भारतीय प्रौद्योगिकी संस्थान हैदराबाद
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Discrete Maths and Algebra

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Contents

***Abstract*—This book provides a computational approach to school algebra and discrete mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

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svn co https://github.com/gadepall/school/trunk/
    ncert/codes
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1 Algebra

1.1 Examples

1. Divide $p(x)$ by $g(x)$, where $p(x) = x + 3x^2 – 1$ and $g(x) = 1 + x$.
2. Divide the polynomial $p(x) = 3x^4 – 4x^3 – 3x – 1$ by $x – 1$.
3. Find the value of k, if $x – 1$ is a factor of $p(x) = 4x^3 + 3x^2 – 4x + k$.
4. Divide $2x^2 + 3x + 1$ by $x + 2$.
5. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.
6. Find all the zeroes of $2x^4 – 3x^3 – 3x^2 + 6x – 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.
7. Find the remainder when $x^3 – ax^2 + 6x – a$ is divided by $x – a$.
8. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
   a) $p(x) = x^2 + x + k$
   b) $p(x) = kx^2 – \sqrt{2}x + 1$
   c) $p(x) = 2x^2 + kx + \sqrt{2}$
   d) $p(x) = kx^2 – 3x + k$
9. Divide the polnyomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:
   a) $p(x) = x^3 – 3x^2 + 5x – 3, g(x) = x^2 – 2$.
   b) $p(x) = x^4 – 3x^2 + 4x + 5, g(x) = x^2 + 1 – x$.
   c) $p(x) = x^4 – 5x + 6, g(x) = 2 – x^2$.
10. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:
    a) $t^2 – x, 2t^4 + 3t^3 – 2t^2 – 9t – 12$.
    b) $x^2 + 3x + 1, 3x^4 + 5x^3 – 7x^2 + 2x + 2$.
    c) $x^3 – 3x + 1, x^5 – 4x^3 + x^2 + 3x + 1$.
11. Obtain all the other zeroes of $3x^4 + 6x^3 – 2x^2 – 10x – 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.
12. On dividing $x^3 – 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x – 2$ and $–2x + 4$respectively. Find $g(x)$.
13. Verify that the numbers given alongside the cubic polynomials below are their zeroes. Also verify if the relationship between the zeroes and the coefficients in each case:
    a) $2x^3 + x^2 – 5x + 2; \frac{1}{2}, 1, –2$
    b) $x^3 – 4x^2 + 5x – 2; 2, 1, 1$
14. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -4 respectively.
15. If two zeroes of the polynomial $x^4 – 6x^3 – 26x^2 + 138x – 35$ are $2 \pm \sqrt{3}$, find the other zeroes.
16. If the polynomial $x^4 – 6x^3 + 16x^2 – 25x + 10$ is

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divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

17. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
18. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.
19. The product of Sunita's age (in years) two years ago and her age four years from now is one more than twice her present age. What is her present age?
20. Find two consecutive odd positive integers, sum of whose squares is 290.
21. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
22. The product of two consecutive positive integers is 306. We need to find the integers.
23. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
24. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.
25. Find two numbers whose sum is 27 and product is 182.
26. Find two consecutive positive integers, sum of whose squares is 365.
27. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.
28. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.
29. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.
30. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
31. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
32. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
33. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.
34. Sum of the areas of two squares is 468 $m^2$2 find the sides of the two squares.
35. Find the values of k for each of the following quadratic equations, so that they have two equal roots:
    a) $2x^2 + kx + 3 = 0$
    b) $kx(x - 2) + 6 = 0$
36. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.
37. If

$$\mathbf{x} = \sqrt{\frac{\begin{pmatrix} a \\ -b \end{pmatrix}}{\begin{pmatrix} c \\ -d \end{pmatrix}}} \tag{1.1.37.1}$$

prove that

$$\|\mathbf{x}\|^2 = \frac{\left\|\begin{pmatrix} a \\ b \end{pmatrix}\right\|}{\left\|\begin{pmatrix} c \\ d \end{pmatrix}\right\|} \tag{1.1.37.2}$$

