Discrete Maths and Algebra

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Contents

***Abstract*—This book provides a computational approach to school algebra and discrete mathematics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    ncert/codes
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1 Algebra

1.1 Examples

1. Divide $p(x)$ by $g(x)$, where $p(x) = x + 3x^2 – 1$ and $g(x) = 1 + x$.
2. Divide the polynomial $p(x) = 3x^4 - 4x^3 - 3x - 1$ by $x - 1$.
3. Find the value of k, if $x–1$ is a factor of $p(x) = 4x^3 + 3x^2 - 4x + k$.
4. Divide $2x^2 + 3x + 1$ by $x + 2$.
5. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.
6. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.
7. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.
8. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
   a) $p(x) = x^2 + x + k$
   b) $p(x) = kx^2 - \sqrt{2}x + 1$
   c) $p(x) = 2x^2 + kx + \sqrt{2}$
   d) $p(x) = kx^2 - 3x + k$
9. Divide the polnyomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:
   a) $p(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$.
   b) $p(x) = x^4 - 3x^2 + 4x + 5, g(x) = x^2 + 1 - x$.
   c) $p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$.
10. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:
    a) $t^2 - x, 2t^4 + 3t^3 - 2t^2 - 9t - 12$.
    b) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$.
    c) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$.
11. Obtain all the other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$and $-\sqrt{\frac{5}{3}}$.
12. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x + 4$respectively. Find $g(x)$.

13. Verify that the numbers given alongside the cubic polynomials below are their zeroes. Also verify if the relationship between the zeroes and the coefficients in each case:
    a) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$
    b) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$
14. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -4 respectively.
15. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find the other zeroes.
16. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.
17. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
18. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.
19. The product of Sunita's age (in years) two years ago and her age four years from now is one more than twice her present age. What is her present age?
20. Find two consecutive odd positive integers, sum of whose squares is 290.
21. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
22. The product of two consecutive positive integers is 306. We need to find the integers.
23. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
24. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.
25. Find two numbers whose sum is 27 and product is 182.
26. Find two consecutive positive integers, sum of whose squares is 365.
27. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.
28. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.
29. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.
30. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
31. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
32. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
33. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.
34. Sum of the areas of two squares is 468 $m^2 2$ find the sides of the two squares.
35. Find the values of k for each of the following quadratic equations, so that they have two equal roots:
    a) $2x^2 + kx + 3 = 0$
    b) $kx(x - 2) + 6 = 0$
36. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

37. If

$$\mathbf{x} = \sqrt{\frac{\begin{pmatrix} a \\ -b \end{pmatrix}}{\begin{pmatrix} c \\ -d \end{pmatrix}}} \tag{1.1.37.1}$$

prove that

$$\|\mathbf{x}\|^2 = \frac{\left\|\begin{pmatrix} a \\ b \end{pmatrix}\right\|}{\left\|\begin{pmatrix} c \\ d \end{pmatrix}\right\|} \tag{1.1.37.2}$$

38. For any two complex numbers $\mathbf{z}_1, \mathbf{z}_2$, prove that

$$\Re \mathbf{z}_1 \mathbf{z}_2 = \Re \mathbf{z}_1 \Re \mathbf{z}_2 - \Im \mathbf{z}_1 \Im \mathbf{z}_2 \tag{1.1.38.1}$$

39. If $\mathbf{x} = \frac{\begin{pmatrix} a \\ b \end{pmatrix}}{\begin{pmatrix} a \\ -b \end{pmatrix}}$, show that $\|\mathbf{x}\| = 1$
40. If $\mathbf{x} = \frac{\begin{pmatrix} x \\ 1 \end{pmatrix}^2}{2x^2+1}$, prove that $\|\mathbf{x}\|^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.
41. If $\begin{pmatrix} x \\ y \end{pmatrix}^3 = \mathbf{uv}$, then show that $\frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$.
42. If $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are different complex numbers with $\|\boldsymbol{\beta}\| = 1$, then find $\left\|\frac{\beta-\alpha}{1-\alpha^*\beta}\right\|$.
43. Find the number of non-zero integral solutions of the equation $\|1 - 1\|^x = 2^x$.
44. If $\begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix}\begin{pmatrix} e \\ f \end{pmatrix}\begin{pmatrix} g \\ h \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix}$, then show that $\left(a^2 + b^2\right)\left(c^2 + d^2\right)\left(e^2 + f^2\right)\left(g^2 + h^2\right) = A^2 + B^2$.
45. If $\left(\frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}\right) = 1$, the find hte least positive integral value of m
46. The length L (in cm) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if $L = 124.942$ when $C = 20$ and $L = 125.134$ when $C = 110$, express L in terms of C.
47. The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?
48. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.
49. The taxi fare in a city is as follows: For the first kilometre, the fare is ₹8 and for the subsequent distance it is ₹5 per km. Taking the distance covered as x km and total fare as ₹y, write a linear equation for this information, and draw its graph.
50. Yamini and Fatima, two students of Class IX of a school, together contributed ₹100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹x and ₹y.) Draw the graph of the same.
51. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \frac{9}{5}C + 32 \tag{1.1.51.1}$$

   a) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.
   b) If the temperature is 30°C, what is the temperature in Fahrenheit?
   c) If the temperature is 95°F, what is the temperature in Celsius?
   d) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?
   e) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.
52. Romila went to a stationery shop and purchased 2 pencils and 3 erasers for ₹9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for ₹18. Represent this situation algebraically and graphically. Find the cost of each pencil and eraser.
53. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and

graphically. Find their respective ages.

