P1 : Problems

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Exercise 1 :

Lateral dynamics :

$$\ddot{y} = -\dot{\psi}\dot{x} + \frac{1}{m}\left(F_{yf}\cos\delta + F_{yr}\right)$$

$$\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

$$\Downarrow$$

$$\ddot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)$$

$$\ddot{\psi} = \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)$$

Linearised System :

$$\dot{s}_1 = A_1 s_1 + B_1 u \qquad u = \begin{bmatrix}\delta \\ F\end{bmatrix} \qquad s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi}\end{bmatrix}$$

Rewriting the system as :

$$\begin{bmatrix}\dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right)\end{bmatrix}$$

Finding equilibria :

$$\left.\begin{aligned} \dot{y} &= 0 \\ \ddot{y} &= 0 \\ \dot{\psi} &= 0 \\ \ddot{\psi} &= 0 \end{aligned}\right\} \quad \begin{aligned} &-\cancelto{0}{\dot{\psi}\dot{x}} + \frac{2C_\alpha}{m}\left(\cos\delta\,(\delta)\right) = 0 \\ &\delta = 0 \\ &\dot{y} = 0 \Rightarrow y = C_1 \;,\; \dot{\psi} = 0 \Rightarrow \psi = C_2 \end{aligned}$$

$$\therefore (s_0, u_0) = (c_1, 0, c_2, 0, 0, 0)$$

Finding Jacobians:

$$J_{A1} = \begin{bmatrix} \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial \dot{y}} & \frac{\partial \dot{y}}{\partial \psi} & \frac{\partial \dot{y}}{\partial \dot{\psi}} \\ \frac{\partial \ddot{y}}{\partial y} & \frac{\partial \ddot{y}}{\partial \dot{y}} & \frac{\partial \ddot{y}}{\partial \psi} & \frac{\partial \ddot{y}}{\partial \dot{\psi}} \\ \frac{\partial \dot{\psi}}{\partial y} & \frac{\partial \dot{\psi}}{\partial \dot{y}} & \frac{\partial \dot{\psi}}{\partial \psi} & \frac{\partial \dot{\psi}}{\partial \dot{\psi}} \\ \frac{\partial \ddot{\psi}}{\partial y} & \frac{\partial \ddot{\psi}}{\partial \dot{y}} & \frac{\partial \ddot{\psi}}{\partial \dot{\psi}} & \frac{\partial \ddot{\psi}}{\partial \dot{\psi}} \end{bmatrix}$$

$$\frac{\partial \ddot{y}}{\partial \dot{y}} = \frac{2C_\alpha}{m}\left[\cos\delta\left[-\frac{1}{\dot{x}}\right] - \frac{1}{\dot{x}}\right] = -\frac{4C_\alpha}{m\dot{x}}$$

$$\frac{\partial \ddot{y}}{\partial \dot{\psi}} = -\dot{x} + \frac{2C_\alpha}{m}\left[\cos\delta\left(\frac{-l_f}{\dot{x}}\right) - \frac{-l_r}{\dot{x}}\right] = -\dot{x} + \frac{2C_\alpha}{m\dot{x}}(l_r - l_f)$$

$$\frac{\partial \ddot{\psi}}{\partial \dot{y}} = \frac{2l_f C_\alpha}{I_z}\left[-\frac{1}{\dot{x}}\right] - \frac{2l_r C_\alpha}{I_z}\left[-\frac{1}{\dot{x}}\right] = \frac{2C_\alpha}{I_z\dot{x}}(l_r - l_f)$$

$$\frac{\partial \ddot{\psi}}{\partial \dot{\psi}} = \frac{2l_f C_\alpha}{I_z}\left(\frac{-l_f}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(\frac{l_r}{\dot{x}}\right) = -\frac{2C_\alpha}{I_z\dot{x}}(l_f^2 + l_r^2)$$

$$\therefore J_{A1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_\alpha}{m\dot{x}}(l_r - l_f) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}(l_r - l_f) & 0 & -\frac{2C_\alpha}{I_z\dot{x}}(l_f^2 + l_r^2) \end{bmatrix}$$

