

# Geometric Constructions through Python

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## CONTENTS

1	Right Triangle	1
2	Circumcircle of Right Triangle	1
3	Tangent	2
4	Incircle	4
5	Exercises	4

**Abstract**—This manual shows how to construct geometric figures using Python. Exercises are based on NCERT math textbooks of Class 9 and 10.

## 1 RIGHT TRIANGLE

1.1 Draw  $\triangle ABC$  right angled at **B** such that  $AB = c = 6, BC = a = 8$ .

**Solution:** The coordinates are

$$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (1)$$

1.2 Let **D, F, F** be the mid points of  $BC, CA$  and  $AB$  respectively in  $\triangle ABC$ . Draw  $AD, BE$  and  $CF$ .

**Solution:**

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} = \frac{1}{2} \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{E} = \frac{\mathbf{C} + \mathbf{A}}{2} = \frac{1}{2} \begin{pmatrix} a \\ c \end{pmatrix} \quad (3)$$

$$\mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{1}{2} \begin{pmatrix} 0 \\ c \end{pmatrix} \quad (4)$$

1.3 Draw  $AD, BE$  and  $CF$ .

1.4 Draw  $\triangle DEF$  in the previous problem.

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## 2 CIRCUMCIRCLE OF RIGHT TRIANGLE

2.1 Show that

$$\mathbf{A} - \mathbf{E} = \begin{pmatrix} -ka \\ c - kc \end{pmatrix} \quad (5)$$

$$\mathbf{B} - \mathbf{E} = -k \begin{pmatrix} a \\ c \end{pmatrix} \quad (6)$$

$$\mathbf{C} - \mathbf{E} = \begin{pmatrix} a - ak \\ -kc \end{pmatrix} \quad (7)$$

where

$$k = \frac{1}{2} \quad (8)$$

**Solution:** Substituting **A** from (1) and **E** from (2)

$$\mathbf{A} - \mathbf{E} = \begin{pmatrix} 0 \\ c \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} - k \begin{pmatrix} a \\ c \end{pmatrix} \quad (9)$$

Thus,

$$\begin{aligned} \mathbf{A} - \mathbf{E} &= \begin{pmatrix} 0 \\ c \end{pmatrix} - k \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} - \begin{pmatrix} ka \\ kc \end{pmatrix} = \begin{pmatrix} 0 - (ka) \\ c - (kc) \end{pmatrix} \\ &= \begin{pmatrix} 0 - ka \\ c - kc \end{pmatrix} = \begin{pmatrix} -ka \\ c - kc \end{pmatrix} \end{aligned} \quad (10)$$

Similarly,  $\mathbf{B} - \mathbf{E}, \mathbf{C} - \mathbf{E}$  can be obtained.

2.2 Find  $EA^2$ .

**Solution:**

$$\begin{aligned}
 EA^2 &= \|\mathbf{A} - \mathbf{E}\|^2 = \left\| \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -ka \\ c - kc \end{pmatrix} \right\|^2 \\
 &= \left\| \begin{pmatrix} 0 - (-ka) \\ 0 - (c - kc) \end{pmatrix} \right\|^2 = \left\| \begin{pmatrix} ka \\ -c + kc \end{pmatrix} \right\|^2 \\
 &= (ka)^2 + (-c + kc)^2 \\
 &= k^2a^2 + (-c + kc)(-c + kc) \\
 &= k^2a^2 + (-c)(-c + kc) + (kc)(-c + kc) \\
 &= k^2a^2 + (-c)(-c) + (-c)(kc) + (kc)(-c) \\
 &\quad + (kc)(kc) \\
 &= k^2a^2 + c^2 - kc^2 - kc^2 + k^2c^2 \\
 &= k^2a^2 + c^2 + (-1 - 1)kc^2 + k^2c^2 \\
 &= k^2a^2 + c^2 - 2kc^2 + k^2c^2
 \end{aligned}$$

$$\because 2k = 1,$$

$$\begin{aligned}
 k^2a^2 + c^2 - 2kc^2 + k^2c^2 &= k^2a^2 + c^2 - c^2 + k^2c^2 \\
 &= k^2a^2 + k^2c^2 \\
 &= k^2(a^2 + c^2)
 \end{aligned}$$

