Computational Approach to School Mathematics

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Contents

Abstract—**This is a problem set related to continuous maths based on JEE question papers**

1 Trigonometry

1. Suppose
$$\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx \tag{1.0.1.1}$$
is an identity in x, where $C_0, C_1, ...C_n$ are constants, and $C_n \neq 0$ then find the value of n.
2. Find the solution set of the system of equations
$$x + y = \frac{2\pi}{3} \tag{1.0.2.1}$$
$$\cos x + \cos y = \frac{3}{2}, \tag{1.0.2.2}$$
where x and y are real.
3. Find the set of all x in the interval$[0,\pi]$ for which
$$2\sin^2 x - 3\sin x + 1 \geq 0 \tag{1.0.3.1}$$
4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. Find the minimum value of the arithmetic mean of $\cos(\alpha + \frac{\pi}{2}), \cos(\beta + \frac{\pi}{2})$ and $\cos(\gamma + \frac{\pi}{2})$.
5. Find the value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$.
6. If
$$K = \sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18}), \tag{1.0.6.1}$$
then find the numerical value of K ?
7. If A>0,B>0 and
$$A + B = \frac{\pi}{3}, \tag{1.0.7.1}$$
then find the maximum value of tan A tan B.
8. Find the general value of θ satisfying the equation
$$\tan^2\theta + \sec 2\theta = 1. \tag{1.0.8.1}$$
9. Find the real roots of the equation
$$\cos^7 x + \sin^4 x = 1 \tag{1.0.9.1}$$
in the interval $(-\pi, \pi)$.
10. If $\tan\theta = -\frac{4}{3}$, then find $\sin\theta$.
11. If $\alpha + \beta + \gamma = 2\pi$ then
    a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
    c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    d) None of these
12. Given
$$A = \sin^2\theta + \cos^4\theta \tag{1.0.12.1}$$
then for all real values of θ
    a) $1 \leq A \leq 2$
    b) $\frac{3}{4} \leq A \leq 1$
    c) $\frac{13}{16} \leq A \leq 1$
    d) $\frac{3}{4} \leq A \leq \frac{13}{16}$
13. The equation
$$2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}; 0 < x < \frac{\pi}{2} \tag{1.0.13.1}$$

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has

a) no real solution
b) One real solution
c) more than the one solution
d) none of these

14. The general solution of the trigonometric equation

$$\sin x + \cos x = 1 \tag{1.0.14.1}$$

is given by :

a) x $= 2n\pi; n = 0, \pm1, \pm2...$
b) x $= 2n\pi + \frac{\pi}{2}; n = 0, \pm1, \pm2...$
c) x $= n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}; n = 0, \pm1, \pm2...$
d) none of these

15. The value of expression $\sqrt{3}cosec 20° - \sec 20°$ is equal to

a) 2
b) $\frac{2\sin 20°}{\sin 40°}$
c) 4
d) $\frac{4\sin 20°}{\sin 40°}$

16. The general solution of

$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x \tag{1.0.16.1}$$

is

a) $n\pi + \frac{\pi}{8}$
b) $\frac{n\pi}{2} + \frac{\pi}{8}$
c) $(-1)^n\frac{n\pi}{2} + \frac{\pi}{8}$
d) $2n\pi + \cos^{-1}\frac{3}{2}$

17. The equation

$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0 \tag{1.0.17.1}$$

In the variable x, has real roots. Then p can take any value in the interval

a) $(0, 2\pi)$
b) $(-\pi, 0)$
c) $(-\frac{\pi}{2}, \frac{\pi}{2})$
d) $(0, \pi)$

18. Number of solutions of the equation

$$\tan x + \sec x = 2\cos x \tag{1.0.18.1}$$

lying in the interval $[0, 2\pi]$ is :

a) 0
b) 1
c) 2
d) 3

19. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals

a) $\tan(x - \frac{\pi}{4})$
b) $\tan(\frac{\pi}{4} - x)$
c) $\tan(x + \frac{\pi}{4})$
d) $\tan^2(x + \frac{\pi}{4})$

20. Let n be a positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$.Then

a) $6 \le n \le 8$
b) $4 < n \le 8$
c) $4 \le n \le 8$
d) $4 < n < 8$

21. If ω is an imaginary cube root of unity then the value of $\sin\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\}$ is

a) $-\frac{\sqrt{3}}{2}$
b) $-\frac{1}{\sqrt{2}}$
c) $\frac{1}{\sqrt{2}}$
d) $\frac{\sqrt{3}}{2}$

22. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$

a) 11
b) 12
c) 13
d) 14

23. The general values of θ satisfying equation

$$2\sin^2\theta - 3\sin\theta - 2 = 0 \tag{1.0.23.1}$$

is

a) $n\pi + (-1)^n\frac{\pi}{6}$
b) $n\pi + (-1)^n\frac{\pi}{2}$
c) $n\pi + (-1)^n\frac{5\pi}{6}$
d) $n\pi + (-1)^n\frac{7\pi}{6}$

24. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if

a) $x + y \ne 0$
b) $x = y, x \ne 0$
c) $x = y$
d) $x \ne 0, y \ne 0$

25. In a triangle PQR, $\angle R = \pi/2$.If $\tan(\frac{P}{2})$ and $\tan(\frac{Q}{2})$ are the roots of the equation

$$ax^2 + bx + c = 0 (a \ne 0) \tag{1.0.25.1}$$

then

a) a+b=c
b) b+c=a
c) a+c=b
d) b=c

26. Let f(θ) $=\sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is

a) ≥ 0 only when $\theta \geq 0$
b) ≤ 0 for all real θ
c) ≥ 0 for all real θ
d) ≤ 0 only when $\theta \leq 0$

27. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is
a) 0
b) 2
c) 1
d) 3

28. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)...(\cos\alpha_n)$, under the restrictions, $0 \leq \alpha_1, \alpha_2, ...\alpha_n \leq \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)...(\cot\alpha_n) = 1$ is
a) $\frac{1}{2^{\frac{n}{2}}}$
b) $\frac{1}{2^n}$
c) $\frac{1}{2n}$
d) 1

29. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$,then $\tan\alpha$ equals
a) $2(\tan\beta + \tan\gamma)$
b) $\tan\beta + \tan\gamma$
c) $\tan\beta + 2\tan\gamma$
d) $2\tan\beta + \tan\gamma$

30. The number of integral values of k for which the equation
$$7\cos x + 5\sin x = 2k + 1 \tag{1.0.30.1}$$
has a solution is
a) 4
b) 8
c) 10
d) 12

31. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}, \cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to
a) $(\frac{\pi}{3}, \frac{\pi}{2}]$
b) $(\frac{\pi}{2}, \frac{2\pi}{3})$
c) $(\frac{2\pi}{3}, \frac{5\pi}{6}]$
d) $(\frac{5\pi}{6}, \pi]$

32. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ where $\alpha, \beta\epsilon[-\pi, \pi]$. Pairs of α, βwhich satisfy both the equations is/are
a) 0
b) 1
c) 2
d) 4

33. The values of $\theta\epsilon(0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are
a) $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$
b) $(\frac{\pi}{8}, \frac{5\pi}{6})$
c) $(0, \frac{\pi}{8}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$
d) $(\frac{41\pi}{48}, \pi)$

34. Let $\theta\epsilon(0, \frac{\pi}{4})$ and $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then
a) $t_1 > t_2 > t_3 > t_4$
b) $t_4 > t_3 > t_1 > t_2$
c) $t_3 > t_1 > t_2 > t_4$
d) $t_2 > t_3 > t_1 > t_4$

35. The number of solutions of the pair of equations
$$2\sin^2\theta - \cos 2\theta = 0 \tag{1.0.35.1}$$
$$2\cos^2\theta - 3\sin\theta = 0 \tag{1.0.35.2}$$
in the interval $[0,2\pi]$ is
a) zero
b) one
c) two
d) four

36. For $x\epsilon(0, \pi)$, the equation
$$\sin x + 2\sin 2x - \sin 3x = 3 \tag{1.0.36.1}$$
has
a) infinitely many solutions
b) three solutions
c) one solution
d) no solution

37. Let $S = \{x\epsilon(-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\}$. The sum of all distinct solutions of the equation
$$\sqrt{3}\sec x + \operatorname{cosec}x + 2(\tan x - \cot x) = 0 \tag{1.0.37.1}$$
in the set S is equal to
a) $-\frac{7\pi}{9}$
b) $-\frac{2\pi}{9}$
c) 0
d) $\frac{5\pi}{9}$

38. The value of $\sum_{k=1}^{13} \frac{1}{\sin(\frac{\pi}{4} + \frac{(k-1)\pi}{6})\sin(\frac{\pi}{4} + \frac{k\pi}{6})}$is equal to
a) $3 - \sqrt{3}$
b) $2(3 - \sqrt{3})$
c) $2(\sqrt{3} - 1)$
d) $2(2 - \sqrt{3})$

39. $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$ is equal to
   a) $\frac{1}{2}$
   b) $\cos(\frac{\pi}{8})$
   c) $\frac{1}{8}$
   d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$
40. The expression $3[\sin^4(\frac{3\pi}{2}-\alpha)+\sin^4(3\pi+\alpha]-2[\sin^6(\frac{\pi}{2}+\alpha)+\sin^6(5\pi-\alpha)]$ is equal to
   a) 0
   b) 1
   c) 3
   d) $\sin 4\alpha+\cos 6\alpha$
   e) none of these
41. The number of all possible triplets (a_1, a_2, a_3) such that
$$a_1+a_2\cos(2x)+a_3\sin^2(x)=0 \quad (1.0.41.1)$$
   for all x is
   a) zero
   b) one
   c) three
   d) infinite
   e) none
42. The values of θ lying between $\theta=0$ and $\theta=\pi/2$ and satisfying the equation
$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix}=0 \quad (1.0.42.1)$$
   are
   a) $\frac{7\pi}{24}$
   b) $\frac{5\pi}{24}$
   c) $\frac{11\pi}{24}$
   d) $\frac{\pi}{24}$
43. Let
$$2\sin^2 x+3\sin x-2>0 \quad (1.0.43.1)$$
$$x^2-x-2<0 \quad (1.0.43.2)$$
   (x is measured in radians). Then x lies in the interval
   a) $(\frac{\pi}{6},\frac{5\pi}{6})$
   b) $(-1,\frac{5\pi}{6})$
   c) (-1 , 2)
   d) $(\frac{\pi}{6},2)$
44. The minimum value of the expression $\sin\alpha+\sin\beta+\sin\gamma$, where$\alpha,\beta,\gamma$are real numbers satisfying $\alpha+\beta+\gamma=\pi$ is
   a) Positive
   b) zero
   c) negative
   d) -3
45. The number of values of x in the interval $[0,\pi]$ satisfying the equation
$$3\sin^2 x-7\sin x+2=0 \quad (1.0.45.1)$$
   is
   a) 0
   b) 5
   c) 6
   d) 10
46. Which of the following number(s) is/are/rational?
   a) $\sin 15°$
   b) $\cos 15°$
   c) $\sin 15°\cos 15°$
   d) $\sin 15°\cos 75°$
47. For a positive integer n, let $f_n(\theta)=\tan(\frac{\theta}{2})(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)......(1+\sec 2^n\theta)$.Then
   a) $f_2(\frac{\pi}{16})=1$
   b) $f_3(\frac{\pi}{32})=1$
   c) $f_4(\frac{\pi}{64})=1$
   d) $f_5(\frac{\pi}{128})=1$
48. If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then
   a) $\tan^2 x=\frac{2}{3}$
   b) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$
   c) $\tan^2 x=\frac{1}{3}$
   d) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$
49. For $0<\theta<\frac{\pi}{2}$. the solution(s) of $\sum_{m=1}^{6}\text{cosec}(\theta+\frac{(m-1)\pi}{4})\text{cosec}(\theta+\frac{m\pi}{4})=4\sqrt{2}$ is(are)
   a) $\frac{\pi}{4}$
   b) $\frac{\pi}{6}$
   c) $\frac{\pi}{12}$
   d) $\frac{5\pi}{12}$
50. Let $\theta,\varphi\epsilon[0,2\pi]$ be such that $2\cos\theta(1-\sin\varphi)=\sin^2\theta(\tan\frac{\theta}{2}+\cot\frac{\theta}{2})\cos\varphi-1, \tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$, then φ can not satisfy
   a) $0<\varphi<\frac{\pi}{2}$
   b) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$
   c) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$
   d) $\frac{3\pi}{2}<\varphi<2\pi$

