The Conic Sections

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Abstract—**Solved problems from JEE mains papers related to Conic Sections in coordinate geometry are available in this document. These problems are solved using linear algebra/matrix analysis.**

1 Find the point of intersection of the tangents at the ends of the latusrectum of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x}. \tag{1.1}$$

2 An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $\mathbf{P} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$. Its one directrix is the common tangent, nearer to the point $\mathbf{P}$, to the circle

$$\mathbf{x}^T \mathbf{x} = 1 \tag{2.1}$$

and the hyperbola

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 1. \tag{2.2}$$

Find the equation of the ellipse.

3 The equation

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{1-r} & 0 \\ 0 & -\frac{1}{1+r} \end{pmatrix} \mathbf{x} = 1, r > 1 \tag{3.1}$$

represents

a) an ellipse
b) a hyperbola
c) a circle
d) none of these

4 Each of the four inequalities given below defines a region in the xy plane. One of these four regions does not have the following property. For any two points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ in the region, the point $\begin{pmatrix} \frac{x_1+x_2}{2} \\ \frac{y_1+y_2}{2} \end{pmatrix}$ is also in the region. Find the inequality defining this region.

a) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} \le 1$
b) $\text{Max} \begin{pmatrix} |x| \\ |y| \end{pmatrix} \le 1$
c) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} \le 1$
d) $\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \le 0$

5 The equation

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -8 & -18 \end{pmatrix} \mathbf{x} + 35 = k \tag{5.1}$$

represents

a) no locus if $k > 0$
b) an ellipse if$k < 0$
c) a point if k=0
d) a hyperbola if $k > 0$

6 Let E be the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \mathbf{x} = 1 \tag{6.1}$$

and C be the circle

$$\mathbf{x}^T \mathbf{x} = 9. \tag{6.2}$$

let $\mathbf{P}$ and $\mathbf{Q}$ be the points $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ respectively. Then

a) Q lies inside C but outside E.
b) Q lies outside both C and E.
c) P lies inside both C and E.
d) P lies inside C but outside E.

7 Consider a circle with its center lying on the focus of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2p & 0 \end{pmatrix} \mathbf{x} \tag{7.1}$$

such that it touches the directrix of the parabola. Then find the point of intersection.

8 Find the radius of the circle passing through

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the foci of the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1, \tag{8.1}$$

and having its centre at $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

9 Let $\mathbf{P} = \begin{pmatrix} a \sec\theta \\ b \tan\theta \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} a \sec\phi \\ b \tan\phi \end{pmatrix}$ where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{-1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{9.1}$$

. If $\begin{pmatrix} h \\ k \end{pmatrix}$ is the point of intersection of the normals at $\mathbf{P}$ and $\mathbf{Q}$, then find k.

10 If

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 9 \tag{10.1}$$

is the chord of contact of the hyperbola

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 9 \tag{10.2}$$

then find the equation of the corresponding pair of tangents.

11 The curve describes parametrically by

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = t^2 + t + 1 \tag{11.1}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = t^2 - t + 1 \tag{11.2}$$

represents

a) a pair of straight lines
b) an ellipse
c) a parabola
d) a hyperbola

12 If

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = k \tag{12.1}$$

is normal to

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 & 0 \end{pmatrix} \mathbf{x}, \tag{12.2}$$

then find k.

13 If the line

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 1 = 0 \tag{13.1}$$

is the directrix of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} k & 0 \end{pmatrix} \mathbf{x} + 8 = 0, \tag{13.2}$$

then find k.

14 Find the equation of the common tangent touching the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{14.1}$$

and the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x}. \tag{14.2}$$

15 Find the equation of the directrix of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 4 & 4 \end{pmatrix} \mathbf{x} + 2 = 0. \tag{15.1}$$

16 If $a > 2b > 0$ then the positive value of m for which

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} m & 0 \end{pmatrix} \mathbf{x} - b\sqrt{1 + m^2} \tag{16.1}$$

is the common tangent to

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = b^2 \tag{16.2}$$

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2a & 0 \end{pmatrix} \mathbf{x} = a^2 - b^2 \tag{16.3}$$

is

a) $\frac{2b}{\sqrt{a^2-4b^2}}$
b) $\frac{\sqrt{a^2-4b^2}}{2b}$
c) $\frac{2b}{a-2b}$
d) $\frac{b}{a-2b}$

17 The locus of the mid-point of the line segment joining the focus to a moving point on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4a & 0 \end{pmatrix} \mathbf{x} \tag{17.1}$$

is another parabola with directrix

a) $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -a$
b) $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = \frac{-a}{2}$
c) $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0$
d) $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = \frac{a}{2}$

18 Find the equation of the common tangent to

the curves

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} \tag{18.1}$$

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}\mathbf{x} = -1 \tag{18.2}$$

19 Find the area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\0 & \frac{1}{5}\end{pmatrix}\mathbf{x} = 1. \tag{19.1}$$

20 The focal chord to

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}16 & 0\end{pmatrix}\mathbf{x} \tag{20.1}$$

is tangent to

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}12 & 0\end{pmatrix}\mathbf{x} + 36 = 0 \tag{20.2}$$

then the possible values of the slope of the chord,are

a) $\begin{pmatrix}-1\\1\end{pmatrix}$
b) $\begin{pmatrix}-2\\2\end{pmatrix}$
c) $\begin{pmatrix}-2\\-\frac{1}{2}\end{pmatrix}$
d) $\begin{pmatrix}2\\-\frac{1}{2}\end{pmatrix}$

21 For hyperbola

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{\cos^2\alpha} & 0\\0 & -\frac{1}{\sin^2\alpha}\end{pmatrix}\mathbf{x} = 1 \tag{21.1}$$

which of the following remains constant with change in $'\alpha'$

a) abscissae of vertices
b) abscissae of foci
c) eccentricity
d) directrix

22 If tangents are drawn to the ellipse

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 2\end{pmatrix}\mathbf{x} = 2 \tag{22.1}$$

then the locus of the mid point of the intercept made by the tangents between the coordinate axes is

a) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$
b) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$
c) $\frac{x^2}{2} + \frac{y^2}{4} = 1$
d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

23 Find the angle between the tangents drawn from the points $\begin{pmatrix}1\\4\end{pmatrix}$ to the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{23.1}$$

24 If the line

$$\begin{pmatrix}2 & \sqrt{6}\end{pmatrix}\mathbf{x} = 2 \tag{24.1}$$

touches the hyperbola

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & -2\end{pmatrix}\mathbf{x} = 4 \tag{24.2}$$

then find the point of contact.

25 The minimum area of the triangle is formed by the tangent to the

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{a^2} & 0\\0 & \frac{1}{b^2}\end{pmatrix}\mathbf{x} = 1 \tag{25.1}$$

the coordinate axes is

a) ab sq.units
b) $\frac{a^2+b^2}{2}$sq.units
c) $\frac{(a+b)^2}{2}$sq.units
d) $\frac{a^2+ab+b^2}{3}$sq.units

26 Tangent to the curve

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} + 6 \tag{26.1}$$

at the points $\begin{pmatrix}1\\7\end{pmatrix}$touches the circle

$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix}16 & 12\end{pmatrix}\mathbf{x} + c = 0 \tag{26.2}$$

at a point $\mathbf{Q}$.Then the coordinates of $\mathbf{Q}$ are

a) $\begin{pmatrix}-6\\-11\end{pmatrix}$
b) $\begin{pmatrix}-9\\-13\end{pmatrix}$
c) $\begin{pmatrix}-10\\-15\end{pmatrix}$
d) $\begin{pmatrix}-6\\-7\end{pmatrix}$

27 The axis of a parabola is along the line

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} \tag{27.1}$$

and the distance of its vertex and focus from the origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively.If vertex and focus both lies in the first quadrant,then find the equation of parabola.

28 A hyperbola, having the transverse axis of length $2\sin\theta$,is confocal with the ellipse

$$\mathbf{x}^T\begin{pmatrix}3 & 0\\0 & 4\end{pmatrix}\mathbf{x} = 12. \tag{28.1}$$

Then find its equation.

