# Computational Approach to School Mathematics

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Contents



***Abstract*—This is a problem set related to continuous maths based on JEE question papers**

## 1 Trigonometry

1. Suppose
$$\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx \tag{1.0.1.1}$$
is an identity in x, where $C_0, C_1, ......C_n$ are constants, and $C_n \neq 0$ then find the value of n.
2. Find the solution set of the system of equations
$$x + y = \frac{2\pi}{3} \tag{1.0.2.1}$$
$$\cos x + \cos y = \frac{3}{2}, \tag{1.0.2.2}$$
where x and y are real.
3. Find the set of all x in the interval[0,$\pi$] for which
$$2\sin^2 x - 3\sin x + 1 \geq 0 \tag{1.0.3.1}$$
4. The sides of a triangle inscribed in a given circle subtend angles $\alpha, \beta$ and $\gamma$ at the centre. Find the minimum value of the arithmetic mean of $\cos(\alpha + \frac{\pi}{2}), \cos(\beta + \frac{\pi}{2})$ and $\cos(\gamma + \frac{\pi}{2})$.
5. Find the value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$.
6. If
$$K = \sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18}), \tag{1.0.6.1}$$
then find the numerical value of K ?
7. If A>0,B>0 and
$$A + B = \frac{\pi}{3}, \tag{1.0.7.1}$$
then find the maximum value of tan A tan B.
8. Find the general value of $\theta$ satisfying the equation
$$\tan^2\theta + \sec 2\theta = 1. \tag{1.0.8.1}$$
9. Find the real roots of the equation
$$\cos^7 x + \sin^4 x = 1 \tag{1.0.9.1}$$
in the interval $(-\pi, \pi)$.
10. If $\tan\theta = -\frac{4}{3}$, then find $\sin\theta$.
11. If $\alpha + \beta + \gamma = 2\pi$ then
    a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
    b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

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c) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

d) None of these

12. Given

$$A=\sin^2\theta+\cos^4\theta \tag{1.0.12.1}$$

then for all real values of $\theta$

a) $1\le A\le 2$
b) $\frac{3}{4}\le A\le 1$
c) $\frac{13}{16}\le A\le 1$
d) $\frac{3}{4}\le A\le\frac{13}{16}$

13. The equation

$$2\cos^2\frac{x}{2}\sin^2x=x^2+x^{-2};0<x<\frac{\pi}{2} \tag{1.0.13.1}$$

has

a) no real solution
b) One real solution
c) more than the one solution
d) none of these

14. The general solution of the trigonometric equation

$$\sin x+\cos x=1 \tag{1.0.14.1}$$

is given by :

a) x $=2n\pi;n=0,\pm1,\pm2...$
b) x $=2n\pi+\frac{\pi}{2};n=0,\pm1,\pm2...$
c) x $=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4};n=0,\pm1,\pm2...$
d) none of these

15. The value of expression $\sqrt{3}cosec20^\circ-\sec 20^\circ$ is equal to

a) 2
b) $\frac{2\sin 20^\circ}{\sin 40^\circ}$
c) 4
d) $\frac{4\sin 20^\circ}{\sin 40^\circ}$

16. The general solution of

$$\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x \tag{1.0.16.1}$$

is

a) $n\pi+\frac{\pi}{8}$
b) $\frac{n\pi}{2}+\frac{\pi}{8}$
c) $(-1)^n\frac{n\pi}{2}+\frac{\pi}{8}$
d) $2n\pi+\cos^{-1}\frac{3}{2}$

17. The equation

$$(\cos p-1)x^2+(\cos p)x+\sin p=0 \tag{1.0.17.1}$$

In the variable x, has real roots. Then p can take any value in the interval

a) $(0,2\pi)$
b) $(-\pi,0)$
c) $(-\frac{\pi}{2},\frac{\pi}{2})$
d) $(0,\pi)$

18. Number of solutions of the equation

$$\tan x+\sec x=2\cos x \tag{1.0.18.1}$$

lying in the interval $[0,2\pi]$ is :

a) 0
b) 1
c) 2
d) 3

19. Let $0<x<\frac{\pi}{4}$ then $(\sec 2x-\tan 2x)$ equals

a) $\tan(x-\frac{\pi}{4})$
b) $\tan(\frac{\pi}{4}-x)$
c) $\tan(x+\frac{\pi}{4})$
d) $\tan^2(x+\frac{\pi}{4})$

20. Let n be a positive integer such that $\sin\frac{\pi}{2n}+\cos\frac{\pi}{2n}=\frac{\sqrt{n}}{2}$.Then

a) $6\le n\le 8$
b) $4<n\le 8$
c) $4\le n\le 8$
d) $4<n<8$

21. If $\omega$ is an imaginary cube root of unity then the value of $\sin\{(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\}$ is

a) $-\frac{\sqrt{3}}{2}$
b) $-\frac{1}{\sqrt{2}}$
c) $\frac{1}{\sqrt{2}}$
d) $\frac{\sqrt{3}}{2}$

22. $3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6x+\cos^6x)=$

a) 11
b) 12
c) 13
d) 14

23. The general values of $\theta$ satisfying equation

$$2\sin^2\theta-3\sin\theta-2=0 \tag{1.0.23.1}$$

is

a) $n\pi+(-1)^n\frac{\pi}{6}$

b) $n\pi + (-1)^n \frac{\pi}{2}$
c) $n\pi + (-1)^n \frac{5\pi}{6}$
d) $n\pi + (-1)^n \frac{7\pi}{6}$

24. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if
a) $x + y \neq 0$
b) $x = y, x \neq 0$
c) $x = y$
d) $x \neq 0, y \neq 0$

25. In a triangle PQR, $\angle R = \pi/2$.If $\tan(\frac{P}{2})$ and $\tan(\frac{Q}{2})$ are the roots of the equation
$$ax^2 + bx + c = 0(a \neq 0) \quad (1.0.25.1)$$
then
a) a+b=c
b) b+c=a
c) a+c=b
d) b=c

26. Let f($\theta$) =$\sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is
a) $\geq 0$ only when $\theta \geq 0$
b) $\leq 0$ for all real $\theta$
c) $\geq 0$ for all real $\theta$
d) $\leq 0$ only when $\theta \leq 0$

27. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$=0 in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is
a) 0
b) 2
c) 1
d) 3

28. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)...(\cos\alpha_n)$, under the restrictions, $0 \leq \alpha_1, \alpha_2, .....\alpha_n \leq \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)...(\cot\alpha_n) = 1$ is
a) $\frac{1}{2^{\frac{n}{2}}}$
b) $\frac{1}{2^n}$
c) $\frac{1}{2n}$
d) 1

29. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$,then $\tan\alpha$ equals
a) $2(\tan\beta + \tan\gamma)$
b) $\tan\beta + \tan\gamma$
c) $\tan\beta + 2\tan\gamma$
d) $2\tan\beta + \tan\gamma$

30. The number of integral values of k for which the equation
$$7\cos x + 5\sin x = 2k + 1 \quad (1.0.30.1)$$
has a solution is
a) 4
b) 8
c) 10
d) 12

31. Given both $\theta$ and $\phi$ are acute angles and $\sin\theta = \frac{1}{2}, \cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to
a) $(\frac{\pi}{3}, \frac{\pi}{2}]$
b) $(\frac{\pi}{2}, \frac{2\pi}{3})$
c) $(\frac{2\pi}{3}, \frac{5\pi}{6}]$
d) $(\frac{5\pi}{6}, \pi]$

32. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ where $\alpha, \beta\epsilon[-\pi, \pi]$. Pairs of $\alpha, \beta$which satisfy both the equations is/are
a) 0
b) 1
c) 2
d) 4

33. The values of $\theta\epsilon(0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are
a) $(0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$
b) $(\frac{\pi}{8}, \frac{5\pi}{6})$
c) $(0, \frac{\pi}{8}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$
d) $(\frac{41\pi}{48}, \pi)$

34. Let $\theta\epsilon(0, \frac{\pi}{4})$ and $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then
a) $t_1 > t_2 > t_3 > t_4$
b) $t_4 > t_3 > t_1 > t_2$
c) $t_3 > t_1 > t_2 > t_4$
d) $t_2 > t_3 > t_1 > t_4$

35. The number of solutions of the pair of equations
$$2\sin^2\theta - \cos 2\theta = 0 \quad (1.0.35.1)$$
$$2\cos^2\theta - 3\sin\theta = 0 \quad (1.0.35.2)$$
in the interval [0,2$\pi$] is
a) zero
b) one
c) two
d) four

36. For $x\epsilon(0, \pi)$, the equation
$$\sin x + 2\sin 2x - \sin 3x = 3 \quad (1.0.36.1)$$

has

a) infinitely many solutions
b) three solutions
c) one solution
d) no solution

37. Let $S = \{x\epsilon(-\pi,\pi) : x \neq 0, \pm\frac{\pi}{2}\}$. The sum of all distinct solutions of the equation

$$\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0 \tag{1.0.37.1}$$

in the set S is equal to

a) $-\frac{7\pi}{9}$
b) $-\frac{2\pi}{9}$
c) 0
d) $\frac{5\pi}{9}$

38. The value of $\sum_{k=1}^{13} \frac{1}{\sin(\frac{\pi}{4}+\frac{(k-1)\pi}{6})\sin(\frac{\pi}{4}+\frac{k\pi}{6})}$ is equal to

a) $3-\sqrt{3}$
b) $2(3-\sqrt{3})$
c) $2(\sqrt{3}-1)$
d) $2(2-\sqrt{3})$

39. $(1+\cos\frac{\pi}{8})(1+\cos\frac{3\pi}{8})(1+\cos\frac{5\pi}{8})(1+\cos\frac{7\pi}{8})$ is equal to

a) $\frac{1}{2}$
b) $\cos(\frac{\pi}{8})$
c) $\frac{1}{8}$
d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

40. The expression $3[\sin^4(\frac{3\pi}{2}-\alpha)+\sin^4(3\pi+\alpha]-2[\sin^6(\frac{\pi}{2}+\alpha)+\sin^6(5\pi-\alpha)]$ is equal to

a) 0
b) 1
c) 3
d) $\sin 4\alpha + \cos 6\alpha$
e) none of these

41. The number of all possible triplets $(a_1, a_2, a_3)$ such that

$$a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0 \tag{1.0.41.1}$$

for all x is

a) zero
b) one
c) three
d) infinite
e) none

42. The values of $\theta$ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \tag{1.0.42.1}$$

are

a) $\frac{7\pi}{24}$
b) $\frac{5\pi}{24}$
c) $\frac{11\pi}{24}$
d) $\frac{\pi}{24}$

43. Let

$$2\sin^2 x + 3\sin x - 2 > 0 \tag{1.0.43.1}$$
$$x^2 - x - 2 < 0 \tag{1.0.43.2}$$

(x is measured in radians). Then x lies in the interval

a) $(\frac{\pi}{6}, \frac{5\pi}{6})$
b) $(-1, \frac{5\pi}{6})$
c) (-1 , 2)
d) $(\frac{\pi}{6}, 2)$

44. The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where$\alpha,\beta,\gamma$are real numbers satisfying $\alpha+\beta+\gamma = \pi$ is

a) Positive
b) zero
c) negative
d) -3

45. The number of values of x in the interval $[0, \pi]$ satisfying the equation

$$3\sin^2 x - 7\sin x + 2 = 0 \tag{1.0.45.1}$$

is

a) 0
b) 5
c) 6
d) 10

46. Which of the following number(s) is/are/rational?

a) $\sin 15°$
b) $\cos 15°$
c) $\sin 15° \cos 15°$
d) $\sin 15° \cos 75°$

47. For a positive integer n, let $f_n(\theta) = \tan(\frac{\theta}{2})(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)......(1+\sec 2^n\theta)$.Then

a) $f_2(\frac{\pi}{16}) = 1$

b) $f_3(\frac{\pi}{32}) = 1$
c) $f_4(\frac{\pi}{64}) = 1$
d) $f_5(\frac{\pi}{128}) = 1$

48. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then
a) $\tan^2 x = \frac{2}{3}$
b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
c) $\tan^2 x = \frac{1}{3}$
d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

49. For $0 < \theta < \frac{\pi}{2}$. the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}(\theta + \frac{(m-1)\pi}{4})\operatorname{cosec}(\theta + \frac{m\pi}{4}) = 4\sqrt{2}$ is(are)
a) $\frac{\pi}{4}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{12}$
d) $\frac{5\pi}{12}$

50. Let $\theta, \varphi \epsilon [0, 2\pi]$ be such that $2\cos\theta(1-\sin\varphi) = \sin^2\theta(\tan\frac{\theta}{2} + \cot\frac{\theta}{2})\cos\varphi - 1, \tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$ , then $\varphi$ can not satisfy
a) $0 < \varphi < \frac{\pi}{2}$
b) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$
c) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$
d) $\frac{3\pi}{2} < \varphi < 2\pi$

51. The number of points in$(-\infty, \infty)$, for which

$$x^2 - x\sin x - \cos x = 0 \tag{1.0.51.1}$$

is
a) 6
b) 4
c) 2
d) 0

52. Let

$$f(x) = x\sin\pi x, x > 0 \tag{1.0.52.1}$$

Then for all natural numbers n, $f'(x)$ vanishes at
a) A unique point in the interval (n,n+$\frac{1}{2}$)
b) A unique point in the interval (n+$\frac{1}{2}$, n+1)
c) A unique point in the interval (n,n+1)
d) Two points in the interval (n,n+1)

53. Let $\alpha$ and $\beta$ be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$.Then which of the following is/are true?
a) $\tan(\frac{\alpha}{2}) + \sqrt{3}\tan(\frac{\beta}{2}) = 0$
b) $\sqrt{3}\tan(\frac{\alpha}{2}) + \tan(\frac{\beta}{2}) = 0$
c) $\tan(\frac{\alpha}{2}) - \sqrt{3}\tan(\frac{\beta}{2}) = 0$
d) $\sqrt{3}\tan(\frac{\alpha}{2}) - \tan(\frac{\beta}{2}) = 0$

54. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, find the possible values of $(\alpha + \beta)$.

55. (a) Draw the graph of

$$y = \frac{1}{\sqrt{2}}(\sin x + \cos x) \tag{1.0.55.1}$$

from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
(b) If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $\alpha, \beta$ lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$

56. Given $\alpha + \beta - \gamma = \pi$, prove that

$$\sin^2\alpha + \sin^2\beta - \sin^2\gamma = 2\sin\alpha\sin\beta\cos\gamma \tag{1.0.56.1}$$

57. Given A= { x: $\frac{\pi}{6} \le x \le \frac{\pi}{3}$} and

$$f(x) = \cos x - x(1 + x); \tag{1.0.57.1}$$

find f(A)

58. For all $\theta$ in $[0, \pi/2]$ show that,

$$\cos(\sin\theta) \ge \sin(\cos\theta) \tag{1.0.58.1}$$

.

59. Without using tables, Prove that $(\sin 12°)(\sin 48°)(\sin 54°) = \frac{1}{8}$

60. Show that $16\cos(\frac{2\pi}{15})\cos(\frac{4\pi}{15})\cos(\frac{8\pi}{15})\cos(\frac{16\pi}{15}) = 1$

61. Find all the solution of $4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$

62. Find the values of x$\epsilon(-\pi, \pi)$ which satisfy the equation

$$8^{(1+|\cos x||+|cos^2 x|+|cos^3 x|+.....)} = 4^3 \tag{1.0.62.1}$$

63. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

64. ABC is a triangle such that $\sin(2A + B) = \sin(C - A) = -\sin(B + 2c) = \frac{1}{2}$ If A,B and C are in arithmetic progression, determine the values of A, B and C.

65. if exp{$(\sin^2 x + \sin^4 x + \sin^6 x + ......\infty)$ In 2 } satisifies the equation

$$x^2 - 9x + 8 = 0 \tag{1.0.65.1}$$

, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$.

66. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3.

67. Determine the smallest positive value of x(in degrees) for which $\tan(x + 100°) = \tan(x + 50°)\tan(x)\tan(x - 50°)$.

68. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a

solution $x\epsilon[0, 2\pi]$

69. Find all values of $\theta$ in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ satisfying the equation

$$(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0 \tag{1.0.69.1}$$

70. Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x.
71. Prove that $\sum_{k=1}^{n-1}(n-k)cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \geq 3$ is an integer
72. If any triangle ABC, Prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{c}{2}$
73. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$.

This section contains 1 paragraph, Based on each paragraph,there are 2 questions. Each question has four options (A),(B),(C) and (D) ONLY ONE of these four options is correct.

**PARAGRAPH 1**

Let O be the origin, and $\mathbf{OX}, \mathbf{OY}, \mathbf{OZ}$ be three unit vectors in the directions of the sides $\mathbf{QR}, \mathbf{RP}, \mathbf{PQ}$ respectively, of a triangle PQR

74. $|\mathbf{OX} \times \mathbf{OY}| =$
    a) $\sin(P + Q)$
    b) $\sin 2R$
    c) $\sin(P + R)$
    d) $\sin(Q + R)$
75. If the triangle PQR varies, then the minimum value of $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$ is
    a) $-\frac{5}{3}$
    b) $-\frac{3}{2}$
    c) $\frac{3}{2}$
    d) $\frac{5}{3}$
76. The number of all possible values of $\theta$ where $0 < \theta < \pi$,for which the system of equations

$$(y + z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$$

have a solution $(x_0, y_0, z_0)$ with $y_0z_0 \neq 0$,is

77. The number of values of $\theta$ in the interval, $(-\frac{\pi}{2}, \frac{\pi}{2})$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0 , ±1,±2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is
78. The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is
79. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of [k] is
    **Note**: [k] denotes the largest integer less than or equal to k.
80. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin(\frac{\pi}{n})} = \frac{1}{\sin(\frac{2\pi}{n})} + \frac{1}{\sin(\frac{3\pi}{n})}$ is
81. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is
82. Let a,b,c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x\epsilon[-\frac{\pi}{2}, \frac{\pi}{2}]$ has two distinct real roots $\alpha$ and $\beta$ with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is
83. The period of $\sin^2\theta$ is
    a) $\pi^2$
    b) $\pi$
    c) $2\pi$
    d) $\pi/2$
84. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
    a) 2
    b) 3
    c) 0
    d) 1
85. Which one is not periodic
    a) $|\sin 3x| + \sin^2 x$
    b) $\cos\sqrt{x} + \cos^2 x$
    c) $\cos 4x + \tan^2 x$
    d) $\cos 2x + \sin x$
86. Let $\alpha$,$\beta$ be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$
    a) $-\frac{6}{65}$
    b) $\frac{3}{\sqrt{130}}$
    c) $\frac{6}{65}$
    d) $-\frac{3}{\sqrt{130}}$
87. If u = $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$then the difference between the maximum and minimum values of $u^2$ is given by
    a) $(a - b)^2$
    b) $2\sqrt{a^2 + b^2}$
    c) $(a + b)^2$
    d) $2(a^2 + b^2)$

88. A line makes the same angle $\theta$, with each of the x and z axis. If the angle $\beta$, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
   a) $\frac{2}{5}$
   b) $\frac{1}{5}$
   c) $\frac{3}{5}$
   d) $\frac{2}{3}$
89. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation
$$2\sin^2 x + 5\sin x - 3 = 0 \qquad (1.0.89.1)$$
is
   a) 4
   b) 6
   c) 1
   d) 2
90. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is
   a) $\frac{(1-\sqrt{7})}{4}$
   b) $\frac{(4-\sqrt{7})}{3}$
   c) $-\frac{(4+\sqrt{7})}{3}$
   d) $\frac{(1+\sqrt{7})}{4}$
91. Let A and B denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$,then :
   a) A is false and B is true
   b) Both A and B are true
   c) both A and B are false
   d) A is true and B is false
92. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$, Then $\tan 2\alpha =$
   a) $\frac{56}{33}$
   b) $\frac{19}{12}$
   c) $\frac{20}{7}$
   d) $\frac{25}{16}$
93. If A = $\sin^2 x + \cos^4 x$ , then for all real x:
   a) $\frac{13}{16} \le A \le 1$
   b) $1 \le A \le 2$
   c) $\frac{3}{4} \le A \le \frac{13}{16}$
   d) $\frac{3}{4} \le A \le 1$
94. In a $\triangle PQR$, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :
   a) $\frac{5\pi}{6}$
   b) $\frac{\pi}{6}$
   c) $\frac{\pi}{4}$
   d) $\frac{3\pi}{4}$
95. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC=p and CD=q, then AB is equal to :
   a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$
   b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$
   c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$
   d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$
96. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as:
   a) $\sin A\cos A + 1$
   b) $\sec A cosec A + 1$
   c) $\tan A + \cot A$
   d) $\sec A + cosec A$
97. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \epsilon R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals
   a) $\frac{1}{4}$
   b) $\frac{1}{12}$
   c) $\frac{1}{6}$
   d) $\frac{1}{3}$
98. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:
   a) 7
   b) 9
   c) 3
   d) 5
99. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$,then the value of cos4x is :
   a) $-\frac{7}{9}$
   b) $-\frac{3}{5}$
   c) $\frac{1}{3}$
   d) $\frac{2}{9}$
100. If sum of all the solutions of the equation 8 $\cos x.(\cos(\frac{\pi}{6}+x)(\cos(\frac{\pi}{6}-x) - \frac{1}{2}) - 1$ in $[0, \pi]$ is $k\pi$. then k is equal to :
   a) $\frac{13}{9}$
   b) $\frac{8}{9}$
   c) $\frac{20}{9}$
   d) $\frac{2}{3}$
101. For any $\theta \epsilon (\frac{\pi}{4}, \frac{\pi}{2})$ the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^2\theta$ equals:

a) $13-4\cos^2\theta+6\sin^2\theta\cos^2\theta$
b) $13-4\cos^6\theta$
c) $13-4\cos^2\theta+6\cos^4\theta$
d) $13-4\cos^4\theta+2\sin^2\theta\cos^2\theta$

102. The value of $\cos^2 10°-\cos 10°\cos 50°+\cos^2 50°$ is:
a) $\frac{3}{4}+\cos 20°$
b) $\frac{3}{4}$
c) $\frac{3}{2}(1+\cos 20°)$
d) $\frac{3}{2}$

103. Let S=$\{\theta\epsilon[-2\pi,2\pi] : 2\cos^{\theta}+3\sin\theta=0\}$. Then the sum of the elements of S is
a) $\frac{13\pi}{6}$
b) $\frac{5\pi}{3}$
c) 2
d) 1

# Match the Following

**DIRECTIONS (Q.1):** *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B , C and D, while the statements in Column-II are labelled p,q,r,s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to theses questions have to be darkened as illustrated in the following example:*
*If the correct matches are A-p, s and t; B-q and r; C-p and q; D -s then the correct darkening of bubbles will look like the given*

---

a) In this question there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.
$\frac{\sin 3\alpha}{\cos 2\alpha}$ is

| **Column-I** | **Column-II** |
|---|---|
| (A) Positive | (p)$(\frac{13\pi}{48},\frac{14\pi}{48})$ |
| (B) Negative | (q)$(\frac{14\pi}{48},\frac{18\pi}{48})$ |
| | (r)$(\frac{18\pi}{48},\frac{23\pi}{48})$ |
| | (s)$(0,\frac{\pi}{2})$ |

b) Let f(x)=$\sin(\pi\cos x)$ and g(x)=$\cos(2\pi\sin x)$be two functions defined for $x>0$. Define the following sets whose elements are written i n the increasing order.

X =$\{x : f(x)=0\}$,Y =$\{x : f'(x)=0\}$

Z =$\{x : g(x)=0\}$,W =$\{x : g'(x)=0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

| **Column-I** | **Column-II** |
|---|---|
| (A)X | (p)$\supseteq\{\frac{\pi}{2},\frac{3\pi}{2},4\pi,7\pi\}$ |
| (B)Y | (q)an arithmetic progression |
| (C)Z | (r)NOT an arithmetic progression |
| (D)W | (s)$\supseteq\{\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\}$ |
| | (t)$\supseteq\{\frac{\pi}{3},\frac{2\pi}{3},\pi\}$ |
| | (u)$\supseteq\{\frac{\pi}{6},\frac{3\pi}{4}\}$ |

Which of the following is the only CORRECT combination?
i) (IV),(P),(R),(S)
ii) (III),(P),(Q),(U)
iii) (III),(R),(U)
iv) (IV),(Q),(T)

c) Let $f(x)=\sin(\pi\cos x)$ and $g(x)=\cos(2\pi\sin x)$ be two functions defined for $x>0$.Define the following sets whose elements are written in the increasing order

X =$\{x : f(x)=0\}$,Y =$\{x : f'(x)=0\}$

Z =$\{x : g(x)=0\}$,W =$\{x : g'(x)=0\}$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

| **Column-I** | **Column-II** |
|---|---|
| (A)X | (p)$\supseteq \{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\}$ |
| (B)Y | (q)an arithmetic progression |
| (C)Z | (r)NOT an arithmetic progression |
| (D)W | (s)$\supseteq \{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\}$ |
| | (t)$\supseteq \{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$ |
| | (u)$\supseteq \{\frac{\pi}{6}, \frac{3\pi}{4}\}$ |

Which of the following is the only CORRECT combination?

i) (I),(Q),(U)
ii) (I),(P),(R)
iii) (II),(R),(S)
iv) (II),(Q),(T)

## 2 Inverse Trigonometric Functions

1. Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b-c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b-c)}{ab}}$$

Then $\tan\theta =$ .............

2. The numerical value of

$$\tan\{2\tan^{-1}(\frac{1}{5}) - \frac{\pi}{4}\}$$

is equal to ............

3. The greater of the two angles

$$A = 2\tan^{-1}(2\sqrt{2}-1)$$

$$B = 3\sin^{-1}(\frac{1}{3}) + \sin^{-1}(\frac{3}{5})$$

is................

**MCQ's with One Correct Answer**

4. The value of $\tan^{-1}[(\cos^{-1}\frac{4}{5}) + \tan^{-1}(\frac{2}{3})]$ is
   a) $\frac{6}{17}$
   b) $\frac{7}{16}$
   c) $\frac{16}{7}$
   d) none of these

5. If we consider only the principle values of the inverse trigonometric functions then the value of $\tan(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}})$ is
   a) $\frac{\sqrt{29}}{3}$
   b) $\frac{29}{3}$
   c) $\frac{\sqrt{3}}{29}$
   d) $\frac{3}{29}$

6. The number of real solutions of

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

   a) zero
   b) one
   c) two
   d) infinite

7. If

$$\sin^{-1}(x - \frac{x^2}{2} + \frac{x^3}{4}......) + \cos^{-1}(x^2 - \frac{x^4}{2} + \frac{x^6}{4}....) = \frac{\pi}{2}$$

for $0 < |x| < \sqrt{2}$, then x equals
   a) 1/2
   b) 1
   c) -1/2
   d) -1

8. The value of x for which

$$\sin(\cot^{-1}(1+x) = \cos(\tan^{-1}x))$$

is
   a) 1/2
   b) 1
   c) 0
   d) -1/2

9. If $0 < x < 1$, then

$$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\} - 1]^{1/2} =$$

   a) $\frac{x}{1+x^2}$
   b) x
   c) $x\sqrt{1+x^2}$
   d) $\sqrt{1+x^2}$

10. The value of

$$\cot(\sum_{n=1}^{23}\cot^{-1}(1 + \sum_{k=1}^{n}2k)) =$$

   a) $\frac{23}{25}$
   b) $\frac{25}{23}$
   c) $\frac{23}{24}$
   d) $\frac{24}{23}$

**MCQs with One or More than One Correct**

11. The principal value of $\sin^{-1}(sin\frac{2\pi}{3})$ is
    a) $-\frac{2\pi}{3}$
    b) $\frac{2\pi}{3}$
    c) $\frac{4\pi}{3}$
    d) none of these
12. If $\alpha = 3\sin^{-1}(\frac{6}{11})$ and $\beta = 3\cos^{-1}(\frac{4}{9})$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)
    a) $\cos\beta > 0$
    b) $\sin\beta < 0$
    c) $\cos(\alpha+\beta) > 0$
    d) $\cos\alpha < 0$
13. For non-negative integers n, let
$$f(n) = \frac{\sum_{k=0}^{n}\sin(\frac{k+1}{n+2}\pi)\sin(\frac{k+2}{n+2}\pi)}{\sum_{k=0}^{n}\sin^2(\frac{k+1}{n+2}\pi)}$$
    Assuming $\cos^{-1}x$ takes values $[0,\pi]$, which of the following options is/are correct?
    a) $[\lim_{n\to\infty} f(n) = \frac{1}{2}]$
    b) $f(4) = \frac{\sqrt{3}}{2}$
    c) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$
    d) $\sin(7\cos^{-1} f(5)) = 0$
14. Find the value of:
$$\cos(2\cos^{-1}x + \sin^{-1}x) at x = \frac{1}{5}$$
    where $0 \le \cos^{-1}x \le \pi$ and $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$.
15. Find all the solutions of
$$4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$$
16. Prove that $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\frac{x^2+1}{x^2+2}}$

**Integer Value Correct Type:**

17. The number of real solutions of the equation
$$\sin^{-1}(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}(\frac{x}{2})^i)$$
$$= \frac{\pi}{2} - \cos^{-1}(\sum_{i=1}^{\infty}(\frac{-x}{2})^i - \sum_{i=1}^{\infty}(-x)^i)$$
    lying in the interval $(\frac{-1}{2}, \frac{1}{2})$ is............
18. The value of
$$\sec^{-1}\frac{1}{4}\sum_{k=0}^{10}\sec(\frac{7\pi}{12} + \frac{k\pi}{2})\sec(\frac{7\pi}{12} + \frac{(k+1)\pi}{2})$$
    in the interval $[\frac{-\pi}{4}, \frac{3\pi}{4}]$ equals..........

