MTH 373/573:
Scientific Computing
Monsoon 2018

Homework 2

IIIT-Delhi
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Due: October 25, 2018 (Friday) by 23.59 **Total: 150 points.**

Before you start, please read General Submission Guidelines on Page 2.

Problem 1: Hilbert matrix conditioning (50 points)

An $n \times n$ Hilbert matrix H has entries:

$$h_{ij} = \frac{1}{(i+j-1)},$$

so that it has the following form:

$$\begin{bmatrix} 1 & 1/2 & 1/3 & \dots \\ 1/2 & 1/3 & 1/4 & \dots \\ 1/3 & 1/4 & 1/5 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

For $n = 2, 3, \dots$, generate the Hilbert matrix of order n and also generate the n-vector $b = H\,x$, where x is the n-vector with all of its components equal to 1. Using the SciPy Linear Algebra routine for LU factorization `scipy.linalg.lu` or Julia's `factorize`, solve the resulting linear system $H\,x = b$. Obtaining the approximate solution $\widehat{x}$. Compute the ∞-norm of the corresponding residual $r = b - H\,\widehat{x}$, and that of the error $\Delta x = \widehat{x} - x$.

How large can you make n until the error is 100% (that is, there are no significant digits in the solution)?

Problem 2: Gaussian elimination and partial pivoting (100 points)

(a) Use Gaussian elimination *without* pivoting to solve the linear system:

$$\begin{bmatrix} \varepsilon & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+\varepsilon \\ 2 \end{bmatrix},$$

for $\varepsilon = 10^{-2k}$, for $k = 1, 2, \dots, 10$. The exact solution is $\begin{bmatrix} 1 & 1 \end{bmatrix}^T$, independent of the value of ε. How does the accuracy of the computed problem behave as the value of ε decreases?

(b) Repeat the above using Gaussian elimination with pivoting, but this time use one iteration of iterative refinement (see page 84 of Chapter 2 in Scientific Computing, Second Edition) to improve the solution, computing the residual in the same precision as the rest of the computations. Now, how does the accuracy of the computed solution behave as the value of ε decreases?

General Submission Guidelines

- **This assignment should be answered individually.**
- **You will be penalized for copying or any plagiarism with an automatic zero.**
- The points for each problem is roughly indicative of effort needed for answering that problem. Your mileage may vary!
- IIIT-Delhi academic policies on honesty and integrity apply to all HWs. This includes not copying from one another, from the internet, a book, or any other online or offline source. A repeat offense will be reported to academic administration.
- If you discuss or read secondary sources (other than class notes), please list all your discussion partners and/or secondary sources in your writeup. Failure to do so will constitute violation of honor code.
- All files should be submitted via Google Classroom.
- If your code generates an output figure or table, please provide all such results in a single PDF file along with your code submission.
- You will need to write a separate code for each problem and sometimes for each subproblem as well. You should name each such file as `problem_`n`.py` where n is the problem number. For example, your files could be named `problem_1.py`, `problem_4a.py` and `problem_4b.py` in this HW.
- *Python tip:* You can import Python modules as follows:

  ```
  from __future__ import division
  import numpy as np
  import scipy as sp
  import matplotlib.pyplot as plt
  import numpy.linalg as npla
  import scipy.linalg as spla
  ```

  Every code you write will have one or more of these import statements.