51. The number of points in$(-\infty, \infty)$, for which

$$x^2 - x\sin x - \cos x = 0 \quad (1.0.51.1)$$

is

a) 6
b) 4
c) 2
d) 0

52. Let

$$f(x) = x\sin \pi x, x > 0 \quad (1.0.52.1)$$

Then for all natural numbers n, $f'(x)$ vanishes at

a) A unique point in the interval (n,n+$\frac{1}{2}$)
b) A unique point in the interval (n+$\frac{1}{2}$, n+1)
c) A unique point in the interval (n,n+1)
d) Two points in the interval (n,n+1)

53. Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$.Then which of the following is/are true?

a) $\tan(\frac{\alpha}{2}) + \sqrt{3}\tan(\frac{\beta}{2}) = 0$
b) $\sqrt{3}\tan(\frac{\alpha}{2}) + \tan(\frac{\beta}{2}) = 0$
c) $\tan(\frac{\alpha}{2}) - \sqrt{3}\tan(\frac{\beta}{2}) = 0$
d) $\sqrt{3}\tan(\frac{\alpha}{2}) - \tan(\frac{\beta}{2}) = 0$

54. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$.

55. (a) Draw the graph of

$$y = \frac{1}{\sqrt{2}}(\sin x + \cos x) \quad (1.0.55.1)$$

from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
(b) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$

56. Given $\alpha + \beta - \gamma = \pi$, prove that

$$\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma \quad (1.0.56.1)$$

57. Given A= { x: $\frac{\pi}{6} \le x \le \frac{\pi}{3}$} and

$$f(x) = \cos x - x(1 + x); \quad (1.0.57.1)$$

find f(A)

58. For all θ in $[0, \pi/2]$ show that,

$$\cos(\sin\theta) \ge \sin(\cos\theta) \quad (1.0.58.1)$$

.

59. Without using tables, Prove that $(\sin 12°)(\sin 48°)(\sin 54°) = \frac{1}{8}$

60. Show that $16\cos(\frac{2\pi}{15})\cos(\frac{4\pi}{15})\cos(\frac{8\pi}{15})\cos(\frac{16\pi}{15}) =$ 1

61. Find all the solution of $4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$

62. Find the values of $x\epsilon(-\pi, \pi)$ which satisfy the equation

$$8^{(1+|\cos x||+|cos^2 x|+|cos^3 x|+.....)} = 4^3 \quad (1.0.62.1)$$

63. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

64. ABC is a triangle such that $\sin(2A + B) = \sin(C - A) = -\sin(B + 2c) = \frac{1}{2}$ If A,B and C are in arithmetic progression, determine the values of A, B and C.

65. if exp{($\sin^2 x + \sin^4 x + \sin^6 x + ...\infty$) In 2 } satisifies the equation

$$x^2 - 9x + 8 = 0 \quad (1.0.65.1)$$

, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$.

66. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3.

67. Determine the smallest positive value of x(in degrees) for which $\tan(x + 100°) = \tan(x + 50°)\tan(x)\tan(x - 50°)$.

68. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x\epsilon[0, 2\pi]$

69. Find all values of θ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ satisfying the equation

$$(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0 \quad (1.0.69.1)$$

70. Prove that the values of the function $\frac{\sin x\cos 3x}{\sin 3x\cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x.

71. Prove that $\sum_{k=1}^{n-1}(n-k)cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \ge 3$ is an integer

72. If any triangle ABC, Prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{c}{2}$

73. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$.

This section contains 1 paragraph, Based on each paragraph,there are 2 questions. Each question has four options (A),(B),(C) and (D) ONLY ONE of these four options is correct.