**Section-B**

19. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$
    a) $\tan^2(\frac{\alpha}{2})$
    b) $\cot^2(\frac{\alpha}{2})$
    c) $\tan\alpha$
    d) $\cot(\frac{\alpha}{2})$
20. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for
    a) $|a| \ge \frac{1}{\sqrt{2}}$
    b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$
    c) all real values of a
    d) $|a| < \frac{1}{2}$
21. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to
    a) $2\sin 2\alpha$
    b) 4
    c) $4\sin^2\alpha$
    d) $-4\sin^2\alpha$
22. If $\sin^{-1}(\frac{x}{5}) + \operatorname{cosec}^{-1}(\frac{5}{4}) = \frac{\pi}{2}$, then the value of x is
    a) 4
    b) 5
    c) 1
    d) 3
23. The value of $\cot(\operatorname{cosec}^{-1}(\frac{5}{3}) + \tan^{-1}(\frac{2}{3}))$ is
    a) $\frac{6}{17}$
    b) $\frac{3}{17}$
    c) $\frac{4}{17}$
    d) $\frac{5}{17}$
24. If x, y, z are in A.P and $\tan^{-1}y$, $\tan^{-1}z$ are also in A.P., then
    a) x = y = z
    b) 2x = 3y = 6z
    c) 6x = 3y = 2z
    d) 6x = 4y = 3z
25. Let
$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}(\frac{2x}{1-x^2})$$
    where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
    a) $\frac{3x-x^3}{1+3x^2}$
    b) $\frac{3x+x^3}{1+3x^2}$
    c) $\frac{3x-x^3}{1-3x^2}$
    d) $\frac{3x+x^3}{1-3x^2}$
26. If $\cos^{-1}(\frac{2}{3x}) + \cos^{-1}(\frac{3}{4x}) = \frac{\pi}{2}(x > \frac{3}{4})$, then x is equal to

a) $\frac{\sqrt{145}}{12}$
b) $\frac{\sqrt{145}}{10}$
c) $\frac{\sqrt{146}}{12}$
d) $\frac{\sqrt{145}}{11}$

27. Match the following:

| Column I | Column II |
|---|---|
| A. $\sum_{n=1}^{23} \tan^{-1}(\frac{1}{2i^2})$ = t, then $\tan t$ = | (p). 1 |
| B. Sides a, b, c of a triangle ABC are in A.P. $\cos\theta_1 = \frac{a}{b+c}$, $\cos\theta_2 = \frac{b}{a+c}$, $\cos\theta_3 = \frac{c}{a+b}$, then $\tan^2(\frac{\theta_1}{2}) + \tan^2(\frac{\theta_3}{2})$ = | (q). $\frac{\sqrt{5}}{3}$ |
| C. A line is perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0) Then the perpendicular distance of this line from the origin is | (r). $\frac{2}{3}$ |

28. Match the following:

| Column I | Column II |
|---|---|
| A. If a = 1 and b = 0, then (x, y) | (p). lies on the circle $x^2 + y^2 = 1$ |
| B. If a = 1 and b = 1, then (x, y) | (q). lies on $(x^2 - 1)(y^2 - 1) = 0$ |
| C. If a = 1 and b = 2, then (x, y) | (r). lies on $y = x$ |
| D. If a = 2 and b = 2, then (x, y) | (s). lies on $(4x^2 - 1)(y^2 - 1) = 0$ |

29. Match the following:

| Column I | Column II |
|---|---|
| P. $(\frac{1}{y^2}(\frac{\cos(\tan^{-1} y)+y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y)+\tan(\sin^{-1} y)})^2 + y^4)^{\frac{1}{2}}$ takes value is | (i) $\frac{1}{2}\sqrt{\frac{5}{3}}$ |
| Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\frac{x-y}{2}$ is | (ii) $\sqrt{2}$ |
| R. If $\cos(\frac{\pi}{4} - x)\cos 2x + \sin x \sin 2 \sec x = \cos x \sin 2x \sec x + \cos(\frac{\pi}{4} + x)$ then possible value of $\sec x$ is | (iii) $\frac{1}{2}$ |
| S. If $\cot(\sin^{-1}\sqrt{1-x^2}) = \sin(\tan^{-1} x\sqrt{6})$, $x \neq 0$ then possible value of $\sec x$ is | (iv) 1 |

**codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 4 | 3 | 1 | 2 |
| (b) | 4 | 3 | 2 | 1 |
| (c) | 3 | 4 | 2 | 1 |
| (d) | 3 | 4 | 1 | 2 |

## 3 Functions

1. The values of $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lie in the interval.......
2. For the function f(x) = $\frac{x}{1+e^{1/x}}$, x ≠ 0 and f(x) = 0, x = 0 the derivative from the right, $f'(0+)$= ........, and the derivative from the left, $f'(0-)$= ..........
3. The domain of the funtion f(x)=$\sin^{-1}\left(log_2\frac{x^2}{2}\right)$ is given by .......
4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is........and out of these.......are onto functions.
5. If
$$f(x) = \sin ln\left(\frac{\sqrt{4-x^2}}{1-x}\right),$$
then domain of f(x) is.... and its range is......
6. There are exactly two distinct linear functions.......,and......which map [-1,1] onto [0,2].
7. If f is an even function defined on the interval (-5,5), then four real values of x satisfying the equation f(x)=$f(\frac{x+1}{x+2})$ are.........., ..........., ........., and......
8. If
$$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + cosxcos\left(x + \frac{\pi}{3}\right)$$
and g($\frac{5}{4}$) = 1, then (gof)(x)= ........
9. If f(x)=$(a - x^n)^{1/n}$ where $a > 0$ and n is a positive integer, then f[f(x)]=x.
10. The function f(x)=$\frac{x^2+4x+30}{x^2-8x+18}$ is not one-to-one.
11. If $f_1(x)$ and $f_2(x)$ are defined on domains $D_1$ and $D_2$ respectively, then $f_1(x)$+$f_2(x)$ is defined on $D_1 \cup D_2$.
12. Let R be the set of real numbers. If f:R → R is a function defined by f(x)=$x^2$, then f is :
   a) Injective but not surjective
   b) Surjective but not injective
   c) Bijective
   d) None of these.
13. The entire graphs of the equation y = $x^2 + kx - x + 9$ is stirctly above the x-axis if and only if
   a) $k < 7$
   b) $-5 < k < 7$
   c) $k > -5$
   d) None of these.
14. Let f(x)= $\left|x - 1\right|$. Then
   a) $f(x^2) = (f(x))^2$
   b) f(x+y)=f(x)+f(y)
   c) $f(\left|x\right|) = \left|f(x)\right|$
   d) None of these
15. If x satisfies $\left|x-1\right| + \left|x-2\right| + \left|x-3\right| \geq 6$,then
   a) $0 \leq x \leq 4$
   b) $x \leq -2$ or $x \geq 4$

c) $x \leq 0$ or $x \geq 4$
d) None of these

16. If f(x)=cos($lnx$), then f(x)f(y) - $\frac{1}{2}\left[f(\frac{x}{y}) + f(xy)\right]$ has the value
a) -1
b) $\frac{1}{2}$
c) -2
d) none of these

17. The domain of definition of the function y=$\frac{1}{log_{10}(1-x)}$ + $\sqrt{x+2}$ is
a) (-3,2) excluding -2.5
b) $[0, 1]$ excluding 0.5
c) $[-2, 1)$ excluding 0
d) none of these

18. Which of the following functions is periodic?
a) f(x)=x-[x] where [x] denotes the largest integer less than or equal to the real number x
b) f(x)=sin $\frac{1}{x}$ for $x \neq 0$, f(0)=0
c) f(x)=xcos $x$
d) none of these

19. Let f(x)=sin $x$ and g(x) = ln $|x|$. If the ranges of the composition functions $fog$ and $gof$ are $R_1$ and $R_2$ respectively, then
a) $R_1 = \{u : -1 \leq u < 1\}, R_2 = \{v : -\infty < v < 0\}$
b) $R_1 = \{u : -\infty < u < 0\}, R_2 = \{v : -1 \leq v \leq 0\}$
c) $R_1 = \{u : -1 < u < 1\}, R_2 = \{v : -\infty < v < 0\}$
d) $R_1 = \{u : -1 \leq u \leq 1\}, R_2 = \{v : -\infty < v \leq 0\}$

20. Let $f(x) = (x + 1)^2 - 1$, x $\geq -1$. Then the set $\{x : f(x) = f^{-1}(x)\}$ is
a) $\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\}$
b) $\{0, 1, -1\}$
c) $\{0, -1\}$
d) empty

21. The function f(x) = $|px - q|$ + r$|x|$, $x \in (-\infty, \infty)$ where p > 0, q > 0, r > 0 assumes its minimum value only on one point if
a) p $\neq$ q
b) r $\neq$ q
c) r $\neq$ p
d) p = q = r

22. Let $f(x)$ be defined for all $x > 0$ and be continuos. Let $f(x)$ satisfy $f(\frac{x}{y}) = f(x) - f(y)$ for all x,y and $f(e) = 1$. Then
a) $f(x)$ is bounded
b) $f(\frac{1}{x}) \to 0$ as $x \to 0$
c) $xf(x) \to 1$ as $x \to 0$
d) $f(x) = lnx$

23. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is
a) $\frac{1}{2}^{x(x-1)}$
b) $\frac{1}{2}(1 + \sqrt{1 + 4log_2 x})$
c) $\frac{1}{2}(1 - \sqrt{1 + 4log_2 x})$
d) not defined

24. Let $f : R \to$ R be any function. Define g: R $\to$ R by $g(x) = |f(x)|$ for all x. Then g is
a) onto if f is onto
b) one-one if f is one-one
c) continuos if f is continuos
d) differentiable if f is differentiable

25. The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is
a) $0 < x \leq 1$
b) $0 \leq x \leq 1$
c) $-\infty < x \leq 0$
d) $-\infty < x < 1$

26. Let $g(x) = 1 + x - [x]$ and

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0. \\ 1, & x > 0 \end{cases}$$

then for all x, $f(g(x))$ is equal to
a) x
b) 1
c) $f(x)$
d) $g(x)$

27. If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals
a) $(x + \sqrt{x^2 - 4})/2$
b) $x/(1 + x^2)$
c) $(x - \sqrt{x^2 - 4})/2$
d) $1 + \sqrt{x^2 - 4}$

28. The domain of definition of $f(x) = \frac{log_2(x+3)}{x^2+3x+2}$ is
a) $R\backslash\{-1, -2\}$
b) $(-2, \infty)$
c) $R\backslash\{-1, -2, -3\}$
d) $(-3, \infty)\backslash\{-1, -2\}$

29. Let E=$\{1, 2, 3, 4\}$ and F=$\{1, 2\}$. Then the number of onto functions from E to F is

a) 14
b) 16
c) 12
d) 8

30. Let $f(x) = \frac{\alpha x}{x+1}$,x ≠ -1. Then, for what value of $\alpha$ is $f(f(x)) = x$?
a) $\sqrt{2}$
b) $-\sqrt{2}$
c) 1
d) -1

31. Suppose $f(x) = (x+1)^2$ for x ≥ -1. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line y=x then $g(x)$ equals
a) $-\sqrt{x}-1, x \geq 0$
b) $\frac{1}{(x+1)^2}, x > -1$
c) $\sqrt{x+1}, x \geq -1$
d) $\sqrt{x}-1, x \geq 0$

32. Let function $f : R \to$ R be defined by $f(x) = 2x + \sin x$ for x ∈ R, then $f$ is
a) one-to-one and onto
b) one-to-one but NOT onto
c) onto but NOT one-to-one
d) neither one-to-one nor onto

33. If $f : [0, \infty) \to [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then $f$ is
a) one-one and onto
b) one-one but not onto
c) onto but not one-one
d) neither one-one nor onto

34. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued $x$ ,is
a) $[-\frac{1}{4}, \frac{1}{2}]$
b) $[-\frac{1}{2}, \frac{1}{2}]$
c) $[-\frac{1}{2}, \frac{1}{9}]$
d) $[-\frac{1}{4}, \frac{1}{4}]$

35. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}; x \epsilon$R is
a) $(1, \infty)$
b) $(1, \frac{11}{7}]$
c) $(1, \frac{7}{3}]$
d) $(1, \frac{7}{5}]$

36. If $f(x) = x^2+2bx+2c^2$ and $g(x) = -x^2-2cx+b^2$ such that min $f(x) > max g(x)$, then the relation between b and c, is
a) no real value of b & c
b) $0 < c < b\sqrt{2}$
c) $|c| < |b|\sqrt{2}$
d) $|c| > |b|\sqrt{2}$

37. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain
a) $[0, \frac{\pi}{2}]$
b) $[-\frac{\pi}{4}, \frac{\pi}{4}]$
c) $[-\frac{\pi}{2}, \frac{\pi}{2}]$
d) $[0, \pi]$

38. If the functions $f(x)$ and $g(x)$ are defined on R→R such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

;

$$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

then $(f-g)(x)$ is
a) one-one & onto
b) neither one-one nor onto
c) one-one but not onto
d) onto but not one-one

39. X and Y are two sets and $f : X \to Y$. If $\{f(c) = y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X\}$, then the true statement is
a) $f(f^{-1}(b)) =$ b
b) $f^{-1}(f(a)) =$ a
c) $f(f^{-1}(b)) =$ b,b ⊂ y
d) $f(f^{-1}(a)) =$ a,a ⊂ x

40. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x) = -f(x)$ and $g(x) = f'(x)$ and given that F(5)=5, then F(10) is equal to
a) 5
b) 10
c) 0
d) 15

41. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for n ≥ 2 and $g(x) = \underbrace{(fofo......of)}_{\text{f occurs n times}}(x)$. Then $\int x^{n-2}g(x)dx$ equals.
a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$
b) $\frac{1}{(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$
c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$
d) $\frac{1}{(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$

42. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2}+e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c

denote, respectively, the absolute maximum of f,g and h on $[0, 1]$, then

a) a = b and c ≠ b
b) a = c and a ≠ b
c) a ≠ b and c ≠ b
d) a = b = c

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in$ R. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is

a) $\pm\sqrt{n\pi}$, n $\in \{0, 1, 2.....\}$
b) $\pm\sqrt{n\pi}$, n $\in \{1, 2.....\}$
c) $\frac{\pi}{2} + 2n\pi$, n $\in \{..... - 2, -1, 0, 1, 2.....\}$
d) $2n\pi$, n $\in \{..... - 2, -1, 0, 1, 2.....\}$

44. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

a) one-one and onto
b) onto but not one-one
c) one-one but not onto
d) neither one-one nor onto

45. If y=$f(x) = \frac{x+2}{x-1}$ then

a) $x = f(y)$
b) $f(1)$=3
c) y increases with $x$ for $x < 1$
d) $f$ is a rational function of $x$

46. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at (0,0) and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is

a) $g(x) = \pm\sqrt{1 - x^2}$
b) $g(x) = \sqrt{1 - x^2}$
c) $g(x) = -\sqrt{1 - x^2}$
d) $g(x) = \sqrt{1 + x^2}$

47. If f(x)=$\cos[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then

a) $f(\frac{\pi}{2}) = -1$
b) $f(\pi) = 1$
c) $f(-\pi) = 0$
d) $f(\frac{\pi}{4}) = 1$

48. If $f(x)$=3x-5, then $f^{-1}(x)$

a) is given by $\frac{1}{3x-5}$
b) is given by $\frac{x+5}{3}$
c) does not exist because $f$ is not one-one
d) does not exist because $f$ is not onto.

49. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then

a) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
b) $f(x) = \sin x, g(x) = \left|x\right|$
c) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$
d) $f$ and g cannot be determined.

50. Let $f$:(0,1) → R be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then

a) $f$ is not invertible on (0,1)
b) $f \neq f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$
c) $f = f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$
d) $f^{-1}$ is differentiable (0,1)

51. Let $f$:(-1,1) → IR be such that $f(\cos 4\Theta) = \frac{2}{2-\sec^2\Theta}$ for $\Theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the values of $f(\frac{1}{3})$ is

a) 1-$\sqrt{\frac{3}{2}}$
b) 1+$\sqrt{\frac{3}{2}}$
c) 1-$\sqrt{\frac{2}{3}}$
d) 1+$\sqrt{\frac{2}{3}}$

52. The funciton $f(x) = 2\left|x\right| + \left|x + 2\right|$ - $\left|\left|x + 2\right| - 2\left|x\right|\right|$ has a local minimum or a local maximum at x=

a) -2
b) $\frac{-2}{3}$
c) 2
d) $\frac{2}{3}$

53. Let $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \to R$ be given by $f(x) = (log(\sec x + \tan x))^3$. Then

a) $f(x)$ is an odd function
b) $f(x)$ is one-one function
c) $f(x)$ is an onto function
d) $f(x)$ is an even function

54. Let a∈R and let $f$: R → R be given by $f(x) = x^5 - 5x + a$. Then

a) $f(x)$ has three real roots if a>4
b) $f(x)$ has only real root if a>4
c) $f(x)$ has three real roots if a<-4
d) $f(x)$ has three real roots if -4<a<4

55. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x$ ∈R and $g(x) = \frac{\pi}{2}\sin x$ for all x∈R. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then which of the following is true?

a) Range of $f$ is $[-\frac{1}{2}, \frac{1}{2}]$
b) Range of fog is $[-\frac{1}{2}, \frac{1}{2}]$
c) $\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$
d) There is an $x$ ∈R such that $(gof)(x) = 1$

56. Find the domain and range of the function

$f(x) = \frac{x^2}{1+x^2}$. Is the function one-to-one?

57. Draw the graph of $y = \left|x\right|^{1/2}$ for $-1 \le x \le 1$.
58. If $f(x) = x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find $f(6)$.
59. Consider the following relations in the set of real numbers R. $R = \{(x, y); x \in R, y \in R, x^2 + y^2 \le 25\}$
$R' = \{(x, y) : x \in R, y \in R, y \ge \frac{4}{9}x^2\}$. Find the domain and the range of $R \cap R'$. Is the relation $R \cap R'$ a function?
60. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B.
61. Let $f$ be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x) = 1$, $f(y) \ne 1$,$f(z) \ne 2$ determine $f^{-1}(1)$.
62. Let R be the set of real numbers and $f$: R → R be such that for all x and y in R$\left|f(x) - f(y)\right| \le \left|x - y\right|^3$. Prove that $f(x)$ is a constant.
63. Find the natural number $'a'$ for which $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, where the function $'f'$ satisfies the relation $f(x+y) = f(x)f(y)$ for all natural numbers x,y and further $f(1) = 2$.
64. Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number $x$ respectively. Solve $4\{x\} = x + [x]$.
65. A function $f : IR \to IR$, where IR is the set of real numbers, defined by $f(x) = \frac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$. Find the interval of values of $\alpha$ for which f is onto. Is the function one-to-one for $\alpha = 3$? Justify your answer.
66. Let $f(x) = Ax^2 + Bx + c$ where A,B,C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A,A+B and C are all integers. Conversely, prove that if the numbers 2A,A+B and C are all integers then $f(x)$ is an integer whenever $x$ is an integer.
67. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is
68. The value of $((log_2 9)^2)^{\frac{1}{log_2(log_2 9)}} \times (\sqrt{7})^{\frac{1}{log_4 7}}$ is ......
69. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If $\alpha$ is the number of one-one functions from X to Y and $\beta$ is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is ......
70. The domain of $\sin^{-1}[log_3(x/3)]$ is
    a) $[1, 9]$
    b) $[-1, 9]$
    c) $[-9, 1]$
    d) $[-9, -1]$
71. The function $f(x) = log(x + \sqrt{x^2 + 1})$, is
    a) neither an even nor an odd function
    b) an even function
    c) an odd function
    d) a periodic function.
72. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + log_{10}(x^3 - x)$, is
    a) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
    b) (a,2)
    c) $(-1, 0) \cup (a, 2)$
    d) $(1, 2) \cup (2, \infty)$.
73. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x$, y ∈ R and f(1)=7, then $\sum_{r=1}^{n} f(r)$ is
    a) $\frac{7n(n+1)}{2}$
    b) $\frac{7n}{2}$
    c) $\frac{7(n+1)}{2}$
    d) $7n + (n + 1)$
74. A function $f$ from the set of natural numbers to integers defined by
$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$
is
    a) neither one-one nor onto
    b) one-one but not onto
    c) onto but not one-one
    d) one-one and onto both.
75. The range of the function $f(x) =^{7-x} P_{x-3}$ is
    a) $\{1, 2, 3, 4, 5\}$
    b) $\{1, 2, 3, 4, 5, 6\}$
    c) $\{1, 2, 3, 4\}$
    d) $\{1, 2, 3, \}$
76. Let $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is
    a) $[-1, 3]$
    b) $[-1, 1]$
    c) $[0, 1]$
    d) $[0, 3]$
77. The graph of the function $y = f(x)$ is symmetrical about the line x=2, then

a) $f(x) = -f(-x)$
b) $f(2+x) = f(2-x)$
c) $f(x) = f(-x)$
d) $f(x+2) = f(x-2)$

78. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is
a) $[1, 2]$
b) $[2, 3)$
c) $[1, 2]$
d) $[2, 3]$

79. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = tan^{-1}\frac{2x}{1-x^2}$, then $f$ is both one-one and onto when B is the interval
a) $(0, \frac{\pi}{2})$
b) $[0, \frac{\pi}{2})$
c) $[-\frac{\pi}{2}, \frac{\pi}{2})$
d) $(-\frac{\pi}{2}, \frac{\pi}{2})$

80. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

| Interval | Function |
|---|---|
| (a). $(-\infty, \infty)$ | $x^3 - 3x^2 + 3x + 3$ |
| (b). $[2, \infty)$ | $2x^3 - 3x^2 - 12x + 6$ |
| (c). $(-\infty, \frac{1}{3}]$ | $3x^2 - 2x + 1$ |
| (d). $(-\infty, -4)$ | $x^3 + 6x^2 + 6$ |

81. A real valued function $f(x)$ satisfies the functional equation

$$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$

where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to
a) $-f(x)$
b) $f(x)$
c) $f(a) + f(a-x)$
d) $f(-x)$

82. The Largest interval lying in $(\frac{-\pi}{2}, \frac{\pi}{2})$ for which the function,

$$f(x) = 4^{-x^2} + cos^{-1}(\frac{x}{2} - 1) + log(\cos x))$$

, is defined, is
a) $[-\frac{\pi}{4}, \frac{\pi}{2})$
b) $[0, \frac{\pi}{2})$
c) $[0, \pi]$
d) $(-\frac{\pi}{2}, \frac{\pi}{2})$

83. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where

$$Y = \{y \in N : y = 4x + 3 for some x \in N\}$$

a) $g(y) = \frac{3y+4}{3}$
b) $g(y) = 4 + \frac{y+3}{4}$
c) $g(y) = \frac{y+3}{4}$
d) $g(y) = \frac{y-3}{4}$

84. Let $f(x) = (x+1)^2 - 1$,x ≥ -1
Statement-1 : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}\}$
Statement-2 : $f$ is a bijection
a) Statement-1 is true,Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
b) Statement-1 is true, Statement-2 is false.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

85. For real x, let $f(x) = x^3 + 5x + 1$, then
a) $f$ is onto R but not one-one
b) $f$ is one-one and onto R
c) $f$ is neither one-one nor onto R
d) $f$ is one-one but not onto R

86. The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is
a) $(0, \infty)$
b) $(-\infty, 0)$
c) $(-\infty, \infty) - \{0\}$
d) $(-\infty, \infty)$

87. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to:
a) $f_3(x)$
b) $f_3(x)$
c) $f_2(x)$
d) $f_1(x)$

88. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:
a) 6
b) 8
c) 4
d) 14

89. If the function $f : R - \{1, -1\}$. A defined by f(x)=$\frac{x^2}{1-x^2}$, is surjective, then A is equal to:
a) R-$\{-1\}$

b) $[0, \infty)$
c) R-$[-1, 0)$
d) R-$(-1, 0)$

90. Let

$$\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1),$$

where the function f satisifies f(x+y)=f(x)f(y) for all natural numbers x,y and f(a) is = 2. Then the natural number $'a'$ is:

a) 2
b) 16
c) 4
d) 3

**Match the following**

91. Let the function defined in Colum 1 have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ and range $(-\infty, \infty)$

| Column I | Column II |
|---|---|
| (A) 1+2x | (p) onto but not one-one |
| (B) $\tan x$ | (q) one-one but not onto |
| | (r) one-one and onto |
| | (s) neither one-one nor onto |

92. Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$
Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

| Column I | Column II |
|---|---|
| (A) If $-1 < x < 1$, then $f(x)$ satisfies | (p) $0 < f(x) < 1$ |
| (B) If $1 < x < 2$, then $f(x)$ satisfies | (q) $f(x) < 0$ |
| (C) If $3 < x < 5$, then $f(x)$ satisfies | (r) $f(x) > 0$ |
| (D) If $x > 5$, then $f(x)$ satisfies | (s) $f(x) < 1$ |

93. Let $E_1 = \{x \in R : x \neq 1 and \frac{x}{x-1} > 0\}$ and $E_2 = \{x \in E_1 : \sin^{-1}\left(log_e(\frac{x}{x-1})\right) is a real number\}$. (Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$).
Let $f : E_1 \rightarrow R$ be the function defined by $f(x) = log_e(\frac{x}{x-1})$ and g: $E_2 \rightarrow$ R be the function defined by $g(x) = sin^{-1}(log_e(\frac{x}{x-1}))$.
The correct option is:

| LIST-I | LIST-II |
|---|---|
| P. The range of $f$ is | 1. $(-\infty, \frac{1}{1-e}] \cup [\frac{e}{e-1}, \infty)$ |
| Q. The range of g contains | 2. (0,1) |
| R. The domain of $f$ contains | 3. $[-\frac{1}{2}, \frac{1}{2}]$ |
| S. The domain of g is | 4. $(-\infty, 0) \cup (0, \infty)$ |
| | 5. $(-\infty, \frac{e}{e-1}]$ |
| | 6. $(-\infty, 0) \cup (\frac{1}{2}, \frac{e}{e-1}]$ |

a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 1$
b) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 6$
c) $P \rightarrow 3; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$
d) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$

## 4 Quadratic Equations and Inequations

1. The coefficient of $x^{99}$ in the polynomial $(x-1)(x-2)$............$(x-100)$ is........
2. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then $(p, q) =$ (..................., ...................)
3. If the product of the roots of the equation $x^2 - 3kx + 2e^{21nk} - 1 = 0$ is 7, then the roots are real for $k =$ ....................
4. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root then, the numerical value of $a$+$b$ is...................
5. The solution of equation $log_7 log_5(\sqrt{x+5} + \sqrt{x}) = 0$ is .......
6. If $x < 0, y < 0, x + y + \frac{x}{y} = \frac{1}{2}$ and $(x+y)\frac{x}{y} = -\frac{1}{2}$, then $x =$ ............and $y =$ ................
7. Let n and k be positive such that $n \geq \frac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, ....., x_k)$, $x_1 \geq 1, x_2 \geq 2, ..., x_k \geq k$, all integers, satisfying $x_1 + x_2 + ... + x_k = n$, is..........
8. The sum of all the real roots of the equation $|x-2|^2 + |x-1| - 2 = 0$ is................
9. For every integer $n > 1$, the inequality $(n!)^{\frac{1}{n}} < \frac{n+1}{2}$ holds.
10. The equation $2x^2 + 3x + 1 = 0$ has an irrational root.
11. If $a < b < c < d$, then the roots of the equation $(x-a)(x-c) + 2(x-b)(x-d) = 0$ are real and distinct.
12. If $n_1, n_2, ......n_p$ are p positive integers, whose sum is an even number, then the number of odd integers among them is odd.
13. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then $P(x)Q(x) = 0$ has at least two real roots.
14. If x and y are positive real numbers and m, n are any positive integers, then $\frac{x^n y^m}{(1+x^{2n})(1+y^{2m})} > \frac{1}{4}$

15. If l,m,n are real, l $\neq$ m, then the roots by the equation $(l-m)x^2-5(l+m)x-2(l-m) = 0$ are:
    a) Real and equal
    b) Complex
    c) Real and unequal
    d) None of these
16. The equation $x+2y+2z = 1$ and $2x+4y+4z = 9$ have
    a) Only one solution
    b) Only two solutions
    c) Infinite number of solutions
    d) None of these
17. If x,y and z are real and different and $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$ then u is always.
    a) non negative
    b) zero
    c) non positive
    d) none of these
18. Let $a > 0, b > 0$ and $c > 0$. Then the roots of the equation $ax^2 + bx + c = 0$
    a) are real and negative
    b) have negative real parts
    c) both (a) and(b)
    d) none of these
19. Both the roots of the equation $(x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b) = 0$ are always
    a) positive
    b) real
    c) negative
    d) none of these
20. The least value of the expression $2log_{10}x - log_x(0.01)$, for $x > 1$, is
    a) 10
    b) 2
    c) $-0.01$
    d) none of these
21. If $(x^2+px+1)$ is a factor of $(ax^3+bx+c)$, then
    a) $a^2 + c^2 = -ab$
    b) $a^2 - c^2 = -ab$
    c) $a^2 - c^2 = ab$
    d) none of these
22. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is
    a) 4
    b) 1
    c) 3
    d) 2
23. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school should be built at
    a) town B
    b) 45 km from town A
    c) town A
    d) 45km from town B
24. If p,q,r are any real numbers, then
    a) max(p,q)<max(p,q,r)
    b) min(p,q)= $\frac{1}{2}(p + q - |p - q|)$
    c) max(p,q)<min(p,q,r)
    d) none of these
25. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is
    a) $-4 < x \leq 0$
    b) $0 < x < 1$
    c) $-100 < x < 100$
    d) $-\infty < x < \infty$
26. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has
    a) no root
    b) one root
    c) two equal roots
    d) infinitely many roots
27. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval
    a) $[\frac{1}{2}, 2]$
    b) $[-1, 2]$
    c) $[-\frac{1}{2}, 1]$
    d) $[-1, \frac{1}{2}]$
28. If $log_{0.3}(x-1) < log_{0.09}(x-1)$, then x lies in the interval
    a) $(2, \infty)$
    b) $(1, 2)$
    c) $(-2, -1)$
    d) none of these
29. If $\alpha$ and $\beta$ are the roots of $x^2 + px + q = 0$ and $\alpha^4, \beta^4$ are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always
    a) two real roots
    b) two positive roots
    c) two negative roots
    d) one positive and one negative root

    *Question has more than one correct option.
30. Let a,b,c be real numbers, $a \neq 0$. If $\alpha$ is a

root of $a^2x^2 + bx + c = 0$. $\beta$ is the root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root $\gamma$ that always satisfies

a) $\gamma = \frac{\alpha+\beta}{2}$
b) $\gamma = \alpha + \frac{\beta}{2}$
c) $\gamma = \alpha$
d) $\alpha < \gamma < \beta$

