Geometric Constructions through Python

G V V Sharma*

Contents

***Abstract*—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.**

1 Right Triangle

1.1 Draw $\triangle ABC$ right angled at $\mathbf{B}$ such that $AB = c = 6, BC = a = 8$.
Solution: The coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{1}$$

1.2 Let $\mathbf{D}, \mathbf{F}, \mathbf{F}$ be the mid points of BC, CA and AB respectively in $\triangle ABC$. Draw AD, BE and CF.
Solution:

$$\mathbf{D} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{E} = \frac{\mathbf{C}+\mathbf{A}}{2} = \frac{1}{2}\begin{pmatrix} a \\ c \end{pmatrix} \tag{3}$$

$$\mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix} 0 \\ c \end{pmatrix} \tag{4}$$

1.3 Draw AD, BE and CF.
1.4 Draw $\triangle DEF$ in the previous problem.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

2 Cirumcircle of Right Triangle

2.1 Find

$$\mathbf{P} = \mathbf{A} - \mathbf{E} \tag{5}$$
$$\mathbf{Q} = \mathbf{B} - \mathbf{E} \tag{6}$$
$$\mathbf{R} = \mathbf{C} - \mathbf{E} \tag{7}$$

Solution: Substituting $\mathbf{A}$ from (24) and $\mathbf{E}$ from (2)

$$\mathbf{P} = \begin{pmatrix} 0 \\ c \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} - k\begin{pmatrix} a \\ c \end{pmatrix} \tag{8}$$

where

$$k = \frac{1}{2} \tag{9}$$

Thus,

$$\mathbf{P} = \begin{pmatrix} 0 \\ c \end{pmatrix} - k\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} - \begin{pmatrix} ka \\ kc \end{pmatrix} = \begin{pmatrix} 0-(ka) \\ c-(kc) \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} 0-ka \\ c-kc \end{pmatrix} = \begin{pmatrix} -ka \\ c-kc \end{pmatrix} \tag{11}$$

Similarly,

$$\mathbf{Q} = -k\begin{pmatrix} a \\ c \end{pmatrix} \tag{12}$$

$$\mathbf{R} = \begin{pmatrix} a-ak \\ -kc \end{pmatrix} \tag{13}$$

2.2 Verify that

$$\mathbf{O} = \frac{\mathbf{P}+\mathbf{R}}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{14}$$

Solution:

$$\mathbf{P}+\mathbf{R} = \begin{pmatrix} -ka \\ c-kc \end{pmatrix} + \begin{pmatrix} a-ka \\ -kc \end{pmatrix} \tag{15}$$

$$= \begin{pmatrix} -ka+a-ka \\ c-kc-kc \end{pmatrix} = \begin{pmatrix} a-2ka \\ c-2kc \end{pmatrix} \tag{16}$$

Since $2k = 1$,

$$\mathbf{P} + \mathbf{R} = \begin{pmatrix} a - 2ka \\ c - 2kc \end{pmatrix} = \begin{pmatrix} a - a \\ c - c \end{pmatrix} = \mathbf{0} \tag{17}$$

2.3 Find OP^2.
Solution: Let

$$\begin{aligned}
OP^2 = \|\mathbf{O} - \mathbf{P}\|^2 &= \left\| \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -ka \\ c - kc \end{pmatrix} \right\|^2 \\
&= \left\| \begin{pmatrix} 0 - (-ka) \\ 0 - (c - kc) \end{pmatrix} \right\|^2 = \left\| \begin{pmatrix} ka \\ -c + kc \end{pmatrix} \right\|^2 \\
&= (ka)^2 + (-c + kc)^2 \\
&= k^2a^2 + (-c + kc)(-c + kc) \\
&= k^2a^2 + (-c)(-c + kc) + (kc)(-c + kc) \\
&= (-c)(-c) + (-c)(kc) + (kc)(-c) \\
&\quad + (kc)(kc) \\
&= k^2a^2 + c^2 - kc^2 - kc^2 + k^2c^2 \\
&= k^2a^2 + c^2 + (-1 - 1)kc^2 + k^2c^2 \\
&= k^2a^2 + c^2 - 2kc^2 + k^2c^2
\end{aligned}$$

