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Abstract—This book provides a computational approach to school mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/ncert/codes
```

1 LINE

1.1 Examples

- Do the points $\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ form a triangle? If so, name the type of triangle formed.

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- Show that the points $\begin{pmatrix} 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 4 \end{pmatrix}$ are the vertices of a square.
- Verify if $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ are points on a line.
- Find the condition for $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ to be equidistant from the points $\begin{pmatrix} 7 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.
- Find a point on the y-axis which is equidistant from the points $\mathbf{A} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.
- Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8.

Solution: Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \quad (1.1.6.1)$$

Then the point \mathbf{C}

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k + 1} \quad (1.1.6.2)$$

divides AB in the ratio $k : 1$. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1.6

```
codes/line/draw_section.py
```

- Find a unit vector in the direction of $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.
- Find the direction vector of PQ , where

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \quad (1.1.8.1)$$

$$\mathbf{Q} = \begin{pmatrix} -1 \\ -2 \\ -4 \end{pmatrix} \quad (1.1.8.2)$$

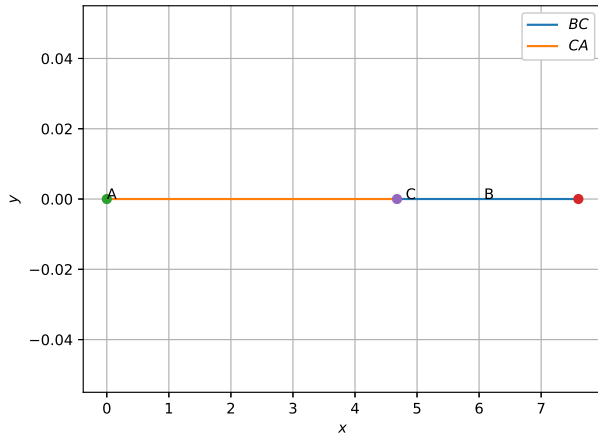


Fig. 1.1.6

1.2 Elementary Exercises

1. Find the distance between the following pairs of points

a)

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad (1.2.1.1)$$

b)

$$\begin{pmatrix} -5 \\ 7 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (1.2.1.2)$$

c)

$$\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} -1 \\ b \end{pmatrix} \quad (1.2.1.3)$$

2. Find the distance between the points

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 36 \\ 15 \end{pmatrix} \quad (1.2.2.1)$$

3. A town B is located 36km east and 15 km north of the town A. How would you find the distance from town A to town B without actually measuring it?

4. Determine if the points

$$\begin{pmatrix} 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ -11 \end{pmatrix} \quad (1.2.4.1)$$

are collinear.

5. Check whether

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix}, \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (1.2.5.1)$$

are the vertices of an isosceles triangle.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

a)

$$\begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.2.6.1)$$

b)

$$\begin{pmatrix} -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -4 \end{pmatrix} \quad (1.2.6.2)$$

c)

$$\begin{pmatrix} 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (1.2.6.3)$$

7. Find the point on the x -axis which is equidistant from

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix}, \begin{pmatrix} -2 \\ 9 \end{pmatrix}, \quad (1.2.7.1)$$

8. Find the values of y for which the distance between the points

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 10 \\ y \end{pmatrix} \quad (1.2.8.1)$$

is 10 units.

9. Find the values of x, y, z such that

$$\begin{pmatrix} x \\ 2 \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ y \\ 1 \end{pmatrix} \quad (1.2.9.1)$$

10. If

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad (1.2.10.1)$$

verify if

a) $\|\mathbf{a}\| = \|\mathbf{b}\|$

b) $\mathbf{a} = \mathbf{b}$

11. Find a vector \mathbf{x} in the direction of $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ such that $\|\mathbf{x}\| = 7$.