54. The coach of a cricket team buys 3 bats and 6 balls for ₹3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and geometrically.Find the cost of each bat and ball.
55. The cost of 2 kg of apples and 1kg of grapes on a day was found to be ₹160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹300. Represent the situation algebraically and geometrically. Find the cost of apples and grape.
56. Form the pair of linear equations in the following problems, and find their solutions.
57. 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
58. 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and that of one pen.
59. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.
60. The difference between two numbers is 26 and one number is three times the other. Find them.
61. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
62. The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.
63. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?
64. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. . Find the fraction.
65. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages
66. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹2000 per month, find their monthly incomes.
67. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?
68. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$. if we only add 1 to the denominator. What is the fraction?
69. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
70. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
71. Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.
72. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.
73. The cost of 5 oranges and 3 apples is ₹35 and the cost of 2 oranges and 4 apples is ₹28. Let us find the cost of an orange and an apple.
74. From a bus stand in Bangalore , if we buy 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, the total cost is ₹46; but if we buy 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur the total cost is ₹74. Find the fares from the bus stand to Malleswaram, and to Yeshwanthpur.
75. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day.
76. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes when 8 is added to its denominator. Find the fraction.
77. Yash scored 40 marks in a test, getting 3

marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

78. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?
79. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.
80. Solve the pair of equations:
$$\begin{aligned} \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= 13 \\ \begin{pmatrix} 5 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= -2 \end{aligned} \tag{1.1.80.1}$$
81. Solve the pair of equations by reducing them to a pair of linear equations
$$\begin{aligned} \begin{pmatrix} 5 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{x-1} \\ \frac{1}{y-2} \end{pmatrix} &= 2 \\ \begin{pmatrix} 6 & -3 \end{pmatrix} \begin{pmatrix} \frac{1}{x-1} \\ \frac{1}{y-2} \end{pmatrix} &= 1 \end{aligned} \tag{1.1.81.1}$$
82. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.
83. Solve the following pairs of equations

a)
$$\begin{aligned} \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= 2 \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} &= \frac{13}{6} \end{aligned} \tag{1.1.83.1}$$

b)
$$\begin{aligned} \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{x}} \\ \frac{1}{\sqrt{y}} \end{pmatrix} &= 2 \\ \begin{pmatrix} 4 & -9 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{x}} \\ \frac{1}{\sqrt{y}} \end{pmatrix} &= -1 \end{aligned} \tag{1.1.83.2}$$

c)
$$\begin{aligned} \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ y \end{pmatrix} &= 14 \\ \begin{pmatrix} 3 & -4 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ y \end{pmatrix} &= 23 \end{aligned} \tag{1.1.83.3}$$

d)
$$\begin{aligned} \begin{pmatrix} 10 & 2 \end{pmatrix} \begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} &= 4 \\ \begin{pmatrix} 15 & -5 \end{pmatrix} \begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} &= -2 \end{aligned} \tag{1.1.83.4}$$

e)
$$\begin{aligned} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{3x+y} \\ \frac{1}{3x-y} \end{pmatrix} &= \frac{3}{4} \\ \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{3x+y} \\ \frac{1}{3x-y} \end{pmatrix} &= -\frac{1}{8} \end{aligned} \tag{1.1.83.5}$$

84. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.
85. 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.
86. Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.
87. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.
88. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other

replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II].

89. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.
90. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.
91. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.
92. Find two numbers whose sum is 24 and whose product is as large as possible.
93. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

2 Arithmetic Progression

2.1 Examples

1. For the AP : $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, ... write the first term a and the common difference d.
2. Which of the following list of numbers form an AP? If they form an AP, write the next two terms :
   a) 4,10,16,22,...
   b) 1,-1,-3,-5,...
   c) -2,2,-2,2,-2,...
   d) 1,1,1,2,2,2,3,3,3,...
3. Find the 10th term of the AP : 2, 7, 12, . . .
4. Which term of the AP : 21, 18, 15, . . . is –81? Also, is any term 0? Give reason for your answer.
5. Determine the AP whose 3rd term is 5 and the 7th term is 9.
6. Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .
7. How many two-digit numbers are divisible by 3?
8. Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.
9. A sum of ₹1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
10. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?
11. Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .
12. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.
13. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?
14. Find the sum of :
    (i) the first 1000 positive integers
    (ii) the first n positive integers.
15. Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3+2n$
16. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :
    (i) the production in the 1st year
    (ii) the production in the 10th year
    (iii) the total production in first 7 years.