38. For any two complex numbers $\mathbf{z}_1, \mathbf{z}_2$, prove that
$$\Re\mathbf{z}_1\mathbf{z}_2 = \Re\mathbf{z}_1\Re\mathbf{z}_2 - \Im\mathbf{z}_1\Im\mathbf{z}_2 \qquad (1.1.38.1)$$
39. If $\mathbf{x} = \frac{\begin{pmatrix}a\\b\end{pmatrix}}{\begin{pmatrix}a\\-b\end{pmatrix}}$, show that $\|\mathbf{x}\| = 1$
40. If $\mathbf{x} = \frac{\begin{pmatrix}x\\1\end{pmatrix}^2}{2x^2+1}$, prove that $\|\mathbf{x}\|^2 = \frac{\left(x^2+1\right)^2}{\left(2x^2+1\right)^2}$.
41. If $\begin{pmatrix}x\\y\end{pmatrix}^3 = \mathbf{uv}$, then show that $\frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$.
42. If $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are different complex numbers with $\|\boldsymbol{\beta}\| = 1$, then find $\left\|\frac{\beta-\alpha}{1-\alpha^*\beta}\right\|$.
43. Find the number of non-zero integral solutions of the equation $\|1-1\|^x = 2^x$.
44. If $\begin{pmatrix}a\\b\end{pmatrix}\begin{pmatrix}c\\d\end{pmatrix}\begin{pmatrix}e\\f\end{pmatrix}\begin{pmatrix}g\\h\end{pmatrix} = \begin{pmatrix}A\\B\end{pmatrix}$, then show that $\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right) = A^2+B^2$.
45. If $\left(\frac{\begin{pmatrix}1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\end{pmatrix}}\right) = 1$, the find hte least positive integral value of m
46. The length L (in cm) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$, express L in terms of C.
47. The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?
48. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.
49. The taxi fare in a city is as follows: For the first kilometre, the fare is ₹8 and for the subsequent distance it is ₹5 per km. Taking the distance covered as x km and total fare as ₹y, write a linear equation for this information, and draw its graph.
50. Yamini and Fatima, two students of Class IX of a school, together contributed ₹100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹x and ₹y.) Draw the graph of the same.
51. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:
$$F = \frac{9}{5}C + 32 \qquad (1.1.51.1)$$
    a) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.
    b) If the temperature is 30°C, what is the temperature in Fahrenheit?
    c) If the temperature is 95°F, what is the temperature in Celsius?
    d) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?
    e) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.
52. Romila went to a stationery shop and purchased 2 pencils and 3 erasers for ₹9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for ₹18. Represent this situation algebraically and graphically. Find the cost of each pencil and eraser.
53. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically. Find their respective ages.
54. The coach of a cricket team buys 3 bats and 6 balls for ₹3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and geometrically.Find the cost of each bat and ball.
55. The cost of 2 kg of apples and 1kg of grapes on a day was found to be ₹160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹300. Represent the situation algebraically and geometrically. Find the cost of apples and grape.

56. Form the pair of linear equations in the following problems, and find their solutions.
57. 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
58. 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and that of one pen.
59. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.
60. The difference between two numbers is 26 and one number is three times the other. Find them.
61. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
62. The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.
63. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?
64. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. . Find the fraction.
65. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages
66. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹2000 per month, find their monthly incomes.
67. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?
68. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$. if we only add 1 to the denominator. What is the fraction?
69. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
70. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
71. Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.
72. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.
73. The cost of 5 oranges and 3 apples is ₹35 and the cost of 2 oranges and 4 apples is ₹28. Let us find the cost of an orange and an apple.
74. From a bus stand in Bangalore , if we buy 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, the total cost is ₹46; but if we buy 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur the total cost is ₹74. Find the fares from the bus stand to Malleswaram, and to Yeshwanthpur.
75. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day.
76. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes when 8 is added to its denominator. Find the fraction.
77. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
78. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

79. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.
80. Solve the pair of equations:

$$\begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\end{pmatrix} = 13$$
$$\begin{pmatrix}5 & 4\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\end{pmatrix} = -2 \tag{1.1.80.1}$$

81. Solve the pair of equations by reducing them to a pair of linear equations

$$\begin{pmatrix}5 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{x-1}\\ \frac{1}{y-2}\end{pmatrix} = 2$$
$$\begin{pmatrix}6 & -3\end{pmatrix}\begin{pmatrix}\frac{1}{x-1}\\ \frac{1}{y-2}\end{pmatrix} = 1 \tag{1.1.81.1}$$

82. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.
83. Solve the following pairs of equations

a)

$$\begin{pmatrix}\frac{1}{2} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\end{pmatrix} = 2$$
$$\begin{pmatrix}\frac{1}{2} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\end{pmatrix} = \frac{13}{6} \tag{1.1.83.1}$$

b)

$$\begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{x}}\\ \frac{1}{\sqrt{y}}\end{pmatrix} = 2$$
$$\begin{pmatrix}4 & -9\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{x}}\\ \frac{1}{\sqrt{y}}\end{pmatrix} = -1 \tag{1.1.83.2}$$

c)

$$\begin{pmatrix}4 & 3\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ y\end{pmatrix} = 14$$
$$\begin{pmatrix}3 & -4\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ y\end{pmatrix} = 23 \tag{1.1.83.3}$$

d)

$$\begin{pmatrix}10 & 2\end{pmatrix}\begin{pmatrix}\frac{1}{x+y}\\ \frac{1}{x-y}\end{pmatrix} = 4$$
$$\begin{pmatrix}15 & -5\end{pmatrix}\begin{pmatrix}\frac{1}{x+y}\\ \frac{1}{x-y}\end{pmatrix} = -2 \tag{1.1.83.4}$$

e)

$$\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{3x+y}\\ \frac{1}{3x-y}\end{pmatrix} = \frac{3}{4}$$
$$\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}\frac{1}{3x+y}\\ \frac{1}{3x-y}\end{pmatrix} = -\frac{1}{8} \tag{1.1.83.5}$$

84. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.
85. 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.
86. Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.
87. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.
88. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II].
89. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.
90. The students of a class are made to stand in rows. If 3 students are extra in a row, there

would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

91. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.
92. Find two numbers whose sum is 24 and whose product is as large as possible.
93. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

2 Trigonometry

2.1 Examples

1. Convert $40°20'$ into radian measure.
2. Convert 6 radians into radian measure.
3. Find the radius of the circle in which a central angle of 60^o intercepts an arc of length 37.4 cm (use $\pi = \frac{22}{7}$).
4. The minute hand of watch is 1.5 cm long. How far does its tip move in 40 minutes? ($\pi = 3.14$)
5. If the arcs of the same lengths in two circles subtend angles 65^o and 110^o at the centre, find the ratio of their radii.
6. If $\cos x = -\frac{3}{5}$, x lies in the third quadrant, find the values of other five trigonometric function.
7. If $\cot x = -\frac{5}{12}$, x lies in the second quadrant, find the values of other five trigonometric function.
8. Find the value of $\sin \frac{31\pi}{3}$.
9. Find the value of $\cos(-1710^o)$.
10. Prove that $3\sin \frac{\pi}{6} \sec \frac{\pi}{3} - 4 \sin \frac{5\pi}{6} \cot \frac{\pi}{4} = 1$.
11. Find the value of $\sin 15^o$.
12. Find the value of $\tan \frac{13\pi}{12}$.
13. Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$
14. Show that