29 Let a and b be non zero real numbers,then the equation

$$(\mathbf{x}^T\begin{pmatrix}a & 0\\0 & b\end{pmatrix}\mathbf{x} + c)(\mathbf{x}^T\begin{pmatrix}1 & 0\\-5 & 6\end{pmatrix}\mathbf{x}) = 0 \tag{29.1}$$

represents

a) four straight lines, when c=0 and a,b are of the same sign
b) two straight lines and a circle ,when a=b,and c is of sign opposite to that of a
c) two straight lines and a hyperbola ,when a and b are of the same sign and c is of sign opposite to that of a
d) a circle and an ellipse , when a and b are of the same sign and c is of sign opposite to that of a

30 Consider a branch of the hyperbola

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & -2\end{pmatrix}\mathbf{x} + \begin{pmatrix}-2\sqrt{2} & -4\sqrt{2}\end{pmatrix}\mathbf{x} - 6 = 0 \tag{30.1}$$

with the vertex at the point **A**.Let **B** be the one of the end points of its latusrectum.If **C** is the focus of the hyperbola nearer to the point **A**,find the area of the triangle ABC.

31 The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 9\end{pmatrix}\mathbf{x} = 9 \tag{31.1}$$

meets its auxiliary circle at the point **M** then the area of the triangle with vertices at A, M and the origin O is

a) $\frac{31}{10}$
b) $\frac{29}{10}$
c) $\frac{21}{10}$
d) $\frac{27}{10}$

32 The normal at a point **P** on the ellipse

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 4\end{pmatrix}\mathbf{x} = 16 \tag{32.1}$$

meets the x-axis at **Q**. If **M** is the mid point of the line segment PQ, then the locus of **M** intersects the latusrectums of the given ellipse at the points

a) $\begin{pmatrix}\pm\frac{3\sqrt{5}}{2}\\ \pm\frac{2}{7}\end{pmatrix}$
b) $\begin{pmatrix}\pm\frac{3\sqrt{5}}{2}\\ \pm\sqrt{\frac{19}{4}}\end{pmatrix}$
c) $\begin{pmatrix}\pm 2\sqrt{3}\\ \pm\frac{1}{7}\end{pmatrix}$
d) $\begin{pmatrix}\pm 2\sqrt{3}\\ \pm\frac{4\sqrt{3}}{7}\end{pmatrix}$

33 The locus of the orthocentre of the triangle formed by the lines

$$\begin{pmatrix}(1+p) & -p\end{pmatrix}\mathbf{x} + p(1+p) = 0 \tag{33.1}$$

$$\begin{pmatrix}(1+q) & -q\end{pmatrix}\mathbf{x} + q(1+q) = 0 \tag{33.2}$$

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 0 \tag{33.3}$$

, where $p \neq q$ is

a) a hyperbola
b) a parabola
c) an ellipse
d) a straight line

34 Let $\mathbf{P} = \begin{pmatrix}6\\3\end{pmatrix}$ be a points on the hyperbola

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{a^2} & 0\\0 & -\frac{1}{b^2}\end{pmatrix}\mathbf{x} = 1. \tag{34.1}$$

If the normal at the points **P** intersects the x-axis at $\begin{pmatrix}9\\0\end{pmatrix}$, then find the eccentricity of the hyperbola.

a) $\sqrt{\frac{5}{2}}$
b) $\sqrt{\frac{3}{2}}$
c) $\sqrt{2}$
d) $\sqrt{3}$

35 Let $\begin{pmatrix}x\\y\end{pmatrix}$ be any point on the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{35.1}$$

. Let $\mathbf{P}$ be the points that divides the lines segment from $\begin{pmatrix}0\\0\end{pmatrix}$ to $\begin{pmatrix}x\\y\end{pmatrix}$ in the ratio 1:3. Then the locus of P is

a) $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} = \begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x}$

b) $\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}2 & 0\end{pmatrix}\mathbf{x}$

c) $\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}$

d) $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} = \begin{pmatrix}0 & 2\end{pmatrix}\mathbf{x}$

36 The ellipse $\mathbf{E_1}$:

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\0 & \frac{1}{4}\end{pmatrix}\mathbf{x} = 1. \tag{36.1}$$

is inscribed in a rectangle R whose sides are parallel to the coordinate axes.Another ellipse $\mathbf{E_2}$ passing through the points $\begin{pmatrix}0\\4\end{pmatrix}$circumscribes the rectangle R. Find the eccentricity of the ellipse $\mathbf{E_2}$.

37 The common tangents to the circle

$$\mathbf{x}^T\mathbf{x} = 2 \tag{37.1}$$

and the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} \tag{37.2}$$

touch the circle at the points $\mathbf{P}, \mathbf{Q}$ and the parabola at the points $\mathbf{R}, \mathbf{S}$. Then find the area of the quadrilateral PQRS.

38 The number of values of c such that the straight line

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} + c \tag{38.1}$$

touches the curve

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{4} & 0\\0 & 1\end{pmatrix}\mathbf{x} = 1 \tag{38.2}$$

is

a) 0
b) 1
c) 2
d) infinite.

39 If $\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix}, \mathbf{F_1} = \begin{pmatrix}3\\0\end{pmatrix}, \mathbf{F_2} = \begin{pmatrix}-3\\0\end{pmatrix}$and

$$\mathbf{x}^T\begin{pmatrix}16 & 0\\0 & 25\end{pmatrix}\mathbf{x} = 400, \tag{39.1}$$

then $\mathbf{PF_1} + \mathbf{PF_2}$equals

a) 8
b) 6
c) 10
d) 12

40 On the ellipse

$$\mathbf{x}^T\begin{pmatrix}4 & 0\\0 & 9\end{pmatrix}\mathbf{x} = 1, \tag{40.1}$$

the points at which the tangents are parallel to the line

$$\begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} = \begin{pmatrix}0 & 9\end{pmatrix}\mathbf{x} \tag{40.2}$$

are

a) $\begin{pmatrix}\frac{2}{5}\\\frac{1}{5}\end{pmatrix}$

b) $\begin{pmatrix}-\frac{2}{5}\\\frac{1}{5}\end{pmatrix}$

c) $\begin{pmatrix}-\frac{2}{5}\\-\frac{1}{5}\end{pmatrix}$

d) $\begin{pmatrix}\frac{2}{5}\\-\frac{1}{5})\end{pmatrix}$

41 The equation of the common tangents to the parabola

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} \tag{41.1}$$

and

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\4 & 1\end{pmatrix}\mathbf{x} = 4 \tag{41.2}$$

is/are

a) $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}-4 & 0\end{pmatrix}\mathbf{x} + 4=0$
b) $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 0$
c) $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} - 4 = 0$
d) $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}30 & 0\end{pmatrix}\mathbf{x} + 50 = 0$

42 Let the hyperbola passes through the focus of the ellipse

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{25} & 0\\0 & \frac{1}{16}\end{pmatrix}\mathbf{x} = 1 \tag{42.1}$$

The transverse and conjugate axes of this hyperbola coincides with the major and minor axis of the given ellipse also the product of eccentricities of given ellipse and hyperbola is 1, then

a) the equation of the hyperbola is $\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{16} \end{pmatrix} \mathbf{x} = 1$
b) the equation of the hyperbola is $\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{25} \end{pmatrix} \mathbf{x} = 1$
c) focus of hyperbola is $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
d) vertex of hyperbola is $\begin{pmatrix} 5\sqrt{3} \\ 0 \end{pmatrix}$

43 Let $\mathbf{P} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} and \mathbf{Q} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, y_1 < 0, y_2 < 0$,be the end point of the latus rectum of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 4 \tag{43.1}$$

.The equation of parabola with latus rectum PQ are

a) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2\sqrt{3} \end{pmatrix} \mathbf{x} = 3 + \sqrt{3}$
b) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -2\sqrt{3} \end{pmatrix} \mathbf{x} = 3 + \sqrt{3}$
c) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2\sqrt{3} \end{pmatrix} \mathbf{x} = 3 - \sqrt{3}$
d) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -2\sqrt{3} \end{pmatrix} \mathbf{x} = 3 - \sqrt{3}$

