Question 1. Marks: 0.0/5.5

Which of the following is/are true for a sequence a_n of real numbers?

- [x] if $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{2n+1} = L$
- [ ] if $a_n > 0$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then $\lim_{n\to\infty} a_n = 0$
- [x] if a_n is decreasing, and $a_n > 0$ for all n, then a_n is convergent
- [ ] if a_n, b_n are both not convergent, then $a_n + b_n$ is not convergent

Question 2. Marks: 5.5/5.5

A function $f : [0,1] \to \mathbb{R}$ is continuous and assumes only rational values. Then which of the following is/are true?

- [x] f is differentiable on $(0, 1)$
- [ ] f is strictly monotonic on $[0, 1]$
- [ ] f is not differentiable at any point of $(0, 1)$ but 0 and 1 are local extrema of f
- [ ] f is differentiable on $(0, 1)$ but no point of $[0, 1]$ is a local extremum of f

Question 3. Marks: 5.5/5.5

Consider the function

$$f(x) = \begin{cases} 0, \text{if } x = 0 \text{ or } x \text{ is irrational} \\ \frac{1}{q^3}, \text{if } x = \frac{p}{q}, \text{ where } p \in \mathbb{Z}, q \in \mathbb{N}, \, gcd(p, q) = 1 \end{cases}$$

.

[Here $\mathbb{Z} =$ the set of all integers, and $\mathbb{N} =$ the set of all natural numbers.]

Then choose the correct option(s).

- f is continuous at 0 but is not differentiable at 0
- f is differentiable at 0 and $f'(0) = 0$
- f is not differentiable at any point in $\mathbb{R}$
- f is not continuous at 0 but the limit $\lim_{x \to 0} f(x)$ exists

Question 4. Marks: 5.5/5.5

Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Which of the following is/are true?

- ( ) if $f'(x) = 0$, then $(x, f(x))$ is an inflection point of the curve $y = f(x)$
- ( ) there exists a function f such that $f(5) = 7, f(0) = 0$ and $f'(x) > 2$ for all x
- (•) there exists a function f such that $f(x) > 0, f'(x) < 0$ and $f''(x) > 0$ for all x
- ( ) if f, g are increasing on some interval I, then fg is increasing on the interval I

Question 5. Marks: 0.0/0.0

If $x \cos(\pi x) = \int_0^{x^4} f(t)dt$, where f is a continuous function, then find $f(1)$.

-0.25

Question 6. Marks: 0.0/5.5

Consider the function

$$f(x) = \begin{cases} \frac{1}{x}, x \neq 0 \\ 0, x = 0 \end{cases}.$$

Then, choose the correct option(s).

- [x] f is not Riemann integrable on $[0, 1]$
- [x] f is Riemann integrable on $[1, b]$ for any $b > 1$
- [ ] f is not Riemann integrable on $[a, b]$ and is integrable on a closed interval $[a, b]$ only if $[a, b] \cap [-1, 1] = \emptyset$
- [ ] f is Riemann integrable on a closed interval $[a, b]$ if and only if $0 \notin [a, b]$

Question 7. Marks: 5.5/5.5

The area of the region common to the insides (i.e., interiors) of the circles $r = \sqrt{3}\cos\theta$ and $r = \sin\theta$ is

- (selected) $\dfrac{5\pi - 6\sqrt{3}}{24}$
- $\dfrac{5\pi + 6\sqrt{3}}{24}$
- $\dfrac{5\pi - 3\sqrt{3}}{24}$
- $\dfrac{5\pi + 3\sqrt{3}}{24}$

Question 8. Marks: 5.5/5.5

Consider $\lim_{(x,y)\to(1,0)} \frac{2(x-1)y^2}{x^2-2x+1-y^4}$. Which is true?

- ○ the above limit equals 1
- ○ the above limit equals 2
- ◉ the above limit does not exist
- ○ the above limit equals $-\frac{8}{3}$

Question 9. Marks: 5.5/5.5

Consider the function

$$f(x,y)=\begin{cases}\frac{y^3}{x^2+y^2}, (x,y)\neq(0,0)\\ 0, (x,y)=(0,0)\end{cases}.$$

Then choose the correct option(s).

- [ ] $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist and are continuous at $(0,0)$
- [x] $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist but they are not continuous at $(0,0)$
- [x] f is not differentiable at $(0,0)$
- [ ] $(0,0)$ is a critical point of f but not a saddle point

Question 10. Marks: 5.5/5.5

Consider a sphere of radius r . The area of the part of the sphere seen by an observer at a height h above the north pole of the sphere is

- $2\pi rh$
- $\dfrac{2\pi r^2 h}{r+h}$
- $\dfrac{\pi r^2 h}{r+h}$
- $\dfrac{2\pi r^2 h}{r+2h}$

Question 11. Marks: 5.5/5.5

Consider the function $f(x, y) = 3xy + \frac{1}{x} - \log y$ in the first quadrant $x > 0, y > 0$. Which of the following is/are true?

- f has one local minimum
- f has one local maximum and one local minimum
- f has one saddle point
- f has one local maximum and one saddle point