31. Let $\alpha,\beta$ be the roots of the equation $(x-a)(x-b) = c, c \neq 0$. Then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are
    a) a,c
    b) b,c
    c) a,b
    d) $a+c, b+c$
32. The number of points of intersection of two curves $y = 2sinx$ and $y = 5x^2 + 2x + 3$ is
    a) 0
    b) 1
    c) 2
    d) $\infty$
33. If p,q,r are +ve and are in A.P., the roots of quadratic equation $px^2 + qx + r = 0$ are all real for
    a) $|\frac{r}{p} - 7| \geq 4\sqrt{3}$
    b) $|\frac{p}{r} - 7| \geq 4\sqrt{3}$
    c) all p nd r
    d) no p and r
34. Let p,q$\in \{1, 2, 3, 4\}$. The number of equations of the form $px^2 + qx + 1 = 0$ having real roots is
    a) 15
    b) 9
    c) 7
    d) 8
35. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then
    a) $a < 2$
    b) $2 \leq a \leq 3$
    c) $3 < a \leq 4$
    d) $a > 4$
36. If $\alpha$ and $\beta$ $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
    a) $0 < \alpha < \beta$
    b) $\alpha < 0 < \beta < |\alpha|$
    c) $\alpha < \beta < 0$
    d) $\alpha < 0 < |\alpha| < \beta$
37. If a,b,c,d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation
    a) $0 \leq M \leq 1$
    b) $1 \leq M \leq 2$
    c) $2 \leq M \leq 3$
    d) $3 \leq M \leq 4$
38. If $b > a$, then the equation $(x-a)(x-b)-1 = 0$ has
    a) both roots in (a,b)
    b) both roots in$(-\infty, a)$
    c) both roots in $(b, +\infty)$
    d) one root in $(-\infty, a)$ and the other in $(b, +\infty)$
39. For the equation $3x^2 + px + 3 = 0, p > 0$, if one of the root is square of the other, then p is equal to
    a) $1/3$
    b) 1
    c) 3
    d) $2/3$
40. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is
    a) $(-\infty, -2) \cup (2, \infty)$
    b) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
    c) $(-\infty, -1) \cup (1, \infty)$
    d) $(\sqrt{2}, \infty)$
41. If $\alpha \in (0, \frac{\pi}{2})$ then $\sqrt{x^2 + x} + \frac{tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to
    a) $2tan\alpha$
    b) 1
    c) 2
    d) $sec^2\alpha$
42. For all 'x',$x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is
    a) $a < -5$
    b) $-5 < a < 2$
    c) $a > 5$
    d) $2 < a < 5$
43. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is
    a) $p^3 - q(3p - 1) + q^2 = 0$
    b) $p^3 - q(3p + 1) + q^2 = 0$
    c) $p^3 + q(3p - 1) + q^2 = 0$
    d) $p^3 + q(3p + 1) + q^2 = 0$
44. Let a,b,c be the sides of a triangle where $a \neq$

$b \neq c$ and $\lambda \in$R. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then

a) $\lambda < \frac{4}{3}$
b) $\lambda > \frac{5}{3}$
c) $\lambda \in (\frac{1}{3}, \frac{5}{3})$
d) $\lambda \in (\frac{4}{3}, \frac{5}{3})$

45. Let $\alpha,\beta$ be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is
a) $\frac{2}{9}(p-q)(2q-p)$
b) $\frac{2}{9}(q-p)(2p-q)$
c) $\frac{2}{9}(q-2p)(2q-p)$
d) $\frac{2}{9}(2p-q)(2q-p)$

46. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If $\alpha$ and $\beta$ are non zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then the quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is
a) $(p^3+q)x^2 - (p^3+2q)x + (p^3+q) = 0$
b) $(p^3+q)x^2 - (p^3-2q)x + (p^3+q) = 0$
c) $(p^3-q)x^2 - (5p^3-2q)x + (p^3-q) = 0$
d) $(p^3-q)x^2 - (5p^3+2q)x + (p^3-q) = 0$

47. Let $(x_0, y_0)$ be the solution of the following equations $(2x)^{ln2} = (3y)^{ln3}$ and $3^{lnx} = 2^{lny}$. Then $x_0$ is
a) $\frac{1}{6}$
b) $\frac{1}{3}$
c) $\frac{1}{2}$
d) 6

48. Let $\alpha$ and $\beta$ be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is
a) 1
b) 2
c) 3
d) 4

49. A value of b for which the equations
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is
a) $-\sqrt{2}$
b) $-i\sqrt{3}$
c) $i\sqrt{5}$
d) $\sqrt{2}$

50. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation p(p(x))=0 has
a) one purely imaginary root
b) all real roots
c) two real and two purely imaginary roots
d) neither real nor purely imaginary roots

51. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose $\alpha_1$ and $\beta_1$ are the roots of the equation $x^2 - 2x(sec\alpha) + 1 = 0$ and $\alpha_2$ and $\beta_2$ are the roots of the equation $x^2 - 2xtan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
a) $2(sec\theta - tan\theta)$
b) $2sec\theta$
c) $-2tan\theta$
d) 0

52. For real x, the function$\frac{(x-a)(x-b)}{x-c}$ will assume all real values provided
a) $a > b > c$
b) $a < b < c$
c) $a > c > b$
d) $a < c < b$

53. If S is the set of all real x such that$\frac{2x-1}{2x^2+3x^2+x}$ is positive, then S contains
a) $[-\infty, -\frac{3}{2}]$
b) $[-\frac{3}{2}, -\frac{1}{4}]$
c) $[-\frac{1}{4}, \frac{1}{2}]$
d) $[\frac{1}{2}, 3]$

54. If a,b and c are distinct positive numbers, then the expression (b+c-a)(c+a-b)(a+b-c)-abc is
a) positive
b) negative
c) non-positive
d) non-negative
e) none of these

55. If a,b,c,d and p are distinct real numbers such that
$(a^2+b^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \leq 0$
then a,b,c,d
a) are in A.P.
b) are in G.P.
c) are in H.P.
d) satisfy ab=cd
e) none of these

56. The equation $x^{3/4(log_2 x)^2 + log_2 x^{-5/4}} = \sqrt{2}$ has
a) at least one real solution
b) exactly three solutions
c) exactly one irrational solution
d) complex roots

57. The product of n positive numbers is unity. Then their sum is
    a) a positive integer
    b) divisible by n
    c) equal to $n + \frac{1}{n}$
    d) never less than n
58. Number of divisor of the form $4n + 2(n \geq 0)$ of the integer 240 is
    a) 4
    b) 8
    c) 10
    d) 3
59. If $3^x = 4^{x-1}$, then $x =$
    a) $\frac{2log_3 2}{2log_3 2-1}$
    b) $\frac{2}{2-2log_2 3}$
    c) $\frac{1}{1-log_4 3}$
    d) $\frac{2log_2 3}{2log_2 3-1}$
60. Let S be the set of all non-zero real numbers $\alpha$ such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots $x_1$ and $x_2$ satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is (are) a subset (s) of S?
    a) $(-\frac{1}{2}, -\frac{1}{\sqrt{5}})$
    b) $(-\frac{1}{\sqrt{5}}, 0)$
    c) $(0, \frac{1}{\sqrt{5}})$
    d) $(\frac{1}{\sqrt{5}}, \frac{1}{2})$
61. Solve for x: $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$
62. If (m,n)= $\frac{(1-x^m)(1-x^{m-1}).......(1-x^{m-n+1})}{(1-x)(1-x^2).........(1-x^n)}$ where m and n are positive integers($n \leq m$), show that $(m, n+1) = (m-1, n+1) + x^{m-n-1}(m-1, n)$.
63. Solve for x: $\sqrt{x+1} - \sqrt{x-1} = 1$.
64. Solve the following equation for x:

$$2log_x a + log_{ax} a + 3log_{a^2 x} a = 0, a > 0. \tag{4.0.64.1}$$

65. Show that the square of $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$,is a rational number.
66. Sketch the solution set of the following system of inequalities:

$$x^2 + y^2 - 2x \geq 0; 3x - y - 12 \leq 0; y - x \leq 0; y \geq 0. \tag{4.0.66.1}$$

67. Find all integers x for which $(5x - 1) < (x + 1)^2 < (7x - 3)$
68. If $\alpha, \beta$ are the roots of $x^2 + px + q = 0$ and $\gamma, \delta$ are the roots of $x^2 + rx + s = 0$, evalute$(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$ in terms of p,q,,r and s. Deduce the condition that the equations have a common root.
69. Given $n^4 < 10^n$ for fixed positive integer $n \geq 2$ prove that $(n + 1)^4 < 10^{n+1}$
70. Let y= $\sqrt{\frac{(x+1)(x-3)}{(x-2)}}$ Find all the real values of x for which y takes real values.
71. For what values of m, does the system of equations $3x+my = m$,$2x-5y = 20$ has solution satisfying the condition $x > 0, y > 0$.
72. Find the solution set of the system $x+2y+z = 1$; $2x - 3y - w = 2$; $x \geq 0; y \geq 0; z \geq 0; w \geq 0$.
73. Show that the equation$e^{sinx} - e^{-sinx} - 4 = 0$ has no real solution.
74. mm squares of equal size are arranged to form a rectangle of dimension m by n, where m and n are natural numbers. Two squares will be called 'neighbours' if they have exactly one common side. A natural number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighbouring squares. Show that this is possible only if all the numbers used are equal.
75. If one root of the quadratic equation $ax^2+bx+c = 0$ is equal to the n-th power of the other, then show that $(ac)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$
76. Find all real values of x which satisfy $x^2-3x+2 > 0$ and $x^2 - 2x - 4 \leq 0$.
77. Solve for x; $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3} = 10$.
78. For $a \leq 0$, determine all real roots of the equation$x^2 - 2a|x - a| - 3a^2 = 0$
79. Find the set of all x for which $\frac{2x}{(2x^2+5x+2)} > \frac{1}{(x+1)}$.
80. Solve $x^2 + 4x + 3 + 2x + 5 = 0$
81. Let a,b,c be real. If $ax^2 + bx + c = 0$ has two real roots $\alpha$ and $\beta$, where $\alpha < -1$ and $\beta > 1$, then show that $1 + \frac{c}{a} + |\frac{b}{a}| < 0$.
82. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a,b,c and d denote the lengths of the sides of the quadrilateral, prove that $2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$.
83. If $\alpha, \beta$ are the roots of $ax^2+bx+c = 0, (a \neq 0)$ and $\alpha+\delta, \beta+\delta$ are the roots of $Ax^2+Bx+C = 0, (A \neq 0)$ for some constant $\delta$, then prove that $\frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$.
84. Let a,b,c be real numbers with $a \neq 0$ and let $\alpha, \beta$ be the roots of the equation $ax^2+bx+c = 0$.

Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of $\alpha, \beta$.

85. If $x^2 + (a - b)x + (1 - a - b) = 0$ where a,b∈R then find the values of a for which equation has unequal real roots for all values of b.
86. If a,b,c are positive real numbers. Then prove that $(a + 1)^7(b + 1)^7(c + 1)^7 > 7^7a^4b^4c^4$.
87. Let a and b be the roots of the equation $x^2 - 10ax - 11b = 0$ are c,d then the value of $a + b + c + d$, when $a \neq b \neq c \neq d$, is.
88. Let p,q be integers and let $\alpha, \beta$ be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n= $0, 1, 2, \ldots\ldots.$,let $a_n = p\alpha^n + q\beta^n$. FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a= 0 =b.s
89. $a_{12} =$
    a) $a_{11} - a_{10}$
    b) $a_{11} + a_{10}$
    c) $2a_{11} + a_{10}$
    d) $a_{11} + 2a_{10}$
90. If $a_4 = 28$, then $p + 2q =\in$
    a) 21
    b) 14
    c) 7
    d) 12
91. Let a,b,c,p,q be real numbers. Suppose $\alpha, \beta$ are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2+2bx+c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$
    STATEMENT-1:$(p^2 - q)(b^2 - ac) \geq 0$ and
    STATEMENT-2:$b \neq pa$ or $c \neq qa$
    a) STATEMENT-1 is True, STATEMENT-2 is True;STATEMENT-2 is a correct explanation for STATEMENT-1
    b) STATEMENT-1 is True,STATEMENT-2 is True;STATEMENT-2 is NOT a correct explanation for STATEMENT-1
    c) STATEMENT-1 is True,STATEMENT-2 is False
    d) STATEMENT-1 is False,STATEMENT-2 is True
92. Let (x,y,z) be points with integer coordinates satisfying the system of homogeneous equations: $3x-y-z = 0$,$-3x+z = 0$,$-3x+2y+z = 0$ Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is
93. The smallest value of k, for which both the roots of the equation $x^2-8kx+16(k^2-k+1) = 0$ are real, distinct and have values at least 4 is
94. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and $a^{10}$ where $a > 0$ is
95. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is
96. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having $\alpha/\beta$ and $\beta/\alpha$ as its roots is
    a) $3x^2 - 19x + 3 = 0$
    b) $3x^2 + 19x - 3 = 0$
    c) $3x^2 - 19x - 3 = 0$
    d) $x^2 - 5x + 3 = 0$
97. Difference between the corresponding roots of $x^2+ax+b = 0$ and $x^2+bx+a = 0$ is same and $a \neq b$, then
    a) $a + b + 4 = 0$
    b) $a + b - 4 = 0$
    c) $a - b - 4 = 0$
    d) $a - b + 4 = 0$
98. Product of real roots of the equation $t^2x^2+|x|+9 = 0$
    a) is always positive
    b) is always negative
    c) does not exist
    d) none of these
99. If p and q are the roots of the equation $x^2 + px + q = 0$, then
    a) $p = 1, q = -2$
    b) $p = 0, q = 1$
    c) $p = -2, q = 0$
    d) $p = -2, q = 1$
100. If a,b,c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
    a) less than 1
    b) equal to 1
    c) greater than 1
    d) any real number
101. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
    a) Arithmetic-Geometric Progression
    b) Arithmetic Progression
    c) Geometric Progression
    d) Harmonic Progression
102. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2 = 0$ is twice as large as the other is
    a) $-\frac{1}{3}$
    b) $\frac{2}{3}$

c) $-\frac{2}{3}$
d) $\frac{1}{3}$

103. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
a) 3
b) 2
c) 4
d) 1

104. The real number x when added to its inverse gives the minimum value of the sum at x equal to
a) $-2$
b) 2
c) 1
d) $-1$

105. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation
a) $x^2 - 18x - 16 = 0$
b) $x^2 - 18x + 16 = 0$
c) $x^2 + 18x - 16 = 0$
d) $x^2 + 18x + 16 = 0$

106. If (1-p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its root are
a) $-1, 2$
b) $-1, 1$
c) $0, -1$
d) $0, 1$

107. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is
a) 4
b) 12
c) 3
d) $\frac{49}{4}$

108. In a triangle PQR,$\angle R = \frac{\pi}{2}$. If $\tan(\frac{P}{2})$ and -$\tan(\frac{Q}{2})$ are roots of $ax^2 + bx + c = 0, a \neq 0$ then
a) $a = b + c$
b) $c = a + b$
c) $b = c$
d) $b = a + c$

109. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval
a) $(5, 6]$
b) $(6, \infty)$
c) $(-\infty, 4)$
d) $[4, 5]$

110. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of 2+q-p is
a) 2
b) 3
c) 0
d) 1

111. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than $-2$ but less than 4, lies in the interval
a) $-2 < m < 0$
b) $m > 3$
c) $-1 < m < 3$
d) $1 < m < 4$

112. If x is real , the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is
a) $\frac{1}{4}$
b) 41
c) 1
d) $\frac{17}{7}$

113. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
a) $(3, \infty)$
b) $(-\infty, -3)$
c) $(-3, 3)$
d) $(-3, \infty)$

114. Statement-1: For every natural number $n \geq 2$, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$ Statement-2: For every natural number $n \geq 2$, $\sqrt{n(n+1)} < n + 1$
a) Statement-1 is false, Statement-2 is true
b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
d) Statement-1 is true, Statement-2 is false

115. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4:3. Then the common root is
a) 1
b) 4
c) 3
d) 2

116. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is:
   a) less than $4ab$
   b) greater than $-4ab$
   c) less than $-4ab$
   d) greater than $4ab$
117. If $|z - \frac{4}{z}| = 2$, then the maximum value of $|Z|$ is equal to:
   a) $\sqrt{5} + 1$
   b) 2
   c) $2 + \sqrt{2}$
   d) $\sqrt{3} + 1$
118. If $\alpha$ and $\beta$ are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
   a) $-1$
   b) 1
   c) 2
   d) $-2$
119. The equation $e^{sinx} - e^{-sinx} - 4 = 0$ has:
   a) infinite number of real roots
   b) no real roots
   c) exactly one real root
   d) exactly four real roots
120. The real number $k$ for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]
   a) lies between 1 and 2
   b) lies between 2 and 3
   c) lies between $-1$ and 0
   d) does not exist
121. The number of values of $k$, for which the system of equations:
   $(k + 1)x + 8y = 4k$
   $kx + (k + 3)y = 3k - 1$
   a) infinite
   b) 1
   c) 2
   d) 3
122. If the equations $x^2+2x+3 = 0$ and $ax^2+bx+c = 0$, a,b,c∈R, have a common root, then a:b:c is
   a) 1 : 2 : 3
   b) 3 : 2 : 1
   c) 1 : 3 : 2
   d) 3 : 1 : 2
123. Is a∈R and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where[x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval:
   a) $(-2, -1)$
   b) $(-\infty, -2) \cup (2, \infty)$
   c) $(-1, 0) \cup (0, 1)$
   d) $(1, 2)$
124. Let $\alpha$ and $\beta$ be the roots of the equation $px^2 + qx + r = 0, p \neq 0$. If $p, q, r$ are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is:
   a) $\frac{\sqrt{34}}{9}$
   b) $\frac{2\sqrt{13}}{9}$
   c) $\frac{\sqrt{61}}{9}$
   d) $\frac{2\sqrt{17}}{9}$
125. Let $\alpha$ and $\beta$ be the roots of equation $x^2-6x-2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to:
   a) 3
   b) $-3$
   c) 6
   d) $-6$
126. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is:
   a) 6
   b) 5
   c) 3
   d) $-4$
127. If $\alpha, \beta \in$ C are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:
   a) 0
   b) 1
   c) 2
   d) $-1$
128. Let p,q ∈R. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then:
   a) $p^2 - 4q + 12 = 0$
   b) $q^2 - 4p - 16 = 0$
   c) $q^2 + 4p + 14 = 0$
   d) $p^2 - 4q - 12 = 0$

## 5 Indefinite integrals

1. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}} dx = Ax + Blog(9e^{2x} - 4) + C$, then A =.........., B =.........., and C =...........

**MCQs with One Correct Answer:**

2. The value of the integral $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x} dx$ is
   a) $\sin x - 6\tan^{-1}(\sin x) + C$
   b) $\sin x - 2(\sin x)^{-1} + C$
   c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$
   d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$

3. If $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x$, then $f(\frac{1}{\sqrt{3}})$ is
   a) $\frac{1}{3}$
   b) $\frac{1}{\sqrt{3}}$
   c) 3
   d) $\sqrt{3}$
4. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx =$
   a) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + C$
   b) $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + C$
   c) $\frac{\sqrt{2x^4-2x^2+1}}{x} + C$
   d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + C$
5. Let I $= \int \frac{e^x}{e^{4x}+e^{2x}+1}dx$, J $= \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. Then for an arbitary constant C, then the value of J - I equals
   a) $\frac{1}{2}log(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}) + C$
   b) $\frac{1}{2}log(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}) + C$
   c) $\frac{1}{2}log(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}) + C$
   d) $\frac{1}{2}log(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}) + C$
6. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals(for some arbitary constant K)
   a) $-\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\} + K$
   b) $\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\} + K$
   c) $-\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\} + K$
   d) $\frac{1}{(\sec x+\tan x)^{11/2}}\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\} + K$

**Subjective Problems:**

7. Evaluate
$$\int \frac{\sin x}{\sin x - \cos x}dx$$
8. Evaluate
$$\int \frac{x^2}{(a+bx)^2}$$
9. Evaluate
$$\int (e^{\log x} + \sin x)\cos x dx$$
10. Evaluate:
$$\int \frac{(x-1)e^x}{(x+1)^3}dx$$
11. Evaluate the following
$$\int \frac{dx}{x^2(x^4+1)^3/4}$$
12. Evaluate the following
$$\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$$
13. Evaluate:
$$\int [\frac{(\cos 2x)^1/2}{\sin x}]dx$$
14. Evaluate
$$\int (\sqrt{\tan x} + \sqrt{\cot x})dx$$
15. Find the indefinite integral
$$\int (\frac{1}{x^{1/3}+4^{1/4}} + \frac{ln(1+x^{1/6})}{x^{1/3}+x^{1/2}})dx$$
16. Find the indefinite integral
$$\int \cos 2\theta ln(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta})d\theta$$
17. Evaluate
$$\int \frac{(x+1)}{x(1+xe^x)^2}dx$$
18. Integrate
$$\frac{x^3+3x+2}{(x^2+1)^2(x+1)}dx$$
19. Evaluate
$$\int \sin^{-1}(\frac{2x+2}{\sqrt{4x^2+8x+13}})dx.$$
20. For any natural number m, evaluate
$$\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{l/m}dx, x > 0$$

**Assertion and Reason Type Questions:**

21. Let F(x) be an indefinite integral of $\sin^2 x$.
   **STATEMENT-1:** The function F(x) satisfies $F(x+\pi) = F(x)$ for all real x.
   **STATEMENT-2:** $\sin^2(x+\pi) = \sin^2 x$ for all real x.
   a) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1.
   b) Statement-1 is True, Statement-2 is True, Statement-2 is NOT a correct explanation for Statement-1.
   c) Statement-1 is True, Statement-2 is False.
   d) Statement-1 is False, Statement-2 is True.

**Section - B:**

22. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + Blog\sin(x - alpha) + C$, then the value of (A, B) is
   a) $(-\cos\alpha, \sin\alpha)$
   b) $(\cos\alpha, \sin\alpha)$
   c) $(-\sin\alpha, \cos\alpha)$
   d) $(\sin\alpha, -\cos\alpha)$
23. $\int \frac{dx}{\cos x - \sin x}$ is equal to
   a) $\frac{1}{\sqrt{2}}log|\tan(\frac{x}{2}) + \frac{3\pi}{8}| + C$
   b) $\frac{1}{\sqrt{2}}log|\cot(\frac{x}{2})| + C$
   c) $\frac{1}{\sqrt{2}}log|\tan(\frac{x}{2}) - \frac{3\pi}{8}| + C$
   d) $\frac{1}{\sqrt{2}}log|\tan(\frac{x}{2}) - \frac{\pi}{8}| + C$
24. $\int\{\frac{(logx-1)}{1+(logx)^2}\}^2dx$ is equal to
   a) $\frac{logx}{(logx)^2+1} + C$
   b) $\frac{x}{x^2+1} + C$
   c) $\frac{xe^x}{1+x^2} + C$
   d) $\frac{x}{(logx)^2+1} + C$
25. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals
   a) $log\tan(\frac{x}{2} + \frac{\pi}{12}) + C$
   b) $log\tan(\frac{x}{2} - \frac{\pi}{12}) + C$
   c) $\frac{1}{2}log\tan(\frac{x}{2} + \frac{\pi}{12}) + C$
   d) $\frac{1}{2}log\tan(\frac{x}{2} - \frac{\pi}{12}) + C$
26. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin(x-\frac{\pi}{4})}$ is
   a) $x + log|\cos(x - \frac{\pi}{4})| + C$
   b) $x - log|\sin(x - \frac{\pi}{4})| + C$
   c) $x + log|\sin(x - \frac{\pi}{4})| + C$
   d) $x - log|\cos(x - \frac{\pi}{4})| + C$
27. If the $\int \frac{5\tan x}{\tan x - 2}dx = x + aln|\sin x - 2\cos x| + k$, then a is equal to:
   a) -1
   b) -2
   c) 1
   d) 2
28. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to
   a) $\frac{1}{3}[x^3\psi(x^3) - \int x^2\psi(x^3)dx] + C$
   b) $\frac{1}{3}[x^3\psi(x^3) - 3\int x^3\psi(x^3)dx] + C$
   c) $\frac{1}{3}[x^3\psi(x^3) - \int x^2\psi(x^3)dx] + C$
   d) $\frac{1}{3}[x^3\psi(x^3) - \int x^3\psi(x^3)dx] + C$
29. The integral $\int(1 + x - \frac{1}{x})e^{x+\frac{1}{x}}dx$ is equal to
   a) $(x + 1)e^{x+\frac{1}{x}} + C$
   b) $(-x)e^{x+\frac{1}{x}} + C$
   c) $(x - 1)e^{x+\frac{1}{x}} + C$
   d) $(x)e^{x+\frac{1}{x}} + C$
30. The integral $\int \frac{dx}{x^2(x^4+1)^3/4}$ equals
   a) $-(x^4 + 1)^{1/4} + C$
   b) $-(\frac{x^4+1}{x^4})^{1/4} + C$
   c) $(\frac{x^4+1}{x^4}) + C$
   d) $(x^4 + 1)^{1/4} + C$
31. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to:
   a) $\frac{x^5}{2(x^5+x^3+1)^2} + C$
   b) $\frac{-x^{10}}{2(x^5+x^3+1)^2} + C$
   c) $\frac{-x^5}{(x^5+x^3+1)^2} + C$
   d) $\frac{x^{10}}{2(x^5+x^3+1)^2} + C$
32. Let $I_n = \int \tan x dx, (n > 1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to:
   a) $(-\frac{1}{5}, 0)$
   b) $(-\frac{1}{5}, 1)$
   c) $(\frac{1}{5}, 0)$
   d) $(\frac{1}{5}, -1)$
33. The integral
$$\frac{\sin^2 x\cos^2 x}{(\sin^5 x + \cos^3 x\sin^3 x\cos^2 + \cos^5 x)^2}dx$$
   is equal to
   a) $\frac{-1}{3(1+\tan^3 x)} + C$
   b) $\frac{1}{1+\cot^3 x} + C$
   c) $\frac{-1}{1+\cot^3 x} + C$
   d) $\frac{1}{3(1+\tan^3 x)} + C$
34. For $x^2 \neq n\pi + 1, n \in N$(the set of natural numbers), the integral
$$\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}dx$$
   is equal to
   a) $log_e|\frac{1}{2}\sec^2(x^2 - 1)| + C$
   b) $\frac{1}{2}log_e|\sec(x^2 - 1)| + C$
   c) $\frac{1}{2}log_e|\sec^2(\frac{x^2-1}{2})| + C$
   d) $log_e|\sec(\frac{x^2-1}{2})| + C$
35. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to
   a) $-3\tan^{-1/3} x + C$
   b) $-\frac{3}{4}\tan^{-4/3} x + C$
   c) $-3\cot^{-1/3} x + C$
   d) $3\tan^{-1/3} x + C$

## 6 Differential Equations

1. A solution of the following diffrential equation is

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$$

a) y = 2
b) y = 2x
c) y = 2x - 4
d) $y = 2x^2 - 4$

2. If $x^2 + y^2 = 1$, then
a) $yy'' - 2(y')^2 + 1 = 0$
b) $yy'' + (y')^2 + 1 = 0$
c) $yy'' + (y')^2 - 1 = 0$
d) $yy'' + 2(y')^2 + 1 = 0$

3. If $y(t)$ is a solution of

$$(1+t)\frac{dy}{dt} - ty = 1, y(0) = -1$$

then $y(1)$ is equal to
a) -1/2
b) e + 1/2
c) e - 1/2
d) 1/2

4. If $y = f(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = \cos x$ y(0) = -1, then $y\left(\frac{\pi}{2}\right)$ equals
a) 1/3
b) 2/3
c) -1/3
d) 1

5. If $y = f(x)$ and it is followsthe relation $x\cos y + y\cos x = \pi$ then $y''(0) =$
a) 1
b) -1
c) $\pi - 1$
d) $\pi$

6. The solution of primitive integral equation $(x^2 + y^2)dy = xydx$ is $y = y(x)$. If $y(1) = 1$ and $(x_0 = e)$, then $x_0$ is equal to
a) $\sqrt{2(e^2-1)}$
b) $\sqrt{2(e^2+1)}$
c) $\sqrt{3}e$
d) $\sqrt{\frac{e^2+1}{2}}$

7. For the primitive integral equation $ydx + y^2dy = x$; $x \in R$, $y > 0$, $y = y(x)$, $y(1) = 1$, then $y(-3)$ is
a) 3
b) 2
c) 1
d) 5

8. The differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$$

determines a family of circles with
a) variable radii and a fixed centre at (0, 1)
b) variable radii and a fixed centre at (0, -1)
c) fixed radius 1 and variables centres along the x-axis
d) fixed radius 1 and variables centres along the y-axis

9. The function $y = f(x)$ is the solution of the differential equation

$$\frac{dy}{dx} = \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$$

in (1, -1) satisfying f(0) = 0. Then

$$\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx$$

is
a) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$
b) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
c) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$
d) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

10. If $y = y(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, x > 0$$

and $y(0) = \sqrt{7}$, then $y(256) =$
a) 3
b) 9
c) 16
d) 80

**MCQs with One or More than One Correct**

11. The order of the differential equation whose general solution is given by

$$y = (C_1 + C_2)\cos(x + C_3) - C_4e^x + C_5$$

where $C_1, C_2, C_3, C_4, C_5$ are arbitrary constants, is
a) 5
b) 4

c) 3
d) 2

12. The differential equation representing the family of curves

$$y = 2c\left(x + \sqrt{x}\right)$$

where c is a positive parameter, is of
a) order 1
b) order 2
c) degree 3
d) degree 4

13. A curve $y = f(x)$ passes through (1, 1) and at P(x, y), tangent cuts the x-axis and y-axis at A and B respectively such that BP : AP = 3 : 1. then
a) equation of curve is $xy' - 3y = 0$
b) normal at (1, 1) is $x + 3y = 4$
c) curve passes through (2, 1/8)
d) equation of curve is $xy' + 3y = 0$

14. If $y(x)$ satisfies the differential equation

$$y' = y\tan x = 2x\sec x$$

and $y(0) = 0$, then
a) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$
b) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$
c) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$
d) $y'\left(\frac{\pi}{3}\right) = y'\left(\frac{4\pi}{3}\right) + \frac{2\pi^2}{3\sqrt{3}}$

15. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be

$$\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$$

Then the equation of the curve is
a) $\sin\left(\frac{y}{x}\right) = logx + \frac{1}{2}$
b) cosec $\left(\frac{y}{x}\right) = logx + \frac{1}{2}$
c) $\sec\left(\frac{2y}{x}\right) = logx + \frac{1}{2}$
d) $\cos\left(\frac{2y}{x}\right) = logx + \frac{1}{2}$

16. Let $f(x)$ be a solution of the differential equation

$$(1 + e^x)y' + ye^x = 1$$

If $y(0) = 2$, then which of the following statementis (are) true?
a) $y(-4) = 0$
b) $y(-2) = 0$
c) $y(x)$ has a critical point in the interval (-1, 0)
d) $y(x)$ has no critical point in the interval (-1, 0)

17. Consider the family of all circles whose centres lie on the straight line y = x. If this family of circle is represented by the differential equation $Py''+Qy'+1 = 0$, where P, Q are functions of x, y and $y'$, then which of the following statement is(are) true?
a) P = y + x
b) P = y - x
c) $P + Q = 1 - x + y + y' + (y')^2$
d) $P - Q = x + y - y' - (y')^2$

18. Let $f : (0,\infty) \to R$ be a differentiable function such that

$$f'(x) = 2 - \frac{f(x)}{x}$$

for all $x \in (0,\infty)$ and $f(1) \neq 1$. Then
a) $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right) = 1$
b) $\lim_{x\to 0^+} xf'\left(\frac{1}{x}\right) = 2$
c) $\lim_{x\to 0^+} x^2 f'(x) = 0$
d) $|f(x)| \leq 2$ for all $x \in (0, 2)$

19. A solution curve of the following differential equation

$$(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0, x > 0$$

passes through the point (1, 3). Then the solution curve
a) intersects y = x + 2 exactly at one point
b) intersects y = x + 2 exactly at two points
c) intersects $y = (x + 2)^2$
d) does NOT intersect $y = (x + 2)^2$

20. Let $f : [0,\infty) \to R$ be a continuous function such that

$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)dt$$

for all $x \in [0,\infty)$. Then which of the following statement(s) is(are) true?
a) The curve $y = f(x)$ passes through the point (1, 2)
b) The curve $y = f(x)$ passes through the point (2, -1)
c) The area of the region

$$\left\{(x, y) \in [0, 1] \times R : f(x) \leq y \leq \sqrt{1 - x^2}\right\}$$

is $\frac{\pi-2}{4}$

d) The area of the region

$$\left\{(x, y) \in [0, 1] \times R : f(x) \leq y \leq \sqrt{1 - x^2}\right\}$$

is $\frac{\pi - 1}{4}$

21. Let $\Gamma$ denotes a curve $y = f(x)$ which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to $\Gamma$ at a point P intersects the y-axis at $Y_p$. If $PY_p$ has length 1 for each point P on $\Gamma$, then which of the following option(s) is(are) correct?

a) $y = -log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1 - x^2}$

b) $xy' - \sqrt{1 - x^2} = 0$

c) $y = log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1 - x^2}$

d) $xy' + \sqrt{1 - x^2} = 0$

**Subjective Problems**

22. If $(a + bx)e^{y/x} = x$, then prove that

$$x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)$$

23. A normal is drawn at apoint P(x, y) of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curve is

$$y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$$

Find the equation of such curve passing through (0, k).