44 In a triangle ABC with fixed base BC,the vertex A moves such that $\cos B + \cos C = 4\sin^2 \frac{A}{2}$. If a, b and c denote the lengths of the triangle A,B and C,respectively,then

a) b+c=4a
b) b+c=2a
c) locus of point**A** is an ellipse
d) locus of point **A** is a pair of straight lines

45 The tangent PT and the normal PN to the parabola

$$\mathbf{x}^T \begin{pmatrix} -4a & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{45.1}$$

at a point **P** on it meet its axis at points **T** and **N**, respectively. The locus of the centroid of the triangle PTN is a parabola whose

a) vertex is $\begin{pmatrix} \frac{2a}{3} \\ 0 \end{pmatrix}$
b) directrix is $\begin{pmatrix} 1 & 0 \end{pmatrix}$=0
c) latus rectum is $\frac{2a}{3}$
d) focus is $\begin{pmatrix} a \\ 0 \end{pmatrix}$

46 An ellipse intersects the hyperbola

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \mathbf{x} = 1 \tag{46.1}$$

orthogonally.The eccentricity of the ellipse is reciprocal of that of the hyperbola.If the axes of the ellipse are along the coordinate axes,then

a) equation of ellipse is$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} = 2$
b) the foci of ellipse are $\begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}$
c) equation of ellipse is $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} = 4$
d) the foci of ellipse are $\begin{pmatrix} \pm\sqrt{2} \\ 0 \end{pmatrix}$

47 Let**A** and **B** two distinct points on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x}. \tag{47.1}$$

If the axis of a parabola touches a circle of radius r, having AB as its diameter, then the slop of the line joining A and B can be

a) $-\frac{1}{r}$
b) $\frac{1}{r}$
c) $\frac{2}{r}$
d) $-\frac{2}{r}$

48 Let the eccentricity of the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & -\frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{48.1}$$

. If the hyperbola passes to that of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 4 \tag{48.2}$$

. If the hyperbola passing through a focus of the ellipse, then

a) the equation of the hyperbola is $\mathbf{x}^T \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \mathbf{x} = 1$
b) the focus of the hyperbola is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
d) the equation of the hyperbola is $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \mathbf{x} = 3$

49 Let L be a normal to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} \tag{49.1}$$

. If $\mathbf{L}$ passes though the point$\begin{pmatrix} 9 \\ 6 \end{pmatrix}$, then $\mathbf{L}$ is given by

a) $\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} + 3 = 0$
b) $\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} - 33 = 0$
c) $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 15 = 0$
d) $\begin{pmatrix} -2 & 1 \end{pmatrix} \mathbf{x} + 12 = 0$

50 Tangents are drawn to the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix} \mathbf{x} = 1, \tag{50.1}$$

parallel to the straight line

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{x} = 1 \tag{50.2}$$

. The point of contact of the tangents on the hyperbola are

a) $\begin{pmatrix} \frac{9}{2\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
b) $\begin{pmatrix} \frac{9}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$
c) $\begin{pmatrix} 3\sqrt{3} \\ -2\sqrt{2} \end{pmatrix}$
d) $\begin{pmatrix} -3\sqrt{3} \\ 2\sqrt{2} \end{pmatrix}$

51 Let $\mathbf{P}$ and $\mathbf{Q}$ be distinct points on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x}. \tag{51.1}$$

such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are)the coordinates of $\mathbf{P}$?

a) $\begin{pmatrix} 4 \\ 2\sqrt{2} \end{pmatrix}$
b) $\begin{pmatrix} 9 \\ 3\sqrt{2} \end{pmatrix}$
c) $\begin{pmatrix} \frac{1}{4} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
d) $\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$

52 Let $\mathbf{E_1}$and $\mathbf{E_2}$be two ellipses whose centers are at the origin. The major axes of $\mathbf{E_1}$ and $\mathbf{E_2}$ lie along the x-axis and the y-axis, respectively. Let S be the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -2 \end{pmatrix} \mathbf{x} = 1 \tag{52.1}$$

. The straight line

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \tag{52.2}$$

touches the curves S. E_1 and E_2 at $\mathbf{P}$,$\mathbf{Q}$ and $\mathbf{R}$ respectively. Suppose that PQ=PR=$\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 andE_2, respectively, Then the correct expression(s) is (are)

a) $e_1^2 + e_2^2 = \frac{43}{40}$
b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$
c) $\mid e_1^2 - e_2^2 \mid= \frac{5}{8}$
d) $e_1 e_2 = \frac{\sqrt{3}}{4}$

53 Consider a hyperbola H:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 1 \tag{53.1}$$

and a circle S with center $\mathbf{N} = \begin{pmatrix} x_2 \\ 0 \end{pmatrix}$. Suppose that H and S touches each other at a point $\mathbf{P} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at $\mathbf{P}$ intersects the x-axis at point $\mathbf{P}$. If$\begin{pmatrix} 1 \\ m \end{pmatrix}$ is the centroid of the triangle PMN, then the correct expression is(are)

a) $\dfrac{dl}{dx_1} = 1 - \frac{1}{3x_1^2} for x_1 > 1$
b) $\dfrac{dm}{dx_1} = \frac{x_1}{3(\sqrt{x_1^2 - 1})}$for $x_1 > 1$
c) $\dfrac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}forx_1 > 1$
d) $\dfrac{dm}{dx_1} = \frac{1}{3}fory_1 > 0$

54 The circle C_1:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 3 \tag{54.1}$$

, with centre at O, intersects the parabola

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 2 \end{pmatrix} \mathbf{x} \tag{54.2}$$

and centres $Q_2 Q_3$, respectively. If $Q_2 Q_3$ lie on the y-axis, then

a) $Q_2Q_3 = 12$
b) $R_2\ R_3$=$4\sqrt{6}$
c) area of the triangle O$R_2R_3$is $6\sqrt{2}$
d) area of the triangle P$Q_2Q_3$is $4\sqrt{2}$

55 Let **P** be the point on the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{55.1}$$

which is at the shortest distance from the center S of the circle $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}+\begin{pmatrix}-4 & -16\end{pmatrix}\mathbf{x}+64 = 0$. Let **Q** be the point on the circle dividing the line segment SP internally. Then

a) SP=$2\sqrt{5}$
b) SQ:QP=($\sqrt{5}+1$) : 2
c) the x-intercept of the normal to the parabola at **P** is 6.
d) the slop of the tangent to the circle at **Q** is $\frac{1}{2}$.

56 If $\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x} + 1 = 0$ is a tangent to the hyperbola

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{a^2} & 0\\0 & -\frac{1}{16}\end{pmatrix}\mathbf{x} = 1. \tag{56.1}$$

then which of the can not be sides of a right angled triangle ?

a) a,4,1
b) a,4,2
c) 2a,8,1
d) 2a,4,1

57 If a chord, which is not a tangent, of the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}16 & 0\end{pmatrix}\mathbf{x} \tag{57.1}$$

has the equation $\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = p$, and midpoint $\begin{pmatrix}h\\k\end{pmatrix}$, then which of the following are possible values of p,h and k?

a) p=-2,h=2,k=-4
b) p=-1,h=1,k=-3
c) p=2,h=3,k=-4
d) p=5,h=4,k=-3

58 Consider two straight lines, each of which is tangents to both the circle

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \frac{1}{2} \tag{58.1}$$

and the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{58.2}$$

. Let these lines intersect at a point **Q**. Consider the ellipse whose centre is at the origin $\mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix}$ and whose semi major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is(are) TRUE?

a) For the ellipse,the eccentricity is $\frac{1}{\sqrt{2}}$and the length of the latus rectum is 1
b) For the ellipse,the eccentricity is $\frac{1}{\sqrt{2}}$and the length of the latus rectum is $\frac{1}{2}$
c) the area of the region bounded by the ellipse between the lines $\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = \frac{1}{\sqrt{2}}$and $\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 1$ is $\frac{1}{4\sqrt{2}}(\Pi-2)$
d) the area of the region bounded by the ellipse between the lines $\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = \frac{1}{\sqrt{2}}$and $\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 1$ is $\frac{1}{16}(\Pi-2)$

Subjective Problems

59 Suppose that the normals drawn at the different points on the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{59.1}$$

pass through the point $\begin{pmatrix}h\\k\end{pmatrix}$. Show that $h > 2$.

