

# JEE Problems in Linear Algebra: 3D

**Abstract**—A collection of problems from JEE mains papers related to 3D geometry are available in this document. Students are expected to solve these using linear algebra.

1.  $\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  is a solution of

$$\begin{pmatrix} 1 & -8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \mathbf{0} \quad (1)$$

such that  $\mathbf{A}$  lies on the plane

$$(1 \ 2 \ 1)\mathbf{x} = 6. \quad (2)$$

Find  $2a_1 + a_2 + a_3$ .

2. For any two  $3 \times 3$  matrices  $A$  and  $B$ , let  $A + B = 2B^T$  and  $3A + 2B = I_3$ . Which of the following is true?

- a)  $5A + 10B = 2I_3$ .
- b)  $10A + 5B = 3I_3$ .
- c)  $2A + B = 3I_3$ .
- d)  $3A + 6B = 2I_3$ .

3. If the line,

$$L_1 : \frac{x_1 - 3}{1} = \frac{x_2 + 2}{-1} = \frac{x_3 + \lambda}{-2} \quad (3)$$

lies in the plane

$$(2 \ -4 \ 3)\mathbf{x} = 2, \quad (4)$$

find the shortest distance between  $L_1$  and

$$L_2 : \frac{x_1 - 1}{12} = \frac{x_2}{9} = \frac{x_3}{4} \quad (5)$$

4. Given

$$\mathbf{A} = (1 \ 1 \ 0)^T \quad (6)$$

$$\mathbf{B} = (0 \ 3 \ 4)^T \quad (7)$$

and  $\mathbf{B}_2$  such that

$$BB_2 \parallel OA \quad (8)$$

$$\mathbf{B}_2^T \mathbf{A} = 0 \quad (9)$$

where  $\mathbf{O}$  is the origin, find  $(\mathbf{B} - \mathbf{B}_2) \times \mathbf{B}_2$ .

5. Find the distance between the point  $(1 \ -5 \ 9)^T$  from the plane

$$(1 \ -1 \ 1)\mathbf{x} = 5, \quad (10)$$

along the line  $x_1 = x_2 = x_3$ .

6. The line

$$L : \frac{x_1 - 3}{2} = \frac{x_2 + 2}{-1} = \frac{x_3 + 4}{3} \quad (11)$$

lies in the plane

$$(l \ m \ -1)\mathbf{x} = 9, \quad (12)$$

Find  $l^2 + m^2$ .

7. Let  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  be three unit vectors such that

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \frac{\sqrt{3}}{2} (\mathbf{B} + \mathbf{C}). \quad (13)$$

If  $\mathbf{B}$  is not parallel to  $\mathbf{C}$ , then find the angle between  $\mathbf{A}$  and  $\mathbf{B}$ .

8. Find the range of the shortest distance between the lines

$$L_1 : \frac{x_1}{2} = \frac{x_2}{2} = \frac{x_3}{1} \quad (14)$$

$$L_2 : \frac{x_1 + 2}{-1} = \frac{x_2 - 4}{8} = \frac{x_3 - 5}{4} \quad (15)$$

9. Find the distance of the point  $(1 \ -2 \ 4)^T$  from the plane passing through the point  $(1 \ 2 \ 2)^T$  and perpendicular to the planes

$$(1 \ -1 \ 2)\mathbf{x} = 3 \quad (16)$$

$$\text{and } (2 \ -2 \ 1)\mathbf{x} = -12. \quad (17)$$

10. In  $\triangle ABC$ , right angled at  $A$ ,

$$\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ p \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ q \\ -4 \end{pmatrix} \quad (18)$$

sketch the point  $\begin{pmatrix} p \\ q \end{pmatrix}$

11. Find the distance of the point  $(1 \ 3 \ -7)$  from the plane passing through the point  $(1 \ -1 \ -1)$ , having normal perpendicular to both the lines

$$L_1 : \frac{x_1 - 1}{1} = \frac{x_2 + 2}{-2} = \frac{x_3 - 4}{3} \quad (19)$$

$$L_2 : \frac{x_1 - 2}{2} = \frac{x_2 + 1}{-1} = \frac{x_3 + 7}{-1} \quad (20)$$

12. If the image of the point

$$\mathbf{P} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad (21)$$

in the plane

$$(2 \ 3 \ -4)\mathbf{x} = -22. \quad (22)$$

measured parallel to the line

$$L : \frac{x_1}{1} = \frac{x_2}{4} = \frac{x_3}{5} \quad (23)$$

is  $Q$ , find  $PQ$ .

