

Discrete: Maths Olympiad

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1. Rama was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divide the result by 3. She got 43 as the answer. What would have been her answer if she had solved the problem correctly?
2. A postman has to deliver five letters to five different houses. Mischievously, he posts one letter through each door without looking to see if it is the correct address. In how many different ways could he do this so that exactly two of the five houses receive the correct letters?
3. Let

$$P(n) = (n + 1)(n + 3)(n + 5)(n + 5)(n + 7)(n + 9)$$
 What is the largest integer that is a divisor of $P(n)$ for all positive even integers n ?
4. Let N be the set of natural numbers. Suppose $f : N \rightarrow N$ is a function satisfying the following conditions.
 - a) $f(m, n) = f(m)f(n)$;
 - b) $f(m) < f(n)$ if $m < n$;
 - c) $f(2) = 2$.
 What is the value of $\sum_{k=1}^{20} f(k)$?
5. Three points X, Y, Z are on a straight line such that $XY = 10$ and $XZ = 3$. What is the product of all possible values of YZ ?
6. There are $n - 1$ red balls, n green balls and $n + 1$ blue balls in a bag. The number of ways of choosing two balls from the bag that have different colours is 299. What is the value of n ?
7. Let $S(M)$ denote the sum of the digits of a positive integer M written in base 10. Let N be the smallest positive integer such that $S(N) = 2013$. What is the value of $S(5N + 2013)$?
8. Let Akbar and Birbal together have n marbles, where $n > 0$. Akbar says to Birbal, "If I give some marbles then you will have twice as many marbles as I will have". Birbal says to Akbar, "If I give you some marbles they you will have thrice as many marbles as I will have". What is the minimum possible value of n for which the above statements are true?
9. Carol was given three numbers and was asked to add the largest of the three to the product of the other two. Instead, she multiplied the largest with the sum of the other two, but still got the right answer. What is the sum of the three numbers?
10. To each element of the set $S = \{1, 2, \dots, 1000\}$ a colour is assigned. Suppose that for any two elements a, b of S , if 15 divides $a + b$ then they are both assigned the same colour. What is the maximum possible number of distinct colours used?
11. Let m be the smallest odd positive integer for which $1 + 2 + \dots + m$ is a square of an integer and let n be the smallest even positive integer for which $1 + 2 + \dots + n$ is a square of an integer. What is the value of $m + n$?
12. What is the maximum possible value of k for which 2013 can be written as a sum of k consecutive positive integers?
13. What is the sum (in base 10) of all natural numbers less than 64 which have exactly three ones in their base 2 representation?
14. The first term of a sequence is 2014. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2014th term of the sequence?
15. In a triangle with integer side lengths, one side is three times as long as second side, and the length of the third side is 17. What is the greatest possible perimeter of the triangle?
16. For how many natural numbers n between 1 and 2014 (both inclusive) is $\frac{8n}{9999-n}$ an integer?
17. One morning, each member of Manjul's family drank an 8-ounce mixture of coffee and milk. The amounts of coffee and milk varied from

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