Computational Approach to
School Mathematics

G V V Sharma*

Abstract—This book provides a computational approach to school mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.
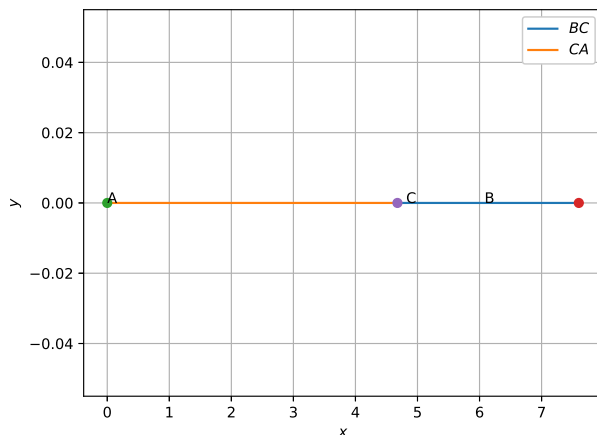
Download python codes using

1 CONSTRUCTIONS

1.1 Triangle Examples

1. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8.
Solution: Let the end points of the line be

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 7.6 \\ 0 \end{pmatrix} \quad (1.1.1.1)$$

Then the point $\mathbf{C}$

$$\mathbf{C} = \frac{k\mathbf{A} + \mathbf{B}}{k + 1} \quad (1.1.1.2)$$

divides AB in the ration $k : 1$. For the given problem, $k = \frac{5}{8}$. The following code plots Fig. 1.1.1

codes/constructions/draw_section.py

2. Draw $\triangle ABC$ where $\angle B = 90°, a = 4$ and $b = 3$.
Solution: The vertices of $\triangle ABC$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (1.1.2.1)$$
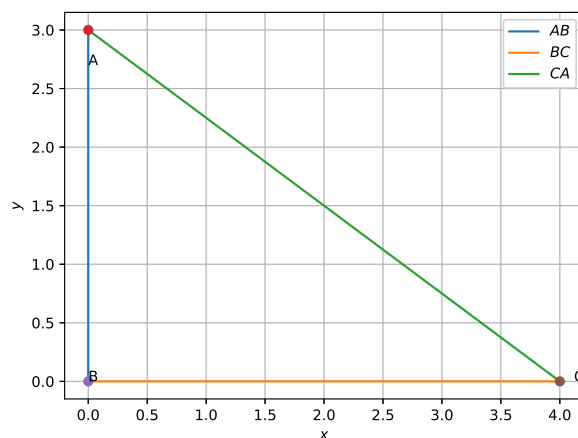
*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

Fig. 1.1.1

The following code plots Fig. 1.1.2

codes/constructions/rt_triangle.py

Fig. 1.1.2

3. Construct a triangle of sides $a = 4$, $b = 5$ and $c = 6$.

Solution: Let the vertices of $\triangle ABC$ be

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1.1.3.1)$$

$$\mathbf{A}^T \triangleq \begin{pmatrix} p & q \end{pmatrix} \quad (1.1.3.2)$$

$$\|\mathbf{A}\|^2 = \mathbf{A}^T \mathbf{A} = \begin{pmatrix} p & q \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} \quad (1.1.3.3)$$

$$= p \times p + q \times q = p^2 + q^2 \quad (1.1.3.4)$$

Then

$$AB \triangleq \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0}$$
$$(1.1.3.5)$$

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \quad (1.1.3.6)$$

$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \quad (1.1.3.7)$$

From (1.1.3.7),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \quad (1.1.3.8)$$

$$= \mathbf{A}^T\mathbf{A} + \mathbf{C}^T\mathbf{C} - \mathbf{A}^T\mathbf{C} - \mathbf{C}^T\mathbf{A} \quad (1.1.3.9)$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \quad (\because \mathbf{A}^T\mathbf{C} = \mathbf{C}^T\mathbf{A})$$
$$(1.1.3.10)$$

$$= a^2 + c^2 - 2ap \quad (1.1.3.11)$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad (1.1.3.12)$$

From (1.1.3.5),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \quad (1.1.3.13)$$

$$\implies q = \pm\sqrt{c^2 - p^2} \quad (1.1.3.14)$$

The following code plots Fig. 1.1.3

codes/constructions/draw_triangle.py

4. Construct a triangle of sides $a = 5$, $b = 6$ and $c = 7$. Construct a similar triangle whose sides are $\frac{7}{5}$ times the corresponding sides of the first triangle.
   Solution: The sides of the similar triangle are $\frac{7}{5}a, \frac{7}{5}b$ and $\frac{7}{5}c$.
5. Construct an isosceles triangle whose base is $a = 8$cm and altitude $AD = h = 4$cm
   Solution: Using Baudhayana's theorem,