2.2 Exercises

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?
   a) The taxi fare after each km when the fare is ₹15 for the first km and ₹8 for each additional km.
   b) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.
   c) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.
   d) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8 % per annum.
2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:
   a) a = 10, d = 10
   b) a = 4, d = –3
   c) a = –2, d = 0

d) a = –1, d = $\frac{1}{2}$
e) a = –1.25, d = –0.25

3. For the following APs, write the first term and the common difference:
   a) 3, 1, –1, –3, . . .
   b) –5, –1, 3, 7, . . .
   c) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$
   d) 0.6, 1.7, 2.8, 3.9....
4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.
   a) 2, 4, 8, 16, . . .
   b) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$
   c) –1.2, –3.2, –5.2, –7.2, . . .
   d) –10, –6, –2, 2, . . .
   e) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, ...$
   f) 0.2, 0.22, 0.222, 0.2222, . . .
   g) 0, –4, –8, –12, . . .
   h) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$
   i) 1, 3, 9, 27,...
   j) a, 2a, 3a, 4a,...
   k) $a, a^2, a^3, a^4, ...$
   l) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$
   m) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$
   n) $1^2, 3^2, 5^2, 7^2$
   o) $1^2, 5^2, 7^2, 73, ...$
5. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the AP:

	a	d	n	a_n
(i)	7	3	8	....
(ii)	-18	....	10	0
(iii)	....	-3	18	-5
(iv)	-18.9	2.5	....	3.6
(v)	3.5	0	105	....

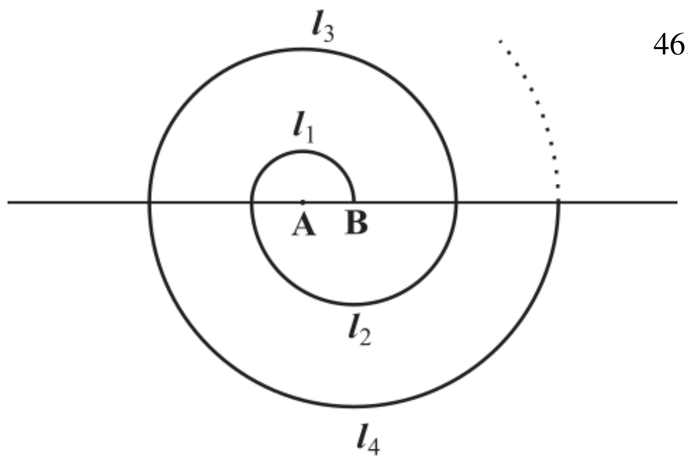
6. Choose the correct choice in the following and justify :
   (i) 30^{th} term of the AP: 10, 7, 4,..., is
   a) 97
   b) 77
   c) -77
   d) -87

   (ii) 11^{th} term of the AP: $-3, -\frac{1}{2}, 2, ...$, is
   a) 28
   b) 22
   c) -38
   d) $-48\frac{1}{2}$

   (iii) In the following APs, find the missing terms in the blanks :
   a) 2, ... , 26
   b) ...., 13. ...., 3
   c) 5, ..., ..., $9\frac{1}{2}$
   d) -4, ..., ..., ..., ..., 6
   e) ...., 38, ..., ..., ..., -22
7. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?
8. Find the number of terms in each of the following APs:
   (i) 7, 13, 19, ...,205.
   (ii) 18, $15\frac{1}{2}$, 13, ..., –47
9. Check whether – 150 is a term of the AP : 11, 8, 5, 2 ....
10. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
11. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
12. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?
13. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.
14. Which term of the AP : 3, 15, 27, 39,... will be 132 more than its 54th term?
15. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
16. How many three-digit numbers are divisible by 7?
17. How many multiples of 4 lies between 10 and 250?
18. For what value of n, are the n^{th} terms of two APs: 63, 65, 67,.... and 3, 10, 17,... equal?
19. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.
20. Find the 20th term from the last term of the AP : 3, 8, 13,..., 253.
21. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
22. Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?
23. Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by

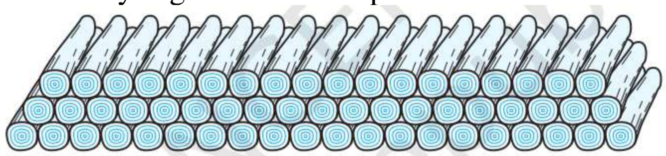
₹1.75. If in the n^{th} week, her weekly savings become ₹20.75, find n.

