# Linear Algebra through Coordinate Geometry

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Contents



***Abstract*—This manual introduces linear algebra through coordinate geometry using a problem solving approach.**

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## 1 The Straight Line

1.1 The equation of the line between two points $\mathbf{A}$ and $\mathbf{B}$ is given by

$$\mathbf{x} = \mathbf{A} + \lambda\left(\mathbf{A} - \mathbf{B}\right) \tag{1.1}$$

Alternatively, it can be expressed as

$$\mathbf{n}^T\left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{1.2}$$

where $\mathbf{n}$ is the solution of

$$\left(\mathbf{A} - \mathbf{B}\right)^T\mathbf{n} = 0 \tag{1.3}$$

1.2 In $\triangle ABC$,

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1.4}$$

and the equations of the medians through $\mathbf{B}$ and $\mathbf{C}$ are respectively

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 5 \tag{1.5}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 4 \tag{1.6}$$

Find the area of $\triangle ABC$.

**Solution:** The centroid $\mathbf{O}$ is the solution of (1.5),(1.6) and is obtained as the solution of the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \tag{1.7}$$

which can be solved using the augmented matrix as follows.

$$\begin{pmatrix} 1 & 1 & 5 \\ 1 & 0 & 4 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 1 & 5 \\ 0 & 1 & 1 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \end{pmatrix} \tag{1.8}$$

Thus,

$$\mathbf{O} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{1.9}$$

Let $AD$ be the median through $\mathbf{A}$. Then,

$$\frac{\mathbf{A} + \mathbf{B} + \mathbf{C}}{3} = \mathbf{O} \tag{1.10}$$

$$\implies \mathbf{B} + \mathbf{C} = 3\mathbf{O} - \mathbf{A} = \begin{pmatrix} 11 \\ 1 \end{pmatrix} \tag{1.11}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{B} + \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{C} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 11 \\ 1 \end{pmatrix} \tag{1.12}$$

From (1.6) and (1.12),

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{B} = 5 \tag{1.13}$$

$$\implies 5 + \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{C} = 12 \tag{1.14}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{C} = 7 \tag{1.15}$$

From (1.15) and (1.6), $\mathbf{C}$ can be obtained by

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solving

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} \tag{1.16}$$

using the augmented matrix as

$$\begin{pmatrix} 1 & 1 & 7 \\ 1 & 0 & 4 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 1 & 7 \\ 0 & 1 & 3 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \end{pmatrix} \tag{1.17}$$

$$\implies \mathbf{C} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{1.18}$$

From (1.11),

$$\mathbf{B} = \begin{pmatrix} 11 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{1.19}$$

Thus,

$$\frac{1}{2} \begin{vmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 7 & 4 \\ 2 & -2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 9 \tag{1.20}$$

## 2 Orthogonality

2.1 $\mathbf{u}^T\mathbf{x} = 0 \implies \mathbf{u} \perp \mathbf{x}$. Show that

$$\mathbf{u}^T\mathbf{x} = \mathbf{P}^T\mathbf{x} = 0 \implies \mathbf{P} = \alpha\mathbf{u} \tag{2.1}$$

2.2 The foot of the perpendicular drawn from the origin on the line

$$AB : \mathbf{u}^T\mathbf{x} = \lambda \tag{2.2}$$

where

$$\mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{2.3}$$

is $\mathbf{P}$. The line meets the $x$-axis at $\mathbf{A}$ and $y$-axis at $\mathbf{B}$. Show that $\mathbf{P} = \alpha\mathbf{u}$ and find $\alpha$.
**Solution:** From (2.2),

$$\mathbf{u}^T\mathbf{A} = \mathbf{u}^T\mathbf{B} = \lambda \tag{2.4}$$

$$\implies \mathbf{u}^T (\mathbf{A} - \mathbf{B}) = 0 \tag{2.5}$$

Since $OP \perp AB$,

$$\mathbf{P}^T (\mathbf{A} - \mathbf{B}) = 0 \tag{2.6}$$

Thus, from (2.1),

$$\mathbf{P} = \alpha\mathbf{u} \tag{2.7}$$

Since $\mathbf{P}$ lies on (2.2),

$$\mathbf{u}^T\mathbf{P} = \alpha\mathbf{u}^T\mathbf{u} = \lambda \tag{2.8}$$

$$\implies \alpha = \frac{\lambda}{\mathbf{u}^T\mathbf{u}} = \frac{\lambda}{10}. \tag{2.9}$$

