Computational Approach to School Mathematics

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Contents

Abstract—**This is a problem set related to continuous maths based on JEE question papers**

1 Trigonometry

1. Suppose
$$\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx \tag{1.0.1.1}$$
is an identity in x, where $C_0, C_1, ...C_n$ are constants, and $C_n \neq 0$ then find the value of n.
2. Find the solution set of the system of equations
$$x + y = \frac{2\pi}{3} \tag{1.0.2.1}$$
$$\cos x + \cos y = \frac{3}{2}, \tag{1.0.2.2}$$
where x and y are real.
3. Find the set of all x in the interval$[0,\pi]$ for which
$$2\sin^2 x - 3\sin x + 1 \geq 0 \tag{1.0.3.1}$$
4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. Find the minimum value of the arithmetic mean of $\cos(\alpha + \frac{\pi}{2}), \cos(\beta + \frac{\pi}{2})$ and $\cos(\gamma + \frac{\pi}{2})$.
5. Find the value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$.
6. If
$$K = \sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18}), \tag{1.0.6.1}$$
then find the numerical value of K ?
7. If A>0,B>0 and
$$A + B = \frac{\pi}{3}, \tag{1.0.7.1}$$
then find the maximum value of tan A tan B.
8. Find the general value of θ satisfying the equation
$$\tan^2\theta + \sec 2\theta = 1. \tag{1.0.8.1}$$
9. Find the real roots of the equation
$$\cos^7 x + \sin^4 x = 1 \tag{1.0.9.1}$$
in the interval $(-\pi, \pi)$.
10. If $\tan\theta = -\frac{4}{3}$, then find $\sin\theta$.
11. If $\alpha + \beta + \gamma = 2\pi$ then
    a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
    c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    d) None of these
12. Given
$$A = \sin^2\theta + \cos^4\theta \tag{1.0.12.1}$$
then for all real values of θ
    a) $1 \leq A \leq 2$
    b) $\frac{3}{4} \leq A \leq 1$
    c) $\frac{13}{16} \leq A \leq 1$
    d) $\frac{3}{4} \leq A \leq \frac{13}{16}$
13. The equation
$$2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}; 0 < x < \frac{\pi}{2} \tag{1.0.13.1}$$
has
    a) no real solution
    b) One real solution
    c) more than the one solution

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d) none of these

14. The general solution of the trigonometric equation

$$\sin x + \cos x = 1 \tag{1.0.14.1}$$

is given by :

a) x $= 2n\pi; n = 0, \pm1, \pm2...$
b) x $= 2n\pi + \frac{\pi}{2}; n = 0, \pm1, \pm2...$
c) x $= n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}; n = 0, \pm1, \pm2...$
d) none of these

15. The value of expression $\sqrt{3}cosec 20° - \sec 20°$ is equal to

a) 2
b) $\frac{2\sin 20°}{\sin 40°}$
c) 4
d) $\frac{4\sin 20°}{\sin 40°}$

16. The general solution of

$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x \tag{1.0.16.1}$$

is

a) $n\pi + \frac{\pi}{8}$
b) $\frac{n\pi}{2} + \frac{\pi}{8}$
c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$
d) $2n\pi + \cos^{-1}\frac{3}{2}$

17. The equation

$$(\cos p - 1)x^2 + (\cos p)x + \sin p = 0 \tag{1.0.17.1}$$

In the variable x, has real roots. Then p can take any value in the interval

a) $(0, 2\pi)$
b) $(-\pi, 0)$
c) $(-\frac{\pi}{2}, \frac{\pi}{2})$
d) $(0, \pi)$

18. Number of solutions of the equation

$$\tan x + \sec x = 2\cos x \tag{1.0.18.1}$$

lying in the interval $[0, 2\pi]$ is :

a) 0
b) 1
c) 2
d) 3

19. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals

a) $\tan(x - \frac{\pi}{4})$
b) $\tan(\frac{\pi}{4} - x)$
c) $\tan(x + \frac{\pi}{4})$
d) $\tan^2(x + \frac{\pi}{4})$

20. Let n be a positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$.Then

a) $6 \le n \le 8$
b) $4 < n \le 8$
c) $4 \le n \le 8$
d) $4 < n < 8$

21. If ω is an imaginary cube root of unity then the value of $\sin\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\}$ is

a) $-\frac{\sqrt{3}}{2}$
b) $-\frac{1}{\sqrt{2}}$
c) $\frac{1}{\sqrt{2}}$
d) $\frac{\sqrt{3}}{2}$

22. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$

a) 11
b) 12
c) 13
d) 14

23. The general values of θ satisfying equation

$$2\sin^2\theta - 3\sin\theta - 2 = 0 \tag{1.0.23.1}$$

is

a) $n\pi + (-1)^n\frac{\pi}{6}$
b) $n\pi + (-1)^n\frac{\pi}{2}$
c) $n\pi + (-1)^n\frac{5\pi}{6}$
d) $n\pi + (-1)^n\frac{7\pi}{6}$

24. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if

a) $x + y \ne 0$
b) $x = y, x \ne 0$
c) $x = y$
d) $x \ne 0, y \ne 0$

25. In a triangle PQR, $\angle R = \pi/2$.If $\tan(\frac{P}{2})$ and $\tan(\frac{Q}{2})$ are the roots of the equation

$$ax^2 + bx + c = 0 (a \ne 0) \tag{1.0.25.1}$$

then

a) a+b=c
b) b+c=a
c) a+c=b
d) b=c

26. Let f(θ) =$\sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is

a) ≥ 0 only when $\theta \ge 0$
b) ≤ 0 for all real θ
c) ≥ 0 for all real θ
d) ≤ 0 only when $\theta \le 0$

27. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$=0 in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is
   a) 0
   b) 2
   c) 1
   d) 3
28. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)...(\cos\alpha_n)$, under the restrictions, $0 \le \alpha_1, \alpha_2, ...\alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)...(\cot\alpha_n) = 1$ is
   a) $\frac{1}{2^{\frac{n}{2}}}$
   b) $\frac{1}{2^n}$
   c) $\frac{1}{2n}$
   d) 1
29. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$,then $\tan\alpha$ equals
   a) $2(\tan\beta + \tan\gamma)$
   b) $\tan\beta + \tan\gamma$
   c) $\tan\beta + 2\tan\gamma$
   d) $2\tan\beta + \tan\gamma$
30. The number of integral values of k for which the equation
$$7\cos x + 5\sin x = 2k + 1 \tag{1.0.30.1}$$
has a solution is
   a) 4
   b) 8
   c) 10
   d) 12
31. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}, \cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to
   a) $(\frac{\pi}{3}, \frac{\pi}{2}]$
   b) $(\frac{\pi}{2}, \frac{2\pi}{3})$
   c) $(\frac{2\pi}{3}, \frac{5\pi}{6}]$
   d) $(\frac{5\pi}{6}, \pi]$
32. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ where $\alpha, \beta\epsilon[-\pi, \pi]$. Pairs of α, βwhich satisfy both the equations is/are
   a) 0
   b) 1
   c) 2
   d) 4
33. The values of $\theta\epsilon(0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are
   a) $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$
   b) $(\frac{\pi}{8}, \frac{5\pi}{6})$
   c) $(0, \frac{\pi}{8}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$
   d) $(\frac{41\pi}{48}, \pi)$
34. Let $\theta\epsilon(0, \frac{\pi}{4})$ and $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then
   a) $t_1 > t_2 > t_3 > t_4$
   b) $t_4 > t_3 > t_1 > t_2$
   c) $t_3 > t_1 > t_2 > t_4$
   d) $t_2 > t_3 > t_1 > t_4$
35. The number of solutions of the pair of equations
$$2\sin^2\theta - \cos 2\theta = 0 \tag{1.0.35.1}$$
$$2\cos^2\theta - 3\sin\theta = 0 \tag{1.0.35.2}$$
in the interval $[0,2\pi]$ is
   a) zero
   b) one
   c) two
   d) four
36. For $x\epsilon(0, \pi)$, the equation
$$\sin x + 2\sin 2x - \sin 3x = 3 \tag{1.0.36.1}$$
has
   a) infinitely many solutions
   b) three solutions
   c) one solution
   d) no solution
37. Let $S = \{x\epsilon(-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\}$. The sum of all distinct solutions of the equation
$$\sqrt{3}\sec x + \text{cosec}x + 2(\tan x - \cot x) = 0 \tag{1.0.37.1}$$
in the set S is equal to
   a) $-\frac{7\pi}{9}$
   b) $-\frac{2\pi}{9}$
   c) 0
   d) $\frac{5\pi}{9}$
38. The value of $\sum_{k=1}^{13} \frac{1}{\sin(\frac{\pi}{4} + \frac{(k-1)\pi}{6})\sin(\frac{\pi}{4} + \frac{k\pi}{6})}$is equal to
   a) $3 - \sqrt{3}$
   b) $2(3 - \sqrt{3})$
   c) $2(\sqrt{3} - 1)$
   d) $2(2 - \sqrt{3})$
39. $(1 + \cos\frac{\pi}{8})(1 + \cos\frac{3\pi}{8})(1 + \cos\frac{5\pi}{8})(1 + \cos\frac{7\pi}{8})$ is equal to
   a) $\frac{1}{2}$

b) $\cos\left(\frac{\pi}{8}\right)$
c) $\frac{1}{8}$
d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

