MA105 - Quiz 1

Div/Tut batch: D _ _ / T _ _ Date: 18/08/2018.

Grading policy

These decisions are arrived at after a discussion among all the three instructors. These decisions are valid only for this examination.

This time we have allowed the answer in decimal expansion as well as in the quotient format.

This decision was taken because the last question for each of the four codes contained decimal expansions. It was decided that the correctness of the decimal expansion will be checked for two decimal places.

Thus, for example, if the correct answer is $1/3$ then we allow 0.33 or 0.333 but not 0.3. Of course, 0.4 is accepted for the answer 0.40.

(1) The statements given were false, for all the four codes.

The correct option, that is FALSE, with a valid counter example gets 2 marks. Only a speculation about the statement being false gets no marks.

If the correct option, that is FALSE, is chosen but no justification is given then again the answer gets no marks.

If a valid counter example is provided but the correct option is not chosen then also the answer gets no marks.

(2) The answer in each of the four codes was a single point.

We have not been fussy about the set notation, if the correct answer is $\{x_0\}$ then simply writing the point x_0 also gets mark.

If the set given by you does not match with the answer, that is a set containing only one point, then the answer gets no marks.

(3) - (5) One again, only the correct answer gets 2 marks.

MA105 - Quiz 1

Div/Tut batch: D _ _ / T _ _ CODE A Date: 18/08/2018.

(1) Let X be a non-empty subset of $\mathbb{R}$ which is bounded above. If $\{x_n\}$ is an increasing sequence of elements in X converging to $L \in \mathbb{R}$ then L is the least upper bound of X.

This is ~~TRUE~~ / **FALSE** because

FOR EXAMPLE $x_n = 1 - 1/n$ gives a counter example in the set $X = [0, 5]$.

(2) Let $g : (0,1) \to \mathbb{R}$ be as given below. Find all points $x \in (0,1)$ where g is continuous:

$$g(x) = \begin{cases} 1 - 5x/4 & x \text{ is rational,} \\ 5x/4 & x \text{ is irrational.} \end{cases}$$

$\{2/5\} = \{0.4\}$

(3) Compute the derivative of the following function at $x = 0$:

$$f(x) = \begin{cases} x^3 \cos(1/x^2) + 5x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

5.

(4) The number of real solutions of $x^3 - 3x^2 + 2 = 0$ is

3.

(5) Consider the interval $[0, 4]$ and the partition $\mathcal{P}$ given by $x_0 = 0$, $x_1 = 1$, $x_2 = 1.5$, $x_3 = 2.25$ and $x_4 = 4$. Let $t_i \in [x_i, x_{i+1}]$ be the mid-point of each interval $[x_i, x_{i+1}]$. If $f(x) = x^2$ then

$$S(f, \dot{\mathcal{P}}) = \frac{2657}{128} = 20.7578125 \approx 20.75 \approx 20.76.$$

MA105 - Quiz 1

Div/Tut batch: D _ _ / T _ _ CODE B Date: 18/08/2018.

(1) Let X be a non-empty subset of $\mathbb{R}$ which is bounded below. If $\{x_n\}$ is a decreasing sequence of elements in X converging to $L \in \mathbb{R}$ then L is the greatest lower bound of X.

This is ~~TRUE~~ / **FALSE** because

FOR EXAMPLE $x_n = 1/n$ gives a counter example in the set $X = [-5, 5]$.

(2) Let $g : (0, 1) \to \mathbb{R}$ be as given below. Find all points $x \in (0, 1)$ where g is continuous:

$$g(x) = \begin{cases} 1 - 3x/2 & x \text{ is rational,} \\ 3x/2 & x \text{ is irrational.} \end{cases}$$

$\{1/3\} = \{0.33\}$.

(3) Compute the derivative of the following function at $x = 0$:

$$f(x) = \begin{cases} x^3 \cos(1/x) + 7x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

7.

(4) The number of real solutions of $x^3 + 4x^2 - 5 = 0$ is

3.

