JEE Problems in Linear Algebra: 2D

Abstract—**A collection of problems from JEE mains papers related to 2D coordinate geometry are available in this document. These problems should be solved using linear algebra.**

1. The sides of a rhombus ABC are parallel to the lines
$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} + 2 = 0 \tag{1}$$
$$\begin{pmatrix}7 & -1\end{pmatrix}\mathbf{x} + 3 = 0. \tag{2}$$
If the diagonals of the rhombus intersect at
$$\mathbf{P} = \begin{pmatrix}1\\2\end{pmatrix} \tag{3}$$
and the vertex **A** (different) from the origin is on the y-axis, then find the ordinate of A.
2. The points
$$\begin{pmatrix}h\\k\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}-3\\4\end{pmatrix} \tag{4}$$
lie on the line L_1. Given that $L_2 \perp L_1$ and passes through
$$\begin{pmatrix}h\\k\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}, \tag{5}$$
find $\frac{k}{h}$.
3. Tangent and normal are drawn at
$$\mathbf{P} = \begin{pmatrix}16\\16\end{pmatrix} \tag{6}$$
on the parabola
$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}16 & 0\end{pmatrix}\mathbf{x} = 0 \tag{7}$$
which intersect the axis of the parabola at **A** and **B** respectively. If **C** is the centre of the circle through the ponts **P A** and **B**, find $\tan CPB$.
4. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, find the equation of the common tangent to the two parabolas.
5. If the tangents drawn to the hyperbola
$$\mathbf{x}^T V\mathbf{x} + 1 = 0 \tag{8}$$
where
$$V = \begin{pmatrix}1 & 0\\0 & -4\end{pmatrix} \tag{9}$$
intersect the coordinate axes at the distinct points **A** and **B**, find the locus of the mid point of AB.
6. β is one of the angles between the normals to the ellipse
$$\mathbf{x}^T V\mathbf{x} = 9 \tag{10}$$
where
$$V = \begin{pmatrix}1 & 0\\0 & 3\end{pmatrix} \tag{11}$$
at the points
$$\begin{pmatrix}3\cos\theta\\ \sqrt{3}\sin\theta\end{pmatrix}, \begin{pmatrix}-3\sin\theta\\ \sqrt{3}\cos\theta\end{pmatrix}, \quad \theta \in \left(0, \frac{\pi}{2}\right), \tag{12}$$
then find $\frac{2\cot\beta}{\sin 2\theta}$.
7. Tangents drawn from the point $\begin{pmatrix}-8\\0\end{pmatrix}$ to the parabola
$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} + \begin{pmatrix}-8 & 0\end{pmatrix}\mathbf{x} = 0 \tag{13}$$
touch the parabola at **P** and **Q**. If **F** is the focus of the parabola, then find the area of $\triangle PFQ$.
8. A normal to the hyperbola
$$\mathbf{x}^T\begin{pmatrix}4 & 0\\0 & -9\end{pmatrix}\mathbf{x} = 36 \tag{14}$$
meets the coordinate axes x and y at **A** and **B** respectively. If the parallelogram $OABP$ is formed, find the locus of **P**.
9. Let **P** be the parabola
$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} + \begin{pmatrix}0 & 4\end{pmatrix}\mathbf{x} = 0 \tag{15}$$

Given that the distance of $\mathbf{P}$ from the centre of the circle

$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix}6\\0\end{pmatrix}\mathbf{x} + 8 = 0 \tag{16}$$

is minimum. Find the equation of the tangent to the parabola at $\mathbf{P}$.

