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Geometry: Pre Regional Maths Olympiad

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***Abstract*—This book provides a collection of the Indian maths olympiad problems in geometry.**

1. In ΔABC, we have AC = BC = 7 and AB = 2. Suppose that D is a point on line AB such that B lies between A and D and CD = 8. What is the length of the segment BD?
2. In rectangle ABCD, AB = 5 and BC = 3. Points F and G are on line segment CD so that DF = 1 and GC = 2. Lines AF and BG intersect at E. What is the area of ΔAEB.
3. ABCD is a square and AB = 1. Equilateral triangles AYB and CXD are drawn such that X and Y are inside the square. What is the length of XY?
4. Let O and I are the circumcentre and incentre of ΔABC respectively. Suppose O lies in the interior of ΔABC and I lies on the circle passing through B, O and C. What is the magnitude of $\angle BAC$ in degrees?
5. PS is a line segment of length 4 and O is the midpoint of PS. A semicircular arc is drawn with PS as diameter. Let X be the midpoint of this arc. Q and R are points on the arc PXS such that QR is parallel to PS and the semicircular arc drawn with QR as diameter is tangent to PS. What is the area of the region QXROQ bounded by the two semicircular arcs?
6. Let AD and BC be the parallel sides of a trapezium ABCD. Let P and Q be the midpoints of the diagonals AC and BD. IF AD = 16 and BC = 20, what is the length of PQ?
7. In a triangle ABC, let H, I and O be the orthocentre, incentre and circumcentre respectively. If the points B, H, I ,C lie on a circle, what is the magnitude of $\angle BOC$ in degrees?
8. Let ABC be an equilateral triangle. Let P and S be points on AB and AC, respectively, and let Q and R be points on BC such that PQRS is a rectangle. If $PQ = \sqrt{3}PS$ and the area of PQRS is $28\sqrt{3}$, what is the length of PC?
9. Let A_1, B_1, C_1, D_1 be the midpoints of the sides of a convex quadrilateral ABCD and let A_2, B_2, C_2, D_2 be the midpoints of the sides of the quadriletaral A_1, B_1, C_1, D_1. If A_2, B_2, C_2, D_2 is a rectangle with sides 4 and 6, then what is the product of the lengths of the diagonals of ABCD?
10. Let S be a circle with centre O. A chord AB, not a diameter, divides S into two regions R_1 and R_2 such that O belongs to R_2. Let S_1 be a xircle with centre in R_1, touching AB at X and S internally. Let S_2 be a circle with centre in R_2, touching AB at Y, the circle S internally and passing through the centre of S. The point X lies on the diameter passing through the centre of S_2 and $\angle YXO = 30^o$. If the radius of S_2 is 100, then what is the radius of S_1?
11. In a triangle ABC with $\angle BCA = 90^o$, the perpendicular bisector of AB intersects segments AB and AC at X and Y, respectively. If the ratio of the area of quadrilateral BXYC to the area of triangle ABC is 13 : 18 and BC = 12 then what is the length of AC?
12. Let ABCD be a convex quadrilateral with perpendicular diagonals. If AB = 20, BC = 70 and CD = 90, then what is the value of DA?
13. In a triangle ABC, X and Y are points on the segments AB and AC, respectively, such that AX : XB = 1 : 2 and AY : YC = 2 : 1. If the area triangle AXY is 10 then what is the area of triangle ABC?
14. Let ABCD be a convex quadrilateral with $\angle DAB = \angle BDC = 90$. Let the incircles of triangles ABD and BCD touch BD at P and

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Q, respectively, with P lying in between B and Q. If AD = 999 and PQ = 200 then what is the sum of the radii of the incircles of triangles ABD and BDC?

