# Geometric Constructions through Python

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Contents



***Abstract*—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.**

Download all codes for this manual from

```
svn co
```

## 1 Triangle

1.1 Draw a line segement of length 7.6 cm and divide it in the ratio 5 : 8.
**Solution:** Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \tag{1}$$

Then the point $\mathbf{C}$

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k+1} \tag{2}$$

divides $AB$ in the ration $k : 1$. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1

```
codes/draw_section.py
```

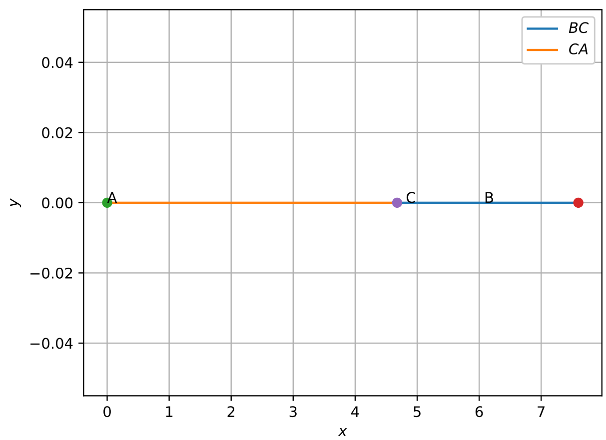


Fig. 1.1

1.2 Draw $\triangle ABC$ where $\angle B = 90°, a = 4$ and $b = 3$.
**Solution:** The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{3}$$

The following code plots Fig. 1.2

```
codes/rt_triangle.py
```

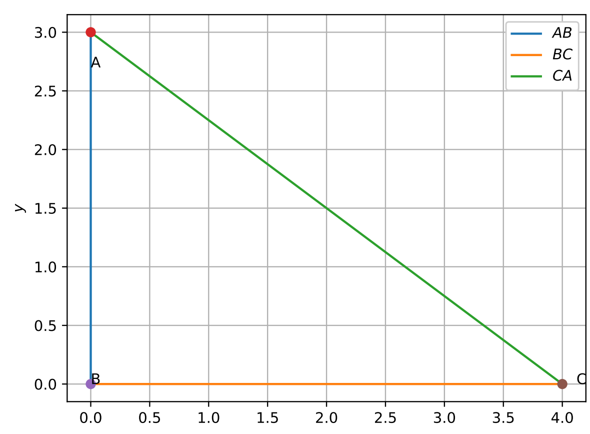


Fig. 1.2

1.3 Construct a triangle of sides $a = 4$cm, $b = 5$ cm and $c = 6$ cm.
**Solution:** Let the vertices of $\triangle ABC$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{4}$$

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Then

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = (p-a)^2 + q^2 \tag{5}$$
$$c^2 = \|\mathbf{A} - \mathbf{C}\|^2 = p^2 + q^2 \tag{6}$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \tag{7}$$
$$q = \sqrt{c^2 - p^2} \tag{8}$$

The following code plots Fig. 1.3

codes/draw_triangle.py

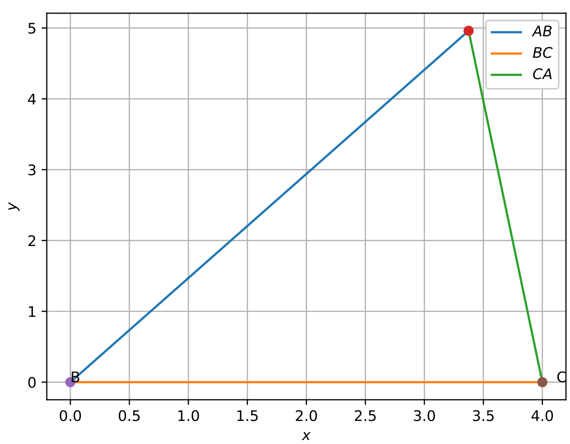


Fig. 1.3

1.4 Construct a triangle of sides $a = 5$cm, $b = 6$ cm and $c = 7$ cm. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.
**Solution:** The sides of the similar triangle are $\frac{7}{5}a, \frac{7}{5}b$ and $\frac{7}{5}c$.