60 **A** is a point on the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4a & 0\end{pmatrix}\mathbf{x}. \tag{60.1}$$

The normal A cuts the parabola again at the point B. if AB subtends a right angle at the vertex of the parabola. find the slop of AB.

61 Three normals are drawn from the point $\begin{pmatrix}c\\0\end{pmatrix}$ to the curve

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} \tag{61.1}$$

. Show that c must be greater than $\frac{1}{2}$. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other.

62 Through the vertex O of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x}, \tag{62.1}$$

chords OQ and OP are drawn at right angles to one other. Show that for all positions of **P**, PQ cuts the axis of the parabola at a fixed point. also find the locus of the middle point of PQ.

63 Show that the locus of point that divides a chord of slop 2 of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} \tag{63.1}$$

internally in the ratio 1:2 is a parabola. Find the vertex of this parabola.

64 Let $'d'$ be the perpendicular distance from the centre of the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{64.1}$$

to the tangent drawn at a point **P** on the ellipse. If F_1 and F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2(1 - \frac{b^2}{d^2})$.

65 Points **A**,**B** and **C** lie on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4a & 0 \end{pmatrix} \mathbf{x}. \tag{65.1}$$

The tangents to the parabola at A,B and C, taken in pairs, intersects at points **P**, **Q** and **R**. Determine the ratio of the areas of the triangles ABC and PQR.

66 From a point **A** common tangents are drawn to the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \frac{a^2}{2} \tag{66.1}$$

and parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4a & 0 \end{pmatrix} \mathbf{x}. \tag{66.2}$$

Find the area of the quadrilateral formed by the common tangents, the chord of the contact of the circle and the chord of the contact of the parabola

67 A tangent to the ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 4 \tag{67.1}$$

meets the ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} = 6 \tag{67.2}$$

at point **P** and **Q**. Prove that the tangents at point **P** and **Q** of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} = 6 \tag{67.3}$$

are at right angles.

68 The angle between a pair of tangents drawn from a point **P** to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4a & 0 \end{pmatrix} \mathbf{x} \tag{68.1}$$

is 45°. Show that the locus of the point **P** is a hyperbola.

69 Consider the family of circles

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = r^2 \tag{69.1}$$

, $2 < r < 5$. If in the first quadrant, the common tangent to the circle of this family and the ellipse

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 25 \end{pmatrix} \mathbf{x} = 100 \tag{69.2}$$

meet the coordinate axes at A and B, then find the equation of the locus of the midpoint of AB.

70 Find co-ordinates of all the points **P** on the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1, \tag{70.1}$$

for which the area of the triangle PON is maximum, where O denotes the origin and N, the foot of the perpendicular from O to the tangent at P.

71 Let ABC be an equilateral triangle inscribed in the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = a^2. \tag{71.1}$$

Suppose perpendiculars from A,B,C to the major axis of the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1, (a > b) \tag{71.2}$$

meets the ellipse respectively, at **P**, **Q**, **R**. So, that P,Q,R lie on the same side of the major axis as ABC respectively. Prove that the normals to the ellipse drawn at the points **P**,**Q** and **R** are concurrent.

72 Let C_1 and C_2 be respectively, the parabola

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} - 1 \tag{72.1}$$

and

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 1. \tag{72.2}$$

Let **P** be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be the reflection of **P** and **Q**, respectively. with respect to the line

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}. \tag{72.3}$$

Prove that P_1 lies on C_2, Q_1 lies on C_1 and $PQ \geq \min \begin{pmatrix} PP_1 \\ QQ_1 \end{pmatrix}$. Hence or otherwise determine points P_0 and Q_0 on parabolas C_1 and C_2 respectively such that $\left(P_0Q_0 \leq PQ\right)$ for all pairs points $\begin{pmatrix} P \\ Q \end{pmatrix}$ with **P** on C_1 and **Q** on C_2.

73 Let**P** be a point on the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1, \tag{73.1}$$

$0 < b < a$. Let the line parallel to y-axis passing through **P** meet the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = a^2 \tag{73.2}$$

at the point **Q** such that **P** and **Q** are on the same side of x-axis. For two positive real numbers r and s, find the locus of the point **R** on PQ such that $PR : RQ = r : s$ as **P** varies over ellipse.

74 Prove that in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding dirctrix.

75 Normals are drawn from the point **P** with slopes m_1, m_2, m_3 to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} \tag{75.1}$$

. If locus of P with m_1,m_2=α. is a part of the parabola it self then find α.

76 Tangents is drawn to parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & -2 \end{pmatrix} \mathbf{x} + 5 = 0 \tag{76.1}$$

at a point **Q**. A point **R** is such that it divides QP externally in the ratio $\frac{1}{2}$: 2. Find the locus of point **R**.

77 Tangents are drawn from any point on the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix} \mathbf{x} = 1 \tag{77.1}$$

to the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 9. \tag{77.2}$$

Find the locus of mid-point of the chord of contact.

78 Find the equation of the common tangents in the 1st quadrant to the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 16 \tag{78.1}$$

and the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \mathbf{x} = 1. \tag{78.2}$$

Also find the length of the intercept of the tangent between the coordinate axes.

Comprehension Based Questions

PASSAGE I

Consider the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{78.3}$$

and the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x}. \tag{78.4}$$

They intersects at **P** and **Q** in the first and fourth quadrants, respectively. Tangents to the circle at **P** and **Q** intersects the x-axis at **R** and tangents to the parabola at **P** and **Q** intersects the x-axis at **S**.

79 The ratio of the areas of the triangles PQS and PQR is

a) $1 : \sqrt{2}$

b) 1 : 2
c) 1 : 4
d) 1 : 8

80 The radius of the circumcircle of the triangle PRS is
a) 5
b) $3\sqrt{3}$
c) $3\sqrt{2}$
d) $2\sqrt{3}$

81 The radius of the incircle of the triangle PQR is
a) 4
b) 3
c) $\frac{8}{3}$
d) 2

PASSAGE 2 The circle

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} - \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} = 0 \tag{81.1}$$

and hyperbola

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\0 & -\frac{1}{4}\end{pmatrix}\mathbf{x} = 1 \tag{81.2}$$

intersect at the points **A** and **B**.

82 Equation of a common tangent with positive slop to the circle as well as to the hyperbola is
a) $\begin{pmatrix}2 & -\sqrt{5}\end{pmatrix}\mathbf{x} - 20 = 0$
b) $\begin{pmatrix}2 & -\sqrt{5}\end{pmatrix}\mathbf{x} + 4 = 0$
c) $\begin{pmatrix}3 & -4\end{pmatrix}\mathbf{x} + 8 = 0$
d) $\begin{pmatrix}4 & -3\end{pmatrix}\mathbf{x} + 4 = 0$

83 Equation of the circle with AB as its diameter is
a) $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}-12 & 0\end{pmatrix}\mathbf{x} + 24 = 0$
b) $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}12 & 0\end{pmatrix}\mathbf{x} + 24 = 0$
c) $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}24 & 0\end{pmatrix}\mathbf{x} - 12 = 0$
d) $\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}-24 & 0\end{pmatrix}\mathbf{x} - 12 = 0$

PASSAGE 3 Tangents are drawn from the point $\mathbf{P} = \begin{pmatrix}3\\4\end{pmatrix}$ to the ellipse

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\0 & \frac{1}{4}\end{pmatrix}\mathbf{x} = 1 \tag{83.1}$$

touches the ellipse at points **A** and **B**.

