

MCT: Project - Part 4

Ashwin Biju Nair
abijnair

Exercise 1:

Discrete time dynamics of the system:

$$x_{t+1} = x_t + \Delta t (\dot{x}_t \cos \varphi_t - \dot{y}_t \sin \varphi_t) + w_t^x$$

$$y_{t+1} = y_t + \Delta t (\dot{x}_t \sin \varphi_t + \dot{y}_t \cos \varphi_t) + w_t^y$$

$$\varphi_{t+1} = \varphi_t + \Delta t \dot{\varphi}_t + w_t^\varphi$$

Input, $u_t = \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{\varphi}_t \end{bmatrix}$

$p_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$

n features in map, $m^j = \begin{bmatrix} m_x^j \\ m_y^j \end{bmatrix}$

Range measurement: $y_{t,dist}^j = \|m^j - p_t\| + \overset{\text{meas. noise}}{v_{t,dist}^j} \quad j = 1 \dots n$

Bearing measurement: $y_{t,bearing}^j = \arctan2[m_y^j - y_t, m_x^j - x_t] - \varphi_t + \overset{\text{meas. noise.}}{v_{t,bearing}^j}$

State vector,

$$x_t = \begin{bmatrix} x_t \\ y_t \\ \varphi_t \\ m_x^1 \\ m_y^1 \\ m_x^2 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix}$$

The measurement,

$$y_t = \begin{bmatrix} \|m^1 - p_t\| \\ \vdots \\ \|m^n - p_t\| \\ \arctan2[m_y^1 - y_t, m_x^1 - x_t] - \varphi_t \\ \vdots \\ \arctan2[m_y^n - y_t, m_x^n - x_t] - \varphi_t \end{bmatrix} + \begin{bmatrix} v_{t,dist}^1 \\ \vdots \\ v_{t,dist}^n \\ v_{t,bearing}^1 \\ \vdots \\ v_{t,bearing}^n \end{bmatrix}$$

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k$$

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$

Let $\mathbf{F}_k = \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k}$, $\mathbf{H}_k = \left. \frac{\partial h}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k|k-1}}$ Replace \mathbf{A} with \mathbf{F}_k , \mathbf{C} with \mathbf{H}_k . We get EKF.

Finding \mathbf{F}_k :

$$\mathbf{F}_k = \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k} = \begin{bmatrix} \frac{\partial x_{t+1}}{\partial x_t} & \frac{\partial x_{t+1}}{\partial y_t} & \dots & \frac{\partial x_{t+1}}{\partial m_y^n} \\ \frac{\partial y_{t+1}}{\partial x_t} & \frac{\partial y_{t+1}}{\partial y_t} & \dots & \vdots \\ \frac{\partial \psi_{t+1}}{\partial x_t} & \vdots & \ddots & \vdots \\ \frac{\partial m'_x}{\partial x_t} & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial m_y^n}{\partial x_t} & \frac{\partial m_y^n}{\partial y_t} & \dots & \frac{\partial m_y^n}{\partial m_y^n} \end{bmatrix}$$

$$\frac{\partial x_{t+1}}{\partial x_t} = 1$$

$$\frac{\partial x_{t+1}}{\partial y_t} = 0$$

$$\frac{\partial x_{t+1}}{\partial \psi_t} = -\ell_t [\dot{x}_t \sin \psi_t + \dot{y}_t \cos \psi_t]$$

$$\frac{\partial y_{t+1}}{\partial x_t} = 0$$

$$\frac{\partial y_{t+1}}{\partial y_t} = 1$$

$$\frac{\partial y_{t+1}}{\partial \psi_t} = \ell_t [\dot{x}_t \cos \psi_t - \dot{y}_t \sin \psi_t]$$

$$\frac{\partial \psi_{t+1}}{\partial x_t} = 0$$

$$\frac{\partial \psi_{t+1}}{\partial y_t} = 0$$

$$\frac{\partial \psi_{t+1}}{\partial \psi_t} = 1$$

$$\frac{\partial m'_x}{\partial x_t} = \frac{\partial m'_x}{\partial y_t} = \frac{\partial m'_x}{\partial \psi_t} = 0 \quad \frac{\partial m'_x}{\partial m'_x} = 1$$