12. Find a unit vector in the direction of $\mathbf{a} + \mathbf{b}$, where

$$\mathbf{a} = \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad (1.2.12.1)$$

1.3 Section Formula

- Find the coordinates of the point which divides the join of

$$\begin{pmatrix} -1 \\ 7 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad (1.3.1.1)$$

in the ratio 2 : 3.

- Find the coordinates of the points of trisection of the line segment joining $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.
- Find the ratio in which the line segment joining the points $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ is divided by $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$.
- Find the ratio in which the line segment joining $\mathbf{A} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ is divided by the x -axis. Also find the coordinates of the point of division.
- If $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ y \end{pmatrix}, \begin{pmatrix} x \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ are the vertices of a parallelogram taken in order, find x and y .
- If $\mathbf{A} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ respectively, find the coordinates of \mathbf{P} such that $AP = \frac{3}{7}AB$ and \mathbf{P} lies on the line segment AB .
- Find the coordinates of the points which divide the line segment joining $\mathbf{A} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$ into four equal parts.
- Find the value of k if the points $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ k \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ are collinear.
- In each of the following, find the value of k for which the points are collinear
 - $\begin{pmatrix} 7 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ k \end{pmatrix}$
 - $\begin{pmatrix} 8 \\ 1 \end{pmatrix}, \begin{pmatrix} k \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \end{pmatrix}$
- Find a condition on \mathbf{x} such that the points $\mathbf{x}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ are collinear.
- If

$$\mathbf{P} = 3\mathbf{a} - 2\mathbf{b} \quad (1.3.11.1)$$

$$\mathbf{Q} = \mathbf{a} + \mathbf{b} \quad (1.3.11.2)$$

find \mathbf{R} , which divides PQ

- internally,
- externally.

1.4 Line Equation

- Determine the ratio in which the line

$$(2 \ x) - 4 = 0 \quad (1.4.1.1)$$

divides the line segment joining the points $\mathbf{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

-

2 TRIANGLE

2.1 Construction

- Draw $\triangle ABC$ where $\angle B = 90^\circ, a = 4$ and $b = 3$.

Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.1.1.1)$$

The following code plots Fig. 2.1.1

```
codes/triangle/rt_triangle.py
```

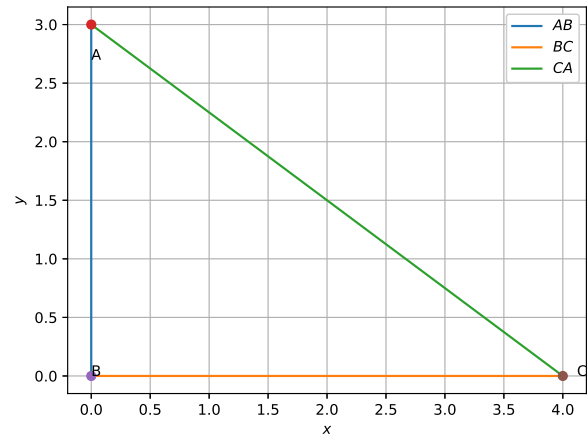


Fig. 2.1.1

- Construct a triangle of sides $a = 4, b = 5$ and $c = 6$.