40. The expression $3[\sin^4(\frac{3\pi}{2}-\alpha)+\sin^4(3\pi+\alpha]-2[\sin^6(\frac{\pi}{2}+\alpha)+\sin^6(5\pi-\alpha)]$ is equal to
a) 0
b) 1
c) 3
d) $\sin 4\alpha+\cos 6\alpha$
e) none of these

41. The number of all possible triplets (a_1, a_2, a_3) such that
$$a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0 \tag{1.0.41.1}$$
for all x is
a) zero
b) one
c) three
d) infinite
e) none

42. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation
$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \tag{1.0.42.1}$$
are
a) $\frac{7\pi}{24}$
b) $\frac{5\pi}{24}$
c) $\frac{11\pi}{24}$
d) $\frac{\pi}{24}$

43. Let
$$2\sin^2 x + 3\sin x - 2 > 0 \tag{1.0.43.1}$$
$$x^2 - x - 2 < 0 \tag{1.0.43.2}$$
(x is measured in radians). Then x lies in the interval
a) $(\frac{\pi}{6}, \frac{5\pi}{6})$
b) $(-1, \frac{5\pi}{6})$
c) (-1 , 2)
d) $(\frac{\pi}{6}, 2)$

44. The minimum value of the expression $\sin\alpha+\sin\beta+\sin\gamma$, where$\alpha,\beta,\gamma$are real numbers satisfying $\alpha+\beta+\gamma=\pi$ is
a) Positive
b) zero
c) negative
d) -3

45. The number of values of x in the interval $[0, \pi]$ satisfying the equation
$$3\sin^2 x - 7\sin x + 2 = 0 \tag{1.0.45.1}$$
is
a) 0
b) 5
c) 6
d) 10

46. Which of the following number(s) is/are/rational?
a) $\sin 15°$
b) $\cos 15°$
c) $\sin 15°\cos 15°$
d) $\sin 15°\cos 75°$

47. For a positive integer n, let $f_n(\theta) = \tan(\frac{\theta}{2})(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)......(1+\sec 2^n\theta)$.Then
a) $f_2(\frac{\pi}{16}) = 1$
b) $f_3(\frac{\pi}{32}) = 1$
c) $f_4(\frac{\pi}{64}) = 1$
d) $f_5(\frac{\pi}{128}) = 1$

48. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then
a) $\tan^2 x = \frac{2}{3}$
b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
c) $\tan^2 x = \frac{1}{3}$
d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

49. For $0 < \theta < \frac{\pi}{2}$. the solution(s) of $\sum_{m=1}^{6}\operatorname{cosec}(\theta+\frac{(m-1)\pi}{4})\operatorname{cosec}(\theta+\frac{m\pi}{4}) = 4\sqrt{2}$ is(are)
a) $\frac{\pi}{4}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{12}$
d) $\frac{5\pi}{12}$

50. Let $\theta, \varphi \epsilon [0, 2\pi]$ be such that $2\cos\theta(1-\sin\varphi) = \sin^2\theta(\tan\frac{\theta}{2}+\cot\frac{\theta}{2})\cos\varphi - 1, \tan(2\pi-\theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, then φ can not satisfy
a) $0 < \varphi < \frac{\pi}{2}$
b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$
d) $\frac{3\pi}{2} < \varphi < 2\pi$

51. The number of points in$(-\infty, \infty)$, for which
$$x^2 - x\sin x - \cos x = 0 \tag{1.0.51.1}$$
is

a) 6
b) 4
c) 2
d) 0

52. Let

$$f(x) = x \sin \pi x, x > 0 \tag{1.0.52.1}$$

Then for all natural numbers n, $f'(x)$ vanishes at

a) A unique point in the interval (n,n+$\frac{1}{2}$)
b) A unique point in the interval (n+$\frac{1}{2}$, n+1)
c) A unique point in the interval (n,n+1)
d) Two points in the interval (n,n+1)

53. Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$.Then which of the following is/are true?

a) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$
c) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$
d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

54. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$.

55. (a) Draw the graph of

$$y = \frac{1}{\sqrt{2}}(\sin x + \cos x) \tag{1.0.55.1}$$

from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
(b) If $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and α,β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$

56. Given $\alpha + \beta - \gamma = \pi$, prove that

$$\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma \tag{1.0.56.1}$$

57. Given A= { x: $\frac{\pi}{6} \le x \le \frac{\pi}{3}$} and

$$f(x) = \cos x - x(1+x); \tag{1.0.57.1}$$

find f(A)

58. For all θ in $[0, \pi/2]$ show that,

$$\cos(\sin\theta) \ge \sin(\cos\theta) \tag{1.0.58.1}$$

.

59. Without using tables, Prove that $(\sin 12°)(\sin 48°)(\sin 54°) = \frac{1}{8}$

60. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right) = 1$

61. Find all the solution of $4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$

62. Find the values of $x\epsilon(-\pi,\pi)$ which satisfy the equation

$$8^{(1+|\cos x||+|cos^2 x|+|cos^3 x|+.....)} = 4^3 \tag{1.0.62.1}$$

63. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

64. ABC is a triangle such that $\sin(2A+B) = \sin(C-A) = -\sin(B+2c) = \frac{1}{2}$ If A,B and C are in arithmetic progression, determine the values of A, B and C.

65. if exp{$(\sin^2 x + \sin^4 x + \sin^6 x + ...\infty)$ In 2 } satisifies the equation

$$x^2 - 9x + 8 = 0 \tag{1.0.65.1}$$

, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$.

66. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3.

67. Determine the smallest positive value of x(in degrees) for which $\tan(x+100°) = \tan(x+50°)\tan(x)\tan(x-50°)$.

68. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x\epsilon[0, 2\pi]$

69. Find all values of θ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ satisfying the equation

$$(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0 \tag{1.0.69.1}$$

70. Prove that the values of the function $\frac{\sin x\cos 3x}{\sin 3x\cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x.

71. Prove that $\sum_{k=1}^{n-1}(n-k)cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \ge 3$ is an integer

72. If any triangle ABC, Prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{c}{2}$

73. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$.

This section contains 1 paragraph, Based on each paragraph,there are 2 questions. Each question has four options (A),(B),(C) and (D) ONLY ONE of these four options is correct.

PARAGRAPH 1

Let O be the origin, and $\mathbf{OX}, \mathbf{OY}, \mathbf{OZ}$ be three unit vectors in the directions of the sides $\mathbf{QR}, \mathbf{RP}, \mathbf{PQ}$ respectively, of a triangle PQR

74. $|\mathbf{OX} \times \mathbf{OY}| =$

a) $\sin(P+Q)$
b) $\sin 2R$
c) $\sin(P+R)$

d) $\sin(Q+R)$

75. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is
a) $-\frac{5}{3}$
b) $-\frac{3}{2}$
c) $\frac{3}{2}$
d) $\frac{5}{3}$

76. The number of all possible values of θ where $0<\theta<\pi$,for which the system of equations

$(y+z)\cos 3\theta = (xyz)\sin 3\theta$

$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$
have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$,is

77. The number of values of θ in the interval, $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0 , ±1,±2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

78. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

79. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k>0$, then the value of [k] is
Note: [k] denotes the largest integer less than or equal to k.