15. Let XOY be a triangle with $\angle XOY = 90^0$. Let M and N be the midpoints of legs OX and OY, respectively. Suppose that XN = 19 and YM = 22. What is XY?
16. In a triangle ABC, let I denote the incenter. Let the lines AI, BI and CI intersect the incircle at P, Q and R, respectively. If $\angle BAC = 40^o$, what is the value of $\angle QPR$ in degrees?
17. In rectangle ABCD, AB = 8 and BC = 20. Let P be a point on AD such that $\angle BPC = 90^o$. If r_1, r_2, r_3 are the radii of the incircles of triangles APB, BPC and CPD, what is the value of $r_1 + r_2 + r_3$?
18. In acute-angled triangle ABC, let D be the foot of the altitude from A, and E be the midpoint of BC. Let F be the midpoint of AC. Suppose $\angle BAE = 40^o$. If $\angle DAE = \angle DFE$, what is the magnitude of $\angle ADF$ in degrees?
19. The circle ω touches the circle Ω internally at P. The centre O of Ω is outside ω. Let XY be a diameter of Ω which is also tangent to ω. Assume $PY > PX$. Let PY intersect ω at Z. If YZ = 2PZ, what is the magnitude of $\angle PYX$ in degrees?
20. In a rectangle ABCD, E is the midpoint of AB; F is a point on AC such that BF is perpendicular to AC; and FE perpendicular to BD. Suppose $BC = \sqrt{3}$, find AB?
21. Suppose in the plane 10 pairwise non-parallel lines intersect one another. What is the maximum possible number of polygons that can be formed?
22. Let P be an interior point of a triangle ABC whose sidelengths are 26, 65, 78. The line through P parallel to BC meets the AB in K and AC in L. The line through P parallel to CA meets BC in M and BA in N. The line through P parallel to AB meets CA in S and CB in T. If KL, MN, ST are of equal lengths, find this common length?
23. Let ABCD be a rectangle and let E and F be points on CD and BC respectively such that are(ADE) = 16, area(CEF) = 9 and area(ABF) = 25. What is the area of triangle AEF?
24. Let AB and CD be two parallel chords in a circle with radius 5 such that the centre O lies between these chords. Suppose AB = 6, CD = 8. Suppose further that the area of the part of the circle lying between the chords AB and CD is $m\pi + n/k$, where m, n, k are positive integers with $gcd(m, n, k) = 1$. What is the value of $m + n + k$?
25. Let Ω_1 be a circle with centre O and let AB be a diameter of Ω_1. Let P be a point on the segment OB different from O. Suppose another circle Ω_2 with centre P lies in the interior of Ω_1. Tangents are drawn form A and B to the circle Ω_2 intersectiong Ω_1 again at A_1 and B_1 respectively such that A_1 and B_1 are on the opposite sides of AB. Given that $A_1B = 5$, $AB_1 = 15$ and $OP = 10$, find the radius of Ω_1.
26. Consider the areas of the 4 triangles obtained by drawing the diagonals AC and BD of a trapezium ABCD. The product of these areas,taken two at time, are computed. If among the 6 products so obtained, 2 products are 1296 and 576, determine the square root of the maximum possible area of the trapezium to the nearest integer.
27. Let D be an interior point of the side BC of a triangle ABC. Let I_1 and I_2 be the incentres of triangles ABD and ACD respectively. Let AI_1 and AI_2 meet BC in E and F respectively. If $\angle BI_1E = 60^o$, what is the measure of $\angle CI_2F$ in degrees?
28. Let ABC ba an acute-angled triangle and let H be its orthocentre. Let G_1, G_2 and G_3 be the centroids of the triangles HBC, HCA and HAB respectively. If the area of the triangle $G_1G_2G_3$ is 7 units, what is the area of the triangle ABC?
29. Triangles ABC and DEF are such that $\angle A = \angle D$, AB = DE = 17, BC = EF = 10 and AC - DF = 12. What is AC + DF?
30. In a triangle ABC, right-angled at A, the altitude through A and the internal bisector of $\angle A$ have lengths 3 and 4 respectively. Find the length of the median through A?
31. In a triangle ABC, the median from B to CA is perpendicular to the median from C to AB. If the median from A to BC is 30, determine $(BC^2 + CA^2 + AB^2)/100$.
32. Let AB be a chord of a circle with centre O. Let C be a point on the circle such that $\angle ABC = 30^o$ and O lies inside the triangle ABC. Let D be a

point on AB such that $\angle DCO = \angle OCB = 20^o$. Find the measure of $\angle CDO$ in degrees.

33. Let ABCD be a trapezium in which AB|| CD and AD $\perp$ AB. Suppose ABCD has an incircle which touches AB at Q and CD at P. Given that PC = 36 and QB = 49, find PQ.
34. In a quadrilateral ABCD, it is given that AB = AD = 13, BC = CD = 20, BD = 24. If r is the radius of the circle inscribable, then what is the integer closest to r?
35. Let ABC be a triangle and let Ω be its circumcircle. The internal bisectors of angles A, B and C intersect Ω at A_1,B_1, and C_1, respectively, and the internal bisectors of angle A_1,B_1, C_1 of the the triangle $A_1B_1C_1$ intersect Ω at A_2,B_2, and C_2, respectively. If the smallest angle of triangle ABC is 40^o, What is the magnitude of the smallest angle of triangle $A_2B_2C_2$.
36. How many distinct triangles ABC are there, upto similarity, such that the magnitudes of angles A, B, and C in degrees are positive ibtegers and satisfy

$$\cos A \cos B + \sin A \sin B \sin kC = 1$$

for some positive integer k, where kC does not exceed 360^o?

37. In how many ways can a pair of parallel diagonals of a regular polygon of 10 sides be selected?
38. Let AB be a diameter of a circle and let C be a point on the segment AB such that AC : CB = 6 : 7. Let D be a point on the circle such that DC is perpendicular to AB. Let DE be the diameter through D. If [XYZ] denote the area of the triangle XYZ, find $[ABD]/[CDE]$to the nearest integer?
39. Let ABCD be a convex cuclic quadrilateral. Suppose P is a point in the plane of the quadrilateral such that the sum of its distances from the vertices ABCD is the least. If

$$\{PA, PB, PC, PD\} = \{3, 4, 6, 8\}$$

What is the maximum possible area of ABCD?

40. Let ABC be a triangle with sides 51, 52, 53. Let Ω denote the incircle of ΔABC. Draw tangents to Ω which are parallel to the sides of ABC. Let r_1, r_2, r_3 be the inradii of the three corner triangles so formed. Find the largest integer that does not exceed $r_1 + r_2 + r_3$?
41. In a triangle ABC, the median, the median AD(with D and BC) and the angle bisector BE(with E on AC) are perpendicular to each other. If AD = 7, and BE = 9, find the integer nearest to the area of triangle ABC.