Solution: Let the vertices of $\triangle ABC$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (2.1.2.1)$$

$$\mathbf{A}^T \triangleq (p \ q) \quad (2.1.2.2)$$

$$\|\mathbf{A}\|^2 = \mathbf{A}^T \mathbf{A} = (p \ q) \begin{pmatrix} p \\ q \end{pmatrix} \quad (2.1.2.3)$$

$$= p \times p + q \times q = p^2 + q^2 \quad (2.1.2.4)$$

Then

$$AB \triangleq \|A - B\|^2 = \|A\|^2 = c^2 \quad \because B = 0 \quad (2.1.2.5)$$

$$BC = \|C - B\|^2 = \|C\|^2 = a^2 \quad (2.1.2.6)$$

$$AC = \|A - C\|^2 = b^2 \quad (2.1.2.7)$$

From (2.1.2.7),

$$b^2 = \|A - C\|^2 = \|A - C\|^T \|A - C\| \quad (2.1.2.8)$$

$$= A^T A + C^T C - A^T C - C^T A \quad (2.1.2.9)$$

$$= \|A\|^2 + \|C\|^2 - 2A^T C \quad (\because A^T C = C^T A) \quad (2.1.2.10)$$

$$= a^2 + c^2 - 2ap \quad (2.1.2.11)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (2.1.2.12)$$

From (2.1.2.5),

$$\|A\|^2 = c^2 = p^2 + q^2 \quad (2.1.2.13)$$

$$\implies q = \pm \sqrt{c^2 - p^2} \quad (2.1.2.14)$$

The following code plots Fig. 2.1.2

codes/triangle/draw_triangle.py

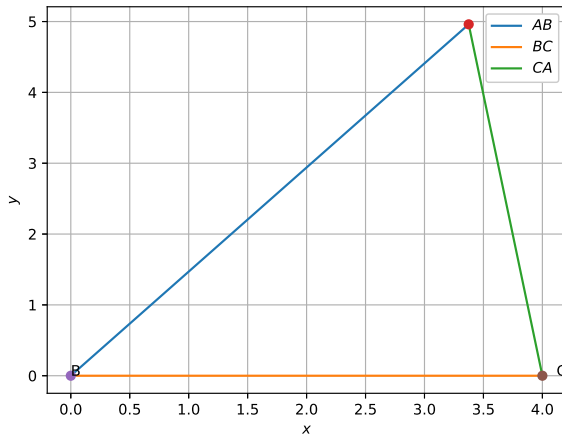


Fig. 2.1.2

3. Construct a triangle of sides $a = 5$, $b = 6$ and $c = 7$. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.

Solution: The sides of the similar triangle are $\frac{7}{5}a$, $\frac{7}{5}b$ and $\frac{7}{5}c$.

4. Construct an isosceles triangle whose base is $a = 8\text{cm}$ and altitude $AD = h = 4\text{cm}$

Solution: Using Baudhayana's theorem,

$$b = c = \sqrt{h^2 + \left(\frac{a}{2}\right)^2} \quad (2.1.4.1)$$

5. In $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c and sketch the triangle.

Solution: From the given information,

$$a + b + c = 11 \quad (2.1.5.1)$$

$$b = c \quad (\because B = C = 45^\circ) \quad (2.1.5.2)$$

$$a^2 = b^2 + c^2 \quad (\because A = 90^\circ) \quad (2.1.5.3)$$

From (2.1.5.1) and (2.1.5.2),

$$a + 2b = 11 \quad (2.1.5.4)$$

From (2.1.5.2) and (2.1.5.3),

$$a^2 = 2b^2 \implies a - b\sqrt{2} = 0 \quad (2.1.5.5)$$

(2.1.5.4) and (2.1.5.5) can be summarized as the matrix equation

$$\begin{pmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix} \quad (2.1.5.6)$$

which can be solved using Cramer's rule as

$$a = \frac{\begin{vmatrix} 11 & 2 \\ 0 & -\sqrt{2} \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{vmatrix}} = \frac{11 \times (-\sqrt{2}) - 2 \times 0}{1 \times (-\sqrt{2}) - 2 \times 1} \quad (2.1.5.7)$$

$$= \frac{11\sqrt{2}}{2 + \sqrt{2}} \quad (2.1.5.8)$$

$$b = \frac{\begin{vmatrix} 11 & 1 \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{vmatrix}} = \frac{11}{2 + \sqrt{2}} \quad (2.1.5.9)$$

by expanding the determinants. The following code may be used to compute a, b and c .

