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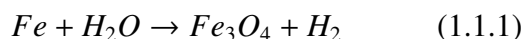
**Abstract**—This manual shows how to balance chemical equations using matrices.

Download python codes using

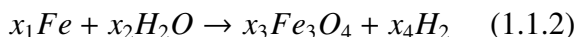
svn co <https://github.com/gadepall/school/trunk/training>

## 1 CHEMISTRY

- Express the problem of balancing the following chemical equation as a matrix equation.



**Solution:** Let the balanced version of (1.1.1) be



which results in the following equations

$$\begin{aligned} (x_1 - 3x_3) Fe &= 0 \\ (2x_2 - 2x_4) H &= 0 \\ (x_2 - 4x_3) O &= 0 \end{aligned} \quad (1.1.3)$$

which can be expressed as

$$\begin{aligned} x_1 + 0.x_2 - 3x_3 + 0.x_4 &= 0 \\ 0.x_1 + 2x_2 + 0.x_3 - 2x_4 &= 0 \\ 0.x_1 + x_2 - 4x_3 + 0.x_4 &= 0 \end{aligned} \quad (1.1.4)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \quad (1.1.5)$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad (1.1.6)$$

- Solve (1.1.2) by row reducing the matrix in (1.1.5).

**Solution:** (1.1.5) can be row reduced as follows

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \quad (1.2.1)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \quad (1.2.2)$$

$$\xrightarrow{R_1 \leftarrow 4R_1 - 3R_3} \begin{pmatrix} 4 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \quad (1.2.3)$$

$$\xrightarrow{\begin{matrix} R_1 \leftarrow \frac{1}{4} \\ R_3 \leftarrow -\frac{1}{4}R_3 \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{4} \end{pmatrix} \quad (1.2.4)$$

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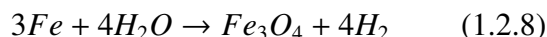
Thus,

$$x_1 = \frac{3}{4}x_4, x_2 = x_4, x_3 = \frac{1}{4}x_4 \quad (1.2.5)$$

$$(1.2.6)$$

$$\Rightarrow \mathbf{x} = x_4 \begin{pmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \quad (1.2.7)$$

upon substituting  $x_4 = 4$ . (1.1.2) then becomes



3. Verify your answer through a python code.

**Solution:** Execute

codes/chembal.py

## 2 MATHEMATICS

1. Find the equation of the plane  $P$  that contains the point  $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$  and is perpendicular to each of the planes

$$P_1 : \begin{pmatrix} 2 & 3 & -2 \end{pmatrix} \mathbf{x} = 5 \quad (2.1.1)$$

$$P_2 : \begin{pmatrix} 1 & 2 & -3 \end{pmatrix} \mathbf{x} = 8 \quad (2.1.2)$$

From (2.1.1), the normals to  $P_1, P_2$  are

$$\begin{aligned} \mathbf{n}_1 &= \begin{pmatrix} 2 & 3 & -2 \end{pmatrix} \\ \mathbf{n}_2 &= \begin{pmatrix} 1 & 2 & -3 \end{pmatrix} \end{aligned} \quad (2.1.3)$$

$\therefore P \perp P_1, P \perp P_2$ , if  $\mathbf{n}$  be the normal to  $P$ ,  $\mathbf{n} \perp \mathbf{n}_1, \mathbf{n} \perp \mathbf{n}_2$ , which can be expressed using (2.1.3) as

$$\begin{pmatrix} 2 & 3 & -2 \\ 1 & 2 & -3 \end{pmatrix} \mathbf{n} = 0 \quad (2.1.4)$$

Obtain  $\mathbf{n}$  using row reduction.

2. Verify your answer through a python code.  
3. Verify that  $\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$ .

## 3 PHYSICS

1. A force of  $\mathbf{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$  acts on a particle whose position vector is  $\mathbf{r} = \hat{i} - \hat{j} + \hat{k}$ . Find the torque about the origin given by  $\mathbf{F} \times \mathbf{r}$  using a matrix equation.  
2. Verify your answer using python.