Computational Approach to School Mathematics

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Contents

***Abstract*—This book provides a computational approach to school mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/codes
```

1 Constructions

1.1 Triangle Examples

1. Draw a line segement of length 7.6 cm and divide it in the ratio 5 : 8.
Solution: Let the end points of the line be
$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \tag{1.1.1.1}$$
Then the point $\mathbf{C}$
$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k+1} \tag{1.1.1.2}$$
divides AB in the ration $k : 1$. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1.1

```
codes/constructions/draw_section.py
```

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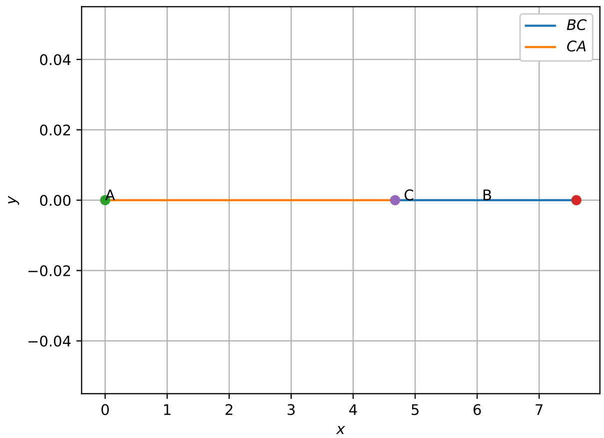

Fig. 1.1.1

2. Draw $\triangle ABC$ where $\angle B = 90°, a = 4$ and $b = 3$.
Solution: The vertices of $\triangle ABC$ are
$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{1.1.2.1}$$
The following code plots Fig. 1.1.2

```
codes/constructions/rt_triangle.py
```

3. Construct a triangle of sides $a = 4$, $b = 5$ and $c = 6$.
Solution: Let the vertices of $\triangle ABC$ be
$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{1.1.3.1}$$
$$\mathbf{A}^T \triangleq \begin{pmatrix} p & q \end{pmatrix} \tag{1.1.3.2}$$
$$\|\mathbf{A}\|^2 = \mathbf{A}^T\mathbf{A} = \begin{pmatrix} p & q \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} \tag{1.1.3.3}$$
$$= p \times p + q \times q = p^2 + q^2 \tag{1.1.3.4}$$

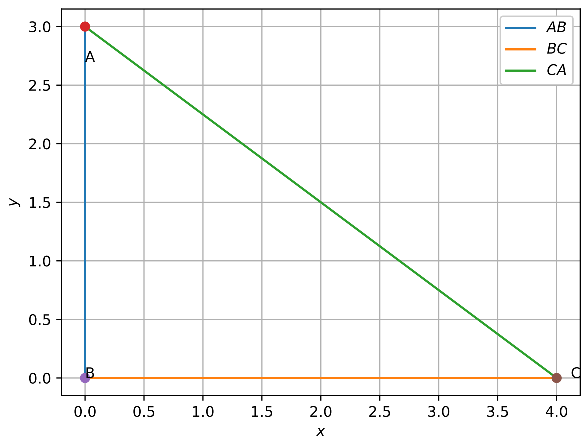

Fig. 1.1.2

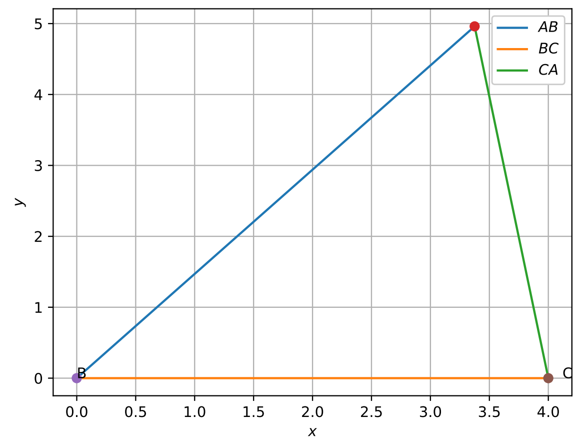

Fig. 1.1.3

Then

$$AB \triangleq \|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \tag{1.1.3.5}$$

$$BC = \|\mathbf{C}-\mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \tag{1.1.3.6}$$

$$AC = \|\mathbf{A}-\mathbf{C}\|^2 = b^2 \tag{1.1.3.7}$$

From (1.1.3.7),

$$b^2 = \|\mathbf{A}-\mathbf{C}\|^2 = \|\mathbf{A}-\mathbf{C}\|^T \|\mathbf{A}-\mathbf{C}\| \tag{1.1.3.8}$$

$$= \mathbf{A}^T\mathbf{A} + \mathbf{C}^T\mathbf{C} - \mathbf{A}^T\mathbf{C} - \mathbf{C}^T\mathbf{A} \tag{1.1.3.9}$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \quad \left(\because \mathbf{A}^T\mathbf{C} = \mathbf{C}^T\mathbf{A}\right) \tag{1.1.3.10}$$

$$= a^2 + c^2 - 2ap \tag{1.1.3.11}$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \tag{1.1.3.12}$$

From (1.1.3.5),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \tag{1.1.3.13}$$

$$\implies q = \pm\sqrt{c^2 - p^2} \tag{1.1.3.14}$$

The following code plots Fig. 1.1.3

```
codes/constructions/draw_triangle.py
```

4. Construct a triangle of sides $a = 5$, $b = 6$ and $c = 7$. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.
