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**Abstract**—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.

Download all codes for this manual from

svn co <https://github.com/gadepall/school/trunk/geometry/constructions/codes>

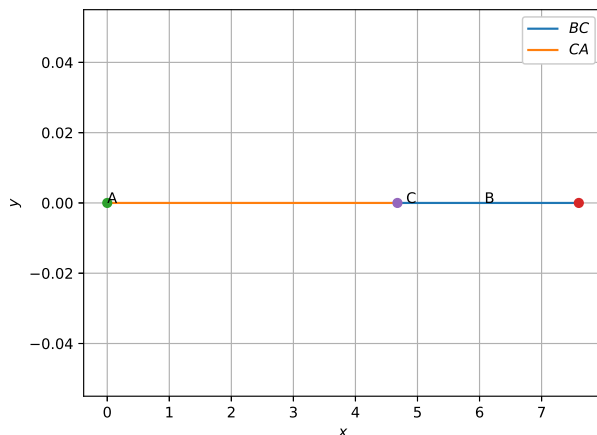


Fig. 1.1

## 1 TRIANGLE

1.1 Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8.

**Solution:** Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \quad (1)$$

Then the point C

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k + 1} \quad (2)$$

divides AB in the ratio  $k : 1$ . For the given problem,  $k = \frac{5}{8}$ . The following code plots Fig. 1.1

codes/draw\_section.py

1.2 Draw  $\triangle ABC$  where  $\angle B = 90^\circ$ ,  $a = 4$  and  $b = 3$ .

**Solution:** The vertices of  $\triangle ABC$  are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (3)$$

The following code plots Fig. 1.2

codes/rt\_triangle.py

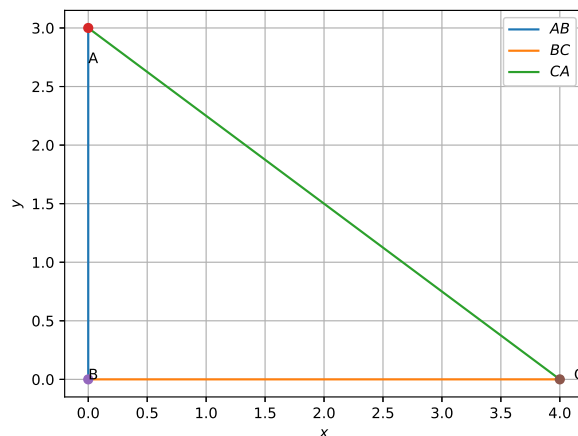


Fig. 1.2

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1.3 Construct a triangle of sides  $a = 4$ ,  $b = 5$  and  $c = 6$ .

**Solution:** Let the vertices of  $\triangle ABC$  be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (4)$$

Then

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad (5)$$

$$\|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \quad (6)$$

$$\|\mathbf{A} - \mathbf{C}\|^2 = b^2 \quad (7)$$

From (7), yielding

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \quad (8)$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (9)$$

$$= a^2 + c^2 - 2ap \quad (10)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (11)$$

From (5),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \quad (12)$$

$$\implies q = \sqrt{c^2 - p^2} \quad (13)$$

The following code plots Fig. 1.3

```
codes/draw_triangle.py
```

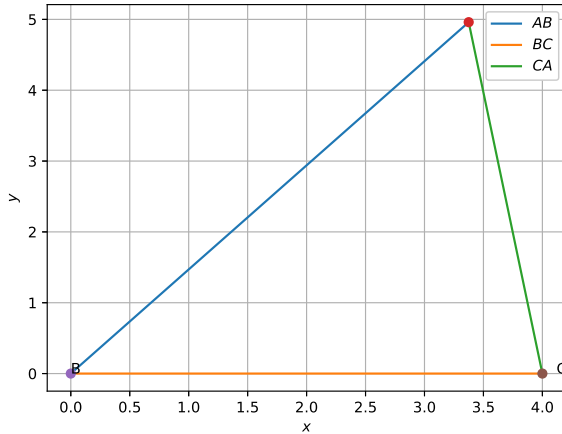


Fig. 1.3

- 1.4 Construct a triangle of sides  $a = 5$ ,  $b = 6$  and  $c = 7$ . Construct a similar triangle whose sides are  $\frac{7}{5}$  times the corresponding sides of the first triangle.