80. The positive integer value of $n>3$ satisfying the equation
$\frac{1}{\sin(\frac{\pi}{n})} = \frac{1}{\sin(\frac{2\pi}{n})} + \frac{1}{\sin(\frac{3\pi}{n})}$ is

81. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

82. Let a,b,c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$ has two distinct real roots α and β with $\alpha+\beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is

83. The period of $\sin^2\theta$ is
a) π^2
b) π
c) 2π
d) $\pi/2$

84. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
a) 2
b) 3
c) 0
d) 1

85. Which one is not periodic
a) $|\sin 3x| + \sin^2 x$
b) $\cos\sqrt{x} + \cos^2 x$
c) $\cos 4x + \tan^2 x$
d) $\cos 2x + \sin x$

86. Let α,β be such that $\pi < \alpha-\beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$
a) $-\frac{6}{65}$
b) $\frac{3}{\sqrt{130}}$
c) $\frac{6}{65}$
d) $-\frac{3}{\sqrt{130}}$

87. If u = $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$then the difference between the maximum and minimum values of u^2 is given by
a) $(a-b)^2$
b) $2\sqrt{a^2+b^2}$
c) $(a+b)^2$
d) $2(a^2+b^2)$

88. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
a) $\frac{2}{5}$
b) $\frac{1}{5}$
c) $\frac{3}{5}$
d) $\frac{2}{3}$

89. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation

$$2\sin^2 x + 5\sin x - 3 = 0 \qquad (1.0.89.1)$$

is
a) 4
b) 6
c) 1
d) 2

90. If $0<x<\pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is
a) $\frac{(1-\sqrt{7})}{4}$
b) $\frac{(4-\sqrt{7})}{3}$
c) $-\frac{(4+\sqrt{7})}{3}$
d) $\frac{(1+\sqrt{7})}{4}$

91. Let A and B denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$,then

:

a) A is false and B is true
b) Both A and B are true
c) both A and B are false
d) A is true and B is false

92. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$, Then $\tan 2\alpha =$
a) $\frac{56}{33}$
b) $\frac{19}{12}$
c) $\frac{20}{7}$
d) $\frac{25}{16}$

93. If A = $\sin^2 x + \cos^4 x$, then for all real x:
a) $\frac{13}{16} \leq A \leq 1$
b) $1 \leq A \leq 2$
c) $\frac{3}{4} \leq A \leq \frac{13}{16}$
d) $\frac{3}{4} \leq A \leq 1$

94. In a $\triangle PQR$, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :
a) $\frac{5\pi}{6}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{4}$
d) $\frac{3\pi}{4}$

95. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC=p and CD=q, then AB is equal to :
a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$
b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$
c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$
d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

96. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as:
a) $\sin A\cos A + 1$
b) $\sec A cosec A + 1$
c) $\tan A + \cot A$
d) $\sec A + cosec A$

97. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \epsilon R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals
a) $\frac{1}{4}$
b) $\frac{1}{12}$
c) $\frac{1}{6}$
d) $\frac{1}{3}$

98. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:
a) 7
b) 9
c) 3
d) 5

99. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$,then the value of cos4x is :
a) $-\frac{7}{9}$
b) $-\frac{3}{5}$
c) $\frac{1}{3}$
d) $\frac{2}{9}$

100. If sum of all the solutions of the equation 8 $\cos x.(\cos(\frac{\pi}{6}+x)(\cos(\frac{\pi}{6}-x)-\frac{1}{2})-1$ in $[0,\pi]$ is $k\pi$. then k is equal to :
a) $\frac{13}{9}$
b) $\frac{8}{9}$
c) $\frac{20}{9}$
d) $\frac{2}{3}$

101. For any $\theta\epsilon(\frac{\pi}{4},\frac{\pi}{2})$ the expression $3(\sin\theta-\cos\theta)^4 + 6(\sin\theta+\cos\theta)^2 + 4\sin^2\theta$ equals:
a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
b) $13 - 4\cos^6\theta$
c) $13 - 4\cos^2\theta + 6\cos^4\theta$
d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

102. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is:
a) $\frac{3}{4} + \cos 20°$
b) $\frac{3}{4}$
c) $\frac{3}{2}(1+\cos 20°)$
d) $\frac{3}{2}$

103. Let S=$\{\theta\epsilon[-2\pi,2\pi] : 2\cos^{\theta} + 3\sin\theta = 0\}$. Then the sum of the elements of S is
a) $\frac{13\pi}{6}$
b) $\frac{5\pi}{3}$
c) 2
d) 1

Match the Following

DIRECTIONS (Q.1): *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B , C and D, while the statements in Column-II are labelled p,q,r,s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to theses*

questions have to be darkened as illustrated in the following example:
If the correct matches are A-p, s and t; B-q and r; C-p and q; D -s then the correct darkening of bubbles will look like the given

---

a) In this question there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.
$\dfrac{\sin 3\alpha}{\cos 2\alpha}$ is

Column-I	**Column-II**
(A) Positive	(p)$(\frac{13\pi}{48}, \frac{14\pi}{48})$
(B) Negative	(q)$(\frac{14\pi}{48}, \frac{18\pi}{48})$
	(r)$(\frac{18\pi}{48}, \frac{23\pi}{48})$
	(s)$(0, \frac{\pi}{2})$

b) Let
f(x)=sin $(\pi \cos x)$ and g(x)=cos $(2\pi \sin x)$be two functions defined for $x > 0$. Define the following sets whose elements are written i n the increasing order.

X $=\{x : f(x) = 0\}$,Y $=\{x : f'(x) = 0\}$

Z $=\{x : g(x) = 0\}$,W $=\{x : g'(x) = 0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

Column-I	**Column-II**
(A)X	(p)$\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$
(B)Y	(q)an arithmetic progression
(C)Z	(r)NOT an arithmetic progression
(D)W	(s)$\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$
	(t)$\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$
	(u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

i) (IV),(P),(R),(S)
ii) (III),(P),(Q),(U)
iii) (III),(R),(U)
iv) (IV),(Q),(T)

c) Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$.Define the following sets whose elements are written in the increasing order

X $=\{x : f(x) = 0\}$,Y $=\{x : f'(x) = 0\}$

Z $=\{x : g(x) = 0\}$,W $=\{x : g'(x) = 0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

Column-I	**Column-II**
(A)X	(p)$\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$
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(C)Z	(r)NOT an arithmetic progression
(D)W	(s)$\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$
	(t)$\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$
	(u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$

Which of the following is the only CORRECT combination?

i) (I),(Q),(U)
ii) (I),(P),(R)
iii) (II),(R),(S)
iv) (II),(Q),(T)

2 Functions

1. The values of $f(x) = 3 \sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lie in the interval.......
2. For the function f(x) = $\frac{x}{1+e^{1/x}}$, x $\neq$ 0 and f(x) = 0, x = 0 the derivative from the right, $f'(0+)$= ...., and the derivative from the left, $f'(0-)$= ....
3. The domain of the funtion f(x)=$\sin^{-1}\left(log_2 \frac{x^2}{2}\right)$ is given by ....
4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is.......and out of these.......are onto functions.