24. Let $y = f(x)$ be a curve passing through (1, 1) such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves.

25. Determine the equation of the curve passing through the origin, in the form $y = f(x)$, which satisfies the differential equation

$$\frac{dy}{dx} = \sin(10x + 6y)$$

26. Let $u(x)$ and $v(x)$ satisfy the differential equation

$$\frac{du}{dx} + p(x)u = f(x)$$

$$\frac{dv}{dx} + p(x)v = g(x)$$

where $p(x)$, $f(x)$ and $g(x)$ are continuous functions. If $u(x_1) > v(x_1)$ for some $x_1$ and $f(x) > g(x)$ for all $x > x_1$, prove that any point (x, y) where $x > x_1$ does not satisfy the equation $y = u(x)$ and $y = v(x)$

27. A curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis. Determine the equation of the curve.

28. A country has a food deficit of 10 percentage. Its population grows continuously at a rate of 3 percentage per year. Its annual food production every year is 4 percentage more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\frac{ln10 - ln9}{ln(1.04) - 0.03}$.

29. A hemispherical tank of radius 2 metres is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law

$$v(t) = 0.6\sqrt{2gh(t)}$$

where $v(t)$ and $h(t)$ are respectively the velocity of the flow through the outlet and height of the water level above the outlet at time t, and g is the accelration due to gravity. Find the time it takes to empty the tank.

30. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air(proportional constant = $k > 0$). Find the time after which the cone is empty.

31. A curve $'C'$ passes through (2, 0) and the slope at (x, y) as

$$\frac{(x + 1)^2 + (y - 3)}{x + 1}$$

Find the equation of the curve. Find the area bounded by the curve and x-axis in fourth quadrant.

32. If length of tangent at any point on the curve $y = f(x)$ intercepted berween the point and the x-axis os of length 1. Find the equation of the curve.

**Section-B**

33. The order and degree of the differential equation

$$\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$$

are

a) $\left(1,\frac{2}{3}\right)$
b) (3, 1)
c) (3, 3)
d) (1, 2)

34. The solution of the equation

$$\frac{d^2y}{dx^2}=2e^{-x}$$

a) $\frac{e^{-2x}}{4}$
b) $\frac{e^{-2x}}{4}+cx+d$
c) $\frac{1}{4}e^{-2x}+cx^2+d$
d) $\frac{1}{4}e^{-4x}+cx+d$

35. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively.

a) 2, 3
b) 2, 1
c) 1, 2
d) 3, 2

36. The solution of the differential equation is

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$$

a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$
b) $(x-2)=ke^{2\tan^{-1}y}$
c) $2xe^{2\tan^{-1}y}=e^{2\tan^{-1}y}+k$
d) $xe^{\tan^{-1}y}=e^{\tan^{-1}y}+k$

37. The differential equation for the family of circle

$$x^2+y^2-2ay=0 \tag{6.0.37.1}$$

where a is an arbitraty constant is

a) $(x^2+y^2)y'=2xy$
b) $2(x^2+y^2)y'=xy$
c) $(x^2-y^2)y'=2xy$
d) $2(x^2-y^2)y'=xy$

38. Solution of the differential equation is

$$ydx+(x+x^2y)dy=0$$

a) $logy=Cx$
b) $-\frac{1}{xy}+logy=C$
c) $\frac{1}{xy}+logy=C$
d) $-\frac{1}{xy}=C$

39. The differential equation representing the family of curves

$$y^2=2c(x+\sqrt{c}),c>0$$

where c is a parameter, is of order and degree follows:

a) order 1, degree 2
b) order 1, degree 1
c) order 1, degree 3
d) order 2, degree 2

40. If

$$x\frac{dy}{dx}=y(logy-logx+1)$$

then the solution of the differential equation is

a) $ylog\left(\frac{x}{y}\right)=cx$
b) $xlog\left(\frac{y}{x}\right)=cy$
c) $log\left(\frac{y}{x}\right)=cx$
d) $log\left(\frac{x}{y}\right)=cy$

41. The diffrential equation whose solution is

$$Ax^2+By^2=1 \tag{6.0.41.1}$$

where A and B are arbitrary constants is of

a) second order and second degree
b) first order and second degree
c) first order and first degree
d) second order and first degree

42. The differential equation of all circles passing through the origin and having their centres on the x-axis is

a) $y^2=x^2+2xy\frac{dy}{dx}$
b) $y^2=x^2-2xy\frac{dy}{dx}$
c) $x^2=y^2+xy\frac{dy}{dx}$
d) $x^2=y^2+3xy\frac{dy}{dx}$

43. The solution of the diffrential equation

$$\frac{dy}{dx}=\frac{x+y}{x}$$

satisfying the condition $y(1)=1$ is

a) $y=lnx+x$
b) $y=xlnx+x^2$
c) $y=xe^{x-1}$
d) $y=xlnx+x$

44. The differential equation which represents the family of curves $y=c_1e^{c_2x}$ where $c_1$ and $c_2$ are arbitrary constants, is

a) $y'' = y'y$
b) $yy'' = y'$
c) $yy'' = (y')^2$
d) $y' = y^2$

45. Solution of the differential equation is

$$\cos x dy = y(\sin x - y)dx, 0 < x < \frac{\pi}{2}$$

a) $y \sec x = \tan x + c$
b) $y \tan x = \sec x + c$
c) $\tan x = (\sec x + c)y$
d) $\sec x = (\tan x + c)y$

46. If

$$\frac{dy}{dx} = y + 3 > 0, y(0) = 2$$

then $yln(2)$ is equal to

a) 5
b) 13
c) -2
d) 7

47. Let I be the purchase value of an equipment ans $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate of given by differential equation

$$\frac{dV(t)}{dt} = -k(T - t), k > 0$$

where k is constant and T is the total life in years of the eqipment. Then the scrap value $V(T)$ of the equipment is

a) $I - \frac{kT^2}{2}$
b) $I - \frac{k(T-t)^2}{2}$
c) $e^{-kT}$
d) $T^2 - \frac{1}{k}$

48. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation

$$\frac{dp(t)}{dt} = 0.5p(t) - 450$$

If $p(0) = 850$, then the time at which the population becomes zero is

a) $2ln18$
b) $ln9$
c) $\frac{1}{2}ln18$
d) $ln18$

49. At present, afirm is manufacturing 2000 items. It is estimated that the rate of chnage of production P w.r.t. additional number of workers x is given by

$$\frac{dP}{dx} = 100 - 12\sqrt{x}$$

If the firm employs 25 more workers, then the new value of production of items is

a) 2500
b) 3000
c) 3500
d) 4500

50. Let the population of rabbits surviving at time t be governed by the differential equation

$$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200.$$

If $p(0) = 100$, then $p(t)$ equals

a) $600 - 500e^{t/2}$
b) $400 - 300e^{-t/2}$
c) $400 - 300e^{t/2}$
d) $300 - 200e^{-t/2}$

51. Let $y(x)$ be the solution of the differential equation

$$(xlogx)\frac{dy}{dx} + y = 2xlogx, (x \geq 1)$$

Then $y(e)$ is equal to

a) 2
b) 2e
c) e
d) 0

52. If a curve $y = f(x)$ passes through the point (1, -1) and satisfies the differential equation

$$y(1 + xy)dx = xdy$$

then, $f\left(-\frac{1}{2}\right)$ is equal to

a) $\frac{2}{5}$
b) $\frac{4}{5}$
c) $-\frac{2}{5}$
d) $-\frac{4}{5}$

53. If

$$(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0, y(0) = 1$$

then $y\left(\frac{\pi}{2}\right)$ is equal to

a) 4/3
b) 1/3
c) -2/3
d) -1/3

54. Let $y - y(x)$ be the solution of the differential

equation

$$\sin x\frac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$$

If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

a) $\frac{-8}{9\sqrt{3}}\pi^2$
b) $\frac{-8}{9}\pi^2$
c) $\frac{-4}{9}\pi^2$
d) $\frac{4}{9\sqrt{3}}\pi^2$

55. If $y=y(x)$ is the solution of the differential equation

$$x\frac{dy}{dx}+2y=x^2$$

satisfying $y(a)=1$, then $y\left(\frac{1}{2}\right)$ is equal to

a) $\frac{7}{64}$
b) $\frac{1}{4}$
c) $\frac{49}{16}$
d) $\frac{13}{16}$

56. The solution of the differential equation

$$x\frac{dy}{dx}+2y=x^2(x\neq 0)$$

with $y(1)=1$ is

a) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$
b) $y=\frac{1}{5}x^3+\frac{1}{5x^2}$
c) $y=\frac{1}{4}x^2+\frac{3}{4x^2}$
d) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

**Assertion and Reason Type Questions**

57. Let a solution $y=y(x)$ of the differential equation

$$x\sqrt{x^2-1}dy-y\sqrt{y^2-1}dx=0$$

satisfy $y(2)=\frac{2}{\sqrt{3}}$.
**Statement-1:**

$$y(x)=\sec\left(\sec^{-1}x-\frac{\pi}{6}\right)$$

**Statement-2:**

$$y(x):\frac{1}{y}=\frac{2\sqrt{3}}{x}-\sqrt{1-\frac{1}{x^2}}$$

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-2
b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-2
c) Statement-1 is true, Statement-2 is false
d) Statement-1 is false, Statement-2 is true

**Integer Value Correct Type**

58. Let

$$y'(x)+y(x)g'(x)=g(x),g'(x),y(0)=0,x\in R$$

where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0)=g(2)=0$. Then the value of $g(2)$ is

59. Let $f:R\to R$ be a differentiable function with $f(0)=0$. If $y=f(x)$ satisfies the differential equation

$$\frac{dy}{dx}=2(2+5y)(5y-2)$$

then the value of $\lim_{x\to\infty}f(x)$ is..........

60. Let $f:R\to R$ be a differentiable function with $f(0)=1$ and satisfies the equation

$$f(x+y)=f(x)f'(y)+f'(x)f(y),y\in R$$

Then, the value of $log_e(f(4))$ is..............

**Match the Following Questions:**

61. Match the following

| **Column I** | **Column II** |
|---|---|
| (A) Interval contained in the domain of non-zero solutions of the differential equation $(x-3)^2 + y' + y = 0$ | (p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |
| (B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)$ $(x-5)dx$ | (q) $\left(0, \frac{\pi}{2}\right)$ |
| (C) Interval in which at least one of the points of local maximum of $\cos^2 + \sin x$ lies | (r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$ |
| (D) The Interval in which $\tan^{-1}(\sin x + \cos x)$ is | (s) $\left(0, \frac{\pi}{8}\right)$ |

## 7 Application of Derivatives

1. The largest of $\cos(ln\theta)$ and $\ln(\cos\theta)$ if $e^{\frac{\pi}{2}} < \theta < \frac{\pi}{2}$ is ..........
2. The function

$$y = 2x^2 - ln|x|$$

is monotonically increasing for values of $x(\neq 0)$ satisfying the inequalities ...... and monotonically decreasing for values of x satisfying the inequalities .....

3. The set of all x for which ln(1 + x) ≤ x is equal to ......
4. Let P be a variable point on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{7.0.4.1}$$

with the foci $F_1$ and $F_2$. If A is the area of the triangle P$F_1$ $F_2$ then the maximum value of A is......

5. Let C be the curve

$$y^2 - 3xy + 2 = 0 \tag{7.0.5.1}$$

If H is the set of points on the curve C where the tangent is horizontal and V is the set of the point on the curve C where the tangent is vertical then H = ....... and V = .......

**True/False:**

6. If x - r is the factor of the polynomial

$$f(x) = a_n x^4 + ..... + a_0,$$

repeated m times ($1 < m < n$), then r is a root of $f'(x) = 0$ repeated m times.

7. For $0 < a < x$, the minimum value of the function $log_a x + log_x$ a is 2.

**MCQs with One Correct Answer:**

8. If a + b + c = 0,Then the quadratic equation

$$3ax^2 + 2bx + c = 0 \tag{7.0.8.1}$$

has

a) at least one root in [0 , 1]
b) one root in [2,3] and the other in [-2,-1]
c) imaginary roots
d) none of these

9. AB is a diameter of a circle and C is any point on the circumference of the circle.Then

a) The area of ΔABC is maximum when it is isosceles
b) The area of ΔABC is maximum when it is isosceles
c) The perimeter of ΔABC is maximum when it is isosceles
d) none of these

10. The normal to the curve

$$x = a(\cos\theta + \theta\sin\theta)$$

$$y = a(\sin\theta - \theta\cos\theta)$$

at any point $'\theta'$ is such that.

a) it makes a constant angle with x-axis
b) it passes through the origin
c) it is at a constant distance from the origin
d) none of these

11. If

$$y = alnx + bx^2 + x \tag{7.0.11.1}$$

has its extremum values at x = -1 and x = 2 then

a) a = 2, b = -1
b) a = 2, b = $\frac{-1}{2}$
c) a = -2, b = $\frac{1}{2}$
d) none of these

12. Which one of the following curves cut the parabola

$$y^2 = 4ax \tag{7.0.12.1}$$

at right angles?

a) $x^2 + y^2 = a^2$

b) $y = e^{\frac{x}{2a}}$
c) y = ax
d) $x^2 = 4ay$

13. The function defined by

$$f(x) = (x+2)e^{-x}$$

is

a) decreasing for all x
b) decreasing in $(-\infty$, -1) and increasing in (-1, $\infty$)
c) increasing for all x
d) decreasing in (-1, $\infty$) and increasing in $(-\infty$, -1)

14. The function

$$f(x) = \frac{ln(\pi + x)}{ln(e + x)}$$

a) increasing on $(0, \infty, )$
b) decreasing on $(0, \infty)$
c) increasing on $(0, \frac{\pi}{e})$, decreasing on $(\frac{\pi}{e}, \infty)$
d) decreasing on $(0, \frac{\pi}{e})$, increasing on $(\frac{\pi}{e}, \infty)$

15. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

a) 0
b) $\frac{1}{4}$
c) $\frac{1}{2}$
d) $\frac{1}{3}$

16. The slope of the tangent to a curve $y = f(x)$ at $[x, f(x)]$ is 2x + 1. If the curve passes through the point (1, 2), then the area bounded by the curve, the x-axis and line x = 1 is

a) $\frac{5}{6}$
b) $\frac{6}{5}$
c) $\frac{1}{6}$
d) 6

17. If

$$f(x) = \frac{x}{\sin x} and$$

$$g(x) = \frac{x}{\tan x}$$

where $0 < x \leq 1$, then in this interval

a) both $f(x)$ and $g(x)$ are increasing functions
b) both $f(x)$ and $g(x)$ are decreasing functions
c) $f(x)$ is an increasing function
d) $g(x)$ is an increasing function

18. The function

$$f(x) = sin^4 x + cos^4 x$$

increasing if

a) $0 < x < \frac{\pi}{8}$
b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$
d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

19. Consider the following statements in S and R
**S:** Both sin x and cos x are decreasing functions in the interval $(\frac{\pi}{2}, \pi)$
**R:** The differentiable function decrease in an interval (a, b), then its derivative also decreases in (a,b).
Which of the following is true

a) Both S and R are wrong
b) Both S and R are correct but R is not the correct explanation for S
c) S is correct and R is correct explanation for S
d) S is Correct and R is wrong

20. Let

$$f(x) = \int e^x(x-1)(x-2)dx$$

Then f decrease in the interval

a) $(-\infty, -2)$
b) $(-2, -1)$
c) (1, 2)
d) $(2, \infty)$

21. If the normal ti the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$

a) -1
b) $\frac{-3}{4}$
c) $\frac{4}{3}$
d) 1

22. Let

$$f(x) = \left\{ \begin{cases} |x| & 0 < |x| \leq 2 \\ 1 & x = 0 \end{cases} \right\}$$

then at x = 0, f has

a) a local maximum
b) no local maximum
c) a local minimum
d) no extremum

23. For all $x \in (0, 1)$

a) $e^x < 1 + x$

b) $log_e(1+x) < x$
c) $\sin x > x$
d) $log_e x > x$

24. If $f(x) = xe^{x(1-x)}$ then f(x) is
a) increasing on $[\frac{-1}{2}, 1]$
b) decreasing on R
c) increasing on R
d) decreasing on $[\frac{-1}{2}, 1]$

25. the triangle formed by the tangents of the curve

$$f(x) = x^2 + bx - b$$

at the point (1, 1) and the coordinate axes, lies in the first quadrant. If its area is 2, Then the value of b is
a) -1
b) 3
c) -3
d) 1

26. Let

$$f(x) = (1+b^2)x^2 + 2bx + 1 \qquad (7.0.26.1)$$

and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is
a) $[0, 1]$
b) $(0, \frac{1}{2}]$
c) $[\frac{1}{2}, 1]$
d) $(0, 1]$

27. The length of the longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing is
a) $\frac{\pi}{3}$
b) $\frac{\pi}{2}$
c) $\frac{3\pi}{2}$
d) $\pi$

28. The points on the curve

$$y^3 + 3x^2 = 12y \qquad (7.0.28.1)$$

where the tangent is vertical, is
a) $(\pm\frac{4}{\sqrt{3}}, -2)$
b) $(\pm\sqrt{\frac{11}{3}}, 1)$
c) (0, 0)
d) $(\pm\frac{4}{\sqrt{3}}, 2)$

29. In [0, 1] Lagranges Mean Value theorem is NOT applicable to
a) f(x) =
$$\begin{cases} \frac{1}{2} - x & x < \frac{1}{2} \\ (\frac{1}{2} - x)^2 & x \geq \frac{1}{2} \end{cases}$$
b) f(x) =
$$\begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$
c) $f(x) = x|x|$
d) $f(x) = |x|$

30. Tangent is drawn to ellipse

$$\frac{x^2}{27} + y^2 = 1 at (3\sqrt{3}\cos\theta, \sin\theta)(where \theta \in (0, \frac{\pi}{2}))$$

Then the value of $\theta$ such that sum of intercepts on axes made by this tangent is minimum is
a) $\frac{\pi}{3}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{8}$
d) $\frac{\pi}{4}$

31. If

$$f(x) = x^3 + bx^2 + cx + d$$

and $0 < b^2 < c$, then in $(-\infty, \infty)$
a) $f(x)$ is strictly increasing function
b) $f(x)$ has local maxima
c) $f(x)$ is a strictly decreasing function
d) $f(x)$ is bounded

32. If

$$f(x) = x^\alpha log x$$

and $f(0) = 0$ then the value of $\alpha$ for which Rolle's theorem can be applied in [0, 1] is
a) -2
b) -1
c) 0
d) $\frac{1}{2}$

33. If $P(x)$ is a polynomial of degree less than or equal to 2 then S is the set of all such polynomials so that p(0) = 0, P(1) = 1 and $P'(x) > 0 \forall x \in [0, 1]$ then
a) $S = \Phi$
b) $S = ax + (1-a)x^2 \forall a \in (0, 2)$
c) $S = ax + (1-a)x^2 \forall a \in (0, \infty)$
d) $S = ax + (1-a)x^2 \forall a \in (0, 1)$

34. The tangent to the curve $y = e^x$ drawn at the point $(c, e^c)$ intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$
a) on the left of x = c
b) on the right of x = c
c) at no point
d) at all points

35. Consider the two curves

$$C_1 : y^2 = 4x$$

$$C_2 : x^2 + y^2 - 6x + 1 = 0$$

then,

a) $C_1$ and $C_2$ touch only each other at one point
b) $C_1$ and $C_2$ touch each other exactly at two point
c) $C_1$ and $C_2$ intersect at exactly two points
d) $C_1$ and $C_2$ neither intersect nor touch each other

36. The total number of local minima and local maxima of the function f(x)=

$$\begin{cases} (2+x)^3 & -3 < x \leq -1 \\ x^{\frac{2}{3}} & -1 < x < 2 \end{cases}$$

is

a) 0
b) 1
c) 2
d) 3

37. Let the function g: $(-\infty, \infty) \rightarrow (\frac{-\pi}{2}, \frac{\pi}{2})$ be given by

$$g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$$

Then g is

a) even and it is strictly increasing in $(0,\infty)$
b) odd and is strictly decreasing in $(-\infty, \infty)$
c) odd and is strictly increasing in $(-\infty, \infty)$
d) neither even nor odd but it is strictly increasing in $(-\infty, \infty)$

38. The least value of $a \in R$ for which $4\alpha x^2 + \frac{1}{x} \leq 1$, for all $x > 0$ is

a) $\frac{1}{64}$
b) $\frac{1}{32}$
c) $\frac{1}{27}$
d) $\frac{1}{25}$

39. If $f : R \rightarrow R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f(\frac{1}{2}) = \frac{1}{2}$, $f(1) = 1$, then

a) $f'(1) \leq 0$
b) $0 < f'(1) \leq \frac{1}{2}$
c) $\frac{1}{2} < f'(1) \leq 1$
d) $f'(1) > 1$

**MCQ's with One or More than One Correct Answer:**

40. Let

$$P(x) = a_0 + a_1x^2 + a_2x^4 .....a_nx^{2n}$$

be a polynomial equation in real variable x with $0 < a_0 < a_1 < a_2 < ..... < a_n$.The function $P(x)$ has

a) neither a maximum nor a minimum
b) only one maximum
c) only one minimum
d) only one minimum and one maximum
e) none of these

41. If the line ax + by + c = 0 is a normal to the curve xy = 1 then

a) $a > 0, b > 0$
b) $a > 0, b < 0$
c) $a < 0, b > 0$
d) $a < 0, b < 0$
e) none of these

42. The smallest positive root of the equation, $\tan x - x = 0$ lies in

a) $(0, \frac{\pi}{2})$
b) $(\frac{\pi}{2}, \pi)$
c) $(\pi, \frac{3\pi}{2})$
d) $(\frac{3\pi}{2}, 2\pi)$

43. Let $f$ and $g$ be the increasing and decreasing functions respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

a) always zero
b) always negative
c) always positive
d) strictly increasing
e) None of these

44. If

$$f(x) = \left\{ \begin{array}{ll} 3x^2 + 12x - 1 & -1 \leq x \leq 2 \\ 37 - x & 2 < x \leq 3 \end{array} \right\}$$

then:

a) $f(x)$ is increasing on [-1, 2]
b) $f(x)$ is continues on [-1, 3]
c) $f(2)$ does not exist
d) $f(x)$ has the maximum value at x = 2

45. If

$$h(x) = f(x) - (f(x))^2 + (f(x))^3$$

for every real number x. Then

a) h is increasing whenever f is increasing
b) h is increasing whenever f is decreasing

c) h is decreasing whenever f is decreasing
d) nothing can be said in general

46. If

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

for every real number then x then the minimum value of f
a) does not exist because f is unbounded
b) is not attained even though f is bounded
c) is equal to 1
d) is equal to -1

47. The number of values of x where function

$$f(x) = \cos x + \cos(\sqrt{2}x)$$

attains its maximum is
a) 0
b) 1
c) 2
d) infinite

48. The function

$$f(x) = \int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 dt$$

has a local minimum at x =
a) 0
b) 1
c) 2
d) 3

49. $f(x)$ is a cubic polynomial with $f(2) = 18$ and $f(1) = -1$. Also $f(x)$ has local maxima at x = -1 and $f'(x)$ has local minima at x = 0, then
a) the distance between (-1, 2) and $(af(a))$, where x = a is the point of local minima is $2\sqrt{2}$
b) $f(x)$ is increasing for $x \in [1, 2\sqrt{5}]$
c) $f(x)$ has local minima at x = 1
d) the value of $f(0) = 15$

50. Let f(x)=

$$\begin{cases} e^x & 1 < x \le 1 \\ 2 - e^{x-1} & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{cases}$$

and $g(x) = \int_0^x f(t)dt,\ x \in [1, 3]$ then $g(x)$ has
a) local maxima at x = 1 + ln2 and local minima at x = e
b) local maxima at x = 1 and local minima at x = 2
c) no local maxima
d) no local minima

51. For the function

$$f(x) = x\cos\frac{1}{x},\ x \ge 1,$$

a) for atleast one x in the interval

$$[1, \infty), f(x + 2) - f(x) < 2$$

b) $\lim_{x\to\infty} f'(x) = 1$
c) for all x in the interval

$$[1, \infty), f(x + 2) - f(x) > 2$$

d) $f'(x)$ is strictly decreasing for the interval $[1, \infty)$

52. If

$$f(x) = \int_0^x e^{x^2}(t - 2)(t - 3)dt$$

for all $x \in (0, \infty)$, then
a) f has local maxima at x = 2
b) f is decreasing on (2, 3)
c) there exist some $c \in (0, \infty)$, such that $f'(c) = 0$
d) f has a local minimum at x = 3

53. A rectangular sheet of fixed perimeter with sides having length in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are
a) 24
b) 32
c) 45
d) 60

54. Let $f : (0, \infty) \to R$ be given by

$$f(x) \int_{\frac{1}{x}}^{x} e^{-(t+\frac{1}{t})\frac{dt}{t}}$$

then
a) $f(x)$ is monotonically increasing on $[1, \infty)$
b) $f(x)$ is monotonically decreasing on(0, 1)
c) $f(x) + f(\frac{1}{x}) = 0$ for all $x \in (0, \infty)$
d) $f(2^x)$ is an odd function of x on R

55. Let $f, g : [-1, 2] \to R$ be continuous functions which are twice differentiable on the interval (-1, 2). Let the values of f and g at the point

-1, 0 and 2 be as given in the following table

| | x = -1 | x = 0 | x = 2 |
|---|---|---|---|
| f(x) | 3 | 6 | 0 |
| g(x) | 0 | 1 | -1 |

in each of the intervals (-1, 0) and (0, 2) the function $(f-3g)''$ never vanishes.Then the correct statement (s) is (are )

a) $f'(x)-3g'(x)=0$ has exactly three solutions in (-1, 0) ∪ (0, 2)
b) $f'(x)-3g'(x)=0$ has exactly one solutions in (-1, 0)
c) $f'(x)-3g'(x)=0$ has exactly one solutions in (0, 2)
d) $f'(x)-3g'(x)=0$ has exactly two solutions in (-1 ,0), exactly two solutions in (0, 2)

56. Let $f: R \to R$ is a differentiable functions such that $f'(x) > 2f(x)$ for all $x \in R$ and $f(0)=1$, then

a) $f(x)$ is increasing in (0, ∞)
b) $f(x)$ is decreasing in (0, ∞)
c) $f(x) > e^{2x}$ in (0, ∞)
d) $f'(x) > e^{2x}$ in (0, ∞)

57. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$ Then

a) $f'(x)=0$ at exactly three points in $(-\pi,\pi)$
b) $f'(x)=0$ at more than three points in $(-\pi,\pi)$
c) $f(x)$ attains its maximum at x = 0
d) $f(x)$ attains its minimum at x = 0

58. Defines collections $\{E_1, E_2, E_3, .....\}$ of ellipses and $\{R_1, R_2, R_3, ....\}$ of rectangles as follows:

$$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1 \tag{7.0.58.1}$$

$R_1$: Rectangle of largest area, with parallel sides to the axes inscribed in $E_1$

$$E_n : Ellipse \frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1 \tag{7.0.58.2}$$

of largest area inscribed in $R_{n-1}$, $n > 1$; $R_n$: Rectangle of largest area with sides parallel to the axes inscribed in $E_n$, $n > 1$.
Then which of the following options are correct?

a) The eccentricities of $E_1 8$ and $E_1 9$ are NOT equal
b) The length of latus rectum of $E_9$ is $\frac{1}{6}$
c) $\sum_{n=1}^{N}$ (area of $R_n$) < 24 for each positive integer N
d) The distance of a focus from the centre in $E_9$ is $\frac{\sqrt{5}}{32}$

59. Let $f: R \to R$ be given by

$$f(x) = (x-1)(x-2)(x-5)$$

Define

$$F(x) = \int_0^x f(t)dt, x > 0$$

Then when of the following options is/are correct?

a) F has a local maximum at x = 2
b) F has a local minimum at x = 1
c) F has two local maximum and one local minimum (0, ∞)
d) F(x) 0 for all x ∈ (0, 5)

60. Let

$$f(x) = \frac{\sin \pi x}{x^2}, x > 0$$

Let $x_1 < x_2 < x_3 ...... < x_n <....$ be all the points of local maximum of f and $y_1 < y_2 < y_3 <....< y_n <....$be all the points of local minimum of f. Then which of the following options is/are correct?

a) $x_{n+1} - x_n > 2$
b) $x_n \in (2n, 2n+\frac{1}{2})$ for every n
c) $|x_n - y_n| > 1$ for every n
d) $x_1 < y_1$

**E.Subjective Problems**

61. Prove that minimum value of

$$\frac{(a+x)(b+x)}{(c+x)}, a, b > c, x > -c$$

is

$$(\sqrt{a-c} + \sqrt{b-c})^2$$

62. Let x and y be two real variables such that $x > 0$ and xy = 1. Find minimum value of x + y.
63. For all x in [0, 1], Let the second derivative $f''(x)$ of a function $f(x)$ exist and satisfy $|f''(x)| < 1$. If f(0) = f(1) Then show that $|f'(x)| < 0$ for all x in [0, 1].
64. Use the function $f(x) = x^{\frac{1}{x}}$, $x > 0$ to determine the biggest of the two numbers $e^{\pi}$ and $\pi^{e}$.
65. If $f(x)$ and $g(x)$ are differentiable function for $0 \le x \le 1$ such that $f(0) = 2$, $g(0) = 0$,

$f(1) = 6$, $g(1) = 2$, then show that there exist c satisfying $0 < c < 1$ and $f'(c) = 2g'(c)$.