24. Find the sum of the following APs:
 (i) 2, 7, 12, . . ., to 10 terms.
 (ii) -37, -33, -29, . . ., to 12 terms.
 (iii) 0.6, 1.7, 2.8, . . ., to 100 terms.
 (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ..., to 11 terms.
25. Find the sums given below :
 (i) $7 + 10\frac{1}{2} + 14 + ... + 84$
 (ii) 34 + 32 + 30 + . . . + 10
 (iii) -5 + (-8) + (-11) + . . . + (-230)
26. In an A.P:
 (i) given a = 5, d = 3, $a_n = 50$, find n and S_n.
 (ii) given a = 7, $a_{13} = 35$, find d and S_{13}.
 (iii) given $a_{12} = 37$, d = 3, find a and S_{12}.
 (iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.
 (v) given d = 5, S_9= 75, find a and a_9.
 (vi) given a = 2, d = 8, $S_n = 90$, find n and a_n.
 (vii) given a = 8, $a_n = 62$, $S_n = 210$, find n and d.
 (viii) given $a_n = 4$, d = 2, $S_n = -14$, find n and a.
 (ix) given a = 3, n = 8, S = 192, find d.
 (x) given l = 28, S = 144, and there are total 9 terms. Find a.
27. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?
28. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
29. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
30. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.
31. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
32. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
33. Show that $a_1, a_2, ..., a_n$, . . . form an AP where a_n is defined as below :
 (i) $a_n = 3 + 4n$
 (ii) $a_n = 9 - 5n$
 Also find the sum of the first 15 terms in each case.
34. If the sum of the first n terms of an AP is 4n - n^2 , what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.
35. Find the sum of the first 40 positive integers divisible by 6.
36. Find the sum of the first 15 multiples of 8.
37. Find the sum of the odd numbers between 0 and 50.
38. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day,₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30days?
39. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.
40. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying,
 e.g., a section of Class I will plant 1 tree,
 a section of Class II will plant 2 trees and so on till Class XII.
 There are three sections of each class. How many trees will be planted by the students?
41. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,... as shown in Fig. What is the total length of such a spiral made up of thirteen consecutive 22 semicircles? (Take $\pi = \frac{22}{7}$)

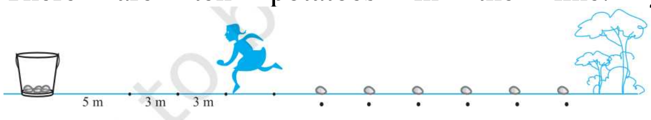


Hint : Length of successive semicircles is $l_1, l_2, l_3, l_4, ...$ with centres at A, B, A, B, . . .,respectively.

42. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig). In how many rows are the 200 logs placed and how many logs are in the top row?



43. In a potato race, a bucket is placed at the starting point, which is 5m from the first potato, and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line.

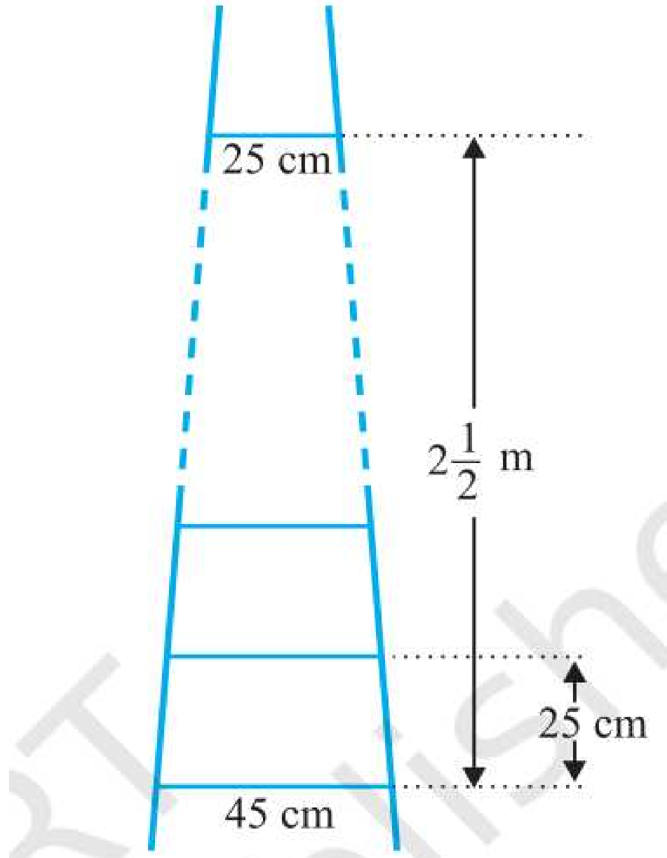


A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run? [**Hint** : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$].

44. Which term of the AP :121, 117, 113,...,is its first negative term?[**Hint:**Find n for a n < 0]

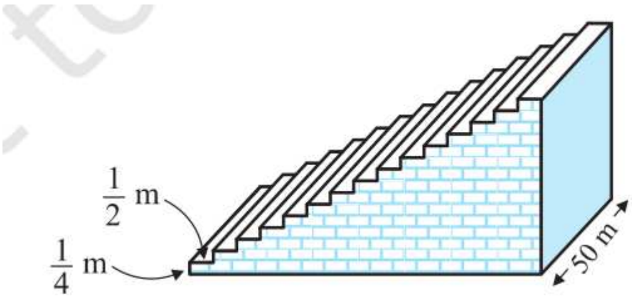
45. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

46. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, which is the length of the wood required for the rungs?[**Hints:** Number of rungs = $\frac{250}{25} + 1$]



47. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.[**Hint:** $S_{X-1} = S_{4}9 - S - x$]

48. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has rise of $\frac{1}{4}$m and a tread of $\frac{1}{2}$m .Caluculate the total volume of concrete required to build the terrace.[**Hint:** Volume of concrete required to build the first step $=\frac{1}{4}X\frac{1}{2}X50m^3$]



3 Principles of Mathematical Induction

3.1 Examples

1. For all n ≥ 1, prove that
$1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
2. Prove that 2^n>n for all positive integers n.
3. For all $n \geq 1$,prove that
$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + .. + \frac{1}{n(n+1)} = \frac{n}{n+1}$.
4. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.
5. Prove that $(1 + x)^n \geq (1 + nx)$, for all natural number n, where x > -1.
6. Prove that
$2.7^n + 3.5^n - 5$ is divisible by 24,for all nϵN.
7. Prove that
$1^2 + 2^2 + ... + n^2 > \frac{n^3}{3}$,n$\epsilon$N.
8. Prove the rule of exponents $(ab)^n = a^n b^n$ by using the principle of mathematical induction for every natural number.