$$b = c = \sqrt{h^2 + \left(\frac{a}{2}\right)^2} \quad (1.1.5.1)$$

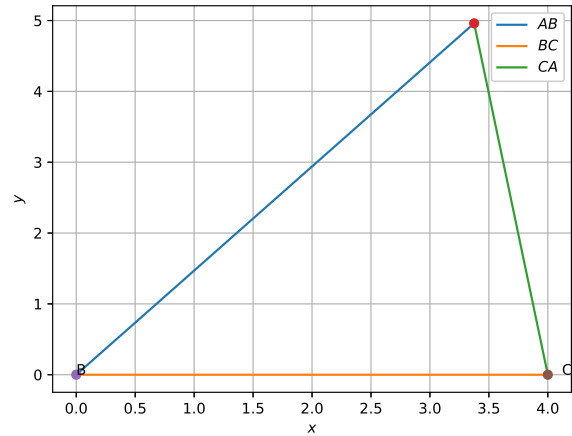6. In $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 45°$ and

Fig. 1.1.3

$\angle C = 45°$, find a, b, c and sketch the triangle.
Solution: From the given information,

$$a + b + c = 11 \quad (1.1.6.1)$$

$$b = c \quad (\because B = C = 45°) \quad (1.1.6.2)$$

$$a^2 = b^2 + c^2 \quad (\because A = 90°) \quad (1.1.6.3)$$

From (1.1.6.1) and (1.1.6.2),

$$a + 2b = 11 \quad (1.1.6.4)$$

From (1.1.6.2) and (1.1.6.3),

$$a^2 = 2b^2 \implies a - b\sqrt{2} = 0 \quad (1.1.6.5)$$

(1.1.6.4) and (1.1.6.5) can be summarized as the matrix equation

$$\begin{pmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix} \quad (1.1.6.6)$$

which can be solved using Cramer's rule as

$$a = \frac{\begin{vmatrix} 11 & 2 \\ 0 & -\sqrt{2} \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{vmatrix}} = \frac{11 \times \left(-\sqrt{2}\right) - 2 \times 0}{1 \times \left(-\sqrt{2}\right) - 2 \times 1}$$
$$(1.1.6.7)$$

$$= \frac{11\sqrt{2}}{2 + \sqrt{2}} \quad (1.1.6.8)$$

$$b = \frac{\begin{vmatrix} 1 & 11 \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -\sqrt{2} \end{vmatrix}} = \frac{11}{2 + \sqrt{2}} \quad (1.1.6.9)$$

The following code may be used to compute a, b and c.

```
codes/constructions/triangle_det.py
```

7. Draw $\triangle ABC$ with $a = 6, c = 5$ and $\angle B = 60°$.
   Solution: In Fig. (1.1.7), $AD \perp BC$.

$$\cos C = \frac{y}{b}, \qquad (1.1.7.1)$$

$$\cos B = \frac{x}{b}, \qquad (1.1.7.2)$$

Thus,

$$a = x + y = b \cos C + c \cos B, \qquad (1.1.7.3)$$
$$b = c \cos A + a \cos C \qquad (1.1.7.4)$$
$$c = b \cos A + a \cos B \qquad (1.1.7.5)$$

The above equations can be expressed in matrix form as

$$\begin{pmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{pmatrix} \begin{pmatrix} \cos A \\ \cos B \\ \cos C \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (1.1.7.6)$$

Using the properties of determinants,

$$\cos A = \frac{\begin{vmatrix} a & c & b \\ b & 0 & a \\ c & a & 0 \end{vmatrix}}{\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}} = \frac{ab^2 + ac^2 - a^3}{abc + abc} \qquad (1.1.7.7)$$

$$= \frac{b^2 + c^2 - a^2}{2bc} \qquad (1.1.7.8)$$

From (1.1.7.8)

$$b^2 = c^2 + a^2 - 2ca \cos B \qquad (1.1.7.9)$$

which is computed by the following code

```
codes/constructions/cos_form.py
```

8. Draw $\triangle ABC$ with $a = 7, \angle B = 45°$ and $\angle A = 105°$.
   Solution: In Fig. (1.1.7),

$$\sin B = \frac{h}{c} \qquad (1.1.8.1)$$

$$\sin C = \frac{h}{b} \qquad (1.1.8.2)$$

which can be used to show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad (1.1.8.3)$$
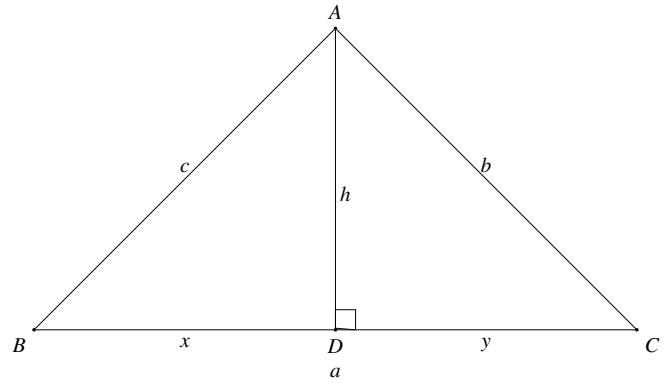