5. If
$$f(x) = \sin ln\Big(\frac{\sqrt{4-x^2}}{1-x}\Big),$$
then domain of f(x) is.... and its range is......
6. There are exactly two distinct linear functions.......,and......which map [-1,1] onto [0,2].
7. If f is an even function defined on the interval (-5,5), then four real values of x satisfying the equation f(x)=$f(\frac{x+1}{x+2})$ are.........., ..., ..., and......
8. If
$$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + cosxcos\Big(x + \frac{\pi}{3}\Big)$$
and g($\frac{5}{4}$) = 1, then (gof)(x)= ....
9. If f(x)=$(a - x^n)^{1/n}$ where $a > 0$ and n is a positive integer, then f[f(x)]=x.
10. The function f(x)=$\frac{x^2+4x+30}{x^2-8x+18}$ is not one-to-one.
11. If $f_1(x)$ and $f_2(x)$ are defined on domains D_1 and D_2 respectively, then $f_1(x)$+$f_2(x)$ is defined on $D_1 \cup D_2$.
12. Let R be the set of real numbers. If f:R $\rightarrow$ R is a function defined by f(x)=x^2, then f is :
    a) Injective but not surjective
    b) Surjective but not injective
    c) Bijective
    d) None of these.
13. The entire graphs of the equation y = $x^2 + kx - x + 9$ is stirctly above the x-axis if and only if
    a) $k < 7$
    b) $-5 < k < 7$
    c) $k > -5$
    d) None of these.
14. Let f(x)= $\left|x-1\right|$. Then
    a) $f(x^2) = (f(x))^2$
    b) f(x+y)=f(x)+f(y)
    c) $f(\left|x\right|) = \left|f(x)\right|$
    d) None of these
15. If x satisfies $\left|x-1\right| + \left|x-2\right| + \left|x-3\right| \geq 6$,then
    a) $0 \leq x \leq 4$
    b) $x \leq -2$ or $x \geq 4$
    c) $x \leq 0$ or $x \geq 4$
    d) None of these
16. If f(x)=cos(lnx), then f(x)f(y) - $\frac{1}{2}\Big[f(\frac{x}{y}) + f(xy)\Big]$ has the value
    a) -1
    b) $\frac{1}{2}$
    c) -2
    d) none of these
17. The domain of definition of the function y=$\frac{1}{log_{10}(1-x)}$ + $\sqrt{x+2}$ is
    a) (-3,2) excluding -2.5
    b) $[0, 1]$ excluding 0.5
    c) $[-2, 1)$ excluding 0
    d) none of these
18. Which of the following functions is periodic?
    a) f(x)=x-[x] where [x] denotes the largest integer less than or equal to the real number x
    b) f(x)=sin $\frac{1}{x}$ for $x \neq 0$, f(0)=0
    c) f(x)=xcos x
    d) none of these
19. Let f(x)=sin x and g(x) = ln $\left|x\right|$. If the ranges of the composition functions fog and gof are R_1 and R_2 respectively, then
    a) $R_1 = \{u : -1 \leq u < 1\}, R_2 = \{v : -\infty < v < 0\}$
    b) $R_1 = \{u : -\infty < u < 0\}, R_2 = \{v : -1 \leq v \leq 0\}$
    c) $R_1 = \{u : -1 < u < 1\}, R_2 = \{v : -\infty < v < 0\}$
    d) $R_1 = \{u : -1 \leq u \leq 1\}, R_2 = \{v : -\infty < v \leq 0\}$
20. Let $f(x) = (x+1)^2 - 1$, x ≥ -1. Then the set $\{x : f(x) = f^{-1}(x)\}$ is
    a) $\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\}$
    b) $\{0, 1, -1\}$
    c) $\{0, -1\}$
    d) empty
21. The function f(x) = $\left|px - q\right|$ + r$\left|x\right|$, $x \in (-\infty, \infty)$ where p > 0, q > 0, r > 0 assumes its minimum value only on one point if
    a) p $\neq$ q
    b) r $\neq$ q
    c) r $\neq$ p
    d) p = q = r
22. Let $f(x)$ be defined for all $x > 0$ and be continuos. Let $f(x)$ satisfy $f(\frac{x}{y}) = f(x) - f(y)$ for all x,y and $f(e) = 1$. Then
    a) $f(x)$ is bounded
    b) $f(\frac{1}{x}) \rightarrow 0$ as $x \rightarrow 0$
    c) $xf(x) \rightarrow 1$ as $x \rightarrow 0$
    d) $f(x) = lnx$
23. If the function $f : [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is

a) $\frac{1}{2}^{x(x-1)}$
b) $\frac{1}{2}(1+\sqrt{1+4log_2 x})$
c) $\frac{1}{2}(1-\sqrt{1+4log_2 x})$
d) not defined

24. Let $f : R \rightarrow$ R be any function. Define g: R $\rightarrow$ R by $g(x) = \left|f(x)\right|$ for all x. Then g is
a) onto if f is onto
b) one-one if f is one-one
c) continuos if f is continuos
d) differentiable if f is differentiable

25. The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is
a) $0 < x \leq 1$
b) $0 \leq x \leq 1$
c) $-\infty < x \leq 0$
d) $-\infty < x < 1$

26. Let $g(x) = 1 + x - [x]$ and

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0. \\ 1, & x > 0 \end{cases}$$

then for all x, $f(g(x))$ is equal to
a) x
b) 1
c) $f(x)$
d) $g(x)$

27. If $f : [1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals
a) $(x+\sqrt{x^2-4})/2$
b) $x/(1+x^2)$
c) $(x-\sqrt{x^2-4})/2$
d) $1+\sqrt{x^2-4}$

28. The domain of definition of $f(x) = \frac{log_2(x+3)}{x^2+3x+2}$ is
a) $R\backslash\{-1,-2\}$
b) $(-2,\infty)$
c) R$\backslash\{-1,-2,-3\}$
d) $(-3,\infty)\backslash\{-1,-2\}$

29. Let E=$\{1, 2, 3, 4\}$ and F=$\{1, 2\}$. Then the number of onto functions from E to F is
a) 14
b) 16
c) 12
d) 8

30. Let $f(x) = \frac{\alpha x}{x+1}$,x $\neq$ -1. Then, for what value of α is $f(f(x)) = x$?
a) $\sqrt{2}$
b) $-\sqrt{2}$
c) 1
d) -1

31. Suppose $f(x) = (x + 1)^2$ for x $\geq$ -1. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line y=x then $g(x)$ equals
a) $-\sqrt{x}-1, x \geq 0$
b) $\frac{1}{(x+1)^2}, x > -1$
c) $\sqrt{x+1}, x \geq -1$
d) $\sqrt{x}-1, x \geq 0$

32. Let function $f : R \rightarrow$ R be defined by $f(x) = 2x + \sin x$ for x $\in$ R, then f is
a) one-to-one and onto
b) one-to-one but NOT onto
c) onto but NOT one-to-one
d) neither one-to-one nor onto

33. If $f : [0, \infty) \rightarrow [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then f is
a) one-one and onto
b) one-one but not onto
c) onto but not one-one
d) neither one-one nor onto

34. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x ,is
a) $[-\frac{1}{4},\frac{1}{2}]$
b) $[-\frac{1}{2},\frac{1}{2}]$
c) $[-\frac{1}{2},\frac{1}{9}]$
d) $[-\frac{1}{4},\frac{1}{4}]$

35. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}$;$x\epsilon$R is
a) $(1,\infty)$
b) $(1,\frac{11}{7}]$
c) $(1,\frac{7}{3}]$
d) $(1,\frac{7}{5}]$

36. If $f(x) = x^2+2bx+2c^2$ and $g(x) = -x^2-2cx+b^2$ such that min $f(x) > maxg(x)$, then the relation between b and c, is
a) no real value of b & c
b) $0 < c < b\sqrt{2}$
c) $\left|c\right| < \left|b\right|\sqrt{2}$
d) $\left|c\right| > \left|b\right|\sqrt{2}$

37. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain
a) $[0,\frac{\pi}{2}]$
b) $[-\frac{\pi}{4},\frac{\pi}{4}]$
c) $[-\frac{\pi}{2},\frac{\pi}{2}]$
d) $[0,\pi]$

38. If the functions $f(x)$ and $g(x)$ are defined on R→R such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

;

$$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

then $(f - g)(x)$ is
a) one-one & onto
b) neither one-one nor onto
c) one-one but not onto
d) onto but not one-one

39. X and Y are two sets and $f : X \to Y$. If $\{f(c) = y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X\}$, then the true statement is
a) $f(f^{-1}(b))$ = b
b) $f^{-1}(f(a))$ = a
c) $f(f^{-1}(b))$ = b,b ⊂ y
d) $f(f^{-1}(a))$ = a,a ⊂ x

40. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x) = -f(x)$ and $g(x) = f'(x)$ and given that F(5)=5, then F(10) is equal to
a) 5
b) 10
c) 0
d) 15

41. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for n ≥ 2 and $g(x) = \underbrace{(fofo......of)}_{\text{f occurs n times}}(x)$. Then $\int x^{n-2} g(x) dx$ equals.
a) $\frac{1}{n(n-1)}(1 + nx^n)^{1-\frac{1}{n}} + K$
b) $\frac{1}{(n-1)}(1 + nx^n)^{1-\frac{1}{n}} + K$
c) $\frac{1}{n(n+1)}(1 + nx^n)^{1+\frac{1}{n}} + K$
d) $\frac{1}{(n+1)}(1 + nx^n)^{1+\frac{1}{n}} + K$

42. Let f, g and h be real-valued functions defined on the interval $[0, 1]$ by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2 e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f,g and h on $[0, 1]$, then
a) a = b and c ≠ b
b) a = c and a ≠ b
c) a ≠ b and c ≠ b
d) a = b = c

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in$ R. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is
a) $\pm\sqrt{n\pi}$, n $\in \{0, 1, 2.....\}$
b) $\pm\sqrt{n\pi}$, n $\in \{1, 2.....\}$
c) $\frac{\pi}{2} + 2n\pi$, n $\in \{..... -2, -1, 0, 1, 2.....\}$
d) $2n\pi$, n $\in \{..... -2, -1, 0, 1, 2.....\}$

44. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is
a) one-one and onto
b) onto but not one-one
c) one-one but not onto
d) neither one-one nor onto

45. If y=$f(x) = \frac{x+2}{x-1}$ then
a) $x = f(y)$
b) $f(1)$=3
c) y increases with x for $x < 1$
d) f is a rational function of x

46. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at (0,0) and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is
a) $g(x) = \pm\sqrt{1 - x^2}$
b) $g(x) = \sqrt{1 - x^2}$
c) $g(x) = -\sqrt{1 - x^2}$
d) $g(x) = \sqrt{1 + x^2}$

47. If f(x)=$\cos[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then
a) $f(\frac{\pi}{2}) = -1$
b) $f(\pi) = 1$
c) $f(-\pi) = 0$
d) $f(\frac{\pi}{4}) = 1$

48. If $f(x)$=3x-5, then $f^{-1}(x)$
a) is given by $\frac{1}{3x-5}$
b) is given by $\frac{x+5}{3}$
c) does not exist because f is not one-one
d) does not exist because f is not onto.

49. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then
a) $f(x) = \sin^2 x$,$g(x) = \sqrt{x}$
b) $f(x) = \sin x$,$g(x) = |x|$
c) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$
d) f and g cannot be determined.

50. Let f:(0,1) → R be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then
a) f is not invertible on (0,1)
b) $f \neq f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$
c) $f = f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$
d) f^{-1} is differentiable (0,1)

51. Let f:(-1,1) $\rightarrow$ IR be such that $f(\cos 4\Theta) = \frac{2}{2-\sec^2\Theta}$ for $\Theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the values of $f(\frac{1}{3})$ is
    a) 1-$\sqrt{\frac{3}{2}}$
    b) 1+$\sqrt{\frac{3}{2}}$
    c) 1-$\sqrt{\frac{2}{3}}$
    d) 1+$\sqrt{\frac{2}{3}}$
52. The funciton $f(x) = 2\left|x\right| + \left|x+2\right|$ - $\left|\left|x+2\right| - 2\left|x\right|\right|$ has a local minimum or a local maximum at x=
    a) -2
    b) $\frac{-2}{3}$
    c) 2
    d) $\frac{2}{3}$
53. Let $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow R$ be given by $f(x) = (log(\sec x + \tan x))^3$. Then
    a) $f(x)$ is an odd function
    b) $f(x)$ is one-one function
    c) $f(x)$ is an onto function
    d) $f(x)$ is an even function
54. Let a∈R and let f: R $\rightarrow$ R be given by $f(x) = x^5 - 5x + a$. Then
    a) $f(x)$ has three real roots if a>4
    b) $f(x)$ has only real root if a>4
    c) $f(x)$ has three real roots if a<-4
    d) $f(x)$ has three real roots if -4<a<4
55. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all x ∈R and $g(x) = \frac{\pi}{2}\sin x$ for all x∈R. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is true?
    a) Range of f is $[-\frac{1}{2}, \frac{1}{2}]$
    b) Range of fog is $[-\frac{1}{2}, \frac{1}{2}]$
    c) $\lim_{x\rightarrow 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$
    d) There is an x ∈R such that $(gof)(x) = 1$
56. Find the domain and range of the function $f(x) = \frac{x^2}{1+x^2}$. Is the function one-to-one?
57. Draw the graph of $y = \left|x\right|^{1/2}$ for $-1 \le x \le 1$.
58. If $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find $f(6)$.
59. Consider the following relations in the set of real numbers R. $R = \{(x, y); x \in R, y \in R, x^2 + y^2 \le 25\}$
    $R' = \{(x, y) : x \in R, y \in R, y \ge \frac{4}{9}x^2\}$. Find the domain and the range of $R \cap R'$. Is the relation $R \cap R'$ a function?
60. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B.
61. Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1$, $f(y) \neq 1$,$f(z) \neq 2$ determine $f^{-1}(1)$.
62. Let R be the set of real numbers and f: R $\rightarrow$ R be such that for all x and y in R$\left|f(x) - f(y)\right| \le \left|x - y\right|^3$. Prove that $f(x)$ is a constant.
63. Find the natural number $'a'$ for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function $'f'$ satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x,y and further $f(1) = 2$.
64. Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$.
65. A function $f : IR \rightarrow IR$, where IR is the set of real numbers, defined by $f(x) = \frac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$. Find the interval of values of α for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer.
66. Let $f(x) = Ax^2 + Bx + c$ where A,B,C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A,A+B and C are all integers. Conversely, prove that if the numbers 2A,A+B and C are all integers then $f(x)$ is an integer whenever x is an integer.
67. Let $f : [0, 4\pi] \rightarrow [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is
68. The value of $((log_2 9)^2)^{\frac{1}{log_2(log_2 9)}} \times (\sqrt{7})^{\frac{1}{log_4 7}}$ is ......
69. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is ......
70. The domain of $\sin^{-1}[log_3(x/3)]$ is
    a) $[1, 9]$
    b) $[-1, 9]$
    c) $[-9, 1]$
    d) $[-9, -1]$
71. The function $f(x) = log(x + \sqrt{x^2 + 1})$, is
    a) neither an even nor an odd function
    b) an even function

c) an odd function
d) a periodic function.

72. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + log_{10}(x^3 - x)$, is
a) $(-1,0) \cup (1,2) \cup (2,\infty)$
b) (a,2)
c) $(-1,0) \cup (a,2)$
d) $(1,2) \cup (2,\infty)$.

73. If $f : R \rightarrow R$ satisfies $f(x+y) = f(x) + f(y)$, for all x, y ∈ R and f(1)=7, then $\sum_{r=1}^{n} f(r)$ is
a) $\frac{7n(n+1)}{2}$
b) $\frac{7n}{2}$
c) $\frac{7(n+1)}{2}$
d) $7n + (n+1)$

74. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$

is
a) neither one-one nor onto
b) one-one but not onto
c) onto but not one-one
d) one-one and onto both.

75. The range of the function $f(x) = ^{7-x} P_{x-3}$ is
a) $\{1,2,3,4,5\}$
b) $\{1,2,3,4,5,6\}$
c) $\{1,2,3,4\}$
d) $\{1,2,3,\}$

76. Let $f : R \rightarrow S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is
a) $[-1,3]$
b) $[-1,1]$
c) $[0,1]$
d) $[0,3]$

77. The graph of the function $y = f(x)$ is symmetrical about the line x=2, then
a) $f(x) = -f(-x)$
b) $f(2+x) = f(2-x)$
c) $f(x) = f(-x)$
d) $f(x+2) = f(x-2)$

78. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is
a) $[1,2]$
b) $[2,3)$
c) $[1,2]$
d) $[2,3]$

79. Let $f : (-1,1) \rightarrow B$, be a function defined by $f(x) = tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval
a) $(0,\frac{\pi}{2})$
b) $[0,\frac{\pi}{2})$
c) $[-\frac{\pi}{2},\frac{\pi}{2})$
d) $(-\frac{\pi}{2},\frac{\pi}{2})$

80. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	Function
(a). $(-\infty,\infty)$	$x^3 - 3x^2 + 3x + 3$
(b). $[2,\infty)$	$2x^3 - 3x^2 - 12x + 6$
(c). $(-\infty,\frac{1}{3}]$	$3x^2 - 2x + 1$
(d). $(-\infty,-4)$	$x^3 + 6x^2 + 6$

81. A real valued function $f(x)$ satisfies the functional equation

$$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$

where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to
a) $-f(x)$
b) $f(x)$
c) $f(a) + f(a-x)$
d) $f(-x)$

82. The Largest interval lying in $(\frac{-\pi}{2},\frac{\pi}{2})$ for which the function,

$$f(x) = 4^{-x^2} + cos^{-1}(\frac{x}{2} - 1) + log(\cos x))$$

, is defined, is
a) $[-\frac{\pi}{4},\frac{\pi}{2})$
b) $[0,\frac{\pi}{2})$
c) $[0,\pi]$
d) $(-\frac{\pi}{2},\frac{\pi}{2})$

83. Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$ where

$$Y = \{y \in N : y = 4x + 3 for some x \in N\}$$

a) $g(y) = \frac{3y+4}{3}$
b) $g(y) = 4 + \frac{y+3}{4}$
c) $g(y) = \frac{y+3}{4}$
d) $g(y) = \frac{y-3}{4}$