**Solution:** The sides of the similar triangle are  $\frac{7}{5}a$ ,  $\frac{7}{5}b$  and  $\frac{7}{5}c$ .

- 1.5 Construct an isosceles triangle whose base is  $a = 8\text{cm}$  and altitude  $AD = p = 4\text{cm}$

**Solution:** Using Baudhayana's theorem,

$$b = c = \sqrt{p^2 + \left(\frac{a}{2}\right)^2} \quad (14)$$

- 1.6 Draw  $\triangle ABC$  with  $a = 6$ ,  $c = 5$  and  $\angle B = 60^\circ$ .

**Solution:** In Fig. (1.6),  $AD \perp BC$ .

$$\cos C = \frac{y}{b}, \quad (15)$$

$$\cos B = \frac{x}{b}, \quad (16)$$

Thus,

$$a = x + y = b \cos C + c \cos B, \quad (17)$$

$$b = c \cos A + a \cos C \quad (18)$$

$$c = b \cos A + a \cos B \quad (19)$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (20)$$

Using the properties of determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \quad (21)$$

$$= \frac{b^2 + c^2 - a^2}{2bc} \quad (22)$$

From (22)

$$b^2 = c^2 + a^2 - 2ca \cos B \quad (23)$$

which is computed by the following code

```
codes/cos_form.py
```

- 1.7 Draw  $\triangle ABC$  with  $a = 7$ ,  $\angle B = 45^\circ$  and  $\angle A = 105^\circ$ .

**Solution:** In Fig. (1.6),

$$\sin B = \frac{h}{c} \quad (24)$$

$$\sin C = \frac{h}{b} \quad (25)$$

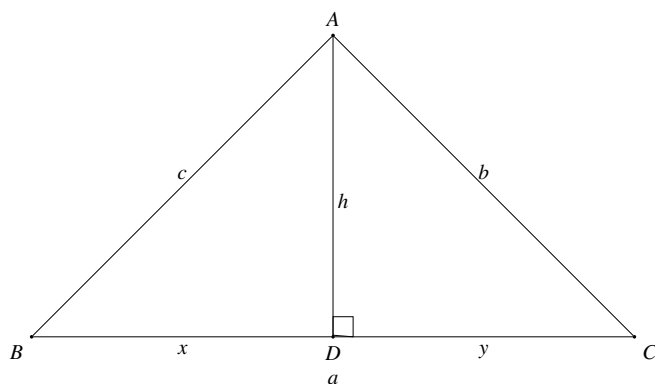


Fig. 1.6: The cosine formula

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (26)$$

Thus,

$$c = \frac{a \sin C}{\sin A} \quad (27)$$

where

$$C = 180 - A - B \quad (28)$$

- 1.8  $\triangle ABC$  is right angled at **B**. If  $a = 12$  and  $b + c = 18$ , find  $b, c$  and draw the triangle.

**Solution:** From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (29)$$

$$\Rightarrow (18 - c)^2 = 12^2 + c^2 \quad (30)$$

which can be simplified to obtain

$$36c - 180 = 0 \quad (31)$$

$$\Rightarrow c = 5 \quad (32)$$

and  $b = 13$

- 1.9 In  $\triangle ABC$ ,  $a = 7$ ,  $\angle B = 75^\circ$  and  $b + c = 13$ . Find  $b$  and  $c$  and sketch  $\triangle ABC$ .

**Solution:** Use cosine formula.

- 1.10 In  $\triangle ABC$ ,  $a = 8$ ,  $\angle B = 45^\circ$  and  $c - b = 3.5$ . Sketch  $\triangle ABC$ .

- 1.11 In  $\triangle ABC$ ,  $a = 6$ ,  $\angle B = 60^\circ$  and  $b - c = 2$ . Sketch  $\triangle ABC$ .

- 1.12 In  $\triangle ABC$ , given that  $a + b + c = 11$ ,  $\angle B = 45^\circ$  and  $\angle C = 45^\circ$ , find  $a, b, c$ .

**Solution:** We have

$$a = b \cos C + c \cos B \quad (33)$$

$$b \sin C = c \sin B \quad (34)$$

$$a + b + c = 11 \quad (35)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (36)$$

Solving the equivalent matrix equation gives the desired answer.

