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JEE Problems in Linear Algebra: 2D

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***Abstract*—A collection of problems from JEE mains papers related to 2D coordinate geometry are available in this document. These problems should be solved using linear algebra.**

1. Tangent and normal are drawn at
$$\mathbf{P} = \begin{pmatrix} 16 \\ 16 \end{pmatrix} \tag{1}$$
on the parabola
$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 16 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$
which intersect the axis of the parabola at **A** and **B** respectively. If **C** is the centre of the circle through the ponts **P** **A** and **B**, find $\tan CPB$.
2. A circle passes through the points $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$. If its centre lies on the line
$$\begin{pmatrix} -1 & 4 \end{pmatrix} \mathbf{x} + 3 = 0 \tag{3}$$
find its radius.
3. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, find the equation of the common tangent to the two parabolas.
4. If the tangents drawn to the hyperbola
$$\mathbf{x}^T V \mathbf{x} + 1 = 0 \tag{4}$$
where
$$V = \begin{pmatrix} 1 & 0 \\ 0 & -4 \end{pmatrix} \tag{5}$$
intersect the coordinate axes at the distinct points **A** and **B**, find the locus of the mid point of AB.
5. β is one of the angles between the normals to the ellipse
$$\mathbf{x}^T V \mathbf{x} = 9 \tag{6}$$
where
$$V = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \tag{7}$$
at the points
$$\begin{pmatrix} 3\cos\theta \\ \sqrt{3}\sin\theta \end{pmatrix}, \begin{pmatrix} -3\sin\theta \\ \sqrt{3}\cos\theta \end{pmatrix}, \quad \theta \in \left(0, \frac{\pi}{2}\right), \tag{8}$$
then find $\frac{2\cot\beta}{\sin 2\theta}$.
6. Tangents drawn from the point $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$ to the parabola
$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -8 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{9}$$
touch the parabola at **P** and **Q**. If **F** is the focus of the parabola, then find the area of $\triangle PFQ$.
7. A normal to the hyperbola
$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & -9 \end{pmatrix} \mathbf{x} = 36 \tag{10}$$
meets the coordinate axes x and y at **A** and **B** respectively. If the parallelogram $OABP$ is formed, find the locus of **P**.
8. If a circle C,whose radius is 3, touches externally the circle
$$\mathbf{x}^T \mathbf{x} + \begin{pmatrix} 2 & -4 \end{pmatrix} \mathbf{x} = 4 \tag{11}$$
at the point $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$, then find the length of the intercept cut by this circle C on the x-axis.
9. Let **P** be the parabola
$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & 4 \end{pmatrix} \mathbf{x} = 0 \tag{12}$$
Given that the distance of **P** from the centre of the circle
$$\mathbf{x}^T \mathbf{x} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} \mathbf{x} + 8 = 0 \tag{13}$$
is minimum. Find the equation of the tangent to the parabola at **P**.

