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MCT: Project - Part 4

Exercise 1:

Discrete time dynamics of the system:

$$x_{t+1} = x_t + \delta t\,(\dot{x}_t \cos\psi_t - \dot{y}_t \sin\psi_t) + w_t^x$$

$$y_{t+1} = y_t + \delta t\,(\dot{x}_t \sin\psi_t + \dot{y}_t \cos\psi_t) + w_t^y$$

$$\psi_{t+1} = \psi_t + \delta t\,\dot{\psi}_t + w_t^\psi$$

Input, $u_t = \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ \psi_t \end{bmatrix}$ $\qquad p_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$ $\qquad$ n features in map, $m^j = \begin{bmatrix} m_x^j \\ m_y^j \end{bmatrix}$

meas. noise

Range measurement: $y^j_{t,dist} = \| m^j - p_t \| + \vartheta^j_{t,dist}$ $\qquad j = 1 \dots n$

Bearing measurement: $y^j_{t,bearing} = \operatorname{atan2}[m_y^j - y_t,\ m_x^j - x_t] - \psi_t + \vartheta^j_{t,bearing}$

meas. noise.

State vector,

$$x_t = \begin{bmatrix} x_t \\ y_t \\ \psi_t \\ m_x^1 \\ m_y^1 \\ m_x^2 \\ \vdots \\ m_x^n \\ m_y^n \end{bmatrix}$$

The measurement,

$$y_t = \begin{bmatrix} \| m^1 - p_t \| \\ \vdots \\ \| m^n - p_t \| \\ \operatorname{atan2}[m_y^1 - y_t,\ m_x^1 - x_t] - \psi_t \\ \vdots \\ \operatorname{atan2}[m_y^n - y_t,\ m_x^n - x_t] - \psi_t \end{bmatrix} + \begin{bmatrix} \vartheta^1_{t,dist} \\ \vdots \\ \vartheta^n_{t,dist} \\ \vartheta^1_{t,bearing} \\ \vdots \\ \vartheta^n_{t,bearing} \end{bmatrix}$$

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k$$
$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$

Let $\mathbf{F}_k = \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k}$, $\mathbf{H}_k = \left.\frac{\partial h}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k-1}}$ Replace $\mathbf{A}$ with $\mathbf{F}_k$, $\mathbf{C}$ with $\mathbf{H}_k$. We get EKF.

Finding F_k :

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}, u_k} = \begin{bmatrix} \frac{\partial x_{t+1}}{\partial x_t} & \frac{\partial x_{t+1}}{\partial y_t} & \cdots & \frac{\partial x_{t+1}}{\partial m_y^n} \\ \frac{\partial y_{t+1}}{\partial x_t} & \frac{\partial y_{t+1}}{\partial y_t} & & \vdots \\ \frac{\partial \psi_{t+1}}{\partial x_t} & \vdots & \ddots & \vdots \\ \frac{\partial m_x^1}{\partial x_t} & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \frac{\partial m_y^n}{\partial x_t} & \frac{\partial m_y^n}{\partial y_t} & \cdots & \frac{\partial m_y^n}{\partial m_y^n} \end{bmatrix}$$

$$\frac{\partial x_{t+1}}{\partial x_t} = 1 \qquad \frac{\partial x_{t+1}}{\partial y_t} = 0 \qquad \frac{\partial x_{t+1}}{\partial \psi_t} = -\delta t \left[\dot{x}_t \sin \psi_t + \dot{y}_t \cos \psi_t \right]$$

$$\frac{\partial y_{t+1}}{\partial x_t} = 0 \qquad \frac{\partial y_{t+1}}{\partial y_t} = 1 \qquad \frac{\partial y_{t+1}}{\partial \psi_t} = \delta t \left[\dot{x}_t \cos \psi_t - \dot{y}_t \sin \psi_t \right]$$

$$\frac{\partial \psi_{t+1}}{\partial x_t} = 0 \qquad \frac{\partial \psi_{t+1}}{\partial y_t} = 0 \qquad \frac{\partial \psi_{t+1}}{\partial \psi_t} = 1$$

$$\frac{\partial m_x^1}{\partial x_t} = \frac{\partial m_x^1}{\partial y_t} = \frac{\partial m_x^1}{\partial \psi_t} = 0 \qquad \frac{\partial m_x^1}{\partial m_x^1} = 1$$

Similarly, the same pattern follows for other terms.