- 1.13 Draw  $\triangle ABC$ , given that  $a + b + c = 11$ ,  $\angle B = 30^\circ$  and  $\angle C = 90^\circ$ .
- 1.14 Construct  $\triangle xyz$  where  $xy = 4.5$ ,  $yz = 5$  and  $zx = 6$ .
- 1.15 Draw an equilateral triangle of side 5.5.
- 1.16 Draw  $\triangle PQR$  with  $PQ = 4$ ,  $QR = 3.5$  and  $PR = 4$ . What type of triangle is this?
- 1.17 Construct  $\triangle ABC$  such that  $AB = 2.5$ ,  $BC = 6$  and  $AC = 6.5$ . Find  $\angle B$ .
- 1.18 Construct  $\triangle PQR$ , given that  $PQ = 3$ ,  $QR = 5.5$  and  $\angle PQR = 60^\circ$ .
- 1.19 Draw  $\triangle ABC$  if  $AB = 3$ ,  $AC = 5$  and  $\angle C = 30^\circ$ .
- 1.20 Construct  $\triangle DEF$  such that  $DE = 5$ ,  $DF = 3$  and  $\angle D = 90^\circ$ .
- 1.21 Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is  $110^\circ$ .
- 1.22 Construct  $\triangle ABC$  with  $BC = 7.5$ ,  $AC = 5$  and  $\angle C = 60^\circ$ .
- 1.23 Construct  $\triangle XYZ$  if  $XY = 6$ ,  $\angle X = 30^\circ$  and  $\angle Y = 100^\circ$ .
- 1.24 If  $AC = 7$ ,  $\angle A = 60^\circ$  and  $\angle B = 50^\circ$ , can you draw the triangle?
- 1.25 Construct  $\triangle ABC$  given that  $\angle A = 60^\circ$ ,  $\angle B = 30^\circ$  and  $AB = 5.8$ .
- 1.26 Construct  $\triangle PQR$  if  $PQ = 5$ ,  $\angle Q = 105^\circ$  and  $\angle R = 40^\circ$ .
- 1.27 Can you construct  $\triangle DEF$  such that  $EF = 7.2$ ,  $\angle E = 110^\circ$  and  $\angle F = 180^\circ$ ?
- 1.28 Construct  $\triangle LMN$  right angled at  $M$  such that  $LN = 5$  and  $MN = 3$ .
- 1.29 Construct  $\triangle PQR$  right angled at  $Q$  such that  $QR = 8$  and  $PR = 10$ .
- 1.30 Construct right angled  $\triangle$  whose hypotenuse is 6 and one of the legs is 4.
- 1.31 Construct an isosceles right angled  $\triangle ABC$  right angled at  $C$  such  $AC = 6$ .

1.32 Construct the triangles in Table 1.32.

S.No	Triangle	Given Measurements		
1	$\triangle ABC$	$\angle A = 85^\circ$	$\angle B = 115^\circ$	$AB = 5$
2	$\triangle PQR$	$\angle Q = 30^\circ$	$\angle R = 60^\circ$	$QR = 4.7$
3	$\triangle ABC$	$\angle A = 70^\circ$	$\angle B = 50^\circ$	$AC = 3$
4	$\triangle LMN$	$\angle L = 60^\circ$	$\angle N = 120^\circ$	$LM = 5$
5	$\triangle ABC$	$BC = 2$	$AB = 4$	$AC = 2$
6	$\triangle PQR$	$PQ = 2.5$	$QR = 4$	$PR = 3.5$
7	$\triangle XYZ$	$XY = 3$	$YZ = 4$	$XZ = 5$
8	$\triangle DEF$	$DE = 4.5$	$EF = 5.5$	$DF = 4$

TABLE 1.32

## 2 CIRCLE

2.1 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents **AC** and **CD** to the circle.

**Solution:** The tangent is perpendicular to the radius. From the given information, in  $\triangle ABC$ ,  $AC \perp AB$ ,  $a = 10$  and  $c = 6$ .