    $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x.$
15. Prove that

    $\cos(\frac{\pi}{4} + x) + \cos(\frac{\pi}{4} - x) = \sqrt{2} \cos x$
16. Prove that $\frac{\cos 7x + \cos 5x}{\cos 7x - \cos 5x} = \cot x$
17. Prove that $\frac{\sin 5x - 2 \sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$
18. Find the principal solutions of the equation $\sin x = \frac{\sqrt{3}}{2}$.
19. Find the principal solutions of the equation $\tan x = -\frac{1}{\sqrt{3}}$.
20. Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$.
21. Solve $\cos x = \frac{1}{2}$.
22. Solve $\tan 2x = -\cot(x + \frac{\pi}{3})$.
23. Solve $\sin 2x - \sin 4x + \sin 6x = 0$.
24. Solve $2\cos^2 x + 3 \sin x = 0$
25. If $\sin x = \frac{3}{5}, \cos y = -\frac{12}{13}$, where x and y both lies in second quadrant, find the value of $\sin(x + y)$.
26. Prove that
    $\cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}$
    .
27. Find the value of $\tan \frac{\pi}{8}$.
28. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find the value of $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$
29. Prove that
    $\cos^2 x + \cos^2(x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3}) = \frac{3}{2}$

2.2 Exercises

1. Find the radian measures corresponding to the following meausres:
   (i) $25°$
   (ii) $-47°30'$
   (iii) $240°$
   (iv) $520°$
2. Find the degree measures corresponding to the following radian measures(use π=3.14)
   (i) $\frac{11}{16}$
   (ii) -4
   (iii) $\frac{5\pi}{3}$

(iv) $\frac{7\pi}{6}$

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?
4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm?
5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.
6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii?
7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length
   (i) 10 cm
   (ii) 15 cm
   (iii) 21 cm
8. Find the values of other five trigonometric functions
   1. $\cos x = -\frac{1}{2}$, x lies in third quadrant.
   2. $\sin x = \frac{3}{5}$, x lies in second quadrant.
   3. $\cot x = \frac{3}{4}$, x lies in third quadrant.
   4. $\sec x = \frac{13}{5}$, x lies in fourth quadrant.
   5. $\tan x = -\frac{5}{12}$, x lies in second quadrant.
9. Find the values of the trigonometric functions
   1. $\sin 765^o$
   2. $cosec(-1410^o)$
   3. $\tan \frac{19\pi}{3}$
   4. $\sin \frac{-11\pi}{3}$
   5. $\cot \frac{-15\pi}{4}$
10. Prove that
    1. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$
    2. $2\sin^2 \frac{\pi}{6} + cosec^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = -\frac{3}{2}$
    3. $\cot^2 \frac{\pi}{6} + cosec^2 \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$
    4. $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} = 10$
11. Find the value of
    (i) $\sin 75^o$
    (ii) $\tan 15^o$
12. Prove that
    $\cos(\frac{\pi}{4} - x)\cos(\frac{\pi}{4} - y) - \sin(\frac{\pi}{4} - x)\sin(\frac{\pi}{4} - y) = \sin(x + y)$
13. Prove that
    $\frac{\tan(\frac{\pi}{4}+x)}{\tan(\frac{\pi}{4}-x)} = (\frac{1+\tan x}{1-\tan x})^2$
14. Prove that
    $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos(\frac{\pi}{2}+x)} = \cot^2 x$
15. Prove that
    $\cos(\frac{3\pi}{2}+x)\cos(2\pi+x)[\cot(\frac{3\pi}{2}-x)+\cot(2\pi+x)] = 1$
16. Prove that
    $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x = \cos x$
17. Prove that
    $\cos(\frac{3\pi}{4} + x) - \cos(\frac{3\pi}{4} - x) = -\sqrt{2}\sin x$
18. Prove that
    $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$
19. Prove that
    $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$
20. Prove that
    $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$
21. Prove that
    $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$
22. Prove that
    $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$
23. Prove that
    $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$
24. Prove that
    $\frac{\sin x + \sin y}{\cos x + \cos y} = \tan(\frac{x-y}{2})$

