CSE 574 Machine Learning
Programming Assignment 2
Classification and Regression

Due Date: April 12th 2017

Group No 67

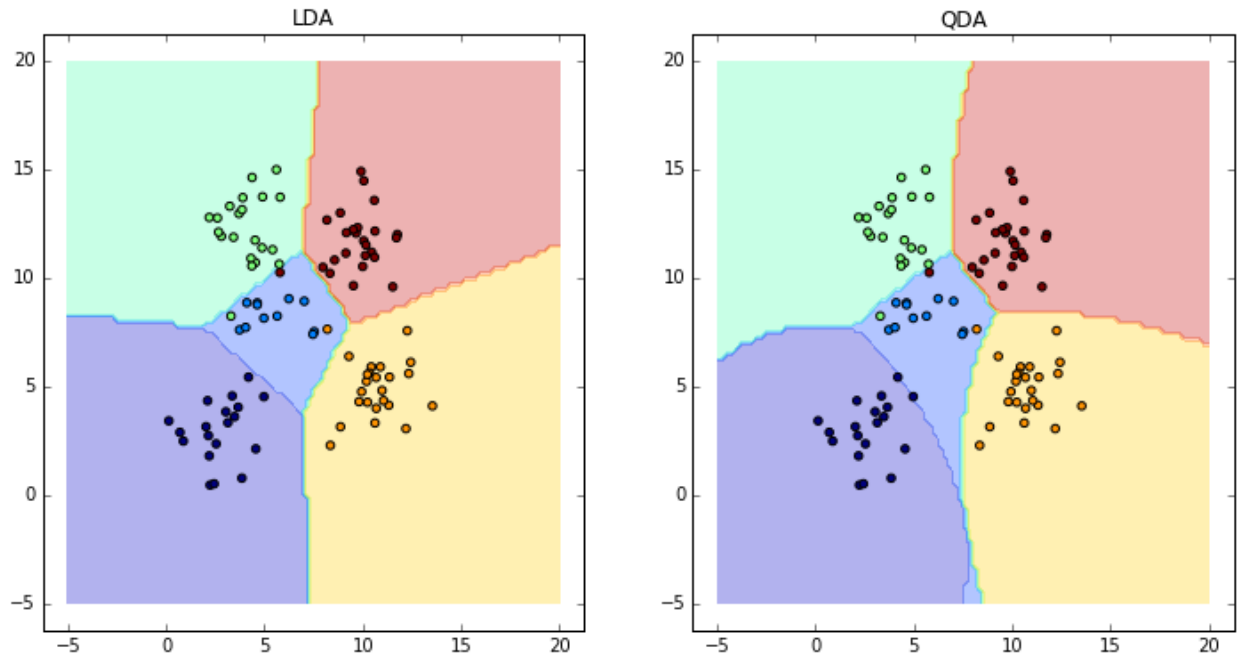
Aniruddh Chaturvedi [5020 6958]
Arnav Ahire [5020 8006]
Ashwin Nikam [5020 7368]

Problem 1:

The obtained accuracies are as follows:

LDA Accuracy	0.97
QDA Accuracy	0.96

The graph plot is as follows:



Observations:

1. In case of LDA, the boundaries that separate all the classes from each other are linear i.e. they are of the form of a straight line.
2. However, with QDA, the boundaries separating the classes are non-linear (quadratic).
3. From the diagram we can observe that LDA is able to learn only lines but QDA is able to learn curves which makes it more flexible compared to LDA and can fit the data better.
4. However, Seeing the accuracies of LDA and QDA, LDA is giving a better accuracy (0.97) than QDA (0.96) since we are obtaining a more reliable estimate of variance between the training data. It is because here the covariance calculation takes all the training data into consideration and not just pieces of training data corresponding to each class thus giving us a better value of covariance and hence accuracy.
5. The difference is because in calculation of LDA no quadratic term is involved and if the test data is not sufficiently large, then this gives a better variance estimation than QDA and thus lead to higher accuracy on small data. Conversely if the test data is big, then

estimating variance would not be much of a concern and hence QDA will have better accuracy in that case.

6. Since all the calculations in LDA are linear, it has linear boundaries and QDA involves quadratic calculations, hence its boundaries are quadratic.