codes/triangle/triangle_det.py

6. Repeat Problem 2.1.5 using a single matrix equation.

Solution: The equations

$$a + 2b = 11 \quad (2.1.6.1)$$

$$a - b\sqrt{2} = 0 \quad (2.1.6.2)$$

$$b - c = 0 \quad (2.1.6.3)$$

can be expressed as a single matrix equation

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 0 \end{pmatrix} \quad (2.1.6.4)$$

and can be solved using Cramer's rule as

$$a = \frac{\begin{vmatrix} 11 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}} \quad (2.1.6.5)$$

$$b = \frac{\begin{vmatrix} 0 & 11 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}} \quad (2.1.6.6)$$

$$c = \frac{\begin{vmatrix} 0 & 2 & 11 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}} \quad (2.1.6.7)$$

The determinant

$$\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0 \times \begin{vmatrix} -\sqrt{2} & 0 \\ 1 & -1 \end{vmatrix} - 2 \times \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & -\sqrt{2} \\ 0 & 1 \end{vmatrix} \quad (2.1.6.8)$$

The determinant can also be expressed as

$$\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0 \times \begin{vmatrix} -\sqrt{2} & 0 \\ 1 & -1 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} + 0 \times \begin{vmatrix} 2 & 0 \\ -\sqrt{2} & 0 \end{vmatrix} \quad (2.1.6.9)$$

The determinants of larger matrices can be

expressed similarly.

7. Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60^\circ$.

Solution: In Fig. (2.1.7), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \quad (2.1.7.1)$$

$$\cos B = \frac{x}{a}, \quad (2.1.7.2)$$

Thus,

$$a = x + y = b \cos C + c \cos B, \quad (2.1.7.3)$$

$$b = c \cos A + a \cos C \quad (2.1.7.4)$$

$$c = b \cos A + a \cos B \quad (2.1.7.5)$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (2.1.7.6)$$

Using Cramer's rule and determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \quad (2.1.7.7)$$

$$= \frac{b^2 + c^2 - a^2}{2bc} \quad (2.1.7.8)$$

From (2.1.7.8)

$$b^2 = c^2 + a^2 - 2ca \cos B \quad (2.1.7.9)$$

which is computed by the following code

```
codes/triangle/cos_form.py
```

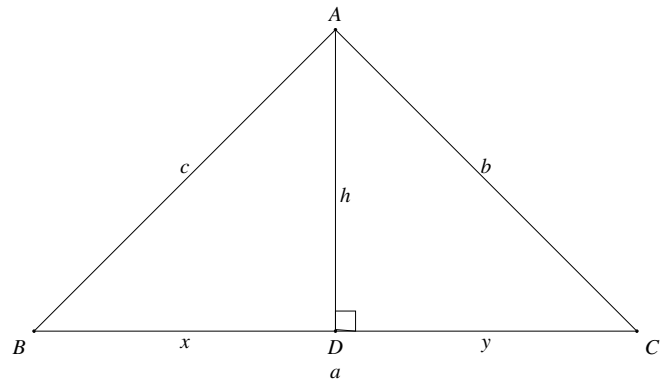


Fig. 2.1.7: The cosine formula

8. Draw $\triangle ABC$ with $a = 7$, $\angle B = 45^\circ$ and $\angle A = 105^\circ$.

Solution: In Fig. (2.1.7),

$$\sin B = \frac{h}{c} \quad (2.1.8.1)$$

$$\sin C = \frac{h}{b} \quad (2.1.8.2)$$

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (2.1.8.3)$$

Thus,

$$c = \frac{a \sin C}{\sin A} \quad (2.1.8.4)$$

where

$$C = 180 - A - B \quad (2.1.8.5)$$

9. Draw $\triangle ABC$ if $AB = 3$, $AC = 5$ and $\angle C = 30^\circ$.

Solution: From (2.1.7.9),

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \quad (2.1.9.1)$$

which can be expressed as

$$a^2 - 2ab \cos C + b^2 - c^2 = 0. \quad (2.1.9.2)$$

$$\therefore (a - b \cos C)^2 = a^2 + b^2 \cos^2 C - 2ab \cos C, \quad (2.1.9.3)$$

(2.1.9.2) can be expressed as

$$(a - b \cos C)^2 - b^2 \cos^2 C + b^2 - c^2 = 0 \quad (2.1.9.4)$$

$$\Rightarrow (a - b \cos C)^2 = b^2 (1 - \cos^2 C) - c^2 \quad (2.1.9.5)$$

$$\text{or, } a = b \cos C \pm \sqrt{b^2 (1 - \cos^2 C) - c^2} \quad (2.1.9.6)$$

Choose the value(s) for which $a > 0$.