(5) Consider the interval $[0, 4]$ and the partition $\mathcal{P}$ given by $x_0 = 0$, $x_1 = 1$, $x_2 = 1.5$, $x_3 = 2.25$ and $x_4 = 4$. Let $t_i \in [x_i, x_{i+1}]$ be the mid-point of each interval $[x_i, x_{i+1}]$. If $f(x) = 3x^2$ then

$$S(f, \dot{\mathcal{P}}) = \frac{7971}{128} = 62.2734375 \approx 62.27.$$

MA105 - Quiz 1

Div/Tut batch: D _ _ / T _ _ CODE C Date: 18/08/2018.

(1) Let X be a non-empty subset of $\mathbb{R}$ which is bounded below. Let $\{x_n\}$ be a decreasing sequence of elements in X converging to $L \in \mathbb{R}$. If $L \notin X$ then L is the greatest lower bound of X.

This is ~~TRUE~~ / **FALSE** because

FOR EXAMPLE $x_n = 1/n$ in the set $X = (-1, 0) \cup (0, 1)$ gives a counter example.

(2) Let $g : (0, 1) \to \mathbb{R}$ be as given below. Find all points $x \in (0, 1)$ where g is continuous:

$$g(x) = \begin{cases} 1 - 4x/5 & x \text{ is rational,} \\ 4x/5 & x \text{ is irrational.} \end{cases}$$

$\{5/8\} = \{0.625\} = \{0.62\} = \{0.63\}.$

(3) Compute the derivative of the following function at $x = 0$:

$$f(x) = \begin{cases} x^3 \cos(1/x^2) - 3x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$-3.$

(4) The number of real solutions of $x^3 + 3x^2 - 2 = 0$ is

3.

(5) Consider the interval $[0, 4]$ and the partition $\mathcal{P}$ given by $x_0 = 0$, $x_1 = 1$, $x_2 = 1.5$, $x_3 = 2.25$ and $x_4 = 4$. Let $t_i \in [x_i, x_{i+1}]$ be the mid-point of each interval $[x_i, x_{i+1}]$. If $f(x) = -2x^2$ then

$$S(f, \dot{\mathcal{P}}) = -\frac{2657}{64} = -41.515625 \approx -41.51 \approx -41.52.$$

MA105 - Quiz 1

Div/Tut batch: D _ _ / T _ _ CODE D Date: 18/08/2018.

(1) Let X be a non-empty subset of $\mathbb{R}$ which is bounded above. Let $\{x_n\}$ be an increasing sequence of elements in X converging to $L \in \mathbb{R}$. If $L \notin X$ then L is the least upper bound of X.

This is ~~TRUE~~ / **FALSE** because

FOR EXAMPLE $x_n = 1 - 1/n$ in the set $(0,1) \cup (1,2)$ gives a counter example.

(2) Let $g : (0,1) \to \mathbb{R}$ be as given below. Find all points $x \in (0,1)$ where g is continuous:

$$g(x) = \begin{cases} 1 - 2x/3 & x \text{ is rational,} \\ 2x/3 & x \text{ is irrational.} \end{cases}$$

$\{3/4\} = \{0.75\}$.

(3) Compute the derivative of the following function at $x = 0$:

$$f(x) = \begin{cases} x^3 \cos(1/x) - 2x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

-2.

(4) The number of real solutions of $x^3 - 4x^2 + 5 = 0$ is

3.

(5) Consider the interval $[0,4]$ and the partition $\mathcal{P}$ given by $x_0 = 0$, $x_1 = 1$, $x_2 = 1.5$, $x_3 = 2.25$ and $x_4 = 4$. Let $t_i \in [x_i, x_{i+1}]$ be the mid-point of each interval $[x_i, x_{i+1}]$. If $f(x) = -x^2$ then

$$S(f, \dot{\mathcal{P}}) = -\frac{2657}{128} = -20.7578125 \approx -20.75 \approx -20.76.$$