84 The coordinates of A and B are
a) $\begin{pmatrix}3\\0\end{pmatrix} and \begin{pmatrix}0\\2\end{pmatrix}$
b) $\begin{pmatrix}-\frac{8}{5}\\\frac{2\sqrt{161}}{15}\end{pmatrix}$and $\begin{pmatrix}-\frac{9}{5}\\\frac{8}{5}\end{pmatrix}$
c) $\begin{pmatrix}-\frac{8}{5}\\\frac{2\sqrt{161}}{15}\end{pmatrix}and\begin{pmatrix}0\\2\end{pmatrix}$
d) $\begin{pmatrix}3\\0\end{pmatrix}and\begin{pmatrix}-\frac{9}{5}\\\frac{8}{5}\end{pmatrix}$

85 The orthocenter of the triangle PAB is
a) $\begin{pmatrix}5\\\frac{8}{7}\end{pmatrix}$
b) $\begin{pmatrix}\frac{7}{5}\\\frac{25}{8}\end{pmatrix}$
c) $\begin{pmatrix}\frac{11}{5}\\\frac{8}{5}\end{pmatrix}$
d) $\begin{pmatrix}\frac{8}{25}\\\frac{7}{5}\end{pmatrix}$

86 The equation of the locus of a point whose distances from the point **P** and the line AB are equal, is
a) $\mathbf{x}^T\begin{pmatrix}9 & 0\\-6 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}-54 & -62\end{pmatrix}\mathbf{x} + 241 = 0$
b) $\mathbf{x}^T\begin{pmatrix}1 & 0\\6 & 9\end{pmatrix}\mathbf{x} + \begin{pmatrix}-54 & 62\end{pmatrix}\mathbf{x} - 241 = 0$
c) $\mathbf{x}^T\begin{pmatrix}9 & 0\\-6 & 9\end{pmatrix}\mathbf{x} + \begin{pmatrix}-54 & -62\end{pmatrix}\mathbf{x} - 241 = 0$
d) $\mathbf{x}^T\begin{pmatrix}1 & 0\\2 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}27 & 31\end{pmatrix}\mathbf{x} - 120 = 0$

PASSAGE 4
Let PQ be the focal chord of the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4a & 0\end{pmatrix}\mathbf{x}. \tag{86.1}$$

The tangents to the parabola at **P** and **Q** meet at a point lying on the line $\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}2 & 0\end{pmatrix}\mathbf{x} + a, a > 0$.

87 Length of chord PQ is
a) 7a
b) 5a
c) 2a
d) 3a

88 If the chord PQ subtends an angle θ at the vertex of

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4a & 0\end{pmatrix}\mathbf{x} \tag{88.1}$$

, then $\tan\theta$=

a) $\frac{2}{3}\sqrt{7}$
b) $-\frac{2}{3}\sqrt{7}$
c) $\frac{2}{3}\sqrt{5}$
d) $-\frac{2}{3}\sqrt{5}$

PASSAGE 5 Let a,r,s,t be the non zero real numbers. Let $\mathbf{P} = \begin{pmatrix} at^2 \\ 2at \end{pmatrix}$,$\mathbf{Q}$, $\mathbf{R} = \begin{pmatrix} ar^2 \\ 2ar \end{pmatrix}$ and $\mathbf{S} = \begin{pmatrix} as^2 \\ 2as \end{pmatrix}$ be distinct points on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4a & 0 \end{pmatrix} \mathbf{x}. \qquad (88.2)$$

Suppose that PQ is the focal chord and lines QR and PK are parallel, where $\mathbf{K} = \begin{pmatrix} 2a \\ 0 \end{pmatrix}$

89 The value of r is
a) $-\frac{1}{t}$
b) $\frac{t^2+1}{t}$
c) $\frac{1}{t}$
d) $\frac{t^2-1}{t}$

90 If st=1, then the tangent at **P** and the normal at **S** to the parabola meet at a point whose ordinate is
a) $\frac{(t^2+1)^2}{2t^3}$
b) $\frac{a(t^2+1)^2}{2t^3}$
c) $\frac{a(t^2+1)^2}{t^3}$
d) $\frac{a(t^2+2)^2}{t^3}$

PASSAGE 6
Let $F_1 = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$ and $F_2 = \begin{pmatrix} x_2 \\ 0 \end{pmatrix}$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{8} \end{pmatrix} \mathbf{x} = 1. \qquad (90.1)$$

Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point **M** in the first quadrant and at point **N** in the fourth quadrant.

91 The orthocentre of the triangle F_1MN is
a) $\begin{pmatrix} -\frac{9}{10} \\ 0 \end{pmatrix}$
b) $\begin{pmatrix} \frac{2}{3} \\ 0 \end{pmatrix}$
c) $\begin{pmatrix} \frac{9}{10} \\ 0 \end{pmatrix}$
d) $\begin{pmatrix} \frac{2}{3} \\ \sqrt{6} \end{pmatrix}$

92 If the tangents of the ellipse at **M** and **N** meet at **R** and the normals to the parabola at **M** meets the x-axis at **Q**, then the ratio of the triangle MQR to area of the quadrilateral MF_1NF_2 is
a) 3:4
b) 4:5
c) 5:8
d) 2:3

Assertion and Reason Type Questions
STATEMENT-I: The curve $\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x}=$

$$\mathbf{x}^T \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 1 \qquad (92.1)$$

. because
STATEMENT-2:A parabola is symmetric about its axis.
a) Statement-1 is True,Statement-2 is True;Statement-2 is a correct explanation for Statement-1
b) Statement-1 is True,Statement-2 is True;Statement-2 is NOT correct explanation for Statement-1
c) Statement-1 is True,Statement-2 is False
d) Statement-1 is False,Statement-2 is True.

I Integer Value Correction Type

93 The line $\begin{pmatrix} 2 & 1 \end{pmatrix}\mathbf{x} = 1$ is tangent to the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & -\frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \qquad (93.1)$$

If this line passes through the point of intersection of the nearest directrix and the x-axis,then the eccentricity of the hyperbola is

94 Consider the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} \qquad (94.1)$$

. Let $\Delta_1$be the area of the triangle formed by the end points of its latus rectum and the point $\mathbf{P} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}$ on the parabola and Δ_2 be the area of the triangle formed by drawing tangents at **P** and at the end of the points of the latus rectum.Then $\frac{\Delta_1}{\Delta_2}$ is

95 Let S be the focus of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} \qquad (95.1)$$

and let PQ be the common chord of the cicle

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}+\begin{pmatrix}-2 & -4\end{pmatrix}\mathbf{x}=0 \tag{95.2}$$

and the given parabola. The area of the triangle PQS is

96 A vertical line passing through the point $\begin{pmatrix}h\\0\end{pmatrix}$ intersects the ellipse

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{4} & 0\\0 & \frac{1}{3}\end{pmatrix}\mathbf{x}=1 \tag{96.1}$$

. at the point **P** and **Q**. Let the tangents to the ellipse at **P** and **Q** meet at the point **R**. If $\Delta(h)$ = area of the triangle PQR, Δ_1 = max $\frac{1}{2} < h < 1\Delta(h)$ and $\Delta_2 = \min\frac{1}{2} < h < 1\Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2$

a) g(x) is continuous but not differentiable at a
b) g(x) is differentiable on R
c) g(x) is continuous but not differentiable at b
d) g(x) is continuous but not differentiable at either (a)or (b) but not both.

