Sensor fusion of GNSS and IMU data using Extended Kalman Filter for localization of vehicles

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Introduction

- Localization is the task of finding the position of an object in its environment.
- It is an essential part of mobile robotic systems like **drones** and **self-driving cars**.
- The most common systems used for localization are the **GNSS** and **IMU** systems.
- These systems have their own individual limitations.
- They are complementary in nature.
- We can fuse data from both of these sensors using an estimator called the **Kalman filter**.

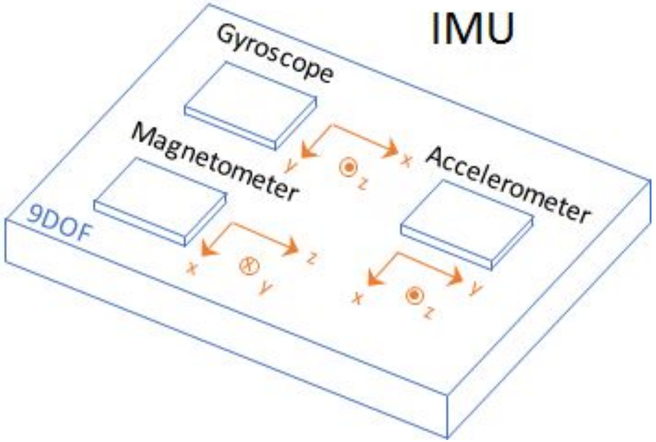
GNSS (Global Navigation Satellite System)

- Satellites in orbit send signals containing their locations in orbit and a timestamp
- Very accurate atomic clocks are used
- Difference in **time-of-flight** from multiple satellites to the receiver is used for trilateration to find position.
- **4** satellites are required to get latitude, longitude and altitude
- **3** satellites are required to get latitude and longitude data.
- Advantages
  - Location data from anywhere around the globe
- Disadvantages
  - Inaccurate (within 5-10m)
  - Low sampling rate (1-10Hz)
  - Unreliable inside tunnels

GNSS utilizes 89 satellites from all 4 satellite systems

IMU (Inertial Measurement Unit)

- Consists of 3 types of sensors
- **Gyroscopes** : Measures angular velocity
- **Accelerometers** : Measures linear acceleration
- **Magnetometers** : Measures magnetic field
- Implemented in silicon chips, as **MEMS** (Micro Electro-mechanical system) devices.
- Advantages
    - High refresh rate
    - Cheap and space-efficient
- Disadvantages
    - Linear position estimates accumulate errors



Double integration of IMU data

- For getting yaw angle, we numerically integrate angular velocity about the z axis.

$$\alpha = \int \dot{\alpha} dt \qquad \alpha_k = \alpha_{k-1} + \dot{\alpha}_k dt$$

- For getting linear positions, we integrate linear acceleration twice.

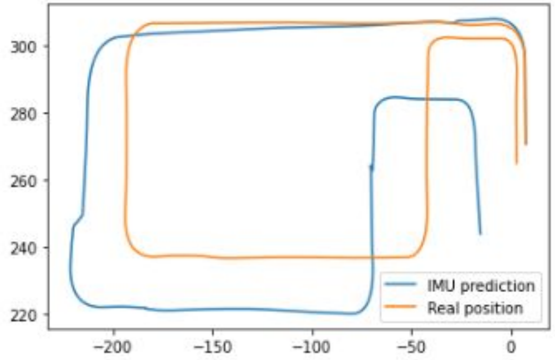
$$x = \int \int \ddot{x} dt \qquad \begin{aligned} \dot{x}_k &= \dot{x}_{k-1} + \ddot{x}_k dt \\ x_k &= x_{k-1} + \dot{x}_k dt \end{aligned}$$

$$y = \int \int \ddot{y} dt \qquad \begin{aligned} \dot{y}_k &= \dot{y}_{k-1} + \ddot{y}_k dt \\ y_k &= y_{k-1} + \dot{y}_k dt \end{aligned}$$

