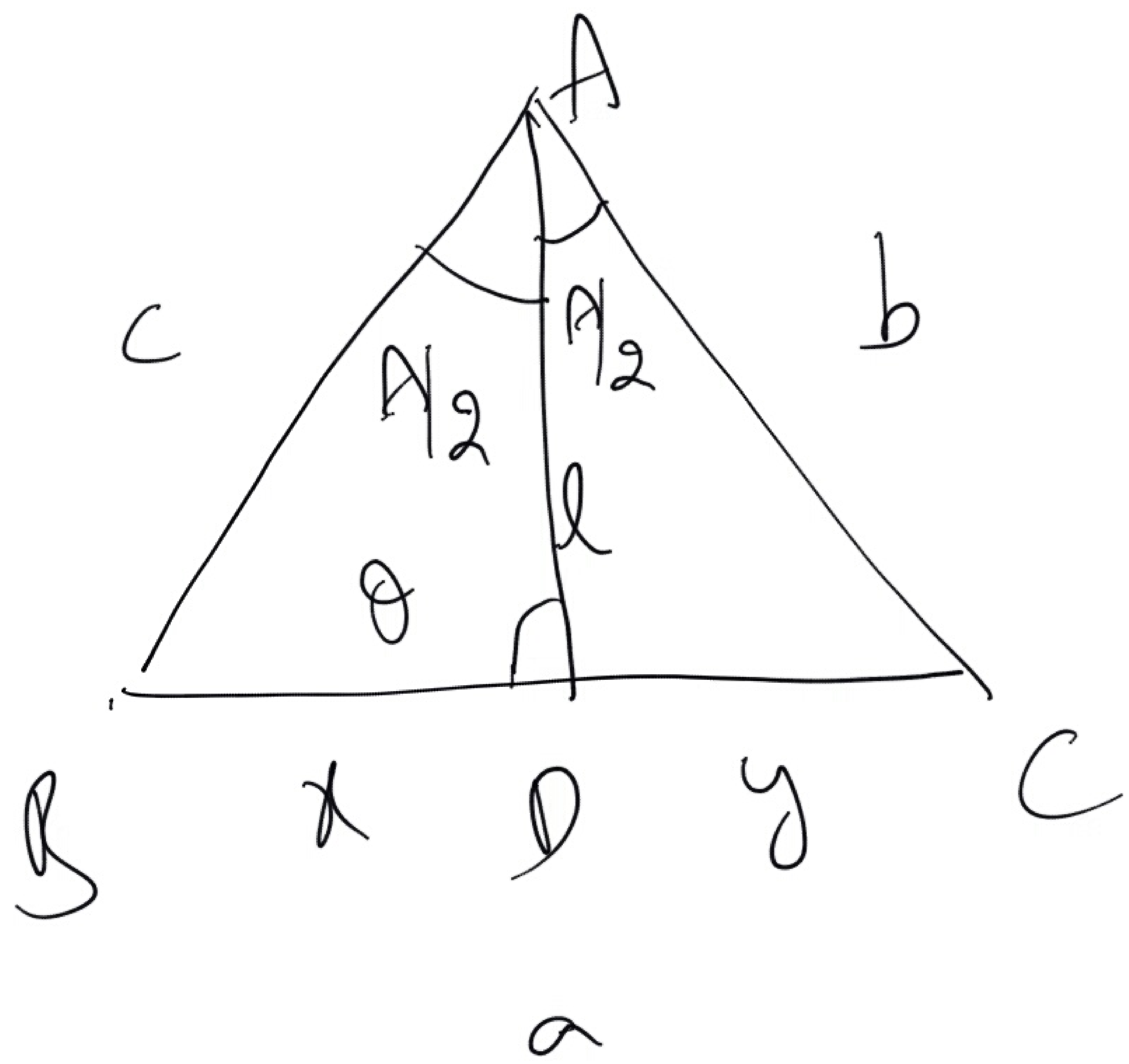
1.



Show that $\frac{x}{y} = \frac{c}{b}$.