10. The solution of a quadratic equation

$$\alpha x^2 + \beta x + \gamma = 0 \quad (2.1.10.1)$$

is given by

$$x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}. \quad (2.1.10.2)$$

Verify (2.1.9.6) using (2.1.10.2).

11. $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.

Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (2.1.11.1)$$

$$\Rightarrow (18 - c)^2 = 12^2 + c^2 \quad (2.1.11.2)$$

which can be simplified to obtain

$$36c - 180 = 0 \quad (2.1.11.3)$$

$$\Rightarrow c = 5 \quad (2.1.11.4)$$

and $b = 13$

12. Find a simpler solution for Problem 2.1.5

Solution: Use cosine formula.

13. In $\triangle ABC$, $a = 7$, $\angle B = 75^\circ$ and $b + c = 13$. Alternatively,

$$a = b \cos C + c \cos B \quad (2.1.13.1)$$

$$b \sin C = c \sin B \quad (2.1.13.2)$$

$$a + b + c = 11 \quad (2.1.13.3)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (2.1.13.4)$$

Solving the equivalent matrix equation gives the desired answer.

2.2 Construction Exercises

1. In $\triangle ABC$, $a = 8$, $\angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.
2. In $\triangle ABC$, $a = 6$, $\angle B = 60^\circ$ and $b - c = 2$. Sketch $\triangle ABC$.
3. Draw $\triangle ABC$, given that $a + b + c = 11$, $\angle B = 30^\circ$ and $\angle C = 90^\circ$.
4. Construct $\triangle xyz$ where $xy = 4.5$, $yz = 5$ and $zx = 6$.
5. Draw an equilateral triangle of side 5.5.
6. Draw $\triangle PQR$ with $PQ = 4$, $QR = 3.5$ and $PR = 4$. What type of triangle is this?
7. Construct $\triangle ABC$ such that $AB = 2.5$, $BC = 6$ and $AC = 6.5$. Find $\angle B$.
8. Construct $\triangle PQR$, given that $PQ = 3$, $QR = 5.5$ and $\angle PQR = 60^\circ$.
9. Construct $\triangle DEF$ such that $DE = 5$, $DF = 3$ and $\angle D = 90^\circ$.
10. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110° .
11. Construct $\triangle ABC$ with $BC = 7.5$, $AC = 5$ and $\angle C = 60^\circ$.

12. Construct $\triangle XYZ$ if $XY = 6$, $\angle X = 30^\circ$ and $\angle Y = 100^\circ$.
13. If $AC = 7$, $\angle A = 60^\circ$ and $\angle B = 50^\circ$, can you draw the triangle?
14. Construct $\triangle ABC$ given that $\angle A = 60^\circ$, $\angle B = 30^\circ$ and $AB = 5.8$.
15. Construct $\triangle PQR$ if $PQ = 5$, $\angle Q = 105^\circ$ and $\angle R = 40^\circ$.
16. Can you construct $\triangle DEF$ such that $EF = 7.2$, $\angle E = 110^\circ$ and $\angle F = 180^\circ$?
17. Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.
18. Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.