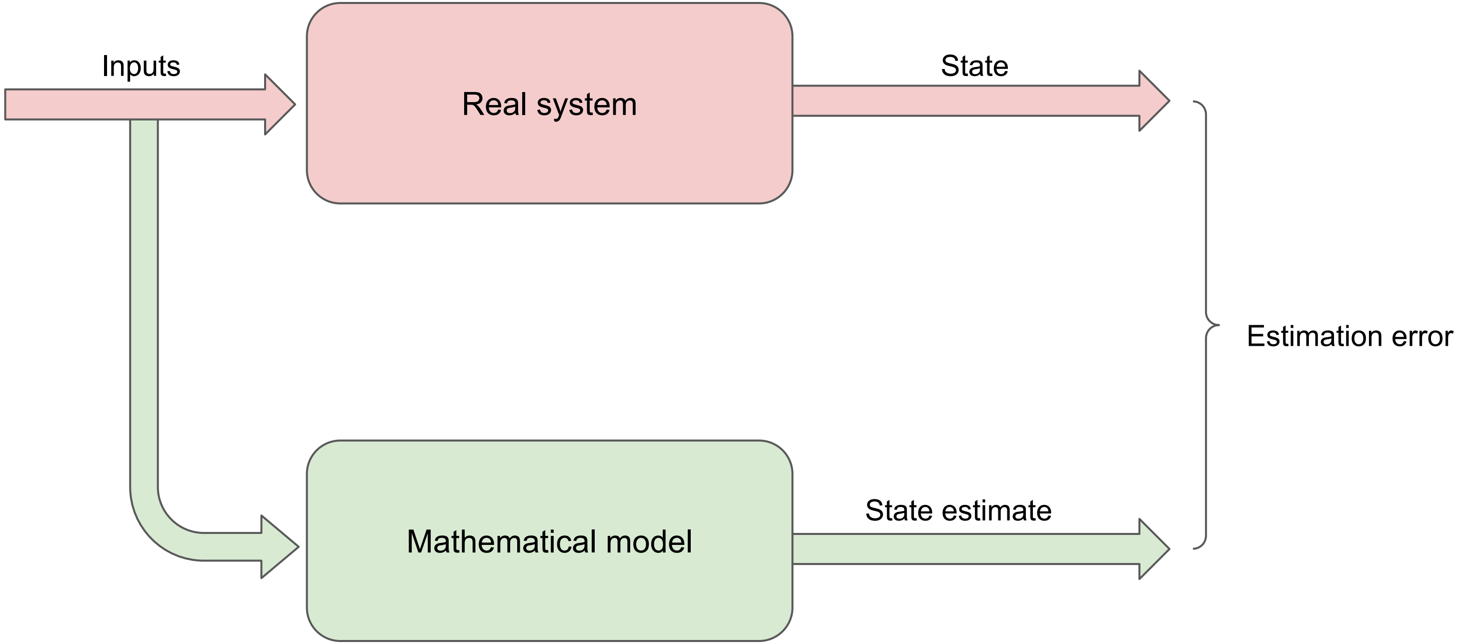
Drift in IMU estimates

$$x = \int \int (\ddot{x} + e)dt \qquad x = \int \int \ddot{x}dt + \frac{et^2}{2}$$