Solution: The sides of the similar triangle are $\frac{7}{5}a, \frac{7}{5}b$ and $\frac{7}{5}c$.

5. Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = h = 4$cm
Solution: Using Baudhayana's theorem,

$$b = c = \sqrt{h^2 + \left(\frac{a}{2}\right)^2} \tag{1.1.5.1}$$

6. In $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c and sketch the triangle.
Solution: From the given information,

$$a + b + c = 11 \tag{1.1.6.1}$$

$$b = c \quad (\because B = C = 45^\circ) \tag{1.1.6.2}$$

$$a^2 = b^2 + c^2 \quad (\because A = 90^\circ) \tag{1.1.6.3}$$

From (1.1.6.1) and (1.1.6.2),

$$a + 2b = 11 \tag{1.1.6.4}$$

From (1.1.6.2) and (1.1.6.3),

$$a^2 = 2b^2 \implies a - b\sqrt{2} = 0 \tag{1.1.6.5}$$

(1.1.6.4) and (1.1.6.5) can be summarized as the matrix equation

$$\begin{pmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix} \tag{1.1.6.6}$$

which can be solved using Cramer's rule as

$$a = \frac{\begin{vmatrix} 11 & 2 \\ 0 & -\sqrt{2} \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{vmatrix}} = \frac{11 \times \left(-\sqrt{2}\right) - 2 \times 0}{1 \times \left(-\sqrt{2}\right) - 2 \times 1} \tag{1.1.6.7}$$

$$= \frac{11\sqrt{2}}{2 + \sqrt{2}} \tag{1.1.6.8}$$

$$b = \frac{\begin{vmatrix} 1 & 11 \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{vmatrix}} = \frac{11}{2 + \sqrt{2}} \tag{1.1.6.9}$$

by expanding the determinants. The following code may be used to compute a, b and c.

```
codes/constructions/triangle_det.py
```

7. Repeat Problem 1.1.6 using a single matrix equation.
Solution: The equations

$$a + 2b = 11 \tag{1.1.7.1}$$

$$a - b\sqrt{2} = 0 \tag{1.1.7.2}$$

$$b - c = 0 \tag{1.1.7.3}$$

can be expressed as a single matrix equation

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 0 \end{pmatrix} \tag{1.1.7.4}$$

and can be solved using Cramer's rule as

$$a = \frac{\begin{vmatrix} 11 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}} \tag{1.1.7.5}$$

$$b = \frac{\begin{vmatrix} 0 & 11 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}} \tag{1.1.7.6}$$

$$c = \frac{\begin{vmatrix} 0 & 2 & 11 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix}} \tag{1.1.7.7}$$

The determinant

$$\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0 \times \begin{vmatrix} -\sqrt{2} & 0 \\ 1 & -1 \end{vmatrix} - 2 \times \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & -\sqrt{2} \\ 0 & 1 \end{vmatrix} \tag{1.1.7.8}$$

The determinant can also be expressed as

$$\begin{vmatrix} 0 & 2 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 1 & -1 \end{vmatrix} = 0 \times \begin{vmatrix} -\sqrt{2} & 0 \\ 1 & -1 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} + 0 \times \begin{vmatrix} 2 & 0 \\ -\sqrt{2} & 0 \end{vmatrix} \tag{1.1.7.9}$$

The determinants of larger matrices can be expressed similarly.

8. Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60°$.
Solution: In Fig. (1.1.8), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \tag{1.1.8.1}$$

$$\cos B = \frac{x}{b}, \tag{1.1.8.2}$$

Thus,

$$a = x + y = b\cos C + c\cos B, \tag{1.1.8.3}$$
$$b = c\cos A + a\cos C \tag{1.1.8.4}$$
$$c = b\cos A + a\cos B \tag{1.1.8.5}$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix}\begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{1.1.8.6}$$

Using Cramer's rule and determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \tag{1.1.8.7}$$

$$= \frac{b^2 + c^2 - a^2}{2bc} \tag{1.1.8.8}$$

From (1.1.8.8)

$$b^2 = c^2 + a^2 - 2ca\cos B \tag{1.1.8.9}$$

which is computed by the following code

```
codes/constructions/cos_form.py
```

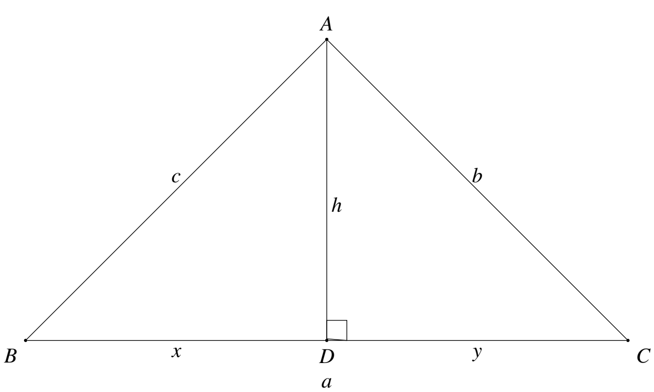

Fig. 1.1.8: The cosine formula

9. Draw $\triangle ABC$ with $a = 7, \angle B = 45°$ and $\angle A = 105°$.
Solution: In Fig. (1.1.8),

$$\sin B = \frac{h}{c} \tag{1.1.9.1}$$
$$\sin C = \frac{h}{b} \tag{1.1.9.2}$$

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \tag{1.1.9.3}$$

Thus,

$$c = \frac{a\sin C}{\sin A} \tag{1.1.9.4}$$

where

$$C = 180 - A - B \tag{1.1.9.5}$$

10. $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.
Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \tag{1.1.10.1}$$
$$\implies (18 - c)^2 = 12^2 + c^2 \tag{1.1.10.2}$$

which can be simplified to obtain

$$36c - 180 = 0 \tag{1.1.10.3}$$
$$\implies c = 5 \tag{1.1.10.4}$$

and $b = 13$

11. Find a simpler solution for Problem 1.1.6
Solution: Use cosine formula.
12. In $\triangle ABC$, $a = 7, \angle B = 75^\circ$ and $b + c = 13$. Alternatively,

$$a = b\cos C + c\cos B \tag{1.1.12.1}$$
$$b\sin C = c\sin B \tag{1.1.12.2}$$
$$a + b + c = 11 \tag{1.1.12.3}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \tag{1.1.12.4}$$

Solving the equivalent matrix equation gives the desired answer.