19. Construct right angled \triangle whose hypotenuse is 6 and one of the legs is 4.
20. Construct an isosceles right angled $\triangle ABC$ right angled at C such $AC = 6$.
21. Construct the triangles in Table 2.2.21.

S.No	Triangle	Given Measurements		
1	$\triangle ABC$	$\angle A = 85^\circ$	$\angle B = 115^\circ$	$AB = 5$
2	$\triangle PQR$	$\angle Q = 30^\circ$	$\angle R = 60^\circ$	$QR = 4.7$
3	$\triangle ABC$	$\angle A = 70^\circ$	$\angle B = 50^\circ$	$AC = 3$
4	$\triangle LMN$	$\angle L = 60^\circ$	$\angle N = 120^\circ$	$LM = 5$
5	$\triangle ABC$	$BC = 2$	$AB = 4$	$AC = 2$
6	$\triangle PQR$	$PQ = 2.5$	$QR = 4$	$PR = 3.5$
7	$\triangle XYZ$	$XY = 3$	$YZ = 4$	$XZ = 5$
8	$\triangle DEF$	$DE = 4.5$	$EF = 5.5$	$DF = 4$

TABLE 2.2.21

2.3 Triangle Geometry

1. Find the area of a triangle whose vertices are $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$.
2. Find the area of a triangle formed by the vertices $\mathbf{A} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$.
3. Find the area of a triangle formed by the points $\mathbf{P} = \begin{pmatrix} -1.5 \\ 3 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.
4. Find the area of the triangle whose vertices are
 - a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$
 - b) $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

5. Find the area of the triangle formed by joining the mid points of the sides of a triangle whose vertices are $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
6. Verify that the median of $\triangle ABC$ with vertices $\mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ divides it into two triangles of equal areas.
7. The vertices of $\triangle ABC$ are $\mathbf{A} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$. A line is drawn to intersect sides AB and AC at D and E respectively, such that

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4} \quad (2.3.7.1)$$

Find

$$\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC}. \quad (2.3.7.2)$$

8. Let $\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ be the vertices of $\triangle ABC$.
 - a) The median from \mathbf{A} meets BC at \mathbf{D} . Find the coordinates of the point \mathbf{D} .
 - b) Find the coordinates of the point \mathbf{P} on AD such that $AP : PD = 2 : 1$.
 - c) Find the coordinates of the points \mathbf{Q} and \mathbf{R} on medians BE and CF respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$.
9. In $\triangle ABC$, Show that the centroid

$$\mathbf{O} = \frac{\mathbf{A} + \mathbf{B} + \mathbf{C}}{3} \quad (2.3.9.1)$$

10. Show that the points

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix} \quad (2.3.10.1)$$

are the vertices of a right angled triangle.

3 QUADRILATERAL

3.1 Construction Examples

1. Draw $ABCD$ with $AB = a = 4.5$, $BC = b = 5.5$, $CD = c = 4$, $AD = d = 6$ and $AC = e = 7$.
Solution: Fig. 3.1.1 shows a rough sketch of $ABCD$. Letting

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (3.1.1.1)$$

it is trivial to sketch $\triangle ABC$ from Problem 2.1.2. $\triangle ACD$ can be obtained by rotating an equivalent triangle with AC on the x -axis by an angle θ with

$$\mathbf{D} = \begin{pmatrix} h \\ k \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} e \\ 0 \end{pmatrix} \quad (3.1.1.2)$$

and

$$\cos \theta = \frac{a^2 + e^2 - b^2}{2ae} \quad (3.1.1.3)$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (3.1.1.4)$$

The coordinates of the rotated triangle ACD are

$$\mathbf{D} = \mathbf{P} \begin{pmatrix} h \\ k \end{pmatrix} \quad (3.1.1.5)$$

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3.1.1.6)$$

$$\mathbf{C} = \mathbf{P} \begin{pmatrix} e \\ 0 \end{pmatrix} \quad (3.1.1.7)$$

where

$$\mathbf{P} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (3.1.1.8)$$

The following code plots quadrilateral $ABCD$

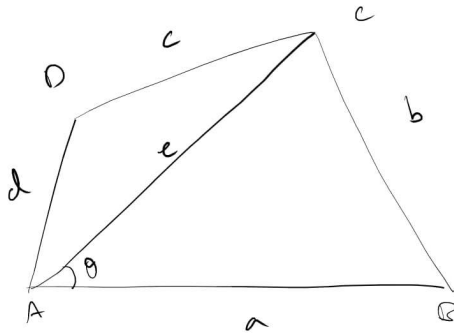


Fig. 3.1.1

in Fig. 3.1.1

```
codes/quad/draw_quad.py
```

2. Draw the parallelogram $MORE$ with $OR = 6$, $RE = 4.5$ and $EO = 7.5$.

Solution: Diagonals of a parallelogram bisect each other. Opposite sides of a parallelogram are equal and parallel.

