# Quantum Tunnelling II

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## 1 Finding the Transmission Coefficient for $\alpha L >> 1$ and $\alpha L << 1$

We know that for a Finite Step Potential of height $V_0$, the Transmission Coefficient for $E < V_0$ is given by:

$$T(E) = [1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2{(\alpha L)}]^{-1} \tag{1}$$

where

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \tag{2}$$

For $\alpha L >> 1$,

$$\sinh{(\alpha L)} = \frac{e^{\alpha L} - e^{-\alpha L}}{2} \approx \frac{e^{\alpha L}}{2} \tag{3}$$

Now we get

$$T(E) \approx [1 + \frac{V_0^2}{16E(V_0 - E)} e^{2\alpha L}]^{-1} \tag{4}$$

Since $e^{2\alpha L} >> 1$

$$T(E) \approx [\frac{V_0^2}{16E(V_0 - E)} e^{2\alpha L}]^{-1} = \frac{16E(V_0 - E)}{V_0^2} e^{-2\alpha L} \tag{5}$$

Therefore, for $\alpha L >> 1$

$$\boxed{T(E) \approx \frac{16E(V_0 - E)}{V_0^2} e^{-2\alpha L}} \tag{6}$$

Now, for $\alpha L << 1$

$$\sinh{(\alpha L)} = \frac{e^{\alpha L} - e^{-\alpha L}}{2} \approx \frac{(1 + \alpha L) - (1 - \alpha L)}{2} = \alpha L = \alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} L \tag{7}$$

Therefore,

$$T(E) \approx [1 + \frac{V_0^2}{4E(V_0 - E)} \frac{2m(V_0 - E)}{\hbar^2} L^2]^{-1} = [1 + \frac{mV_0^2 L^2}{2\hbar^2 E}]^{-1} \tag{8}$$

Let $k = \frac{2mE}{\hbar}$. Expressing $E$ in terms of $k$ ($E = \frac{\hbar^2 k^2}{2m}$), we get

$$\boxed{T(E) \approx [1 + \frac{m^2 V_0^2 L^2}{\hbar^4 k^2}]^{-1}} \tag{9}$$

## 2 Quantum Tunnelling of an electron

Say an electron with total energy $E = 6\,\text{eV}$ approaches a potential barrier with height $V_0 = 12\,\text{eV}$ and width $L = 0.18\,\text{nm}$. Let $P$ represent the probability that the electron (which classically has insufficient Energy to cross the barrier) tunnels through it.

In this case $E < V_0$, and therefore the Transmission Coefficient $T(E)$, which represents the Probability that the electron tunnels through the barrier is given by

$$T(E) = [1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(\alpha L)]^{-1} \tag{10}$$

where

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \tag{11}$$

Substituting $E = 9.6 \times 10^{-19}\,\text{J}$, $V_0 = 1.92 \times 10^{-18}\,\text{J}$, $L = 1.8 \times 10^{-10}\,\text{m}$, $m = 9.11 \times 10^{-31}\,\text{kg}$ and $\hbar = 1.05 \times 10^{-34}\,\text{Js}$, we get $\alpha L = 2.267$, and therefore

$$\boxed{P = T(E) = 0.042} \tag{12}$$