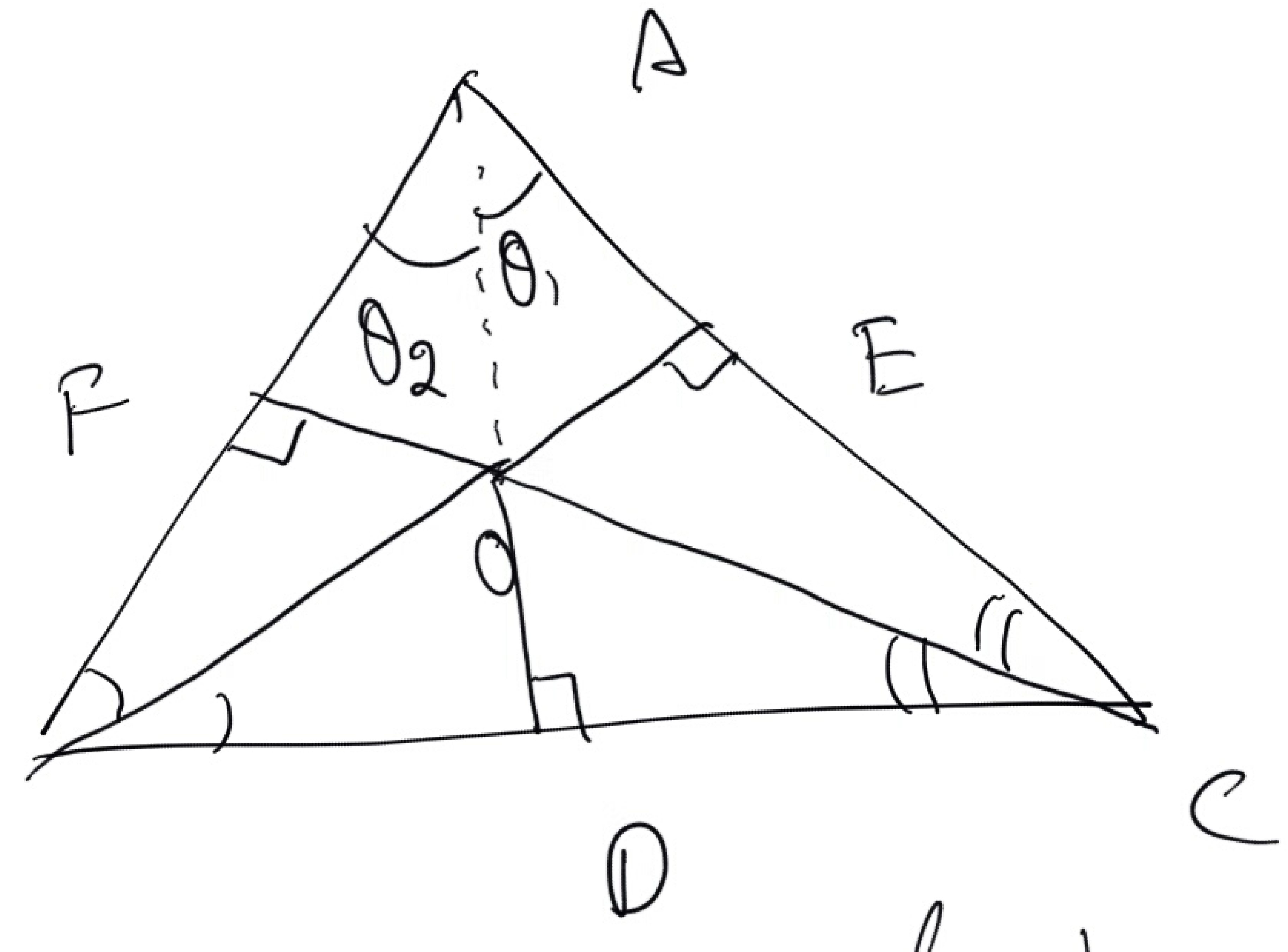
Proof:

$$\frac{y}{\sin\frac{A}{2}} = \frac{b}{\sin\theta}$$

$$\frac{x}{\sin A/2} = \frac{c}{\sin\theta}.$$

$$\Rightarrow \quad cy = bx \Rightarrow \frac{x}{y} = \frac{c}{b}.$$

2.



To show that AD is an angle bisector.

Proof: $OD = OC \sin \frac{C}{2}$ — (1)

$OE = OC \sin \frac{C}{2}$ — (2)

$\Rightarrow OD = OE$.

Similarly, $OD = OF$.

$\Rightarrow OD = OE = OF$.