97 If the normals of the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}=\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{97.1}$$

drawn at the end points of its latus rectum are tangents to the circle

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}+\begin{pmatrix}-6 & 4\end{pmatrix}\mathbf{x}-5=r^2, \tag{97.2}$$

then the value of r^2 is

98 Let the curve C be the mirror image of the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}=\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{98.1}$$

with respect to the line $\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x}+4=0$. If **A** and **B** are the points of intersecting of **C** with the line $\begin{pmatrix}0 & 1\end{pmatrix}=-5$, then the distance between A and B is

99 Suppose that the foci of the ellipse

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{9} & 0\\0 & \frac{1}{5}\end{pmatrix}\mathbf{x}=1 \tag{99.1}$$

are $\begin{pmatrix}f_1\\0\end{pmatrix}$ and $\begin{pmatrix}f_2\\0\end{pmatrix}$ where $f_1>0$ and $f_2<0$. Let P_1 and P_2 be two parabolas with a common vertex at $\begin{pmatrix}0\\0\end{pmatrix}$ and with foci at $\begin{pmatrix}f_1\\0\end{pmatrix}$ and $\begin{pmatrix}2f_2\\0\end{pmatrix}$ respectively. Let T_1 be a tangent to P_1 which passes through $\begin{pmatrix}2f_2\\0\end{pmatrix}$ and T_2 be a tangent to P_2 which passes through $\begin{pmatrix}f_1\\0\end{pmatrix}$. If m_1 is the slope of the T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is

Section-B

JEE Main/AIEEE

100 Two common tangents to the circle

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}=2a^2 \tag{100.1}$$

and parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}=\begin{pmatrix}8a & 0\end{pmatrix}\mathbf{x} \tag{100.2}$$

are

a) $\begin{pmatrix}1 & 0\end{pmatrix}=\pm(\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x}+2a)$
b) $\begin{pmatrix}0 & 1\end{pmatrix}=\pm(\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}+2a)$
c) $\begin{pmatrix}1 & 0\end{pmatrix}=\pm(\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x}+a)$
d) $\begin{pmatrix}0 & 1\end{pmatrix}=\pm(\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}+a)$

101 The normals at the point $\begin{pmatrix}bt_1^2\\2bt_1\end{pmatrix}$ on a parabola meets the parabola again in the point $\begin{pmatrix}bt_2^2\\2bt_2\end{pmatrix}$, then

a) $t_2=t_1+\frac{2}{t_1}$
b) $t_2=-t_1-\frac{2}{t_1}$
c) $t_2=-t_1+\frac{2}{t_1}$
d) $t_2=t_1-\frac{2}{t_1}$

102 The foci of the ellipse

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{16} & 0\\0 & \frac{1}{b^2}\end{pmatrix}\mathbf{x}=1 \tag{102.1}$$

and the hyperbola

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{144} & 0\\0 & -\frac{1}{81}\end{pmatrix}\mathbf{x}=\frac{1}{25} \tag{102.2}$$

coincide.Then the value of b^2 is

a) 9
b) 1
c) 5
d) 7

103 If $a \neq 0$ and the line

$$\mathbf{x}^T \begin{pmatrix} 2b & 0 \\ 0 & 3c \end{pmatrix} \mathbf{x} + 4d = 0 \tag{103.1}$$

passes through the point of intersection of the parabolas

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4a & 0 \end{pmatrix} \mathbf{x} \tag{103.2}$$

and

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 4a \end{pmatrix} \mathbf{x} \tag{103.3}$$

, then

a) $d^2 + (3b - 2c)^2 = 0$
b) $d^2 + (3b + 2c)^2 = 0$
c) $d^2 + (2b - 3c)^2 = 0$
d) $d^2 + (2b + 3c)^2 = 0$

104 The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$.If one of the directrices is $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 4$, then the equation of the ellipse is:

a) $\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix} \mathbf{x} = 1$
b) $\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 12$
c) $\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix} \mathbf{x} = 12$
d) $\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 1$

105 Let $\mathbf{P}$ be the point $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{Q}$ a point on the locus

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} \tag{105.1}$$

the locus of mid point of PQ is

a) $\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 2 = 0$
b) $\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} + 2 = 0$
c) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & 4 \end{pmatrix} \mathbf{x} + 2 = 0$
d) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 2 = 0$

106 The locus of a point $\mathbf{P} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ moving under the condition that the line $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \alpha & 0 \end{pmatrix} \mathbf{x} + \beta$ is a tangent to the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & -\frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{106.1}$$

is

a) an ellipse
b) a circle
c) a parabola
d) a hyperbola

107 An ellipse has OB as semi minor axis, F and F' it's foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

a) $\frac{1}{\sqrt{2}}$
b) $\frac{1}{2}$
c) $\frac{1}{4}$
d) $\frac{1}{\sqrt{3}}$

108 The locus of the vertices of the family of parabolas

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \mathbf{x}^T \begin{pmatrix} \frac{a^3}{3} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} \frac{a^2}{2} & 0 \end{pmatrix} \mathbf{x} - 2a \tag{108.1}$$

is

a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \frac{105}{64}$
b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \frac{3}{4}$
c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \frac{35}{16}$
d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \frac{64}{105}$

109 In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is

a) $\frac{3}{5}$
b) $\frac{1}{2}$
c) $\frac{4}{5}$
d) $\frac{1}{\sqrt{5}}$

110 Angle between the tangents to the curve $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -5 & 0 \end{pmatrix} \mathbf{x} + 6$ at the points $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ is

a) π
b) $\frac{\pi}{2}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{4}$

111 For the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{\cos^2 \alpha} & 0 \\ 0 & -\frac{1}{\sin^2 \alpha} \end{pmatrix} \mathbf{x} = 1 \tag{111.1}$$

, which of the following remains constant when α varies=?

a) abscissae of vertices
b) abscissae of foci
c) eccentricity
d) directrix.

112 The equation of a tangent to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} \tag{112.1}$$

is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} + 2. \tag{112.2}$$

The point on this line from which the other tangents to the parabola is perpendicular to the given tangent is

a) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$
b) $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
c) $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$
d) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

113 The normal to a curve at $\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$ meets the x-axis at G. If the distance G from the origin is twice the abscissa of $\mathbf{P}$, then the curve is a

a) circle
b) hyperbola
c) ellipse
d) parabola.

114 A focus of an ellipse is at the origin. The directrix is the line

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 4 \tag{114.1}$$

and the eccentricity is $\frac{1}{2}$. Then the length of the semi major axis is

a) $\frac{8}{3}$
b) $\frac{2}{3}$
c) $\frac{4}{3}$
d) $\frac{5}{3}$

115 A parabola has the origin as its focus and the line

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \tag{115.1}$$

as directrix. Then the vertex of the parabola is at

a) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
b) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
c) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
d) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

116 The ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 4 \tag{116.1}$$

is inscribed in a rectangular aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$. Then the equation of the ellipse is :

a) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 12 \end{pmatrix} \mathbf{x} = 16$
b) $\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 48 \end{pmatrix} \mathbf{x} = 48$
c) $\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 64 \end{pmatrix} \mathbf{x} = 48$
d) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 16 \end{pmatrix} \mathbf{x} = 16$

117 If two tangents drawn from a point $\mathbf{P}$ to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} \tag{117.1}$$

are at right angles, then the locus of P is

a) $\begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} + 1 = 0$
b) $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -1$
c) $\begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} - 1 = 0$
d) $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 1$

118 Equation of the ellipse whose axes are the axes of coordinates and which passes through the point $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and has eccentricity $\sqrt{\frac{2}{5}}$ is

a) $\mathbf{x}^T \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix} \mathbf{x} - 48 = 0$

b) $\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix} \mathbf{x} - 15 = 0$
c) $\mathbf{x}^T \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix} \mathbf{x} - 32 = 0$
d) $\mathbf{x}^T \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix} \mathbf{x} - 32 = 0$

119 **Statement-1:**An equation of a common tangent to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 16\sqrt{3} & 0 \end{pmatrix} \mathbf{x} \tag{119.1}$$

and the ellipse

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 4 \tag{119.2}$$

is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} + 2\sqrt{3} \tag{119.3}$$

Statement-2: If the line

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} m & 0 \end{pmatrix} \mathbf{x} + \frac{4\sqrt{3}}{m}(m \neq 0) \tag{119.4}$$

, is a common tangent to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 16\sqrt{3} & 0 \end{pmatrix} \mathbf{x} \tag{119.5}$$

and the ellipse

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 4 \tag{119.6}$$

, then m satisfies $m^4 + 2m^2 = 24$

a) Statement-1 is false, Statement-2 is true.
b) Statement-1 is true, Statement-2 is true;Statement-2 is correct explanation for Statement-1.
c) Statement-1 is true, Statement-2 is true;Statement-2 is NOT correct explanation for Statement-1.
d) Statement-1 is true, Statement-2 is false.