66. Find the shortest distance of the points(0, c) from the parabola $y = x^2$ where $0 \leq c \leq 5$.
67. If $ax^2 + \frac{b}{x} \geq c$ for all positive x where $a > 0$ and $b > 0$ show that $27ab^2 \geq 4c^3$.
68. Show that $1 + xln(x + \sqrt{x^2+1}) \geq \sqrt{1+x^2}$ for all $x \geq 0$.
69. Find the coordinates of the points on the curve $y = \frac{x}{1+x^2}$ where the tangent to the curve has the greatest slope.
70. Find all the tangents to the curve $y = \cos(x+y)$, $-2\pi \leq x \leq 2\pi$ that are parallel to the line x + 2y = 0.
71. Let $f(x) = \sin^3 x + \lambda \sin^2 x$, $\frac{-\pi}{2} < x < \frac{\pi}{2}$ find the intervals in which $\lambda$ should lie in order that $f(x)$ has exactly one minumum and one maximum.
72. Find the point on the curve

$$4x^2 + a^2y^2 = 4a^2, 4 < a^2 < 8$$

that is farthest from the point (0, -2).

73. Investigate maxima and minima the function

$$f(x) = \int_1^x [2(t-1)(t-2)^2 + 3(t-1)^2(t-2)^2]dt$$

74. Find all maxima and minima of the function $y = x(x-1)^2, 0 \leq x \leq 2$ also determine the area bounded by the curve $y = x(x-1)^2$ the y-axis and the line y = 2.
75. Show that $2\sin x + \tan x \geq 3x$ where $0 \leq x < \frac{\pi}{2}$.
76. A point P is given on the circumference of a circle of radius r. Chord QR is parallel to the tangent at P. Determine the maximum possible area of the triangle PQR.
77. A window of perimeter P is in the form of rectangle surrounded by a semicircle. The semicicular portion is fitted with coloured glass while the rectangular portion is fitted with the clear glass transmits three times as much light per square meter as the colour glass does. What is the ratio for the sides of rectangle so that the window transmits the maximum light?
78. A cubic $f(x)$ vanishes at x = 2 and has relative minimum and maximum at x = -1 and $x = \frac{1}{3}$ if

$$\int_{-1}^{1} f dx = \frac{14}{3}$$

find the cubic f(x).

79. What normal to the curve $y = x^2$ forms the shortest chord?
80. Find the equation of normal to the curve

$$y = (1+x)^y + \sin^{-1}(\sin^2 x)$$

at x=0.

81. Let f(x) =

$$\begin{cases} -x^3 + \frac{(b^3-b^2+b-1)}{(b^2+3b+2)} & 0 \leq x < 1 \\ 2x-3 & 1 \leq x \leq 3 \end{cases}$$

Find all possible real values of b such that $f(x)$ has the smallest value at x = 1.

82. The curve

$$y = ax^3 + bx^2 + cx + 5 \qquad (7.0.82.1)$$

touches the x-axis at P(-2, 0) and cuts the y axis at a point Q where its gradient is 3 . Find a, b, c

83. The circle

$$x^2 + y^2 = 1 \qquad (7.0.83.1)$$

cuts the x-axis at Pand Q another circle with centre at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S Find the maximum area of the triangle QSR.

84. Let (h, k) be a fixed point where $h > 0, k > 0$. A stright line passing through this point cuts the positive direction of the coordinate axis at points P and Q. Find the minimum area of triangle OPQ, O being the origin.
85. A curve $y = f(x)$ passes through the point p(1, 1). The normal to the curve at P is $a(y-1)+(x-1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, determine th equation of the curve also obtain the area bounded by the y-axis the curve and the normal to the curve at P.
86. Determine the points of maxima and minima of the function

$$f(x) = \frac{1}{8}lnx - bx + x^2$$

x > 0 where $b \geq 0$ is a constant.

87. Let f(x)=

$$\begin{cases} xe^{ax} & x \leq 0 \\ x + ax^2 - x^3 & x > 0 \end{cases}$$

where a is positive constant. Find the interval in which $f'(x)$ is increasing.

**Match the Following Questions:**

88. In this questions there are entries in column I and column II. Each entry in column I ia related to exactly one entry in column II.Write the correct letter from column II againest the entry number in column I in your answer book.Let the functions defined in column I have domain $(-\frac{\pi}{2}, \frac{\pi}{2})$

| Column I | Column II |
|---|---|
| (A) X + $\sin X$ | (p) increasing |
| (B) $\sec x$ | (q) decreasing |
| | (r) neither increasing nor decreasing |

By appropriately matching the matching the information given in the three columns of the following table. Let f(x)=x+$log_e x - xlog_e x$ $x \in (0, \infty)$

a) Column 1 contains information about zeros of f(x),f'(x) and f''(x).
b) Column 2 contains information about the limiting behaviour of f(x),f'(x) and f''(x) at infity
c) Column 3 contains information about the increasing/decreasing nature of f(x) and f'(x).

| Column-1 | Column-2 | Column-3 |
|---|---|---|
| (I) f(x)=0 for some $x \in (1, e^2)$ | (i) $lim_{x\to\infty} f(x) = 0$ | (P) f is increasing on (0, 1) |
| (II)f'(x) for $x \in (1, e)$ | (ii)$lim_{x\to\infty} f(x) = -\infty$ | (Q) f is increasing in $(e, e^2)$ |
| (III)f'(x) for $x \in (0, 1)$ | (iii)$lim_{x\to\infty} f'(x) = -\infty$ | (R) f' is increasing in (0, 1) |
| (IV)f''(x) for $x \in (1, e)$ | (iv)$lim_{x\to\infty} f''(x) = 0$ | (S) f' is decreasing in $(e, e^2)$ |

89. Which of the following option is the only correct combination
    a) (I)(i)(P)
    b) (II)(ii)(Q)
    c) (III)(iii)(R)
    d) (IV)(iv)(S)
90. Which of the following option is the only correct combination
    a) (I)(ii)(R)
    b) (II)(iii)(S)
    c) (III)(iv)(P)
    d) (IV)(i)(S)
91. Which of the following option is the only incorrect combination
    a) (I)(iii)(P)
    b) (II)(iv)(Q)
    c) (III)(i)(R)
    d) (II)(iii)(P)

**Comprehension Based Questions:**

**PASSAGE-1**

If a continuous function f defined on the real line R,assume positive and negative values in R then the equation f(x)=0 has a root in R.For example ,if it is known that a continuous function f on R is positive at some point and its minimum value is negative then the equation f(x)=0 has a root in R.consider f(x)=$ke^x - x$ for all real x where k is a real constant.

92. The line y = x meets $y = ke^x$ for $k \le 0$ at
   a) no point
   b) one point
   c) two points
   d) more than two points

93. The positive value of k for $ke^x - x = 0$ has only one root is
   a) $\frac{1}{e}$
   b) 1
   c) e
   d) $log_e 2$

94. for $k > 0$ the set of all values of k for which $ke^x - x = 0$ has two distinct roots
   a) (0, $\frac{1}{e}$)
   b) ($\frac{1}{e}$, 1)
   c) ($\frac{1}{e}$, ∞)
   d) (0, 1)

**PASSAGE-2**

Let f(x) = $(1+x)^2 \sin^2 x + x^2$ for all $x\ inIR$ and let g(x) = $\int_1^x (\frac{2(t-1)}{t+1} - lnt)$ f(t)dt for all $x \in (1, \infty)$

95. Consider the following statements:
   **P:** There exist some x ∈ R such that f(x) + 2x = 2(1 + $x^2$)
   **Q:** There exist some $x \in R$ such that 2f(x) + 1 = 2x(1 + x) Then
   a) Both P and Q are true
   b) P is true and Q is false
   c) P is false and Q is true
   d) Both P and Q are false

96. Which of the following is true?
   a) g is increasing on (0, ∞)
   b) g is decreasing on (1, ∞)
   c) g is increasing on (1, 2) and decreasing on(2, ∞)
   d) g is decreasing on (1, 2) and increasing on (2, ∞)

**PASSAGE-3**

Let f:[0, 1] → R be a function suppose the function f is twice differenciable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, x ∈ [0, 1].

97. Which of the following is true for $0 < x < 1$?
   a) $0 < f(x) < \infty$
   b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
   c) $-\frac{1}{4} < f(x) < 1$
   d) $-\infty < f(x) < 0$

98. If function $e^{-x}$ f(x) assumes its minimum in the interval [0, 1] at x = $\frac{1}{4}$ which of the following is true?
   a) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$
   b) $f'(x) > f(x), 0 < x < \frac{1}{4}$
   c) $f'(x) < f(x), 0 < x < \frac{1}{4}$
   d) $f'(x) < f(x), \frac{3}{4} < x < 1$

**Integer Value Correct Type:**

99. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set A = $\{|x|x^2 + 20 \le 9x\}$
100. Let p(x) be the polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}(1 + \frac{p(x)}{x^2}) = 2$. then the value of p(2) is
101. Let f be a real valued differential function on R such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P then find the value of f(-3).
102. Let f be a function defined on R such that $f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$ forall $x \in R$. If g is a function defined on R with values in the interval (0, ∞) such that f(x) = ln(g(x)),for all $x \in R$. Then the number of points at which g has a local maximum is
103. Let $f : IR \to IR$ be defined as f(x) = $|x| + |x^2 - 1|$. The total number of points at which f attains either local maximum or local minimum is
104. Let p(x) be a real polynomials of least degree which has a local maximum at x = 1 and local minimum at x = 3. If p(1) = 6 and p(3) = 2, Then $p'(0)$ is
105. A vertical line passing through the point(h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangent to the ellipse at P and Q meet at the point R. If Δ(h) = area of the triangle PQR,$\Delta_1 = \max_{\frac{1}{2}\le h\le 1}$ Δ(h) and $\Delta_2 = \min_{\frac{1}{2}\le h\le 1}$ Δ(h), then $\frac{8}{\sqrt{5}}\Delta_1$ - $8\Delta_2$ =
106. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is
107. A cylindrical container is to be made from certain solid material with the following con-

straints. It has a fixed inner volume of V $mm^3$ has a 2mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the values of $\frac{v}{250\pi}$ is

108. Let $-1 \leq p \leq 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval $[\frac{1}{2}, 1]$ and identify it.
109. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line x + y = 7, is minimum.
110. Using the relation $2(1 - \cos x) < x^2$, $x \neq 0$ or otherwise prove that $\sin(\tan x) \geq x \forall x \in [0, \frac{\pi}{4}]$.
111. If the function $f : [0, 4] \rightarrow R$ is differentiable then show that
    a) for $a, b \in (0, 4)$, $(f(4))^2 - (f(0))^2 = 8f'(a)f(b)$
    b) $\int_0^4 f(t)dt = 2[\alpha f(\alpha^2) + \beta f(\beta^2)] \forall 0 < \alpha, \beta < 2$
112. If $p(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all x ≥ 1 then prove that $P(x) > 0$, for all x > 1.
113. Using Rolle's theorem prove that there is at least one root for $(45^{\frac{1}{100}}, 46)$ of polynomial
$$P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035$$
114. Prove that for $x \in [0, \frac{\pi}{2}]$, $\sin x + 2x \geq \frac{3x(x+1)}{\pi}$ explain the identity if any used in the proof.
115. $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$ for $x_1, x_2 \in R$. Find the equation of the tangent to the curve $y = f(x)$ at the point (1, 2).
116. If p(x) be the polynomial of degree 3 satisfying p(-1) = 10, p(1) = -6 and p(x) has maxima at x = -1 and $p'(x)$ has minima at x = 1. Find the distance between the local maxima and local minima of the curve.
117. For a twice differenciable function f(x), g(x) is defined as g(x) = $(f'(x)^2 + f''(x))$ f(x) on [a, e] If for $a < b < c < d < e$, f(a) = 0, f(b) = 2, f(c) = -1, f(d) = 2, f(e) = 0 then find the minimum number of zeros of g(x).
118. Let a + b = 4, where a < 2 and let $g(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x Prove that $\int_0^a g(x)dx + \int_0^b$ g(x) dx increases as (b - a) increasing.
119. Suppose $f(x)$ is a function statisfying th following conditions
    a) f(0) = 2, f(1) = 1
    b) f has a minimum value at x = $\frac{5}{2}$ and
    c) for all x
$$f'(X) = \begin{vmatrix} 2ax & 2ax - 1 & 2ax + b + 1 \\ b & b + 1 & -1 \\ 2(ax + b) & 2ax + 2b + 1 & 2ax + b \end{vmatrix}$$
    where a, b be are some constants. Determine the constants a, b and the function f(x).
120. A curve C has the property that if the tangent drawn at any point P on C the co-ordinate axes at A and B then P is the mid point of AB. The curve passes through the point (1, 1). Determine the equation of the curve.
121. Suppose
$$p(x) = a_0 + a_1x + ..... + a_nx^n$$
    If $|p(x)| \leq |e^{x-1} - 1|$ for all x ≥ 0. Prove that
$$|a_1 + 2a_2 + .... + na_n| \leq 1.$$

**Section-B:**

122. The maximum distance from origin of a point on the curve
$$x = a\sin t - b\sin\frac{at}{b}$$
$$y = a\cos t - b\cos\frac{at}{b}, a, b > 0$$
    a) a - b
    b) a + b
    c) $\sqrt{a^2 + b^2}$
    d) $\sqrt{a^2 - b^2}$
123. If 2a + 3b + 6c = 0,$(a, b, c \in R)$ then the quadratic equation $ax^2 + bx + c = 0$ has
    a) at least one root in [0, 1]
    b) at least one root in [2, 3]
    c) at least one root in [4, 5]
    d) none of these
124. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2 + 1$, where $a > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$ then a equals
    a) $\frac{1}{2}$
    b) 3
    c) 1
    d) 2
125. A point on the parabola $y^2 = 18x$ at which the ordinate increase at twice the rate of the abscissa is
    a) $(\frac{9}{8}, \frac{9}{2})$

b) (2, -4)
c) $(-\frac{9}{8}, \frac{9}{2})$
d) (2, 4)

126. A function y = f(x) has a second order derivative $f''(x) = 6(1-x)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is y = 3x - 5 then the function is
a) $(x+1)^2$
b) $(x-1)^3$
c) $(x+1)^3$
d) $(x-1)^2$

127. The normal to the curve x = (1 + $\cos\theta$), y = $a\sin\theta$ at $\theta$ always passes through the fixed point
a) (a, a)
b) (0, a)
c) (0, 0)
d) (a, 0)

128. If 2a + 3b + 6c = 0 then at least one root of the equation $ax^2+bx+c=0$ lies in the interval
a) (1, 3)
b) (1, 2)
c) (2, 3)
d) (0, 1)

129. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
a) 2ab
b) ab
c) $\sqrt{ab}$
d) $\frac{a}{b}$

130. The normal to the curve x = a($\cos\theta + \theta\sin\theta$), y = a($\sin\theta - \theta\cos\theta$) at any point $'\theta'$ is such that
a) it passes through the origin
b) it makes an angle $\frac{\pi}{2} + \theta$ with the x-axis
c) it passes through (a$\frac{\pi}{2}$, -a)
d) it is at constant distance from the origin

131. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness of ice is 5cm then the rate at which the thickness of ice decrease is
a) $\frac{1}{36\pi} cm/min$
b) $\frac{1}{18\pi} cm/min$
c) $\frac{1}{54\pi} cm/min$
d) $\frac{5}{6\pi} cm/min$

132. if the equation $a_n x^n + a_{n-1}x^{n-1}$....... + $a_1 x = 0$, $a_1 \neq 0, n \geq 2$, has a positive root x = $\alpha$ then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ....... + a_1$ = 0 has a positive root which is
a) greater than $\alpha$
b) smaller than $\alpha$
c) greater than or equal to $\alpha$
d) equal to $\alpha$

133. The function f(x) = $\frac{x}{2} + \frac{2}{x}$ has a local minimum at
a) x = 2
b) x = -2
c) x = 0
d) x = 1

134. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank.The two sides having fence are of same length x,the maximum area enclosed by the park is
a) $\frac{3}{2}x^2$
b) $\sqrt{\frac{x^3}{8}}$
c) $\frac{1}{2}$
d) $\pi x^2$

135. A value of C for which conclusion of Mean Value Theorem holds for the function f(x) = $\log_e^x$ on the interval [1, 3] is
a) $\log_3 e$
b) $\log_e 3$
c) $2\log_3 e$
d) $\frac{1}{2}\log_3 e$

136. The function f(x) = $\tan^{-1}(\sin x + \cos x)$ is an increasing function in
a) $(0, \frac{\pi}{2})$
b) $(-\frac{\pi}{2}, \frac{\pi}{2})$
c) $(\frac{\pi}{4}, \frac{\pi}{2})$
d) $(-\frac{\pi}{2}, \frac{\pi}{4})$

137. If p and q are positive real numbers such that $p^2 + q^2 = 1$ then the maximum value of (p + q) is
a) $\frac{1}{2}$
b) $\frac{1}{\sqrt{2}}$
c) $\sqrt{2}$
d) 2

138. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$ Then which one of the following holds?
a) the cubic has minimum at $\sqrt{\frac{p}{3}}$ and maximum at $-\sqrt{\frac{p}{3}}$

b) the cubic has minimum at $-\sqrt{\frac{p}{3}}$ and maximum at $\sqrt{\frac{p}{3}}$
c) the cubic has minimum at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
d) the cubic has maximum at $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

139. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
a) 7
b) 1
c) 3
d) 5

140. Let f(x) = x$|x|$ and g(x) = $\sin x$.
a) Statement 1: gof is differentiable at x = 0 and its derivative is continuous at that point.
b) Statement 2: gof is twice differentiable at x = 0.
a) statement-1 is true,statement-2 is true statement-2 is not correct explination for statement-1
b) statement-1 is true,statement-2 is false
c) statement-1 is false and statement-2 is true
d) statement-1 is true,statement-2 is true statement-2 is correct explination of statement-1

141. Given P(x) = $x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of $P'(x) = 0$. If P(-1) < P(1), Then in the interval [-1, 1]:
a) P(-1) is not a minimum but P(1) is the maximum of P
b) P(-1) is the minimum but P(1) is not the maximum of P
c) Neither P(-1) is a minimum nor P(1) is the maximum of P
d) P(-1) is a minimum but P(1) is the maximum of P

142. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$ that is parallel to the x-axis is
a) y = 1
b) y = 2
c) y = 3
d) y = 0

143. Let $f : R \to R$ be defined by f(x) =

$$\begin{cases} k - 2x & if x \leq -1 \\ 2x + 3 & if x > -1 \end{cases}$$

if f has a local minimum at x = -1 then a possible value of k is
a) 0
b) $\frac{-1}{2}$
c) -1
d) 1

144. Let $f : R \to R$ be a continuous function defined by f(x) = $\frac{1}{e^x + 2e^{-x}}$
a) Statement-1: f(c) = $\frac{1}{3}$ for some $c \in R$
b) Statement -2: $0 < f(x) \leq \frac{1}{2\sqrt{2}}$ for all $x \in R$
a) statement-1 is true,statement-2 is true statement-2 is not correct explination for statement-1
b) statement-1 is true,statement-2 is false
c) statement-1 is false and statement-2 is true
d) statement-1 is true,statement-2 is true statement-2 is correct explination of statement-1

145. The shortest distance between line y - x = 1 and curve x = $y^2$ is
a) $\frac{3\sqrt{2}}{8}$
b) $\frac{8}{3\sqrt{2}}$
c) $\frac{4}{\sqrt{3}}$
d) $\frac{\sqrt{3}}{4}$

146. For $x \in (0, \frac{5\pi}{2})$ define f(x) = $\int_0^x \sqrt{t}\sin t dt$ then f has
a) local minimum at $\pi and 2\pi$
b) local minimum at $\pi$ and local maximum at $2\pi$
c) local maximum at $\pi$ and local minimum at $2\pi$
d) local maximum at $\pi$ and $2\pi$

147. A spherical balloon filled with $4500\pi$ cubic meters of helium gas. If a leak in balloon causes the gas to escape at the rate of $72\pi$ cubic meters per minute then then rate at which the radius of balloon decreases 49 minutes after the leakage began is:
a) $\frac{9}{7}$
b) $\frac{7}{9}$
c) $\frac{2}{9}$
d) $\frac{9}{2}$

148. Let a, b $\in$ R be such that the function f given by f(x) = ln$|x|$ + b$x^2$ + ax, $x \neq 0$ has extreme values at x = -1 and at x = 2.
a) Statement-1: f has local maximum at x = -1 and at x = 2
b) Statement-2: a = $\frac{1}{2}$ and $b = -\frac{1}{4}$
a) Statement-1 is false,Statement-2 is true
b) Statement-1 is true,statement-2 is true statement-2 is a correct explanation of statement -1
c) Statement-1 is true,statement-2 is true statement-2 is not a correct explanation of statement -1
d) Statement-1 is true and statement-2 is false

149. A line is drawn through the point [1, 2] to meet

the coordinates axes at P and Q such that it forms a triangle OPQ where O is the origin. If the area of the triangle OPQ is least then the slope of the line PQ is:

a) $\frac{-1}{4}$
b) -4
c) -2
d) $\frac{-1}{2}$

150. The intercepts on the axis made by tangents to the curve y = $\int_0^x |t|$ dt, $x \in R$ which are parallel to the line y = 2x are equal to
a) ±1
b) ±2
c) ±3
d) ±4

151. If f and g are differentiable functions in [0, 1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, Then for some c ∈ [0, 1]
a) $f'(c) = g'(c)$
b) $f'(c) = 2g'(c)$
c) $2f'(c) = g'(c)$
d) $2f'(c) = 3g'(c)$

152. Let f(x) be the polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}$ [1 + $\frac{f(x)}{x^2}$] = 3, then f(2) is equal to:
a) 0
b) 4
c) -8
d) -4

153. Consider:

$$f(x) = \tan^{-1}(\sqrt{\frac{1+\sin x}{1-\sin x}})x \in (0, \frac{\pi}{2})$$

A normal to y = f(x) at x = $\frac{p}{6}$ also passes through the point:
a) ($\frac{\pi}{6}$, 0)
b) ($\frac{\pi}{4}$, 0)
c) (0, 0)
d) (0, $\frac{2\pi}{3}$)

154. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle os radius = r units. If sum of the areas of the squares and the circle so formed is minimum then,
a) x = 2r
b) 2x = r
c) 2x = $(\pi + 4)$r
d) $(4 - \pi)$x = $\pi r$

155. The function $f : R \to [\frac{-1}{2}, \frac{1}{2}]$ defined as f(x) = $\frac{x}{1+x^2}$ is
a) neither injective nor surjective
b) invertible
c) injective but not surjective
d) surjective but not injective

156. The Normal to the curve y(x - 2)(x - 3) = x + 6 at the point where the curve intersects the y-axis passes through the point:
a) $(\frac{1}{2}, \frac{1}{3})$
b) $(-\frac{1}{2}, -\frac{1}{2})$
c) $(\frac{1}{2}, \frac{1}{2})$
d) $(\frac{1}{2}, -\frac{1}{3})$

157. Twenty meter of wire is available for fencing off a flower bed in the form of circular sector Then the maximum area of flower bed is:
a) 30
b) 12.5
c) 10
d) 25

158. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$ If one of its directices is x = -4 then the equation of normal to it at (1, $\frac{3}{2}$) is ;
a) x + 2y = 4
b) 2y - x = 2
c) 4x - 2y = 1
d) 4x + 2y = 7

159. Let f(x) = $x^2 + \frac{1}{x^2}$ and g(x) = x - $\frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If h(x) = $\frac{f(x)}{g(x)}$ then local minimum value of h(x) is:
a) -3
b) -2$\sqrt{2}$
c) 2$\sqrt{2}$
d) 3

160. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersects each other at right angles then the value of b is:
a) $\frac{1}{2}$
b) 4
c) $\frac{9}{2}$
d) 6

161. The maximum volume of the right circular cone having slant height 3m is
a) $6\pi$
b) $3\sqrt{3}\pi$
c) $\frac{4}{3}\pi$

d) $2\sqrt{3}\pi$

162. If q denotes the acute angle between the curves, y = $10 - x^2$ and y = $2 + x^2$ at a point of their intersection then $|\tan\theta|$ is equal to
   a) $\frac{4}{9}$
   b) $\frac{8}{15}$
   c) $\frac{7}{17}$
   d) $\frac{8}{17}$
163. If f(x) is a non-zero polynomial of degree four having local extreme points at x = -1, 0, 1 then the set S = {x R: f(x) = f(0)} contains exactly
   a) four irrational numbers.
   b) four rational numbers.
   c) two irrational and two rational number.
   d) two irrational and one rational number.
164. If the tangent to the curve y = x3 + ax - b at the point (1, -5) is perpendicular to the line -x + y + 4 = 0, then which one of the following points lie on the curve?
   a) (-2, 1)
   b) (-2, 2)
   c) (2, -1)
   d) (2, -2)
165. Let S be the set of all values of x for which the tangent to the curves y = f(x) = $x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(a)) and (-1, f(-1)) then S is equal to:
   a) $\{\frac{1}{3}$, 1}
   b) $\{-\frac{1}{3}$, -1}
   c) $\{\frac{1}{3}$, -1}
   d) $\{-\frac{1}{3}$, 1}

## 8 Differentiation

1. If y=f$\left(\frac{2x-1}{x^2+1}\right)$ and f'(x) = $\sin x^2$, then $\frac{dy}{dx}$ = ........
2. $f_r(x)$,$g_r(x)$, $h_r(x)$, r = 1,2,3 are polynomials in x such that $f_r(a) = g_r(a) = h_r(a)$ r = 1,2,3 and
$$F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$
Then F'(x) at x = a is .............
3. If f(x) = $\log_x$(ln x), then $f'(x)$ at x = e is ...........
4. The derevative of $sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at x = $\frac{1}{2}$ is ...........
5. If f(x) = $|x-2|$ and g(x) = f[f(x)], then g'(x) = ............ for x>20
6. If x$e^{xy}$ = y + $\sin^2$x, then at x=0, $\frac{dy}{dx}$ = ...........
7. The derivavtive of an even function is always an odd function.
8. If $y^2$ = P(x), a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$, equals .....

| | |
|---|---|
| (a) P'''(x)+P'(x) | (c) P(x)P'''(x) |
| (b) P''(x)P'''(x) | (d) constant |

9. Let f(x) be a quadratic expression which is positive for all the real values of x. If g(x) = f(x)+f'(x)+f''(x), then for any real x,

| | |
|---|---|
| (a) g(x)<0 | (c) g(x) = 0 |
| (b) g(x)>0 | (d) g(x)≥0 |

10. If y = $\sin x^{\tan x}$ then $\frac{dy}{dx}$ is equals to
   (a) $\sin x^{\tan x}(1+\sec^2 x \log\sin(x)$
   (b) $\tan x(\sin x)^{\tan x - 1}.\cos x$
   (c) $\sin x^{\tan x}\sec^2 x\log\sin x$
   (d) $\tan x(\sin x)^{\tan x-1}$
11. $x^2 + y^2 = 1$

| | |
|---|---|
| (a) yy''-2$y'^2$+1 = 0 | (c) yy''+$y'^2$-1 = 0 |
| (b) yy''+$y'^2$+1 = 0 | (d) yy''+2$y'^2$+1=0 |

12. Let f:(0 ∞) → R and F(x) = $\int_0^x$ f(t)dt. If F($x^2$) = $x^2$(1+x), then f(4) equals

| | |
|---|---|
| (a) $\frac{5}{4}$ | (c) 4 |
| (b) 7 | (d) 0 |

13. If y is a function of x and $\log(x+y)$-2xy = 0, then the value of y'(0) is equals to

| | |
|---|---|
| (a) 1 | (c) 2 |
| (b) -1 | (d) 0 |

14. If f(x) is a twice diffferentiable function and given that
f(1) = 1;f(2) = 4,f(3) = 9, then
(a) f''(x)=2 for ∀ x ∈ (1,3)
(b) f''(x)=f'(x) = 5 for some x ∈ (2,3)
(c) f''(x)=3 for ∀ x ∈ (1,3)
(d) f''(x) =2 for some x ∈ (1,3)

15. $\frac{d^2x}{dy^2}$ equals

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$
(b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

16. Let g(x)=logf(x) where f(x) is twice differentiable positive function on (0, ∞) such that f(x+1) = xf(x). Then, for N=1,2,3,.......
g''$\left(N+\frac{1}{2}\right)$-g''$\left(\frac{1}{2}\right)$ =
g''$\left(N+\frac{1}{2}\right)$ - g''($\frac{1}{2}$ =
(a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N-1)^2}\right\}$
(b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N-1)^2}\right\}$
(c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N+1)^2}\right\}$
(d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N+1)^2}\right\}$

17. Let f:[0, 2] → R be a function which is continuous on [0,2] and is differntiable on(0,2) with f(0) = 1. Let
F(x) = $\int_0^{x^2}$f( $\sqrt{t}$)dt for x ∈ [0,2]. If F'(x) = f'(x) for all x ∈ (0,2), then F(2) equals

a $e^2$ - 1 (c) e - 1
(b) $e^4$ - 1 (d) $e^4$

18. Let f:R → R, g:R → R and h : R → R be differentiable functions such that f(x)=$x^2$+3x+2, g(f(x))=x and h(g(g(x)))=x for all x ∈ R. Then

(a) g'(2) = $\frac{1}{15}$ (c) h(0) = 16
(b) h'(1) = 666 (d) h(g(3)) = 36

19. For every twice differentiable function f:R → [-2,2] with $(f(0))^2 + (f(0))^2 = 85$, which of the following statement(s) is True?