3.2 Exercises

1. $1 + 3 + 3^2 + ... + 3^{n-1} = \frac{(3^n-1)}{2}$.
2. $1^3 + 2^3 + 3^3 + ... + n^3 = (\frac{n(n+1)}{2})^2$
3. $1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + ... + \frac{1}{(1+2+3+...n)} = \frac{2n}{(n+1)}$.
4. $1.2.3+2.3.4+...+n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.
5. $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$
6. $1.2 + 2.3 + 3.4 + ... + n.(n + 1) = [\frac{n(n+1)(n+2)}{3}]$.
7. $1.3+3.5+5.7+...+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$
8. $1.2 + 2.2^2 + 3.2^3 + ... + n.2^n = (n - 1)2^{n+1} + 2$.
9. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + .. + \frac{1}{2^n} = 1 - \frac{1}{2^n}$
10. $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$.
11. $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + .. + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$.
12. $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n-1)}{r-1}$.
13. $(1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9})...(1 + \frac{(2n+1)}{n^2}) = (n + 1)^2$.
14. $(1 + \frac{1}{1})(1 + \frac{1}{2})(1 + \frac{1}{3})...(1 + \frac{1}{n}) = (n + 1)$.
15. $1^2 + 3^2 + 5^2 + ... + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$
16. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3(2n+1)}$
17. $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$.
18. $1 + 2 + 3 + .. + n < \frac{1}{8}(2n + 1)^2$.
19. n(n+1)(n+5) is a multiple of 3.
20. 10^{2n-1}+1 is divisible by 11.
21. $x^{2n} - y^{2n}$ is divisible by x+y.
22. $3^{2n+2} - 8n - 9$ is divisible by 8.
23. $41^n - 14^n$ is a multiple of 27
24. $(2n + 7) < (n + 3)^2$.

4 Binomial Theorem

4.1 Examples

1. Expand $(x^2 + \frac{3}{4})^4$,x≠ 0
2. Compute $(98)^5$
3. Which is larger $(1.01)^{1000000}$or 10,000?
4. Using binomial theorem,prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

5. Find a if the 17^{th} and 18^{th} terms of expansiom $(2+a)^{50}$ are equal.

6. Show that the middile term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5..(2n-1)}{n!}2nx^n$, where n is a positive integer

7. Find the coefficient of x^6y^3 in the expansion of $(x+2y)^9$.
8. The second,third and fourth terms in the binomial expansion $(x+a)^n$ are 240,720 and 1080, respectively.Find x,a and n.

9. The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42.Find n.

10. Find the term independent of x in the expansion of $(\frac{3}{2}x^2 - \frac{1}{3x})^6$.

11. If the coefficients of a^{r-1}, a^r and a^{r+1} in the expansion of $(1+a)^n$ are in arithmetic progression , prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

12. Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$.

13. Find the coefficient of a^4 in the product $(1+2a)^4(2-a)^5$ using binomial theorem.

14. Find thr r^{th} term from the end in the expansion of $(x+a)^n$.

15. Find the term independent of x in the expansion of $(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}})^{18}$, x>0.

16. The sum of the coefficients of the first three terms in the expansion of $(x - \frac{3}{x^2})^m$, x≠0, m being a natural number, is 559. Find the term of the expansion containing x^3.

17. If the coefficients of $(r-5)^{th}$ and $(2r-1)^{th}$ terms in the expansion of $(1+x)^{34}$ are equal , find r.

4.2 Exercises

1. $(1-2x)^5$

2. $(\frac{2}{x} - \frac{x}{2})^5$

3. $(2x-3)^6$

4. $(\frac{x}{3} + \frac{1}{x})^5$

5. $(x + \frac{1}{x})^6$

Using binomial theorem,evaluate each of the following:

6. $(96)^3$

7. $(102)^5$

8. $(101)^4$

9. $(99)^5$

10. Using Binomial Theorem ,indicate which number is larger $(1.1)^{10000}$ or 1000.

11. Find $(a+b)^4 - (a-b)^4$. Hence evaluate $(\sqrt{3} + \sqrt{2})^4$ - $(\sqrt{3} - \sqrt{2})^4$.

12. Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$.

13. Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

14. Prove that $\sum_{r=0}^{n} 3^r \, {}^nC_r = 4^n$.

Find the coefficient of

15. x^5 in $(x+3)^8$

16. a^5b^7 in $(a-2b)^{12}$.

Write the general term in the expansion of

17. $(x^2 - y)^6$

18. $(x^2 - yx)^{12}, x \neq 0$.

