Geometry: Maths Olympiad

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Abstract—**This book provides a collection of the international maths olympiad problems in geometry.**

1. Construct a right triangle with given hypotenuse c such that the median drawn to the hypotenuse is the geometric mean of the two legs of the triangle.
2. An arbitrary point M is selected in the interior of the segment AB. The squares AMCD and MBEF are constructed on the same side of AB, with the segments AM and MB as their respective bases. The circles circumscribed about these squares, with centers P and Q, intersect at M and also at another point N. Let N0 denote the point of intersection of the straight lines AF and BC.
   a) Prove that the points N and N0 coincide.
   b) Prove that the straight lines MN pass through a fixed point S independent of the choice of M.
   c) Find the locus of the midpoints of the segments P Q as M varies between A and B.
3. Two planes, P and Q, intersect along the line p. The point A is given in the plane P, and the point C in the plane Q; neither of these points lies on the straight line p. Construct an isosceles trapezoid ABCD (with AB parallel to CD) in which a circle can be inscribed, and with vertices B and D lying in the planes P and Q respectively.
4. Point A and segment BC are given. Determine the locus of points in space which are vertices of right angles with one side passing through A, and the other side intersecting the segment BC.
5. Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$

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