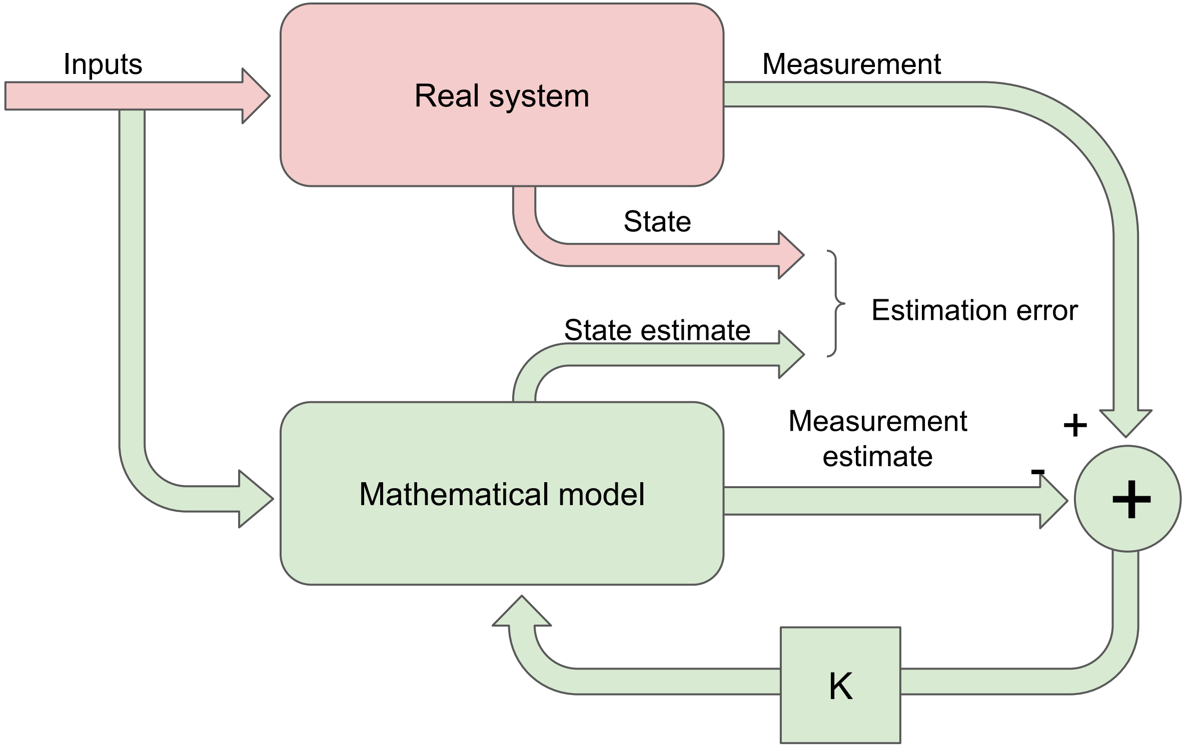
$$y = \int \int (\ddot{y} + e)dt \qquad y = \int \int \ddot{y}dt + \frac{et^2}{2}$$



Simple estimator



With feedback from measurements



What is a kalman filter?

- Invented by Rudolph E.Kalman in 1960
- Kalman filter is a stochastic estimator. Estimate the state x $\in$ R^n of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_k = A \cdot x_{k-1} + B \cdot U_k + w_{k-1}$$

with a measurement y $\in$ R^m that is

$$y_k = H \cdot x_k + v_k$$

- The system is assumed to be linear and the measurement and process noise is assumed to be gaussian and independent of each other, with probability distributions

$$p(w) = N(0, Q) \qquad p(v) = N(0, R)$$

Modeling our system

- For vehicles, we assume they are on a 2-D plane. So, we include the global x and y position and velocity and the yaw angle as our state vector.
- We need to transform acceleration from local to global coordinates.

$$
\begin{bmatrix} x_k \\ y_k \\ \alpha_k \\ \dot{x}_k \\ \dot{y}_k \end{bmatrix}
= f\left(\begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \alpha_{k-1} \\ \dot{x}_{k-1} \\ \dot{y}_{k-1} \end{bmatrix},
\begin{bmatrix} \ddot{x}_k \\ \ddot{y}_k \\ \dot{\alpha}_k \end{bmatrix}\right) =
\begin{bmatrix} x_{k-1} + \dot{x}_k dt \\ y_{k-1} + \dot{y}_k dt \\ \alpha_{k-1} + \dot{\alpha}_k dt \\ \dot{x}_{k-1} + (\ddot{x}_k cos(\alpha_{k-1}) - \ddot{y}_k sin(\alpha k - 1))dt \\ \dot{y}_{k-1} + (\ddot{x}_k sin(\alpha_{k-1}) + \ddot{y}_k cos(\alpha k - 1))dt \end{bmatrix}
$$

Next state (x_k) | Previous state (x_{k-1}) | Input vector (u_k)