84. Let $f(x) = (x+1)^2 - 1, x \geq -1$
Statement-1 : The set $\{x : f(x) = f^{-1}(x) = \{0,-1\}\}$
Statement-2 : f is a bijection

a) Statement-1 is true,Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

85. For real x, let $f(x) = x^3 + 5x + 1$, then
a) f is onto R but not one-one
b) f is one-one and onto R
c) f is neither one-one nor onto R
d) f is one-one but not onto R

86. The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is
a) $(0, \infty)$
b) $(-\infty, 0)$
c) $(-\infty, \infty) - \{0\}$
d) $(-\infty, \infty)$

87. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 oJof_1)(x) = f_3(x)$ then J(x) is equal to:
a) $f_3(x)$
b) $f_3(x)$
c) $f_2(x)$
d) $f_1(x)$

88. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:
a) 6
b) 8
c) 4
d) 14

89. If the function $f : R - \{1, -1\}$. A defined by f(x)=$\frac{x^2}{1-x^2}$, is surjective, then A is equal to:
a) R-$\{-1\}$
b) $[0, \infty)$
c) R-$[-1, 0)$
d) R-$(-1, 0)$

90. Let

$$\sum_{k=1}^{10} f(a + k) = 16(2^{10} - 1),$$

where the function f satisifies f(x+y)=f(x)f(y) for all natural numbers x,y and f(a) is = 2. Then the natural number $'a'$ is:
a) 2
b) 16
c) 4
d) 3

Match the following

91. Let the function defined in Colum 1 have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ and range $(-\infty, \infty)$

Column I	Column II
(A) 1+2x	(p) onto but not one-one
(B) $\tan x$	(q) one-one but not onto
	(r) one-one and onto
	(s) neither one-one nor onto

92. Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$
Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

93. Let $E_1 = \{x \in R : x \neq 1 and \frac{x}{x-1} > 0\}$ and $E_2 = \{x \in E_1 : \sin^{-1}\left(log_e(\frac{x}{x-1})\right) isarealnumber\}$. (Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$).
Let $f : E_1 \to R$ be the function defined by $f(x) = log_e(\frac{x}{x-1})$ and g: $E_2 \to$ R be the function defined by $g(x) = sin^{-1}(log_e(\frac{x}{x-1}))$.

LIST-I	LIST-II
P. The range of f is	1. $(-\infty, \frac{1}{1-e}] \cup [\frac{e}{e-1}, \infty)$
Q. The range of g contains	2. (0,1)
R. The domain of f contains	3. $[-\frac{1}{2}, \frac{1}{2}]$
S. The domain of g is	4. $(-\infty, 0) \cup (0, \infty)$
	5. $(-\infty, \frac{e}{e-1}]$
	6. $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1}]$

The correct option is:
a) $P \to 4; Q \to 2; R \to 1; S \to 1$
b) $P \to 4; Q \to 2; R \to 1; S \to 6$
c) $P \to 3; Q \to 3; R \to 6; S \to 5$
d) $P \to 4; Q \to 3; R \to 6; S \to 5$

3 Quadratic Equations and Inequations

1. The coefficient of x^{99} in the polynomial $(x-1)(x-2)............(x-100)$ is........
2. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where p and q are real, then $(p,q) =$ (..................., ...)
3. If the product of the roots of the equation $x^2-3kx+2e^{21nk}-1=0$ is 7, then the roots are real for $k =$ ....
4. If the quadratic equations $x^2+ax+b=0$ and $x^2+bx+a=0$ $(a \neq b)$ have a common root then, the numerical value of $a+b$ is...................
5. The solution of equation $log_7 log_5(\sqrt{x+5}+\sqrt{x}) = 0$ is ....
6. If $x<0, y<0, x+y+\frac{x}{y}=\frac{1}{2}$ and $(x+y)\frac{x}{y}=-\frac{1}{2}$, then $x =$ ....and $y =$ ....
7. Let n and k be positive such that $n \geq \frac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, ..., x_k)$, $x_1 \geq 1, x_2 \geq 2, ..., x_k \geq k$, all integers, satisfying $x_1+x_2+...+x_k = n$, is...........
8. The sum of all the real roots of the equation $\left|x-2\right|^2+\left|x-1\right|-2=0$ is................
9. For every integer $n>1$, the inequality $(n!)^{\frac{1}{n}} < \frac{n+1}{2}$ holds.
10. The equation $2x^2+3x+1=0$ has an irrational root.
11. If $a<b<c<d$, then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are real and distinct.
12. If $n_1, n_2, ...n_p$ are p positive integers, whose sum is an even number, then the number of odd integers among them is odd.
13. If $P(x) = ax^2+bx+c$ and $Q(x) = -ax^2+dx+c$, where $ac \neq 0$, then $P(x)Q(x) = 0$ has at least two real roots.
14. If x and y are positive real numbers and m, n are any positive integers, then $\frac{x^n y^m}{(1+x^{2n})(1+y^{2m})} > \frac{1}{4}$
15. If l,m,n are real, $l \neq m$, then the roots by the equation $(l-m)x^2-5(l+m)x-2(l-m)=0$ are:
    a) Real and equal
    b) Complex
    c) Real and unequal
    d) None of these
16. The equation $x+2y+2z=1$ and $2x+4y+4z=9$ have
    a) Only one solution
    b) Only two solutions
    c) Infinite number of solutions
    d) None of these
17. If x,y and z are real and different and $u = x^2+4y^2+9z^2-6yz-3zx-2xy$ then u is always.
    a) non negative
    b) zero
    c) non positive
    d) none of these
18. Let $a>0, b>0$ and $c>0$. Then the roots of the equation $ax^2+bx+c=0$
    a) are real and negative
    b) have negative real parts
    c) both (a) and(b)
    d) none of these
19. Both the roots of the equation $(x-b)(x-c)+(x-a)(x-c)+(x-a)(x-b)=0$ are always
    a) positive
    b) real
    c) negative
    d) none of these
20. The least value of the expression $2log_{10}x - log_x(0.01)$, for $x>1$, is
    a) 10
    b) 2
    c) -0.01
    d) none of these
21. If (x^2+px+1) is a factor of (ax^3+bx+c), then
    a) $a^2+c^2=-ab$
    b) $a^2-c^2=-ab$
    c) $a^2-c^2=ab$
    d) none of these
22. The number of real solutions of the equation $|x|^2-3|x|+2=0$ is
    a) 4
    b) 1
    c) 3
    d) 2
23. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school should be built at
    a) town B
    b) 45 km from town A
    c) town A
    d) 45km from town B
24. If p,q,r are any real numbers, then
    a) max(p,q)<max(p,q,r)

b) min(p,q)= $\frac{1}{2}(p+q-|p-q|)$
c) max(p,q)<min(p,q,r)
d) none of these

25. The largest interval for which $x^{12}-x^9+x^4-x+1>0$ is
a) $-4<x\leq 0$
b) $0<x<1$
c) $-100<x<100$
d) $-\infty<x<\infty$

26. The equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ has
a) no root
b) one root
c) two equal roots
d) infinitely many roots

27. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in the interval
a) $[\frac{1}{2},2]$
b) $[-1,2]$
c) $[-\frac{1}{2},1]$
d) $[-1,\frac{1}{2}]$

28. If $log_{0.3}(x-1)<log_{0.09}(x-1)$, then x lies in the interval
a) $(2,\infty)$
b) $(1,2)$
c) $(-2,-1)$
d) none of these

29. If α and β are the roots of $x^2+px+q=0$ and α^4,β^4 are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always
a) two real roots
b) two positive roots
c) two negative roots
d) one positive and one negative root

*Question has more than one correct option.