$$J_{B_1} = \begin{bmatrix} \frac{\partial \dot{y}}{\partial \delta} & \frac{\partial \dot{y}}{\partial F} \\ \frac{\partial \ddot{y}}{\partial \delta} & \frac{\partial \ddot{y}}{\partial F} \\ \frac{\partial \dot{\psi}}{\partial \delta} & \frac{\partial \dot{\psi}}{\partial F} \\ \frac{\partial \ddot{\psi}}{\partial \delta} & \frac{\partial \ddot{\psi}}{\partial F} \end{bmatrix}$$

$$\frac{\partial \ddot{y}}{\partial \delta} = \frac{2C_\alpha}{m}\left[-\sin\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) + \cos\delta\right]$$

$$= \frac{2C_\alpha}{m}$$

$$\frac{\partial \ddot{\psi}}{\partial \delta} = \frac{2 l_f C_\alpha}{I_z}$$

$$J_{B_1} = \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2 l_f C_\alpha}{I_z} & 0 \end{bmatrix}$$

∴ The linearised system is:

$$\underbrace{\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}}_{\dot{s}_1} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_\alpha}{m\dot{x}}(l_r - l_f) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}(l_r - l_f) & 0 & -\frac{2C_\alpha}{I_z\dot{x}}(l_f^2 + l_r^2) \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix}}_{s_1} +$$

$$\underbrace{\begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2 l_f C_\alpha}{I_z} & 0 \end{bmatrix}}_{B_1} \underbrace{\begin{bmatrix} \delta \\ F \end{bmatrix}}_{u}$$

Longitudinal dynamics:

$$\ddot{x} = \dot{\psi}\dot{y} + \frac{1}{M}(F - fmg)$$

After linearization:

$$\dot{s}_2 = A_2 s_2 + B_2 u \qquad u = \begin{bmatrix}\delta \\ F\end{bmatrix} \qquad s_2 = \begin{bmatrix}x \\ \dot{x}\end{bmatrix}$$

$$\begin{bmatrix}\dot{x} \\ \ddot{x}\end{bmatrix} = A_2 \begin{bmatrix}x \\ \dot{x}\end{bmatrix} + B_2 \begin{bmatrix}\delta \\ F\end{bmatrix}$$

Rewriting system as:

$$\begin{bmatrix}\dot{x} \\ \ddot{x}\end{bmatrix} = \begin{bmatrix}\dot{x} \\ \dot{\psi}\dot{y} + \frac{1}{M}(F - fmg)\end{bmatrix}$$

Finding equilibria:

$$\left.\begin{matrix}\dot{x} = 0 \\ \ddot{x} = 0\end{matrix}\right\} \quad \begin{matrix}x = c \text{ (constant)} \\ \dot{x} = 0\end{matrix} \quad \& \quad \begin{matrix}F = -\dot{\psi}\dot{y}m + fmg \\ \delta = 0\end{matrix}$$

$$(s_0, u_0) = (c, 0, 0, -\dot{\psi}\dot{y}m + fmg)$$

$$J_{A_2} = \begin{bmatrix}\frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \dot{x}} \\ \frac{\partial \ddot{x}}{\partial x} & \frac{\partial \ddot{x}}{\partial \dot{x}}\end{bmatrix} = \begin{bmatrix}\ddot{x} & 1 \\ 0 & 0\end{bmatrix}$$

$$J_{B_2} = \begin{bmatrix}\frac{\partial \dot{x}}{\partial \delta} & \frac{\delta \dot{x}}{\partial F} \\ \frac{\partial \ddot{x}}{\partial \delta} & \frac{\partial \ddot{x}}{\partial F}\end{bmatrix} = \begin{bmatrix}0 & 0 \\ 0 & \frac{1}{m}\end{bmatrix}$$

$\therefore$ Linerised system is:

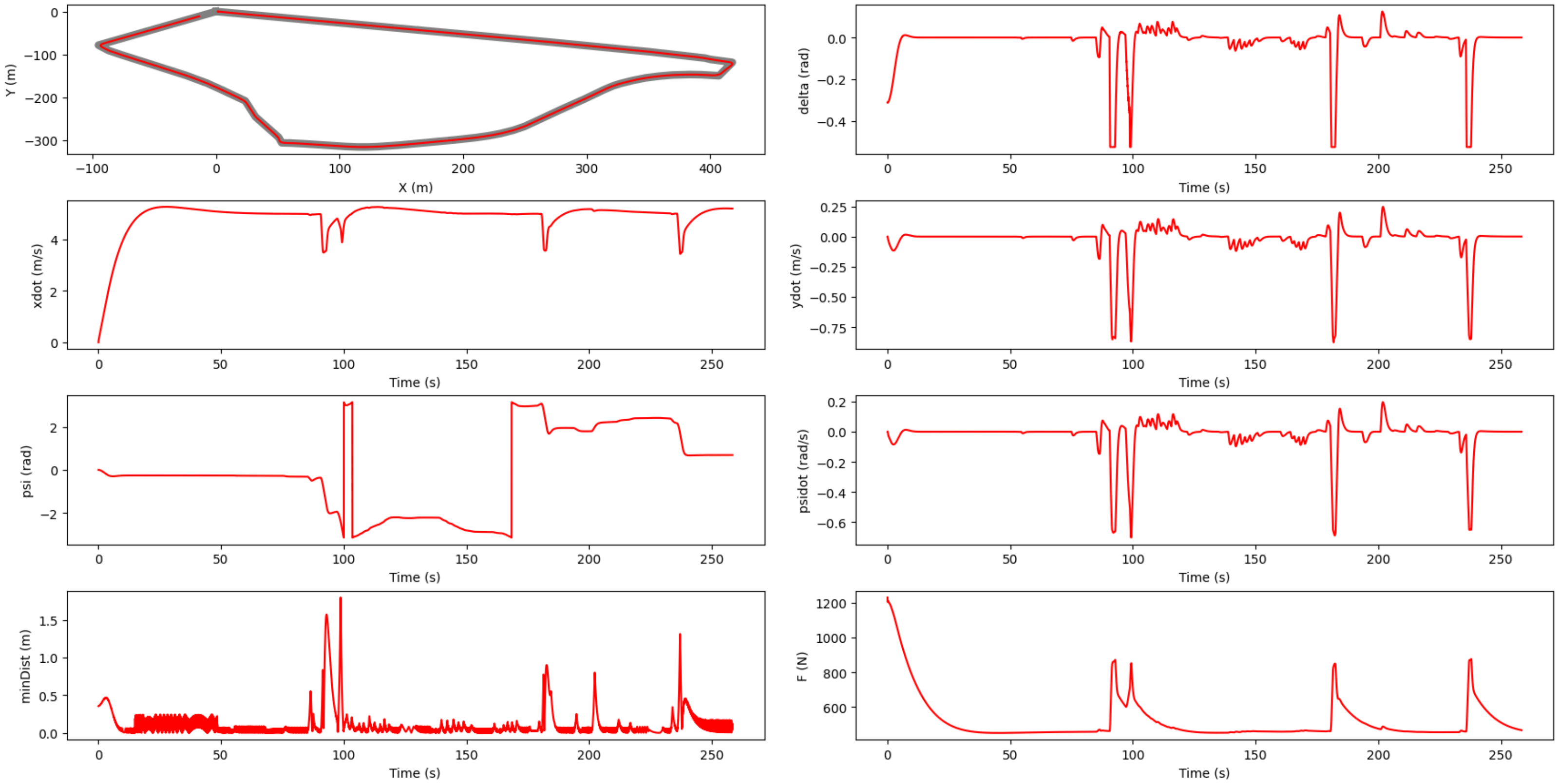
$$\begin{bmatrix}\dot{x} \\ \ddot{x}\end{bmatrix} = J_{A_2}\Big|_{(s_0,u_0)} (s_2 - s_0) + J_{B_2}\Big|_{(s_0,u_0)} (u - u_0)$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x - c \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix} \begin{bmatrix} \delta \\ F - (-\dot{\psi}\dot{y}m + fmg) \end{bmatrix}$$

$$\underset{\dot{s}_2}{\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}} = \underset{A_2}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}} \underset{s_2}{\begin{bmatrix} x \\ \dot{x} \end{bmatrix}} + \underset{B_2}{\begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}} \underset{u}{\begin{bmatrix} \delta \\ F \end{bmatrix}} + \underset{d}{\begin{bmatrix} 0 \\ \dot{\psi}\dot{y} - fg \end{bmatrix}}$$

disturbance term.

Exercise 2



Y (m)
X (m)
delta (rad)
xdot (m/s)
ydot (m/s)
psi (rad)
psidot (rad/s)
minDist (m)
F (N)
Time (s)