$$b = \sqrt{a^2 - c^2} \quad (37)$$

The following code plots Fig. 2.1

```
codes/draw_circle_eg.py
```

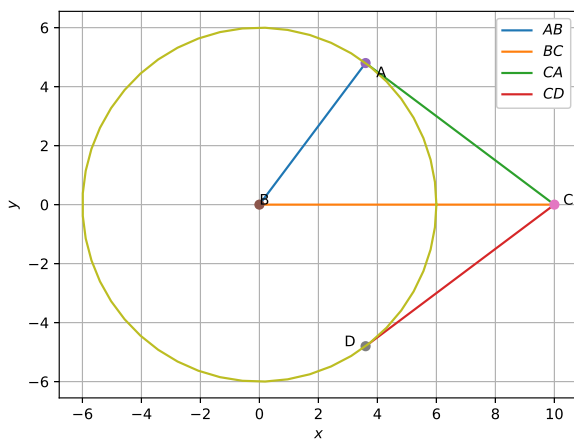


Fig. 2.1

2.2 Draw a circle of diameter 6.1

2.3 Draw a circle of radius 3. Mark any point **A** on the circle, point **B** inside the circle and point

**C** outside the circle.

**Solution:** For any angle  $\theta$ , a point on the circle with radius 3 has coordinates

$$3 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (38)$$

2.4 With the same centre **O**, draw two circles of radii 4 and 2.5

2.5 Draw a circle of radius 3 and any two of its diameters. draw the ends of these diameters. What figure do you get?

2.6 Let **A** and **B** be two circles of equal radii 3 such that each one of them passes through the centre of the other. Let them intersect at **C** and **D**. Is  $AB \perp CD$ ?

2.7 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

**Solution:** Take the centre of both circles to be at the origin.

2.8 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

**Solution:** Take the diameter to be on the  $x$ -axis.

2.9 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of  $60^\circ$ .

**Solution:** The tangent is perpendicular to the radius.

2.10 Draw a line segment **AB** of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

**Solution:** Let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}. \quad (39)$$

2.11 Let **ABC** be a right triangle in which  $a = 8$ ,  $c = 6$  and  $\angle B = 90^\circ$ . **BD** is the perpendicular from **B** on **AC**. The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.

2.12 Draw a circle with centre **C** and radius 3.4. Draw any chord. Construct the perpendicular bisector of the chord and examine if it passes through **C**

### 3 QUADRILATERALS

3.1 Draw  $ABCD$  with  $AB = a = 4.5$ ,  $BC = b = 5.5$ ,  $CD = c = 4$ ,  $AD = d = 6$  and  $AC = e = 7$ .

**Solution:** Fig. 3.1 shows a rough sketch of  $ABCD$ . Letting

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (40)$$

it is trivial to sketch  $\triangle ABC$  from Problem 1.3.  $\triangle ACD$  is can be obtained by rotating an equivalent triangle with  $AC$  on the  $x$ -axis by an angle  $\theta$  with

$$\mathbf{D} = \begin{pmatrix} h \\ k \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} e \\ 0 \end{pmatrix} \quad (41)$$

and

$$\cos \theta = \frac{a^2 + e^2 - b^2}{2ae} \quad (42)$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (43)$$

The coordinates of the rotated triangle  $ACD$  are

$$\mathbf{D} = \mathbf{P} \begin{pmatrix} h \\ k \end{pmatrix} \quad (44)$$

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (45)$$

$$\mathbf{C} = \mathbf{P} \begin{pmatrix} e \\ 0 \end{pmatrix} \quad (46)$$

where

$$\mathbf{P} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (47)$$

The following code plots quadrilateral  $ABCD$

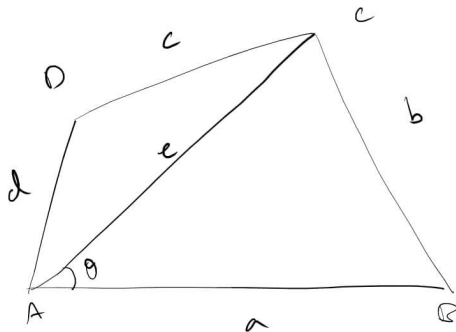


Fig. 3.1

in Fig. 3.1

codes/draw\_quad.py

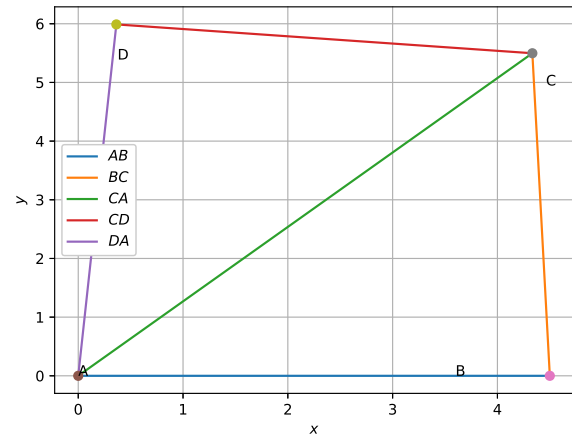


Fig. 3.1

3.2 Construct a quadrilateral  $ABCD$  such that  $AB = 5$ ,  $\angle A = 50^\circ$ ,  $AC = 4$ ,  $BD = 5$  and  $AD = 6$ .