2.3 Find $\mathbf{A}$.
**Solution:** Let

$$\mathbf{A} = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.10}$$

From (2.2),

$$\mathbf{u}^T\mathbf{A} = a\begin{pmatrix} 3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda \tag{2.11}$$

$$\implies a = \frac{\lambda}{3} \tag{2.12}$$

$$\text{and } \mathbf{A} = \frac{\lambda}{3}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.13}$$

2.4 Find the ratio $BP : PA$.
**Solution:** Let

$$\frac{BP}{PA} = k \tag{2.14}$$

Then,

$$k\mathbf{A} + \mathbf{B} = (k+1)\,\mathbf{P} \tag{2.15}$$

$$\implies k\mathbf{A}^T\mathbf{A} + \mathbf{A}^T\mathbf{B} = (k+1)\,\mathbf{P}^T\mathbf{A} \tag{2.16}$$

$$\implies ka^2 = \alpha\,(k+1)\,\lambda \tag{2.17}$$

using (2.7), (2.10), (2.2) and $\mathbf{A} \perp \mathbf{B}$. Substituting from (2.9) and (2.12),

$$\implies k\frac{\lambda^2}{9} = (k+1)\,\frac{\lambda^2}{10} \tag{2.18}$$

$$\implies k = 9 \tag{2.19}$$

## 3 Locus

3.1 The line through

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3.1}$$

intersects the coordinate axes at $\mathbf{P}$ and $\mathbf{Q}$. $\mathbf{O}$ is the origin and rectangle $OPRQ$ is completed as shown in Fig. (3.1),

3.2 Show that

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{R} \tag{3.2}$$

$$\mathbf{Q} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{R} \tag{3.3}$$

$$\mathbf{P} + \mathbf{Q} = \mathbf{R} \tag{3.4}$$

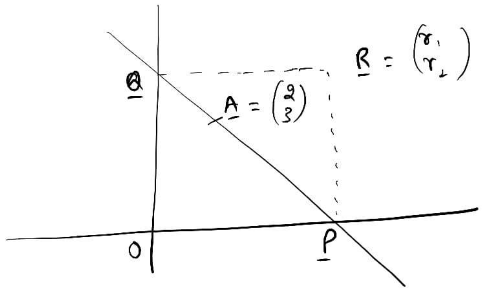


Fig. 3.1

3.3 Show that

$$\begin{aligned}(\mathbf{A}-\mathbf{P})^T\mathbf{n} &= 0\\ (\mathbf{A}-\mathbf{Q})^T\mathbf{n} &= 0 \qquad (3.5)\\ (\mathbf{P}-\mathbf{Q})^T\mathbf{n} &= 0\end{aligned}$$

**Solution:** Trivial using (1.2) and (1.3).

3.4 Show that

$$(2\mathbf{A}-\mathbf{R})^T\mathbf{n} = 0 \qquad (3.6)$$

$$\mathbf{R}^T\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\mathbf{n} = 0 \qquad (3.7)$$

**Solution:** From (3.5) and (3.4)

$$[2\mathbf{A}-(\mathbf{P}+\mathbf{Q})]^T\mathbf{n} = 0 \qquad (3.8)$$

resulting in (3.6). From (3.5) and (3.2),(3.3), (3.7) is obtained.