PARAGRAPH 1

Let O be the origin, and $\mathbf{OX}, \mathbf{OY}, \mathbf{OZ}$ be three unit vectors in the directions of the sides $\mathbf{QR}, \mathbf{RP}, \mathbf{PQ}$ respectively, of a triangle PQR

74. $|\mathbf{OX} \times \mathbf{OY}| =$
 a) $\sin(P+Q)$
 b) $\sin 2R$
 c) $\sin(P+R)$
 d) $\sin(Q+R)$
75. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is
 a) $-\frac{5}{3}$
 b) $-\frac{3}{2}$
 c) $\frac{3}{2}$
 d) $\frac{5}{3}$
76. The number of all possible values of θ where $0 < \theta < \pi$,for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$,is
77. The number of values of θ in the interval, $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0 , ±1,±2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is
78. The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is
79. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of [k] is
 Note: [k] denotes the largest integer less than or equal to k.
80. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin(\frac{\pi}{n})} = \frac{1}{\sin(\frac{2\pi}{n})} + \frac{1}{\sin(\frac{3\pi}{n})}$ is
81. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x = 2$ in the interval $[0, 2\pi]$ is
82. Let a,b,c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$ has two distinct real roots α and β with $\alpha+\beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is
83. The period of $\sin^2\theta$ is
 a) π^2
 b) π
 c) 2π
 d) $\pi/2$
84. The number of solution of $\tan x+\sec x = 2\cos x$ in $[0, 2\pi)$ is
 a) 2
 b) 3
 c) 0
 d) 1
85. Which one is not periodic
 a) $|\sin 3x| + \sin^2 x$
 b) $\cos\sqrt{x} + \cos^2 x$
 c) $\cos 4x + \tan^2 x$
 d) $\cos 2x + \sin x$
86. Let α,β be such that $\pi < \alpha-\beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$
 a) $-\frac{6}{65}$
 b) $\frac{3}{\sqrt{130}}$
 c) $\frac{6}{65}$
 d) $-\frac{3}{\sqrt{130}}$
87. If u = $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta}$+ $\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$then the difference between the maximum and minimum values of u^2 is given by
 a) $(a-b)^2$
 b) $2\sqrt{a^2+b^2}$
 c) $(a+b)^2$
 d) $2(a^2+b^2)$
88. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
 a) $\frac{2}{5}$
 b) $\frac{1}{5}$
 c) $\frac{3}{5}$
 d) $\frac{2}{3}$
89. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

$$2\sin^2 x + 5\sin x - 3 = 0 \tag{1.0.89.1}$$

 is
 a) 4
 b) 6
 c) 1
 d) 2
90. If $0 < x < \pi$ and $\cos x+\sin x = \frac{1}{2}$, then $\tan x$ is
 a) $\frac{(1-\sqrt{7})}{4}$
 b) $\frac{(4-\sqrt{7})}{3}$
 c) $-\frac{(4+\sqrt{7})}{3}$
 d) $\frac{(1+\sqrt{7})}{4}$

91. Let A and B denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta) = -\frac{3}{2}$,then :
 a) A is false and B is true
 b) Both A and B are true
 c) both A and B are false
 d) A is true and B is false
92. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha,\beta \le \frac{\pi}{4}$, Then $\tan 2\alpha =$
 a) $\frac{56}{33}$
 b) $\frac{19}{12}$
 c) $\frac{20}{7}$
 d) $\frac{25}{16}$
93. If A = $\sin^2 x + \cos^4 x$, then for all real x:
 a) $\frac{13}{16} \le A \le 1$
 b) $1 \le A \le 2$
 c) $\frac{3}{4} \le A \le \frac{13}{16}$
 d) $\frac{3}{4} \le A \le 1$
94. In a $\triangle PQR$, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :
 a) $\frac{5\pi}{6}$
 b) $\frac{\pi}{6}$
 c) $\frac{\pi}{4}$
 d) $\frac{3\pi}{4}$
95. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC=p and CD=q, then AB is equal to :
 a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$
 b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$
 c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$
 d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$
96. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as:
 a) $\sin A\cos A + 1$
 b) $\sec A cosec A + 1$
 c) $\tan A + \cot A$
 d) $\sec A + cosec A$
97. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \epsilon R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals
 a) $\frac{1}{4}$
 b) $\frac{1}{12}$
 c) $\frac{1}{6}$
 d) $\frac{1}{3}$
98. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:
 a) 7
 b) 9
 c) 3
 d) 5
99. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$,then the value of cos4x is :
 a) $-\frac{7}{9}$
 b) $-\frac{3}{5}$
 c) $\frac{1}{3}$
 d) $\frac{2}{9}$
100. If sum of all the solutions of the equation 8 $\cos x.(\cos(\frac{\pi}{6}+x)(\cos(\frac{\pi}{6}-x)-\frac{1}{2})-1$ in $[0,\pi]$ is $k\pi$. then k is equal to :
 a) $\frac{13}{9}$
 b) $\frac{8}{9}$
 c) $\frac{20}{9}$
 d) $\frac{2}{3}$
101. For any $\theta\epsilon(\frac{\pi}{4},\frac{\pi}{2})$ the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta+\cos\theta)^2 + 4\sin^2\theta$ equals:
 a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
 b) $13 - 4\cos^6\theta$
 c) $13 - 4\cos^2\theta + 6\cos^4\theta$
 d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
102. The value of $\cos^2 10° - \cos 10°\cos 50° + \cos^2 50°$ is:
 a) $\frac{3}{4} + \cos 20°$
 b) $\frac{3}{4}$
 c) $\frac{3}{2}(1+\cos 20°)$
 d) $\frac{3}{2}$
103. Let S=$\{\theta\epsilon[-2\pi,2\pi] : 2\cos^\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is
 a) $\frac{13\pi}{6}$
 b) $\frac{5\pi}{3}$
 c) 2
 d) 1

Match the Following

DIRECTIONS (Q.1): *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B , C and D, while the statements in Column-II are labelled p,q,r,s*

and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to theses questions have to be darkened as illustrated in the following example:
If the correct matches are A-p, s and t; B-q and r; C-p and q; D -s then the correct darkening of bubbles will look like the given

---

a) In this question there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.
$\dfrac{\sin 3\alpha}{\cos 2\alpha}$ is

Column-I	**Column-II**
(A) Positive	(p)$\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
(B) Negative	(q)$\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
	(r)$\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$
	(s)$\left(0, \frac{\pi}{2}\right)$

b) Let
f(x)=$\sin(\pi \cos x)$ and g(x)=$\cos(2\pi \sin x)$be two functions defined for $x > 0$. Define the following sets whose elements are written i n the increasing order.

