MTH 373/573:
Scientific Computing
Monsoon 2018

Homework 1

IIIT-Delhi
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Due: October 25, 2018 (Friday) by 23.59 **Total: 150 points.**

Before you start, please read General Submission Guidelines on Page 2.

Problem 1: Errors in polynomial evaluation (25 points)

Consider the polynomial: $p(x) = (x - 2)^9$. Consider its expansion:

$$p(x) = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512.$$

Write a program (Python/Julia) to compute and plot $p(x)$ for $x = 1.920, 1.921, 1.922, \dots, 2.080$ evaluating p directly and via its expansion. Your plot should therefore have two different colored graphs. Clearly label your graphs.

Problem 2: Errors in Stirling's approximation (25 points)

Write a program to compute the evaluation of factorial directly, via the approximation as well as plot absolute and relative errors for Stirling's approximation:

$$n! \approx \sqrt{2\pi n}(n/e)^n,$$

for $n = 1, 2, \dots, 10$.

Notice that in this case all your graphs should be discrete points and not joined by curves. Clearly label all your graphs again.

Problem 3: Analysis of Computational Errors (20 points)

In Problems 1 and 2 above, comment on the absolute and relative errors you obtain for each of the two different computations based on your generated plots.

(a) For the polynomial evaluation, which method of evaluation would you choose based on your understanding of floating point computations?

(b) For Stirling's approximation, do the absolute and relative errors increase or decrease with n?

Problem 4: Finite differences and ε_M (80 points)

(a) Write a program to compute an approximate value for the derivative of a function using the finite-difference formula:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Test your program using the function $\tan(x)$ for $x = 1$. Determine the absolute error by comparing with the inverse square of in-built (in NumPy or Julia Base) $\cos(x)$. Plot the absolute error as a function of h, for $h = 10^{-k}$, $k = 0, 1, \ldots, 16$. You should use a logarithmic scale for h as well as the magnitude of the error (that is, make a log-log plot). Is there a minimum value for the magnitude of the error? How does the corresponding value for h compare with $\sqrt{\varepsilon_{mach}}$?

(b) Repeat the above exercise but using the centered difference approximation:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

General Submission Guidelines

- **This assignment should be answered individually.**
- **You will be penalized for copying or any plagiarism with an automatic zero.**
- The points for each problem is roughly indicative of effort needed for answering that problem. Your mileage may vary!
- IIIT-Delhi academic policies on honesty and integrity apply to all HWs. This includes not copying from one another, from the internet, a book, or any other online or offline source. A repeat offense will be reported to academic administration.
- If you discuss or read secondary sources (other than class notes), please list all your discussion partners and/or secondary sources in your writeup. Failure to do so will constitute violation of honor code.
- All files should be submitted via Google Classroom.
- If your code generates an output figure or table, please provide all such results in a single PDF file along with your code submission.
- You will need to write a separate code for each problem and sometimes for each subproblem as well. You should name each such file as `problem_`n`.py` where n is the problem number. For example, your files could be named `problem_1.py`, `problem_4a.py` and `problem_4b.py` in this HW.
- *Python tip:* You can import Python modules as follows:

```python
from __future__ import division
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
import numpy.linalg as npla
import scipy.linalg as spla
```

Every code you write will have one or more of these import statements.