3.5 Show that

$$\mathbf{R}^T\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\mathbf{R} = 0. \qquad (3.9)$$

3.6 Find the locus of $\mathbf{R}$.

**Solution:** For $\mathbf{n}$ to be unique in (3.6),(3.7),

$$\begin{aligned}(2\mathbf{A}-\mathbf{R}) &= k\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\mathbf{R}\\ \implies \mathbf{R}^T\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}(2\mathbf{A}-\mathbf{R}) &\\ = k\mathbf{R}^T\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\mathbf{R} &\\ = k\mathbf{R}^T\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\mathbf{R} &= 0 \qquad (3.10)\end{aligned}$$

where $k$ is some constant. Thus, the desired locus is

$$\mathbf{R}^T\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}(2\mathbf{A}-\mathbf{R}) = 0 \qquad (3.11)$$

$$\implies \mathbf{R}^T\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\mathbf{R} - 2\mathbf{A}^T\mathbf{R} = 0 \qquad (3.12)$$

## 4 Conics

4.1 The equation of a quadratic curve is given by

$$Ax_1^2 + Bx_1x_2 + Cx_2^2 + Dx_1 + Ex_2 + F = 0 \quad (4.1)$$

Show that (4.1) can be expressed as

$$\mathbf{x}^T V\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + F = 0 \qquad (4.2)$$

Find the matrix $V$ and vector $\mathbf{u}$.

4.2 The tangent to (4.1) at a point $\mathbf{p}$ on the curve is given by

$$\begin{pmatrix}\mathbf{p}^T & 1\end{pmatrix}\begin{pmatrix}V & \mathbf{u}\\ \mathbf{u}^T & F\end{pmatrix}\begin{pmatrix}\mathbf{x}\\ 1\end{pmatrix} = 0 \qquad (4.3)$$

Show that (4.3) can be expressed as

$$\left(\mathbf{p}^T V + \mathbf{u}^T\right)\mathbf{x} + \mathbf{p}^T\mathbf{u} + F = 0 \qquad (4.4)$$

4.3 Classify the various conic sections based on (4.2).

**Solution:**

| Curve | Property |
|---|---|
| Circle | $V = kI$ |
| Parabola | $\det(V) = 0$ |
| Ellipse | $\det(V) > 0$ |
| Hyperbola | $\det(V) < 0$ |

TABLE 4.3

## 5 Circle

5.1 The tangent to the circle

$$C_1 : \mathbf{x}^T\mathbf{x} - \begin{pmatrix}2 & 0\end{pmatrix}\mathbf{x} + -1 = 0 \qquad (5.1)$$

at the point $\begin{pmatrix}2\\ 1\end{pmatrix}$, cuts off a chord of length 4 from a circle $C_2$ whose centre is $\begin{pmatrix}3\\ -2\end{pmatrix}$. Find the radius of $C_2$.

## 6 Parabola

6.1 Find the tangent at $\begin{pmatrix}1\\7\end{pmatrix}$ to the parabola

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x}+\begin{pmatrix}0 & -1\end{pmatrix}\mathbf{x}+6=0 \tag{6.1}$$

**Solution:** Substituting

$$\mathbf{p}=\begin{pmatrix}1\\7\end{pmatrix}, V=\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \mathbf{u}=\frac{1}{2}\begin{pmatrix}0\\-1\end{pmatrix} \tag{6.2}$$

in (4.4), the desired equation is

$$\left[\begin{pmatrix}1 & 7\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}+\frac{1}{2}\begin{pmatrix}0 & -1\end{pmatrix}\right]\mathbf{x} + \frac{1}{2}\begin{pmatrix}1 & 7\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}+6=0 \tag{6.3}$$

resulting in

$$\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x}=5 \tag{6.4}$$

6.2 The line in (6.4) touches the circle

$$\mathbf{x}^T\mathbf{x}+4\begin{pmatrix}4 & 3\end{pmatrix}\mathbf{x}+c=0 \tag{6.5}$$

Find $c$.
**Solution:** Comparing (4.2) and (6.5),

$$V=I, \quad \mathbf{u}=2\begin{pmatrix}4\\3\end{pmatrix} \tag{6.6}$$

Comparing (4.4) and (6.4),

$$\mathbf{p}+2\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}2\\-1\end{pmatrix} \tag{6.7}$$

$$\implies \mathbf{p}=-\begin{pmatrix}6\\7\end{pmatrix} \tag{6.8}$$

and

$$c+\mathbf{p}^T\mathbf{u}=5 \tag{6.9}$$

$$\implies c=5+2\begin{pmatrix}6 & 7\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix} \tag{6.10}$$