(a) There exist r,s ∈ R, where r<s, such that f is on the open interval (r,s)
(b) There exists $x_0$ →(-4,0) such that $|f'(x_0)| \leq 1$
(c) $\lim_{x\to\infty}$ = 1
(d) There exists $\alpha$ → (-4, 4) such that f($\alpha$)+f'($\alpha$) = 0 and f'($\alpha$) ≠ 0

20. For any positive integer n, define $f_n$:(0, ∞)→R as $f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$ for all x∈ (0,∞).
Here, the inverse trignometric function $\tan^{-1}(x)$ assumes values in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
Then, which of the following statements are True?
(a) $\sum_{j=1}^{5} \tan^2 f_j(0)) = 55$
(b) $\sum_{j=1}^{10}(1+f_j'(0))\sec^2(f_j'(0)) = 10$
(c) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$
(d) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

21. Let f:(0,$\pi$)→R be a twice differentiable function such that $\lim_{t\to\infty} \frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$ for all x∈ $(0,\pi)$. If $\frac{\pi}{6}$ = $-\frac{\pi}{12}$, then which of the following statement(s) are True?

(a) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$
(b) f(x)< $\frac{x^4}{6} - x^2$ for all x∈ $(0,\pi)$
(c) There exist $\alpha \in (0,\pi)$ such that f'($\alpha$) = 0

(d) f’$\left(\frac{\pi}{2}\right)$+f$\left(\frac{\pi}{2}\right)$ = 0

22. Find the derivative of $\sin(x^2+1)$ with respect to x from first principle.

23. Find the derivative of

$$F(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{when } x \neq 1 \\ -\frac{1}{3}, & \text{when } x = 1 \end{cases}$$

24. Given y = $\frac{5x}{3\sqrt{(1-x)^2}}$ + $\cos^2(2x+1)$; Find $\frac{dy}{dx}$.

25. $y = e^{x\sin x^3} + (\tan x)^x$. Find $\frac{dy}{dx}$

26. Let f be a twice differentiable function such that f’(x) = -f(x) and f’(x) = g(x), h(x) = $[f(x)]^2$ + $[g(x)]^2$ find h(10) if h(5) =11

27. If $\alpha$ be a repeated root of a quadratic equation f(x) = 0 and A(x), B(x) and C(x) be polynomials of degree 3,4 and 5 respectively, then show that

$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

is divisible by f(x), where prime denotes the derivatives.

28. If x = $\sec\theta$ - $\cos\theta$ and y= $\sec^n\theta$ - $\cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$.

29. Find $\frac{dy}{dx}$ at x = -1, when

$(\sin y)^{\sin\left(\frac{\pi}{2}x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x\tan(ln(x+2))$ = 0

30. $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)}$+1, prove that

$$\frac{y'}{y} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right).$$

31. Let f(x) = 2+$\cos x$ for all real x.

STATEMENT-1: For each real t, there exist a point c in [t,t+$\pi$] such that f’(c) = 0 because
STATEMENT-2 f(t) = f(t+$2\pi$) for each real t.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation of Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation of Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

32. Let f and g be real valued functions defined on interval (-1,1) such that g”(x) is continuous, g(0) ≠ 0. g’(0) = 0, g” ≠ 0, and f(x) = g(x)$\sin x$

STATEMENT-1: $\lim_{x\to 0}$ [g(x)cot x - g(0)cosec x] = f’(0) and
STATEMENT-2: f’(0) = g(0)

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation of Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation of Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

33. If the function f(x) =$x^3 + e^{\frac{x}{2}}$ and g(x) =$f^{-1}$(x), then the value of g’(1) is

34. Let f($\theta$) = $\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, Then the value of $\frac{d}{d(\tan\theta)}$(f($\theta$)) is

35. If y = (x+$\sqrt{1+x^2}$)$^n$, then $(1+x^2)\frac{d^2y}{dx^2}$+x$\frac{dy}{dx}$ is
(a) $n^2y$
(b) $-n^2y$

(a) 24 (b) 36 (c) 12 (d) 18

43. The set of points where f(x) $=\frac{x}{1+|x|}$ is differentiable is

(a) $(-\infty, 0) \cup (0, \infty)$ (c) $(-\infty, \infty)$
(b) $(-\infty, -1) \cup (-1, \infty)$(d) $(0, \infty)$

44. If $x^m.y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is
(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$ (c) xy (d) $\frac{x}{y}$

45. Let y be an implicit function of x defined by $x^{2x} - 2x^x coty - 1 = 0$. Then y'(1) equals

(a) 1 (b) log 2 (c) -log 2 (d) -1

46. Let f:(-1,1)→R be a differentiable function with f(0) = -1 and f'(0) = 1. Let g(x)=$[f(2f(x)+2))]^2$ Then g'(0) =

(a) -4 (b) 0 (c) -2 (d) 4

47. $\frac{d^2x}{dx^y}$ equals:
(a) -$\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (c) -$\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$
(b) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

48. If y= $\sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at x = 1 is equals to:

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

49. If g is the inverse of a function f and $f^{-1}(x) = \frac{1}{1+x^5}$ then g'(x) is equals to:
(a) $\frac{1}{1+(g(x))^5}$ (c) 1+$x^5$
(b) 1+$(g(x))^5$ (d) $5x^4$

50. If x = -1 and x = 2 are extreme points of f(x) $=\alpha log|x| + \beta x^2 + x$ then

(a) $\alpha = 2, \beta = -\frac{1}{2}$ (b) $\alpha = 2, \beta = \frac{1}{2}$

(c) -y
(d) $2x^2y$

36. If f(y) =$e^y$, g(y) = y; y>0 and

F(t) = $\int_0^t$ f(t-y)g(y)dy, then
(a) F(t) = t$e^{-t}$
(b) F(t) = 1-t$e^{-t}$(1+t)
(c) F(t) = $e^t$-(1+t)
(a) F(t) = t$e^t$

37. f(x) = $x^n$, then the value of

f(1)-$\frac{f'(1)}{1!}+\frac{f''(1)}{2!}+\frac{f'''}{3!}+.........\frac{(-1)^n f^n((1)}{n!}$ is

(a) 1 (b) $2^n$ (c) $2^n$-1 (d) 0

38. Let f(x) be a polynomial function of second degree. If f(1) = f(-1) and a,b,c are in A.P then f'(a),f'(b),f'(c) are in

(a)Arthemetic-Geometric progression
(b)A.P
(c)G.P
(d)H.P

39. If $e^{y+e^y+e^{y+...\infty}}$, x>0, then $\frac{dy}{dx}$

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

40. The value of a for which sum of the squares of the roots of the equation $x^2$-(a-2)x-a-1 = 0 assume the least value is

(a) 1 (b) 0 (c) 3 (d) 2

41. If the roots of the equation $x^2$-bx+c = 0 be two consecutive integers, then $b^2$-4ac equals

(a) -2 (b) 3 (c) 2 (d) 1

42. let f:R→R be a differentiable function having f(2) = 6, f'(2) = $\frac{1}{48}$ Then $\lim_{x\to f(x)} \int_6^{f(x)} \frac{4t^3}{x-2}$dt equals

(c) $\alpha = -6, \beta = \frac{1}{2}$ (d) $\alpha = -6, \beta = -\frac{1}{2}$

51. If for x∈ $\left(0, \frac{1}{4}\right)$, the derivative of $tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}$.g(x), then g(x) equals:
(a) $\frac{3}{1+9x^3}$ (c) $\frac{3x\sqrt{x}}{1-9x^3}$
(b) $\frac{9}{1+9x^3}$ (d) $\frac{3x}{1-9x^3}$

## 9 Limits, Continuity and Differentiability

1. Let $f(x)=\begin{cases}(x-1)^2\sin\frac{1}{(x-1)}-|x|, & \text{if } x\neq 1.\\ -1, & \text{if x=1.}\end{cases}$

be real-valued function. Then find the set of points where $f(x)$ is not differentiable ?

2. Let
$$f(x)=\begin{cases}\frac{\left(x^3+x^2-16x+20\right)}{(x-2)^2}, & \text{if } x\neq 2\\ k, & \text{if x=2}\end{cases}$$
If $f(x)$ is continuous for all x, then find k ?

3. A discontinuous function $y = f(x)$ satisfying $x^2 + y^2 = 4$ is given by $f(x) =$ .......

4. $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$= ........

5. If f(x)=$\begin{cases}\sin x, & x\neq n\pi, n=0,\pm1,\pm2,\pm3,.....\\ 2, & \text{otherwise}\end{cases}$

and g(x)=$\begin{cases}x^2+1, & x\neq 0,2\\ 4, & x=0\\ 5, & x=2\end{cases}$ then

$\lim_{x\to 0} g\,[f(x)]$ is ...........

6. $\lim_{x\to-\infty}\left[\frac{x^4\sin\left(\frac{1}{x}\right)+x^2}{(1+|x|^3)}\right]$ = .........

7. If $f(9) = 9, f'(9) = 4$, then $\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ equals ........

8. $ABC$ is an isosceles triangle inscribed in a circle of radius $r$. If $AB = AC$ and $h$ is the

altitude from $A$ to $BC$ then the triangle $ABC$ has perimeter $P = \left(2\left(\sqrt{2hr - h^2}\right) + \sqrt{2hr}\right)$ and area $A=$ ........ also $\lim_{h\to 0} \frac{A}{P^3} =$ ........

9. $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} =$ .......

10. Let $f(x) = x|x|$. The set of points where $f(x)$ is twice differentiable is .........

11. Let $f(x) = [x]\sin\left(\frac{\pi}{[x+1]}\right)$, where

[•] denotes the greatest integer function. The domain of $f$ is ........ and the points of discontinuity of $f$ in the domain are .........

12. $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2} =$ ..........

13. Let $f(x)$ be a continuous function defined for $1 \le x \le 3$. If $f(x)$ takes rational values for all $x$ and $f(2) = 10$, then $f(1.5) =$ ........

14. If $\lim_{x\to a}[f(x)g(x)]$ exists then both $\lim_{x\to a} f(x)$

and $\lim_{x\to a} g(x)$ exist. (True / False)

15. If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ is

(a) 0 (b) $\infty$

(c) 1 (d) none of these

16. For a real number $y$, let $[y]$ denotes the greatest integer less than or equal to $y$ : Then

the function $f(x) = \frac{\tan(\pi[x - \pi])}{1 + [x]^2}$ is

(a) discontinuous at some $x$
(b) continuous at all $x$, but the derivative $f'(x)$ does not exist for some $x$
(c) $f'(x)$ exists for all $x$, but the second derivative $f''(x)$ does not exist for some $x$
(d) $f'(x)$ exists for all $x$

17. There exists a function $f(x)$, satisfying $f(0) = 1$, $f'(0) = -1$, $f(x) > 0$ for all $x$, and

(a) $f''(x) > 0$ for all $x$

(b) $-1 < f''(x) < 0$ for all $x$

(c) $-2 \le f''(x) \le -1$ for all $x$

(d) $f''(x) < -2$ for all $x$

18. If G(x)= $-\sqrt{25 - x^2}$ then $\lim_{x\to 1}\frac{G(x) - G(1)}{x - 1}$ has the value

(a) $\frac{1}{24}$ (b) $\frac{1}{5}$

(c) $-\sqrt{24}$ (d) none of these

19. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2,$

then the value of $\lim_{x\to a}\frac{g(x)f(a) - g(a)f(x)}{x - a}$ is

(a) -5 (b) $\frac{1}{5}$

(c) 5 (d) none of these

20. The function

$$f(x) = \frac{ln(1 + ax) - ln(1 - bx)}{x}$$

is not defined at $x = 0$. The value which should be assigned to $f$ at $x = 0$ so that it is continuous at $x = 0$, is

(a) $a - b$ (b) $a + b$

(c) $lna - lnb$ (d) none of these

21. $\lim_{n\to\infty}\left\{\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots.. + \frac{n}{1-n^2}\right\}$

is equal to

(a) 0 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) none of these

22. If $f(x) = \begin{cases} =\frac{\sin[x]}{[x]}, & [x] \ne 0 \\ 0, & [x] = 0 \end{cases}$

Where [x] denotes the greatest integer less than or equal to $x$, then $\lim_{x\to 0} f(x)$ equals

(a) 1 (b) 0

(c) -1 (d) none of these

23. Let $f : R \to R$ be differentiable function and $f(1) = 4$. Then the value of $\lim_{x\to 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

(a) $8f'(1)$ (b) $4f'(1)$

(c) $2f'(1)$ (d) $f'(1)$

24. Let [•] denote the greatest integer function and $f(x) = [\tan^2 x]$, then
(a) $\lim_{x\to 0} f(x)$ does not exist
(b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$
(d) $f'(0) = 1$

25. The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, [•] denotes the greatest integer function, is discontinuous at
(a) All x
(b) All integer points
(c) No x
(d) x which is not an integer

26. $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals

(a) $1 + \sqrt{5}$ (b) $-1 + \sqrt{5}$

(c) $-1 + \sqrt{2}$ (d) $1 + \sqrt{2}$

27. The function $f(x) = [x^2] - [x^2]$ (where $[y]$ is the greatest integer less than or equal to $y$), is discontinuous at
(a) all integers
(b) all integers except 0 and 1
(c) all integers except 0
(d) all integers except 1

28. The function $f(x)=\left(x^2-1\right)|x^2-3x+2|+\cos(|x|)$ is NOT differentiable at

(a) -1 (b) 0 (c) 1 (d) 2

29. $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) -2 (c) 1/2 (d) -1/2

30. For x ∈ R, $\lim_{x\to\infty} \left(\frac{x-3}{x+2}\right)^x =$

(a) $e$ (b) $e^{-1}$ (c) $e^{-5}$ (d) $e^5$

31. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ equals

(a) $-\pi$ (b) $\pi$ (c) $\pi/2$ (d) 1

32. The left-hand derivative of $f(x)=[x]sin(\pi x)$ at $x = k$, k and integer, is

(a) $(-1)^k(k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$

(c) $(-1)^k k\pi$ (d) $(-1)^{k-1}k\pi$

33. Let $f : R \to R$ be a function defined by $f(x) = max\{x, x^3\}$. The set fo all points where $f(x)$ is NOT differentiable is

(a) {-1,1} (b) {-1,0}

(c) {0,1} (d) {-1,0,1}

34. Which of the following functions is differentiable at x = 0 ?

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$

(c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$

35. The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x & \text{if } |x| \le 1 \\ \frac{1}{2}(|x|-1) & |x| > 1 \end{cases}$ is

(a) $R - \{0\}$ (b) $R - \{-1\}$

(c) $R - \{1\}$ (d) $R - \{-1, 1\}$

36. The integer for which

$\lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite

non-zero number is

(a) 1 (b) 2 (c) 3 (d) 4

37. Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ eqauls

(a) 1 (b) $e^{1/2}$ (c) $e^2$ (d) $e^3$

38. If $\lim_{x\to 0}\frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to

(a) 0 (b) $\frac{n+1}{n}$
(c) $n$ (d) $n + \frac{1}{n}$

39. $\lim_{h\to 0}\frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$, given that $f'(2) = 6$ and $f'(1) = 4$

(a) does not exist (b) is equal to -3/2
(c) is equal to 3/2 (d) is equal to 3

40. If (x) is differentiable and strictly increasing function, then the value of

$\lim_{x\to 0}\frac{f(x^2) - f(x)}{f(x) - f(0)}$ is

(a) 1 (b) 0 (c) -1 (d) 2

41. The function given by $y = ||x| - 1|$ is differentiable for all real numbers except the points

(a) {0,1,-1} (b) ±1
(c) 1 (d) -1

42. If $f(x)$ is continuous and differentiable function and $f(1/n) = 0 \forall n \geq 1$ and $n \in$ I, then
(a) $f(x) = 0, x \in (0, 1]$
(b) $f(0) = 0, f'(0) = 0$
(c) $f(0) = 0 = f'(0), x \in (0, 1]$
(d) $f(0) = 0$ and $f'(0)$ need not to be zero

43. The value of $\lim x \to 0\left((\sin x)^{1/x} + (1+x)^{\sin x}\right)$, where $x > 0$ is

(a) 0 (b) -1 (c) 1 (d) 2

44. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and

$\lim_{t\to x}\frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$.

Then $f(x)$ is

(a) $\frac{1}{3x} + \frac{2x^2}{3}$ (b) $\frac{-1}{3x} + \frac{4x^2}{3}$
(c) $\frac{-1}{x} + \frac{2}{x^2}$ (d) $\frac{1}{x}$

45. $\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec x^2} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ equals

(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$
(c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$

46. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; $0 < x < 2$,

m and n are integers, $m \neq 0, n > 0$, let $p$ be the left hand derivative of $|x-1|$ at x = 1. If $\lim_{x\to 1} g(x) = p$, then

(a) $n = 1, m = 1$ (b) $n = 1, m = 1$

(c) $n = 2, m = 2$ (d) $n > 2, m = n$

47. If $\lim_{x\to 0}\left[1 + xln(1+b^2)\right]^{1/x} = 2bsin^2\theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of $\theta$ is

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

48. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right) = 4$, then
(a) $a = 1, b = 4$ (b) $a = 1, b = -4$
(c) $a = 2, b = -3$ (d) $a = 2, b = 3$

49. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$,

$x \in R$ then $f$ is
(a) differentiable both at x = 0 and at x = 2
(b) differentiable at x= 0 but not differentiable at x = 2

(c) not differentiable at x = 0 but differentiable at x = 2
(d) differentiable neither at x = 0 nor at x = 2

50. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where a>-1. Then $\lim_{a\to 0^+}\alpha(a)$ and $\lim_{x\to 0^+}\beta(a)$ are
(a) $-\frac{5}{2}$ and 1
(b) $-\frac{1}{2}$ and -1
(c) $-\frac{7}{2}$ and 2
(d) $-\frac{9}{2}$ and 3

51. If x+$|y|$ = 2y, then y as a function of x is
(a) defined for all real x
(b) continuous at x = 0
(c) differentiable for all x
(d) such that $\frac{dy}{dx}=\frac{1}{3}$ for $x<0$

52. If $f(x)=x(\sqrt{x}-\sqrt{x+1})$, then
(a) f(x) is continuous but not differentiable at x = 0
(b) f(x) is differentiable at x = 0
(c) f(x) is not differentiable at x = 0
(d) none of these

53. The function $f(x)=1+|\sin x|$ is
(a) continuous nowhere
(b) continuous everywhere
(c) differentiable nowhere
(d) not differentiable at $x=0$
(e) not differentiable at infinite number of points

54. Let [x] denote the greatest integer less than or equal to x. If $f(x)=[x\sin\pi x]$, then $f(x)$ is
(a) continuous at $x=0$
(b) continuous in (-1,0)
(c) differentiable at $x=1$
(d) differentiable in (-1,1)
(e) none of these

55. The set of all points where the function $f(x)=\frac{x}{(1+|x|)}$ is differentiable, is
(a) $(-\infty,\infty)$
(b) $[0,\infty)$
(c) $(-\infty,0)\cup(0,\infty)$
(d) $(0,\infty)$
(e) None

56. The function
$$f(x)=\begin{cases}|x-3|, & x\ge 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, & x<1\end{cases}$$
is
(a) continuous at $x=1$
(b) differentiable at $x=1$
(c) continuous at $x=3$
(d) differentiable at $x=3$

57. If $f(x)=\frac{1}{2}x-1$, then on the interval $[0,\pi]$
(a) $\tan[f(x)]$ and $1/f(x)$ are both continuous
(b) $\tan[f(x)]$ and $1/f(x)$ are both discontinuous
(c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous
(d) $\tan[f(x)]$ is continuous but $1/f(x)$ is not

58. The value of $\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$
(a) 1
(b) -1
(c) 0
(d) none of these

59. The following functions are continuous on $(0,\pi)$
(a) $\tan x$
(b) $\int_0^x t\sin\frac{1}{t}dt$
(c) $\begin{cases}1, & 0<x\le\frac{3\pi}{4}\\ 2\sin\frac{2}{9}x, & \frac{3\pi}{4}<x<\pi\end{cases}$
(d) $\begin{cases}x\sin x, & 0<x\le\frac{\pi}{2}\\ \frac{\pi}{2}\sin(\pi+x), & \frac{\pi}{2}<x<\pi\end{cases}$

60. Let $f(x)=\begin{cases}0, & x<0\\ x^2, & x\ge 0\end{cases}$ then for all $x$
(a) $f'$ is differentiable
(b) $f$ is differentiable
(c) $f'$ is continuous
(d) $f$ is continuous

61. Let $g(x)=xf(x)$, where
$$f(x)=\begin{cases}x\sin\frac{1}{x}, & x\ne 0\\ 0, & x=0\end{cases}.$$ At $x=0$
(a) $g$ is differentiable but $g'$ is not contionuous

(b) $g$ is differentiable while $f$ is not
(c) both $f$ and $g$ are differentiable
(d) $g$ is differentiable and $g'$ is continuous

62. The function $f(x)$=max$\{(1-x),(1+x),2\}$, $x \in (-\infty,\infty)$ is
(a) continuous at all points
(b) diffrentiable at all points
(c) diffrentiable at all points except at $x = 1$ and $x = -1$
(d) continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous

63. Let $h(x)$=min$\{x, x^2\}$, for every real number of $x$, then
(a) $h$ is continuous for all $x$
(b) $h$ is differentiable for all $x$
(c) $h'(x) = 1$, for all $x > 1$
(d) $h$ is differentiable at two values of $x$

64. $\lim\limits_{x\to 1} \dfrac{\sqrt{1-\cos 2(x-1)}}{x-1}$
(a) exists and it equals $\sqrt{2}$
(b) exists and it equals $-\sqrt{2}$
(c) does not exist because $x - 1 \to 0$
(d) does not exist because the left hand limit is not equal to the right hand limit

65. If $f(x)$=min$\{1, x^2, x^3\}$, then
(a) $f(x)$ is continuous $\forall x \in R$
(b) $f(x)$ is continuous and differentiable everywhere
(c) $f(x)$ is not differentiable at two points
(d) $f(x)$ is not differentiable at one point

66. Let $L = \lim\limits_{x\to 0} \dfrac{a - \sqrt{a^2-x^2} - \dfrac{x^2}{4}}{x^4}, a > 0$. If $l$ is finite, then

(a) $a = 2$ (b) $a = 1$

(c) $L = \dfrac{1}{64}$ (d) $L = \dfrac{1}{32}$

67. Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y), \forall x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then
(a) $f(x)$ is differentiable only in a finite interval containing zero
(b) $f(x)$ is continuous $\forall x \in R$
(c) $f(x)$ is constant $\forall x \in R$
(d) $f(x)$ is differentiable except at finitely many points

68. If $f(x) = \begin{cases} -x - \dfrac{\pi}{2}, & x \le -\dfrac{\pi}{2} \\ -\cos x, & -\dfrac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ lnx, & x > 1 \end{cases}$, then
(a) $f(x)$ is continuous at $x = -\dfrac{\pi}{2}$
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x)$ is differentiable at $x = 1$
(d) $f(x)$ is differentiable at $x = -\dfrac{3}{2}$

69. For every integer $n$, let $a_n$ and $b_n$ be real numbers. Let function $f : IR \to IR$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & x \in [2n, 2n+1] \\ b_n + \cos \pi x, & x \in (2n-1, 2n) \end{cases}$$

for all integers $n$. If $f$ is continuous, then which of the flowing hold(s) for all $n$ ?

(a) $a_{n-1} - b_{n-1} = 0$ (b) $a_n - b_n = 1$
(c) $a_n - b_{n+1} = 1$ (d) $a_{n-1} - b_n = -1$

70. For $a \in R$(the set of all real numbers), a≠-1, $\lim\limits_{x\to\infty} \dfrac{(1^a + 2^a + .. + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+..+(na+n)]} = \dfrac{1}{60}$. Then $a =$
(a) 5 (b) 7 (c) $\dfrac{-15}{2}$ (d) $\dfrac{-17}{2}$

71. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as
$$g(x) = \begin{cases} 0, & \text{if } x < a \\ \int_a^x f(t)dt, & \text{if } a \le x \le b \\ \int_a^b f(t)dt, & \text{if } x > b \end{cases}; \text{ then}$$
(a) $g(x)$ is continuous but not differentiable at a
(b) $g(x)$ is differentiable on $R$
(c) $g(x)$ is continuous but not differentiable at b
(d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

72. For every pair of continuous functions $f, g : [0,1] \to R$ such that max$\{f(x) : x \in [0,1]\}$=max$\{g(x) : x \in [0,1]\}$, the correct statement(s) is(are)

(a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$

(d) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

73. Let $g : R \to R$ be a differentiable function with $g(0) = 0, g'(0) = 0$ and $g'(1) \neq 0$. Let
$$f(x) = \begin{cases} \frac{x}{|x|}g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$
and $h(x) = e^{|x|}$ for all $x \in R$. Let $(foh)(x)$ denote $f(h(x))$ and $(hof)(x)$ denote $h(f(x))$. Then which of the following is(are) true ?
(a) $f$ is differentiable at $x = 0$
(b) $h$ is differentiable at $x = 0$
(c) $foh$ is differentiable at $x = 0$
(d) $hof$ is differentiable at $x = 0$

74. Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$.
Then f is
(a) differentiable at $x = 0$ if $a = 0$ and $b = 1$
(b) differentiable at $x = 1$ if $a = 1$ and $b = 0$
(c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
(d) NOT differentiable at $x = 1$ if $a = 1$ and $b = 1$

75. Let $f : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where [y] denotes the greatest integer less than or equal to y for y $\in R$. Then
(a) $f$ is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
(b) $f$ is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
(c) $g$ is NOT differentiable excatly at four points in $\left(-\frac{1}{2}, 2\right)$
(d) $g$ is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

76. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?

(a) $x = -1$ (b) $x = 0$
(c) $x = 1$ (d) $x = 2$

77. Let $f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|}\cos\left(\frac{1}{1 - x}\right)$ for $x \neq 1$. Then
(a) $\lim_{x \to 1^-} f(x) = 0$

(b) $\lim_{x \to 1^-} f(x)$ does not exist

(c) $\lim_{x \to 1^+} f(x) = 0$

(d) $\lim_{x \to 1^+} f(x)$ does not exist

78. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If
$f'(x) = \left(e^{(f(x) - g(x))}\right)g'(x)$ for all $x \in \mathbb{R}$,
and $f(1) = g(2) = 1$, then which of the following statement(s) is(are) TRUE ?

(a) $f(2) < 1 - log_e 2$ (b) $f(2) > 1 - log_e 2$

(c) $g(1) > 1 - log_e 2$ (d) $g(1) < 1 - log_e 2$

79. Let $f : R \to R$ given by
$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1, & 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3 \\ (x - 2)\log_e(x - 2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$
then which of the following options is/are correct ?
(a) $f'$ has a local maximum at $x = 1$
(b) $f$ is increasing on $(-\infty, 0)$
(c) $f'$ is NOT differentiable at $x = 1$
(d) $f$ is onto

80. Let $f : R \to R$ be a function. We say that $f$ has
**PROPERTY 1** if $\lim_{h \to 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and

is finite, and

**PROPERTY 2** if $\lim_{h\to 0}\dfrac{f(h)-f(0)}{h^2}$ exists and is finite

(a) $f(x) = x^{2/3}$ has **PROPERTY 1**
(b) $f(x) = \sin x$ has **PROPERTY 2**
(c) $f(x) = |x|$ has **PROPERTY 1**
(d) $f(x) = x|x|$ has **PROPERTY 2**

81. Evaluate $\lim_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, $(a \neq 0)$

82. $f(x)$ is the integral of $\dfrac{2\sin x-\sin 2x}{x^3}, x \neq 0$, find $\lim_{x\to 0} f'(x)$.

83. Evaluate $\lim_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

84. Let $f(x+y) = f(x)+f(y)$ for all $x$ and $y$. If the function $f(x)$ is continuous at $x = 0$, then show that $f(x)$ is continuous at all $x$.

85. Use the formula $\lim_{x\to 0}\dfrac{a^x-1}{x} = lna$ to find $\lim_{x\to 0}\dfrac{2^x-1}{(1+x)^{1/2}-1}$

86. Let $f(x) = \begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 \le x \le 3 \end{cases}$

Determine the form of $g(x) = fIf(x)$ and hence find the points of discontinuity of $g$, if any.

87. Let $f(x) = \begin{cases} \dfrac{x^2}{2}, & 0 \le x < 1 \\ 2x^2-3x+\dfrac{3}{2}, & 1 \le x \le 2 \end{cases}$

Discuss the continuity of $f, f'$ and $f''$ on [0,2].

88. Let $f(x) = x^3-x^2+x+1$ and $g(x) = \begin{cases} \max\{f(t); 0 \le t \le x\}, & 0 \le x \le 1 \\ 3-x, & 0 \le x \le 2 \end{cases}$

Discuss the continuity and differentiability f the function $g(x)$ in the interval (0,2).

89. Let $f(x)$ be defined in the interval [-2,2] such that $f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$ and $g(x) = f(|x|)+|f(x)|$. Test the differentiability of $g(x)$ in (-2,2).

90. Let $f(x)$ be a continuous and $g(x)$ be a discontinuous function. Prove that $f(x)+g(x)$ is a discontinuous function.

91. Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real $x$. If $f'(0)$ exists, find its value.

92. Find the values of a and b so that the function

$$f(x) = \begin{cases} x+a\sqrt{2}\sin x, & 0 \le x \le \pi/4 \\ 2x\cot x+b, & \pi/4 \le x \le \pi/2 \\ a\cos 2x-b\sin x, & \pi/2 < x \le \pi \end{cases}$$

is continuous for $0 \le x \le \pi$.