Similarly, the same pattern follows for other terms.

$$\therefore F_R = \begin{bmatrix} 1 & 0 & -\delta t \begin{bmatrix} \dot{x}_t \sin \psi_t + \dot{y}_t \cos \psi_t \\ \dot{x}_t \cos \psi_t - \dot{y}_t \sin \psi_t \end{bmatrix} & 0 & \dots & 0 \\ 0 & 1 & \delta t \begin{bmatrix} \dot{x}_t \sin \psi_t + \dot{y}_t \cos \psi_t \\ \dot{x}_t \cos \psi_t - \dot{y}_t \sin \psi_t \end{bmatrix} & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_R = \left. \frac{\partial h}{\partial x} \right|_{\hat{x} \in \{k-1\}} = \begin{bmatrix} \frac{\partial \|m^i - p_t\|}{\partial x_t} & \frac{\partial \|m^i - p_t\|}{\partial y_t} & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \|m^n - p_t\|}{\partial x_t} & \frac{\partial \|m^n - p_t\|}{\partial y_t} & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \text{atan2}[m_y^n - y_t, m_x^n - x_t]}{\partial x_t} & \dots & \dots & \dots \end{bmatrix}$$

$$\|m^i - p_t\| = \sqrt{(x_t - m_x^i)^2 + (y_t - m_y^i)^2}$$

$$\therefore \frac{\partial \|m^i - p_t\|}{\partial x_t} = \frac{2(x_t - m_x^i)}{2\sqrt{(x_t - m_x^i)^2 + (y_t - m_y^i)^2}} = \frac{(x_t - m_x^i)}{\|m^i - p_t\|}$$

$$\frac{\partial \|m^i - p_t\|}{\partial y_t} = \frac{(y_t - m_y^i)}{\|m^i - p_t\|}$$

$$\frac{\partial \|m^i - p_t\|}{\partial m_x^i} = -\frac{(x_t - m_x^i)}{\|m^i - p_t\|}$$

$$\frac{\partial \|m^i - p_t\|}{\partial m_y^i} = -\frac{(y_t - m_y^i)}{\|m^i - p_t\|}$$

$$\frac{\partial \text{atan2}[m_y^n - y_t, m_x^n - x_t]}{\partial x_t} = -\frac{(m_y^n - y_t)}{\|m^n - p_t\|^2} (-1) = \frac{(m_y^n - y_t)}{\|m^n - p_t\|^2}$$

$$\frac{\partial \text{atan2}[m_y^n - y_t, m_x^n - x_t]}{\partial y_t} = \frac{(m_x^n - x_t)}{\|m^n - p_t\|^2} (-1) = -\frac{(m_x^n - x_t)}{\|m^n - p_t\|^2}$$