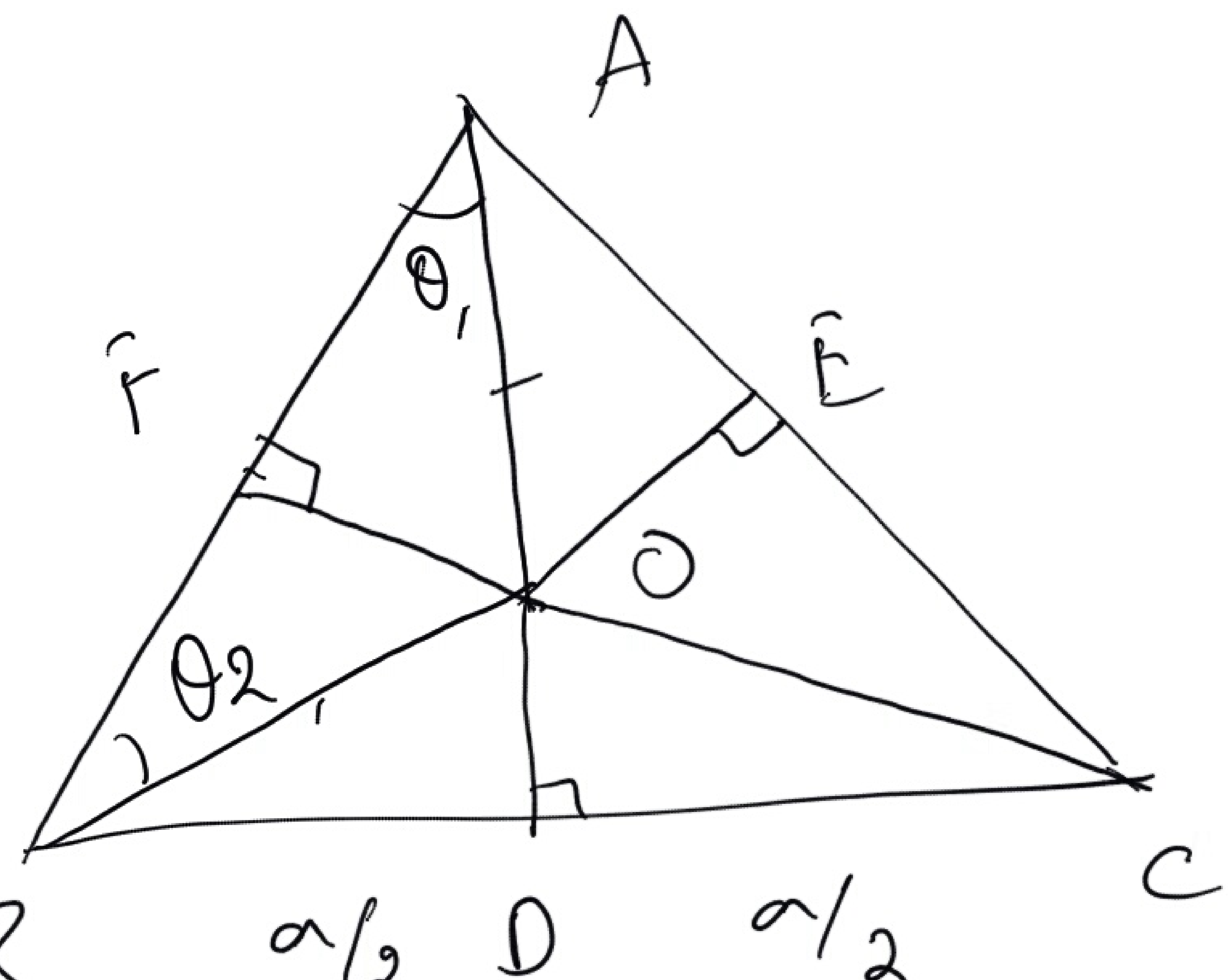
In $\triangle OEA$, $\sin \theta_1 = \dfrac{OE}{OA}$.

$\sin \theta_2 = \dfrac{OF}{OA} = \dfrac{OE}{OA}$ $(\because OE = OF)$

$\Rightarrow \sin \theta_1 = \sin \theta_2$ or $\underline{\theta_1 = \theta_2}$

Hence OA is the angle bisector.

3.



Perpendicular bisectors meet at a point

<u>Proof</u>: OD is a perp. bisector of BC.

$$OC^2 = OD^2 + \left(\frac{a}{2}\right)^2 \quad -(1)$$

$$OB^2 = OD^2 + \left(\frac{a}{2}\right)^2 \quad -(2)$$

using <u>Budhayana's</u> theorem.

Hence, $OB = OC$.

Similarly OE is the perp. bisector for AC. Hence, $OA = OC$.

Thus, $OA = OC = OB$.