$$=95 \tag{6.11}$$

## 7 Hyperbola

7.1 Tangents are drawn to the hyperbola

$$\mathbf{x}^TV\mathbf{x}=36 \tag{7.1}$$

where

$$V=\begin{pmatrix}4 & 0\\0 & -1\end{pmatrix} \tag{7.2}$$

at points $\mathbf{P}$ and $\mathbf{Q}$. If these tangents intersect at

$$\mathbf{T}=\begin{pmatrix}0\\3\end{pmatrix}, \tag{7.3}$$

find the equation of $PQ$.
**Solution:** The equations of the two tangents are obtained using (4.4) as

$$\mathbf{P}^TV\mathbf{x}=36 \tag{7.4}$$

$$\mathbf{Q}^TV\mathbf{x}=36. \tag{7.5}$$

Since both pass through $\mathbf{T}$

$$\mathbf{P}^TV\mathbf{T}=36 \implies \mathbf{P}^T\begin{pmatrix}0\\-3\end{pmatrix}=36 \tag{7.6}$$

$$\mathbf{Q}^TV\mathbf{T}=36 \implies \mathbf{Q}^T\begin{pmatrix}0\\-3\end{pmatrix}=36 \tag{7.7}$$

Thus, $\mathbf{P},\mathbf{Q}$ satisfy

$$\begin{pmatrix}0 & -3\end{pmatrix}\mathbf{x}=-36 \tag{7.8}$$

$$\implies \begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x}=-12 \tag{7.9}$$

which is the equation of $PQ$.

7.2 In $\triangle PTQ$, find the equation of the altitude $TD \perp PQ$.
**Solution:** Since

$$\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=0 \tag{7.10}$$

using (1.2) and (7.9), the equation of $TD$ is

$$\begin{pmatrix}1 & 0\end{pmatrix}(\mathbf{x}-\mathbf{T})=0 \tag{7.11}$$

$$\implies \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}=0 \tag{7.12}$$

7.3 Find $D$.
**Solution:** From (7.9) and (7.12),

$$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{D}=\begin{pmatrix}0\\-12\end{pmatrix} \tag{7.13}$$

$$\implies \mathbf{D}=\begin{pmatrix}0\\-12\end{pmatrix} \tag{7.14}$$

7.4 Show that the equation of $PQ$ can also be expressed as

$$\mathbf{x}=\mathbf{D}+\lambda\mathbf{m} \tag{7.15}$$

where

$$\mathbf{m}=\begin{pmatrix}1\\0\end{pmatrix} \tag{7.16}$$

7.5 Show that for $\mathbf{V}^T = \mathbf{V}$,

$$(\mathbf{D} + \lambda\mathbf{m})^T V (\mathbf{D} + \lambda\mathbf{m}) + F = 0 \tag{7.17}$$

can be expressd as

$$\lambda^2\mathbf{m}^T V\mathbf{m} + 2\lambda\mathbf{m}^T V\mathbf{D} + \mathbf{D}^T V\mathbf{D} + F = 0 \tag{7.18}$$

7.6 Find $\mathbf{P}$ and $\mathbf{Q}$.
**Solution:** From (7.15) and (7.1) (7.18) is obtained. Substituting from (7.16), (7.2) and (7.14)

$$\mathbf{m}^T V\mathbf{m} = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 4 \tag{7.19}$$

$$\mathbf{m}^T V\mathbf{D} = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ -12 \end{pmatrix} = 0 \tag{7.20}$$

$$\mathbf{D}^T V\mathbf{D} = \begin{pmatrix} 0 & -12 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ -12 \end{pmatrix} = -144 \tag{7.21}$$

Substituting in (7.18)

$$4\lambda^2 - 144 = 36 \tag{7.22}$$

$$\implies \lambda = \pm 3\sqrt{5} \tag{7.23}$$

Substituting in (7.15),

$$\mathbf{P} = \mathbf{D} + 3\sqrt{5}\mathbf{m} = 3\begin{pmatrix} \sqrt{5} \\ -4 \end{pmatrix} \tag{7.24}$$

$$\mathbf{Q} = \mathbf{D} - 3\sqrt{5}\mathbf{m} = -3\begin{pmatrix} \sqrt{5} \\ 4 \end{pmatrix} \tag{7.25}$$