Extended Kalman filter

- Prediction

$$x_k = f(x_{k-1}, u_k)$$

$$P_k = A_k P_{k-1} A_{k-1}^T + Q_k$$

$$A_k = \left.\frac{\partial f}{\partial x}\right|_x$$

- Measurement

$$v_k = y_k - h(x_k)$$

$$K_k = \frac{P_{k-1} H_k^T}{S_k}$$

$$S_k = H_k P_{k-1} H_k^T + R_k$$

$$H_k = \left.\frac{\partial h}{\partial x}\right|_x$$

$$x_k = x_{k-1} + K_k v_k$$

$$P_k = P_{k-1} - K_k S_k K_k^T$$

Linearization

$$A_k = \frac{\partial f}{\partial x}\bigg|_x = \begin{bmatrix} 1 & 0 & 0 & dt & 0 \\ 0 & 1 & 0 & 0 & dt \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -dt(\ddot{x}_k sin(\alpha_{k-1}) + \ddot{y}_k cos(\alpha_{k-1})) & 1 & 0 \\ 0 & 0 & dt(\ddot{x}_k cos(\alpha_{k-1}) - \ddot{y}_k sin(\alpha_{k-1})) & 0 & 1 \end{bmatrix}$$

$$B_k = \frac{\partial f}{\partial u}\bigg|_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & dt \\ dtcos(\alpha_{k-1}) & -dtsin(\alpha_{k-1}) & 0 \\ dtsin(\alpha_{k-1}) & dtcos(\alpha_{k-1}) & 0 \end{bmatrix}$$