13. Let

$$A = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \quad (24)$$

If

$$|\mathbf{C} - \mathbf{A}| = 3, \quad (25)$$

$$|(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}| = 3, \quad (26)$$

$$\frac{\mathbf{C}^T (\mathbf{A} \times \mathbf{B})}{|\mathbf{C}| |\mathbf{A} \times \mathbf{B}|} = \frac{\sqrt{3}}{2}, \quad (27)$$

then find  $\mathbf{A}^T \mathbf{C}$ .

14. Find  $b$  such that the planes

$$(1 \ 1 \ 1)\mathbf{x} = 1 \quad (28)$$

$$(1 \ a \ 1)\mathbf{x} = 1 \quad (29)$$

$$(a \ b \ 1)\mathbf{x} = 0 \quad (30)$$

do not intersect.

15. If the shortest distance between the lines

$$x + 2\lambda = 2y = -12z \quad (31)$$

$$x = y + 4\lambda = 6z - 12\lambda \quad (32)$$

is  $4\sqrt{2}$ , find  $\lambda$ .

16. Find the perpendicular distance from the point

$$\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \quad (33)$$

on the plane passing through the point

$$\mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad (34)$$

and containing the line

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad (35)$$

17. If

$$|\mathbf{a}| = 1 \quad (36)$$

$$|\mathbf{b}| = 2 \quad (37)$$

$$|\mathbf{c}| = 4 \quad (38)$$

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0. \quad (39)$$

Find

$$4\mathbf{a}^T \mathbf{b} + 3\mathbf{b}^T \mathbf{c} + 3\mathbf{c}^T \mathbf{a} \quad (40)$$

18. Find the coordinates of the foot of the perpendicular from the point

$$\mathbf{B} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad (41)$$

on the plane containing the lines

$$L_1 : \frac{x_1 + 1}{6} = \frac{x_2 - 1}{7} = \frac{x_3 - 3}{8} \quad (42)$$

$$L_2 : \frac{x_1 - 1}{3} = \frac{x_2 - 2}{5} = \frac{x_3 - 3}{7} \quad (43)$$

19. Find the intersection of the planes

$$(3 \ -1 \ 1)\mathbf{x} = 1 \quad (44)$$

$$(1 \ 4 \ -2)\mathbf{x} = 2 \quad (45)$$

20. If  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$  are the vertices of a parallelogram such that

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 8 \\ -6 \\ 0 \end{pmatrix} \quad (46)$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} 3 \\ 4 \\ -12 \end{pmatrix} \quad (47)$$

Find its area.

21. Find  $\lambda$  for which the planes

$$\begin{pmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{pmatrix} \mathbf{x} = 0 \quad (48)$$

do not intersect at the origin.

22. In  $\triangle ABC$ ,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \mathbf{C} = \begin{pmatrix} \lambda \\ 5 \\ \mu \end{pmatrix} \quad (49)$$

The median through  $\mathbf{A}$  is equally inclined to the coordinate axes. Find  $(\lambda^3 + \mu^3 + 5)$

23. If

$$L_1 : \frac{x_1 - 1}{1} = \frac{x_2 - 2}{2} = \frac{x_3 + 3}{\lambda^2} \quad (50)$$

$$L_2 : \frac{x_1 - 3}{1} = \frac{x_2 - 2}{\lambda^2} = \frac{x_3 - 1}{2} \quad (51)$$

are coplanar, find the number of distinct real values of  $\lambda$ .

24. The circumcentre of  $\triangle ABC$  is

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B} + \mathbf{C}}{4} \quad (52)$$

Find its orthocentre.