{\sqrt{3}}$
b) $\frac{20}{3}$
c) $3\sqrt{10}$
d) $10\sqrt{3}$

230. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that

$$|\vec{c} - \vec{a}| = 3, |(\vec{a} \times \vec{b}) \times \vec{c}| = 3$$

and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30^o. Then $\vec{a}.\vec{c}$ is equal to:
a) $\frac{1}{8}$
b) $\frac{25}{8}$
c) 2
d) 5

231. If the image of the point P(1,-2,3) in the plane 2x+3y-4z+22=0 measured parallel to line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to
a) $6\sqrt{5}$
b) $3\sqrt{5}$
c) $2\sqrt{42}$
d) $\sqrt{42}$

232. The distance of the point (1,3,-7) from the plane passing through the point (1,-1,-1) having normal perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$$

and

$$\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$$

is
a) $\frac{10}{\sqrt{74}}$
b) $\frac{20}{\sqrt{74}}$
c) $\frac{10}{\sqrt{83}}$
d) $\frac{5}{\sqrt{83}}$

233. Let $\vec{u}$ be a vector coplanr with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} - 24$, then $|\vec{u}|^2$ is equal to:
a) 315
b) 256
c) 84
d) 336

234. The length of the projection of the line segment joining the points (5,-1,4) and (4,-1,3) on the plane x+y+z=7 is:
a) $\frac{2}{3}$
b) $\frac{1}{3}$
c) $\sqrt{\frac{2}{3}}$
d) $\frac{2}{\sqrt{3}}$

235. If L_1 is the line of intersection of the planes

$$2x - 2y + 3z - 2 = 0$$

$$x - y + z + 1 = 0$$

and L_2 is the line of intersection of the planes

$$x + 2y - z - 3 = 0$$

$$3x - y + 2z - 1 = 0$$

then the distance of the origin from the plane, containing the lines L_1 and L_2 is

a) $\frac{1}{3\sqrt{2}}$
b) $\frac{1}{2\sqrt{2}}$
c) $\frac{1}{\sqrt{2}}$
d) $\frac{1}{4\sqrt{2}}$

236. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to

a) $\frac{19}{2}$
b) 9
c) 8
d) $\frac{17}{2}$

237. The equation of the line passing through the point (-4,3,1), parallel to the plane x+2y-z-5=0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is

a) $\frac{x-4}{-3} = \frac{y+3}{1} = \frac{z+1}{4}$
b) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$
c) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$
d) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

238. The plane through the inersection of the planes x+y+z=1 and 2x+3y-z+4=0 and parallel to y-axis also passes through the point:

a) (-3,0,-1)
b) (-3,1,1)
c) (3,3,-1)
d) (3,2,1)

239. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, x+2y+3z=15 at a point P, then the distance of P from the origin is

a) $\frac{\sqrt{5}}{2}$
b) $2\sqrt{5}$
c) $\frac{9}{2}$
d) $\frac{7}{2}$

240. A plane passing through the points (0, -1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y - z + 5 = 0, also passes through the point:

a) $(-\sqrt{2}, 1, -4)$
b) $(\sqrt{2}, -1, 4)$
c) $(-\sqrt{2}, -1, -4)$
d) $(\sqrt{2}, 1, 4)$

241. Let $\overrightarrow{\alpha} = 3i + \hat{j}$ and $\overrightarrow{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\overrightarrow{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$, where $\overrightarrow{\beta_1}$ is parallel to $\overrightarrow{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\overrightarrow{\alpha}$, then $\overrightarrow{\beta_1} \times \overrightarrow{\beta_2}$ is equal to:
   a) $-3\hat{i} + 9\hat{j} + 5\hat{k}$
   b) $3\hat{i} - 9\hat{j} - 5\hat{k}$
   c) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$
   d) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$