MA105 - Quiz 2'

Div/Tut batch: D _ _ / T _ _ CODE **A** Date: 12/11/2018.

Name: _ Roll Number: _ _ _ _ _ _ _ _ _ _ _ _

(1) Define $f(0,0)$ (without explanation) in a way that makes f continuous in a neighborhood of $(0,0)$. Here

$$f(x,y) = \log\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right) \qquad \text{for } (x,y) \neq (0,0).$$

Answer: $f(0,0) = \log 3 = 0.477$.

(2) Find the directional derivative $D_{\mathbf{u}}f$ at $(0,0)$ along $\mathbf{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ of the function

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^2+y^2}, & \text{if } (x,y) \neq 0 \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Answer: $D_{\mathbf{u}}f(0,0) = \dfrac{1}{2\sqrt{2}} = 0.35355$.

(3) Find the maximum value of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

Answer: 30.

(4) Let W be the subset of $\mathbb{R}^3$ bounded by the three planes $x = 0$, $y = 0$, $z = 2$, the surface $z = x^2 + y^2$, and lying in the quadrant $x \geq 0$, $y \geq 0$. Then

$$\iiint_W x\,dx\,dy\,dz = \frac{8\sqrt{2}}{15} = 0.7542.$$

(5) The volume of the region below the plane $z + x = 1$ and inside the cylinder $x^2 + y^2 \leq 1$,

$0 \leq z \leq 1$, is $\pi - \dfrac{2}{3} = 2.4749$.

MA105 - Quiz 2'

Div/Tut batch: D _ _ / T _ _ CODE **B** Date: 12/11/2018.

Name: _ Roll Number: _ _ _ _ _ _ _ _ _ _ _ _

(1) Define $f(0,0)$ (without explanation) in a way that makes f continuous in a neighborhood of $(0,0)$. Here

$$f(x,y) = \log\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right) \qquad \text{for } (x,y) \neq (0,0).$$

Answer: $f(0,0) = \log 3 = 0.477$.

(2) Find the directional derivative $D_{\mathbf{u}}f$ at $(0,0)$ along $\mathbf{u} = (\frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ of the function

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^2+y^2}, & \text{if } (x,y) \neq 0 \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Answer: $D_{\mathbf{u}}f(0,0) = \dfrac{2}{3\sqrt{3}} = 0.3849$.

(3) Find the minimum value of $f(x,y) = x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 = 4y$.

Answer: 0.

(4) Let W be the subset of $\mathbb{R}^3$ bounded by the three planes $x = 0$, $y = 0$, $z = 2$, the surface $z = x^2 + y^2$, and lying in the quadrant $x \geq 0$, $y \geq 0$. Then

$$\iiint_W y\,dxdydz = \frac{8\sqrt{2}}{15} = 0.7542.$$

(5) The volume of the region below the plane $z + y = 1$ and inside the cylinder $x^2 + y^2 \leq 1$,

$0 \leq z \leq 1$, is $\pi - \dfrac{2}{3} = 2.4749$.

MA105 - Quiz 2'

Div/Tut batch: D _ _ / T _ _ CODE **C** Date: 12/11/2018.

Name: _ Roll Number: _ _ _ _ _ _ _ _ _ _ _ _

(1) Define $f(0,0)$ (without explanation) in a way that makes f continuous in a neighborhood of $(0,0)$. Here

$$f(x,y) = \log\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right) \qquad \text{for } (x,y) \neq (0,0).$$

Answer: $f(0,0) = \log 3 = 0.477$.

(2) Find the directional derivative $D_{\mathbf{u}}f$ at $(0,0)$ along $\mathbf{u} = (\frac{\sqrt{3}}{2}, \frac{1}{2})$ of the function

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^2+y^2}, & \text{if } (x,y) \neq 0 \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Answer: $D_{\mathbf{u}}f(0,0) = \dfrac{3}{8} = 0.375$.

(3) Find the maximum value of $f(x,y) = x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 = 4y$.

Answer: 20.

(4) Let W be the subset of $\mathbb{R}^3$ bounded by the three planes $x = 0$, $y = 0$, $z = 2$, the surface $z = x^2 + y^2$, and lying in the quadrant $x \geq 0$, $y \geq 0$. Then

$$\iiint_W x\,dx\,dy\,dz = \frac{8\sqrt{2}}{15} = 0.7542.$$

(5) The volume of the region below the plane $z + x = 1$ and inside the cylinder $x^2 + y^2 \leq 1$,

$0 \leq z \leq 1$, is $\pi - \dfrac{2}{3} = 2.4749$.

MA105 - Quiz 2'

Div/Tut batch: D _ _ / T _ _ CODE **D** Date: 12/11/2018.

Name: _ Roll Number: _ _ _ _ _ _ _ _ _ _ _ _

(1) Define $f(0,0)$ (without explanation) in a way that makes f continuous in a neighborhood of $(0,0)$. Here

$$f(x,y) = \log\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right) \qquad \text{for } (x,y) \neq (0,0).$$

Answer: $f(0,0) = \log 3 = 0.477$.

(2) Find the directional derivative $D_{\mathbf{u}}f$ at $(0,0)$ along $\mathbf{u} = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$ of the function

$$f(x,y) = \begin{cases} \dfrac{x^2y}{x^2+y^2}, & \text{if } (x,y) \neq 0 \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Answer: $D_{\mathbf{u}}f(0,0) = \dfrac{4}{5\sqrt{5}} = 0.35777$.

(3) Find the minimum value of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2+y^2+z^2 = 30$.

Answer: -30.

(4) Let W be the subset of $\mathbb{R}^3$ bounded by the three planes $x=0$, $y=0$, $z=2$, the surface $z = x^2+y^2$, and lying in the quadrant $x \geq 0$, $y \geq 0$. Then

$$\iiint_W y\,dx\,dy\,dz = \frac{8\sqrt{2}}{15} = 0.7542.$$

(5) The volume of the region below the plane $z+y=1$ and inside the cylinder $x^2+y^2 \leq 1$,

$0 \leq z \leq 1$, is $\pi - \dfrac{2}{3} = 2.4749$.