19. Find the 4^{th} term in the expansion of $(x-2y)^{12}$.

20. Find the 13^{th} term in the expansion of $(9x - \frac{1}{3\sqrt{x}})^{18}, x \neq 0$.

Find the middle terms in the expansions of

21. $(3 - \frac{x^3}{6})^7$

22. $(\frac{x}{3} + 9y)^{10}$

23. In the expansion of $(1 + a)^{m+n}$,prove that coefficients of a^m and a^n are equal.

24. The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3: 5.Find n and r.

25. Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

26. Find the positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

27. Find a,b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729,7290 and 30375,respectively.

28. Find the coefficient of x^5 in the product $(1 + 2x)^6(1 - x)^7$ using binomial theorem.

29. Find a if the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal.

30. If a and b are distinct integers, prove that a - b is a factor of $a^n - b^n$,whenever n is a positive integer.
Hint write $a^n = (a - b + b)^n$ and expand

31. Evaluate $(\sqrt{3} + \sqrt{2})^6$ - $(\sqrt{3} - \sqrt{2})^6$

32. Find the value of $(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4$.

33. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

34. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}})^n$ is $\sqrt{6} : 1$

35. Expand using BinomialTheorem $(1 + \frac{x}{2} - \frac{2}{x})^4$, $x \neq 0$.

36. Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.

5 Permutations and Combinations

5.1 Examples

1. How many 3 digit numbers can be formed from the digits 1,2,3,4 and 5 assuming that
(i) repetition of the digits is allowed?

(ii) repetition of the digits is not allowed?

2. How many 3 digit even numbers can be formed from the digits 1,2,3,4,5,6 if the digits can be repeated?

3. How many 4 letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

4. How many 5 digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

5. A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

6. Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

7. Evaluate
(i) 8!

(ii) 4!-3!

8. Is 3!+4!=7!?

9. Compute $\frac{8!}{6! \times 2!}$.

10. If $\frac{1}{6!} + \frac{1}{71} = \frac{x}{8!}$, find x?

11. Evaluate $\frac{n!}{(n-1)!}$, when
(i) n=6, r=2

(ii) n=9, r=5.

12. How many 3 digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

13. How many 4 digit numbers are there with no digit repeated?

14. How many 3 digit even numbers can be made using the digits 1,2,3,4,5,6,7, if no digit is repeated?

15. Find the number of 4 digit numbers that can be formed using the digits 1,2,3,4,5, if no digit is repeated. How many of these will be even?

16. From a committee of 8 persons, in how many ways can we choose a chairman and vice chairman assuming one person can not hold more than one position?

17. Find n if ${}^{n-1}P_3 : {}^{n}P_4$= 1:9

18. Find r if
(i) ${}^5P_r = 2\ {}^6P_{r-1}$

(ii) ${}^5P_r = {}^6P_{r-1}$.

19. How many words, with or without meaning cane be formed using all the letters of the word EQUATION, using each letter exactly once?

20. How many words with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if
(i) 4 letters are used at a time

(ii) all letters are used at a time

(iii) all the letters used but first letter is a vowel?

21. In how many ways can the letters of the word PERMUTATIONS be arranged if the
(i) words start with P and end with S

(ii) vowels are together

(iii) there are always 4 letters between P and S?

22. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

23. If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$.
24. Determine n if
(i) ${}^{2n}C_3 : {}^nC_3 = 12{:}1$

(ii) ${}^{2n}C_3 : {}^nC_3 = 11{:}1$

25. How many chords can be drawn through 21 points on a circle?

26. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

27. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour?

28. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

29. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

30. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

31. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

32. How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

33. How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

34. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of
(i) exactly 3 girls?

(ii) at least 3 girls?

(iii) at most 3 girls?

35. If the different permutations of all the letters of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

36. How many 6 digit numbers can be formed from the digits 0,1,3,5,7 and 9 which are divisible by 10 and no digit is repeated?

37. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

38. In an examination, a question paper consists of 12 questions divided into two parts i.e., Part-I and Part-II, containing 5 and 7 questions respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

39. Determine the number of 5 card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king?

40. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

41. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will be join or none of them will join. In how many ways can the excursion party be chosen?

42. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

5.2 Exercises

1. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed.

2. Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

3. How many 2 digit even numbers can be formed from the digits 1,2,3,4,5 if the digits can be repeated?

4. Find the number of different signals that can be generated by arranging at least 2 flags in order(one below the other) on a vertical staff, if five different flags are available.

5. Evaluate
(i) 5!

(ii) 7!

(iii) 7!-5!

6. Compute the
(i) $\frac{7!}{5!}$

(ii) $\frac{12!}{(10!)(2!)}$

7. Evaluate the
(i) $\frac{n!}{r!(n-r)!}$, when n=5, r=2.

8. If $\frac{1}{8!} + \frac{1}{9!} = \frac{x}{10!}$, find x?

9. Find the number of permutations of the letters of the word ALLAHABAD.

10. How many 4 digit numbers can be formed by using the digits 1 to 9 if repetition of digits is not allowed?

11. How many numbers lying between 100 and 1000 can be formed with the digits 0,1,2,3,4,5, if the repetition of the digits is not allowed?

