Assignment_3

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```
#Assignment 3->MODULE 6
```

```
#THE TRANSPORTATION MODEL
```

```
#solving transportation problem in R
```

```
library(lpSolve)
```

```
## Warning: package 'lpSolve' was built under R version 4.1.3
```

```
library(tinytex)
```

```
## Warning: package 'lpSolve' was built under R version 4.1.3
```

```
#set the matrix
AEDs<-matrix(c(22,14,30,600,100,
               16,20,24,625,120,
               80,60,70,"-","-"),ncol=5,byrow=TRUE)

colnames(AEDs)<-c("Warehouse1","Warehouse2","Warehouse3","Production cost","ProductionCapacity")
rownames(AEDs)<-c("Plant A","Plant B","Monthly Demand")
AEDs<-as.table(AEDs)
AEDs
```

```
##                Warehouse1 Warehouse2 Warehouse3 Production cost
## Plant A        22         14         30         600
## Plant B        16         20         24         625
## Monthly Demand 80         60         70         -
##                Production Capacity
## Plant A        100
## Plant B        120
## Monthly Demand -
```

Objective Function:

The transportation problem can be formulated in the LP format as :
$$\text{Miminize} \hspace{.2cm} TC = 622 x_{A1} + 614 x_{A2} + 630 x_{A3} + 641 x_{B1} + 645 x_{B2} + 649 x_{B3} $$Subject to:

Subject to:

Supply constraints

$$x_{A1}+x_{A2}+x_{A3} \leq 100$$

$$x_{B1}+x_{B2}+x_{B3} \leq 120$$

Demand Constraints:

$$x_{A1} + x_{B1} \geq 80$$
$$x_{A2} + x_{B2} \geq 60$$
$$x_{A3}+x_{B3} \geq 70$$

Non-negativity of the variables:

$$x_{ij} \geq 0$$

$$Xij$$

- where represents the number of AEDs shipped from plant i to warehouse j.

$$i = A, B$$

and

$$j = 1, 2, 3$$

I have used R programming language to solve the above transpiration cost minimization problem.
This transportation problem is unbalanced one (demand is not equal to supply), that is demand is less than supply by 10, so I create a dummy variable in column 4 with transportation cost zero and demand 10.

```
#Set up AEDs_Costs matrix
AEDs_Costs<-matrix(c(622,614,630,0,
                     641,645,649,0),ncol =4,byrow=TRUE)
## Set the names of the rows (constraints) and columns (decision variables)
colnames(AEDs_Costs)<-c("Warehouse1","Warehouse2","Warehouse3","Dummy")
rownames(AEDs_Costs)<-c("Plant A","Plant B")
AEDs_Costs
```

```
##         Warehouse1 Warehouse2 Warehouse3 Dummy
## Plant A        622        614        630     0
## Plant B        641        645        649     0
```

```
#setting up constraint signs and right-hand sides(supply side)
row.signs<-rep("<=",2)
row.rhs<-c(100,120)
#Supply function cannot be greater than the specified units

#Demand side constraints
col.signs<-rep(">=",4)
col.rhs<-c(80,60,70,10)
#Demand function can be greater than the specified units
```

```
#solve the model
lptrans<-lp.transport(AEDs_Costs,"min", row.signs, row.rhs, col.signs, col.rhs)
```

Following that, we will return the decision variable values to determine how many units should be produced and transported from each plant.

```
## Get the optimum decision variables (6)values
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

Plant A Units Shipped to Warehouse 1: 0 units Plant A Units Shipped to Warehouse 2: 60 units Plant A Units Shipped to Warehouse 3: 40 units Plant B Units Shipped to Warehouse 1: 80 units Plant B Units Shipped to Warehouse 2: 0 units Plant B Units Shipped to Warehouse 3: 30 units and “10” shows up in the 4th variable it is a “throw-away variable”(dummy).

The function below will give the minimum value for the objective function

```
lptrans$objval
```

```
## [1] 132790
```

The minimum combined shipping and production costs will be 132,790 dollars based on the given information and constraints.

```
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

2)Formulating the dual of the transportation problem

In primal, the number of variables equals the number of constants in dual. First, we must discover the LP's primal . We will take the minimization from the primal and maximize it in the dual. With the variables m and n, the dual issue can be solved.