30. Let a,b,c be real numbers, $a\neq 0$. If α is a root of $a^2x^2+bx+c=0$. β is the root of $a^2x^2-bx-c=0$ and $0<\alpha<\beta$, then the equation $a^2x^2+2bx+2c=0$ has a root γ that always satisfies
a) $\gamma=\frac{\alpha+\beta}{2}$
b) $\gamma=\alpha+\frac{\beta}{2}$
c) $\gamma=\alpha$
d) $\alpha<\gamma<\beta$

31. Let α,β be the roots of the equation $(x-a)(x-b)=c, c\neq 0$. Then the roots of the equation $(x-\alpha)(x-\beta)+c=0$ are
a) a,c
b) b,c
c) a,b
d) $a+c,b+c$

32. The number of points of intersection of two curves $y=2sinx$ and $y=5x^2+2x+3$ is
a) 0
b) 1
c) 2
d) ∞

33. If p,q,r are +ve and are in A.P., the roots of quadratic equation $px^2+qx+r=0$ are all real for
a) $|\frac{r}{p}-7|\geq 4\sqrt{3}$
b) $|\frac{p}{r}-7|\geq 4\sqrt{3}$
c) all p nd r
d) no p and r

34. Let p,q$\in\{1,2,3,4\}$. The number of equations of the form $px^2+qx+1=0$ having real roots is
a) 15
b) 9
c) 7
d) 8

35. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then
a) $a<2$
b) $2\leq a\leq 3$
c) $3<a\leq 4$
d) $a>4$

36. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$, where $c<0<b$, then
a) $0<\alpha<\beta$
b) $\alpha<0<\beta<|\alpha|$
c) $\alpha<\beta<0$
d) $\alpha<0<|\alpha|<\beta$

37. If a,b,c,d are positive real numbers such that $a+b+c+d=2$, then $M=(a+b)(c+d)$ satisfies the relation
a) $0\leq M\leq 1$
b) $1\leq M\leq 2$
c) $2\leq M\leq 3$
d) $3\leq M\leq 4$

38. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has
a) both roots in (a,b)
b) both roots in$(-\infty,a)$

c) both roots in $(b, +\infty)$
d) one root in $(-\infty, a)$ and the other in $(b, +\infty)$

39. For the equation $3x^2 + px + 3 = 0, p > 0$, if one of the root is square of the other, then p is equal to
a) 1/3
b) 1
c) 3
d) 2/3

40. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is
a) $(-\infty, -2) \cup (2, \infty)$
b) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
c) $(-\infty, -1) \cup (1, \infty)$
d) $(\sqrt{2}, \infty)$

41. If $\alpha \in (0, \frac{\pi}{2})$ then $\sqrt{x^2 + x} + \frac{tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to
a) $2tan\alpha$
b) 1
c) 2
d) $sec^2\alpha$

42. For all 'x',$x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is
a) $a < -5$
b) $-5 < a < 2$
c) $a > 5$
d) $2 < a < 5$

43. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is
a) $p^3 - q(3p - 1) + q^2 = 0$
b) $p^3 - q(3p + 1) + q^2 = 0$
c) $p^3 + q(3p - 1) + q^2 = 0$
d) $p^3 + q(3p + 1) + q^2 = 0$

44. Let a,b,c be the sides of a triangle where $a \neq b \neq c$ and $\lambda \in$R. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then
a) $\lambda < \frac{4}{3}$
b) $\lambda > \frac{5}{3}$
c) $\lambda \in (\frac{1}{3}, \frac{5}{3})$
d) $\lambda \in (\frac{4}{3}, \frac{5}{3})$

45. Let α,β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is
a) $\frac{2}{9}(p - q)(2q - p)$
b) $\frac{2}{9}(q - p)(2p - q)$
c) $\frac{2}{9}(q - 2p)(2q - p)$
d) $\frac{2}{9}(2p - q)(2q - p)$

46. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then the quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is
a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

47. Let (x_0, y_0) be the solution of the following equations $(2x)^{ln2} = (3y)^{ln3}$ and $3^{lnx} = 2^{lny}$. Then x_0 is
a) $\frac{1}{6}$
b) $\frac{1}{3}$
c) $\frac{1}{2}$
d) 6

48. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is
a) 1
b) 2
c) 3
d) 4

49. A value of b for which the equations
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is
a) $-\sqrt{2}$
b) $-i\sqrt{3}$
c) $i\sqrt{5}$
d) $\sqrt{2}$

50. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation p(p(x))=0 has
a) one purely imaginary root
b) all real roots
c) two real and two purely imaginary roots
d) neither real nor purely imaginary roots

51. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x(sec\alpha) + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 - 2xtan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
a) $2(sec\theta - tan\theta)$
b) $2sec\theta$
c) $-2tan\theta$

d) 0

52. For real x, the function $\frac{(x-a)(x-b)}{x-c}$ will assume all real values provided
 a) $a > b > c$
 b) $a < b < c$
 c) $a > c > b$
 d) $a < c < b$
53. If S is the set of all real x such that $\frac{2x-1}{2x^2+3x^2+x}$ is positive, then S contains
 a) $[-\infty, -\frac{3}{2}]$
 b) $[-\frac{3}{2}, -\frac{1}{4}]$
 c) $[-\frac{1}{4}, \frac{1}{2}]$
 d) $[\frac{1}{2}, 3]$
54. If a,b and c are distinct positive numbers, then the expression (b+c-a)(c+a-b)(a+b-c)-abc is
 a) positive
 b) negative
 c) non-positive
 d) non-negative
 e) none of these
55. If a,b,c,d and p are distinct real numbers such that $(a^2+b^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2) \leq 0$ then a,b,c,d
 a) are in A.P.
 b) are in G.P.
 c) are in H.P.
 d) satisfy ab=cd
 e) none of these
56. The equation $x^{3/4(log_2 x)^2+log_2 x^{-5/4}} = \sqrt{2}$ has
 a) at least one real solution
 b) exactly three solutions
 c) exactly one irrational solution
 d) complex roots
57. The product of n positive numbers is unity. Then their sum is
 a) a positive integer
 b) divisible by n
 c) equal to $n + \frac{1}{n}$
 d) never less than n
58. Number of divisor of the form $4n + 2 (n \geq 0)$ of the integer 240 is
 a) 4
 b) 8
 c) 10
 d) 3
59. If $3^x = 4^{x-1}$, then $x =$
 a) $\frac{2log_3 2}{2log_3 2-1}$
 b) $\frac{2}{2-2log_2 3}$
 c) $\frac{1}{1-log_4 3}$
 d) $\frac{2log_2 3}{2log_2 3-1}$
60. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is (are) a subset (s) of S?
 a) $(-\frac{1}{2}, -\frac{1}{\sqrt{5}})$
 b) $(-\frac{1}{\sqrt{5}}, 0)$
 c) $(0, \frac{1}{\sqrt{5}})$
 d) $(\frac{1}{\sqrt{5}}, \frac{1}{2})$
61. Solve for x: $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$
62. If (m,n)= $\frac{(1-x^m)(1-x^{m-1}).......(1-x^{m-n+1})}{(1-x)(1-x^2).........(1-x^n)}$ where m and n are positive integers$(n \leq m)$, show that $(m, n+1) = (m-1, n+1) + x^{m-n-1}(m-1, n)$.
63. Solve for x: $\sqrt{x+1} - \sqrt{x-1} = 1$.
64. Solve the following equation for x:
$$2log_x a + log_{ax} a + 3log_{a^2 x} a = 0, a > 0. \tag{3.0.64.1}$$
65. Show that the square of $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$,is a rational number.
66. Sketch the solution set of the following system of inequalities:
$$x^2 + y^2 - 2x \geq 0; 3x - y - 12 \leq 0; y - x \leq 0; y \geq 0. \tag{3.0.66.1}$$
67. Find all integers x for which $(5x - 1) < (x + 1)^2 < (7x - 3)$
68. If α, β are the roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + rx + s = 0$, evalute$(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$ in terms of p,q,,r and s. Deduce the condition that the equations have a common root.
69. Given $n^4 < 10^n$ for fixed positive integer $n \geq 2$ prove that $(n+1)^4 < 10^{n+1}$
70. Let y= $\sqrt{\frac{(x+1)(x-3)}{(x-2)}}$ Find all the real values of x for which y takes real values.
71. For what values of m, does the system of equations $3x+my = m$,$2x-5y = 20$ has solution satisfying the condition $x > 0, y > 0$.
72. Find the solution set of the system $x+2y+z = 1$; $2x - 3y - w = 2$; $x \geq 0; y \geq 0; z \geq 0; w \geq 0$.