93. Draw a graph of the function $y = [x]+|1-x|, -1 \le x \le 3$. Determine the points, if any, where this function is not differentiable.

94. Let $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x > 0 \end{cases}$

Determine the value of $a$, if possible, so that the function is continuous at $x = 0$.

95. A function $f : R \to R$ satisfies the equation $f(x+y) = f(x)f(y)$ for all $x, y$ in $R$ and $f(x) \neq 0$ for any $x$ in $R$. Let the function be differentiable at $x = 0$ and $f'(0) = 2$. Show that $f'(x) = 2f(x)$ for all $x$ in $R$. hence, determine $f(x)$.

96. Find $\lim_{x\to 0}\{\tan(\pi/4+x)\}^{1/x}$.

97. Let

$$f(x) = \begin{cases} \{1+|\sin x|\}^{a/|\sin x|}; & \dfrac{\pi}{6} < x < 0 \\ b: & x = 0 \\ e^{\tan 2x/\tan 3x}; & 0 < x < \dfrac{\pi}{6} \end{cases}$$

Determine $a$ and $b$ such that $f(x)$ is continuous at $x = 0$.

98. Let $f\left(\dfrac{x+y}{2}\right) = \dfrac{f(x)+f(y)}{2}$ for all real

$x$ and $y$. If $f'(0)$ exists and equal $-1$ and $f(0) = 1$, find $f(2)$.

99. Determine the values of $x$ for which the following function fails to be continuous or differentiable :

$$f(x) = \begin{cases} 1 - x, & x < 1 \\ (1 - x)(2 - x), & 1 \le x \le 2 \\ 3 - x, & x > 2 \end{cases}$$

Justify your answer.

100. Let $f(x), x \ge 0$, be non-negative continuous function, and let $F(x) = \int\limits_0^x f(t)dt, x \ge 0$. If for some $c > 0, f(x) \le cF(x)$ for all $x \ge 0$, then show that $f(x) = 0$ for all $x \ge 0$.

101. Let $\alpha \in R$. Prove that a function $f : R \to R$ is differentiable at $\alpha$ if and only if there is a function $g : R \to R$ which is continuous at $\alpha$ and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha)$ for all $x \in R$.

102. Let $f(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ |x - 1|, & \text{if } x \ge 0 \end{cases}$ and $g(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ (x - 1)^2 + b, & \text{if } x \ge 0 \end{cases}$ where $a$ and $b$ are non-negative numbers. Determine the composite function $gof$. If $(gof)(x)$ is continuous for all real $x$, determine the values of $a$ and $b$. Further, for these values of $a$ and $b$, is $gof$ differentiable at $x = 0$ ? Justify your answer.

103. If a function $f : [-2a, 2a] \to R$ is an odd function such that $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left hand derivative at $x = a$ is 0 then find the left hand derivative at $x = -a$.

104. $f'(0) = \lim\limits_{n\to\infty} nf\left(\frac{1}{n}\right)$ and $f(0) = 0$. Using this find

$$\lim_{n\to\infty}\left((n + 1)\frac{2}{\pi}\cos^{-1}\left(\frac{1}{n}\right) - n\right),$$

$$\left|cos^{-1}\frac{1}{n}\right| < \frac{\pi}{2}$$

105. if $|c| \le \frac{1}{2}$ and $f(x)$ is a differentiable function at $x = 0$ given by

$$f(x) = \begin{cases} b\sin^{-1}\left(\frac{c + x}{2}\right), & -\frac{1}{2} < x < 0 \\ \frac{1}{2}, & x = 0 \\ \frac{e^{ax/2} - 1}{x}, & 0 < x < \frac{1}{2} \end{cases}$$

Find the value of 'a' and prove that $64b^2 = 4 - c^2$.

106. If $f(x - y) = f(x)$ffl$g(y) - f(y)$ffl$g(x)$ and $g(x - y) = g(x)$ffl$g(y) - f(x)$ffl$f(y)$ for all $x, y \in R$. If right hand derivative at $x = 0$ exists for $f(x)$. Find derivative of $g(x)$ at $x = 0$.

107. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable functions such that $f(1) = 2$. If $6\int\limits_1^x f(t)dt = 3xf(x) - x^3$ for all $x \ge 14$, then the value of $f(2)$ is ?

108. The largest value of non-negative integer a for which

$$\lim_{x\to1}\left\{\frac{-ax + \sin(x - 1) + a}{x + \sin(x - 1) - 1}\right\}^{\frac{1 - x}{1 - \sqrt{x}}} = \frac{1}{4}$$

is

109. Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} max\{f(x).g(x)\}, & \text{if } x \le 0 \\ min\{f(x).g(x)\}, & \text{if } x > 0 \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

110. Let $m$ and $n$ be two positive integers greater than 1. If $\lim\limits_{\alpha\to0}\left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is

111. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim\limits_{x\to0}\frac{x^2\sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

112. $\lim\limits_{x\to0}\frac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) -1
(c) zero (d) does not exist

113. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^x$

(a) $e^4$ (b) $e^2$ (c) $e^3$ (d) 1

114. Let $f(x)=4$ and $f'(x)=4$. Then

$\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by

(a) 2 (b) -2 (c) -4 (d) 3

115. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+...+n^p}{n^{p+1}}$

(a) $\frac{1}{p+1}$ (b) $\frac{1}{p}-\frac{1}{p-1}$
(c) $\frac{1}{1-p}$ (d) $\frac{1}{p+2}$

116. $\lim_{x\to 0}\frac{\log x^n-[x]}{[x]}, n\in N$, ([x] denotes greatest integer less than or equal to x)

(a) has value -1 (b) has value 0
(c) has value 1 (d) does not exist

117. If $f(1)=1, f'(1)=2$, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(a) 2 (b) 4 (c) 1 (d) 1/2

118. $f$ is defined in [-5,5] as
$f(x)=\begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational}\end{cases}$. Then

(a) $f(x)$ is continuous at every $x$, except $x=0$
(b) $f(x)$ is discontinuous at every $x$, except $x=0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere

119. $f(x)$ and $g(x)$ are two differentiable functions on [0,2] such that $f''(x)-g''(x)=0, f'(1)=2g'(1)=4f(2)=3g(2)=9$ then $f(x)-g(x)$ at x = 3/2 is

(a) 0 (b) 2 (c) 10 (d) 5

120. If $(x+y)=f(x).f(y)\forall x,y$ and $f(5)=2$,

$f'(0)=3$, then $f'(5)$ is

(a) 0 (b) 1 (c) 6 (d) 2

121. $\lim_{n\to\infty}\frac{1+2^4+3^4+...+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+..+n^3}{n^5}$

(a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

122. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of $k$ is

(a) $-\frac{2}{3}$ (b) 0 (c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

123. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(a) 0 (b) 3 (c) 2 (d) 1

124. Let $f(a)=g(a)=k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some $n$. Further if

$\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+f(a)}{g(x)-f(x)}=4$

then the value of $k$ is

(a) 0 (b) 4 (c) 2 (d) 1

125. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is

(a) $\infty$ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

126. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0 \\ 0, & x=0\end{cases}$ then $f(x)$ is

(a) discontinuous everywhere
(b) continuous as well as differentiable for all $x$
(c) continuous for all $x$ but not differentiable at $x=0$
(d) neither differentiable nor continuous at $x=0$

127. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the

values of $a$ and $b$, are

(a) $a = 1$ and $b = 2$ (b) $a = 1, b \in R$
(c) $a \in R, b = 2$ (d) $a \in R, b \in R$

128. Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}, x \neq \dfrac{\pi}{4}, x \in \left[0, \dfrac{\pi}{2}\right]$.
If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$ is

(a) -1 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) 1

129. $\lim_{n\to\infty}\left[\dfrac{1}{n^2}\sec^2\dfrac{1}{n^2} + \dfrac{2}{n^2}\sec^2\dfrac{4}{n^2}... + \dfrac{1}{n}\sec^2 1\right]$
equals

(a) $\dfrac{1}{2}\sec 1$ (b) $\dfrac{1}{2}cosec 1$

(c) $\tan 1$ (d) $\dfrac{1}{2}\tan 1$

130. Let $\alpha$ and $\beta$ be the distinct roots of $ax^2+bx+c = 0$, then $\lim_{x\to\alpha}\dfrac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(a) $\dfrac{\alpha^2}{2}(\alpha - \beta)^2$ (b) 0

(c) $\dfrac{-\alpha^2}{2}(\alpha - \beta)^2$ (d) $\dfrac{1}{2}(\alpha - \beta)^2$

131. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\dfrac{1}{h}f(1 + h) = 5$, then $f'(1)$ equals

(a) 3 (b) 4 (c) 5 (d) 6

132. Let $f$ be differentiable for all $x$. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(a) $f(6) \geq 8$ (b) $f(6) < 8$
(c) $f(6) < 5$ (d) $f(6) = 5$

133. If $f$ is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2, x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(a) -1 (b) 0 (c) 2 (d) 1

134. Let $f : R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$, Then which of the following is true ?
(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x) \geq 1$ for all $x \in R$
(d) $f(x)$ is not differentiable at $x = 1$

135. The function $f : R/\{0\} \to R$ is given by $f(x) = \dfrac{1}{x} - \dfrac{2}{e^{2x} - 1}$ can be made continuous at $x = 0$ by defininig $f(0)$ as

(a) 0 (b) 1 (c) 2 (d) -1

136. Let $f(x) = \begin{cases} (x - 1)\sin\dfrac{1}{x - 1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$
Then which of the following is true ?
(a) $f$ is neither differentiable at $x = 0$ nor at $x = 1$
(b) $f$ is differentiable at $x = 0$ and at $x = 1$
(c) $f$ is differentiable at $x = 0$ but not at $x = 1$
(d) $f$ is differentiable at $x = 1$ but not at $x = 0$

137. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty}\dfrac{f(3x)}{f(x)} = 1$. then $\lim_{x\to\infty}\dfrac{f(2x)}{f(x)} =$
(a) $\dfrac{2}{3}$ (b) $\dfrac{3}{2}$ (c) 3 (d) 1

138. $\lim_{x\to 2}\left(\dfrac{\sqrt{1 - \cos\{2(x - 2)\}}}{x - 2}\right)$
(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$
(c) equals $\dfrac{1}{\sqrt{2}}$ (d) does not exist

139. The values of $p$ and $q$ for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p + 1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all $x$ in $R$, are

(a) $p = \dfrac{5}{2}, q = \dfrac{1}{2}$ (b) $p = \dfrac{1}{2}, q = \dfrac{3}{2}$

(c) $p = -\dfrac{3}{2}, q = \dfrac{1}{2}$ (d) $p = \dfrac{1}{2}, q = -\dfrac{3}{2}$

140. Let $f : R \to [0, \infty)$ be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5}\dfrac{(f(x))^2 - 9}{\sqrt{|x - 5|}} = 0$. Then $\lim_{x\to 5} f(x)$ equals

(a) 0 (b) 1 (c) 2 (d) 3

141. If $f : R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then $f$ is
(a) continuous for every real $x$
(b) discontinuous only at $x = 0$
(c) discontinuous only at non-zero integral values of $x$
(d) continuous only at $x = 0$

142. Consider the function, $f(x) = |x-2|+|x-5|, x \in R$.
**Statement-1:** $f'(4) = 0$
**Statement-2:** $f$ is continuous in [2,5], differentiable in (2,5) and $f(2) = f(5)$
(a) Statement-1 is false, Statement-2 is true
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is false

143. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to
(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

144. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to
(a) $-\pi$ (b) $\pi$ (c) $\frac{\pi}{2}$ (d) 1

145. If the function,

$$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases} \text{ is}$$

differentiable, then the value of k + m is
(a) $\frac{10}{3}$ (b) 4 (c) 2 (d) $\frac{16}{5}$

146. For $x \in R, f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then
(a) $g'(0) = -\cos(\log 2)$
(b) $g$ is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$
(c) $g$ is not differentiable at $x = 0$
(d) $g'(0) = \cos(\log 2)$

147. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to
(a) $\frac{9}{e^2}$ (b) $3\log 3 - 2$
(c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

148. Let $p = \lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then $\log p$ is equal to
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) 2 (d) 1

149. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals
(a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

150. For each $t \in R$, let [t] be the greatest integer less than or equal to t/. Then

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+...+\left[\frac{15}{x}\right]\right)$$

(a) is equal to 15 (b) is equal to 120
(c) does not exist(in R) (d) is equal to 0

151. Let $S = t \in R : f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$ is not differentiable at t. Then the set S is equal to
(a) $\{0\}$ (b) $\{\pi\}$
(c) $\{0,\pi\}$ (d) $\phi$(an empty set)

152. $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$
(a) exists and equals $\frac{1}{4\sqrt{2}}$
(b) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$
(c) exists and equals $\frac{1}{2\sqrt{2}}$
(d) does not exist

153. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Then f is

(a) continuous if a = 5 and b = 5
(b) continuous if a = -5 and b = 10
(c) continuous if a = 0 and b = 5
(d) not continuous for any values of a and b

154. if the function f is defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \ne \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$$

is continuous, then k is equal to

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

155. Let $f(x) = 15 - |x - 10|; x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is

(a) {5,10,15} (b) {10,15}
(c) {5,10,15,20} (d) {10}

156. In this questions there are entries in columns I and II. Each entry in column I is related to exactly one entry in column II.

| **Column I** | **Column II** |
|---|---|
| (A) $\sin(\pi[x])$ | (p) differentiable everywhere |
| (B) $\sin(\pi(x - [x]))$ | (q) nowhere differentiable |
| | (r) not differentiable at 1 and -1 |

157. In the following [x] denotes the greatest integer less than or equal to $x$.Match the functions in **Column I** with the properties in **Column II**

| **Column I** | **Column II** |
|---|---|
| (A) $x\|x\|$ | (p) continuous in (-1,1) |
| (B) $\sqrt{\|x\|}$ | (q) differentiable in (-1,1) |
| (C) $x + [x]$ | (r) strictly increasing in (-1,1) |
| (D) $\|x - 1\| + \|x + 1\|$ | (s) not differentiable at least at one point (-1,1) |

158. Let $f_1 : R \to R, f_2 : [0, \infty) \to R, f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by $f_1(x) = \begin{cases} |x|, & \text{if } x < 0 \\ e^x, & \text{if } x \ge 0 \end{cases}$ ; $f_2(x) = x^2$; $f_3(x) = \begin{cases} \sin x, & \text{if } x < 0 \\ x, & \text{if } x \ge 0; \end{cases}$ and

$$f_4(x) = \begin{cases} f_2(f_1(x)), & \text{if } x < 0 \\ f_2(f_1(x)) - 1, & \text{if } x \geq 0; \end{cases}$$

| **List-I** | **List-II** |
|---|---|
| (P) $f_4$ is | 1. onto but not one-one |
| (Q) $f_3$ is | 2. Neither continuous nor one-one |
| (R) $f_2 o f_1$ is | 3. differentiable but not one-one |
| (S) $f_2$ is | 4. continuous and one-one |

(a) P → 3; Q → 1; R → 4; S → 2
(b) P → 1; Q → 3; R → 4; S → 2
(c) P → 3; Q → 1; R → 2; S → 4
(d) P → 1; Q → 3; R → 2; S → 4

159. let $f_1 : \mathbb{R} \to \mathbb{R}, f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$
$f_3 : \left(-1, e^{\frac{\pi}{2}-2}\right)$ and $f_4 : \mathbb{R} \to \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{tan^{-1} x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$, where the inverse trignometric function

$\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where for $t \in \mathbb{R}, [t]$ denotes the greatest integer less than or equal to $t$

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

| **List-I** | **List-II** |
|---|---|
| (P) The function $f_1$ is | 1. **NOT** continuous at $x = 0$ |
| (Q) The function $f_2$ is | 2. Continuous at $x = 0$ and **NOT** differentiable at $x = 0$ |
| (R) The function $f_3$ is | 3. differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$ |
| (S) The function $f_4$ is | 4. differentiable at $x = 0$ and its derivative is continuous at $x = 0$ |

(a) P → 2; Q → 3; R → 1; S → 4
(b) P → 4; Q → 1; R → 2; S → 3
(c) P → 4; Q → 2; R → 1; S → 3
(d) P → 2; Q → 1; R → 4; S → 3

## 10 Definite integrals

1. 
$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$
Then $\int_0^{\pi/2} f(x)dx$=......

2. The integral $\int_0^{1.5} [x^2]dx$, Where [ ] denotes the greatest integer function, equals.......

3. The value of
$$\int_{-2}^{2} |1 - x^2| dx =$$

4. The value of
$$\int_{\pi/4}^{3\pi/4} \frac{\phi}{1 + \sin\phi} d\phi =$$

5. The value of
$$\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx =$$

6. If for non-zero x,
$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$$
where $a \neq b$, then $\int_1^2 f(x)dx =$ ...............

7. For $n > 0$,
$$\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx =$$

8. The value of
$$\int_1^{e^{37}} \frac{\pi \sin(\pi ln x)}{x} dx =$$

9. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If
$$\int_1^4 \frac{2e^{\sin x^2}}{x} = F(k) - F(1)$$
then one of the possible values of k is..........

**True/False**

10. The value of the integral
$$\int_0^{2a} \frac{f(x)}{f(x) + f(2a - x)} dx$$
is equal to a.

**MCQs with One Correct Answer**

11. The value of the definite integral $\int_0^1 (1 + e^{-x^2}) dx$ is

a) -1
b) 2
c) $1 + e^{-1}$
d) None of these

12. Let a, b, c be non-zero real numbers such that

$$\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx$$

Then the quadratic equation $ax^2 + bx + c = 0$ has

a) no root in (0, 2)
b) at least one root in (0, 2)
c) a double root in (0, 2)
d) two imaginary roots

13. The area bounded by the curves $y = f(x)$, the x-axis and the ordinates x = 1 and x = b is $(b - 1)\sin(3b + 4)$. Then $f(x)$ is

a) $(x - 1)\cos(3x + 4)$
b) $\sin(3x + 4)$
c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
d) None of these

14. The value of the integral

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx =$$

a) $\pi/4$
b) $\pi/2$
c) $\pi$
d) None of these

15. For any integer n the integral

$$\int_0^{\pi} e^{\cos^2 x} \cos^3(2n + 1)dx$$

has the value

a) $\pi$
b) 1
c) 0
d) None of these

16. Let $f : R \to R$ and $g : R \to R$ be continuous functions. Then the value of the integral

$$\int_{-\pi/2}^{\pi/2} [f(x) + f(-x)][g(x) - g(-x)]dx =$$

a) $\pi$
b) 1
c) -1
d) 0

17. The value of

$$\int_0^{\pi/2} \frac{dx}{1 + \tan^3 x} =$$

a) 0
b) 1
c) $\pi/2$
d) $\pi/4$

18. If $f(x) = A\sin(\frac{\pi x}{2}) + B$, $f'(\frac{1}{2}) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then constants A and B are

a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$
b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$
c) 0 and $\frac{-4}{\pi}$
d) $\frac{4}{\pi}$ and 0

19. The value of $\int_{\pi}^{2\pi}[2\sin x]dx$ where [ ] represents the greatest integer funcion is

a) $\frac{-5\pi}{3}$
b) $-\pi$
c) $\frac{5\pi}{3}$
d) $-2\pi$

20. If

$$g(x) = \int_0^x \cos^4 t dt$$

then $g(x + \pi)$ equals

a) $g(x) + g(\pi)$
b) $g(x) - g(\pi)$
c) $g(x)g(\pi)$
d) $\frac{g(x)}{g(\pi)}$

21.

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x} =$$

a) 2
b) -2
c) 1/2
d) -1/2

22. If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral

$$\int_{\pi/2}^{3\pi/2} [2\sin x]dx =$$

a) -$\pi$
b) 0
c) $\pi/2$
d) $\pi/2$

23. Let

$$g(x) = \int_0^x f(t)dt$$

where f is such that $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0,1]$ and $0 \leq f(t) \leq \frac{1}{2}$, for $t \in [1,2]$. Then g(2) satisfies the inequality

a) $\frac{3}{2} \leq g(2) < \frac{1}{2}$
b) $0 \leq g(2) < 2$
c) $\frac{3}{2} < g(2) \leq \frac{5}{2}$
d) $2 < g(2) < 4$

24. If

$$f(x) = \left\{ \begin{array}{ll} e^{\cos x} \sin x & for|x| \leq 2 \\ 2 & otherwise \end{array} \right\}$$

then $\int_{-2}^{3} f(x)dx =$

a) 0
b) 1
c) 2
d) 3

25. The value of the integral

$$\int_{e^{-1}}^{e^2} \left|\frac{log_e x}{x}\right| dx =$$

a) 3/2
b) 5/2
c) 3
d) 5

26. The value of

$$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx =$$

a) $\pi$
b) $a\pi$
c) $\pi/2$
d) $2\pi$

27. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

a) 1
b) 2
c) $2\sqrt{2}$
d) 4

28. Let

$$f(x) = \int_1^x \sqrt{2-t^2}dt$$

Then the real roots of the equation $x^2 - f'(x) = 0$ are

a) $\pm 1$
b) $\pm\frac{1}{\sqrt{2}}$
c) $\pm\frac{1}{2}$
d) 0 and 1

29. Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$ then the value of $\int_3^{3+3T} f(2x)dx$ is

a) 3/2I
b) 2I
c) 3I
d) 6I

30. The integral

$$\int_{-1/2}^{1/2} ([x] + ln(\frac{1+x}{1-x}))dx =$$

a) $\frac{-1}{2}$
b) 0
c) 1
d) $2ln(\frac{1}{2})$

31. If $l(m,n) = \int_0^1 t^m(1+t)^n dt$ then the expression for l(m, n) in terms of l(m + 1, n - 1) is

a) $\frac{2^n}{m+1} - \frac{n}{m+1}l(m+1, n-1)$
b) $\frac{n}{m+1}l(m+1, n-1)$
c) $\frac{2^n}{m+1} + \frac{n}{m+1}l(m+1, n-1)$
d) $\frac{m}{m+1}l(m+1, n-1)$

32. If

$$f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$$

then, f(x) increases in

a) (-2, 2)
b) No value of x
c) $(0, \infty)$
d) $(-\infty, 0)$

33. The area bounded by the curves $y = \sqrt{x}$, 2y + 3 = x and x-axis in the $1^{st}$ quadrant is

a) 9
b) 27/4
c) 36
d) 18

34. If f(x) is differentiable and

$$\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5$$

then $f(\frac{4}{25})$ equals

a) 2/5

b) -5/2
c) 1
d) 5/2

35. The value of the integral

$$\int_0^1 \sqrt{\frac{1-x}{1+x}}dx =$$

a) $\frac{\pi}{2}+1$
b) $\frac{\pi}{2}-1$
c) -1
d) 1

36. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq. unit, then the value of a is

a) $1/\sqrt{3}$
b) 1/2
c) 1
d) 1/3

37.

$$\int_{-2}^{0}\{x^3+3x^2+3x+3+(x+1)\cos(x+1)\}dx =$$

a) -4
b) 0
c) 4
d) 6

38. The area bounded by the parabolas $y = (x+1)^2$ and $y = (x-1)^2$ and the line $y = 1/4$ is

a) 4 sq.units
b) 1/6 sq.units
c) 4/3 sq.units
d) 1/3 sq.units

39. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines x = 0 and $x = \frac{\pi}{4}$ is

a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$
b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$
c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$
d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

40. Let f be a non-negative function defined on the interval [0, 1]. If

$$\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt,$$

$0 \le x \le 1$, and f(0) = 0, then

a) $f(\frac{1}{2}) < \frac{1}{2}$ and $f(\frac{1}{3}) > \frac{1}{3}$
b) $f(\frac{1}{2}) > \frac{1}{2}$ and $f(\frac{1}{3}) > \frac{1}{3}$
c) $f(\frac{1}{2}) < \frac{1}{2}$ and $f(\frac{1}{3}) < \frac{1}{3}$
d) $f(\frac{1}{2}) > \frac{1}{2}$ and $f(\frac{1}{3}) < \frac{1}{3}$

41. The value of

$$\lim_{x\to 0}\frac{1}{x^3}\int_0^x \frac{tln(1+t)}{t^4+4}dt$$

a) 0
b) $\frac{1}{12}$
c) $\frac{1}{24}$
d) $\frac{1}{64}$

42. Let f be a real-valued function defined on the interval (-1, 1) such that

$$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}dt$$

for all $x \in (-1,1)$, and let $f^{-1}$ be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

a) 1
b) 1/3
c) 1/2
d) 1/e

43. The value of

$$\int_{\sqrt{\ln 2}}^{\sqrt{ln3}} \frac{x\sin x^2}{\sin x^2+\sin(ln6-x^2)}dx =$$

a) $\frac{1}{4}ln\frac{3}{2}$
b) $\frac{1}{2}ln\frac{3}{2}$
c) $ln\frac{3}{2}$
d) $\frac{1}{6}ln\frac{3}{2}$

44. Let the straight line x = b divide the area enclosed by $y = (1-x)^2$, y = 0 and x = 0 into two parts $R_1 (0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equlas

a) 3/4
b) 1/2
c) 1/3
d) 1/4

45. Let $f : [-1,2] \to [0,\infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1,2]$. Let $R_1 = \int_{-1}^{2} xf(x)dx$, x = -1, x = 2 and the x-axis. Then

a) $R_1 = 2R_2$
b) $R_1 = 3R_2$
c) $2R_1 = R_2$
d) $3R_1 = R_2$

46. The value of the integral

$$\int_{-\pi/2}^{\pi/2}(x^2 + ln\frac{\pi + x}{\pi - x})\cos x dx$$

a) 0
b) $\frac{\pi^2}{2} - 4$
c) $\frac{\pi^2}{4} + 4$
d) $\frac{\pi^2}{2}$

47. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $[0, \frac{\pi}{2}]$ is
a) $4(\sqrt{2} - 1)$
b) $2\sqrt{2}(\sqrt{2} - 1)$
c) $2(\sqrt{2} + 1)$
d) $2\sqrt{2}(\sqrt{2} + 1)$

48. Let $f : [\frac{1}{2}, 1] \to R$(the set of all real number) be a positive, non-constant and diffrentiable function such that $f'(x) < 2f(x)$ and $f(\frac{1}{2}) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval
a) (2e-1, 2e)
b) (e - 1, 2e - 1)
c) $(\frac{e-1}{2}, e - 1)$
d) $(0, \frac{e-1}{2})$

49. The following integral

$$\int_{\pi/4}^{\pi/2}(2\,\text{cosec}\, x)^{17}dx$$

is equal to
a) $\int_0^{log(1+\sqrt{2})} 2(e^u + e^{-u})^{16}dx$
b) $\int_0^{log(1+\sqrt{2})} (e^u + e^{-u})^{17}dx$
c) $\int_0^{log(1+\sqrt{2})} 2(e^u - e^{-u})^{17}dx$
d) $\int_0^{log(1+\sqrt{2})} 2(e^u - e^{-u})^{16}dx$

50. The value of

$$\int_{\pi/2}^{\pi/2} \frac{x^2 \cos x}{1 + e^x}dx$$

is equal to
a) $\frac{\pi^2}{4} - 2$
b) $\frac{\pi^2}{4} + 2$
c) $\pi^2 - e^{\frac{\pi}{2}}$
d) $\pi^2 + e^{\frac{\pi}{2}}$

51. Area of the region

$$\{(x, y) \in R^2 : y \geq \sqrt{|x + 3|}, 5y \leq x + 9 \leq 15\}$$

is equal to
a) $\frac{1}{6}$
b) $\frac{4}{3}$
c) $\frac{3}{2}$
d) $\frac{5}{3}$

52. The area of the region

$$\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\} =$$

a) $8log_e2 - \frac{14}{3}$
b) $16log_e2 - \frac{14}{3}$
c) $8log_e2 - \frac{7}{3}$
d) $16log_e2 - 6$

**MCQs with One or More than One Correct Answer**

53. If

$$\int_0^x f(t)dt = x + \int_x^t tf(t)dt$$

then the value of f(1) is
a) 1/2
b) 0
c) 1
d) -1/2

54. Let $f(x) = x - [x]$, for every real number x, where [x] is the integral part of x. Then

$$\int_{-1}^{1} f(x)dx =$$

a) 1
b) 2
c) 0
d) 1/2

55. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2?
a) -4
b) -2
c) 2
d) 4

56. Let $f(x)$ be a non-constant twice diffrentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1 - x)$ and $f'(\frac{1}{4}) = 0$. Then,
a) $f''(x)$ vanishes at least twice on [0, 1]
b) $f'(\frac{1}{2}) = 0$
c) $\int_{-1/2}^{1/2} f(x + \frac{1}{2})\sin x dx = 0$
d) $\int_0^{1/2} f(t)e^{\sin \pi t}dt = \int_{1/2}^{1} f(1 - t)e^{\sin \pi t}dt$

57. Area of the region bounded by the curve $y = e^x$ and lines x = 0 and y = e is

a) e - 1
b) $\int_1^e ln(e+1-y)dy$
c) $e - \int_0^1 e^x dx$
d) $\int_1^e lnydy$

58. If

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx$$

where n = 0, 1 , 2..... then

a) $I_n = I_{n+2}$
b) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$
c) $\sum_{m=1}^{10} I_{2m} = 0$
d) $I_n = I_{n+1}$

59. The value(s) of

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx =$$

a) $\frac{22}{7} - \pi$
b) $\frac{2}{105}$
c) 0
d) $\frac{71}{15} - \frac{3\pi}{2}$

60. Let f be a real-valued function defined on the interval $(0,\infty)$ by

$$f(x) = lnx + \int_0^x \sqrt{1+\sin t}dt$$

Then which of the following statement(s) is(are) true?

a) $f''(x)$ exists for all $x \in (0,\infty)$
b) $f'(x)$ exists for all $x \in (0,\infty)$ and $f'$ is continuous on $(0,\infty)$, but not differentiable on $(0,\infty)$
c) There exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha,\infty)$
d) There exists $\beta > 1$ such that $|f'(x)| + |f(x)| \leq \beta$ for all $x \in (0,\infty)$

61. Let S be the area of the region enclosed by $y = e^{x^2}$, y = 0, x = 0 and x = 1: then

a) $S \geq \frac{1}{e}$
b) $S \geq 1 - \frac{1}{e}$
c) $S \leq \frac{1}{4}(1 + \frac{1}{\sqrt{e}})$
d) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}(1 - \frac{1}{\sqrt{2}})$

62. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)dt} = L?$$

a) a = 2, $L = \frac{e^{4\pi}-1}{e^{\pi}-1}$
b) a = 2, $L = \frac{e^{4\pi}+1}{e^{\pi}+1}$
c) a = 4, $L = \frac{e^{4\pi}-1}{e^{\pi}-1}$
d) a = 4, $L = \frac{e^{4\pi}+1}{e^{\pi}+1}$

63. Let

$$f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$$

for all $x \in (\frac{\pi}{2}, \frac{\pi}{2})$. Then the correct expression(s) is(are)

a) $\int_0^{\pi/4} xf(x)dx = \frac{1}{12}$
b) $\int_0^{\pi/4} f(x)dx = 0$
c) $\int_0^{\pi/4} xf(x)dx = \frac{1}{6}$
d) $\int_0^{\pi/4} f(x)dx = 1$

64. Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f(\frac{1}{2}) = 0$. If

$$m <\leq \int_{1/2}^1 f(x)dx \leq M$$

then the possible values of m and M are

a) m = 13, M = 24
b) m = 0.25, M = 0.5
c) m = -11, M = 0
d) m = 1, M = 12

65. Let

$$f(x) = \lim_{x\to\infty}\left(\frac{n^n(x+n)(x+\frac{n}{2}).....(x+\frac{n}{n})}{n!(x^2+n^2)(x^2+\frac{n^2}{4}).......(x^2+\frac{n^2}{n^2})}\right)^{\frac{x}{n}}$$

for all $x > 0$. Then

a) $f(\frac{1}{2}) \geq f(1)$
b) $f(\frac{1}{3}) \leq f(\frac{2}{3})$
c) $f'(2) \leq 0$
d) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

66. Let $f : R \to (0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

a) $x^9 - f(x)$
b) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos tdt$
c) $e^x - \int_0^x f(t)\sin tdt$
d) $f(x) + \int_0^{\pi/2} f(t)\sin tdt$

67. If

$$g(x) = \int \sin x^{\sin(2x)} \sin^{-1}(t)dt$$

then,

a) $g'(\frac{\pi}{2}) = -2\pi$

b) $g'(\frac{-\pi}{2}) = 2\pi$
c) $g'(\frac{\pi}{2}) = 2\pi$
d) $g'(\frac{-\pi}{2}) = -2\pi$

68. If the line $sx = \alpha$ divides the area of the region

$$R = \{(x, y) \in R^2 : x^3 \leq y \leq x, 0 \leq x \leq 1\}$$

into two equal parts, then

a) $0 < \alpha \leq \frac{1}{2}$
b) $\frac{1}{2} < \alpha < 1$
c) $2\alpha^4 - 4\alpha^2 + 1 = 0$
d) $\alpha^4 + 4\alpha^2 - 1 = 0$

69. If

$$I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx, then$$

a) $1 > log_e 99$
b) $1 < log_e 99$
c) $1 < \frac{49}{50}$
d) $1 > \frac{49}{50}$

70. For, $a \in R$, $|a| > 1$, let

$$\lim_{x\to\infty} \frac{1 + 2^{3/2} + ....... + n^{3/2}}{n^{7/3}(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + ....... + \frac{1}{(an+n)^2})} = 54$$

Then the possible value(s) of a is/are

a) -9
b) 7
c) 6
d) 8

**Subjective Problems**

71. Find the area bounded by the curve $x^2 = 4y$ and the straight line x = 4y - 2.

72. Show that:

$$\lim_{n\to\infty}(\frac{1}{n+1} + \frac{1}{n+2} + ...... + \frac{1}{6n}) = log 6$$

73. Show that

$$\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$$

74. Find the value of

$$\int_{-1}^{3/2} |x \sin \pi x| dx$$

75. For any real t, $x = \frac{e^t + e^{-t}}{2}$, $y = \frac{e^t - e^{-t}}{2}$ is a point on the hyperbola $x^2 - y^2 = 1$. Show that the area bounded by the this hyperbola and the line joining its centre to the points corresponding to $t_1$ and $-t_1$ is $t_1$.