10. The length of the latus rectum of an ellipse is 4 ad the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$. Find its eccentricity.
11. Find the eccentricity of an ellipse having centre at the origin, axes along the coordinate axes and passing through the points
$$\mathbf{P} = \begin{pmatrix}4\\-1\end{pmatrix}, \mathbf{Q} = \begin{pmatrix}-2\\2\end{pmatrix}. \tag{17}$$
12. $\begin{pmatrix}m & -1\end{pmatrix}\mathbf{x} + c = 0$ is the normal at a point on the parabola
$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} - \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} = 0 = 0 \tag{18}$$
whose focal distance is 8. Find $|c|$.
13. The common tangents to the parabola
$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} - \begin{pmatrix}0\\4\end{pmatrix}\mathbf{x} = 0 = 0 \tag{19}$$
intersect at the point $\mathbf{P}$. Find the distance of $\mathbf{P}$ from the origin.
14. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then find the area of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse.
15. A hyperbola passes through the point
$$\mathbf{P} = \begin{pmatrix}\sqrt{2}\\\sqrt{3}\end{pmatrix} \tag{20}$$
and has foci at $\begin{pmatrix}\pm 2\\0\end{pmatrix}$. Find the equation of the tangent to this hyperbola at $\mathbf{P}$.
16. Find the product of the perpendiculars drawn from the foci of the ellipse
$$\mathbf{x}^T\begin{pmatrix}25 & 0\\0 & 9\end{pmatrix}\mathbf{x} = 225 \tag{21}$$
upon the tangent to it at the point
$$\frac{1}{2}\begin{pmatrix}3\\5\sqrt{3}\end{pmatrix} \tag{22}$$
17. Find the equation of the normal to the hyperbola
$$\mathbf{x}^T\begin{pmatrix}9 & 0\\0 & -16\end{pmatrix}\mathbf{x} = 144 \tag{23}$$
drawn at the point
$$\begin{pmatrix}8\\3\sqrt{3}\end{pmatrix} \tag{24}$$
18. Let P be the point on the parabola
$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} - \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} = 0 \tag{25}$$
which is at a minimum distance from the centre C of the circle
$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix}0 & 12\end{pmatrix}\mathbf{x} = 1 \tag{26}$$
Find the equation of the circle passing through C and having its centre at $\left(P\right)$.
19. Find the eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half the distance between its foci.
20. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation
$$9e^2 - 18e + 5 = 0 \tag{27}$$
If
$$\mathbf{S} = \begin{pmatrix}5\\0\end{pmatrix} \tag{28}$$
is a focus and
$$\begin{pmatrix}5 & 0\end{pmatrix}\mathbf{x} = 9 \tag{29}$$
is the corresponding directrix of this hyperbola, then find $a^2 - b^2$.
21. $\mathbf{P}$ and $\mathbf{Q}$ are two distinct points on the parabola
$$\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x} - \begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} = 0 \tag{30}$$
with parameters t and t_1 respectively. If the normal at $\mathbf{P}$ passes through $\mathbf{Q}$, then find the minimum value of t_1^2.
22. A hyperbola whose transverse axis is along the

major axis of the conic

$$\mathbf{x}^T V \mathbf{x} = 51 \tag{31}$$

where

$$V = \begin{pmatrix} 3 & 0 \\ 0 & 27 \end{pmatrix} \tag{32}$$

and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, which of the following points doesnot lie on it?

a) $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
b) $\begin{pmatrix} \sqrt{5} \\ 2\sqrt{2} \end{pmatrix}$
c) $\begin{pmatrix} \sqrt{10} \\ 2\sqrt{3} \end{pmatrix}$
d) $\begin{pmatrix} 5 \\ 2\sqrt{3} \end{pmatrix}$

23. A tangent at a point on the ellipse

$$\mathbf{x}^T V \mathbf{x} = 51 \tag{33}$$

where

$$V = \begin{pmatrix} 3 & 0 \\ 0 & 27 \end{pmatrix} \tag{34}$$

meets the coordinate axes at **A** and **B**. If **O** be the origin, find the minimum area of $\triangle OAB$.

24. The eccentricity of the standard hyperbola passing through the point

$$\mathbf{P} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{35}$$

is 2. Find the equation of the tangent to the hyperbola at **P**.

25. The tangent and normal lines at the point

$$\mathbf{P} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \tag{36}$$

to the circle

$$\|\mathbf{x}\| = 2 \tag{37}$$

and the $x-$axis form a triangle. Find the area of this triangle.

26. An ellipse has centre at the origin and the difference of the lengths of its major and minor axis is 10. If one of its foci is at

$$\mathbf{F} = \begin{pmatrix} 0 \\ 5\sqrt{3} \end{pmatrix}, \tag{38}$$

find the length of its latus rectum.

27. The tangent to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{39}$$

at the point where it intersects the circle

$$\|\mathbf{x}\| = \sqrt{5} \tag{40}$$

in the first quadrant, passes through the point

a) $\begin{pmatrix} -\frac{1}{3} \\ \frac{4}{3} \end{pmatrix}$
b) $\begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$
c) $\begin{pmatrix} \frac{3}{4} \\ \frac{7}{4} \end{pmatrix}$
d) $\begin{pmatrix} -\frac{1}{4} \\ \frac{1}{2} \end{pmatrix}$

28. Find the shortest distance between the line

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 0 \tag{41}$$

and the curve

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} + 2 = 0 \tag{42}$$

29. Find the sum of the squares of the lengths of the chords intercepted on the circle

$$\|\mathbf{x}\| = 4 \tag{43}$$

by the lines

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = n \tag{44}$$

30. The tangents on the ellipse

$$\mathbf{x}^T \mathbf{V} \mathbf{x} = 8 \tag{45}$$

where

$$\mathbf{V} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \tag{46}$$

at the points

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix} \tag{47}$$

are perpendicular to each other. Find a^2.