73. Show that the equation$e^{sinx} - e^{-sinx} - 4 = 0$ has no real solution.
74. mm squares of equal size are arranged to form a rectangle of dimension m by n, where m and n are natural numbers. Two squares will be called 'neighbours' if they have exactly one common side. A natural number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighbouring squares. Show that this is possible only if all the numbers used are equal.
75. If one root of the quadratic equation $ax^2+bx+c = 0$ is equal to the n-th power of the other, then show that $(ac)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$
76. Find all real values of x which satisfy $x^2-3x+2 > 0$ and $x^2 - 2x - 4 \leq 0$.
77. Solve for x; $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3} = 10$.
78. For $a \leq 0$, determine all real roots of the equation$x^2 - 2a|x - a| - 3a^2 = 0$
79. Find the set of all x for which $\frac{2x}{(2x^2+5x+2)} > \frac{1}{(x+1)}$.
80. Solve $x^2 + 4x + 3 + 2x + 5 = 0$
81. Let a,b,c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \frac{c}{a} + |\frac{b}{a}| < 0$.
82. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a,b,c and d denote the lengths of the sides of the quadrilateral, prove that $2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$.
83. If α,β are the roots of $ax^2+bx+c = 0, (a \neq 0)$ and $\alpha+\delta,\beta+\delta$ are the roots of $Ax^2+Bx+C = 0, (A \neq 0)$ for some constant δ, then prove that $\frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$.
84. Let a,b,c be real numbers with $a \neq 0$ and let α,β be the roots of the equation $ax^2+bx+c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α,β.
85. If $x^2 + (a - b)x + (1 - a - b) = 0$ where a,b∈R then find the values of a for which equation has unequal real roots for all values of b.
86. If a,b,c are positive real numbers. Then prove that $(a + 1)^7(b + 1)^7(c + 1)^7 > 7^7a^4b^4c^4$.
87. Let a and b be the roots of the equation $x^2 - 10ax - 11b = 0$ are c,d then the value of $a + b + c + d$, when $a \neq b \neq c \neq d$, is.
88. Let p,q be integers and let α,β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n= $0, 1, 2,.......$,let $a_n = p\alpha^n + q\beta^n$. FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a= 0 =b.s
89. $a_{12} =$
    a) $a_{11} - a_{10}$
    b) $a_{11} + a_{10}$
    c) $2a_{11} + a_{10}$
    d) $a_{11} + 2a_{10}$
90. If $a_4 = 28$, then $p + 2q =\in$
    a) 21
    b) 14
    c) 7
    d) 12
91. Let a,b,c,p,q be real numbers. Suppose α,β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2+2bx+c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$
    STATEMENT-1:$(p^2 - q)(b^2 - ac) \geq 0$ and
    STATEMENT-2:$b \neq pa$ or $c \neq qa$
    a) STATEMENT-1 is True, STATEMENT-2 is True;STATEMENT-2 is a correct explanation for STATEMENT-1
    b) STATEMENT-1 is True,STATEMENT-2 is True;STATEMENT-2 is NOT a correct explanation for STATEMENT-1
    c) STATEMENT-1 is True,STATEMENT-2 is False
    d) STATEMENT-1 is False,STATEMENT-2 is True
92. Let (x,y,z) be points with integer coordinates satisfying the system of homogeneous equations: $3x-y-z = 0$,$-3x+z = 0$,$-3x+2y+z = 0$ Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is
93. The smallest value of k, for which both the roots of the equation $x^2-8kx+16(k^2-k+1) = 0$ are real, distinct and have values at least 4 is
94. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} where $a > 0$ is
95. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is
96. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is
    a) $3x^2 - 19x + 3 = 0$
    b) $3x^2 + 19x - 3 = 0$
    c) $3x^2 - 19x - 3 = 0$
    d) $x^2 - 5x + 3 = 0$
97. Difference between the corresponding roots of $x^2+ax+b = 0$ and $x^2+bx+a = 0$ is same and

$a \neq b$, then

a) $a+b+4=0$
b) $a+b-4=0$
c) $a-b-4=0$
d) $a-b+4=0$

98. Product of real roots of the equation $t^2x^2+|x|+9=0$

a) is always positive
b) is always negative
c) does not exist
d) none of these

99. If p and q are the roots of the equation $x^2+px+q=0$, then

a) $p=1, q=-2$
b) $p=0, q=1$
c) $p=-2, q=0$
d) $p=-2, q=1$

100. If a,b,c are distinct +ve real numbers and $a^2+b^2+c^2=1$ then $ab+bc+ca$ is

a) less than 1
b) equal to 1
c) greater than 1
d) any real number

101. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in

a) Arithmetic-Geometric Progression
b) Arithmetic Progression
c) Geometric Progression
d) Harmonic Progression

102. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other is

a) $-\frac{1}{3}$
b) $\frac{2}{3}$
c) $-\frac{2}{3}$
d) $\frac{1}{3}$

103. The number of real solutions of the equation $x^2-3|x|+2=0$ is

a) 3
b) 2
c) 4
d) 1

104. The real number x when added to its inverse gives the minimum value of the sum at x equal to

a) -2
b) 2
c) 1
d) -1

105. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

a) $x^2-18x-16=0$
b) $x^2-18x+16=0$
c) $x^2+18x-16=0$
d) $x^2+18x+16=0$

106. If (1-p) is a root of quadratic equation $x^2+px+(1-p)=0$ then its root are

a) $-1, 2$
b) $-1, 1$
c) $0, -1$
d) $0, 1$

107. If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots, then the value of 'q' is

a) 4
b) 12
c) 3
d) $\frac{49}{4}$

108. In a triangle PQR,$\angle R=\frac{\pi}{2}$. If tan$(\frac{P}{2})$ and -tan$(\frac{Q}{2})$ are roots of $ax^2+bx+c=0, a \neq 0$ then

a) $a=b+c$
b) $c=a+b$
c) $b=c$
d) $b=a+c$

109. If both the roots of the quadratic equation $x^2-2kx+k^2+k-5=0$ are less than 5, then k lies in the interval

a) $(5, 6]$
b) $(6, \infty)$
c) $(-\infty, 4)$
d) $[4, 5]$

110. If the roots of the quadratic equation $x^2+px+q=0$ are tan30° and tan15°, respectively, then the value of 2+q-p is

a) 2
b) 3
c) 0
d) 1

111. All the values of m for which both roots of the equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4, lies in the interval

a) $-2 < m < 0$
b) $m > 3$
c) $-1 < m < 3$
d) $1 < m < 4$

112. If x is real , the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is
a) $\frac{1}{4}$
b) 41
c) 1
d) $\frac{17}{7}$

113. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
a) $(3, \infty)$
b) $(-\infty, -3)$
c) $(-3, 3)$
d) $(-3, \infty)$

114. Statement-1: For every natural number $n \geq 2$, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$ Statement-2: For every natural number $n \geq 2$, $\sqrt{n(n+1)} < n+1$
a) Statement-1 is false, Statement-2 is true
b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
d) Statement-1 is true, Statement-2 is false

115. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4:3. Then the common root is
a) 1
b) 4
c) 3
d) 2

116. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is:
a) less than $4ab$
b) greater than $-4ab$
c) less than $-4ab$
d) greater than $4ab$

117. If $|z - \frac{4}{z}| = 2$, then the maximum value of $|Z|$ is equal to:
a) $\sqrt{5} + 1$
b) 2
c) $2 + \sqrt{2}$
d) $\sqrt{3} + 1$

118. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
a) -1
b) 1
c) 2
d) -2

119. The equation $e^{sinx} - e^{-sinx} - 4 = 0$ has:
a) infinite number of real roots
b) no real roots
c) exactly one real root
d) exactly four real roots

120. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]
a) lies between 1 and 2
b) lies between 2 and 3
c) lies between -1 and 0
d) does not exist

121. The number of values of k, for which the system of equations:
$(k+1)x + 8y = 4k$
$kx + (k+3)y = 3k - 1$
a) infinite
b) 1
c) 2
d) 3

122. If the equations $x^2+2x+3 = 0$ and $ax^2+bx+c = 0$, a,b,c∈R, have a common root, then a:b:c is
a) 1 : 2 : 3
b) 3 : 2 : 1
c) 1 : 3 : 2
d) 3 : 1 : 2

123. Is a∈R and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where[x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval:
a) $(-2, -1)$
b) $(-\infty, -2) \cup (2, \infty)$
c) $(-1, 0) \cup (0, 1)$
d) $(1, 2)$

124. Let α and β be the roots of the equation $px^2 + qx + r = 0, p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is:
a) $\frac{\sqrt{34}}{9}$
b) $\frac{2\sqrt{13}}{9}$
c) $\frac{\sqrt{61}}{9}$
d) $\frac{2\sqrt{17}}{9}$

125. Let α and β be the roots of equation $x^2-6x-2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to:
   a) 3
   b) -3
   c) 6
   d) -6
126. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is:
   a) 6
   b) 5
   c) 3
   d) -4
127. If $\alpha, \beta \in$ C are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:
   a) 0
   b) 1
   c) 2
   d) -1
128. Let p,q $\in$R. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then:
   a) $p^2 - 4q + 12 = 0$
   b) $q^2 - 4p - 16 = 0$
   c) $q^2 + 4p + 14 = 0$
   d) $p^2 - 4q - 12 = 0$