1.5 Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = p = 4$cm
**Solution:** Using Baudhayana's theorem,

$$b = c = \sqrt{p^2 + \left(\frac{a}{2}\right)^2} \tag{9}$$

1.6 Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60°$.
**Solution:** In Fig. (1.6), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \tag{10}$$
$$\cos B = \frac{x}{b}, \tag{11}$$

Thus,

$$a = x + y = b\cos C + c\cos B, \tag{12}$$
$$b = c\cos A + a\cos C \tag{13}$$
$$c = b\cos A + a\cos B \tag{14}$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{15}$$

Using the properties of determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \tag{16}$$
$$= \frac{b^2 + c^2 - a^2}{2bc} \tag{17}$$

From (17)

$$b^2 = c^2 + a^2 - 2ca\cos B \tag{18}$$

which is computed by the following code

codes/cos_form.py

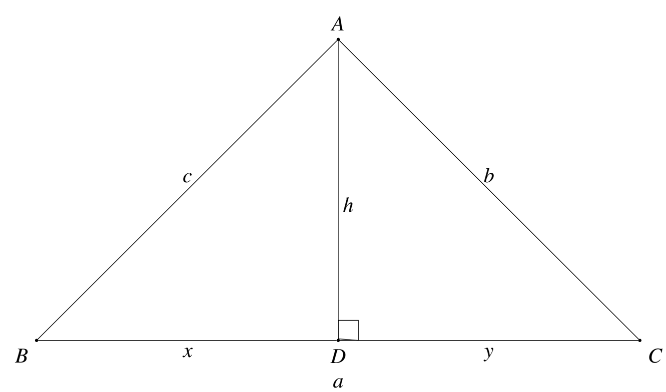


Fig. 1.6: The cosine formula

1.7 Draw $\triangle ABC$ with $a = 7, \angle B = 45°$ and $\angle A = 105°$.
**Solution:** In Fig. (1.6),

$$\sin B = \frac{h}{c} \tag{19}$$
$$\sin C = \frac{h}{b} \tag{20}$$

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \tag{21}$$

Thus,

$$c = \frac{a \sin C}{\sin A} \tag{22}$$

where

$$C = 180 - A - B \tag{23}$$

1.8 $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find $b, c$ and draw the triangle.
**Solution:** From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \tag{24}$$

$$\implies (18 - c)^2 = 12^2 + c^2 \tag{25}$$

which can be simplified to obtain

$$c^2 + 36c^2 - 180 = 0 \tag{26}$$

$$\implies (c + 18)^2 - 18^2 - 180 = 0 \tag{27}$$

$$\tag{28}$$

which can be simplified as

$$\implies (c + 18)^2 = \left(18^2 + 180\right) \tag{29}$$

$$\implies c = -18 \pm \sqrt{18^2 + 180} \tag{30}$$

1.9 In $\triangle ABC$, $a = 7, \angle B = 75^\circ$ and $b + c = 13$. Find $b$ and $c$ and sketch $\triangle ABC$.

1.10 In $\triangle ABC$, $a = 8, \angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.

1.11 In $\triangle ABC$, $a = 6, \angle B = 60^\circ$ and $b - c = 2$. Sketch $\triangle ABC$.

1.12 In $\triangle ABC$, given that $a + b + c = 11, \angle B = 45^\circ$ and $\angle C = 45^\circ$, find $a, b, c$.
**Solution:** We have

$$a = b \cos C + c \cos B \tag{31}$$

$$b \sin C = c \sin B \tag{32}$$

$$a + b + c = 11 \tag{33}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \tag{34}$$

Solving the equivalent matrix equation gives the desired answer.

1.13 Draw $\triangle ABC$, given that $a+b+c = 11, \angle B = 30^\circ$ and $\angle C = 90^\circ$, find $a, b, c$.

## 2 Circle

2.1 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents $AC$ and $CD$ to the circle.
**Solution:** From the given information, in $\triangle ABC, AC \perp AB, a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \tag{35}$$

The following code plots Fig. 2.1

```
codes/draw_circle_eg.py
```

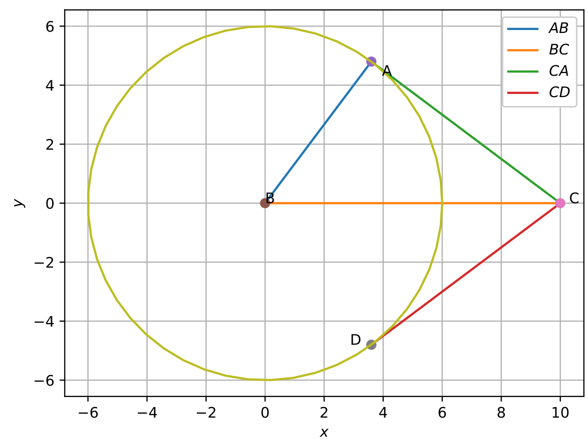


Fig. 2.1

2.2 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

2.3 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

2.4 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of $60^\circ$.

2.5 Draw a line segment $AB$ of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

2.6 Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^\circ$. $BD$ is the perpendicular from **B** on $AC$. The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.