The Binomial Distribution

- A Binomial Random Variable
  - n identical trials
  - Two outcomes: **S**uccess or **F**ailure
  - P(**S**) = p; P(**F**) = $q = 1 - p$
  - Trials are independent
  - x is the number of **S**uccesses in n trials

The Binomial Distribution-Example

- A Binomial Random Variable
  - *n* identical trials

    Flip a coin 3 times
  - Two outcomes: **S**uccess or **F**ailure

    Outcomes are Heads or Tails
  - P(**S**) = *p*; P(**F**) = *q* = 1 – *p*

    P(H) = .5; P(F) = 1-.5 = .5
  - Trials are independent
  - *x* is the number of **S**'s in *n* trials

The Binomial Distribution

- The Binomial Probability Distribution

  p = P(**S**) on a single trial

  $q = 1 – p$

  n = number of trials

  x = number of successes

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

The Binomial Distribution

- The Binomial Probability Distribution

The number of ways of getting the desired results

The probability of getting the required number of successes

The probability of getting the required number of failures

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

The Binomial Distribution-Example

- Say 40% of the class is female.
- What is the probability that 6 of the first 10 students walking in will be female?

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

$$= \binom{10}{6} (.4^6)(.6^{10-6})$$

$$= 210(.004096)(.1296)$$

$$= .1115$$

The Binomial Distribution

- A Binomial Random Variable has

Mean	$\mu = np$
Variance	$\sigma^2 = npq$
Standard Deviation	$\sigma = \sqrt{npq}$

Example 1

What is the probability of getting 3 heads out of 6 trials in a coin tossing experiment

```
p <- 1/2
choose(6,3)*(p^3)*((1-p)^(6-3))
[1] 0.3125
```

Alternatively, we could use the *dbinom* function as follows:

```
dbinom(3, 6, 1/2)
[1] 0.3125
```

R- Binomial Distribution

R has four in-built functions to generate binomial distribution.

rbinom(n, x, p)
dbinom(v, x, p)
pbinom(v, x, p)
qbinom(u, x, p)

Following is the description of the parameters used −

- **v** is a vector of numbers.
- **u** is a vector of probabilities.
- **n** is number of observations.
- **x** is the number of trials.
- **p** is the probability of success of each trial.

Example 2

- Calculate the probability of getting 26 or less heads from a 51 tosses of a coin.

pbinom()

This function gives the cumulative probability of an event. It is a single value representing the probability.

```
# Probability of getting 26 or less heads from a 51 tosses of a coin.
 x <- pbinom(26,51,0.5)
print(x)
```

0.610116

qbinom()

This function takes the probability value and gives a number whose cumulative value matches the probability value.

```
# If the probability of x successes is 0.25, what is the value of x.
 x <- qbinom(0.25,51,1/2)
print(x)
```

Example 4

Find 8 random values from a sample of 150 with probability of 0.4

rbinom()

This function generates required number of random values of given probability from a given sample.

```
# Find 8 random values from a sample of 150 with probability of 0.4.

x <- rbinom(8,150,.4)
print(x)
```

Example 5

- Hospital records show that, out of the patients suffering from a certain disease, 75% die of it. What is the probability that 6 randomly selected patients, 4 will recover?

Example 5

Hospital records show that, out of the patients suffering from a certain disease, 75% die of it. What is the probability that 6 randomly selected patients, 4 will recover?

dbinom(4, 6, .25)

0.03295898

Example 6

- A professional shooter finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of

- (a) more than 2 hits?

- (b) at least 3 misses?

Example 6

- a) 1-pbinom(2,4,4/5)

- b) 3 misses means 1 hit,

  and 4 misses means 0 hits.

  pbinom(1,4,4/5)

  0.0272

The Poisson Distribution

- Evaluates the probability of a (usually small) number of occurrences out of many opportunities in a …
  - Period of time
  - Area
  - Volume
  - Weight
  - Distance
  - Other units of measurement

The Poisson Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- lambda = mean number of occurrences in the given unit of time, area, volume, etc.
- e = 2.71828….

The Poisson Distribution

- Say in a given stream there are an average of 3 Blackfin Tuna fishes per 100 yards. What is the probability of seeing 5 Blackfin Tuna fishes in the next 100 yards, assuming a Poisson distribution?

$$P(x=5)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{3^5 e^{-3}}{5!}=.1008$$

The Poisson Distribution

- How about in the next 50 yards, assuming a Poisson distribution?
  - Since the distance is only half as long, lambda is only half as large.

$$P(x=5)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{1.5^5 e^{-1.5}}{5!}=.0141$$

Probability Calculation for poison distribution

```
> dpois(5, lambda=3)
```

Uniform Distributions:

- A uniform distribution is used to describe equiprobable outcomes over a closed interval [a,b]. Because each outcome has the same probability, the R function runif is quite useful for creating a sample of random variables between some minimum and maximum value. For example, we could use the following R code to create a sample of 20 random variables in the interval between 1 and 10:
- > runif(20,1,10)
- [1] 6.734666 9.334942 4.718385 9.982222 1.912516 1.865814 2.001767 8.011947 1.643035 8.711432
- [11] 3.584781 7.679167 3.254184 1.273075 9.030344 6.860417 6.125817 4.510118 3.724532 7.548615

Uniform Distributions:

- R also provides probability density, cumulative probability, and quantile functions for the uniform probability distribution (dunif, punif, and qunif respectively).

```
hist(round(runif(2000,1,10)))
```