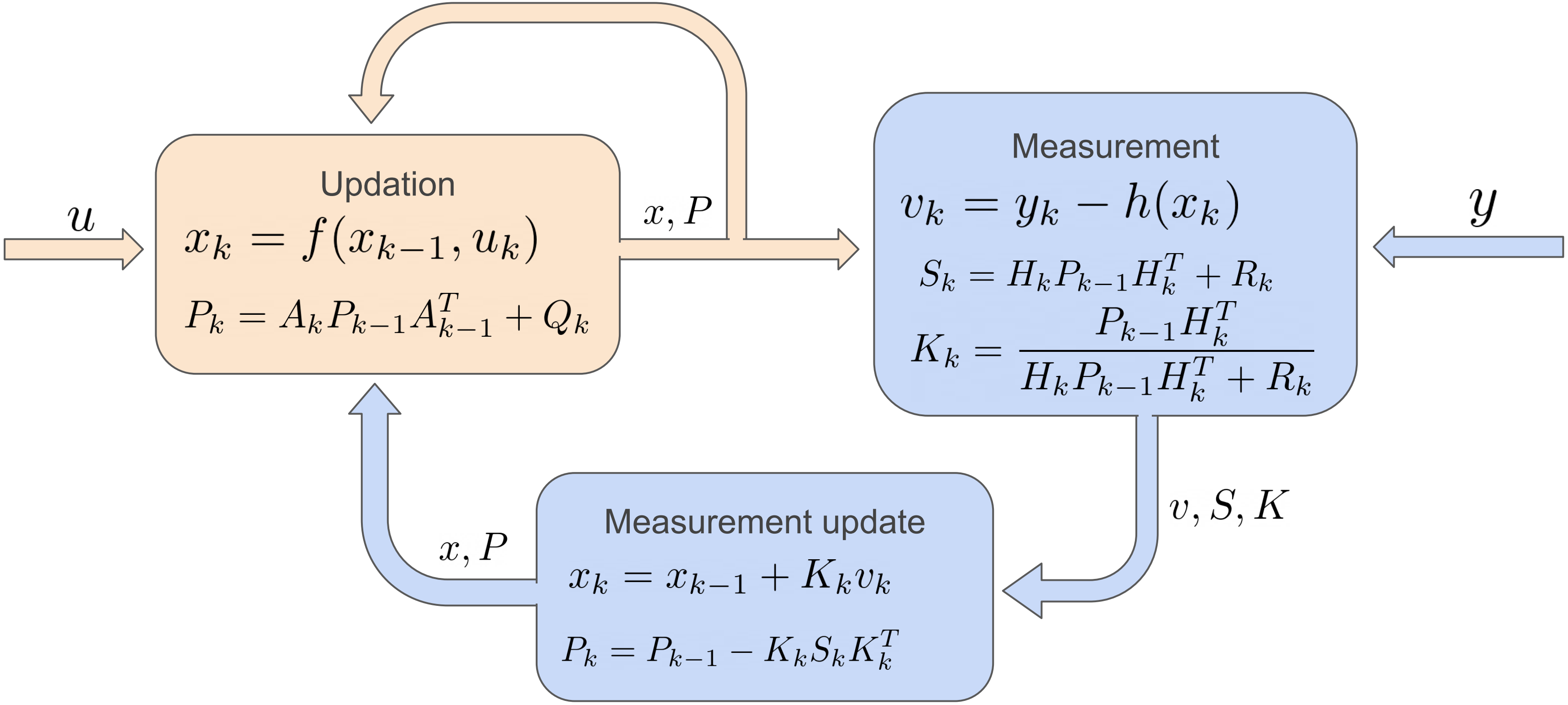
Measurement model

- The measurements are direct. We get global x and y coordinates from GNSS system

$$y_k = \begin{bmatrix} x_{meas} \\ y_{meas} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \alpha_k \\ \dot{x}_k \\ \dot{y}_k \end{bmatrix}$$

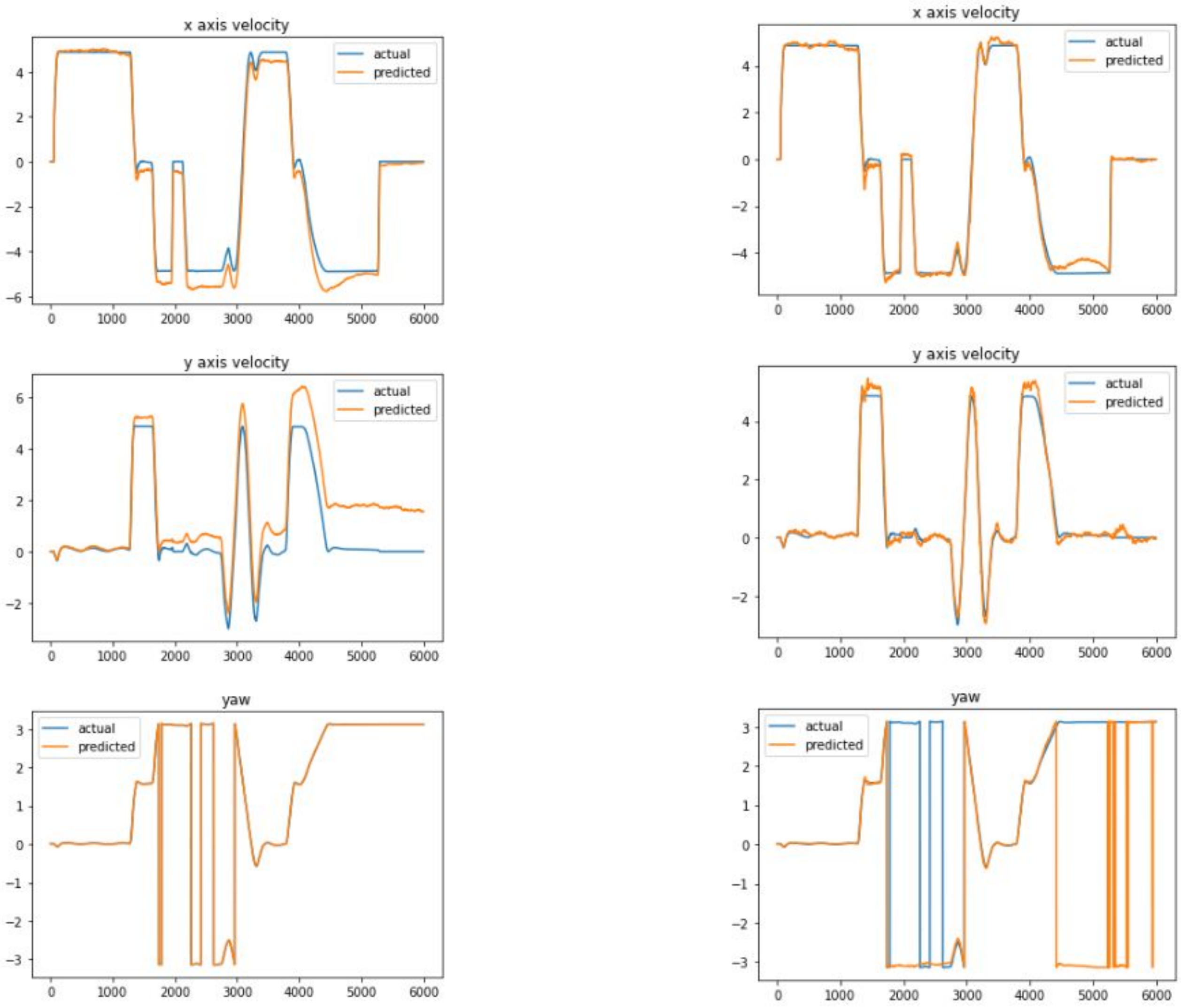
$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Extended kalman filter flowchart



