

# Geometric Constructions through Python

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**Abstract**—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.

### 1 TRIANGLE

1.1 Draw  $\triangle ABC$  right angled at **B** such that  $AB = c = 6, BC = a = 8$ .

**Solution:** The coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1)$$

1.2 Let **D, F, F** be the mid points of  $BC, CA$  and  $AB$  respectively in  $\triangle ABC$ . Draw  $AD, BE$  and  $CF$ .

**Solution:**

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (2)$$

$$\mathbf{E} = \frac{\mathbf{C} + \mathbf{A}}{2} \quad (3)$$

$$\mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (4)$$

1.3 Draw  $AD, BE$  and  $CF$ .

1.4 Draw  $\triangle DEF$  in the previous problem.

### 2 CIRCLE

2.1 Now redraw  $\triangle ABC$  as  $\triangle PQR$  with

$$\mathbf{O} = \frac{\mathbf{P} + \mathbf{R}}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5)$$

**Solution:** The coordinates of the  $\triangle PQR$  are

$$\mathbf{P} = \mathbf{A} - \mathbf{E} \quad (6)$$

$$\mathbf{Q} = \mathbf{B} - \mathbf{E} \quad (7)$$

$$\mathbf{R} = \mathbf{A} - \mathbf{E} \quad (8)$$

2.2 Find  $r$  such that **O** is equidistant from **P, Q** and **R**.

**Solution:** We have

2.3 Draw the circumcircle of  $\triangle ABC$  with centre **O**.

**Solution:** The radius of the circumcircle is

$$r = \frac{b}{2} = \frac{\sqrt{a^2 + c^2}}{2} \quad (9)$$

2.4 Draw a circle with centre

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (10)$$

and radius  $c$ .

2.5 For

$$\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \quad (11)$$

find  $p, q$  such that

$$\mathbf{B} = \begin{pmatrix} p \\ q \end{pmatrix}, \quad (12)$$

2.6 Redraw  $\triangle ABC$  with centre **A** and radius  $c$ .

2.7 Draw the tangent  $CD$  to the circle.

**Solution:** The coordinate

$$\mathbf{D} = \begin{pmatrix} p \\ -q \end{pmatrix} \quad (13)$$

The following code draws the circle and tangents in Fig. 2.6

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
```

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```

import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
    len = 10
    x_AB = np.zeros((2,len))
    lam_1 = np.linspace(0,1,len)
    for i in range(len):
        temp1 = A + lam_1[i]*(B-A)
        x_AB[:,i]= temp1.T
    return x_AB

#Triangle sides
a = 10
c = 6
b = np.sqrt(a**2-c**2)

p = (a**2 + c**2-b**2)/(2*a)
q = np.sqrt(c**2-p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])
D = np.array([p,-q])

#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)
x_CD = line_gen(C,D)

#Plotting all lines
plt.plot(x_AB[0,:],x_AB[1:],label='$AB$')
plt.plot(x_BC[0,:],x_BC[1:],label='$BC$')
plt.plot(x_CA[0,:],x_CA[1:],label='$CA$')
plt.plot(x_CD[0,:],x_CD[1:],label='$CD$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) ,
        A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')

```

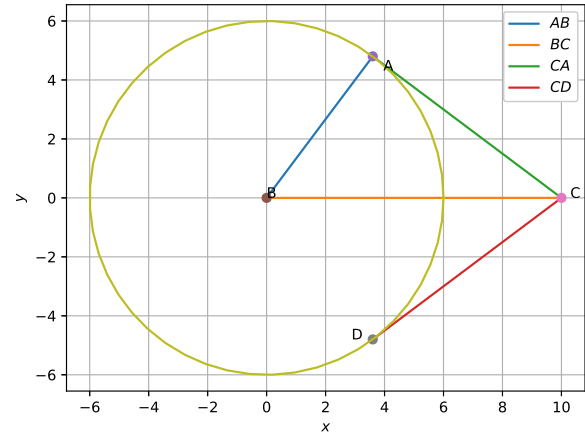


Fig. 2.7

```

plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) ,
        'C')
plt.plot(D[0], D[1], 'o')
plt.text(D[0] * (1 - 0.2), D[1] * (1) , 'D')

#Plotting the circle

theta = np.linspace(0,2*np.pi,50)
x = c*np.cos(theta)
y = c*np.sin(theta)

plt.plot(x,y)

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')
#if using termux
plt.savefig('../figs/circle.pdf')
plt.savefig('../figs/circle.eps')
subprocess.run(shlex.split("termux-open ../figs/circle.pdf"))
#else
plt.show()

```

2.8 Consider  $\triangle ABC$  with  $BC = a$ ,  $CA = b$  and  $AB = c$ . Let

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (14)$$

Find  $p$  and  $q$ .

**Solution:** Since

$$p^2 + q^2 = c^2 \quad (15)$$

$$(p - a)^2 + q^2 = b^2, \quad (16)$$

we obtain

$$p = \frac{a^2 + c^2 - b^2}{2a}, q = \sqrt{c^2 - p^2} \quad (17)$$

2.9 Plot  $\triangle ABC$  for  $a = 8, b = 11$  and  $c = 13$ .

**Solution:** The following program plots  $\triangle ABC$  in Fig. 2.8

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
    len = 10
    x_AB = np.zeros((2,len))
    lam_1 = np.linspace(0,1,len)
    for i in range(len):
        temp1 = A + lam_1[i]*(B-A)
        x_AB[:,i]= temp1.T
    return x_AB

#Triangle sides
a = 8
b = 11
c = 13
p = (a**2 + c**2 - b**2)/(2*a)
q = np.sqrt(c**2 - p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])