Problem 2:

MSE without intercept	106775.3615663
MSE with intercept	3707.84018141

Observations:

1. MSE - Mean Squared Error.
   Since this is an error estimate, higher the value, higher will be the error and lower will be the accuracy of the linear regression. Conversely, lower the value, lower will be the error and it would mean that the linear regression model fits the data appropriately.
2. Hence taking that into consideration MSE with intercept (3707.84018141) is better than MSE without intercept (106775.3615663), since this gives an improvised regression line that fits the data very well as opposed to the regression line that always passes through the origin.
3. This bias that is introduced in the form of intercept facilitates flexibility in linear regression and hence gives a better MSE value.

Problem 3:

In this problem we need to calculate and report the MSE for training and test data using ridge regression parameters. The training and test data used in this case consists of an intercept which means that the regression line won't have to necessarily pass through the origin thus generating better accuracy. Errors on test and train data had to be plot for different values of lambda (λ) ranging from 0 to 1 in increments of 0.01.
Let us compare both approaches in terms of error on test and train data

Train data:

1. learnOLERegression

MSE without intercept	19099.44684457
MSE with intercept	2187.16029493

2. learnRidgeRegression (with intercept)

MSE	Lambda (λ)
2187.16029493	0.00
2306.83221793	**0.01**
2354.07134393	**0.02**
2386.7801631	**0.03**
2412.119043	**0.04**
2433.1744367	**0.05**
…	**…**

Test Data:

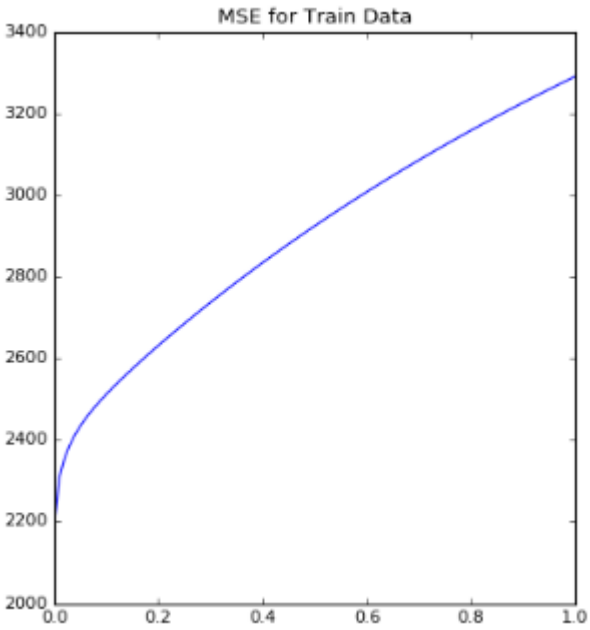
1. learnOLERegression

MSE without intercept	106775.3615663
MSE with intercept	3707.84018141

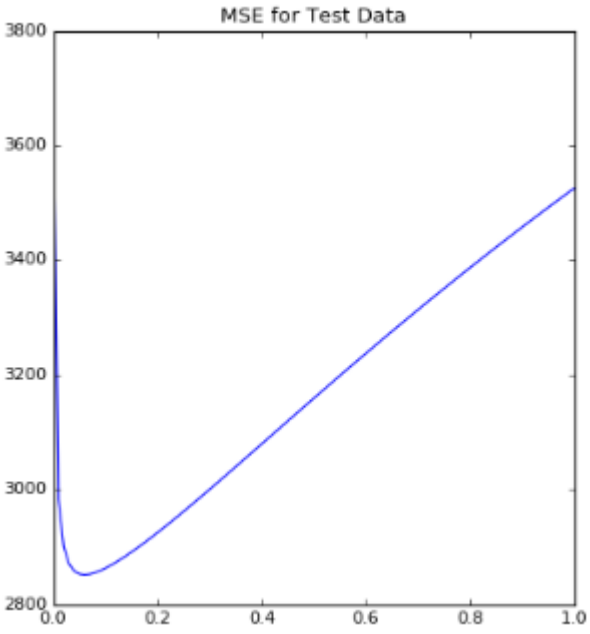
2. learnRidgeRegression (with intercept)