$\because 2k = 1$,

$$k^2a^2 + c^2 - 2kc^2 + k^2c^2 = k^2a^2 + c^2 - c^2 + k^2c^2 \tag{18}$$

$$= k^2a^2 + k^2c^2 \tag{19}$$

2.4 Find OQ^2.
2.5 Find OR^2.
Solution: We have
2.6 Draw the circumcircle of $\triangle ABC$ with centre $\mathbf{O}$.
Solution: The radius of the circumcircle is

$$r = \frac{b}{2} = \frac{\sqrt{a^2 + c^2}}{2} \tag{20}$$

2.7 Draw a circle with centre

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{21}$$

and radius c.
2.8 For

$$\mathbf{C} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \tag{22}$$

find p, q such that

$$\mathbf{B} = \begin{pmatrix} p \\ q \end{pmatrix}, \tag{23}$$

2.9 Redraw $\triangle ABC$ with centre $\mathbf{A}$ and radius c.
2.10 Draw the tangent CD to the circle.
Solution: The coordinate

$$D = \begin{pmatrix} p \\ -q \end{pmatrix} \tag{24}$$

The following code draws the circle and tangents in Fig. 2.10

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
  len =10
  x_AB = np.zeros((2,len))
  lam_1 = np.linspace(0,1,len)
  for i in range(len):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB


#Triangle sides
a = 10
c = 6
b = np.sqrt(a**2-c**2)

p = (a**2 + c**2-b**2 )/(2*a)
q = np.sqrt(c**2-p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])
D = np.array([p,-q])


#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)
x_CD = line_gen(C,D)
```

```
#Plotting all lines
plt.plot(x_AB[0,:],x_AB[1,:],label='$AB$')
plt.plot(x_BC[0,:],x_BC[1,:],label='$BC$')
plt.plot(x_CA[0,:],x_CA[1,:],label='$CA$')
plt.plot(x_CD[0,:],x_CD[1,:],label='$CD$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) , '
    A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) ,
    'C')
plt.plot(D[0], D[1], 'o')
plt.text(D[0] * (1 - 0.2), D[1] * (1) , 'D')

#Plotting the circle

theta = np.linspace(0,2*np.pi,50)
x = c*np.cos(theta)
y = c*np.sin(theta)

plt.plot(x,y)

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')
#if using termux
plt.savefig('../figs/circle.pdf')
plt.savefig('../figs/circle.eps')
subprocess.run(shlex.split("termux-open ../
    figs/circle.pdf"))
#else
#plt.show()
```

2.11 Consider $\triangle ABC$ with $BC = a, CA = b$ and $AB = c$. Let

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ a \end{pmatrix} \tag{25}$$

Find p and q.
Solution: Since

$$p^2 + q^2 = c^2 \tag{26}$$
$$(p - a)^2 + q^2 = b^2, \tag{27}$$

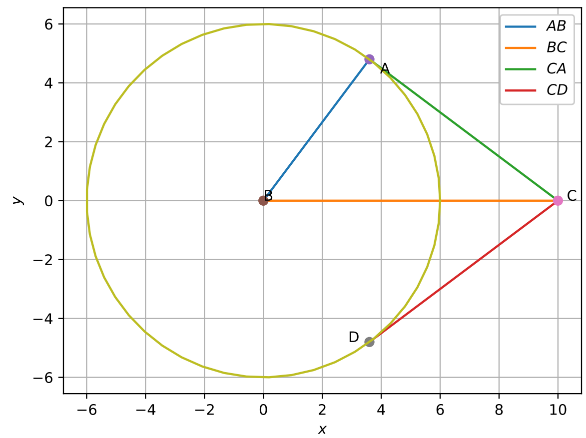

Fig. 2.10

we obtain

$$p = \frac{a^2 + c^2 - b^2}{2a}, q = \sqrt{c^2 - p^2} \tag{28}$$

2.12 Plot $\triangle ABC$ for $a = 8, b = 11$ and $c = 13$.
Solution: The following program plots $\triangle ABC$ in Fig. 2.12

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
  len =10
  x_AB = np.zeros((2,len))
  lam_1 = np.linspace(0,1,len)
  for i in range(len):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB


#Triangle sides
a = 8
b = 11
c = 13
```

```
p = (a**2 + c**2−b**2 )/(2*a)
q = np.sqrt(c**2−p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])


#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)

#Plotting all lines
plt.plot(x_AB[0,:],x_AB[1,:],label='$AB$')
plt.plot(x_BC[0,:],x_BC[1,:],label='$BC$')
plt.plot(x_CA[0,:],x_CA[1,:],label='$CA$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 − 0.1) , '
    A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 − 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 − 0.1) ,
    'C')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor

#if using termux
plt.savefig('../figs/triangle.pdf')
plt.savefig('../figs/triangle.eps')
subprocess.run(shlex.split("termux−open ../
    figs/triangle.pdf"))
#else
#plt.show()
```

2.13 Find $\mathbf{O}$ and R such that

$$R = OA = OB = OC \tag{29}$$

2.14 Let

$$x + y = ay + z = bz + x = c \tag{30}$$

Find x, y, z.

