Numerical Analysis-ME542
Assignment-03

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Gaussian elimation with scaled partial pivoting

Scaled partial pivoting is a technique used to improve Gaussian elimination.Ideally pivot entry should be maximum of all rows and columns of submatrix.However,this involves complicated column and row exchange which is computationlly expensive.Instand of full pivoting scaled partial pivoting is more used .
Frist a scale vector S_i is constructed comprising of the maximuum in each of the row of the matrix.

$$S_i = max_{1\leq j\leq n}|a_{ij}| \quad where, 1 \leq i \leq n$$

$$S_i = \begin{bmatrix} S_1 & S_2 & \cdots & s_n \end{bmatrix}$$

Pivot row is slected on the basis of hightest ratio

$$\{\frac{|a'_{ik}|}{S_i} \quad k \leq i \leq n\}$$

where k reprasent elimination step and a'_{ik} submatrix element after$(k-1)$ step of elimination.

Algorithm 1 Scaling Vector Extrection

```
for i = 1 to n do
  for j = 1 to n do
    if |a_ij| > S_i then
      S_i ← a_ij
    end if
  end for
end for
```

Algorithm 2 Gauss Elimination with Scaled Partial Pivoting

```
for k = 1 to n − 1 do
    Finding pivot row
    f ← 0
    for j = k to n do
        if f < a_jk / S_i then
            f ← a_jk / S_i
            p ← j
        end if
    end for
    //Swaping of Row for pivoting
    for j = k to n do
        f ← a_kj
        a_kj ← a_pj
        a_pj ← f
    end for
    f ← b_k
    b_k ← b_p
    b_p ← f
    //Gauss elimination
    for i = k + 1 to n do
        for j = k + 1 to n do
            a_ij ← a_ij − (a_kj / a_kk) a_ik
        end for
        b_i ← b_i − (a_kj / a_kk) b_k
    end for
end for
```

Algorithm 3 Back Substitution

```
x_n ← b_n / a_nn
sum ← 0
for k = n − 1 to 1 do
    sum ← b_k
    for j = k + 1 to n do
        sum ← sum − a_kj * x_j
    end for
    x_k ← sum / a_kk
end for
```

Problem

$$\mathbf{A.x} = \mathbf{b}$$

where

$$\mathbf{b} = \begin{bmatrix} 0.4043 \\ 0.1550 \\ 0.4240 \\ 0.2557 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0.4096 & 0.1234 & 0.3678 & 0.2943 \\ 0.2246 & 0.3872 & 0.4015 & 0.1129 \\ 0.3645 & 0.1921 & 0.3781 & 0.0643 \\ 0.1784 & 0.4002 & 0.2786 & 0.3927 \end{bmatrix}$$

Solution of above problem is

$$\mathbf{x} = \begin{bmatrix} 3.45804 \\ 1.5594 \\ -2.93155 \\ -0.429226 \end{bmatrix}$$