Question 1. Marks: 9.0/9.0

Let $f(x) = \cos\frac{1}{x}$. Then,

- $\lim_{x \to 0} f(x)$ does not exist
- $\lim_{x \to 0} f(x)$ exists and is equal to $0$
- $\lim_{x \to 0} f(x)$ exists and is equal to $1$
- $\lim_{x \to 0} f(x)$ exists and is equal to $\frac{1}{\sqrt{2}}$.

Question 2. Marks: 7.0/7.0

Consider the sequence

$$x_n = \frac{a^{2021}}{\sqrt{n^2+1}} + \frac{a^{2021}}{\sqrt{n^2+2}} + \ldots + \frac{a^{2021}}{\sqrt{n^2+n}},$$

where $a$ is a real number. Then, which of the following options are true?

- [ ] $x_n$ is divergent for every $a > 0$.
- [x] $x_n$ is convergent if $a \geq 1$
- [x] $x_n$ converges if $0 < a < 1$
- [x] $x_n$ converges if $a > 1$ and $\lim_{n\to\infty} x_n = a^{2021}$.

Question 3. Marks: 8.0/8.0

If $f(x) = \begin{cases} \frac{1-\cos mx}{x \sin x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

is continuous at $x = 0$, then the value of $m$ is equal to

- ○ $0$
- ○ $-\frac{1}{2}$
- ○ $\pm 1$
- ◉ $\pm 2$

Question 4. Marks: 7.0/7.0

Consider the following statement: for a function $f : \mathbb{R} \to \mathbb{R}$, between any two consecutive local minima, there is a local maximum. This statement is

- always true
- true, if $f$ is continuous
- never true
- true, only if $f$ is a polynomial.

Question 5. Marks: 9.0/9.0

Let $a$ be some real number and $f(x) = x^3 - 27x + a$. Then, which of the following options is necessarily true?

- $f(x)$ has exactly one root in $[-2, 2]$
- $f(x)$ cannot have two distinct roots in $[-2, 2]$
- $f(x)$ has two roots in $[-2, 2]$
- $f(x)$ has no root in $[-2, 2]$.