$$\therefore F_k = \begin{bmatrix} 1 & 0 & -\delta t\left[\dot{x}_t \sin\psi_t + \dot{y}_t \cos\psi_t\right] & 0 & \cdots & & 0 \\ 0 & 1 & \delta t\left[\dot{x}_t \cos\psi_t - \dot{y}_t \sin\psi_t\right] & 0 & \cdots & & 0 \\ 0 & 0 & 1 & 0 & \cdots & & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & 0 & 1 & 0 & 0 \\ & & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_k = \frac{\partial h}{\partial x}\bigg|_{\hat{x}_{k|k-1}} = \begin{bmatrix} \frac{\partial \|m^1 - p_t\|}{\partial x_t} & \frac{\partial \|m^1 - p_t\|}{\partial y_t} & \cdots \\ \vdots & & \\ \frac{\partial}{\partial x_t}\|m^n - p_t\| & \frac{\partial}{\partial y_t}\|m^n - p_t\| & \\ \vdots & & \ddots \\ \frac{\partial}{\partial x_t}\operatorname{atan2}\left[m_y^n - y_t,\ m_x^n - x_t\right] & \cdots & \end{bmatrix}$$

$$\|m^i - p_t\| = \sqrt{(x_t - m_x^i)^2 + (y_t - m_y^i)^2}$$

$$\therefore \frac{\partial \|m^i - p_t\|}{\partial x_t} = \frac{2(x_t - m_x^i)}{2\sqrt{(x_t - m_x^i)^2 + (y_t - m_y^i)^2}} = \frac{(x_t - m_x^i)}{\|m^i - p_t\|}$$

$$\frac{\partial \|m^i - p_t\|}{\partial y_t} = \frac{(y_t - m_y^i)}{\|m^i - p_t\|}$$

$$\frac{\partial \|m^i - p_t\|}{\partial m_x^i} = -\frac{(x_t - m_x^i)}{\|m^i - p_t\|} \qquad \frac{\partial \|m^i - p_t\|}{\partial m_y^i} = -\frac{(y_t - m_y^i)}{\|m^i - p_t\|}$$

$$\frac{\partial}{\partial x_t}\operatorname{atan2}\left[m_y^i - y_t,\ m_x^i - x_t\right] = -\frac{(m_y^i - y_t)}{\|m^i - p_t\|^2}(-1) = \frac{(m_y - y_t)}{\|m^i - p_t\|^2}$$

$$\frac{\partial}{\partial y_t}\operatorname{atan2}\left[m_y^i - y_t,\ m_x^i - x_t\right] = \frac{(m_x^i - x_t)}{\|m^i - p_t\|^2}(-1) = -\frac{(m_x - x_t)}{\|m^i - p_t\|^2}$$

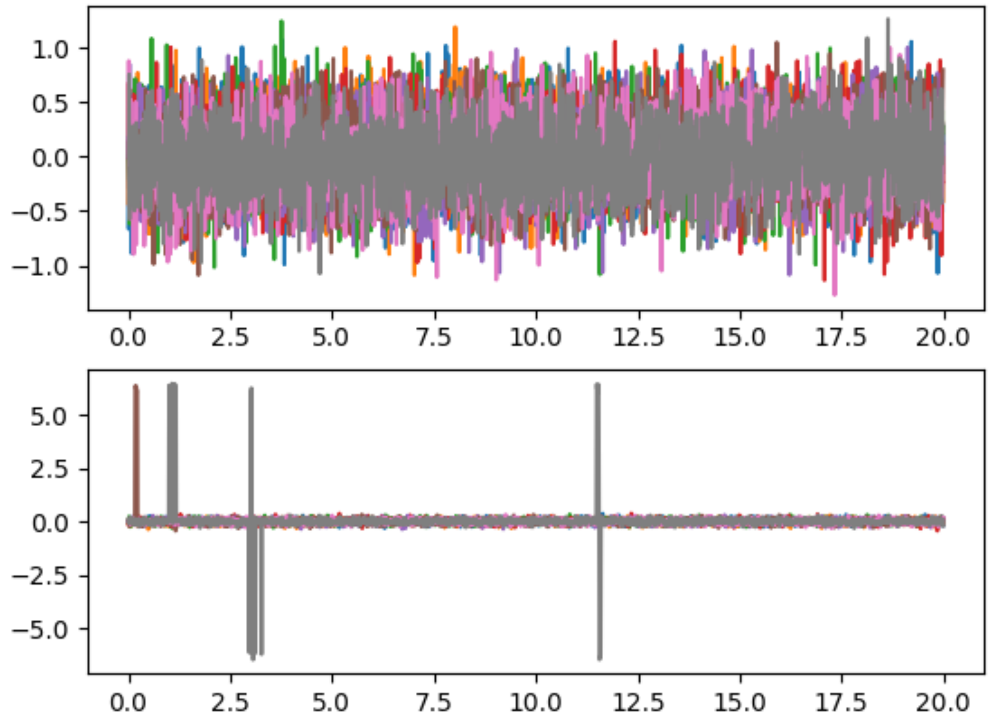
$$\frac{\partial}{\partial m_x^i}\operatorname{atan2}\left[m_y^i - y_t,\ m_x^i - x_t\right] = -\frac{(m_y^i - y_t)}{\|m^i - p_t\|^2}$$

