Linear Algebra through Coordinate Geometry

G V V Sharma*

Contents

***Abstract*—This manual introduces linear algebra through coordinate geometry using a problem solving approach.**

1 The Straight Line

1.1 The equation of the line between two points $\mathbf{A}$ and $\mathbf{B}$ is given by

$$\mathbf{x} = \mathbf{A} + \lambda (\mathbf{A} - \mathbf{B}) \tag{1.1}$$

Alternatively, it can be expressed as

$$\mathbf{m}^T (\mathbf{x} - \mathbf{A}) = 0 \tag{1.2}$$

where $\mathbf{m}$ is the solution of

$$(\mathbf{A} - \mathbf{B})^T \mathbf{m} = 0 \tag{1.3}$$

2 Locus

2.1 The line through

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.1}$$

intersects the coordinate axes at $\mathbf{P}$ and $\mathbf{Q}$. $\mathbf{O}$ is the origin and rectangle $OPRQ$ is completed as shown i Fig. (2.1),

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

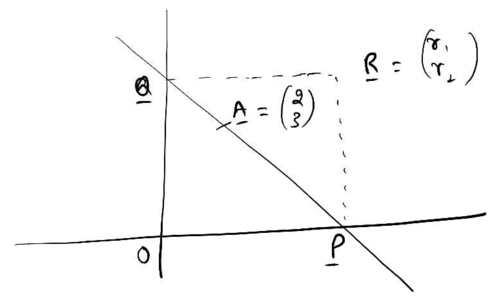

Fig. 2.1

2.2 Show that

$$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{R} \tag{2.2}$$

$$\mathbf{Q} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{R} \tag{2.3}$$

$$\mathbf{P} + \mathbf{Q} = \mathbf{R} \tag{2.4}$$

2.3 Show that

$$\begin{aligned} (\mathbf{A} - \mathbf{P})^T \mathbf{m} &= 0 \\ (\mathbf{A} - \mathbf{Q})^T \mathbf{m} &= 0 \\ (\mathbf{P} - \mathbf{Q})^T \mathbf{m} &= 0 \end{aligned} \tag{2.5}$$

Solution: Trivial using (1.2) and (1.3).

2.4 Show that

$$(2\mathbf{A} - \mathbf{R})^T \mathbf{m} = 0 \tag{2.6}$$

$$\mathbf{R}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{m} = 0 \tag{2.7}$$

Solution: From (2.5) and (2.4)

$$[2\mathbf{A} - (\mathbf{P} + \mathbf{Q})]^T \mathbf{m} = 0 \tag{2.8}$$

resulting in (2.6). From (2.5) and (2.2),(2.3), (2.7) is obtained.

2.5 Show that

$$\mathbf{R}^T \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{R} = 0. \tag{2.9}$$

2.6 Find the locus of $\mathbf{R}$.

Solution: For $\mathbf{m}$ to be unique in (2.6),(2.7),

$$(\mathbf{2A} - \mathbf{R}) = k \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{R}$$

$$\implies \mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbf{2A} - \mathbf{R})$$

$$= k\mathbf{R}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{R}$$

$$= k\mathbf{R}^T \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{R} = 0 \tag{2.10}$$

where k is some constant.

3 Conics

3.1 The tangent at $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$ to the curve

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} = 6 \tag{3.1}$$

touches the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 4 \begin{pmatrix} 4 & 3 \end{pmatrix} \mathbf{x} + c = 0 \tag{3.2}$$

Find c.