Fig. 1.1.7: The cosine formula

Thus,

$$c = \frac{a \sin C}{\sin A} \qquad (1.1.8.4)$$

where

$$C = 180 - A - B \qquad (1.1.8.5)$$

9. $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find b, c and draw the triangle.
   Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \qquad (1.1.9.1)$$
$$\implies (18 - c)^2 = 12^2 + c^2 \qquad (1.1.9.2)$$

which can be simplified to obtain

$$36c - 180 = 0 \qquad (1.1.9.3)$$
$$\implies c = 5 \qquad (1.1.9.4)$$

and $b = 13$

10. Find a simpler solution for Problem 1.1.6
    Solution: Use cosine formula.

11. In $\triangle ABC$, $a = 7, \angle B = 75°$ and $b + c = 13$. Alternatively,

$$a = b \cos C + c \cos B \qquad (1.1.11.1)$$
$$b \sin C = c \sin B \qquad (1.1.11.2)$$
$$a + b + c = 11 \qquad (1.1.11.3)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \qquad (1.1.11.4)$$

Solving the equivalent matrix equation gives the desired answer.

1.2 Triangle Exercises

1. In $\triangle ABC$, $a = 8, \angle B = 45°$ and $c - b = 3.5$. Sketch $\triangle ABC$.
2. In $\triangle ABC$, $a = 6$, $\angle B = 60°$ and $b-c = 2$. Sketch $\triangle ABC$.
3. Draw $\triangle ABC$, given that $a+b+c = 11$, $\angle B = 30°$ and $\angle C = 90°$.
4. Construct $\triangle xyz$ where $xy = 4.5, yz = 5$ and $zx = 6$.
5. Draw an equilateral triangle of side 5.5.
6. Draw $\triangle PQR$ with $PQ = 4, QR = 3.5$ and $PR = 4$. What type of triangle is this?
7. Construct $\triangle ABC$ such that $AB = 2.5, BC = 6$ and $AC = 6.5$. Find $\angle B$.
8. Construct $\triangle PQR$, given that $PQ = 3, QR = 5.5$ and $\angle PQR = 60°$.
9. Draw $\triangle ABC$ if $AB = 3, AC = 5$ and $\angle C = 30°$.
10. Construct $\triangle DEF$ such that $DE = 5, DF = 3$ and $\angle D = 90°$.
11. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°.
12. Construct $\triangle ABC$ with $BC = 7.5, AC = 5$ and $\angle C = 60°$.
13. Construct $\triangle XYZ$ if $XY = 6, \angle X = 30°$ and $\angle Y = 100°$.
14. If $AC = 7, \angle A = 60°$ and $\angle B = 50°$, can you draw the triangle?
15. Construct $\triangle ABC$ given that $\angle A = 60°, \angle B = 30°$ and $AB = 5.8$.
16. Construct $\triangle PQR$ if $PQ = 5, \angle Q = 105°$ and $\angle R = 40°$.
17. Can you construct $\triangle DEF$ such that $EF = 7.2, \angle E = 110°$ and $\angle F = 180°$?
18. Construct $\triangle LMN$ right angled at M such that $LN = 5$ and $MN = 3$.
19. Construct $\triangle PQR$ right angled at Q such that $QR = 8$ and $PR = 10$.
20. Construct right angled $\triangle$ whose hypotenuse is 6 and one of the legs is 4.
21. Construct an isosceles right angled $\triangle ABC$ right angled at C such $AC = 6$.
22. Construct the triangles in Table 1.2.22.

S.No	Triangle	Given Measurements		
1	$\triangle ABC$	$\angle A = 85°$	$\angle B = 115°$	$AB = 5$
2	$\triangle PQR$	$\angle Q = 30°$	$\angle R = 60°$	$QR = 4.7$
3	$\triangle ABC$	$\angle A = 70°$	$\angle B = 50°$	$AC = 3$
4	$\triangle LMN$	$\angle L = 60°$	$\angle N = 120°$	$LM = 5$
5	$\triangle ABC$	$BC = 2$	$AB = 4$	$AC = 2$
6	$\triangle PQR$	$PQ = 2.5$	$QR = 4$	$PR = 3.5$
7	$\triangle XYZ$	$XY = 3$	$YZ = 4$	$XZ = 5$
8	$\triangle DEF$	$DE = 4.5$	$EF = 5.5$	$DF = 4$

TABLE 1.2.22