An electron in a one-dimensional Box

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We know that, for a particle moving in one-dimension, confined to a box of length L, the Energy Levels of the stationary states of the particle are given by:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \tag{1}$$

where n is a positive Integer.

Hence the lowest Energy (the Ground State Energy) occurs when $n = 1$ and

$$\boxed{E_1 = \frac{\pi^2\hbar^2}{2mL^2}} \tag{2}$$

The second lowest energy (the energy of the particle in the first excited state) occurs when $n = 2$ and

$$E_2 = \frac{2\pi^2\hbar^2}{mL^2} \tag{3}$$

The difference in energy between these two states (i.e. the energy needed to excite the particle from its ground state to its first excited state is:

$$\boxed{\Delta E = E_2 - E_1 = \frac{3\pi^2\hbar^2}{2mL^2}} \tag{4}$$

Given that the particle is an electron, and the size of the box is $0.5\,\mathrm{nm}$, we use $m = 9.11 \times 10^{-31}\,\mathrm{kg}$ and $L = 5 \times 10^{-10}\,\mathrm{m}$, $\hbar = 1.05 \times 10^{-34}\,\mathrm{Js}$ and we get:

$$\boxed{E_1 = 2.39 \times 10^{-19}\,\mathrm{J} = 1.49\,\mathrm{eV}} \tag{5}$$

and

$$\boxed{\Delta E = 7.17 \times 10^{-19}\,\mathrm{J} = 4.48\,\mathrm{eV}} \tag{6}$$