Implementation and simulation

- Implemented in **python**, and jupyter notebooks
- Used **numpy** library for linear algebra
- Used open-source self-driving simulator, **Carla** for simulation
- Code is available at : https://github.com/Ashwin-Rajesh/Kalman_filter_carla
- GNSS system with
  - 3.33 standard deviation
  - 5Hz sampling rate
- IMU with
  - 0.1 standard deviation
  - 60Hz sampling rate


x axis velocity
actual
predicted
y axis velocity
actual
predicted
yaw
actual
predicted
x axis velocity
actual
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y axis velocity
actual
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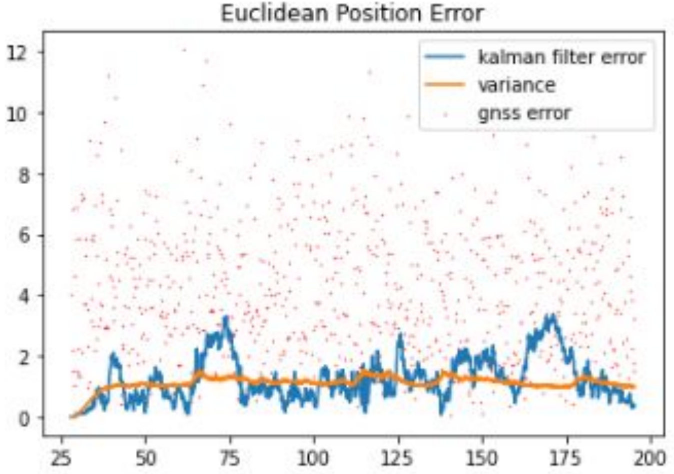
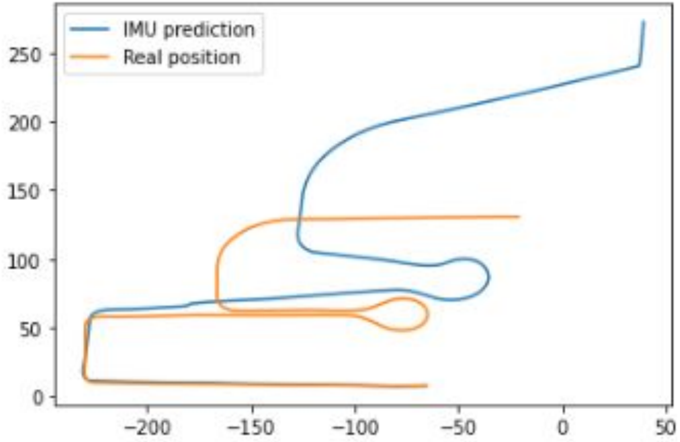
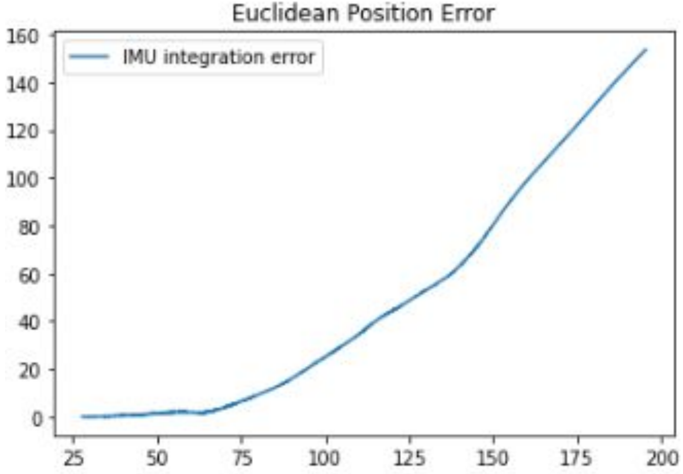
yaw
actual
predicted



