

Geometric Constructions through Python

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Abstract—This manual shows how to construct geometric figures using Python. The problems are based on NCERT math textbooks of Class 9 and 10.

1 TRIANGLE

- 1.1 Draw the rectangle $ABCD$ where $AB = 5, BC = 8$.
- 1.2 Draw a square of side 3.
- 1.3 Draw a parallelogram with sides 12 and 5.
- 1.4 Draw $\triangle ABC$ right angled at B such that $a = 5, c = 13$.
- 1.5 Consider $\triangle ABC$ with $BC = a, CA = b$ and $AB = c$. Let

$$A = \begin{pmatrix} p \\ q \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, C = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1)$$

Find the p and q .

Solution: Since

$$p^2 + q^2 = c^2 \quad (2)$$

$$(p - a)^2 + q^2 = b^2, \quad (3)$$

we obtain

$$p = \frac{a^2 + c^2 - b^2}{2a}, q = \sqrt{c^2 - p^2} \quad (4)$$

- 1.6 Write a program to compute p and q when $a = 8, b = 11$ and $c = 13$.
- 1.7 Plot $\triangle ABC$ for $a = 8, b = 11$ and $c = 13$.

Solution: The following program plots $\triangle ABC$ in Fig. 1.7

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```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
    len = 10
    x_AB = np.zeros((2,len))
    lam_1 = np.linspace(0,1,len)
    for i in range(len):
        temp1 = A + lam_1[i]*(B-A)
        x_AB[:,i]= temp1.T
    return x_AB

#Triangle sides
a = 8
b = 11
c = 13
p = (a**2 + c**2 - b**2)/(2*a)
q = np.sqrt(c**2 - p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])

#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)
```

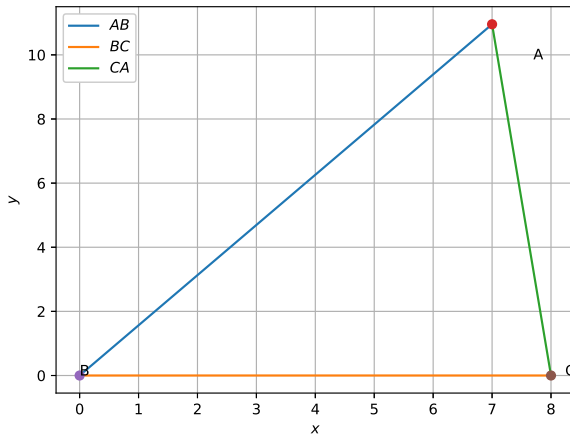


Fig. 1.7

```
#Plotting all lines
plt.plot(x_AB[0:],x_AB[1:],label='$AB$')
plt.plot(x_BC[0:],x_BC[1:],label='$BC$')
plt.plot(x_CA[0:],x_CA[1:],label='$CA$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) , '
    A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) ,
    'C')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor

#if using termux
plt.savefig('../figs/triangle.pdf')
plt.savefig('../figs/triangle.eps')
subprocess.run(shlex.split("termux-open ../
    figs/triangle.pdf"))
#else
plt.show()
```

- 1.8 Let **D**, **F**, **F** be the mid points of BC , CA and AB respectively in $\triangle ABC$. Draw AD , BE and CF .
- 1.9 Draw $\triangle DEF$ in the previous problem.
- 1.10 In $\triangle ABC$, a and $\angle B$ are known and $b + c = k$.

If

$$b^2 = a^2 + c^2 - 2ac \cos B \quad (5)$$

find b and c .

Solution: From (5),

$$(k - c)^2 = a^2 + c^2 - 2ac \cos B \quad (6)$$

$$\Rightarrow k^2 - 2kc + c^2 = a^2 + c^2 - 2ac \cos B \quad (7)$$

$$\Rightarrow -2kc + 2ac \cos B = a^2 - k^2 \quad (8)$$

$$\Rightarrow 2c(a \cos B - k) = a^2 - k^2 \quad (9)$$

$$\text{or, } c = \frac{a^2 - k^2}{2(a \cos B - k)} \quad (10)$$

1.11 In $\triangle ABC$, $a = 7$, $\angle B = 75^\circ$ and $b + c = 13$. Find b and c and sketch $\triangle ABC$.

1.12 In $\triangle ABC$, $a = 8$, $\angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.

1.13 In $\triangle ABC$, $a = 6$, $\angle B = 60^\circ$ and $b - c = 2$. Sketch $\triangle ABC$.

1.14 $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b + c = 18$, find a, b, c and draw the triangle.

Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (11)$$

1.15 In $\triangle ABC$, given that $a + b + c = 11$, $\angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c .

Solution: We have

$$a = b \cos C + c \cos B \quad (12)$$

$$b \sin C = c \sin B \quad (13)$$

$$a + b + c = 11 \quad (14)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (15)$$

Solving the equivalent matrix equation gives the desired answer.

1.16 Draw $\triangle ABC$, given that $a + b + c = 11$, $\angle B = 30^\circ$ and $\angle C = 90^\circ$, find a, b, c .

2 CIRCLE

- 2.1 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.

Solution: From the given information, in $\triangle ABC$, $AC \perp AB$, $a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \quad (16)$$

The following code draws the circle and tangents in Fig. 2.1

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
    len =10
    x_AB = np.zeros((2,len))
    lam_1 = np.linspace(0,1,len)
    for i in range(len):
        temp1 = A + lam_1[i]*(B-A)
        x_AB[:,i]= temp1.T
    return x_AB

#Triangle sides
a = 10
c = 6
b = np.sqrt(a**2-c**2)

p = (a**2 + c**2-b**2)/(2*a)
q = np.sqrt(c**2-p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])
D = np.array([p,-q])

#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)
x_CD = line_gen(C,D)
```

```
#Plotting all lines
plt.plot(x_AB[:,0],x_AB[:,1],label='$AB$')
plt.plot(x_BC[:,0],x_BC[:,1],label='$BC$')
plt.plot(x_CA[:,0],x_CA[:,1],label='$CA$')
plt.plot(x_CD[:,0],x_CD[:,1],label='$CD$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) , 'A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) , 'C')
plt.plot(D[0], D[1], 'o')
plt.text(D[0] * (1 - 0.2), D[1] * (1) , 'D')

#Plotting the circle

theta = np.linspace(0,2*np.pi,50)
x = c*np.cos(theta)
y = c*np.sin(theta)

plt.plot(x,y)

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')
#if using termux
plt.savefig('../figs/circle.pdf')
plt.savefig('../figs/circle.eps')
subprocess.run(shlex.split("termux-open ../figs/circle.pdf"))
#else
plt.show()
```

2.2 Draw a circle with centre **O** and diameter $AC = 6$. Choose any point **B** on the circle and draw $\triangle ABC$.

2.3 In $\triangle ABC$, $a = 8$, $b = 11$, $c = 13$. Find

$$R = \frac{a}{2 \sin A}. \quad (17)$$

Let **D** be the mid point of BC . Find the point **O** such that $\triangle ODB$ is right angled at **D** and $OD = R$. Draw the circle with centre **O** and radius R .

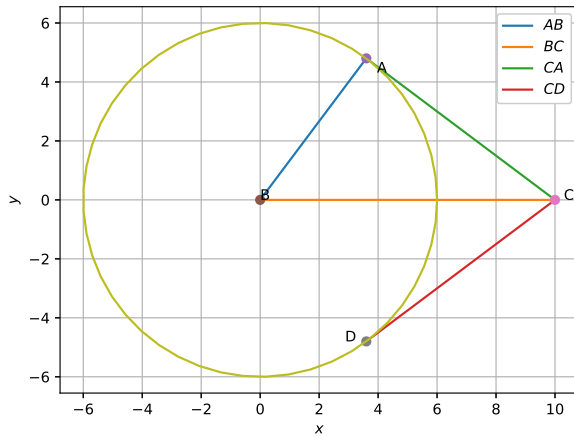


Fig. 2.1

2.4 Let

$$r = \frac{abc}{2(a+b+c)}. \quad (18)$$

and

$$IB = r \sqrt{\frac{2}{1 - \cos B}}. \quad (19)$$

Draw a circle with centre **I** and radius r .

- 2.5 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.
- 2.6 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
- 2.7 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60° .
- 2.8 Draw a line segment AB of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
- 2.9 Let ABC be a right triangle in which $a = 8$, $c = 6$ and $\angle B = 90^\circ$. BD is the perpendicular from **B** on AC . The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.