120 An ellipse is drawn by taking a diameter of the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{120.1}$$

as its semi-minor axis and a diameter of the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & 4 \end{pmatrix} \mathbf{x} = 0 \tag{120.2}$$

is semi-major axis. If the center of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

a) $\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 4$
b) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 8$
c) $\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = 8$
d) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \mathbf{x} = 16$

121 The equation of the circle passing through the foci of the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1, \tag{121.1}$$

and having centre at $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is

a) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -6 \end{pmatrix} \mathbf{x} - 7 = 0$
b) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -6 \end{pmatrix} \mathbf{x} + 7 = 0$
c) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -6 \end{pmatrix} \mathbf{x} - 5 = 0$
d) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -6 \end{pmatrix} \mathbf{x} + 5 = 0$

122 **Given:** A circle,

$$\mathbf{x}^T \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} = 5 \tag{122.1}$$

and a parabola,

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4\sqrt{5} & 0 \end{pmatrix} \mathbf{x} \tag{122.2}$$

. **Statement-I:** An equation of a common tangent to these curve is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} + \sqrt{5} \tag{122.3}$$

. **Statement-2:** If the line,

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} m & 0 \end{pmatrix} \mathbf{x} + \frac{\sqrt{5}}{m}(m \neq 0) \tag{122.4}$$

is their common tangent,then m satisfies $m^4 - 3m^2 + 2 = 0$.

a) Statement-1 is true,Statement-2 is true;Statement-2 is correct explanation for Statement-1.
b) Statement-1 is true,Statement-2 is

true;Statement-2 is not correct explanation for Statement-1.
c) Statement-1 is true,Statement-2 is false.
d) Statement-1 is false,Statement-2 is true.

123 The locus of the foot of perpendicular drawn from the centre of the ellipse

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \mathbf{x} = 6 \quad (123.1)$$

on any tangent to it is
a) $(x^2 + y^2)^2 = 6x^2 + 2y^2$
b) $(x^2 + y^2)^2 = 6x^2 - 2y^2$
c) $(x^2 - y^2)^2 = 6x^2 + 2y^2$
d) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

124 The slop of the line touching both the parabolas

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} \quad (124.1)$$

and

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & -32 \end{pmatrix} \mathbf{x} \quad (124.2)$$

is
a) $\frac{1}{8}$
b) $\frac{2}{3}$
c) $\frac{1}{2}$
d) $\frac{3}{2}$

125 Let O be the vertex and Q be any point on the parabola,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{x}. \quad (125.1)$$

If the point **P** divides the lines segments OQ internally in the ratio 1:3, then locus of P is:
a) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x}$
b) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 2 \end{pmatrix} \mathbf{x}$
c) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x}$
d) $\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}$

126 The normal to the curve,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix} \mathbf{x} = 0, \quad (126.1)$$

at$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
a) meets the curve again in the third quadrant.
b) meets the curve again in the fourth quadrant.
c) does not meet the curve again.
d) meets the curve again in the second quadrant.

127 The area(in sq.units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{5} \end{pmatrix} \mathbf{x} = 1 \quad (127.1)$$

is
a) $\frac{27}{2}$
b) 27
c) $\frac{27}{4}$
d) 18

128 Let **P** be the point on the parabola,

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} \quad (128.1)$$

which is at a minimum distance from the centre C of the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & 12 \end{pmatrix} \mathbf{x} + 36 = 1, \quad (128.2)$$

Then the equation of the circle, passing through C and having its centre at **P** is:
a) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -\frac{1}{4} & 2 \end{pmatrix} \mathbf{x} - 24 = 0$
b) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & 9 \end{pmatrix} \mathbf{x} + 18 = 0$
c) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & 8 \end{pmatrix} \mathbf{x} + 12 = 0$
d) $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} - 12 = 0$

129 The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :
a) $\frac{2}{\sqrt{3}}$
b) $\sqrt{3}$
c) $\frac{4}{3}$
d) $\frac{4}{\sqrt{3}}$

130 A hyperbola passes through the point $\mathbf{P} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix}$ and has foci at $\begin{pmatrix} \pm 2 \\ 0 \end{pmatrix}$. Then the tangent to this hyperbola at **P** also passes through the point:

a) $\begin{pmatrix} -\sqrt{2} \\ -\sqrt{3} \end{pmatrix}$
b) $\begin{pmatrix} 3\sqrt{2} \\ 2\sqrt{3} \end{pmatrix}$
c) $\begin{pmatrix} 2\sqrt{3} \\ 3\sqrt{3} \end{pmatrix}$
d) $\begin{pmatrix} \sqrt{3} \\ \sqrt{2} \end{pmatrix}$

131 The radius of a circle, having minimum area, which touches the curve

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 4 - \mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} \tag{131.1}$$

and the lines,

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} |1| & 0 \end{pmatrix} \mathbf{x} \tag{131.2}$$

is:
a) $4(\sqrt{2}+1)$
b) $2(\sqrt{2}+1)$
c) $2(\sqrt{2}-1)$
d) $4(\sqrt{2}-1)$

132 Tangents are drawn to the hyperbola

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 36 \tag{132.1}$$

at the points **P** and **Q**. if these tangents intersect at the point $\mathbf{T} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ then the area(in sq.units) of the ΔPTQ is :
a) $54\sqrt{3}$
b) $60\sqrt{3}$
c) $36\sqrt{5}$
d) $45\sqrt{5}$

133 Tangents are normal are drawn at $\mathbf{P} = \begin{pmatrix} 16 \\ 16 \end{pmatrix}$ on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 16 & 0 \end{pmatrix} \mathbf{x}, \tag{133.1}$$

which intersect the axis of the parabola at **A** and **B**, respectively. If C is the centre of the circle through the points P,A and B and $\angle CPB = \theta$, then the value of $\tan\theta$ is :
a) 2
b) 3
c) $\frac{4}{3}$
d) $\frac{1}{2}$

134 Two sets A and B are as under :

$$A = \begin{pmatrix} a \\ b \end{pmatrix} \in RxR : |a-5| < 1 and |b-5| < 1 \tag{134.1}$$

$$B = \begin{pmatrix} a \\ b \end{pmatrix} \in RxR : 4(a-6)^2 + 9(b-5)^2 \leq 36. \tag{134.2}$$

Then:
a) $A \subset B$
b) $A \cap B = \phi$(an empty set)
c) neither $A \subset B$ nor $B \subset A$
d) $B \subset A$

135 If the tangent at $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$ to the curve

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} - 6 \tag{135.1}$$

touches the circle

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 16 & 12 \end{pmatrix} \mathbf{x} + c = 0 \tag{135.2}$$

then the value of c is :
a) 185
b) 85
c) 95
d) 195

136 Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?
a) $\begin{pmatrix} 5 \\ 2\sqrt{6} \end{pmatrix}$
b) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$
c) $\begin{pmatrix} 6 \\ 4\sqrt{2} \end{pmatrix}$
d) $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

137 Let $0 < \theta < \frac{\pi}{2}$. If The eccentricity of the hyperbola

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{\cos^2\theta} & 0 \\ 0 & \frac{1}{\sin^2\theta} \end{pmatrix} \mathbf{x} = 1 \tag{137.1}$$

is greater than 2, then the length of its latus rectum lies in the interval:
a) $\begin{pmatrix} 3 \\ \infty \end{pmatrix}$

b) $\begin{pmatrix}\frac{3}{2}\\2\end{pmatrix}$
c) $\begin{pmatrix}2\\3\end{pmatrix}$
d) $\begin{pmatrix}1\\\frac{3}{2}\end{pmatrix}$

138 Equation of a common tangent to the circle,

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x}-\begin{pmatrix}6 & 0\end{pmatrix}\mathbf{x}=0 \tag{138.1}$$

and the parabola,

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}=\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} \tag{138.2}$$

is :
a) $\begin{pmatrix}0 & 2\sqrt{3}\end{pmatrix}\mathbf{x}=\begin{pmatrix}12 & 0\end{pmatrix}\mathbf{x}+1$
b) $\begin{pmatrix}0 & \sqrt{3}\end{pmatrix}\mathbf{x}=\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}+3$
c) $\begin{pmatrix}0 & 2\sqrt{3}\end{pmatrix}\mathbf{x}=\begin{pmatrix}-1 & 0\end{pmatrix}\mathbf{x}-12$
d) $\begin{pmatrix}0 & \sqrt{3}\end{pmatrix}\mathbf{x}=\begin{pmatrix}3 & 0\end{pmatrix}\mathbf{x}+1$