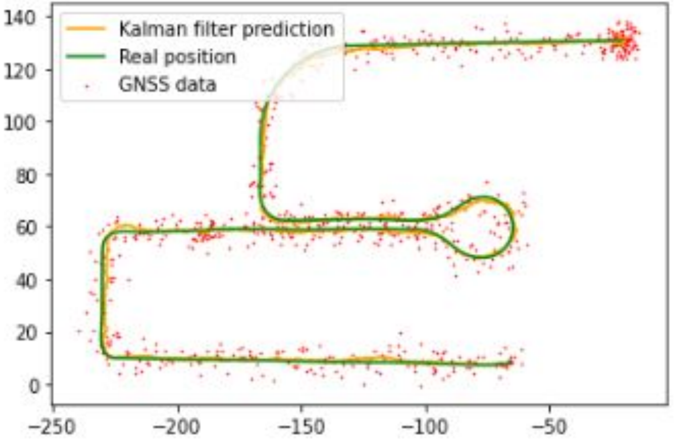
Euclidean Position Error
kalman filter error
variance
gnss error



IMU prediction
Real position



Euclidean Position Error
IMU integration error



Kalman filter prediction
Real position
GNSS data

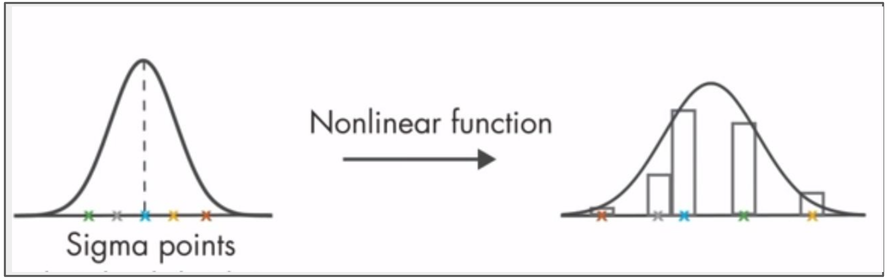
Improvements

- More sensors can be added to improve estimate, including
  - Depth cameras for visual odometry
  - Sensors reading values from the drivetrain and steering systems of the vehicle
  - LIDAR data

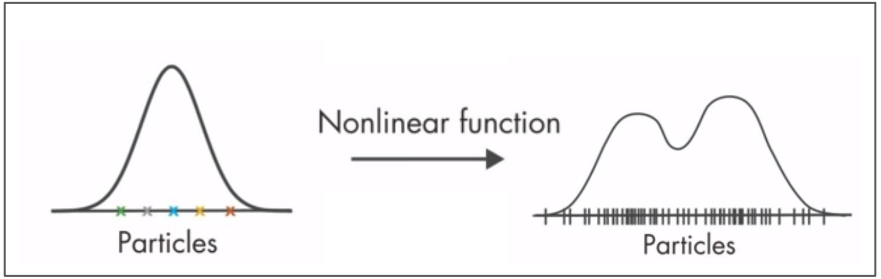
- LIDAR and depth data can be used in particle filters for localization. This is used in SLAM (simultaneous localization and mapping) systems.