3. Construct a kite $EASY$ if $AY = 8$, $EY = 4$ and $SY = 6$.

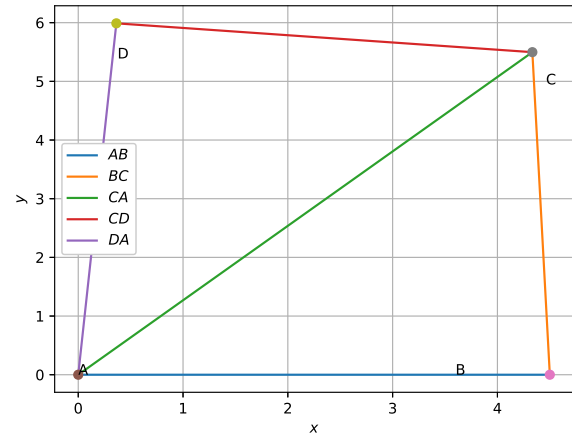


Fig. 3.1.1

Solution: The diagonals of a kite are perpendicular to each other.

4. Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.

Solution: Diagonals of a rhombus bisect each other at right angles.

3.2 Construction Exercises

1. Construct a quadrilateral $ABCD$ such that $AB = 5$, $\angle A = 50^\circ$, $AC = 4$, $BD = 5$ and $AD = 6$.
2. Construct $PQRS$ where $PQ = 4$, $QR = 6$, $RS = 5$, $PS = 5.5$ and $PR = 7$.
3. Draw $JUMP$ with $JU = 3.5$, $UM = 4$, $MP = 5$, $PJ = 4.5$ and $PU = 6.5$.
4. Construct a quadrilateral $ABCD$ such that $BC = 4.5$, $AC = 5.5$, $CD = 5$, $BD = 7$ and $AD = 5.5$.
5. Can you construct a quadrilateral $PQRS$ with $PQ = 3$, $RS = 3$, $PS = 7.5$, $PR = 8$ and $SQ = 4$?
6. Construct $LIFT$ such that $LI = 4$, $IF = 3$, $TL = 2.5$, $LF = 4.5$, $IT = 4$.
7. Draw $GOLD$ such that $OL = 7.5$, $GL = 6$, $GD = 6$, $LD = 5$, $OD = 10$.
8. Draw rhombus $BEND$ such that $BN = 5.6$, $DE = 6.5$.
9. Construct a quadrilateral $MIST$ where $MI = 3.5$, $IS = 6.5$, $\angle M = 75^\circ$, $\angle I = 105^\circ$ and $\angle S = 120^\circ$.
10. Can you construct the above quadrilateral $MIST$ if $\angle M = 100^\circ$ instead of 75° .

11. Can you construct the quadrilateral *PLAN* if $PL = 6, LA = 9.5, \angle P = 75^\circ, \angle L = 150^\circ$ and $\angle A = 140^\circ$?