76. Evaluate

$$\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

77. Find the area bounded by the x-axis, part of the curve $y = (1 + \frac{8}{x^2})$ and the ordinates at x = 2 and x = 4. If the ordinate at x = a divides the area into two equal parts, find a.

78. Evaluate the follwing

$$\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx.$$

79. Find the area of the region bounded by the x-axis and the curves defined by

$$y = \tan x, \frac{-\pi}{3} \leq x \leq \frac{\pi}{3}$$

$$y = \cot x, \frac{\pi}{6} \leq x \leq \frac{3\pi}{2}$$

80. Given a function $f(x)$ such that

a) it is integratable over every interval on the real line and
b) $f(t + x) = f(x)$, for every x and a real t, then Show that the integral $\int_a^{a+1} f(x) dx$ is independent of a.

81. Evaluate the following:

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx$$

82. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

83. Evaluate

$$\int_0^{\pi} \frac{x dx}{1 + \cos \alpha \sin x}, 0 < \alpha < \pi$$

84. Find the area bounded by the curves

$$x^3 + y^2 = 25, \qquad (10.0.84.1)$$

$4y = |4 - x^2|$ and x = 0 above the x-axis.

85. Find the area of the region bounded by the curves $C : y = \tan x$, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis.

86. Evaluate

$$\int_0^1 log[\sqrt{1 - x} + \sqrt{1 + x}] dx$$

87. If $f$ and $g$ are continuous function on [0, a] satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$,

then Show that

$$\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$$

88. Show that

$$\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx.$$

89. Prove that for any positive integer k,

$$\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + ......... + \cos(2k-1)x]$$

Hence prove that

$$\int_0^{\pi/2} \sin 2kx \cot x dx = \frac{\pi}{2}$$

90. Compute the area of the region bounded by the curves $y = exlnx$ and $y = \frac{lnx}{ex}$ where $lne = 1$.
91. Sketch the curves and identify the region bounded by the $x = \frac{1}{2}$, x = 2, $y = lnx$ and $y = 2^x$. Find the area of the region.
92. Evaluate

$$\int_0^{\pi} \frac{x\sin 2x \sin(\frac{\pi}{2}\cos x)}{2x-\pi}dx$$

93. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$. Find the area.
94. Determine a positive integer $n \le 5$, such that

$$\int_0^1 e^x(x-1)^n dx = 16 - 6e$$

95. Evaluate

$$\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)}dx$$

96. Show that

$$\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$$

where n is a positive integer and $0 \le v < \pi$.
97. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x-x^2$ and $y = x^2 - x$?
98. Let

$$I_m = \int_0^{\pi} \frac{1-\cos mx}{1-\cos x}dx$$

Use the mathematical induction to prove that $I_m = m\pi$, m = 0, 1, 2........

99. Evaluate the definite integral

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

100. Consider a square with vertices at (1, 1), (-1, 1), (-1, -1) and (1, -1). Let S be the region consisting of all points inside the square which are near to the origin than to any edge. Sketch the region S and find its area.
101. Let $A_n$ be the area bounded by the curve $y = (\tan x)^n$ and the lines x = 0, y = 0 and $x = \frac{\pi}{4}$. Prove that $n > 2$,

$$A_n + A_{n-2} = \frac{1}{n-1}$$

and deduce

$$\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$$

102. Determine the value of

$$\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx.$$

103. Let

$$f(x) = Maximum\{x^2, (1-x^2), 2x(1-x)\}$$

where $0 \le x \le 1$. Determine the area of the region bounded by the curves $y = f(x)$ x-axis x = 0 and x = 1.
104. Prove that

$$\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = 2\int_0^1 \tan^{-1} x dx$$

Hence or otherwise, evaluate the integral

$$\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$$

105. Integrate

$$\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}dx.$$

106. Let $f(x)$ be a continuous function given by

$$f(x) = \left\{ \begin{array}{ll} 2x & |x| \le 1 \\ x^2 + ax + b & |x| > 1 \end{array} \right\}$$

Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$.

107. For $x > 0$, let
$$f(x) = \int_e^x \frac{lnt}{1+t}dt$$
Find the function $f(x) + f(\frac{1}{x})$ and show that $f(e) + f(\frac{1}{e}) = \frac{1}{2}$.

108. Let $b \neq 0$ and for j = 0, 1, 2, ......n, let $S_j$ be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin by$, $\frac{jr}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0$, $S_1$, $S_2$,.......$S_n$ are in geometric progression. Also, find their sum for a = -1 and $b = \pi$.

109. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and y = 2, which lies to the right of the line x = 1.

110. If f is an even function then prove that
$$\int_0^{\pi/2} f(\cos 2x)\cos x dx = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x dx$$

111. Find the value of
$$\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}dx$$

112. If
$$y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x \cos\sqrt{\theta}}{1 + \sin^2\sqrt{\theta}}d\theta$$
find $\frac{dy}{dx}$ at $x = \pi$

113. Evaluate
$$\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x dx$$

114. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

115. $f(x)$ is a differentiable function and $g(x)$ is a double differentiable function such that $|f(x)| \leq 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exists some $c \in (-3, 3)$ such that $g(c).g''(c) < 0$.

116. If
$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$
$f(x)$ is a quadrant function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB.

117. Find the value of
$$5050\frac{\int_0^1 (1 - x^{50})^{100}dx}{\int_0^1 (1 - x^{50})^{101}dx}$$

118. Let $f : R \to R$ be a function defined by
$$f(x) = \left\{ \begin{array}{ll} [x] & x \leq 2 \\ 0 & x > 2 \end{array} \right\}$$
where [x] is the greatest integer less than or equal to x, if
$$I = \int_{-1}^{2} \frac{xf(x^2)}{2 + f(x+1)}dx$$
then the value of (4I - 1) is

119. Let
$$F(x) = \int_x^{x^2 + \frac{\pi}{6}} 2\cos^2 t dt$$
for all $x \in R$ and $f : [0, 0.5]$, if $F'(a) + 2$ is the area of the region bounded by x = 0, y = 0, y = f(x) and x = a, then f(0) is

120. If
$$\alpha = \int_0^1 (e^{9x + 3\tan^{-1}x})\left(\frac{12 + 9x^2}{1 + x^2}\right)dx$$
where $\tan^{-1}x$ takes only polynomial values, then the value of $\left(log_e|1 + \alpha| - \frac{3\pi}{4}\right)$ is

121. Let $f : R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose
$$F(x) = \int_{-1}^{x} f(t)dt$$
for all $x \in [-1, 2]$ and
$$G(x) = \int_{-1}^{x} t|f(f(t))|dt$$
for all $x \in [-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f(\frac{1}{2})$ is

122. The total number of distinct $x \in [0, 1]$ for which
$$\int_{t^2}^{1+t^4} dt = 2x - 1$$
is

123. Let $f : R \to R$ be a differentiable function such

that f(0)=0, $f(\frac{\pi}{2}) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_{x}^{\pi/2} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)]dt$$

for $x \in (0, \frac{\pi}{2}]$, then $\lim_{x\to 0} g(x) =$

124. For positive integer n, let

$$y_n = \frac{1}{n}(n+1)(n+2).....(n+n)^{\frac{1}{n}}$$

For $x \in R$, let [x] be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $f(L) =$

125. A farmer $F_1$ has a land in the shape of triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer $F_2$ takes away the region which lies between the side PQ and a curve of the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer $F_2$ is exactly 30 percentage of the area of $\Delta PQR$, then the value of n is...........

**Match the Following**

126. Match the following

| **Column I** | **Column II** |
|---|---|
| (A) $\int_0^{\pi/2}(\sin x)^{\cos x}\left(\cos x\cot x - log(\sin x)^{\sin x}\right)dx$ | (p) 1 |
| (B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ | (q) 0 |
| (C) Cosine of angle of intersection of curves $y = 3^{x-1}logx$ and $y = x^x - 1$ is | (r) $6ln2$ |
| (D) Let $\frac{dy}{dx} = \frac{6}{x+y}$ where y(0) = 0 then value of y when x + y = 6 is | (s) $\frac{4}{3}$ |

127. Match the following

| **Column I** | **Column II** |
|---|---|
| (A) $\int_{-1}^{1}\frac{dx}{1+x^2}$ | (p) $\frac{1}{2}log\left(\frac{2}{3}\right)$ |
| (B) $\int_0^1\frac{dx}{\sqrt{1-x^2}}$ | (q) $2log\left(\frac{2}{3}\right)$ |
| (C) $\int_2^3\frac{dx}{1-x^2}$ | (r) $\frac{\pi}{3}$ |
| (D) $\int_1^2\frac{dx}{x\sqrt{x^2-1}}$ | (s) $\frac{\pi}{2}$ |

128. Match the following

| **Column I** | **Column II** |
|---|---|
| (A) The number of polynomials $f(x)$ with non-negative integer coefficients of degree $\leq 2$, satisfying f(0) = 0 and $\int_0^1 f(x)dx = 1$, is | (p) 8 |
| (B) The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ of the maximum value is | (q) 2 |
| (C) $\int_{-2}^{2}\frac{3x^2}{(1+e^x)}dx$ equals | (r) 4 |
| (D) $\frac{\int_{-1/2}^{1/2}\cos 2x log\left(\frac{1+x}{1-x}\right)dx}{\int_0^{1/2}\cos 2x log\left(\frac{1+x}{1-x}\right)dx}$ | (s) 0 |

**codes:**

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 3 | 2 | 4 | 1 |
| (b) | 2 | 3 | 4 | 1 |
| (c) | 3 | 2 | 1 | 4 |
| (d) | 2 | 3 | 1 | 4 |

**Comprehension Based Questions**

**PASSAGE-1**

Let the definite integral be defined by the fomula

$$\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$$

For more acurate results for $c \in (a,b)$ we can use

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$$

so that for $c = \frac{a+b}{2}$, we get

$$\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c)).$$

129. $\int_0^{\pi/2} \sin x dx =$
 a) $\frac{\pi}{8}(1+\sqrt{2})$
 b) $\frac{\pi}{4}(1+\sqrt{2})$
 c) $\frac{\pi}{8\sqrt{2}}$
 d) $\frac{\pi}{4\sqrt{2}}$
130. If

$$\lim_{x\to a} \frac{\int_a^x f(x)dx - \left(\frac{x-a}{2}\right)(f(x)+f(a))}{(x-a)^3} = 0$$

then $f(x)$ is of maximum degree
 a) 4
 b) 3
 c) 2
 d) 1
131. If $f''(x) < 0$ for all $x \in (a,b)$ and c is a point such that $a < c < b$ and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum, then $f'(c)$ is equal to
 a) $\frac{f(b)-f(a)}{b-a}$
 b) $2\frac{f(b)-f(a)}{b-a}$
 c) $2\frac{f(b)-f(a)}{2b-a}$
 d) 0

**PASSAGE-2**

Consider the functions defined implicity by the equation

$$y^3 - 3y + x = 0 \qquad (10.0.131.1)$$

on various intervals in the real line. If $x \in (-\infty,-2) \cup (2,\infty)$, the equation implicity defines a unique real valued differentiable function $y = f(x)$. If $x \in (-2,2)$, the equation implicity defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

132. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$
 a) $\frac{4\sqrt{2}}{7^3 3^2}$
 b) $-\frac{4\sqrt{2}}{7^3 3^2}$
 c) $\frac{4\sqrt{2}}{7^3 3}$
 d) $-\frac{4\sqrt{2}}{7^3 3}$
133. The area of the region bounded by the curve $y = f(x)$, the x-axis and the lines x = a and x = b, where $-\infty < a < b < -2$ is
 a) $\int_a^b \frac{x}{3(f(x))^2-1}dx + bf(b) - af(a)$
 b) $-\int_a^b \frac{x}{3(f(x))^2-1}dx + bf(b) - af(a)$
 c) $\int_a^b \frac{x}{3(f(x))^2-1}dx - bf(b) + af(a)$
 d) $-\int_a^b \frac{x}{3(f(x))^2-1}dx - bf(b) + af(a)$
134.

$$\int_{-1}^{1} g'(x)dx =$$

 a) $2g(-1)$
 b) 0
 c) $-2g(1)$
 d) $2g(1)$

**PASAAGE-3**

Consider the function $f : (-\infty,\infty) \to (-\infty,\infty)$ defined by

$$f(x) = \frac{x^2-ax+1}{x^2+ax+1}, 0 < a < 2.$$

135. Which of the following is True?
 a) $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$
 b) $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$
 c) $f'(1)f'(-1) = (2-a)^2$
 d) $f'(1)f'(-1) = -(2-a)^2$
136. Which of the following is True?
 a) $f(x)$ is decreasing on (-1, 1) and has a local minimum at x = 1
 b) $f(x)$ is increasing on (-1, 1) and has a local minimum at x = 1
 c) $f(x)$ is increasing on (-1, 1) but has a neither local maximum nor local minimum at x = 1
 d) $f(x)$ is decreasing on (-1, 1) but has a neither local maximum nor local minimum at x = 1
137. Let

$$g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2}dt$$

Which of the following is True?

a) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
b) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$
c) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
d) $g'(x)$ does not change sign on $(-\infty, \infty)$

**PASSAGE-4**

Consider the polynomial

$$f(x) = 1 + 2x + 3x^2 + 4x^3 \qquad (10.0.137.1)$$

Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$

138. The real numbers lies in the interval
a) $\left(-\frac{1}{4}, 0\right)$
b) $\left(-11, -\frac{3}{4}\right)$
c) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$
d) $\left(0, \frac{1}{4}\right)$

139. The area bounded by the curve $y = f(x)$ and the lines x = 0, y = 0 and x = t lies in the interval
a) $\left(\frac{3}{4}, 3\right)$
b) $\left(\frac{21}{64}, \frac{11}{16}\right)$
c) (9, 10)
d) $\left(0, \frac{21}{64}\right)$

140. The function $f'(x)$ is
a) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$
b) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$
c) increasing in (-t, t)
d) decreasing in (-t, t)

**PASSAGE-5**

Given that for each $a \in (0, 1)$,

$$\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1} dt$$

exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on (0, 1).

141. The value of $g\left(\frac{1}{2}\right)$ is
a) $\pi$
b) $2\pi$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4}$

142. The value of $g'\left(\frac{1}{2}\right)$ is
a) $\frac{\pi}{2}$
b) $\pi$
c) $-\frac{\pi}{2}$
d) 0

**PASSAGE-6**

Let $F : R \to R$ be a thrice differentiable function. Suppose that F(1) = 0, F(3) = -4 and $F(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in R$.

143. The correct statement(s) is(are)
a) $f'(1) < 0$
b) $f(2) < 0$
c) $f'(x) \neq 0$ for any $x \in (1, 3)$
d) $f'(x) = 0$ for any $x \in (1, 3)$

144. If

$$\int_1^3 x^2 F'(x) dx = -12$$

and

$$\int_1^3 x^3 F''(x) dx = 40$$

then the correct expression(s) is(are)
a) $9f'(3) + f'(1) - 32 = 0$
b) $\int_1^3 f(x)dx = 12$
c) $9f'(3) - f'(1) + 32 = 0$
d) $\int_1^3 f(x)dx = -12$

**Integer Value Correct Type**

145. Let $f : R \to R$ be a continuous function which satisfies

$$f(x) = \int_0^x f(t)dt$$

Then the value of $f(ln5)$ is

146. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [-10, 10] by

$$f(x) = \begin{Bmatrix} x - [x] & if[x]isodd \\ 1 + [x] - x & if[x]iseven \end{Bmatrix}$$

Then the value of

$$\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x dx =$$

147. The value of

$$\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2}(1 - x^2)^5 \right\} dx =$$

148. The value of the integral

$$\int_0^{1/2} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{1/4}}dx$$

is

149. If

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$$

then $27I^2$ equals........

150. The value of the integral

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$$

equals............

**Section - B**

151. $\int_0^{10\pi} |\sin x| dx$ is

a) 20
b) 8
c) 10
d) 18

152. $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals

a) 1/2
b) 1
c) $\infty$
d) 0

153. $\int_0^2 [x^2] dx$ is

a) $2-\sqrt{2}$
b) $2+\sqrt{2}$
c) $\sqrt{2}-1$
d) $-\sqrt{2}-\sqrt{3}+5$

154. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

a) $\frac{\pi^2}{4}$
b) $\pi^2$
c) 0
d) $\frac{\pi}{2}$

155. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is

a) 3/2
b) 1
c) 5/4
d) -3/4

156. The area of bounded by the curves $y = lnx$, $y = ln|x|$, $y = |lnx|$ and $y = |ln|x||$ is

a) 4 sq.units
b) 6 sq.units
c) 10 sq.units
d) None of these

157. If $f(a+b+-x) = f(x)$, then

$$\int_a^b xf(x)dx$$

is equal to

a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$
b) $\frac{a+b}{2}\int_a^b f(b-x)dx$
c) $\frac{a+b}{2}\int_a^b f(x)dx$
d) $\frac{b-a}{2}\int_a^b f(x)dx$

158. The area of the region bounded by the curves $y = |x-1|$ and $y = 3-|x|$ is

a) 6 sq.units
b) 2 sq.units
c) 3 sq.units
d) 4 sq.units

159. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral

$$\int_0^1 f(x)g(x)dx =$$

a) $e + \frac{e^2}{2} + \frac{5}{2}$
b) $e - \frac{e^2}{2} - \frac{5}{2}$
c) $e + \frac{e^2}{2} - \frac{3}{2}$
d) $e - \frac{e^2}{2} - \frac{5}{2}$

160. The value of the integral $I = \int_0^1 x(1-x)^n dx$

a) $\frac{1}{n+1} + \frac{1}{n+2}$
b) $\frac{1}{n+1}$
c) $\frac{1}{n+2}$
d) $\frac{1}{n+1} - \frac{1}{n+2}$

161. $\lim_{n\to\infty} \sum_{r=1}^{n} e^{\frac{r}{n}} =$

a) e + 1
b) e - 1
c) 1 - e
d) e

162. The value of

$$\int_{-2}^{3} |1-x^2| dx =$$

a) $\frac{1}{3}$
b) $\frac{14}{3}$

c) $\frac{7}{3}$
d) $\frac{28}{3}$

163. The value of

$$I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx =$$

a) 3
b) 1
c) 2
d) 0

164. If $f(x) = \frac{e^x}{1+e^x}$,

$$I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$$

and

$$I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$$

then the value of $\frac{I_2}{I_1}$ is

a) 1
b) -3
c) -1
d) 2

165. The area of the region bounded by the curves $y = |x-2|$, x = 1, x = 3 and the x-axis is

a) 4
b) 2
c) 3
d) 1

166. If

$$I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx$$

$$I_3 = \int_1^2 2^{x^2} dx, I_4 = \int_1^2 2^{x^3} dx$$

then

a) $I_2 > I_1$
b) $I_2 < I_1$
c) $I_3 = I_4$
d) $I_3 > I_4$

167. The area enclosed between the curve $y = log_e(x+e)$ and the coordinate axes is

a) 1
b) 2
c) 3
d) 4

168. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If $S_1$, $S_2$, $S_3$ are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

a) 1:2:1
b) 1:2:3
c) 2:1:2
d) 1:1:1

169. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f(\frac{\pi}{2})$ is

a) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$
b) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$
c) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$
d) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

170. The value of

$$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx =$$

a) $a\pi$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{a}$
d) $2\pi$

171. The value of integral

$$\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx =$$

a) 1/2
b) 3/2
c) 2
d) 1

172. $\int_0^{\pi} xf(\sin x)dx =$

a) $\pi \int_0^{\pi} f(\cos x)dx$
b) $\pi \int_0^{\pi} f(\sin x)dx$
c) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x)dx$
d) $\pi \int_0^{\pi/2} f(\cos x)dx$

173.

$$\int_{-3\pi/2}^{-\pi/2} [(x+\pi^3) + \cos^2(x+3\pi)]dx =$$

a) $\frac{\pi^4}{32}$
b) $\frac{\pi^4}{32} + \frac{\pi}{2}$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4} - 1$

174. The value of

$$\int_1^a [x]f'(x)dx$$

$a > 1$ where [x] denotes the greatest integer not exceeding x is

a) $af(a) - \{f(1) + f(2) + .........f([a])\}$
b) $[a]f(a) - \{f(1) + f(2) + .......f([a])\}$
c) $[a]f([a]) - \{f(1) + f(2) + ........f(a)\}$
d) $af([a]) - \{f(1) + f(2) + ......f(a)\}$

175. Let $F(x) = f(x) + f(\frac{1}{x})$, where

$$f(x) = \int_l^t \frac{logt}{1+t}dt$$

Then $F(e)$ equals

a) 1
b) 2
c) 1/2
d) 0

176. The solution for x of the equation

$$\int_{\sqrt{2}}^x \frac{dt}{\sqrt{t^2-1}} = \frac{\pi}{2} =$$

a) $\frac{\sqrt{3}}{2}$
b) $2\sqrt{2}$
c) 2
d) None

177. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

a) 1/6
b) 1/3
c) 2/3
d) 1

178. Let

$$I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx, J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$$

Then which one of the following is True?

a) $I > \frac{2}{3}$ and $J > 2$
b) $I < \frac{2}{3}$ and $J < 2$
c) $I < \frac{2}{3}$ and $J > 2$
d) $I > \frac{2}{3}$ and $J < 2$

179. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent of the parabola at the point (2, 3) and the x-axis is

a) 6
b) 9
c) 12
d) 3

180. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is eqaul to

a) 5/3
b) 1/3
c) 2/3
d) 4/3

181. $\int_0^\pi [\cot x]$, where [ ] denotes the greatest integer function is equal to

a) 1
b) -1
c) $\frac{\pi}{2}$
d) $\frac{\pi}{2}$

182. The area of bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

a) $4\sqrt{2} + 2$
b) $4\sqrt{2} - 1$
c) $4\sqrt{2} + 1$
d) $4\sqrt{2} - 2$

183. Let $p(x)$ be a function defined on $R$ such that $p'(x) = p'(1-x)$, for all $x \in [0,1]$, p(0) = 1 and p(1) = 41. Then

$$\int_0^1 p(x)dx =$$

a) 21
b) 41
c) 42
d) $\sqrt{41}$

184. The value of

$$\int_0^1 \frac{8log(1+x)}{1+x^2}dx =$$

a) $\frac{\pi}{8}log2$
b) $\frac{\pi}{2}log2$
c) $log2$
d) $\pi log2$

185. The area of the region enclosed by the curves y = x, x = e, $y = \frac{1}{x}$ and the positive x-axis is

a) 1 sq.units
b) $\frac{3}{3}$ sq.units
c) $\frac{5}{3}$ sq.units
d) $\frac{1}{2}$ sq.units

186. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is

a) $20\sqrt{2}$
b) $\frac{10\sqrt{2}}{3}$

c) $\frac{20\sqrt{2}}{2}$
d) $10\sqrt{2}$

187. If

$$g(x) = \int_0^x \cos 4t dt$$

then $g(x+\pi)$ is equal to

a) $\frac{g(x)}{g(\pi)}$
b) $g(x) + g(\pi)$
c) $g(x) - g(\pi)$
d) $g(x).g(\pi)$

188. **Statement-1:** The value of the integral

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

is equal to $\pi/6$
**Statement-2:**

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-2
b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-2
c) Statement-1 is true, Statement-2 is false
d) Statement-1 is false, Statement-2 is true

189. The area(in square units) bounded by the curves $y = \sqrt{x}$, 2y - x + 3 = 0, x-axis and lying in the first quadrant is

a) 9
b) 36
c) 18
d) 27/4

190. The integral

$$\int_0^\pi \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx =$$

a) $4\sqrt{3} - 4$
b) $4\sqrt{3} - 4 - \frac{\pi}{3}$
c) $\pi - 4$
d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

191. The area of the region bounded by

$$\{(x,y) : y^2 \le 2x, y \ge 4x - 1\} =$$

a) $\frac{15}{64}$
b) $\frac{9}{32}$
c) $\frac{7}{32}$
d) $\frac{5}{64}$

192. The area of the region bounded by

$$A = \{(x,y) : x^2 + y^2 \le 1, y^2 \le 1 - x\} =$$

a) $\frac{\pi}{2} - \frac{2}{3}$
b) $\frac{\pi}{2} + \frac{2}{3}$
c) $\frac{\pi}{2} + \frac{4}{3}$
d) $\frac{\pi}{2} - \frac{4}{3}$

193. The integral

$$\int_2^4 \frac{logx^2}{logx^2 + log(36 - 12x + x^2)}dx$$

is equal to

a) 1
b) 6
c) 2
d) 4

194. The area(in square units) of the region

$$\{(x,y) : y^2 \ge 2x, x^2 + y^2 \le 4x, x \ge 0, y \ge 0\} =$$

a) $\pi - \frac{4\sqrt{2}}{\text{ﬃ}3}$
b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
c) $\pi - \frac{4}{3}$
d) $\pi - \frac{8}{3}$

195. The area(in square units) of the region

$$\{(x,y) : x \ge 0, x + y \le 3, x^2 \le 4y, y \ge 1 + \sqrt{x}\} =$$

a) 5/2
b) 59/12
c) 3/2
d) 7/3

196. The integral

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} =$$

a) -1
b) -2
c) 2
d) 4

197. Let $g(x) = \cos^2 x$, $f(x) = \sqrt{x}$ and $\alpha, \beta(\alpha < \beta)$ be the roots of the quadratic equation

$$18x^2 - 9\pi x + \pi^2 = 0 \qquad (10.0.197.1)$$

Then the area(in sq.units) bounded by the curve $y = (gof)(x)$ and he lines $x = \alpha$, $x = \beta$ and y = 0 is

a) $\frac{1}{2}(\sqrt{3}+1)$
b) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$
c) $\frac{1}{2}(\sqrt{2}-1)$
d) $\frac{1}{2}(\sqrt{3}-1)$

198. The value of

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx =$$

a) $\frac{\pi}{2}$
b) $4\pi$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{8}$

199. The value of

$$\int_0^{\pi} |\cos x| dx =$$

a) 0
b) 4/3
c) 2/3
d) -4/3

200. The area(in sq.units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is

a) 8/3
b) 32/3
c) 56/3
d) 14/3

201. The value of

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx =$$

a) $\frac{\pi-2}{8}$
b) $\frac{\pi-1}{4}$
c) $\frac{\pi-2}{4}$
d) $\frac{\pi-1}{2}$

202. The area(in sq.units) of the region

$$A = \{(x, y) : x^2 \leq y \leq x + 2\} =$$

a) 10/3
b) 9/2
c) 31/6
d) 13/6