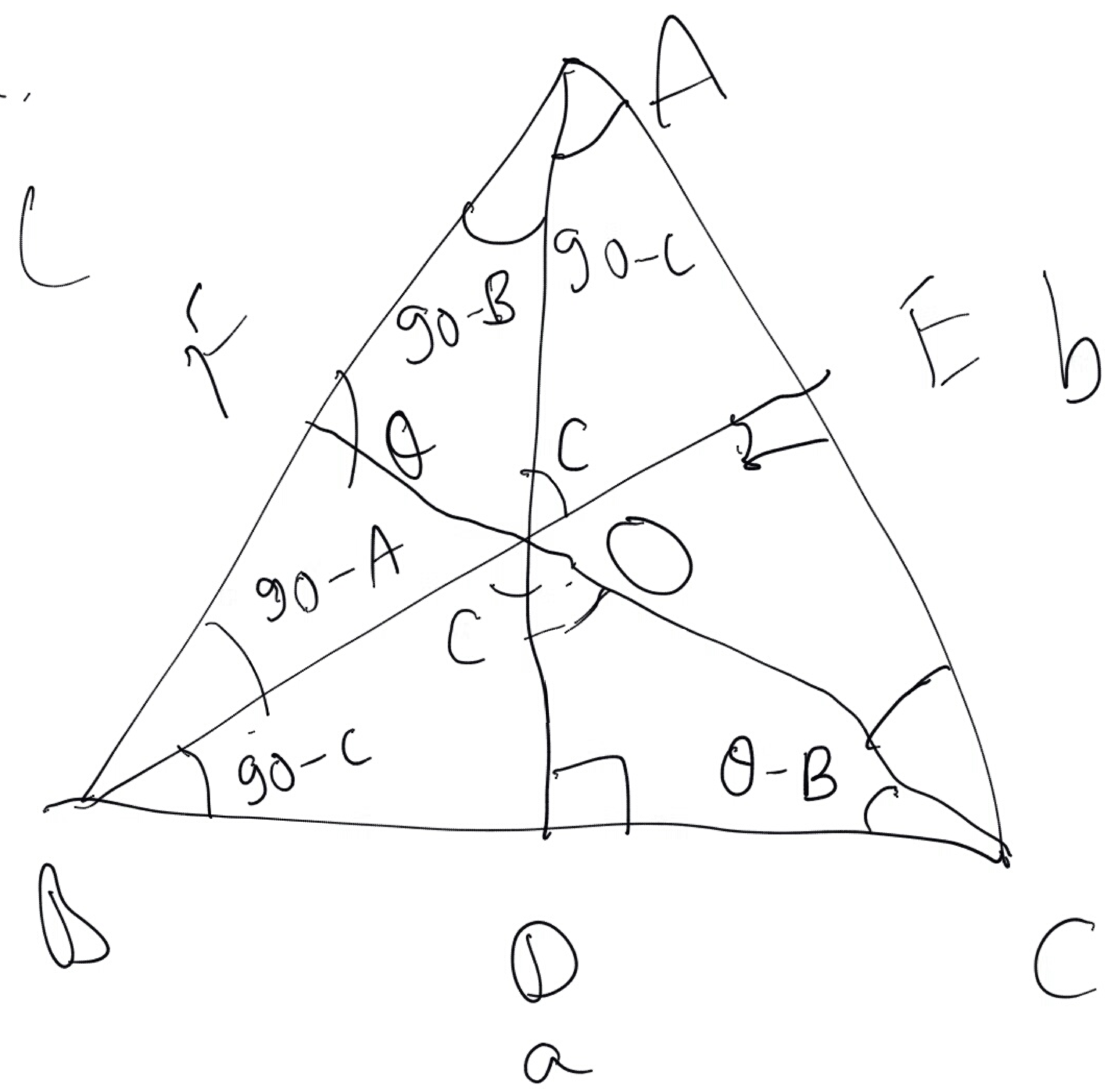
Draw $OF \perp AB$. Using the definition of Sine,

$OA = OB \Rightarrow \theta_1 = \theta_2$.

$AF = OA \cos\theta = OB \cos\theta = BF$.

proved.

4.



$$AO = \frac{a}{\tan A} \qquad \frac{CF}{\sin A} = \frac{b}{\sin\theta}$$

$$BO = \frac{b}{\tan B}$$

$$OD = OB\cos C = \frac{b\cos C}{\tan B}$$

$$OC = \frac{b\cos C}{\tan B}\operatorname{cosec}(\theta - B)$$

$$ar(\triangle AOB) = \frac{1}{2}\,\frac{ab\sin C}{\tan A\tan B}$$

$$= \frac{\triangle}{\tan A\tan B}$$

$$\frac{1}{2} AO\ BO \sin C$$

$$= \frac{1}{2} \frac{a}{\tan A} \frac{b}{\tan B} \sin C$$

$$= \frac{\Delta}{\tan A \tan B}.$$

$$c = OF' (\tan A + \tan B)$$

$$OF' = \frac{c}{\tan A + \tan B}.$$

$$\frac{1}{2} c\ OF \sin\theta = \frac{\Delta}{\tan A \tan B}. \qquad \text{— (1)}$$