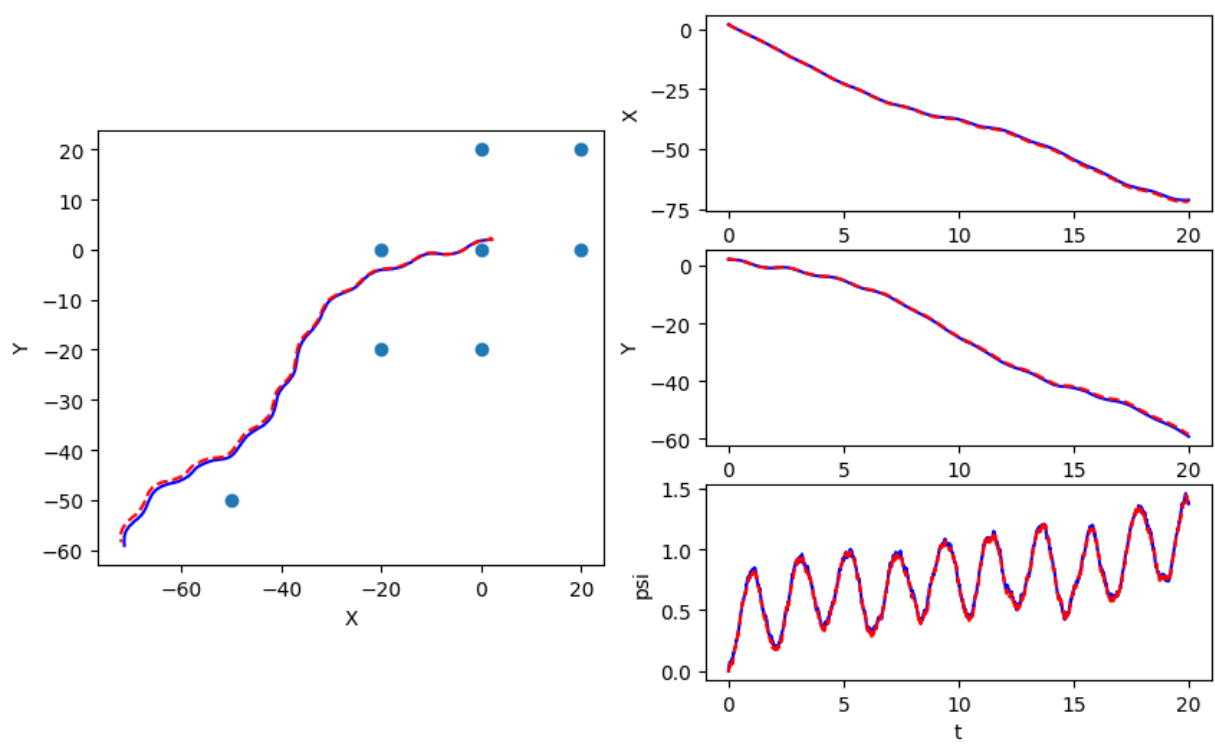
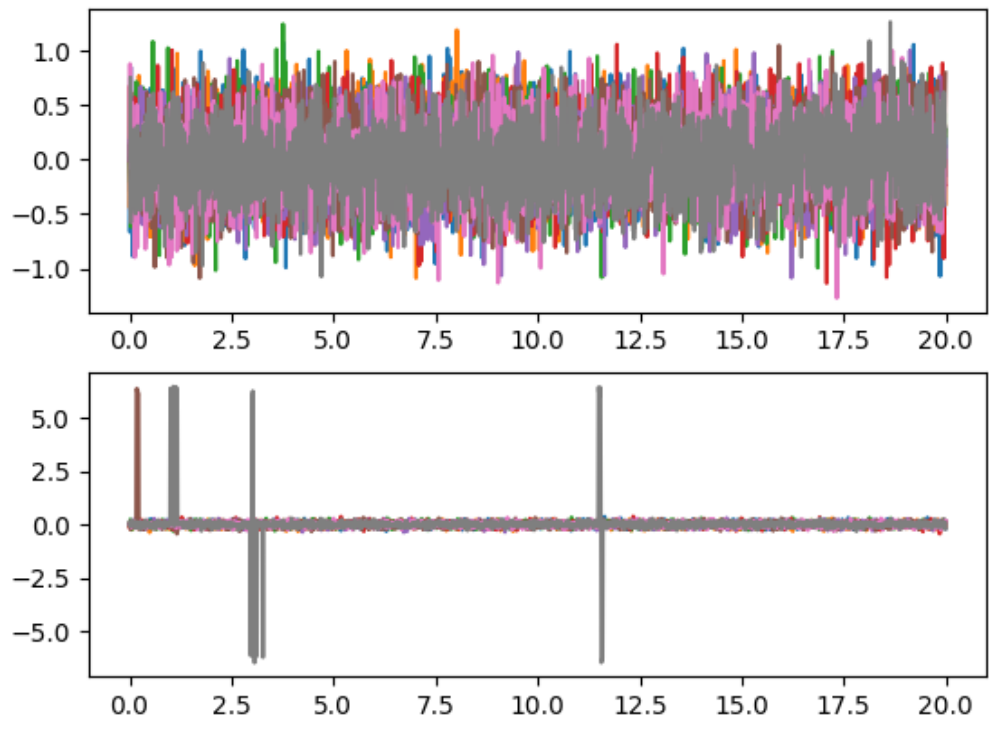
$$\frac{\partial \text{atan2}[m_y^n - y_t, m_x^n - x_t]}{\partial m_x^n} = -\frac{(m_x^n - x_t)}{\|m^n - p_t\|^2}$$