MSE	Lambda (λ)
3707.84018141	**0.00**
2982.44611971	**0.01**
2900.97358708	**0.02**
2870.94158888	**0.03**
2858.00040957	**0.04**
2852.66573517	**0.05**
…	**…**

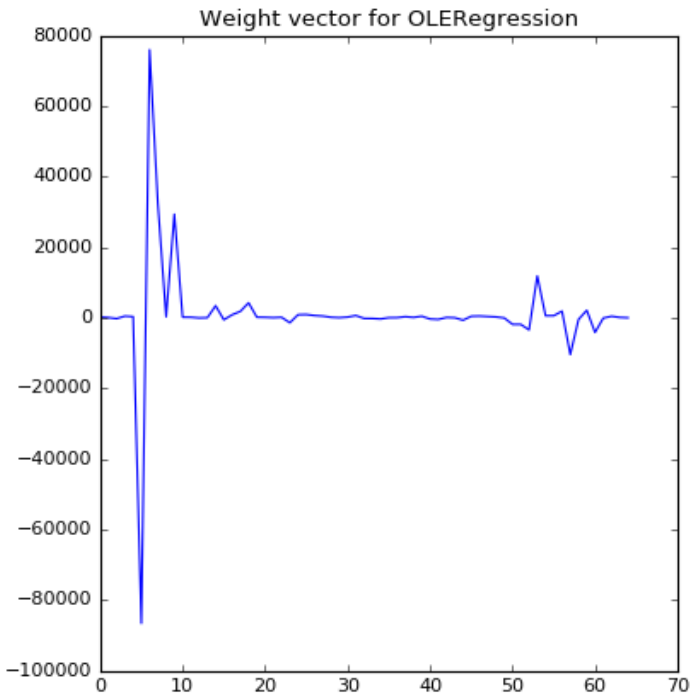
- From the above observations it can be clearly seen that for training data the MSE computed using learnOLERegression is the same as MSE computed using learnRidgeRegression for Lambda (λ) = 0, which is pretty obvious since lambda=0 indicates no regularization. However as we increase the lambda the error kept on increasing.
- This is also shown in the plot for train data.

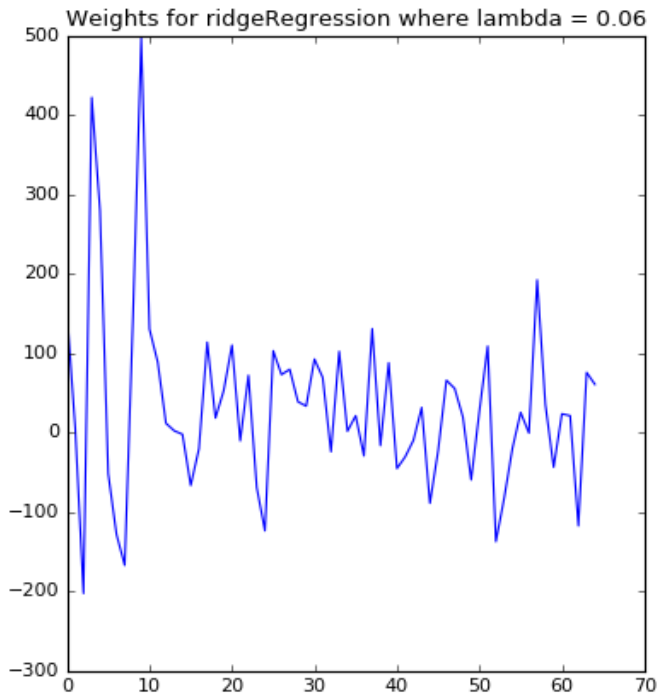


- In case of test data too, the MSE for lambda (λ) =0 when computed using learnRidgeRegression is the same as the MSE which is computed using learnOLERegression.
- However, as we increase the lambda the error decreases up to a certain value of lambda which in our case is 0.06.
- After this point the error gradually increases as we increase the lambda. This has also been shown in the plot for test data.
- Thus, the optimal value chosen for lambda (λ) is 0.06 as it gives the least MSE.



Weight vectors for OLERegression (no regularization) and ridgeRegression (regularization where lambda = 0.06) are shown in the below graph. The x-axis indicates the feature to which the weight corresponds to. The y-axis indicates the actual weight. Each data point consists of 64 features hence we get a *(65x1)* weight vector (bias term included).