10. The length of the latus rectum of an ellipse is 4 ad the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$. Find its eccentricity.
11. A line drawn through the point
$$\mathbf{P} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{14}$$
cuts the circle
$$\mathbf{x}^T\mathbf{x} = 9 \tag{15}$$
at the points **A** and **B**. Find $PA.PB$.
12. Find the eccentricity of an ellipse having centre at the origin, axes along the coordinate axes and passing through the points
$$\mathbf{P} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}. \tag{16}$$
13. $\begin{pmatrix} m & -1 \end{pmatrix}\mathbf{x} + c = 0$ is the normal at a point on the parabola
$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} - \begin{pmatrix} 8 & 0 \end{pmatrix}\mathbf{x} = 0 = 0 \tag{17}$$
whose focal distance is 8. Find $|c|$.
14. The common tangents to the parabola
$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x} - \begin{pmatrix} 0 \\ 4 \end{pmatrix}\mathbf{x} = 0 = 0 \tag{18}$$
intersect at the point **P**. Find the distance of **P** from the origin.
15. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then find the area of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse.
16. A hyperbola passes through the point
$$\mathbf{P} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \tag{19}$$
and has foci at $\begin{pmatrix} \pm 2 \\ 0 \end{pmatrix}$. Find the equation of the tangent to this hyperbola at **P**.
17. Find the equation of the circle, which is the mirror image of the circle
$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 2 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{20}$$
in the line
$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 3. \tag{21}$$
18. Find the product of the perpendiculars drawn from the foci of the ellipse
$$\mathbf{x}^T \begin{pmatrix} 25 & 0 \\ 0 & 9 \end{pmatrix}\mathbf{x} = 225 \tag{22}$$
upon the tangent to it at the point
$$\frac{1}{2}\begin{pmatrix} 3 \\ 5\sqrt{3} \end{pmatrix} \tag{23}$$
19. Find the equation of the normal to the hyperbola
$$\mathbf{x}^T \begin{pmatrix} 9 & 0 \\ 0 & -16 \end{pmatrix}\mathbf{x} = 144 \tag{24}$$
drawn at the point
$$\begin{pmatrix} 8 \\ 3\sqrt{3} \end{pmatrix} \tag{25}$$
20. Find the locus of the centres of those circles which touch the circle
$$\mathbf{x}^T\mathbf{x} - 8\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 4 \tag{26}$$
and also touch the x-axis.
21. One of the diameters of the circle, given by
$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -2 & 3 \end{pmatrix}\mathbf{x} = 12 = 0 \tag{27}$$
is a chord of a circle S, whose centre is at
$$\begin{pmatrix} -3 \\ 2 \end{pmatrix}. \tag{28}$$
Find the radius of S.
22. Let P be the point on the parabola
$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} - \begin{pmatrix} 8 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{29}$$
which is at a minimum distance from the centre C of the circle
$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix} 0 & 12 \end{pmatrix}\mathbf{x} = 1 \tag{30}$$
Find the equation of the circle passing through C and having its centre at $\begin{pmatrix} P \end{pmatrix}$.
23. Find the eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half the distance between its foci.
24. A circle passes through
$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} \tag{31}$$

and touches the y-axis at

$$\begin{pmatrix}0\\2\end{pmatrix}. \tag{32}$$

Which one of the following equations can represent a diameter of this circle?

a) $\begin{pmatrix}4 & 5\end{pmatrix}\mathbf{x} = 6$

b) $\begin{pmatrix}2 & -3\end{pmatrix}\mathbf{x} + 10 = 0$

c) $\begin{pmatrix}3 & 4\end{pmatrix}\mathbf{x} = 3$

d) $\begin{pmatrix}5 & 2\end{pmatrix}\mathbf{x} + 4 = 0$

25. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation

$$9e^2 - 18e + 5 = 0 \tag{33}$$

If

$$\mathbf{S} = \begin{pmatrix}5\\0\end{pmatrix} \tag{34}$$

is a focus and

$$\begin{pmatrix}5 & 0\end{pmatrix}\mathbf{x} = 9 \tag{35}$$

is the corresponding directrix of this hyperbola, then find $a^2 - b^2$.

26. Find the equation of the tangent to the circle, at the point

$$\begin{pmatrix}1\\-1\end{pmatrix}, \tag{36}$$

whose centre is the point of intersection of the straight lines

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 3 \tag{37}$$

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 1 \tag{38}$$

27. **P** and **Q** are two distinct points on the parabola

$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} - \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} = 0 \tag{39}$$

with parameters t and t_1 respectively. If the normal at **P** passes through **Q**, then find the minimum value of t_1^2.

28. A hyperbola whose transverse axis is along the major axis of the conic

$$\mathbf{x}^T V\mathbf{x} = 51 \tag{40}$$

where

$$V = \begin{pmatrix}3 & 0\\0 & 27\end{pmatrix} \tag{41}$$

and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, which of the following points doesnot lie on it?

a) $\begin{pmatrix}0\\2\end{pmatrix}$

b) $\begin{pmatrix}\sqrt{5}\\2\sqrt{2}\end{pmatrix}$

c) $\begin{pmatrix}\sqrt{10}\\2\sqrt{3}\end{pmatrix}$

d) $\begin{pmatrix}5\\2\sqrt{3}\end{pmatrix}$

29. A tangent at a point on the ellipse

$$\mathbf{x}^T V\mathbf{x} = 51 \tag{42}$$

where

$$V = \begin{pmatrix}3 & 0\\0 & 27\end{pmatrix} \tag{43}$$

meets the coordinate axes at **A** and **B**. If **O** be the origin, find the minimum area of $\triangle OAB$.