$$\frac{\partial \text{abun2} [m_y' - y_t, m_x' - x_t]}{\partial m_y'} = \frac{(m_x' - x_t)}{\|m' - p_t\|^2}$$

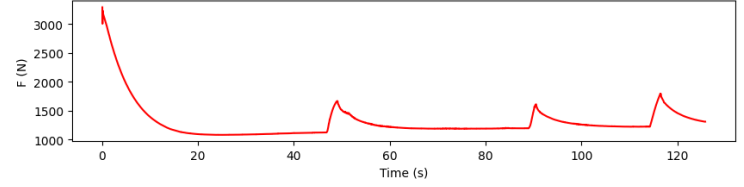
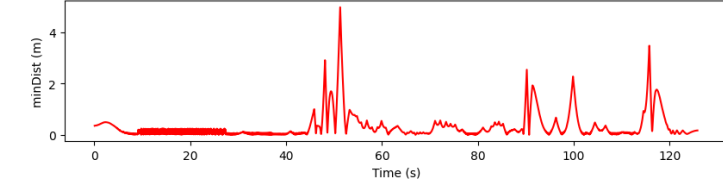
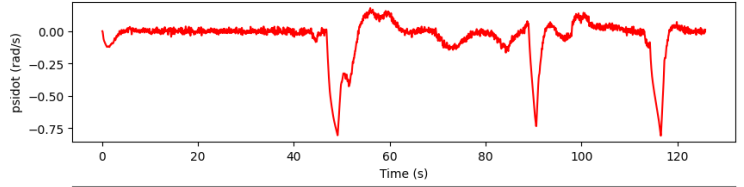
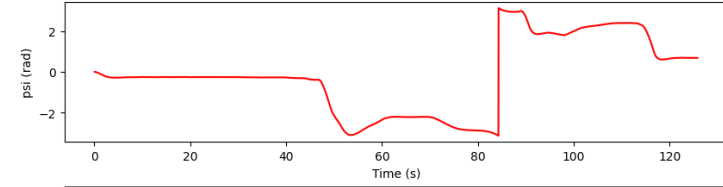
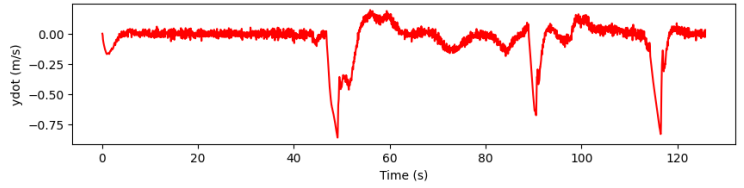
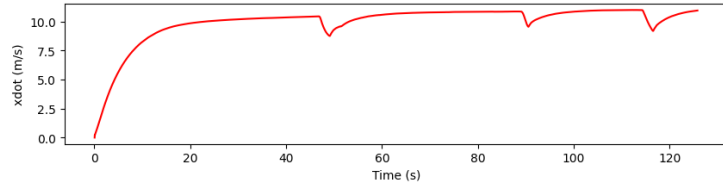
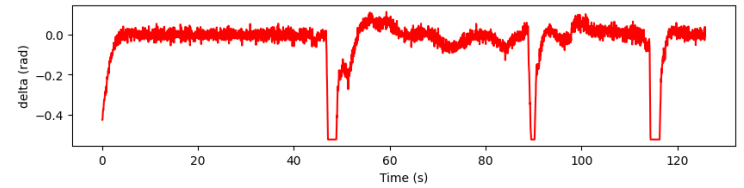
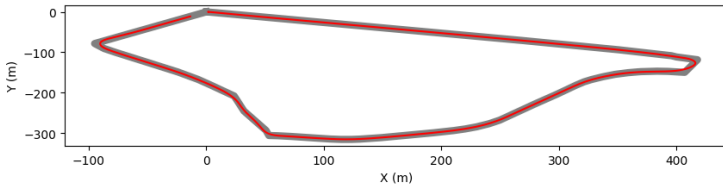
Similarly, the same pattern follows for the rest of the variables.

