Answer to the in-video Quiz (Week 2, Lecture 1)

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1 From De Broglie's wave condition, it can be shown that the Energy levels in a Hydrogen atom are discrete (separate)

According to De Broglie, electrons also display wave properties. An electron in a Hydrogen atom can be present in a stationary state, only if the electron wave exists as a standing wave in the orbit. For this to occur, the following condition should be satisfied:

$$n\lambda = 2\pi r \tag{1}$$

Here λ represents the wavelength of the electron wave and r represents the radius of the orbit of the electron around the nucleus. According to De Broglie, the wavelength of a matter wave and its momentum are related as:

$$\lambda = \frac{h}{p} \tag{2}$$

From equations (1) and (2), we can derive Bohr's postulate that the Angular Momentum of an electron in a stationary state is quantized.

$$p = mv$$

$$\lambda = \frac{h}{mv}$$

$$\frac{nh}{mv} = 2\pi r$$

Rearranging the terms, we see that:

$$mvr = \frac{nh}{2\pi} \tag{3}$$

Now, since the electron is orbiting the nucleus in a circular orbit due the influence of the Coulombic Force between the two:

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \tag{4}$$

From (3) and (4), we can show that:

$$r = \frac{\epsilon_0 h^2}{\pi m e^2} n^2 \tag{5}$$

From (3) and (5), we can conclude that:

$$v = \frac{e^2}{2\epsilon_0 h} \frac{1}{n} \tag{6}$$

The energy of the electron is the sum of its Kinetic and Potential energies.

$$KE = \frac{1}{2} m v^2$$

$$PE = -\frac{e^2}{4\pi\epsilon_0 r}$$

From (4), we see that:

$$KE = \frac{e^2}{8\pi\epsilon_0 r}$$

And therefore,

$$E = -\frac{e^2}{8\pi\epsilon_0 r} \tag{7}$$

Now, from equations (5) and (7), it can be seen that:

$$E = -\frac{m e^4}{8\epsilon_0^2 h^2} \frac{1}{n^2} \tag{8}$$

Hence, it is seen that the Energy levels in the Hydrogen atom are discrete (separate).