12. Find the value of n such that
(i) ${}^nP_5 = 42\ {}^nP_3, n > 4$

(ii) $\frac{{}^nP_4}{{}^{n-1}P_5} = \frac{5}{3}, n > 4$

13. Find r, if $5\ {}^4P_r = 6\ {}^5P_{r-1}$

14. Find the number of different 8 letter arrangements that can be made from the letters of the word DAUGHTER so that
(i) all vowels occur together

(ii) all vowels do not occur together

15. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

16. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,
(i) do the words start with P

(ii) do all the vowels always occur together

(iii) do the vowels never occur together

(iv) do the words begin with I and end in P?

17. If ${}^nC_9 = {}^nC_8$, find ${}^nC_{17}$.
18. A committee of 3 persons is to be constituted form a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

19. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these
(i) four cards are of the same suit

(ii) four cards belong to four different suits

(iii) are face cards

(iv) two are red cards and two are black cards

(v) cards are of the same colour?

20. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?

21. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
(i) no girl

(ii) at least one boy and one girl

(iii) at least 3 girls

22. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50^{th} word?

23. How many numbers greater than 1000000 can be formed by using the digits 1,2,0,2,4,2,4?

24. In how many ways can 5 girls and 3 boys be seated in a row so that no 2 boys are together?

6 Sequences and Series

6.1 Examples

1. Write the first three terms in each of the following sequences defined by the following:
(i) $an = 2n + 5$,
(ii) $a_n = \frac{n-3}{4}$
2. What is the 20^{th} term of the sequence defined by $a_n = (n-1)(2-n)(3+n)$?
3. Let the sequence a_n be defined as follows:
$a_1 = 1$,
$a_n = a_{n-1} + 2 for n \geq 2$.
Find first five terms and write corresponding series.
4. In an A.P. if m^{th} term is n and the n^{th} term is m, where $m \neq n$, find the p^{th} term.

5. If the sum of n terms of an A.P. is nP + $\frac{1}{2}n(n-1)Q$, where P and Q are constants, find the common differences.
6. The sum of n terms of two arithmetic progressions are in the ratio (3n + 8) : (7n + 15). Find the ratio of their 12^{th} terms.
7. The income of a person is Rs. 3,00,000, in the first year and he receives an increase of Rs.10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.
8. Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.
9. Find the 10 and n terms of the G.P. 5, 25,125,... .
10. Which term of the G.P., 2,8,32, ... up to n terms is 131072?
11. In a G.P., the 3^{rd} term is 24 and the 6^{th} term is 192.Find the 10^{th} term.
12. Find the sum of first n terms and the sum of first 5 terms of the geometric series $1 + \frac{2}{3} + \frac{4}{9} +$ ......
13. How many terms of the G.P.$3, \frac{3}{2}, \frac{3}{4}$, ... are needed to give the sum $\frac{3069}{512}$?
14. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the common ratio and the terms.
15. Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.
16. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.
17. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.
18. If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.
19. Find the sum to n terms of the series: 5 + 11 + 19 + 29 + 41...
20. Find the sum to n terms of the series whose n^{th} term is n(n+3).
21. If p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P. are in G.P, then show that (p - q), (q - r), (r - s) are also in G.P.
22. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, prove that x, y, z are in A.P.
23. If a, b, c, d and p are different real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$, then show that a, b, c and d are in G.P.
24. If p, q, r are in G.P. and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

6.2 Exercises

1. $a_n = n(n+2)$
2. $a_n = \frac{n}{n+1}$
3. $a_n = 2^n$
4. $a_n = \frac{2n-3}{6}$
5. $a_n = (-1)^{n-1}5^{n+1}$
6. $a_n = n\frac{n^2+5}{4}$

   Find the indicated terms in each of the sequences whose n^{th} terms are:
7. $a_n = 4n - 3; a_{17}, a_{24}$
8. $a_n = \frac{n^2}{2^n}; a_7$
9. $a_n = (-1)^{n-1}n^3; a_9$
10. $a_n = \frac{n(n-2)}{n+3}; a_{20}$.

    Write the first five terms of each of the sequences and obtain the corresponding series:
11. $a_1 = 3, a_n = 3a_{n-1} + 2$ for all n > 1
12. $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$
13. $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$
14. The fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}, n > 2$ Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4,5.
15. Find the sum of odd integers from 1 to 2001.
16. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.
17. In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.
18. How many terms of the A.P. -6,$-\frac{11}{2}, -5$....are needed to give the sum -25?
19. In an A.P., If p^{th} term is $\frac{1}{q}$ q^{th} term is $\frac{1}{p}$, prove that the sum of first pq $\frac{1}{2}$(pq+1), where $p \neq q$.
20. If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116. Find the last term.
21. Find the sum to n terms of the A.P., whose k^{th} term is 5k + 1.
22. If the sum of n terms of an A.P. is (pn + qn^2), where p and q are constants, find the common difference.
23. The sums of n terms of two arithmetic progressions are in the ratio 5n + 4 : 9n + 6. Find the ratio of their 18^{th} terms.

24. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first (p + q) terms.
25. Sum of the first p, q and r terms of an A.P. are a, b and c, respectively. Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$
26. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is (2m - 1) : (2n - 1).
27. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.
28. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.
29. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$is the A.M. between a and b, then find the value of n.
30. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m-1)^{th}$ numbers is 5 : 9. Find the value of m.
31. A man starts repaying a loan as first instalment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30^{th} instalment?
32. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.
33. Find the 20^{th} and n^{th} terms of the G.P.$\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, ...$
34. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.
35. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s, respectively. Show that $q^2 =$ ps.
36. The 4^{th} term of a G.P. is square of its second term, and the first term is -3. Determine its 7^{th} term.
37. Which term of the following sequences:
    (a) 2, 2, $\sqrt{2}$, 4,... is 128 ?
    (b) $\sqrt{3}, 3, 3\sqrt{3}, ...is 729$
    (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}...is\frac{1}{19683}$
38. For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.? Find the sum to indicated number of terms in each of the geometric progressions.
39. 0.15, 0.015, 0.0015,... 20 terms.
40. $\sqrt{7}$, $\sqrt{21}$, $\sqrt[3]{7}, ...n$ terms.
41. $1, -a, a^2, -a^2, a^3, ...n$ terms (if $a \neq -1$).
42. $x^3, x^5, x^7, ...n$ terms (if x $\neq \pm$ 1).
43. Evaluate $\sum_{k=1}^{11}(2+3^k)$.
44. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the 10 common ratio and the terms.
45. How many terms of G.P. 3, $3^2, 3^3$, ... are needed to give the sum 120?
46. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.
47. Given a G.P. with a = 729 and 7^{th} term 64, determine S_7.
48. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.
49. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.
50. Find the sum to n terms of the sequence, 8, 88, 888, 8888...
51. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32, and 128, 32, 8, 2, $\frac{1}{2}$
52. Show that the products of the corresponding terms of the sequences a, ar, $ar^2, ...ar^{n-1}$ and $A, AR, AR^2, ...AR^{n-1}$ form a G.P, and find the common ratio.
53. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4^{th} by 18.
54. If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}c^{p-q} = 1$.
55. If the first and the n^{th} term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.
56. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th} to (2n)^{th}$term is $\frac{1}{r^n}$
57. If a, b, c and d are in G.P. show that $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$.
58. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.
59. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.
60. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}) : (3-2\sqrt{2})$.
61. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.
62. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria

present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour ?

63. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?
64. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Find the sum to n terms of each of the series

65. $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$
66. $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + ...$
67. $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + ...$
68. $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + ...$
69. $5^2 + 6^2 + 7^2 + ... + 20^2$
70. $3 \times 8 + 6 \times 11 + 9 \times 14 + ...$
71. $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + ...$

Find the sum to n terms of the series

72. n (n+1) (n+4).
73. $n^2 + 2^n$
74. $(2n - 1)^2$
75. Show that the sum of $(m + n)^{th}$ and $(m - n)^{th}$ terms of an A.P. is equal to twice the m^{th} term.
76. If the sum of three numbers in A.P., is 24 and their product is 440, find the numbers.
77. Let the sum of n, 2n, 3n terms of an A.P. be S_1, S_2 and S_3 , respectively, show that $S_3 = 3(S_2 - S_1)$
78. Find the sum of all numbers between 200 and 400 which are divisible by 7.
79. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.
80. Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.
81. If f is a function satisfying f(x +y) = f(x) f(y) for all x, $y \in N$ such that f(1) = 3 and $\sum_{x=1}^{n}$ f(x) = 120 , find the value of n.
82. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.
83. The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of G.P.
84. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.
85. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.
86. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.
87. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}(x \neq 0)$, then show that a, b, c and d are in G.P.
88. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.
89. The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c, respectively. Show that (q - r)a + (r - p)b + (p - q)c = 0
90. If $a(\frac{1}{b} + \frac{1}{c}), b(\frac{1}{c} + \frac{1}{a}), c(\frac{1}{a} + \frac{1}{b})$ are in A.P., prove that a, b, c are in A.P.
91. If a, b, c, d are in G.P, prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.
92. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that (q + p) : (q - p) = 17:15.
93. The ratio of the A.M. and G.M. of two positive numbers a and b, is m : n. Show that a:b = $(m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$.
94. If a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that a, c, e are in G.P.
95. Find the sum of the following series up to n terms:
    (i) 5 + 55 +555 + ...
    (ii) .6 + .66 + .666+...
96. Find the 20^{th} term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + ... + n$ terms.
97. Find the sum of the first n terms of the series: 3 + 7 + 13 + 21 + 31 + ...
98. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.
99. Find the sum of the following series up to n terms: $\frac{1^3}{1} + \frac{1^3+2^2}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$
100. Show that $\frac{1\times2^2+2\times3^2+.....+n\times(n+1)^2}{1^2\times2+2^2\times3+.....+n^2\times(n+1)} = \frac{3n+5}{3n+1}$.
101. A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agrees to pay the balance in annual instalments of Rs 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?
102. Shamshad Ali buys a scooter for Rs 22000. He pays Rs 4000 cash and agrees to pay the

balance in annual instalment of Rs 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

103. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8^{th} set of letter is mailed.

104. A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually.Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

105. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

106. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on.It took 8 more days to finish the work. Find the number of days in which the work was completed.