2.3 Show that

$$EB^2 = k^2(a^2 + c^2) \quad (11)$$

2.4 Show that

$$EC^2 = k^2(a^2 + c^2) \quad (12)$$

2.5 Draw the circumcircle of  $\triangle ABC$  with centre  $\mathbf{E}$  and radius

$$R = EA = EB = EC = k\sqrt{a^2 + c^2} \quad (13)$$

### 3 TANGENT

3.1 In the right  $\triangle ABC$ , right angled at  $\mathbf{A}$ ,  $AC = b = 8$ ,  $AB = c = 6$  and

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad (14)$$

find

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix} \quad (15)$$

**Solution:**  $\because$

$$AB^2 = \|\mathbf{A} - \mathbf{B}\|^2, \quad (16)$$

$$\Rightarrow AB^2 = \left\| \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\|^2 \quad (17)$$

$$= \left\| \begin{pmatrix} p - 0 \\ q - 0 \end{pmatrix} \right\|^2 = \left\| \begin{pmatrix} p \\ q \end{pmatrix} \right\|^2 \quad (18)$$

$$\Rightarrow c^2 = p^2 + q^2 \quad (19)$$

$$\text{or, } c^2 - q^2 = p^2 \quad (20)$$

Similarly,

$$AC^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \left\| \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} \right\|^2 \quad (21)$$

$$= \left\| \begin{pmatrix} p - a \\ q - 0 \end{pmatrix} \right\|^2 = \left\| \begin{pmatrix} p - a \\ q \end{pmatrix} \right\|^2 \quad (22)$$

Thus,

$$\begin{aligned}
 AC^2 &= b^2 = (p - a)^2 + q^2 \\
 &= (p - a)(p - a) + q^2 \\
 &= (p)(p - a) + (-a)(p - a) + q^2 \\
 &= p^2 + p(-a) + (-a)(p) \\
 &\quad + (-a)(-a) + q^2
 \end{aligned} \quad (23)$$

$$\begin{aligned}
 &= p^2 - pa - pa + a^2 + q^2 \\
 &= p^2 + pa(-1 - 1) + a^2 + q^2 \\
 \Rightarrow b^2 &= p^2 - 2pa + a^2 + q^2 \\
 \Rightarrow b^2 - (-2pa + a^2 + q^2) &= p^2 \\
 \Rightarrow b^2 + 2pa - a^2 - q^2 &= p^2 \quad (24)
 \end{aligned}$$

From (20) and (24), equating the L.H.S,

$$\begin{aligned}
 c^2 - q^2 &= b^2 + 2pa - a^2 - q^2 \\
 \Rightarrow c^2 - q^2 - (b^2 + 2pa - a^2 - q^2) &= 0 \\
 \Rightarrow c^2 - q^2 - b^2 - 2pa + a^2 + q^2 &= 0 \\
 \Rightarrow c^2 - q^2 + q^2 - b^2 - 2pa + a^2 &= 0 \\
 \Rightarrow c^2 + q^2(-1 + 1) - b^2 - 2pa + a^2 &= 0 \\
 \Rightarrow c^2 + q^2(0) - b^2 - 2pa + a^2 &= 0 \\
 \Rightarrow c^2 - b^2 - 2pa + a^2 &= 0 \\
 \Rightarrow c^2 - b^2 + a^2 &= 2pa \\
 \Rightarrow \frac{c^2 - b^2 + a^2}{2a} &= p \quad (25)
 \end{aligned}$$

From (25),  $p$  is obtained.  $q$  can be obtained from (20) as

$$\begin{aligned} c^2 &= p^2 + q^2 \\ \Rightarrow c^2 - p^2 &= q^2 \\ \Rightarrow \sqrt{c^2 - p^2} &= q \end{aligned} \quad (26)$$

3.2 Draw a circle with centre **B** and radius  $c = 6$ .

3.3 Let

$$D = \begin{pmatrix} p \\ -q \end{pmatrix} \quad (27)$$

Draw  $\triangle ABC$  and  $\triangle ADC$ .

**Solution:** The following code draws the circle and tangents in Fig. 3.3

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
    len =10
    x_AB = np.zeros((2,len))
    lam_1 = np.linspace(0,1,len)
    for i in range(len):
        temp1 = A + lam_1[i]*(B-A)
        x_AB[:,i]= temp1.T
    return x_AB

#Triangle sides
a = 10
c = 6
b = np.sqrt(a**2-c**2)

p = (a**2 + c**2-b**2)/(2*a)
q = np.sqrt(c**2-p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])
D = np.array([p,-q])
```

```
#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)
x_CD = line_gen(C,D)

#Plotting all lines
plt.plot(x_AB[0:],x_AB[1:],label='$AB$')
plt.plot(x_BC[0:],x_BC[1:],label='$BC$')
plt.plot(x_CA[0:],x_CA[1:],label='$CA$')
plt.plot(x_CD[0:],x_CD[1:],label='$CD$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1) ,
        A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1) , 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1) ,
        'C')
plt.plot(D[0], D[1], 'o')
plt.text(D[0] * (1 - 0.2), D[1] * (1) , 'D')

#Plotting the circle

theta = np.linspace(0,2*np.pi,50)
x = c*np.cos(theta)
y = c*np.sin(theta)

plt.plot(x,y)

