An estimation of the minimum Kinetic Energy of a confined, non-relativistic electron

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As given in the question, the electron is confined in a region of width $L = 0.1\,\mathrm{nm}$. Hence, the uncertainty in the position of the electron would be $\Delta x = \frac{L}{2} = 0.05\,\mathrm{nm}$. Heisenberg's Uncertainty principle states that:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{1}$$

Substituting the value of Δx, and using $\hbar = 1.05 \times 10^{-34}\,\mathrm{J\,s}$, we get:

$$\Delta p \geq 1.05 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}} \tag{2}$$

Note that in the non-relativistic regime, we have

$$E = \frac{p^2}{2m} \tag{3}$$

,where E represents the Kinetic Energy of the electron, and p represents its momentum.

To use this formula for a Quantum particle, such as an electron, we must replace p^2 with its expectation value $\langle p^2 \rangle$. Note that, by definition:

$$\Delta p^2 = \sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2 \tag{4}$$

Therefore:

$$\langle p^2 \rangle = \Delta p^2 + \langle p \rangle^2 \geq \Delta p^2 \tag{5}$$

Substituting all this into the expression for Kinetic Energy, we get:

$$E = \frac{\langle p^2 \rangle}{2m} \geq \frac{\Delta p^2}{2m} \tag{6}$$

Now, using $\Delta p \geq 1.05 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}}$, and $m = 9.11 \times 10^{-31}\,\mathrm{kg}$, we get:

$$E \geq 6.05 \times 10^{-19}\,\mathrm{J} \tag{7}$$

Therefore:

$$\boxed{E_{min} = 6.05 \times 10^{-19}\,\mathrm{J} = 3.78\,\mathrm{eV}} \tag{8}$$