Problem 4:

- In this problem we are using gradient descent for ridge regression learning instead of directly calculating the regression parameters.
- Here, we will minimize the error function and update the weights accordingly.
- Following is the expression for regularized square error:

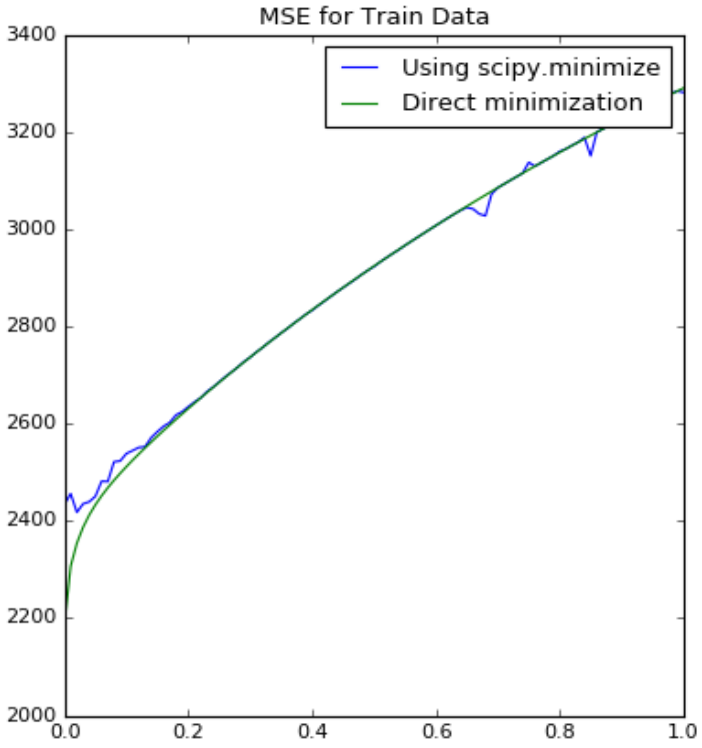
$$J(\mathbf{w}) = \frac{1}{2}(\mathbf{y} - \mathbf{X}\mathbf{w})^\top(\mathbf{y} - \mathbf{X}\mathbf{w}) + \frac{1}{2}\lambda\mathbf{w}^\top\mathbf{w}$$

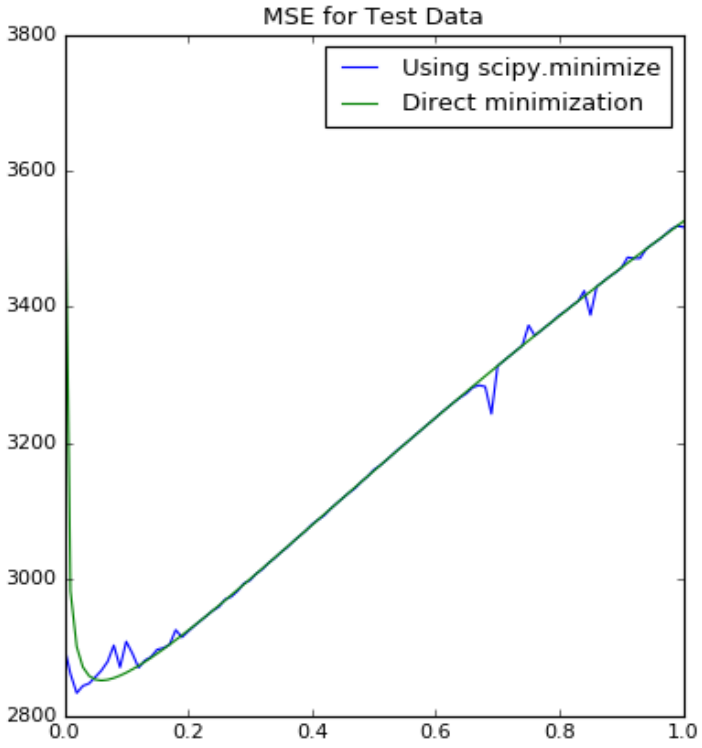
- Following is the expression for calculating gradient of square error:

$$\sum_{i=1}^{N}(\mathbf{w}^\top\mathbf{x}_i - y_i)x_{ij}$$

- A term w * lambda is added to the expression given above to get the gradient of regularized square error.