X $=\{x : f(x) = 0\}$,Y $=\{x : f'(x) = 0\}$

Z $=\{x : g(x) = 0\}$,W $=\{x : g'(x) = 0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

Column-I	**Column-II**
(A)X	(p)$\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$
(B)Y	(q)an arithmetic progression
(C)Z	(r)NOT an arithmetic progression
(D)W	(s)$\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$
	(t)$\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$
	(u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

i) (IV),(P),(R),(S)
ii) (III),(P),(Q),(U)
iii) (III),(R),(U)
iv) (IV),(Q),(T)

c) Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$.Define the following sets whose elements are written in the increasing order

X $=\{x : f(x) = 0\}$,Y $=\{x : f'(x) = 0\}$

Z $=\{x : g(x) = 0\}$,W $=\{x : g'(x) = 0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

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	(u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

i) (I),(Q),(U)
ii) (I),(P),(R)
iii) (II),(R),(S)
iv) (II),(Q),(T)

2 Functions

1. The values of $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lie in the interval.......
2. For the function f(x) = $\frac{x}{1+e^{1/x}}$, x $\neq$ 0 and f(x) = 0, x = 0 the derivative from the right, $f'(0+)$= ...., and the derivative from the left, $f'(0-)$= ....
3. The domain of the funtion f(x)=$\sin^{-1}\left(log_2 \frac{x^2}{2}\right)$ is given by ....
4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is.......and out of these.......are onto functions.
5. If
$$f(x) = \sin ln\left(\frac{\sqrt{4-x^2}}{1-x}\right),$$
then domain of f(x) is.... and its range is......
6. There are exactly two distinct linear functions.......,and......which map [-1,1] onto [0,2].
7. If f is an even function defined on the interval (-5,5), then four real values of x satisfying the equation f(x)=$f(\frac{x+1}{x+2})$ are.........., ..., ..., and......
8. If
$$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + cosxcos\left(x + \frac{\pi}{3}\right)$$
and g($\frac{5}{4}$) = 1, then (gof)(x)= ....
9. If f(x)=$(a - x^n)^{1/n}$ where $a > 0$ and n is a positive integer, then f[f(x)]=x.
10. The function f(x)=$\frac{x^2+4x+30}{x^2-8x+18}$ is not one-to-one.
11. If $f_1(x)$ and $f_2(x)$ are defined on domains D_1 and D_2 respectively, then $f_1(x)$+$f_2(x)$ is defined on $D_1 \cup D_2$.
12. Let R be the set of real numbers. If f:R $\rightarrow$ R is a function defined by f(x)=x^2, then f is :
    a) Injective but not surjective
    b) Surjective but not injective
    c) Bijective
    d) None of these.
13. The entire graphs of the equation y = $x^2 + kx - x + 9$ is stirctly above the x-axis if and only if
    a) $k < 7$
    b) $-5 < k < 7$
    c) $k > -5$
    d) None of these.
14. Let f(x)= $\left|x - 1\right|$. Then
    a) $f(x^2) = (f(x))^2$
    b) f(x+y)=f(x)+f(y)
    c) $f(\left|x\right|) = \left|f(x)\right|$
    d) None of these
15. If x satisfies $\left|x-1\right| + \left|x-2\right| + \left|x-3\right| \geq 6$,then
    a) $0 \leq x \leq 4$
    b) $x \leq -2$ or $x \geq 4$
    c) $x \leq 0$ or $x \geq 4$
    d) None of these
16. If f(x)=cos(lnx), then f(x)f(y) - $\frac{1}{2}\left[f(\frac{x}{y}) + f(xy)\right]$ has the value
    a) -1
    b) $\frac{1}{2}$
    c) -2
    d) none of these
17. The domain of definition of the function y=$\frac{1}{log_{10}(1-x)}$ + $\sqrt{x+2}$ is
    a) (-3,2) excluding -2.5
    b) $[0, 1]$ excluding 0.5
    c) $[-2, 1)$ excluding 0
    d) none of these
18. Which of the following functions is periodic?
    a) f(x)=x-[x] where [x] denotes the largest integer less than or equal to the real number x
    b) f(x)=sin $\frac{1}{x}$ for $x \neq 0$, f(0)=0
    c) f(x)=xcos x
    d) none of these
19. Let f(x)=sin x and g(x) = ln $\left|x\right|$. If the ranges of the composition functions fog and gof are R_1 and R_2 respectively, then
    a) $R_1 = \{u : -1 \leq u < 1\}, R_2 = \{v : -\infty < v < 0\}$
    b) $R_1 = \{u : -\infty < u < 0\}, R_2 = \{v : -1 \leq v \leq 0\}$
    c) $R_1 = \{u : -1 < u < 1\}, R_2 = \{v : -\infty < v < 0\}$
    d) $R_1 = \{u : -1 \leq u \leq 1\}, R_2 = \{v : -\infty < v \leq 0\}$
20. Let $f(x) = (x + 1)^2 - 1$, x ≥ -1. Then the set $\{x : f(x) = f^{-1}(x)\}$ is
    a) $\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\}$
    b) $\{0, 1, -1\}$
    c) $\{0, -1\}$
    d) empty
21. The function f(x) = $\left|px - q\right|$ + r$\left|x\right|$, $x \in (-\infty, \infty)$ where p > 0, q > 0, r > 0 assumes its minimum value only on one point if

a) p $\neq$ q
b) r $\neq$ q
c) r $\neq$ p
d) p = q = r

22. Let $f(x)$ be defined for all $x > 0$ and be continuos. Let $f(x)$ satisfy $f(\frac{x}{y}) = f(x) - f(y)$ for all x,y and $f(e) = 1$. Then
a) $f(x)$ is bounded
b) $f(\frac{1}{x}) \to 0$ as $x \to 0$
c) $xf(x) \to 1$ as $x \to 0$
d) $f(x) = lnx$

23. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is
a) $\frac{1}{2}^{x(x-1)}$
b) $\frac{1}{2}(1 + \sqrt{1 + 4log_2 x})$
c) $\frac{1}{2}(1 - \sqrt{1 + 4log_2 x})$
d) not defined

24. Let $f : R \to$ R be any function. Define g: R $\to$ R by $g(x) = \left|f(x)\right|$ for all x. Then g is
a) onto if f is onto
b) one-one if f is one-one
c) continuos if f is continuos
d) differentiable if f is differentiable

25. The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is
a) $0 < x \leq 1$
b) $0 \leq x \leq 1$
c) $-\infty < x \leq 0$
d) $-\infty < x < 1$

26. Let $g(x) = 1 + x - [x]$ and
$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0. \\ 1, & x > 0 \end{cases}$$
then for all x, $f(g(x))$ is equal to
a) x
b) 1
c) $f(x)$
d) $g(x)$

27. If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals
a) $(x + \sqrt{x^2 - 4})/2$
b) $x/(1 + x^2)$
c) $(x - \sqrt{x^2 - 4})/2$
d) $1 + \sqrt{x^2 - 4}$

28. The domain of definition of $f(x) = \frac{log_2(x+3)}{x^2+3x+2}$ is
a) $R\backslash\{-1, -2\}$
b) $(-2, \infty)$
c) R$\backslash\{-1, -2, -3\}$
d) $(-3, \infty)\backslash\{-1, -2\}$

29. Let E=$\{1, 2, 3, 4\}$ and F=$\{1, 2\}$. Then the number of onto functions from E to F is
a) 14
b) 16
c) 12
d) 8

30. Let $f(x) = \frac{\alpha x}{x+1}$,x $\neq$ -1. Then, for what value of α is $f(f(x)) = x$?
a) $\sqrt{2}$
b) $-\sqrt{2}$
c) 1
d) -1

31. Suppose $f(x) = (x + 1)^2$ for x $\geq$ -1. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line y=x then $g(x)$ equals
a) $-\sqrt{x} - 1, x \geq 0$
b) $\frac{1}{(x+1)^2}, x > -1$
c) $\sqrt{x + 1}, x \geq -1$
d) $\sqrt{x} - 1, x \geq 0$

32. Let function $f : R \to$ R be defined by $f(x) = 2x + \sin x$ for x $\in$ R, then f is
a) one-to-one and onto
b) one-to-one but NOT onto
c) onto but NOT one-to-one
d) neither one-to-one nor onto

33. If $f : [0, \infty) \to [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then f is
a) one-one and onto
b) one-one but not onto
c) onto but not one-one
d) neither one-one nor onto

34. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x ,is
a) $[-\frac{1}{4}, \frac{1}{2}]$
b) $[-\frac{1}{2}, \frac{1}{2}]$
c) $[-\frac{1}{2}, \frac{1}{9}]$
d) $[-\frac{1}{4}, \frac{1}{4}]$

35. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}; x \epsilon$R is
a) $(1, \infty)$
b) $(1, \frac{11}{7}]$
c) $(1, \frac{7}{3}]$
d) $(1, \frac{7}{5}]$

36. If $f(x) = x^2+2bx+2c^2$ and $g(x) = -x^2-2cx+b^2$ such that min $f(x) > maxg(x)$, then the relation between b and c, is
   a) no real value of b & c
   b) $0 < c < b\sqrt{2}$
   c) $|c| < |b|\ \sqrt{2}$
   d) $|c| > |b|\ \sqrt{2}$
37. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain
   a) $[0, \frac{\pi}{2}]$
   b) $[-\frac{\pi}{4}, \frac{\pi}{4}]$
   c) $[-\frac{\pi}{2}, \frac{\pi}{2}]$
   d) $[0, \pi]$
38. If the functions $f(x)$ and $g(x)$ are defined on R→R such that
$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$
;
$$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$
then $(f - g)(x)$ is
   a) one-one & onto
   b) neither one-one nor onto
   c) one-one but not onto
   d) onto but not one-one
39. X and Y are two sets and $f : X \to Y$. If $\{f(c) = y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X\}$, then the true statement is
   a) $f(f^{-1}(b))$ = b
   b) $f^{-1}(f(a))$ = a
   c) $f(f^{-1}(b))$ = b,b ⊂ y
   d) $f(f^{-1}(a))$ = a,a ⊂ x
40. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x) = -f(x)$ and $g(x) = f'(x)$ and given that F(5)=5, then F(10) is equal to
   a) 5
   b) 10
   c) 0
   d) 15
41. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for n ≥ 2 and $g(x) = \underbrace{(fofo......of)}_{\text{f occurs n times}}(x)$. Then $\int x^{n-2} g(x) dx$ equals.
   a) $\frac{1}{n(n-1)}(1 + nx^n)^{1-\frac{1}{n}} + K$
   b) $\frac{1}{(n-1)}(1 + nx^n)^{1-\frac{1}{n}} + K$
   c) $\frac{1}{n(n+1)}(1 + nx^n)^{1+\frac{1}{n}} + K$
   d) $\frac{1}{(n+1)}(1 + nx^n)^{1+\frac{1}{n}} + K$
42. Let f, g and h be real-valued functions defined on the interval $[0, 1]$ by $f(x) = e^{x^2}+e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f,g and h on $[0, 1]$, then
   a) a = b and c ≠ b
   b) a = c and a ≠ b
   c) a ≠ b and c ≠ b
   d) a = b = c
43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in$ R. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is
   a) $\pm\sqrt{n\pi}$, n ∈ {0, 1, 2.....}
   b) $\pm\sqrt{n\pi}$, n ∈ {1, 2.....}
   c) $\frac{\pi}{2} + 2n\pi$, n ∈ {..... − 2, −1, 0, 1, 2.....}
   d) $2n\pi$, n ∈ {..... − 2, −1, 0, 1, 2.....}
44. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is
   a) one-one and onto
   b) onto but not one-one
   c) one-one but not onto
   d) neither one-one nor onto
45. If y=$f(x) = \frac{x+2}{x-1}$ then
   a) $x = f(y)$
   b) $f(1)$=3
   c) y increases with x for $x < 1$
   d) f is a rational function of x
46. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at (0,0) and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is
   a) $g(x) = \pm\sqrt{1 - x^2}$
   b) $g(x) = \sqrt{1 - x^2}$
   c) $g(x) = -\sqrt{1 - x^2}$
   d) $g(x) = \sqrt{1 + x^2}$
47. If f(x)=$\cos[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then
   a) $f(\frac{\pi}{2}) = -1$
   b) $f(\pi) = 1$
   c) $f(-\pi) = 0$
   d) $f(\frac{\pi}{4}) = 1$
48. If $f(x)$=3x-5, then $f^{-1}(x)$
   a) is given by $\frac{1}{3x-5}$
   b) is given by $\frac{x+5}{3}$
   c) does not exist because f is not one-one
   d) does not exist because f is not onto.

49. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then
    a) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
    b) $f(x) = \sin x, g(x) = |x|$
    c) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$
    d) f and g cannot be determined.
50. Let f:(0,1) $\rightarrow$ R be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then
    a) f is not invertible on (0,1)
    b) $f \neq f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$
    c) $f = f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$
    d) f^{-1} is differentiable (0,1)
51. Let f:(-1,1) $\rightarrow$ IR be such that $f(\cos 4\Theta) = \frac{2}{2-\sec^2\Theta}$ for $\Theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the values of $f(\frac{1}{3})$ is
    a) 1-$\sqrt{\frac{3}{2}}$
    b) 1+$\sqrt{\frac{3}{2}}$
    c) 1-$\sqrt{\frac{2}{3}}$
    d) 1+$\sqrt{\frac{2}{3}}$
52. The funciton $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at x=
    a) -2
    b) $\frac{-2}{3}$
    c) 2
    d) $\frac{2}{3}$
53. Let $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow R$ be given by $f(x) = (log(\sec x + \tan x))^3$. Then
    a) $f(x)$ is an odd function
    b) $f(x)$ is one-one function
    c) $f(x)$ is an onto function
    d) $f(x)$ is an even function
54. Let a∈R and let f: R $\rightarrow$ R be given by $f(x) = x^5 - 5x + a$. Then
    a) $f(x)$ has three real roots if a>4
    b) $f(x)$ has only real root if a>4
    c) $f(x)$ has three real roots if a<-4
    d) $f(x)$ has three real roots if -4<a<4
55. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all x ∈R and $g(x) = \frac{\pi}{2}\sin x$ for all x∈R. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is true?
    a) Range of f is $[-\frac{1}{2}, \frac{1}{2}]$
    b) Range of fog is $[-\frac{1}{2}, \frac{1}{2}]$
    c) $\lim_{x\to 0}\frac{f(x)}{g(x)} = \frac{\pi}{6}$
    d) There is an x ∈R such that $(gof)(x) = 1$
56. Find the domain and range of the function $f(x) = \frac{x^2}{1+x^2}$. Is the function one-to-one?
57. Draw the graph of $y = |x|^{1/2}$ for $-1 \leq x \leq 1$.
58. If $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find $f(6)$.
59. Consider the following relations in the set of real numbers R. $R = \{(x, y); x \in R, y \in R, x^2 + y^2 \leq 25\}$
    $R' = \{(x, y) : x \in R, y \in R, y \geq \frac{4}{9}x^2\}$. Find the domain and the range of $R \cap R'$. Is the relation $R \cap R'$ a function?
60. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B.
61. Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1$, $f(y) \neq 1, f(z) \neq 2$ determine $f^{-1}(1)$.
62. Let R be the set of real numbers and f: R $\rightarrow$ R be such that for all x and y in R$|f(x) - f(y)| \leq |x - y|^3$. Prove that $f(x)$ is a constant.
63. Find the natural number $'a'$ for which $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, where the function $'f'$ satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x,y and further $f(1) = 2$.
64. Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$.
65. A function $f : IR \rightarrow IR$, where IR is the set of real numbers, defined by $f(x) = \frac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$. Find the interval of values of α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer.
66. Let $f(x) = Ax^2 + Bx + c$ where A,B,C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A,A+B and C are all integers. Conversely, prove that if the numbers 2A,A+B and C are all integers then $f(x)$ is an integer whenever x is an integer.
67. Let $f : [0, 4\pi] \rightarrow [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is