3.3 Construct  $PQRS$  where  $PQ = 4$ ,  $QR = 6$ ,  $RS = 5$ ,  $PS = 5.5$  and  $PR = 7$ .

3.4 Draw  $JUMP$  with  $JU = 3.5$ ,  $UM = 4$ ,  $MP = 5$ ,  $PJ = 4.5$  and  $PU = 6.5$

3.5 Draw the parallelogram  $MORE$  with  $OR = 6$ ,  $RE = 4.5$  and  $EO = 7.5$ .

**Solution:** Diagonals of a parallelogram bisect each other. Opposite sides of a parallelogram are equal and parallel.

3.6 Draw the rhombus  $BEST$  with  $BE = 4.5$  and  $ET = 6$ .

**Solution:** Diagonals of a rhombus bisect each other at right angles.

3.7 Construct a quadrilateral  $ABCD$  such that  $BC = 4.5$ ,  $AC = 5.5$ ,  $CD = 5$ ,  $BD = 7$  and  $AD = 5.5$ .

3.8 Can you construct a quadrilateral  $PQRS$  with  $PQ = 3$ ,  $RS = 3$ ,  $PS = 7.5$ ,  $PR = 8$  and  $SQ = 4$ ?

3.9 Construct  $LIFT$  such that  $LI = 4$ ,  $IF = 3$ ,  $TL = 2.5$ ,  $LF = 4.5$ ,  $IT = 4$ .

3.10 Draw  $GOLD$  such that  $OL = 7.5$ ,  $GL = 6$ ,  $GD = 6$ ,  $LD = 5$ ,  $OD = 10$ .

3.11 DRAW rhombus  $BEND$  such that  $BN = 5.6$ ,  $DE = 6.5$ .

3.12 construct a quadrilateral  $MIST$  where  $MI = 3.5$ ,  $IS = 6.5$ ,  $\angle M = 75^\circ$ ,  $\angle I = 105^\circ$  and  $\angle S = 120^\circ$ .

- 3.13 Can you construct the above quadrilateral MIST if  $\angle M = 100^\circ$  instead of  $75^\circ$ .
- 3.14 Can you construct the quadrilateral PLAN if  $PL = 6, LA = 9.5, \angle P = 75^\circ, \angle L = 150^\circ$  and  $\angle A = 140^\circ$ ?
- 3.15 Construct MORE where  $MO = 6, OR = 4.5, \angle M = 60^\circ, \angle O = 105^\circ, \angle R = 105^\circ$ .
- 3.16 Construct PLAN where  $PL = 4, LA = 6.5, \angle P = 90^\circ, \angle A = 110^\circ$  and  $\angle N = 85^\circ$ .
- 3.17 Construct parallelogram HEAR where  $HE = 5, EA = 6, \angle R = 85^\circ$ .
- 3.18 Draw rectangle OKAY with  $OK = 7$  and  $KA = 5$ .
- 3.19 Construct ABCd, where  $AB = 4, BC = 5, Cd = 6.5, \angle B = 105^\circ$  and  $\angle C = 80^\circ$ .
- 3.20 Construct DEAR with  $DE = 4, EA = 5, AR = 4.5, \angle E = 60^\circ$  and  $\angle A = 90^\circ$ .
- 3.21 Construct TRUE with  $TR = 3.5, RU = 3, UE = 4, \angle R = 75^\circ$  and  $\angle U = 120^\circ$ .
- 3.22 Draw a square of side 4.5.
- 3.23 Can you construct a rhombus ABCD with  $AC = 6$  and  $BD = 7$ ?
- 3.24 Construct a kite EASY if  $AY = 8, EY = 4$  and  $SY = 6$ .
- Solution:** The diagonals of a kite are perpendicular to each other.
- 3.25 Draw a square READ with  $RE = 5.1$ .
- 3.26 Draw a rhombus whose diagonals are 5.2 and 6.4.
- 3.27 Draw a rectangle with adjacent sides 5 and 4.
- 3.28 Draw a parallelogram OKAY with  $OK = 5.5$  and  $KA = 4.2$ .

#### 4 PROPERTIES OF A TRIANGLE

##### 4.1 Cosine Formula:

$$c^2 = a^2 + b^2 - 2ab \cos C \quad (48)$$

##### 4.2 Sine Formula:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (49)$$

##### 4.3 Sum of two sides is always greater than the third side.

$$a + b > c \quad (50)$$

$$b + c > a \quad (51)$$

$$c + a > b \quad (52)$$