```
AEDs_2<-matrix(c(622,614,630,100,"m1", 641,645,649,120,"m2",
80,60,70,220,"-",
"n1","n2","n3","-","-"),ncol=5,nrow=4,byrow=TRUE)
colnames(AEDs_2)<-c("Warehouse1","Warehouse2","Warehouse3","Production Capacity","Supply (Dual)")
rownames(AEDs_2)<-c("PlantA","PlantB","Monthly Demand","Demand (Dual)")
AEDs_2<-as.table(AEDs_2)
AEDs_2
```

```
##                Warehouse1 Warehouse2 Warehouse3 Production Capacity
## PlantA         622        614        630        100
## PlantB         641        645        649        120
```

```
## Monthly Demand    80       60       70       220
## Demand (Dual)     n1       n2       n3       -
##                   Supply   (Dual)
## PlantA            m1
## PlantB            m2
## Monthly Demand    -
## Demand (Dual)     -
```

$$\text{Max } Z = 100m_1 + 120m_2 + 80n_1 + 60n_2 + 70n_3$$

Subject to the following constraints

$$m_1 + n_1 \leq 622$$
$$m_1 + n_2 \leq 614$$
$$m_1 + n_3 \leq 630$$
$$m_2 + n_1 \leq 641$$

$$m_2 + n_2 \leq 645$$
$$m_2 + n_3 \leq 649$$

Where n1 = Warehouse_1

n2 = Warehouse_2

n3 = Warehouse_3 m1

= Plant_1

m2 = Plant_2

These constants are provided by transposed matrix of the Primal of Linear Programming function. you can double-check your work by transposing f.con into the matrix and comparing it to the constants listed above. where

$$m_k, n_l$$

where m= 1,2 and n=1,2,3

```
#Objective function

f.obj<-c(100,120,80,60,70)

#transposed from the constraints matrix in the primal
f.con<-matrix(c(1,0,1,0,0,
                1,0,0,1,0,
                1,0,0,0,1,
                0,1,1,0,0,
                0,1,0,1,0,
                0,1,0,0,1),nrow =6,byrow =TRUE)

f.dir<-c("<=",
         "<=",
         "<=",
         "<=",
```

```
                 "<=",
                 "<=")

f.rhs<-c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633        8     0    16
```

So Z=139,120 dollars and variables are:

m_1 =614

which represents Plant A

m_2 =633

which represents Plant B which

n_1 = 8

represents Warehouse 1 which

n_3 = 16

represents Warehouse 3

3)Economic Interpretation of thedual

Based on the information and limits provided, the maximum combined shipping and production costs will be 139,120 dollars.
There is a minimum value of Z=132790 (Primal) and a maximum value of Z=139120 (Dual).
The goal of this challenge is to discover a maximum and a minimum.
As a result, we realized that we shouldn't ship from Plant(A/B) to all three warehouses at the same time. We should be able to ship from:

$60x_{12}$

which is 60 Units from Plant A to Warehouse 2.

$40x_{13}$

which is 40 Units from Plant A to Warehouse 3.

$80x_{21}$

which is 80 Units from Plant B to Warehouse 1.

$30x_{23}$

which is 30 Units from Plant B to Warehouse 3. We will Max the profit from each distribution to the respective capacity.

We have the following:

$$m_1^0 - n_1^0 \leq 622$$

then we subtract

$$n_1^0$$

to the other side to get

$$m_1^0 \leq 622 - n_1^0$$

To compute it would be $614 <= (-8+622) which is correct. we would continue to evaluate these equations:

$$m_1 \leq 622 - n_1 => 614 \leq 622 - 8 = 614 => correct$$

$$m_1 \leq 614 - n_2 => 614 \leq 614 - 0 = 614 => correct\ m_1 \leq 630 - n_3 => 614 \leq 630 - 16 = 614 => correct\ m_2 \leq 641 - n_1 => 633 \leq 614 - 8 = 633 => correct\ m_2 \leq 645 - n_2 => 633 \leq 645 - 0 = 645 => Incorrect\ m_2 \leq 649 - n_3 => 633 \leq 649 - 16 = 633 => correct$$

Using the Duality-and-Sensitivity, we may test for the shadow price by updating each of the columns. In our LP Transportation problem, we modify 100 to 101 and 120 to 121. R. can be seen here.

```
row.rhs1<-c(101,120)
row.signs1<-rep("<=",2)
col.rhs1<-c(80,60,70,10)
col.signs1<-rep(">=",4)
row.rhs2<-c(100,121)
row.signs2<-rep("<=",2)
col.rhs2<-c(80,60,70,10)
col.signs2<-rep(">=",4)

lp.transport(AEDs_Costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```
lp.transport(AEDs_Costs,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```
lp.transport(AEDs_Costs,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

By taking the minimum of this specific function, seeing the number go down by 19 means the shadow price is 19, which was calculated by adding 1 to every plant.The Plant B does not have a shadow price. We also

From the dual variable

$$n_2$$

where Marginal Revenue <= Marginal Cost. The equation was

$$m_2 \leq 645 - n_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$m_1^0 - n_1^0 \leq 622$$

then we subtract

$$n_1^0$$

to the other side to get

$$m_1^0 \leq 622 - n_1^0$$

```
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633     8     0    16
```

$$n_2 = 0$$

.

The interpretation from above: from theprimal:

$$60x_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

which is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

which is 60 Units from Plant B to Warehouse 3.

Due to the dual.Our goal is to get MR=MC.

MR=MC in five of the six examples.

Only Plant B to Warehouse 2 do not meet this criteria.

We can see from the primal that no AED gadgets will be transported there.