25. Prove that

$\frac{\sin x+\sin 3x}{\cos x+\cos 3x} = \tan 2x$

26. Prove that

$\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x} = 2\sin x$

27. Prove that

$\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x} = \cot 3x$

28. Prove that

$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

29. Prove that

$\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

30. Prove that

$\cos 4x = 1 - 8\sin^2 x \cos^2 x$

31. Prove that

$\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

32. Find the principle and general solutions of the following equations:
    1. $\tan x = \sqrt{3}$
    2. $\sec x = 2$
    3. $\cot x = -\sqrt{3}$
    4. $cosecx = -2$

33. Find the general solution for each of the following equations:
    1. $\cos 4x = \cos 2x$
    2. $\cos 3x + \cos x - \cos 2x = 0$
    3. $\sin 2x + \cos x = 0$
    4. $\sec^2 2x = 1 - \tan 2x$
    5. $\sin x + \sin 3x + \sin 5x = 0$

34. Prove that
    1. $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$
    2. $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x = 0$
    3. $(\cos x+\cos y)^2+(\sin x-\sin y)^2 = 4\cos^2(\frac{x+y}{2})$
    4. $(\cos x-\cos y)^2+(\sin x-\sin y)^2 = 4\sin^2(\frac{x-y}{2})$
    5. $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x\cos 2x\sin 4x$
    6. $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$
    7. $\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2\cos\frac{3x}{2}}$

35. Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the following:
    1. $\tan x = -\frac{4}{3}$, x in second quadrant.
    2. $\sin x = \frac{1}{4}$, x in second quadrant.
    3. $\cos x = -\frac{1}{3}$, x in third quadrant.

3 Arithmetic Proression

3.1 Examples

1. For the AP : $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, ... write the first term a and the common difference d.
2. Which of the following list of numbers form an AP? If they form an AP, write the next two terms :
    a) 4,10,16,22,...
    b) 1,-1,-3,-5,...
    c) -2,2,-2,2,-2,...
    d) 1,1,1,2,2,2,3,3,3,...
3. Find the 10th term of the AP : 2, 7, 12, . . .
4. Which term of the AP : 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.
5. Determine the AP whose 3rd term is 5 and the 7th term is 9.
6. Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .
7. How many two-digit numbers are divisible by 3?
8. Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.
9. A sum of ₹1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
10. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?
11. Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .
12. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

13. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?
14. Find the sum of :
    (i) the first 1000 positive integers
    (ii) the first n positive integers.
15. Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3+2n$
16. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :
    (i) the production in the 1st year
    (ii) the production in the 10th year
    (iii) the total production in first 7 years.

3.2 Exercises

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?
   a) The taxi fare after each km when the fare is ₹15 for the first km and ₹8 for each additional km.
   b) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.
   c) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.
   d) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8 % per annum.
2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:
   a) a = 10, d = 10
   b) a = 4, d = –3
   c) a = –2, d = 0
   d) a = –1, d = $\frac{1}{2}$
   e) a = –1.25, d = –0.25
3. For the following APs, write the first term and the common difference:
   a) 3, 1, –1, –3, . . .
   b) –5, –1, 3, 7, . . .
   c) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$
   d) 0.6, 1.7, 2.8, 3.9....
4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.
   a) 2, 4, 8, 16, . . .
   b) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$
   c) –1.2, –3.2, –5.2, –7.2, . . .
   d) –10, –6, –2, 2, . . .
   e) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$
   f) 0.2, 0.22, 0.222, 0.2222, . . .
   g) 0, –4, –8, –12, . . .
   h) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$
   i) 1, 3, 9, 27,...
   j) a, 2a, 3a, 4a,...
   k) $a, a^2, a^3, a^4, ...$
   l) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$
   m) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$
   n) $1^2, 3^2, 5^2, 7^2$
   o) $1^2, 5^2, 7^2, 73, ...$
5. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the AP:

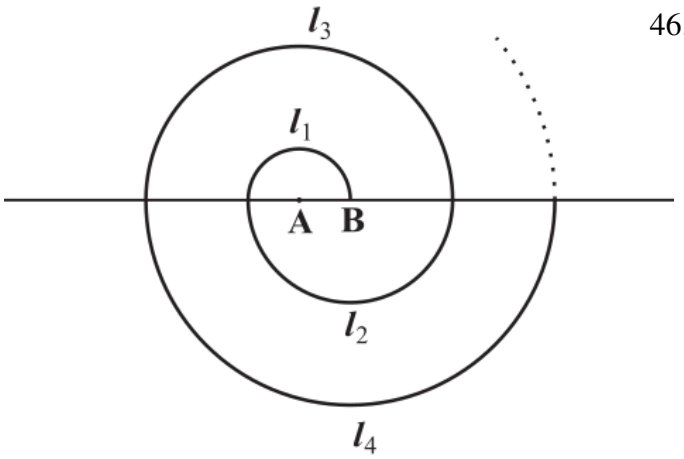
	a	d	n	a_n
(i)	7	3	8	....
(ii)	-18	....	10	0
(iii)	....	-3	18	-5
(iv)	-18.9	2.5	....	3.6
(v)	3.5	0	105	....

6. Choose the correct choice in the following and justify :
   (i) 30^{th} term of the AP: 10, 7, 4,..., is
   a) 97
   b) 77
   c) -77
   d) -87
   (ii) 11^{th} term of the AP: $-3, -\frac{1}{2}, 2, ..., $ is
   a) 28
   b) 22
   c) -38
   d) $-48\frac{1}{2}$
   (iii) In the following APs, find the missing terms in the blanks :
   a) 2, ... , 26
   b) ...., 13. ...., 3
   c) 5, ..., ..., $9\frac{1}{2}$
   d) -4, ..., ..., ..., ..., 6
   e) ...., 38, ..., ..., ..., -22
7. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

8. Find the number of terms in each of the following APs:
   (i) 7, 13, 19, ...,205.
   (ii) 18, $15\frac{1}{2}$, 13, ..., –47
9. Check whether – 150 is a term of the AP : 11, 8, 5, 2 ....
10. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
11. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
12. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?
13. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.
14. Which term of the AP : 3, 15, 27, 39,... will be 132 more than its 54th term?
15. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
16. How many three-digit numbers are divisible by 7?
17. How many multiples of 4 lies between 10 and 250?
18. For what value of n, are the n^{th} terms of two APs: 63, 65, 67,.... and 3, 10, 17,... equal?
19. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.
20. Find the 20th term from the last term of the AP : 3, 8, 13,..., 253.
21. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
22. Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?
23. Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the n^{th} week, her weekly savings become ₹20.75, find n.
24. Find the sum of the following APs:
   (i) 2, 7, 12, . . ., to 10 terms.
   (ii) -37, -33, -29, . . ., to 12 terms.
   (iii) 0.6, 1.7, 2.8, . . ., to 100 terms.
   (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ..., to 11 terms.
25. Find the sums given below :
   (i) $7 + 10\frac{1}{2} + 14 + ... + 84$
   (ii) 34 + 32 + 30 + . . . + 10
   (iii) -5 + (-8) + (-11) + . . . + (-230)
26. In an A.P:
   (i) given a = 5, d = 3, a_n = 50, find n and S_n.
   (ii) given a = 7, a_{13} = 35, find d and S_{13}.
   (iii) given a_{12} = 37, d = 3, find a and S_{12}.
   (iv) given a_3 = 15, S_{10} = 125, find d and a_{10}.
   (v) given d = 5, S_9= 75, find a and a_9.
   (vi) given a = 2, d = 8, S_n = 90, find n and a_n.
   (vii) given a = 8, a_n = 62, S_n = 210, find n and d.
   (viii) given a_n = 4, d = 2, S_n = –14, find n and a.
   (ix) given a = 3, n = 8, S = 192, find d.
   (x) given l = 28, S = 144, and there are total 9 terms. Find a.
27. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?
28. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
29. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
30. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.
31. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
32. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
33. Show that $a_1, a_2, ..., a_n$, . . . form an AP where a_n is defined as below :
   (i) a_n = 3 + 4n
   (ii) a_n = 9 – 5n
   Also find the sum of the first 15 terms in each case.
34. If the sum of the first n terms of an AP is 4n - n^2 , what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.
35. Find the sum of the first 40 positive integers divisible by 6.
36. Find the sum of the first 15 multiples of 8.
37. Find the sum of the odd numbers between 0 and 50.
38. A contract on construction job specifies a

