Balancing a Chemical Equation

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***Abstract*—This manual shows how to balance chemical equations using matrices.**

Download python codes using

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svn co https://github.com/gadepall/school/trunk/
    training
```

1 Chemistry

1. Express the problem of balancing the following chemical equation as a matrix equation.

$$Fe + H_2O \to Fe_3O_4 + H_2 \tag{1.1.1}$$

Solution: Let the balanced version of (1.1.1) be

$$x_1Fe + x_2H_2O \to x_3Fe_3O_4 + x_4H_2 \tag{1.1.2}$$

which results in the following equations

$$\begin{aligned}(x_1 - 3x_3)\,Fe &= 0\\ (2x_2 - 2x_4)\,H &= 0\\ (x_2 - 4x_3)\,H &= 0\end{aligned} \tag{1.1.3}$$

which can be expressed as

$$\begin{aligned}x_1 + 0.x_2 - 3x_3 + 0.x_4 &= 0\\ 0.x_1 + 2x_2 + 0.x_3 - 2x_4 &= 0\\ 0.x_1 + x_2 - 4x_3 + 0.x_4 &= 0\end{aligned} \tag{1.1.4}$$

resulting in the matrix equation

$$\begin{pmatrix}1 & 0 & -3 & 0\\ 0 & 2 & 0 & -2\\ 0 & 1 & -4 & 0\end{pmatrix}\mathbf{x} = \mathbf{0} \tag{1.1.5}$$

where

$$\mathbf{x} = \begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} \tag{1.1.6}$$

2. Solve (1.1.2) by row reducing the matrix in (1.1.5).

Solution: (1.1.5) can be row reduced as follows

$$\begin{pmatrix}1 & 0 & -3 & 0\\ 0 & 2 & 0 & -2\\ 0 & 1 & -4 & 0\end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{2}} \begin{pmatrix}1 & 0 & -3 & 0\\ 0 & 1 & 0 & -1\\ 0 & 1 & -4 & 0\end{pmatrix} \tag{1.2.1}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix}1 & 0 & -3 & 0\\ 0 & 1 & 0 & -1\\ 0 & 0 & -4 & 1\end{pmatrix} \tag{1.2.2}$$

$$\xleftrightarrow{R_1 \leftarrow 4R_1 - 3R_3} \begin{pmatrix}4 & 0 & 0 & -3\\ 0 & 1 & 0 & -1\\ 0 & 0 & -4 & 1\end{pmatrix} \tag{1.2.3}$$

$$\xleftrightarrow[R_3 \leftarrow -\frac{1}{4}R_3]{R_1 \leftarrow \frac{1}{4}} \begin{pmatrix}1 & 0 & 0 & -\frac{3}{4}\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & -\frac{1}{4}\end{pmatrix} \tag{1.2.4}$$

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Thus,

$$x_1 = \frac{3}{4}x_4, x_2 = x_4, x_3 = \frac{1}{4}x_4 \quad (1.2.5)$$

$$\implies \mathbf{x} = x_4 \begin{pmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \quad (1.2.6)$$

upon substituting $x_4 = 4$. (1.1.2) then becomes

$$3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2 \quad (1.2.7)$$

3. Verify your answer through a python code.
Solution: Execute

```
codes/chem_bal.py
```

2 Mathematics

1. Find the equation of the plane P that containes the point $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and is perpendicular to each of the planes

$$P_1 : \quad \begin{pmatrix} 2 & 3 & -2 \end{pmatrix}\mathbf{x} = 5 \quad (2.1.1)$$

$$P_2 : \quad \begin{pmatrix} 1 & 2 & -3 \end{pmatrix}\mathbf{x} = 8 \quad (2.1.2)$$

From (2.1.1), the normals to P_1, P_2 are

$$\mathbf{n}_1 = \begin{pmatrix} 2 & 3 & -2 \end{pmatrix}$$
$$\mathbf{n}_2 = \begin{pmatrix} 1 & 2 & -3 \end{pmatrix} \quad (2.1.3)$$

$\because P \perp P_1, P \perp P_2$, if $\mathbf{n}$ be the normal to P, $\mathbf{n} \perp \mathbf{n}_1, \mathbf{n} \perp \mathbf{n}_2$, which can be expressed using (2.1.3) as

$$\begin{pmatrix} 2 & 3 & -2 \\ 1 & 2 & -3 \end{pmatrix}\mathbf{n} = 0 \quad (2.1.4)$$

Obtain $\mathbf{n}$ using row reduction.
2. Verify your answer through a python code.
3. Verify that $\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$.

3 Physics

1. A force of $\mathbf{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$ acts on a particle whose position vector is $\mathbf{r} = \hat{i} - \hat{j} + \hat{k}$. Find the torque about the origin given by $\mathbf{F} \times \mathbf{r}$ using a matrix equation.
2. Verify your answer using python.