139 If the line

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x}=\begin{pmatrix}m & 0\end{pmatrix}\mathbf{x}+7\sqrt{3} \tag{139.1}$$

is normal to the hyperbola

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{24} & 0\\0 & -\frac{1}{18}\end{pmatrix}\mathbf{x}=1, \tag{139.2}$$

then a value of m is:
a) $\frac{\sqrt{5}}{2}$
b) $\frac{\sqrt{15}}{2}$
c) $\frac{2}{\sqrt{5}}$
d) $\frac{3}{\sqrt{5}}$

140 If one end of a focal chord of the parabola,

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}=\begin{pmatrix}16 & 0\end{pmatrix}\mathbf{x} \tag{140.1}$$

is a $\begin{pmatrix}1\\4\end{pmatrix}$.Then the length of this focal chord is:
a) 25
b) 22
c) 24
d) 20

Match the Following DIRECTIONS(Q. 1-3) Each question contains statements given in two columns, which have to be matched. the statement in column-1 is labelled can A, B, C and D. while the three statements in column-2 are labelled p, q, r, s and t. any given statement in column-1 can have correct matching with ONE or MORE statements in column-2.

141 Match the following: $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ is the pt, from which three normals are drawn to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} \tag{141.1}$$

which meet the parabola in the points **P**, **Q** and **R**. Then

Column-I	**Column-II**
(A) Area of ΔPQR	(p)2
(B) Radius of circum circle of ΔPQR	(q)$\frac{5}{2}$
(C) Centroid of ΔPQR	(r)$\begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix}$
(D) circumcentre of ΔPQR	(s)$\begin{pmatrix} \frac{2}{3} \\ 0 \end{pmatrix}$

142 Match statements in the column I with the properties in Column II and indicate your answer by darkening the bubbles in 4 x 4 matrix given in the ORS.

Column-I	**Column-II**
(A) Two intersecting circles	(p) have a common tangents
(B) Two mutually external circles	(q) have a common normals
(C) Two circles,one strictly inside the other	(r) do not have a common tangents
(D) Two branches of a hyperbola	(s) do not have a common normals

143 Match the conics in Column I with the statement/expression in Column II

Column-I	Column-II
(A) Circle	(p) The focus of point $\begin{pmatrix} h \\ k \end{pmatrix}$for which the line $\begin{pmatrix} h & k \end{pmatrix}\mathbf{x} = 1$touches the circle $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} = 4$
(B) Parabola	(q) Point **z** in the complex plane satisfying $\lvert z+2\rvert - \lvert z-2\rvert = \pm 3$
(C) Ellipse	(r) Points of the conic have paramatric representation $\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = \sqrt{3}(\frac{1-t^2}{1+t^2}),\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = \frac{2t}{1+t^2}$
(D) Hyperbola	(s) The eccentricity of the conic lies in the interval $1 \le x < \infty$

DIRECTIONS(Q.4) Following questions are matching lists.The codes for the list have choices(a),(b),(c)and (d)out of which ONLY ONE is correct.

144 A line L:$\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} m & 0 \end{pmatrix}+3$meets y-axis at $\mathbf{E} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ and the are of the parabola $\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 16 & 0 \end{pmatrix}\mathbf{x}$, $0 \le y \le 6$at the point $\mathbf{F} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$. The tangent to the parabola at $\mathbf{F} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$intersects the y-axis at $\mathbf{G} = \begin{pmatrix} 0 \\ y_1 \end{pmatrix}$.The slope m of the line L is chosen such that the area of the triangle EGF has a local maximum. Match the List I with List II and select the correct answer using the code given below the lists:

List-I	List-II
P. m=	1. $\frac{1}{2}$
Q. Maximum area of ΔEFG is	2. 4
R. y_0=	3. 2
S. y_1=	4. 1

codes:

	P	Q	R	S
(a)	4	1	2	3
(b)	3	4	1	2
(c)	1	3	2	4
(d)	1	3	4	2

Qs.5-7: By appropriately matching the information given in the three columns of the following table Column1,2 and 3 contains conics,equations of the tangents to the conics and points of contact,respectively.

Column-I	Column-II	Column-III
(I) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\mathbf{x} = a^2$	(i) $\begin{pmatrix}0 & m\end{pmatrix}\mathbf{x} = \begin{pmatrix}m^2 & 0\end{pmatrix}\mathbf{x} + a$	(P) $\begin{pmatrix}\frac{a}{m^2}\\ \frac{2a}{m}\end{pmatrix}$
(II) $\begin{pmatrix}1 & 0\\ 0 & a^2\end{pmatrix}\mathbf{x} = a^2$	(ii) $\begin{pmatrix}0 & m\end{pmatrix}\mathbf{x} = \begin{pmatrix}m & 0\end{pmatrix}\mathbf{x} + a\sqrt{m^2+1}$	(Q) $\begin{pmatrix}-\frac{ma}{\sqrt{m^2+1}}\\ \frac{a}{\sqrt{m^2+1}}\end{pmatrix}$
(III) $\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}4a & 0\end{pmatrix}\mathbf{x}$	(iii) $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}m & 0\end{pmatrix}\mathbf{x} + \sqrt{a^2m^2-1}$	(R) $\begin{pmatrix}-\frac{a^2m}{\sqrt{a^2m^2+1}}\\ \frac{1}{\sqrt{a^2m^2+1}}\end{pmatrix}$
(IV) $\begin{pmatrix}1 & 0\\ 0 & -a^2\end{pmatrix}\mathbf{x} = a^2$	(iv) $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}m & 0\end{pmatrix}\mathbf{x} + \sqrt{a^2m^2+1}$	(S) $\begin{pmatrix}-\frac{a^2m}{\sqrt{a^2m^2-1}}\\ -\frac{1}{\sqrt{a^2m^2-1}}\end{pmatrix}$

145 For $\mathbf{a} = \sqrt{2}$,if a tangent is drawn to a suitable conic(Column 1)at the point of contact$\begin{pmatrix}-1\\ 1\end{pmatrix}$,then which of the following options is the only correct combination for obtaining its equation?

a) (I) (i) (P)
b) (I) (ii) (Q)
c) (II) (ii) (Q)
d) (III) (i) (P)

146 If a tangent to a suitable conic(Column 1)is founded to be $\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} + 8$ and its point of contact is $\begin{pmatrix}8\\ 16\end{pmatrix}$,then which of the following options is the only correct combination?

a) (I) (ii) (Q)
b) (II) (iv) (R)
c) (III) (i) (P)
d) (III) (ii) (Q)

147 The tangent to a suitable conic(Column1)at $\begin{pmatrix}\sqrt{3}\\ \frac{1}{2}\end{pmatrix}$ is found to be $\begin{pmatrix}\sqrt{3} & 2\end{pmatrix}\mathbf{x} = 4$,then which of the following options is the only correct combination?

a) (IV) (iii) (S)
b) (IV) (iv) (S)
c) (II) (iii) (R)
d) (II) (iii) (R)

148 Let $\mathbf{H}$:

$$\mathbf{x}^T\begin{pmatrix}\frac{1}{a^2} & 0\\ 0 & -\frac{1}{b^2}\end{pmatrix}\mathbf{x} = 1, \tag{148.1}$$

where $a > b > 0$,be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of $60°$ at one of its vertices

N.Let the area of the triangle LMN be $4\sqrt{3}$.

List-I	**List-II**
P. The length of the conjugate axis of H is	1. 8
Q. The eccentricity of H is	2. $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	3. $\frac{2}{\sqrt{3}}$
P. The length of the latus rectum of H is	4. 4

a) $P \rightarrow 4 Q \rightarrow 2 R \rightarrow 1 S \rightarrow 3$
b) $P \rightarrow 4 Q \rightarrow 3 R \rightarrow 1 S \rightarrow 2$
c) $P \rightarrow 4 Q \rightarrow 1 R \rightarrow 3 S \rightarrow 2$
d) $P \rightarrow 3 Q \rightarrow 4 R \rightarrow 2 S \rightarrow 1$