

# Numerical Analysis-ME542

## Assignment-04

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## Problem

System of linear equation

$$\mathbf{Ax} = \mathbf{b}$$

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$$\mathbf{A} = \begin{bmatrix} 0.2 & 0.1 & 1 & 1 & 0 \\ 0.1 & 4 & -1 & 1 & -1 \\ 1 & -1 & 60 & 0 & -2 \\ 1 & 1 & 0 & 8 & 4 \\ 0 & -1 & -2 & 4 & 700 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

## Solution

Solution of above equation found out by two method Gauss-Seidel method and Conjugate gradient method. This both method require initial  $x = 0$  vector and a termination criteria

For both the method we are assuming same initial value and termination criteria.

**Initial solution vectore**

$$\mathbf{X_0} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

**termination criteria**

$$\|AX_k - b\| < 10^{-12}$$

## Result of Gauss-Seidel method

It.	X1	X2	X3	X4	X5	AXk - b
1	-12.7	1.0175	0.311958	1.46031	0.0011431	5.1499
2	-4.3701	0.32245	0.128247	1.00539	0.00222486	0.758707
3	-0.829388	0.302006	0.0689307	0.56481	0.00454375	0.631806
4	1.68029	0.335159	0.0277326	0.245797	0.00629634	0.453425
5	3.46477	0.360439	-0.001529	0.0187	0.00754654	0.322639
6	4.73392	0.378481	-0.0223392	-0.142824	0.00843586	0.229474
7	5.63658	0.391316	-0.0371398	-0.257704	0.00906836	0.163208
8	6.27856	0.400444	-0.0476664	-0.33941	0.00951822	0.116078
9	6.73516	0.406936	-0.0551531	-0.397521	0.00983816	0.0825572
10	7.0599	0.411554	-0.0604779	-0.438851	0.0100657	0.0587167
11	7.29087	0.414838	-0.064265	-0.468246	0.0102276	0.0417608
12	7.45514	0.417174	-0.0669585	-0.489153	0.0103427	0.0297013
13	7.57197	0.418835	-0.0688741	-0.504022	0.0104245	0.0211243
14	7.65506	0.420016	-0.0702366	-0.514597	0.0104828	0.0150241
15	7.71416	0.420857	-0.0712056	-0.522119	0.0105242	0.0106855
16	7.75619	0.421454	-0.0718948	-0.527468	0.0105536	0.00759981
17	7.78609	0.42188	-0.072385	-0.531273	0.0105746	0.00540517
18	7.80735	0.422182	-0.0727336	-0.533979	0.0105895	0.00384429
19	7.82247	0.422397	-0.0729816	-0.535903	0.0106001	0.00273416
20	7.83322	0.42255	-0.0731579	-0.537272	0.0106076	0.0019446

21	7.84087	0.422659	-0.0732833	-0.538245	0.010613	0.00138305
22	7.84631	0.422736	-0.0733725	-0.538938	0.0106	0.000983657
23	7.85018	0.422791	-0.073436	-0.53943	0.0106195	0.000699601
24	7.85294	0.42283	-0.0734811	-0.53978	0.0106214	0.000497574
25	7.85489	0.422858	-0.0735132	-0.54003	0.0106228	0.000353887
26	7.85628	0.422878	-0.073536	-0.540207	0.0106238	0.000251693
27	7.85727	0.422892	-0.0735523	-0.540333	0.0106245	0.00017901
28	7.85798	0.422902	-0.0735638	-0.540422	0.0106249	0.000127316
29	7.85848	0.422909	-0.073572	-0.540486	0.0106253	9.05506e-05
30	7.85884	0.422914	-0.0735779	-0.540531	0.0106255	6.44018e-05
31	7.85909	0.422918	-0.073582	-0.540564	0.0106257	4.58042e-05
32	7.85927	0.42292	-0.073585	-0.540587	0.0106259	3.25771e-05
33	7.8594	0.422922	-0.0735871	-0.540603	0.0106259	2.31696e-05
34	7.85949	0.422923	-0.0735886	-0.540614	0.010626	1.64788e-05
35	7.85955	0.422924	-0.0735896	-0.540623	0.0106261	1.17201e-05
36	7.8596	0.422925	-0.0735904	-0.540629	0.0106261	8.33564e-06
37	7.85963	0.422925	-0.0735909	-0.540633	0.0106261	5.92851e-06
38	7.85966	0.422926	-0.0735913	-0.540636	0.0106261	4.21651e-06
39	7.85967	0.422926	-0.0735916	-0.540638	0.0106261	2.99888e-06
40	7.85968	0.422926	-0.0735918	-0.540639	0.0106261	2.13288e-06
41	7.85969	0.422926	-0.0735919	-0.54064	0.0106261	1.51696e-06
42	7.8597	0.422926	-0.073592	-0.540641	0.0106262	1.0789e-06
43	7.8597	0.422926	-0.0735921	-0.540642	0.0106262	7.67338e-07
44	7.85971	0.422926	-0.0735921	-0.540642	0.0106262	5.4575e-07

45	7.85971	0.422926	-0.0735922	-0.540642	0.0106262	3.88151e-07
46	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	2.76062e-07
47	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.96342e-07
48	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.39643e-07
49	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	9.93179e-08
50	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	7.06373e-08
51	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	5.0239e-08
52	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	3.57312e-08
53	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	2.54129e-08
54	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.80743e-08
55	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.28549e-08
56	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	9.14272e-09
57	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	6.50252e-09
58	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	4.62476e-09
59	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	3.28924e-09
60	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	2.33939e-09
61	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.66383e-09
62	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.18336e-09
63	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	8.41633e-10
64	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	5.9859e-10
65	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	4.25732e-10
66	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	3.02791e-10
67	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	2.15352e-10
68	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.53164e-10

69	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.08934e-10
70	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	7.74769e-11
71	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	5.51034e-11
72	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	3.91912e-11
73	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	2.78733e-11
74	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.98247e-11
75	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.40991e-11
76	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.00281e-11
77	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	7.13241e-12
78	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	5.07282e-12
79	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	3.60752e-12
80	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	2.56649e-12
81	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.82485e-12
82	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.29794e-12
83	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	9.22986e-13

solution to the problem

value of  $x_0$  is = 7.85971

value of  $x_1$  is = 0.422926

value of  $x_2$  is = -0.0735922

value of  $x_3$  is = -0.540643

value of  $x_4$  is = 0.0106262

Final  $\|AX_k - b\| = 9.22986e-13$

Number of iteration = 83

## Result of Conjugate gradient method

Itra.	X1	X2	X3	X4	X5	AXk-b
1	0.998131	0.998419	0.920941	0.985626	-0.000459793	52.8138
2	0.97776	0.962074	0.0383673	0.885957	0.00353172	5.80733
3	0.94917	0.613144	0.0450204	0.252426	0.00670983	0.957789
4	1.0865	0.358672	0.038119	0.330283	0.00587558	0.429599
5	7.85971	0.422926	-0.0735922	-0.540643	0.0106261	2.50695e-05
6	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	4.10071e-10
7	7.85971	0.422926	-0.0735922	-0.540643	0.0106262	1.28336e-13

solution to the problem

value of x0 is = 7.85971

value of x1 is = 0.422926

value of x2 is = -0.0735922

value of x3 is = -0.540643

value of x4 is = 0.0106262

Final  $||\mathbf{AXk}-\mathbf{b}|| = 1.28336\text{e-}13$

Number of iteration = 7

## Observation

Conjugate gradient method require less number of iteration.