1.2 Triangle Exercises

1. In $\triangle ABC$, $a = 8, \angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.
2. In $\triangle ABC$, $a = 6, \angle B = 60^\circ$ and $b - c = 2$. Sketch $\triangle ABC$.
3. Draw $\triangle ABC$, given that $a+b+c = 11, \angle B = 30^\circ$ and $\angle C = 90^\circ$.
4. Construct $\triangle xyz$ where $xy = 4.5, yz = 5$ and $zx = 6$.
5. Draw an equilateral triangle of side 5.5.

6. Draw $\triangle PQR$ with $PQ = 4, QR = 3.5$ and $PR = 4$. What type of triangle is this?
7. Construct $\triangle ABC$ such that $AB = 2.5, BC = 6$ and $AC = 6.5$. Find $\angle B$.
8. Construct $\triangle PQR$, given that $PQ = 3, QR = 5.5$ and $\angle PQR = 60°$.
9. Draw $\triangle ABC$ if $AB = 3, AC = 5$ and $\angle C = 30°$.
10. Construct $\triangle DEF$ such that $DE = 5, DF = 3$ and $\angle D = 90°$.
11. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°.
12. Construct $\triangle ABC$ with $BC = 7.5, AC = 5$ and $\angle C = 60°$.
13. Construct $\triangle XYZ$ if $XY = 6, \angle X = 30°$ and $\angle Y = 100°$.
14. If $AC = 7, \angle A = 60°$ and $\angle B = 50°$, can you draw the triangle?
15. Construct $\triangle ABC$ given that $\angle A = 60°, \angle B = 30°$ and $AB = 5.8$.
16. Construct $\triangle PQR$ if $PQ = 5, \angle Q = 105°$ and $\angle R = 40°$.
17. Can you construct $\triangle DEF$ such that $EF = 7.2, \angle E = 110°$ and $\angle F = 180°$?
18. Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.
19. Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.
20. Construct right angled $\triangle$ whose hypotenuse is 6 and one of the legs is 4.
21. Construct an isosceles right angled $\triangle ABC$ right angled at C such $AC = 6$.
22. Construct the triangles in Table 1.2.22.

S.No	Triangle	Given Measurements		
1	△ABC	$\angle A = 85°$	$\angle B = 115°$	AB = 5
2	△PQR	$\angle Q = 30°$	$\angle R = 60°$	QR = 4.7
3	△ABC	$\angle A = 70°$	$\angle B = 50°$	AC = 3
4	△LMN	$\angle L = 60°$	$\angle N = 120°$	LM = 5
5	△ABC	BC = 2	AB = 4	AC = 2
6	△PQR	PQ = 2.5	QR = 4	PR = 3.5
7	△XYZ	XY = 3	YZ = 4	XZ = 5
8	△DEF	DE = 4.5	EF = 5.5	DF = 4

TABLE 1.2.22

1.3 Quadrilateral Examples

1. Draw $ABCD$ with $AB = a = 4.5, BC = b = 5.5, CD = c = 4, AD = d = 6$ and $AC = e = 7$.
Solution: Fig. 1.3.1 shows a rough sketch of $ABCD$. Letting

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{1.3.1.1}$$

it is trivial to sketch $\triangle ABC$ from Problem 1.1.3. $\triangle ACD$ is can be obtained by rotating an equivalent triangle with AC on the x-axis by an angle θ with

$$\mathbf{D} = \begin{pmatrix} h \\ k \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} e \\ 0 \end{pmatrix} \tag{1.3.1.2}$$

and

$$\cos\theta = \frac{a^2 + e^2 - b^2}{2ae} \tag{1.3.1.3}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} \tag{1.3.1.4}$$

The coordinates of the rotated triangle ACD are

$$\mathbf{D} = \mathbf{P}\begin{pmatrix} h \\ k \end{pmatrix} \tag{1.3.1.5}$$

$$\mathbf{A} = \mathbf{P}\begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.3.1.6}$$

$$\mathbf{C} = \mathbf{P}\begin{pmatrix} e \\ 0 \end{pmatrix} \tag{1.3.1.7}$$

where

$$\mathbf{P} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{1.3.1.8}$$

The following code plots quadrilateral $ABCD$

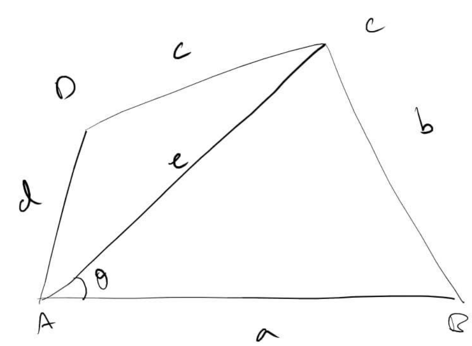

Fig. 1.3.1

in Fig. 1.3.1

```
codes/draw_quad.py
```

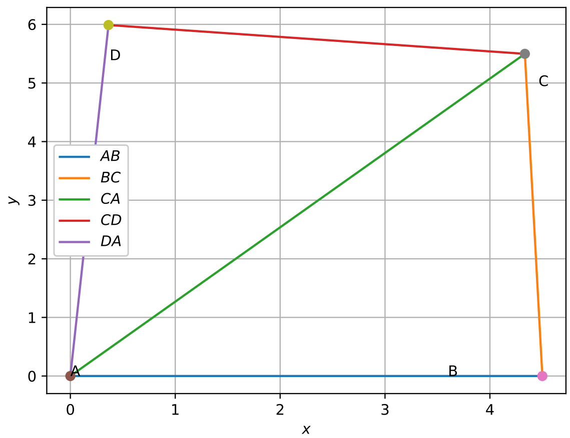

Fig. 1.3.1