Improvements

- Kalman filters are the optimal state estimators in the ideal world (linear systems, gaussian noise). But, for non-linear systems and outliers, they have terrible performance
- **Unscented kalman filters** work better for highly non-linear systems
- **Particle filters** work by generating random "particles" sampled from an initial distribution and passing each through the non-linear system model and getting the distribution for the next step.



Unscented kalman filter [4]

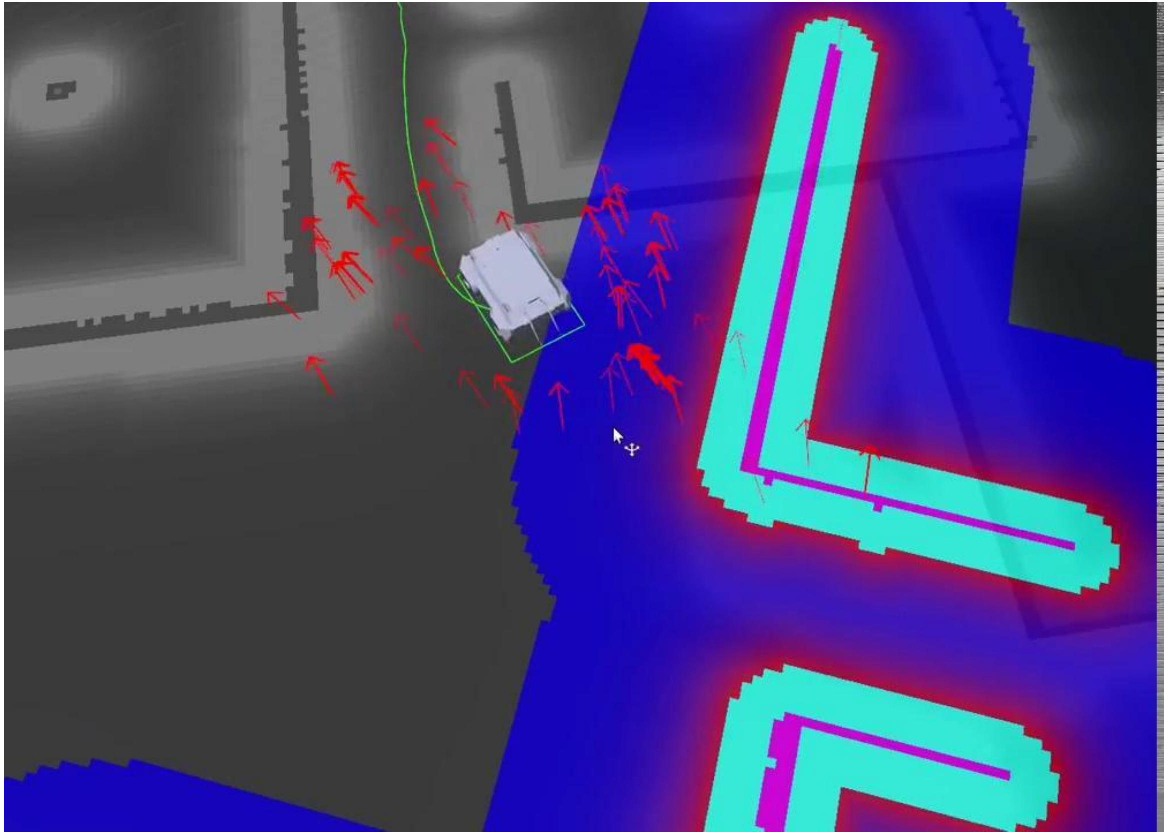


Particle filter [4]

Estimators compared

Filter	Model	Assumed distribution	Computational cost
Kalman filter	Linear	Gaussian	Low
Extended kalman filter	Linearized	Gaussian	Low - Medium
Unscented kalman filter	Non-linear	Gaussian	Medium
Particle filter	Non-linear	Non-gaussian	High

Particle filter localization in ROS - amcl



References and related work

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Thank You!