31. Let

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{48}$$

Find the locus of **P** such that the perimeter of $\triangle AOP = 4$.

32. Find the area of the smaller of the two circles

that touch the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{49}$$

at the point

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{50}$$

and the x-axis.

33. The lines

$$\begin{pmatrix} 1 & a-1 \end{pmatrix} \mathbf{x} = 1 \tag{51}$$

$$\begin{pmatrix} 2 & a^2 \end{pmatrix} \mathbf{x} = 1 \tag{52}$$

are perpendicular. Find the distance of their point of intersection from the origin.

34. The common tangent to the circles

$$\|\mathbf{x}\| = 2 \tag{53}$$

$$\left\| \mathbf{x} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\| = 7 \tag{54}$$

passes through the point

a) $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

b) $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$

c) $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

d) $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

35. The tangent to the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{55}$$

at a point

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \beta > 0 \tag{56}$$

is also a tangent to the ellipse

$$\mathbf{x}^T \mathbf{V} \mathbf{x} = 1 \tag{57}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}. \tag{58}$$

Find α.

36. A rectangle is inscribed in a circle with a diameter lying along the line

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = -7 \tag{59}$$

If two adjacent vertices of the rectangle are

$$\begin{pmatrix} -8 \\ 5 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{60}$$

Find the area of the rectangle.

37. Find the direction vector of a line passing through

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \tag{61}$$

and intersecting the line

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 7 \tag{62}$$

at a distance of 4 units from $\mathbf{P}$.

38. The line

$$\begin{pmatrix} -m & 1 \end{pmatrix} \mathbf{x} = 7\sqrt{3} \tag{63}$$

is normal to the hyperbola

$$\mathbf{x}^T \mathbf{V} \mathbf{x} = 72 \tag{64}$$

where

$$V = \begin{pmatrix} 3 & 0 \\ 0 & -4 \end{pmatrix} \tag{65}$$

Find m.

39. One end of the focal chord of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 16 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{66}$$

is at

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix}. \tag{67}$$

Find the length of this focal chord.

40. Let $\mathbf{P}$ be the point on the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{68}$$

such that OP makes an angle $\frac{\pi}{6}$ with the $x-$axis, where $\mathbf{O}$ is the origin. A normal is drawn to the parabola intersecting the axis of the parabolat $\mathbf{Q}$. If $\mathbf{S}$ is the focus of the parabola, then find SQ.

41. Let ellipse

$$\mathbf{x}^T \mathbf{V} \mathbf{x} = 16 \tag{69}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 16 \end{pmatrix} \tag{70}$$

be inscribed in a rectangle whose sides are parallel to the coordinate axes. If the rectangle is inscribed in another ellipse that passes through the point

$$\begin{pmatrix} 16 \\ 0 \end{pmatrix}, \tag{71}$$

find the equation of the outer ellipse.

42. Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \tag{72}$$

be the opposite vertices of a rectangle. The other two vertices

$$\mathbf{B} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{D} = \begin{pmatrix} c \\ d \end{pmatrix}, \tag{73}$$

lie on the line

$$\begin{pmatrix} -2 & 1 \end{pmatrix} \mathbf{x} = k \tag{74}$$

for some k. Find the value of

$$(a + b)(c + d) \tag{75}$$

43. The abscissae of $\mathbf{A}, \mathbf{B}$ are roots of

$$x^2 + 2x - 4 = 0 \tag{76}$$

and their ordinates are roots of

$$y^2 + 4y - 16 = 0 \tag{77}$$

If AB is a diameter of a circle, find its radius.

44. A line is drawn from a point

$$\mathbf{P} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \tag{78}$$

to cut the circle

$$\|\mathbf{x}\| = 2 \tag{79}$$

at the points $\mathbf{A}$ and $\mathbf{B}$. Find $PA.PB$.

45. The normal to the ellipse

$$\mathbf{x}^T \mathbf{V} \mathbf{x} = 144 \tag{80}$$

where

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix} \tag{81}$$

at a point $\mathbf{P}$ has direction vector

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}. \tag{82}$$

If this normal intersects the major axis of the ellipse at at a point $\mathbf{A}$, find PA^2.