#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
```

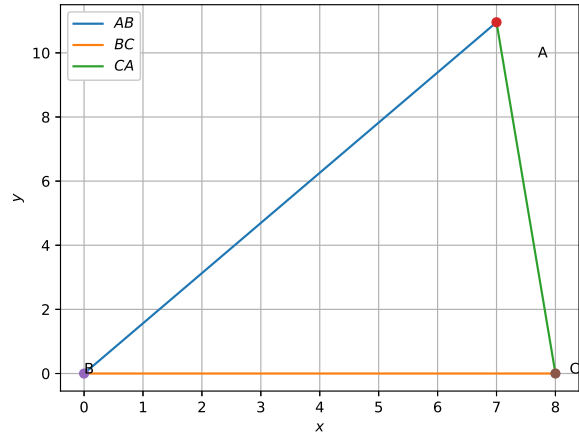


Fig. 2.9

```
x_CA = line_gen(C,A)

#Plotting all lines
plt.plot(x_AB[0:],x_AB[1:],label='$AB$')
plt.plot(x_BC[0:],x_BC[1:],label='$BC$')
plt.plot(x_CA[0:],x_CA[1:],label='$CA$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) , 'A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) , 'C')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor

#if using termux
plt.savefig('../figs/triangle.pdf')
plt.savefig('../figs/triangle.eps')
subprocess.run(shlex.split("termux-open ../figs/triangle.pdf"))
#else
plt.show()
```

2.10 Find  $O$  and  $R$  such that

$$R = OA = OB = OC \quad (18)$$

2.11 Let

$$x + y = ay + z = bz + x = c \quad (19)$$

Find  $x, y, z$ .

**Solution:** The given information can be expressed as the matrix equation

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (20)$$

which can be solved to obtain  $x, y, z$ .

2.12 Find **D, E, F** such that

$$AE = AF = zBE = BD = xCD = CF = y \quad (21)$$

2.13 Find **I** such that

$$ID = IE = IF = r \quad (22)$$

### 3 EXERCISES

3.1 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents  $AC$  and  $CD$  to the circle.

**Solution:** From the given information, in  $\triangle ABC$ ,  $AC \perp AB$ ,  $a = 10$  and  $c = 6$ .

$$b = \sqrt{a^2 - c^2} \quad (23)$$

3.2 Write a program to compute  $p$  and  $q$  when  $a = 8, b = 11$  and  $c = 13$ .

3.3 In  $\triangle ABC$ ,  $a$  and  $\angle B$  are known and  $b + c = k$ . If

$$b^2 = a^2 + c^2 - 2ac \cos B \quad (24)$$

find  $b$  and  $c$ .

**Solution:** From (24),

$$(k - c)^2 = a^2 + c^2 - 2ac \cos B \quad (25)$$

$$\Rightarrow k^2 - 2kc + c^2 = a^2 + c^2 - 2ac \cos B \quad (26)$$

$$\Rightarrow -2kc + 2ac \cos B = a^2 - k^2 \quad (27)$$

$$\Rightarrow 2c(a \cos B - k) = a^2 - k^2 \quad (28)$$

$$\text{or, } c = \frac{a^2 - k^2}{2(a \cos B - k)} \quad (29)$$

3.4 In  $\triangle ABC$ ,  $a = 7, \angle B = 75^\circ$  and  $b + c = 13$ . Find  $b$  and  $c$  and sketch  $\triangle ABC$ .

3.5 In  $\triangle ABC$ ,  $a = 8, \angle B = 45^\circ$  and  $c - b = 3.5$ . Sketch  $\triangle ABC$ .

3.6 In  $\triangle ABC$ ,  $a = 6, \angle B = 60^\circ$  and  $b - c = 2$ . Sketch  $\triangle ABC$ .

3.7  $\triangle ABC$  is right angled at **B**. If  $a = 12$  and  $b + c = 18$ , find  $a, b, c$  and draw the triangle.

**Solution:** From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (30)$$

3.8 In  $\triangle ABC$ , given that  $a + b + c = 11, \angle B = 45^\circ$  and  $\angle C = 45^\circ$ , find  $a, b, c$ .

**Solution:** We have

$$a = b \cos C + c \cos B \quad (31)$$

$$b \sin C = c \sin B \quad (32)$$

$$a + b + c = 11 \quad (33)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (34)$$

Solving the equivalent matrix equation gives the desired answer.

3.9 Draw  $\triangle ABC$ , given that  $a + b + c = 11, \angle B = 30^\circ$  and  $\angle C = 90^\circ$ , find  $a, b, c$ .

3.10 Draw a square of side 3.

3.11 Draw a parallelogram with sides 12 and 5.

3.12 Draw a circle with centre **O** and diameter  $AC = 6$ . Choose any point  $B$  on the circle and draw  $\triangle ABC$ .

3.13 In  $\triangle ABC$ ,  $a = 8, b = 11, c = 13$ . Find

$$R = \frac{a}{2 \sin A}. \quad (35)$$

Let **D** be the mid point of  $BC$ . Find the point **O** such that  $\triangle ODB$  is right angled at **D** and  $OD = R$ . Draw the circle with centre **O** and radius  $R$ .

3.14 Let

$$r = \frac{abc}{2(a + b + c)}. \quad (36)$$

and

$$IB = r \sqrt{\frac{2}{1 - \cos B}}. \quad (37)$$

Draw a circle with centre **I** and radius  $r$ .

3.15 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

- 3.16 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
- 3.17 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of  $60^\circ$ .
- 3.18 Draw a line segment  $AB$  of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
- 3.19 Let  $ABC$  be a right triangle in which  $a = 8$ ,  $c = 6$  and  $\angle B = 90^\circ$ .  $BD$  is the perpendicular from **B** on  $AC$ . The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.