12. Construct *MORE* where $MO = 6, OR = 4.5, \angle M = 60^\circ, \angle O = 105^\circ, \angle R = 105^\circ$.
13. Construct *PLAN* where $PL = 4, LA = 6.5, \angle P = 90^\circ, \angle A = 110^\circ$ and $\angle N = 85^\circ$.
14. Construct parallelogram *HEAR* where $HE = 5, EA = 6, \angle R = 85^\circ$.
15. Draw rectangle *OKAY* with $OK = 7$ and $KA = 5$.
16. Construct *ABCD*, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105^\circ$ and $\angle C = 80^\circ$.
17. Construct *DEAR* with $DE = 4, EA = 5, AR = 4.5, \angle E = 60^\circ$ and $\angle A = 90^\circ$.
18. Construct *TRUE* with $TR = 3.5, RU = 3, UE = 4, \angle R = 75^\circ$ and $\angle U = 120^\circ$.
19. Draw a square of side 4.5.
20. Can you construct a rhombus *ABCD* with $AC = 6$ and $BD = 7$?
21. Draw a square *READ* with $RE = 5.1$.
22. Draw a rhombus whose diagonals are 5.2 and 6.4.
23. Draw a rectangle with adjacent sides 5 and 4.
24. Draw a parallelogram *OKAY* with $OK = 5.5$ and $KA = 4.2$.

3.3 Quadrilateral Geometry

1. Find the area of a rhombus if its vertices are $\begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ taken in order.
2. If $\mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, find the area of the quadrilateral *ABCD*.
3. Find the area of the quadrilateral whose vertices, taken in order, are $\begin{pmatrix} -4 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.
4. The two opposite vertices of a square are $\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Find the coordinates of the other two vertices.
5. *ABCD* is a rectangle formed by the points $\mathbf{A} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$. $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ are the mid points of *AB, BC, CD, DA* respectively. Is the quadrilateral *PQRS* a
 - a) square?
 - b) rectangle?
 - c) rhombus?

4 CIRCLE

4.1 Construction Examples

1. Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents *AC* and *CD* to the circle.

Solution: The tangent is perpendicular to the radius. From the given information, in $\triangle ABC, AC \perp AB, a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \quad (4.1.1.1)$$

The following code plots Fig. 4.1.1

codes/circle/draw_circle_eg.py

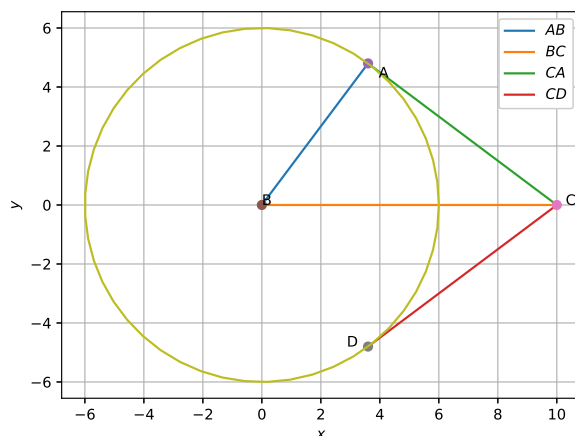


Fig. 4.1.1

2. Draw a circle of radius 3. Mark any point **A** on the circle, point **B** inside the circle and point **C** outside the circle.

Solution: For any angle θ , a point on the circle with radius 3 has coordinates

$$3 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (4.1.2.1)$$

4.2 Construction Exercises

1. Draw a circle of diameter 6.1
2. With the same centre **O**, draw two circles of radii 4 and 2.5
3. Draw a circle of radius 3 and any two of its diameters. draw the ends of these diameters. What figure do you get?
4. Let **A** and **B** be two circles of equal radii 3 such that each one of them passes through

the centre of the other. Let them intersect at **C** and **D**. Is $AB \perp CD$?

5. Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

Solution: Take the centre of both circles to be at the origin.

6. Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

Solution: Take the diameter to be on the x -axis.

7. Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60° .

Solution: The tangent is perpendicular to the radius.

8. Draw a line segment AB of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \quad (4.2.2.1)$$

9. Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^\circ$. BD is the perpendicular from **B** on AC (altitude). The circle through **B, C, D** (circumcircle of $\triangle BCD$) is drawn. Construct the tangents from **A** to this circle.

10. Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through **C**

4.3 Circle Geometry

- Find the coordinates of a point **A**, where AB is the diameter of a circle whose centre is $(2, -3)$ and $\mathbf{B} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.
- Find the centre of a circle passing through the points $\begin{pmatrix} 6 \\ -6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$.