  Following is the plot of errors on test and train data obtained by using gradient descent and varying the regularization parameter lambda.





Observations:

- The blue curve is the one that is obtained by using scipy.minimize function and the green curve is the one that is obtained using direct minimization.
- We can clearly see that both the curves are fairly close to each other but there are some outliers in the curve obtained by using scipy.minimize function.
- These outliers are eliminated from the curve of direct minimization as the curve is made smooth by increasing the number of iterations.
- For training data, the error increases as the value of lambda increases.
- The MSE values for increasing value of lambda using scipy.minimize are shown below:

MSE	Lambda (λ)
2433.6685790	**0.00**
2455.82465919	**0.01**
2417.4976429	**0.02**
2434.9044833	**0.03**
2439.18958407	**0.04**
2450.62674676	**0.05**

- From the above table and the given graph we can observe that even though there is some inconsistency in the curve for scipy.minimize the MSE gradually increases as we increase the lambda.
- If we consider test data however, the error decreases up to a certain value of lambda and then it increases with lambda. In test data too there is some inconsistency in the smoothness of the curve for MSE when plotted using scipy.minimize compared to direct minimization. MSE values for test data using scipy.minimize have been shown below.

MSE	Lambda (λ)
2900.54490778	**0.00**
2860.8535915	**0.01**
2832.85210624	**0.02**
2843.32265021	**0.03**
2846.96589511	**0.04**
2856.20058136	**0.05**

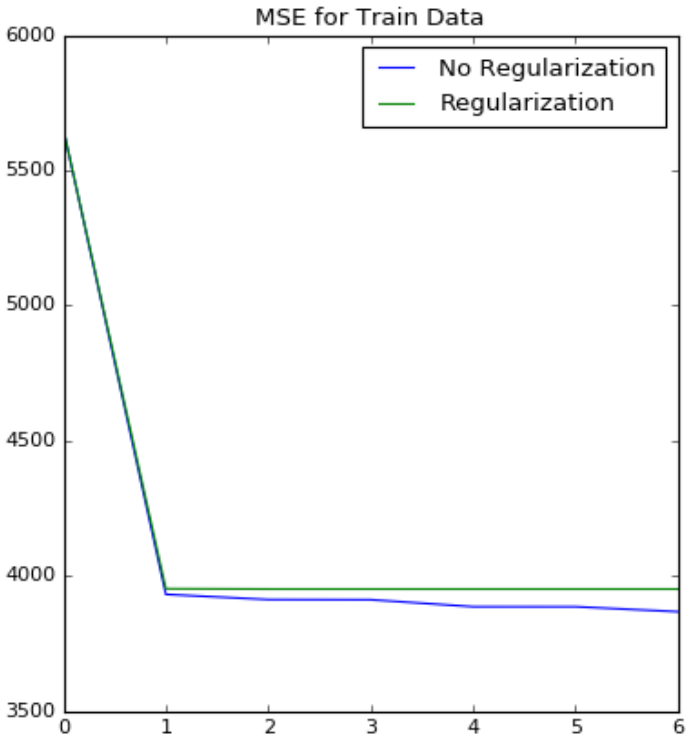
- After this point the error gradually increases as we increase the lambda. This has also been shown in the plot for test data.

Comparison with problem 3:

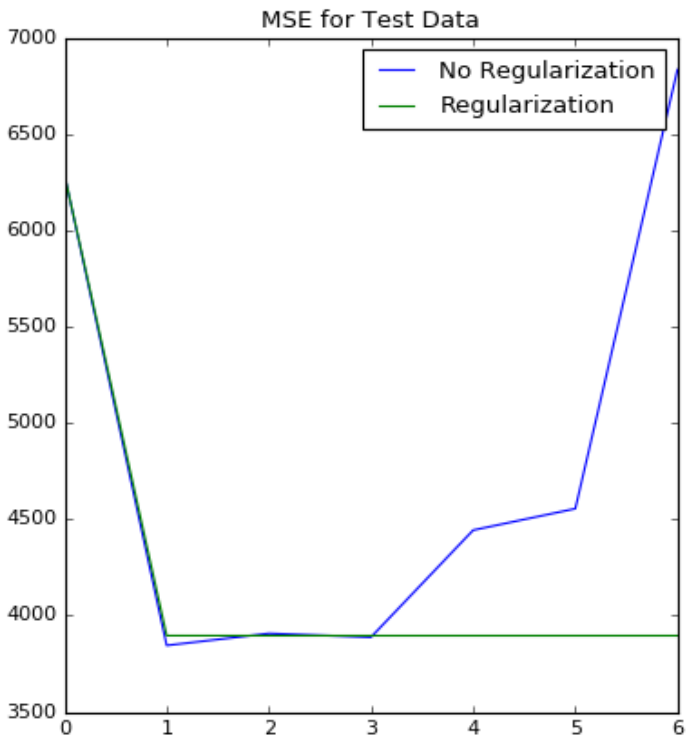
- By comparing the curves obtained in Problem 3 and Problem 4, we can clearly see that the curves are fairly the same and we can say that the result obtained by approaches used in Problem 3 and 4 are same.
- The only difference is that the curve plotted in Problem 3 is much smoother than the curve which is plotted in Problem 4.

Problem 5:

- For this problem we're using only the third variable as the input variable to study impact of using higher order polynomials for the input features. Weights are computed using two lambda values 0 and optimal value. Optimal lambda value in our case is 0.06 which has been computed in Problem 3.
- Regression line has been plotted for various values of p ranging from 0 to 6. For each value of p two lines are plotted corresponding to regression weights generated by the two different lambda values. In this case, p is a number which converts a single attribute x into a vector of p attributes.
- Thus as we increase the value of p we get a regression line that tries to pass through each data point. Increasing the value of p beyond a certain limit may lead to an overfitting problem which we may need to avoid.



- In the plot for train data the blue line indicates MSE for various values of p ranging from 0 to 6 when regression weights have been calculated using lambda = 0.
- The green line indicates the MSE for various values of p when regression weights have been calculated using the optimal lambda value which in our case is 0.06.
- This has been done as we want to minimize our error but also want to penalize the complexity of our model. Hence we use regularization. Regularization forces w terms to be closer to 0 thus minimizing the weight vector.
- Hence the regression line becomes nonlinear but not too much. Due to this, the MSE without regularization would decrease as we increase p up to a certain point. This is because the regression line is trying to fit through each data point.
- Applying regularization however tries to reduce this non linearity hence we get MSE which is greater than that without regularization which can be seen from the green line.



- The effect of this regularization can be seen on the test data where the MSE keeps on increasing as we increase p however if the weight vector has been regularized using optimal value of lambda, the MSE is almost constant and has a low value.
- The optimal value of p can be calculated for test error using both regularization and no regularization from the above plot.
- If we consider the plots for no regularization, we take a look at the blue line. We can clearly see from the plot that for p = 1 we have the least MSE thus the optimal value of p is 1 for no regularization.
- In case of regularization as we increase the p, the MSE slowly but surely decreases in the plot hence the optimal value of p in case of regularization can be stated as p = 6. This is because at p = 6 we have the least MSE.

p	No_Regularization	Regularization
1	3845.035	3895.856
6	6833.459	3895.583

Problem 6:

Final Observations:

For Train Data:

1) Using Linear Regression: MSE = 3707.84018141

2) Using Ridge Regression: MSE = 2187.16029493, with lambda = 0

3) Using Gradient Descent: MSE = 2433.6685790, with lambda = 0

4) Using Non Linear Regression: MSE = 3895.33, with regularization
MSE = 6833.459, without regularization

For Test Data:

1) Using Linear Regression: MSE = 3707.84018141

2) Using Ridge Regression: MSE = 2851.33021344, with lambda = 0.06

3) Using Gradient Descent: MSE = 2832.85210624, with lambda = 0.02

4) Using Non Linear Regression: MSE = 3895.856, with regularization
MSE = 3845.035, without regularization

From the observations given above, we can clearly see that the value of MSE is more when we use Non Linear Regression or Simple Linear Regression, however, the value of MSE is comparatively less when we use Ridge Regression and Gradient Descent.

The value of MSE is least when we use Ridge Regression and it takes lesser computations as compared to gradient descent, hence ***Ridge Regression*** would the best approach amongst all the other linear models.