7.7 Find the area of $\triangle PTQ$.
**Solution:** Since

$$PQ = \|\mathbf{P} - \mathbf{Q}\| = 6\sqrt{5} \tag{7.26}$$

$$TD = \|\mathbf{T} - \mathbf{D}\| = 15, \tag{7.27}$$

the desired area is

$$\frac{1}{2}PQ \times TD = 45\sqrt{5} \tag{7.28}$$

7.8 Repeat the previous exercise using determinants.

## 8 JEE

8.1 Tangent and normal are drawn at

$$\mathbf{P} = \begin{pmatrix} 16 \\ 16 \end{pmatrix} \tag{8.1}$$

on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 16 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{8.2}$$

which intersect the axis of the parabola at $\mathbf{A}$ and $\mathbf{B}$ respectively. If $\mathbf{C}$ is the centre of the circle through the ponts $\mathbf{P}$ $\mathbf{A}$ and $\mathbf{B}$, find $\tan CPB$.

8.2 A circle passes through the points $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$. If its centre lies on the line

$$\begin{pmatrix} -1 & 4 \end{pmatrix}\mathbf{x} + 3 = 0 \tag{8.3}$$

find its radius.

8.3 Two parabolas with a common vertex and with axes along $x$-axis and $y$-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, find the equation of the common tangent to the two parabolas.

8.4 If the tangents drawn to the hyperbola

$$\mathbf{x}^T V\mathbf{x} + 1 = 0 \tag{8.4}$$

where

$$V = \begin{pmatrix} 1 & 0 \\ 0 & -4 \end{pmatrix} \tag{8.5}$$

intersect the coordinate axes at the distinct points $\mathbf{A}$ and $\mathbf{B}$, find the locus of the mid point of $AB$.

8.5 $\beta$ is one of the angles between the normals to the ellipse

$$\mathbf{x}^T V\mathbf{x} = 9 \tag{8.6}$$

where

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \tag{8.7}$$

at the points

$$\begin{pmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{pmatrix}, \begin{pmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{pmatrix}, \quad \theta \in \left(0, \frac{\pi}{2}\right), \tag{8.8}$$

then find $\frac{2\cot\beta}{\sin 2\theta}$.

8.6 The sides of a rhombus $ABC$ are parallel to the lines

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} + 2 = 0 \tag{8.9}$$

$$\begin{pmatrix} 7 & -1 \end{pmatrix}\mathbf{x} + 3 = 0. \tag{8.10}$$

If the diagonals of the rhombus intersect at

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{8.11}$$

and the vertex $\mathbf{A}$ (different) from the origin is on the $y$-axis, then find the ordinate of $A$.

8.7 Tangents drawn from the point $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$ to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -8 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{8.12}$$

touch the parabola at $\mathbf{P}$ and $\mathbf{Q}$. If $\mathbf{F}$ is the focus of the parabola, then find the area of $\triangle PFQ$.

8.8 A normal to the hyperbola

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & -9 \end{pmatrix} \mathbf{x} = 36 \tag{8.13}$$

meets the coordinate axes $x$ and $y$ at $\mathbf{A}$ and $\mathbf{B}$ respectively. If the parallelogram $OABP$ is formed, find the locus of $\mathbf{P}$.

8.9 Find the locus of the point of intersection of the lines

$$\begin{pmatrix} \sqrt{2} & -1 \end{pmatrix} \mathbf{x} + 4\sqrt{2}k = 0 \tag{8.14}$$

$$\begin{pmatrix} \sqrt{2}k & k \end{pmatrix} \mathbf{x} - 4\sqrt{2} = 0 \tag{8.15}$$

8.10 If a circle $C$,whose radius is 3, touches externally the circle

$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix} 2 & -4 \end{pmatrix} \mathbf{x} = 4 \tag{8.16}$$

at the point $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$, then find the length of the intercept cut by this circle $C$ on the $x$-axis.