MA109: Main Quiz (30 marks)

08:15 - 09:15, Friday, December 18

Read these instructions carefully before proceeding.

1. All answers must be supported by proper reasoning. Otherwise, they may not be awarded marks. In "True or False" questions, you must give adequate justification if your answer is "True", and provide a counter-example if your answer is "False".
2. **Very important:** Let a denote the last digit of your roll number and let b be the second last digit of your roll number. Let $A = 10 - a$ and $B = 10 - b$. Note that $0 \leq a, b \leq 9$ and $1 \leq A, B \leq 10$. Record your values for a, A, b, B below. You must use these values of a, b, A, B in your quiz. If you use any other values, you will be immediately awarded 0 marks for that question.

Q. *(6 marks) First declare your values of a, A, b and B. Using the $\epsilon - \delta$ definition of the limit of a function, determine whether the following function is continuous at $x = 0$.*

$$f(x) = \begin{cases} A & \text{if } x \leq 0 \\ x & \text{if } x > 0. \end{cases}$$

First write down the function with your value of A and then proceed.

Q. *(4 marks) First declare your values of a, A, b and B. True or False (if true give reasons, if false give an explicit counter-example): Let $f(x)$ be a twice differentiable function on $(0, 2B)$. If $f''(x) > 0$ on $(0, B)$ and $f''(B) = 0$, then B must be an inflection point for the curve $y = f(x)$.*

Q. *(4 marks) First declare your values of a, A, b and B. Let $f(x) = x^3 + Ax + B$. Show that $f(x)$ has exactly one real root. First write down the function with your values of A and B and then proceed.*

Q. *(4 marks) First declare your values of a, A, b and B. Let (x_n) be a convergent sequence of non-negative real numbers. Let*

$$x = \lim_{n \to \infty} b + x_n.$$

Prove (using the $\epsilon - N$ definition of limits) that $x \geq b$.

Q. *(6 marks) First declare your values of a, A, b and B. Let*

$$S_n = \frac{1}{n}\sum_{k=1}^{n} \cos\left(\frac{(a+1)k\pi}{(a+2)n}\right).$$

Evaluate $\lim_{n\to\infty} S_n$ by identifying it as a Riemann sum for a certain continuous function on a certain interval, and with respect to a certain (tagged/marked) partition. First write down S_n with your value of a and then proceed. You must explicitly give the continuous function, the interval and the tagged/marked partition that you are using. You must then justify all further steps.

Q. *(6 marks) First declare your values of a, A, b and B. Let $C = \frac{B}{B+1}$. Write down C (as a fraction) for your value of B. Let $f(x) = \cos x$ on the interval $[0, \pi]$. Let $P_3(x)$ be the Taylor polynomial of $f(x)$ of degree* 3 **around the point** *$\pi/2$.*

Write down the Taylor polynomial $P_3(x)$ and state if the following statement is true or false:

$$|f(x) - P_3(x)| < \frac{2C}{3}$$

for all $x \in [\pi/2, \pi]$. Justify your answer. You may use the fact that $\pi/2 < 1.6$.