

NUMERICAL ANALYSIS - ME 542

Assignment-6

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1. . Solve the following ODE with $y(0) = 1$ using Euler method with step size $h = 0.1$. Make a table to compare the results with the same obtained analytically. Consider two perturbed initial conditions $y(0) = 0.99$ and $y(0) = 1.01$ and solve numerically with same step size.

$$\begin{aligned} y' &= -100y - 100t + 101 \\ y(0) &= 1, y(0) = 0.99, y(0) = 1.1 \end{aligned} \quad (1)$$

Solution The analytical solution of the (1) is following.

$$y(t) = C_1 e^{-100t} + t + 1 \quad (2)$$

Using the initial condition $y(0) = 1$ solution of (2) becomes

$$y(t) = t + 1 \quad (3)$$

For initial condition $y(0) = 0.99$ solution of (2) becomes

$$y(t) = -0.1e^{-100t} + t + 1 \quad (4)$$

For initial condition $y(0) = 1.01$ solution of (2) becomes

$$y(t) = 0.1e^{-100t} + t + 1 \quad (5)$$

Numerical solution:

Equ. (1) using Euler's method shows following result.

In table (Table 1), represent the numerical solution of (1) at 10 points equally spaced between the t interval 0 and 1 are used to compare the results obtained from three different initial conditions.

Table 1: Compare the results for $y(0) = 1, 0.99$ and 1.1

t	Analytical solu.	y(0)=1	y(0)=0.99	y(0)=1.1
0	1	1	0.99	1.01
0.1	1.1	1.1	1.19	1.01
0.2	1.2	1.2	0.39	2.01
0.3	1.3	1.3	8.59	-5.99
0.4	1.4	1.4	-64.21	67.01
0.5	1.5	1.5	591.99	-588.99
0.6	1.6	1.6	-5312.81	5316.01
0.7	1.7	1.7	47831.4	-47828
0.8	1.8	1.8	-430465	430469
0.9	1.9	1.9	3.87421e+06	-3.8742e+06
1	2	2	-3.48678e+07	3.48678e+07

Result Discussion: Result for initial condition $y(0) = 0.99$ and $y(1) = 1.1$ is coming different because these two initial conditions cannot satisfy equation (3) which is the solution for $y(0) = 1$. Results of $y(0) = 0.99$ and $y(1) = 1.1$ representing solutions of equations (4) and (5) respectively.