68. The value of $((log_2 9)^2)^{\frac{1}{log_2(log_2 9)}} \times (\sqrt{7})^{\frac{1}{log_4 7}}$ is ......
69. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is ......
70. The domain of $\sin^{-1}[log_3(x/3)]$ is
    a) $[1, 9]$
    b) $[-1, 9]$
    c) $[-9, 1]$
    d) $[-9, -1]$
71. The function $f(x) = log(x + \sqrt{x^2 + 1})$, is
    a) neither an even nor an odd function
    b) an even function
    c) an odd function
    d) a periodic function.
72. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + log_{10}(x^3 - x)$, is
    a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
    b) (a,2)
    c) $(-1, 0) \cup (a, 2)$
    d) $(1, 2) \cup (2, \infty)$.
73. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all x, y $\in$ R and f(1)=7, then $\sum_{r=1}^{n} f(r)$ is
    a) $\frac{7n(n+1)}{2}$
    b) $\frac{7n}{2}$
    c) $\frac{7(n+1)}{2}$
    d) $7n + (n + 1)$
74. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$

    is
    a) neither one-one nor onto
    b) one-one but not onto
    c) onto but not one-one
    d) one-one and onto both.
75. The range of the function $f(x) =^{7-x} P_{x-3}$ is
    a) $\{1, 2, 3, 4, 5\}$
    b) $\{1, 2, 3, 4, 5, 6\}$
    c) $\{1, 2, 3, 4\}$
    d) $\{1, 2, 3, \}$
76. Let $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is
    a) $[-1, 3]$
    b) $[-1, 1]$
    c) $[0, 1]$
    d) $[0, 3]$
77. The graph of the function $y = f(x)$ is symmetrical about the line x=2, then
    a) $f(x) = -f(-x)$
    b) $f(2 + x) = f(2 - x)$
    c) $f(x) = f(-x)$
    d) $f(x + 2) = f(x - 2)$
78. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is
    a) $[1, 2]$
    b) $[2, 3)$
    c) $[1, 2]$
    d) $[2, 3]$
79. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval
    a) $(0, \frac{\pi}{2})$
    b) $[0, \frac{\pi}{2})$
    c) $[-\frac{\pi}{2}, \frac{\pi}{2})$
    d) $(-\frac{\pi}{2}, \frac{\pi}{2})$
80. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	Function
(a). $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b). $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c). $(-\infty, \frac{1}{3}]$	$3x^2 - 2x + 1$
(d). $(-\infty, -4)$	$x^3 + 6x^2 + 6$

81. A real valued function $f(x)$ satisfies the functional equation

$$f(x - y) = f(x)f(y) - f(a - x)f(a + y)$$

    where a is a given constant and $f(0) = 1, f(2a - x)$ is equal to
    a) $-f(x)$
    b) $f(x)$
    c) $f(a) + f(a - x)$
    d) $f(-x)$
82. The Largest interval lying in $(\frac{-\pi}{2}, \frac{\pi}{2})$ for which the function,

$$f(x) = 4^{-x^2} + cos^{-1}(\frac{x}{2} - 1) + log(\cos x))$$

    , is defined, is
    a) $[-\frac{\pi}{4}, \frac{\pi}{2})$
    b) $[0, \frac{\pi}{2})$

c) $[0, \pi]$
d) $(-\frac{\pi}{2}, \frac{\pi}{2})$

83. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where

$$Y = \{y \in N : y = 4x + 3 for some x \in N\}$$

a) $g(y) = \frac{3y+4}{3}$
b) $g(y) = 4 + \frac{y+3}{4}$
c) $g(y) = \frac{y+3}{4}$
d) $g(y) = \frac{y-3}{4}$

84. Let $f(x) = (x + 1)^2 - 1$,x ≥ -1
Statement-1 : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}\}$
Statement-2 : f is a bijection

a) Statement-1 is true,Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

85. For real x, let $f(x) = x^3 + 5x + 1$, then

a) f is onto R but not one-one
b) f is one-one and onto R
c) f is neither one-one nor onto R
d) f is one-one but not onto R

86. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is

a) $(0, \infty)$
b) $(-\infty, 0)$
c) $(-\infty, \infty) - \{0\}$
d) $(-\infty, \infty)$

87. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to:

a) $f_3(x)$
b) $f_3(x)$
c) $f_2(x)$
d) $f_1(x)$

88. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:

a) 6
b) 8
c) 4
d) 14

89. If the function $f : R - \{1, -1\}$. A defined by f(x)=$\frac{x^2}{1-x^2}$, is surjective, then A is equal to:

a) R-$\{-1\}$
b) $[0, \infty)$
c) R-$[-1, 0)$
d) R-$(-1, 0)$

90. Let

$$\sum_{k=1}^{10} f(a + k) = 16(2^{10} - 1),$$

where the function f satisifies f(x+y)=f(x)f(y) for all natural numbers x,y and f(a) is = 2. Then the natural number $'a'$ is:

a) 2
b) 16
c) 4
d) 3

Match the following

91. Let the function defined in Colum 1 have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ and range $(-\infty, \infty)$

Column I	Column II
(A) 1+2x	(p) onto but not one-one
(B) $\tan x$	(q) one-one but not onto
	(r) one-one and onto
	(s) neither one-one nor onto

92. Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$.
Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

93. Let $E_1 = \{x \in R : x \neq 1 and \frac{x}{x-1} > 0\}$ and $E_2 = \{x \in E_1 : \sin^{-1}\left(log_e(\frac{x}{x-1})\right) is a real number\}$. (Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$).
Let $f : E_1 \to R$ be the function defined by $f(x) = log_e(\frac{x}{x-1})$ and g: $E_2 \to$ R be the function defined by $g(x) = sin^{-1}(log_e(\frac{x}{x-1}))$.
The correct option is:

LIST-I	LIST-II
P. The range of f is	1. $(-\infty, \frac{1}{1-e}] \cup [\frac{e}{e-1}, \infty)$
Q. The range of g contains	2. (0,1)
R. The domain of f contains	3. $[-\frac{1}{2}, \frac{1}{2}]$
S. The domain of g is	4. $(-\infty, 0) \cup (0, \infty)$
	5. $(-\infty, \frac{e}{e-1}]$
	6. $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1}]$

a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 1$
b) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 6$
c) $P \rightarrow 3; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$
d) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$

3 Quadratic Equations and Inequations