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Algebra: Maths Olympiad

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***Abstract*—This book provides a collection of the international maths olympiad problems in algebra.**

1. For what real values of x is

$$\sqrt{(x+\sqrt{2x-1}}+\sqrt{(x-\sqrt{2x-1}}=A \quad (1.1)$$

given
a) $A=\sqrt{2}$,
b) $A=1$,
c) $A=2$

where only non-negative real numbers are admitted for square roots?

2. Let a,b,c be real numbers. Consider the quadratic equation in $\cos x$:

$$a\cos^2 x+b\cos x+c=0. \quad (2.1)$$

Using the numbers a,b,c, form a quadratic equation in $\cos 2x$, whose roots are the same as those of the original equation. Compare the equations in $\cos x$ and $\cos 2x$ for $a=4,b=2,c=-1$.

3. Find all real roots of the equation

$$\sqrt{x^2-p}+2\sqrt{x^2-1}=x \quad (3.1)$$

where p is a real parameter.

4. Find all solutions x_1,x_2,x_3,x_4,x_5 of the system

$$x_5+x_2=yx_1 \quad (4.1)$$
$$x_1+x_3=yx_2 \quad (4.2)$$
$$x_2+x_4=yx_3 \quad (4.3)$$
$$x_3+x_5=yx_4 \quad (4.4)$$
$$x_4+x_1=yx_5 \quad (4.5)$$

where y is a parameter.

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