# Optimization

## Problem 6

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# Quadratically constrained quadratic optimization problem

- Find the shortest distance between the line,

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \bar{x} = 0 \tag{1}$$

and the curve

$$\bar{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bar{x} - \begin{pmatrix} 1 & 0 \end{pmatrix} \bar{x} + 2 = 0 \tag{2}$$

# Distance of a point from a line

- Let the point be x, the distance of the point to line (1) is given by,

$$\frac{\left| \begin{pmatrix} 1 & -1 \end{pmatrix} x \right|}{\sqrt{2}} \tag{3}$$

# Frame as optimization problem

- $$\min_{x} \frac{\left| \begin{pmatrix} 1 & -1 \end{pmatrix} x \right|}{\sqrt{2}} \tag{4}$$

  with constraints

  $$x^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x - \begin{pmatrix} 1 & 0 \end{pmatrix} x + 2 = 0 \tag{5}$$

# Frame as convex optimization problem

- $$\min_x \frac{|\begin{pmatrix}1 & -1\end{pmatrix} x|}{\sqrt{2}} \tag{6}$$

  is similar to

  $$\min_x \frac{[\begin{pmatrix}1 & -1\end{pmatrix} x]^2}{2} \tag{7}$$

# Frame as convex optimization problem

- $$((1 \quad -1)\, x)^T ((1 \quad -1)\, x) = x^T \begin{pmatrix} 1 \\ -1 \end{pmatrix} (1 \quad -1)\, x \tag{8}$$

  thus,

  $$\min_x \frac{1}{2} x^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} x \tag{9}$$

# Frame as convex optimization problem

For using cvxpy, the given problem has to be reformulated as,

- $$\min_x \frac{1}{2} x^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} x \tag{10}$$

  with constraints

  $$x^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x - \begin{pmatrix} 1 & 0 \end{pmatrix} x + 2 \leq 0 \tag{11}$$

# Frame as convex optimization problem

The constraint is a convex constraint, because we are optimizing over a convex set which also includes the boundary.

# CVXPY code

```
#QCQP example
import cvxpy as cvx
from numpy import matrix, round, eye

#Create Variable
vect = cvx.Variable((2))

#Create constant vectors/matrices
P = matrix([[1,-1],[-1,1]])
Q = matrix([[0,0],[0,1]])
q = matrix([[1,0]])
c = 2;
```

# CVXPY code

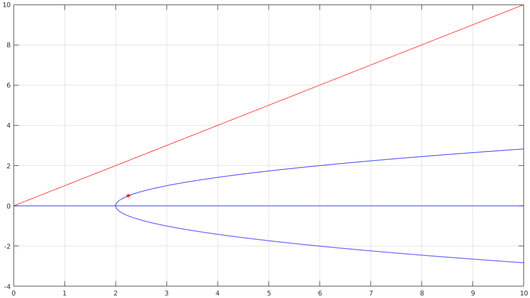
```
#Define the problem
f = 0.5*cvx.quad_form(vect, P)
obj = cvx.Minimize(f)
constraints =
[cvx.quad_form(vect, Q)- q*vect + c <= 0]
# #solution
cvx.Problem(obj, constraints).solve()
```

# Answer

Answer:

$X = [2.25, 0.5]$

Shortest distance $= 1.2374$



# Lagrangian Multipliers

- $$\min_x \frac{1}{2} x^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} x \tag{12}$$

  with constraints

  $$x^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x - \begin{pmatrix} 1 & 0 \end{pmatrix} x + 2 \leq 0 \tag{13}$$

# Lagrangian Multipliers

- Lagrangian is,

$$L(x,\lambda) = \frac{1}{2}x^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} x + \lambda \left( x^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x - \begin{pmatrix} 1 & 0 \end{pmatrix} x + 2 \right) \tag{14}$$

# Lagrangian Multipliers

- Lagrangian is,

$$\frac{\partial L(x,\lambda)}{\partial x} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} x + \lambda \left( 2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x - \begin{pmatrix} 1 & 0 \end{pmatrix} \right) = 0 \tag{15}$$

solving,

$$x = \begin{pmatrix} \lambda + 0.5 \\ 0.5 \end{pmatrix} \tag{16}$$

# Lagrangian Multipliers

- Lagrangian is,

$$\frac{\partial L(x,\lambda)}{\partial \lambda} = x^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x - \begin{pmatrix} 1 & 0 \end{pmatrix} x + 2 = 0 \tag{17}$$

Sub (16) in (17), and solving,

$$\frac{7}{4} - \lambda = 0 \tag{18}$$

$$\lambda = \frac{7}{4} \tag{19}$$

$$x = \begin{pmatrix} \frac{7}{4} + 0.5 \\ 0.5 \end{pmatrix} \tag{20}$$

X = [2.25,0.5]
Shortest distance = 1.2374