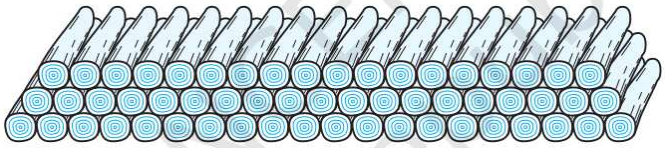
penalty for delay of completion beyond a certain date as follows: ₹200 for the first day,₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30days?

39. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.
40. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying,
    e.g., a section of Class I will plant 1 tree,
    a section of Class II will plant 2 trees and so on till Class XII.
    There are three sections of each class. How many trees will be planted by the students?
41. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,... as shown in Fig. What is the total length of such a spiral made up of thirteen consecutive 22 semicircles? (Take $\pi = \frac{22}{7}$)

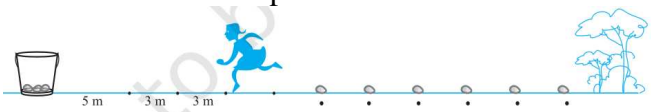


Hint : Length of successive semicircles is $l_1, l_2, l_3, l_4, ...$ with centres at A, B, A, B, . . .,respectively.

42. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig). In how many rows are the 200 logs placed and how many logs are in the top row?

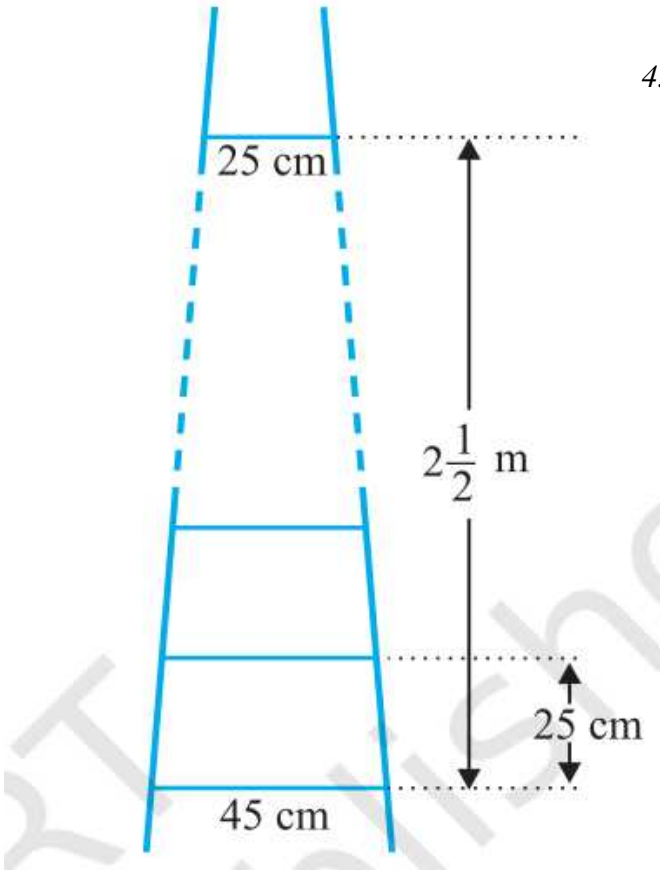


43. In a potato race, a bucket is placed at the starting point, which is 5m from the first potato, and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line.