$$\therefore H_K = \begin{bmatrix} \frac{(x_t - m_x')}{\|m' - p_t\|} & \frac{(y_t - m_y')}{\|m' - p_t\|} & 0 & -\frac{(x_t - m_x')}{\|m' - p_t\|} & -\frac{(y_t - m_y')}{\|m' - p_t\|} & 0 & 0 \dots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{(x_t - m_x'')}{\|m'' - p_t\|} & \frac{(y_t - m_y'')}{\|m'' - p_t\|} & 0 & 0 & 0 & -\frac{(x_t - m_x'')}{\|m'' - p_t\|} & -\frac{(y_t - m_y'')}{\|m'' - p_t\|} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{(m_y' - y_t)}{\|m' - p_t\|^2} & -\frac{(m_x' - x_t)}{\|m' - p_t\|^2} & -1 & -\frac{(m_y' - y_t)}{\|m' - p_t\|^2} & \frac{(m_x' - x_t)}{\|m' - p_t\|^2} & 0 & 0 \dots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{(m_y' - y_t)}{\|m' - p_t\|^2} & -\frac{(m_x' - x_t)}{\|m' - p_t\|^2} & -1 & 0 & \dots & -\frac{(m_y'' - y_t)}{\|m'' - p_t\|^2} & \frac{(m_x'' - x_t)}{\|m'' - p_t\|^2} \end{bmatrix}$$

Result of `erf_slurm.py`



Result of Webots Simulation



Simulation View

0:15:52.096 - 0.00x

ss-track error: 0.17383
rest waypoint: 8153
cent complete: 100.0%
ile point passed:
tination reached! :)

your_controller_ekf_slam.py

```
53 [0],  
54 [2*self.Ca*self.lf/self.lz]  
55 ]]  
56  
57  
58 self.Q = np.diagflat([1, 0.1, 0.1, 0.01])  
59 self.R = np.array([50]).reshape(1,1)  
60  
61 # Add additional member variables according to your need here.  
62  
63 def getStates(self, timestep, use_slam=False):  
64  
65     delX, X, Y, xdot, ydot, psi, psidot = super().getStates(timestep)  
66  
67     # Initialize the EKF SLAM estimation  
68     if self.counter == 0:  
69         # Load the map  
70         minX, maxX, minY, maxY = -120., 450., -500., 50.  
71         map_x = np.linspace(minX, maxX, 7)  
72         map_y = np.linspace(minY, maxY, 7)  
73         map_X, map_Y = np.meshgrid(map_x, map_y)  
74         map_X = map_X.reshape(-1,1)  
75         map_Y = map_Y.reshape(-1,1)  
76         self.map = np.hstack((map_X, map_Y)).reshape((-1))  
77  
78         # Parameters for EKF SLAM  
79         self.n = int(len(self.map)/2)  
80         X_est = X + 0.5  
81         Y_est = Y + 0.5  
82         psi_est = psi - 0.02
```

Estimated X, Y, psi: -13.480857939156067 -11.11490370818885 0.6903396143288927
True X, Y, psi: -13.429536788427654 -11.022602379072024 0.6875356261335517
Estimated X, Y, psi: -13.216581597952237 -10.890507430946064 0.6822529249227421
Evaluating...
Score for completing the loop: 30.0/30.0
Score for average distance: 30.0/30.0
Score for maximum distance: 30.0/30.0
Your time is 125.792
Your total score is : 100.0/100.0
total steps: 125792
maxMinDist: 4.972473633451433
avgMinDist: 0.3363968108959321
INFO: 'main' controller exited successfully.