STEP 2 Preparation

# Curve Sketching

Advanced Mathematics Support Programme

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# Curve Sketching : 1

The curve $C$ has equation $y = \dfrac{x}{\sqrt{x^2 - 2x + a}}$, where the square root is positive. Show that, if $a > 1$, then $C$ has exactly one stationary point.

Sketch $C$ when (i) $a = 2$ and (ii) $a = 1$.

[STEP2 1999/7]

# Curve Sketching : 2

The curve $y = \left(\dfrac{x-a}{x-b}\right)e^x$, where $a$ and $b$ are constants, has two stationary points. Show that $a - b < 0$ or $a - b > 4$.

(i) Show that, in the case $a = 0$ and $b = \frac{1}{2}$, there is one stationary point on either side of the curve's vertical asymptote, and sketch the curve.

(ii) Sketch the curve in the case $a = \frac{9}{2}$ and $b = 0$.

[STEP1 2010/2]

# Curve Sketching : 3

Given that the cubic equation $x^3 + 3ax^2 + 3bx + c = 0$ has three distinct real roots and $c < 0$, show with the help of sketches that either exactly one of the roots is positive or all three of the roots are positive.

(i) Given that the equation $x^3 + 3ax^2 + 3bx + c = 0$ has three distinct real positive roots show that $a^2 > b > 0, a < 0, c < 0$. $(*)$

[ Hint: Consider the turning points.]

(ii) Given that the equation $x^3 + 3ax^2 + 3bx + c = 0$ has three distinct real roots and that $ab < 0, c > 0$ determine, with the help of sketches, the signs of the roots.

(iii) Show by means of an explicit example (giving values for $a$, $b$ and $c$) that it is possible for the conditions $(*)$ to be satisfied even though the corresponding cubic equation has only one real root.

[STEP2 2013/3]