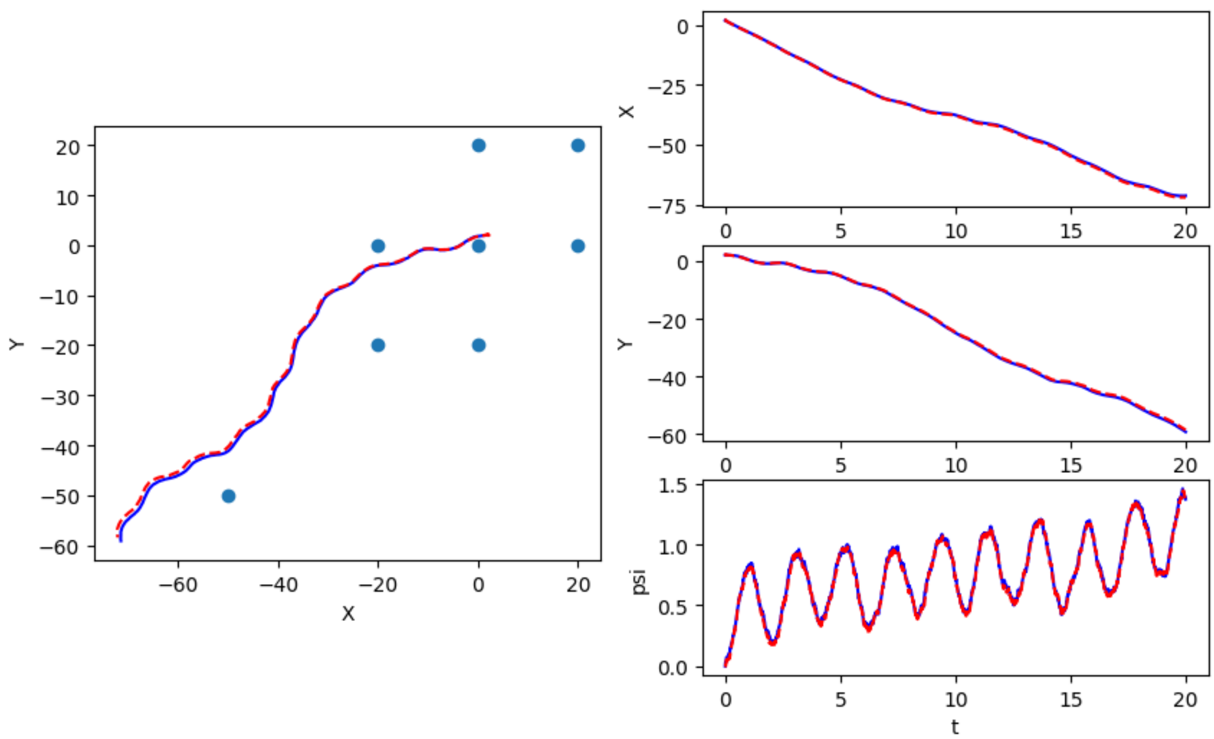
$$\frac{\partial}{\partial m'_y} \operatorname{atan2}\left[m'_y - y_t,\ m'_x - x_t\right] = \frac{(m'_x - x_t)}{\| m' - p_t \|^2}$$

Similarly, the same pattern follows for the rest of the variables.

$$\therefore\ H_R = \begin{bmatrix}
\frac{(x_t - m'_x)}{\|m' - p_t\|} & \frac{(y_t - m'_y)}{\|m' - p_t\|} & 0 & -\frac{(x_t - m'_x)}{\|m' - p_t\|} & -\frac{(y_t - m'_y)}{\|m' - p_t\|} & 0 & 0 \cdots 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{(x_t - m^n_x)}{\|m^n - p_t\|} & \frac{(y_t - m^n_y)}{\|m^n - p_t\|} & 0 & 0 & 0 & -\frac{(x_t - m^n_x)}{\|m^n - p_t\|} & -\frac{(y_t - m^n_y)}{\|m^n - p_t\|} \\
\frac{(m'_y - y_t)}{\|m' - p_t\|^2} & -\frac{(m'_x - x_t)}{\|m' - p_t\|^2} & -1 & -\frac{(m'_y - y_t)}{\|m' - p_t\|^2} & \frac{(m'_x - x_t)}{\|m' - p_t\|^2} & 0 & 0 \cdots 0 \\
\vdots & \vdots & & \vdots & \ddots & \vdots & \vdots \\
\frac{(m^n_y - y_t)}{\|m^n - p_t\|^2} & -\frac{(m^n_x - x_t)}{\|m^n - p_t\|^2} & -1 & 0 \cdots & -\frac{(m^n_y - y_t)}{\|m^n - p_t\|^2} & & \frac{(m^n_x - x_t)}{\|m - p_t\|^2}
\end{bmatrix}$$

Result of ekf_slam.py





Result of Webots Simulation