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')
#if using termux
plt.savefig('../figs/circle.pdf')
plt.savefig('../figs/circle.eps')
subprocess.run(shlex.split("termux-open ../figs/circle.pdf"))
#else
plt.show()
```

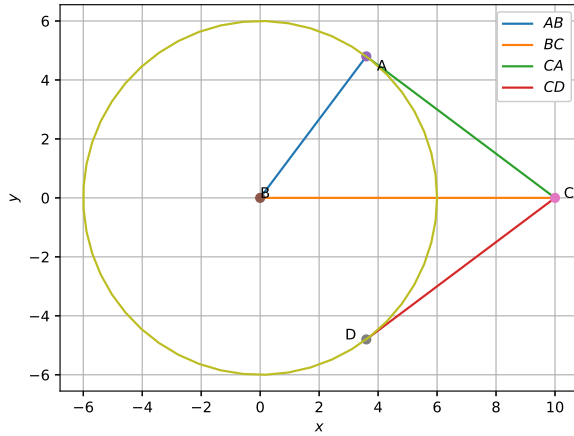


Fig. 3.3

## 4 INCIRCLE

4.1 Consider the right angled  $\triangle ABC$ , right angled at **B** with  $a = 8, b = 10, c = 6$ . Let

$$\begin{aligned} x + y &= a \\ y + z &= b \\ z + x &= c \end{aligned} \quad (28)$$

Show that

$$\begin{aligned} x &= \frac{a + c - b}{2} \\ y &= \frac{b + a - c}{2} \\ z &= \frac{c + b - a}{2} \end{aligned} \quad (29)$$

4.2 Find **D, E, F** such that

$$\begin{aligned} \mathbf{D} &= \frac{x\mathbf{C} + y\mathbf{B}}{x + y} \\ \mathbf{E} &= \frac{y\mathbf{A} + z\mathbf{C}}{y + z} \\ \mathbf{F} &= \frac{z\mathbf{B} + x\mathbf{A}}{z + x} \end{aligned} \quad (30)$$

4.3 Let

$$\mathbf{I} = \begin{pmatrix} p \\ q \end{pmatrix} \quad (31)$$

If

$$\mathbf{D} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}, \quad (32)$$

and

$$ID = IE \quad (33)$$

show that

$$p(d_1 - e_1) + q(d_2 - e_2) = \frac{e_1^2 + e_2^2 - d_1^2 - d_2^2}{2} \quad (34)$$

4.4 If

$$IE = IF, \quad (35)$$

show that

$$p(e_1 - f_1) + q(e_2 - f_2) = \frac{f_1^2 + f_2^2 - e_1^2 - e_2^2}{2} \quad (36)$$

4.5 Find **I** and  $r$  if

$$ID = IE = IF = r \quad (37)$$

## 5 EXERCISES

5.1 Plot  $\triangle ABC$  for  $a = 8, b = 11$  and  $c = 13$ .

**Solution:** The following program plots  $\triangle ABC$  in Fig. 5.1

```
#Code by GVV Sharma
#March 26, 2019
#released under GNU GPL
import numpy as np
import matplotlib.pyplot as plt

#if using termux
import subprocess
import shlex
#end if

#Generate line points
def line_gen(A,B):
    len =10
    x_AB = np.zeros((2,len))
    lam_1 = np.linspace(0,1,len)
    for i in range(len):
        temp1 = A + lam_1[i]*(B-A)
        x_AB[:,i]= temp1.T
    return x_AB

#Triangle sides
a = 8
b = 11
c = 13
```

```

p = (a**2 + c**2 - b**2)/(2*a)
q = np.sqrt(c**2 - p**2)

#Triangle vertices
A = np.array([p,q])
B = np.array([0,0])
C = np.array([a,0])

#Generating all lines
x_AB = line_gen(A,B)
x_BC = line_gen(B,C)
x_CA = line_gen(C,A)

#Plotting all lines
plt.plot(x_AB[0:],x_AB[1:],label='$AB$')
plt.plot(x_BC[0:],x_BC[1:],label='$BC$')
plt.plot(x_CA[0:],x_CA[1:],label='$CA$')

plt.plot(A[0], A[1], 'o')
plt.text(A[0] * (1 + 0.1), A[1] * (1 - 0.1), 'A')
plt.plot(B[0], B[1], 'o')
plt.text(B[0] * (1 - 0.2), B[1] * (1), 'B')
plt.plot(C[0], C[1], 'o')
plt.text(C[0] * (1 + 0.03), C[1] * (1 - 0.1), 'C')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor

#if using termux
plt.savefig('./figs/triangle.pdf')
plt.savefig('./figs/triangle.eps')
subprocess.run(shlex.split("termux-open ./figs/triangle.pdf"))
#else
plt.show()

```

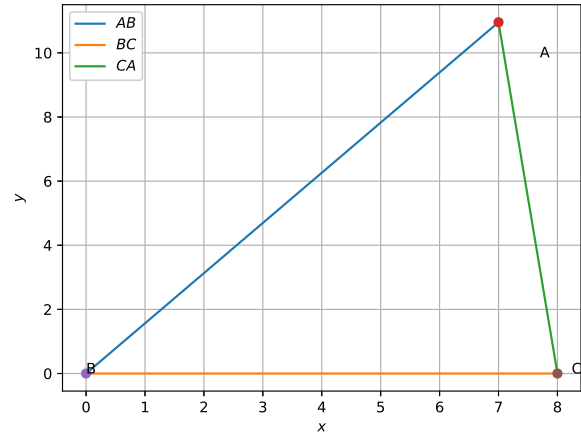


Fig. 5.1

$\triangle ABC, AC \perp AB, a = 10$  and  $c = 6$ .

$$b = \sqrt{a^2 - c^2} \quad (39)$$

5.4 Write a program to compute  $p$  and  $q$  when  $a = 8, b = 11$  and  $c = 13$ .

5.5 In  $\triangle ABC$ ,  $a$  and  $\angle B$  are known and  $b + c = k$ . If

$$b^2 = a^2 + c^2 - 2ac \cos B \quad (40)$$

show that

$$c = \frac{a^2 - k^2}{2(a \cos B - k)} \quad (41)$$

5.6 In  $\triangle ABC$ ,  $a = 7, \angle B = 75^\circ$  and  $b + c = 13$ . Find  $b$  and  $c$  and sketch  $\triangle ABC$ .

5.7 In  $\triangle ABC$ ,  $a = 8, \angle B = 45^\circ$  and  $c - b = 3.5$ . Sketch  $\triangle ABC$ .

5.8 In  $\triangle ABC$ ,  $a = 6, \angle B = 60^\circ$  and  $b - c = 2$ . Sketch  $\triangle ABC$ .

5.9  $\triangle ABC$  is right angled at **B**. If  $a = 12$  and  $b + c = 18$ , find  $a, b, c$  and draw the triangle.

**Solution:** From Baudhayana's theorem,

$$b^2 = a^2 + c^2 \quad (42)$$

5.10 In  $\triangle ABC$ , given that  $a + b + c = 11, \angle B = 45^\circ$  and  $\angle C = 45^\circ$ , find  $a, b, c$ .

**Solution:** We have

$$a = b \cos C + c \cos B \quad (43)$$

$$b \sin C = c \sin B \quad (44)$$

$$a + b + c = 11 \quad (45)$$

5.2 Find **O** and  $R$  such that

$$R = OA = OB = OC \quad (38)$$

5.3 Draw a circle with centre **B** and radius 6. If **C** be a point 10 units away from its centre, construct the pair of tangents  $AC$  and  $CD$  to the circle.

**Solution:** From the given information, in

resulting in the matrix equation

$$\begin{pmatrix} 1 & -\cos C & -\cos B \\ 0 & \sin C & -\sin B \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix} \quad (46)$$

Solving the equivalent matrix equation gives the desired answer.

- 5.11 Draw  $\triangle ABC$ , given that  $a+b+c = 11$ ,  $\angle B = 30^\circ$  and  $\angle C = 90^\circ$ , find  $a, b, c$ .
- 5.12 Draw a square of side 3.
- 5.13 Draw a parallelogram with sides 12 and 5.
- 5.14 Draw a circle with centre **O** and diameter  $AC = 6$ . Choose any point  $B$  on the circle and draw  $\triangle ABC$ .
- 5.15 In  $\triangle ABC$ ,  $a = 8, b = 11, c = 13$ . Find

$$R = \frac{a}{2 \sin A}. \quad (47)$$

Let **D** be the mid point of  $BC$ . Find the point **O** such that  $\triangle ODB$  is right angled at **D** and  $OD = R$ . Draw the circle with centre **O** and radius  $R$ .

- 5.16 Let

$$r = \frac{abc}{2(a+b+c)}. \quad (48)$$

and

$$IB = r \sqrt{\frac{2}{1 - \cos B}}. \quad (49)$$

Draw a circle with centre **I** and radius  $r$ .

- 5.17 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.
- 5.18 Draw a circle of radius 3 units. Take two points **P** and **Q** on one of its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points **P** and **Q**.
- 5.19 Draw a pair of tangents to a circle of radius 5 units which are inclined to each other at an angle of  $60^\circ$ .
- 5.20 Draw a line segment  $AB$  of length 8 units. Taking **A** as centre, draw a circle of radius 4 units and taking **B** as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
- 5.21 Let  $ABC$  be a right triangle in which  $a = 8, c = 6$  and  $\angle B = 90^\circ$ .  $BD$  is the perpendicular from **B** on  $AC$ . The circle through **B, C, D** is drawn. Construct the tangents from **A** to this circle.