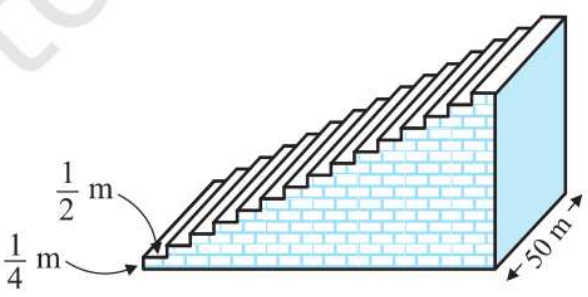


A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run? [**Hint** : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$].

44. Which term of the AP :121, 117, 113,...,is its first negative term?[**Hint:**Find n for a $n < 0$]
45. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.
46. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, which is the length of the wood required for the rungs?[**Hints:** Number of rungs = $\frac{250}{25} + 1$]



47. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.[**Hint:** $S_{X-1} = S_49 - S - x$]
48. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has rise of $\frac{1}{4}$m and a tread of $\frac{1}{2}$m .Caluculate the total volume of concrete required to build the terrace.[**Hint:** Volume of concrete required to build the first step $=\frac{1}{4}X\frac{1}{2}X50m^3$]



4 Principles of Mathematical Induction

4.1 Examples

1. For all n ≥ 1, prove that $1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
2. Prove that 2^n>n for all positive integers n.
3. For all $n \geq 1$,prove that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + .. + \frac{1}{n(n+1)} = \frac{n}{n+1}$.
4. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.
5. Prove that $(1 + x)^n \geq (1 + nx)$, for all natural number n, where x > -1.
6. Prove that $2.7^n + 3.5^n - 5$ is divisible by 24,for all nϵN.
7. Prove that $1^2 + 2^2 + ... + n^2 > \frac{n^3}{3}$,n$\epsilon$N.
8. Prove the rule of exponents $(ab)^n = a^n b^n$ by using the principle of mathematical induction for every natural number.

4.2 Exercises

1. $1 + 3 + 3^2 + ... + 3^{n-1} = \frac{(3^n-1)}{2}$.
2. $1^3 + 2^3 + 3^3 + ... + n^3 = (\frac{n(n+1)}{2})^2$
3. $1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + ... + \frac{1}{(1+2+3+...n)} = \frac{2n}{(n+1)}$.
4. $1.2.3+2.3.4+...+n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.
5. $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$
6. $1.2 + 2.3 + 3.4 + ... + n.(n + 1) = [\frac{n(n+1)(n+2)}{3}]$.
7. $1.3+3.5+5.7+...+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$
8. $1.2 + 2.2^2 + 3.2^3 + ... + n.2^n = (n - 1)2^{n+1} + 2$.
9. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + .. + \frac{1}{2^n} = 1 - \frac{1}{2^n}$
10. $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$.

11. $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + .. + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$.

12. $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n-1)}{r-1}$.

13. $(1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9})...(1 + \frac{(2n+1)}{n^2}) = (n + 1)^2$.

14. $(1 + \frac{1}{1})(1 + \frac{1}{2})(1 + \frac{1}{3})...(1 + \frac{1}{n}) = (n + 1)$.

15. $1^2 + 3^2 + 5^2 + ... + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$

16. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3(2n+1)}$

17. $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$.

18. $1 + 2 + 3 + .. + n < \frac{1}{8}(2n + 1)^2$.

19. n(n+1)(n+5) is a multiple of 3.

20. $10^{2n-1}+1$ is divisible by 11.

21. $x^{2n} - y^{2n}$ is divisible by x+y.

22. $3^{2n+2} - 8n - 9$ is divisible by 8.

23. $41^n - 14^n$ is a multiple of 27

24. $(2n + 7) < (n + 3)^2$.