Numerical Analysis-ME542
Assignment-05

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1. **Problem:** Write down the main steps in setting up a program to solve this two-point boundary value problem using the finite-difference method.

$$x'' = x \sin t + x' \cos t - \exp t$$
$$x(0) = 0, x(1) = 1; \quad (1)$$

All the preliminary work before programming should be neatly reported. Exploit the linearity of the differential equation. Make a program and compare the results when different values of n are used, say, $n = 10; 100; and 1000$. Use the solvers you have already developed as a part of ME542.

Solution The central difference formulas (2) and (3),

$$x'(t) = \frac{1}{2h}[x(t+h) - x(t-h)] \quad (2)$$
$$x''(t) = \frac{1}{h^2}[x(t+h) - 2x(t) + x(t-h)] \quad (3)$$

where, h is the step size .

$$h = \frac{(b-a)}{n}$$

where, n is the number of intervals
Standred diffrential equastion.

$$y'' + u(x)y' + v(x)y = f(x)$$

By compering to the problem we can written as

$$v(t) = -cos(t)$$

$$u(t) = -sin(t)$$

$$f(x) = -exp(t)$$

.

$$(\frac{1}{h^2} + \frac{u(t)}{2h})x(t+h) + (\frac{2}{h^2} - v(x))x(t) + (\frac{1}{h^2} - \frac{v(t)}{2h})x(t-h) = f(t) \quad (4)$$

Multiplying the equation with h^2 and writing $x(t+h) as x_{i+1}$, $x(t) as x_i$ and $x(t-h) as x_{i-1}$, we get (5)

$$(1 - \frac{hu(t)}{2})x_{i+1} - (2 - h^2 v(t))x_i - (1 - \frac{hv(t)}{2})x_{i-1} = h^2 f(x) \quad (5)$$

If the coefficients of $x_{i+1}, x_i and x_{i-1}$ are taken as $a_i, b_i and c_i$ respectively, we get (6).

$$a_i x_{i+1} + d_i x_i + c_i x_{i-1} = h^2 b_i \quad (6)$$

$$a_i = 1 - \frac{hu_i}{2}$$
$$d_i = (h^2 v_i - 2)$$
$$c_i = 1 + \frac{hu_i}{2}$$
$$b_i = h^2 f_i$$

From the given boundary condition we get .

$$x(0) = 0 \quad (7a)$$
$$x(1) = 1 \quad (7b)$$

For $i = 1$ and can be written as .

$$a_1 x(0) + d_1 x(h) + c_1 x(2h) = h^2 b_1$$
$$d_1 x(h) + c_1 x(2h) = h^2 b_1 - c_1 x(0)$$
$$(8)$$

For $i = 2, 3, ..., n-2$ can be written as .

$$a_2 x(h) + d_2 x(2h) + c_2 x(3h) = -h^2 \exp(2h) \tag{9a}$$

$$a_3 x(2h) + d_3 x(3h) + c_3 x(4h) = -h^2 \exp(3h) \tag{9b}$$

$$...$$

$$a_{n-2} x([n-3]h) + d_{n-2} x([n-2]h) + c_{n-2} x([n-1]h) = -h^2 \exp([n-2]h) \tag{9c}$$

For $i = n-1$ can be written as using .

$$\begin{aligned} a_{n-1} x([n-2]h) + d_{n-1} x([n-1]h) + c_{n-1} x(nh) &= -h^2 \exp{(n-1)h} \\ a_{n-1} x([n-2]h) + d_{n-1} x([n-1]h) &= -h^2 \exp{(n-1)h} - c_{n-1} x(1) \\ a_{n-1} x([n-2]h) + d_{n-1} x([n-1]h) &= -h^2 \exp{(n-1)h} - c_{n-1} \end{aligned} \tag{10}$$

The above set of equation, , can be written in the matrix form as (11).

$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & \dots & 0 \\ a_2 & d_2 & c_2 & 0 & \dots & 0 \\ 0 & a_3 & d_3 & c_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & a_{n-1} & d_{n-1} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{Bmatrix} = \begin{bmatrix} b_1 - a_1 t_0 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} - c_{n-1} t_n \end{bmatrix} \tag{11}$$

The above tridiagonal matrix is solved by modified Gauss elimination method called Thomsoms algorithm.

The results for $n = 10$ is as shown in (Fig.1).

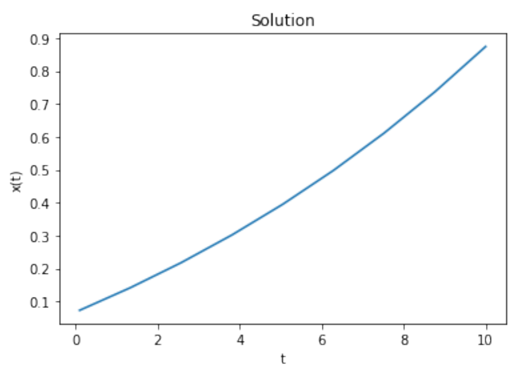

Figure 1: For n = 10

The results for $n = 100$ is as shown in (Fig.2).

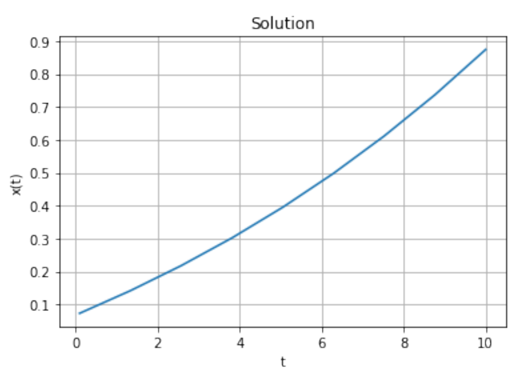

Figure 2: For n = 100

The results for $n = 1000$ is as shown in (Fig.3).

Conclusion It has been observved that for $n = 100$ and $n = 1000$ the values are same.

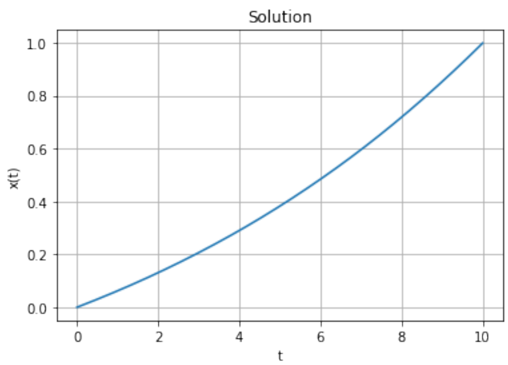

Figure 3: For n = 1000

Table 1: results for n = 10

t	n=10
0	0
0.1	0.113591
0.2	0.227566
0.3	0.340918
0.4	0.452525
0.5	0.561117
0.6	0.665246
0.7	0.763251
0.8	0.853224
0.9	0.932975
1	1