Solution: The given information can be expressed as the matrix equation

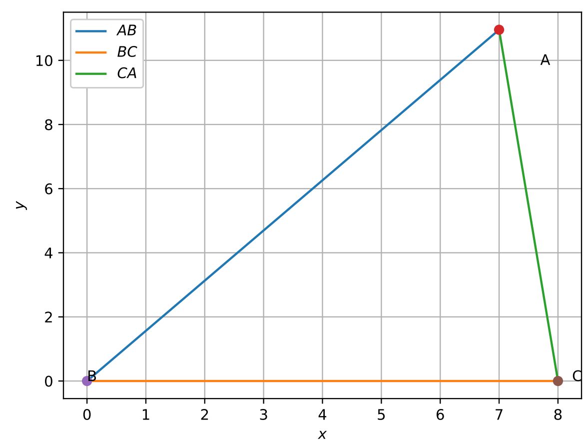

Fig. 2.12

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{31}$$

which can be solved to obtain x, y, z.

2.15 Find $\mathbf{D}, \mathbf{E}, \mathbf{F}$ such that

$$AE = AF = zBE = BD = xCD = CF = y \tag{32}$$

2.16 Find $\mathbf{I}$ such that

$$ID = IE = IF = r \tag{33}$$

3 Exercises

3.1 Draw a circle with centre $\mathbf{B}$ and radius 6. If $\mathbf{C}$ be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.
Solution: From the given information, in $\triangle ABC, AC \perp AB, a = 10$ and $c = 6$.

$$b = \sqrt{a^2 - c^2} \tag{34}$$

3.2 Write a program to compute p and q when $a = 8, b = 11$ and $c = 13$.

3.3 In $\triangle ABC$, a and $\angle B$ are known and $b + c = k$. If

$$b^2 = a^2 + c^2 - 2ac\cos B \tag{35}$$

find b and c.

Solution: From (35),

$$(k-c)^2 = a^2 + c^2 - 2ac\cos B \tag{36}$$

$$\implies k^2 - 2kc + c^2 = a^2 + c^2 - 2ac\cos B \tag{37}$$

$$\implies -2kc + 2ac\cos B = a^2 - k^2 \tag{38}$$

$$\implies 2c\,(a\cos B - k) = a^2 - k^2 \tag{39}$$

$$\text{or, } c = \frac{a^2 - k^2}{2\,(a\cos B - k)} \tag{40}$$

3.4 In $\triangle ABC$, $a = 7, \angle B = 75^\circ$ and $b+c = 13$. Find b and c and sketch $\triangle ABC$.

3.5 In $\triangle ABC$, $a = 8, \angle B = 45^\circ$ and $c - b = 3.5$. Sketch $\triangle ABC$.

3.6 In $\triangle ABC$, $a = 6, \angle B = 60^\circ$ and $b-c = 2$. Sketch $\triangle ABC$.

3.7 $\triangle ABC$ is right angled at **B**. If $a = 12$ and $b+c = 18$, find a, b, c and draw the triangle.
Solution: From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \tag{41}$$

3.8 In $\triangle ABC$, given that $a + b + c = 11, \angle B = 45^\circ$ and $\angle C = 45^\circ$, find a, b, c.
Solution: We have

$$a = b\cos C + c\cos B \tag{42}$$

$$b\sin C = c\sin B \tag{43}$$

$$a + b + c = 11 \tag{44}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \tag{45}$$

Solving the equivalent matrix equation gives the desired answer.

3.9 Draw $\triangle ABC$, given that $a+b+c = 11, \angle B = 30^\circ$ and $\angle C = 90^\circ$, find a, b, c.

3.10 Draw a square of side 3.

3.11 Draw a parallelogram with sides 12 and 5.

3.12 Draw a circle with centre **O** and diameter $AC = 6$. Choose any point B on the circle and draw $\triangle ABC$.

3.13 In $\triangle ABC$, $a = 8, b = 11, c = 13$. Find

$$R = \frac{a}{2\sin A}. \tag{46}$$

Let **D** be the mid point of BC. Find the point **O** such that $\triangle ODB$ is right angled at **D** and $OD = R$. Draw the circle with centre **O** and radius R.

3.14 Let

$$r = \frac{abc}{2\,(a+b+c)}. \tag{47}$$

and

$$IB = r\sqrt{\frac{2}{1-\cos B}}. \tag{48}$$

Draw a circle with centre **I** and radius r.

3.15 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

3.16 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

3.17 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of 60°.

3.18 Draw a line segment AB of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

3.19 Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^\circ$. BD is the perpendicular from **B** on AC. The circle through **B**, **C**, **D** is drawn. Construct the tangents from **A** to this circle.