2. Draw the parallelogram $MORE$ with $OR = 6, RE = 4.5$ and $EO = 7.5$.
   Solution: Diagonals of a parallelogram bisect each other. Opposite sides of a parallelogram are equal and parallel .
3. Construct a kite $EASY$ if $AY = 8, EY = 4$ and $SY = 6$.
   Solution: The diagonals of a kite are perpendicular to each other.
4. Draw the rhombus $BEST$ with $BE = 4.5$ and $ET = 6$.
   Solution: Diagonals of a rhombus bisect each other at right angles.
5. Construct a quadrilateral $ABCD$ such that $AB = 5, \angle A = 50°, AC = 4, BD = 5$ and $AD = 6$.
6. Construct $PQRS$ where $PQ = 4, QR = 6, RS = 5, PS = 5.5$ and $PR = 7$.
7. Draw $JUMP$ with $JU = 3.5, UM = 4, MP = 5, PJ = 4.5$ and $PU = 6.5$
8. Construct a quadrilateral $ABCD$ such that $BC = 4.5, AC = 5.5, CD = 5, BD = 7$ and $AD = 5.5$.
9. Can you construct a quadrilateral $PQRS$ with $PQ = 3, RS = 3, PS = 7.5, PR = 8$ and $SQ = 4$?
10. Construct $LIFT$ such that $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.
11. Draw $GOLD$ such that $OL = 7.5, GL = 6, GD = 6, LD = 5, OD = 10$.
12. DRAW rhombus $BEND$ such that $BN = 5.6$, $DE = 6.5$.
13. construct a quadrilateral MIST where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.
14. Can you construct the above quadrilateral MIST if $\angle M = 100°$ instead of $75°$.
15. Can you constrcut the quadrilateral PLAN if $PL = 6, LA = 9.5, \angle P = 75°, \angle L = 150°$ and $\angle A = 140°$?
16. Construct $MORE$ where $MO = 6, OR = 4.5, \angle M = 60°, \angle O = 105°, \angle R = 105°$.
17. Construct $PLAN$ where $PL = 4, LA = 6.5, \angle P = 90°, \angle A = 110°$ and $\angle N = 85°$.
18. Constrcut parallelogram $HEAR$ where $HE = 5, EA = 6, \angle R = 85°$.
19. Draw rectangle $OKAY$ with $OK = 7$ and $KA = 5$.
20. Construct $ABCd$, where $AB = 4, BC = 5, Cd = 6.5, \angle B = 105°$ and $\angle C = 80°$.
21. Construct $DEAR$ with $DE = 4, EA = 5, AR = 4.5, \angle E = 60°$ and $\angle A = 90°$.
22. Construct $TRUE$ with $TR = 3.5, RU = 3, UE = 4\angle R = 75°$ and $\angle U = 120°$.
23. Draw a square of side 4.5.
24. Can you construct a rhombus $ABCD$ with $AC = 6$ and $BD = 7$?
25. Draw a square $READ$ with $RE = 5.1$.
26. Draw a rhombus who diagonals are 5.2 and 6.4.
27. Draw a rectangle with adjacent sides 5 and 4.
28. Draw a parallelogram $OKAY$ with $OK = 5.5$ and $KA = 4.2$.