Experiment - 6.

Aim: To study CDF & PDF functions of different Continous Random Variables & the effect of parametric changes.

Software Used: Matlab.

Continuous Random Variable: Discrete random variable appear in experiments in which we count. Continous Random Variable appear in experiment in which we measure. By Definition, a random variable X and is distribution function can be given are of continous, if its distribution function can be given by an integral:

$$F(x) = \int_{-\infty}^{\infty} f(v)\,dv$$

Uniform Random Variable: The distribution describes an experiment where there is an arbitiary outcome, that lies b/w certain bounds. The bounds are defined by the parameters a & b, which are maximum & minimum values.

PDF $f_X(x) = \begin{cases} \frac{1}{b-a} & \text{for } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$

CDF $F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a,b] \\ 0 & x > b \end{cases}$

Mean $\bar{x} = \frac{1}{2}(a+b)$

Variance $\sigma_x^2 = \frac{1}{12}(b-a)^2$

Gaussian/Normal Random Variable:

This is the most important continous distribution because in application, many random variable are normal random variable. Furthermore, the normal distribution is a useful approximation of more complicated distribution & it also occurs in the proofs of various statistical tests.

PDF $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

CDF $F_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} d\vartheta$

Mean $\bar{x} = \mu$ **Variance** $\sigma_x^2 = \sigma^2$

Rayleigh Random Variable

The Rayleigh random variable is a continous probability distribution for non negative valued random variable. A Rayleigh Distribution is often observed when the overall magnitude of a vector is related to its directional component.

PDF: $f_X(x) = \frac{x}{\sigma^2} e^{-\left(\frac{x^2}{2\sigma^2}\right)}$

CDF: $F_X(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}$

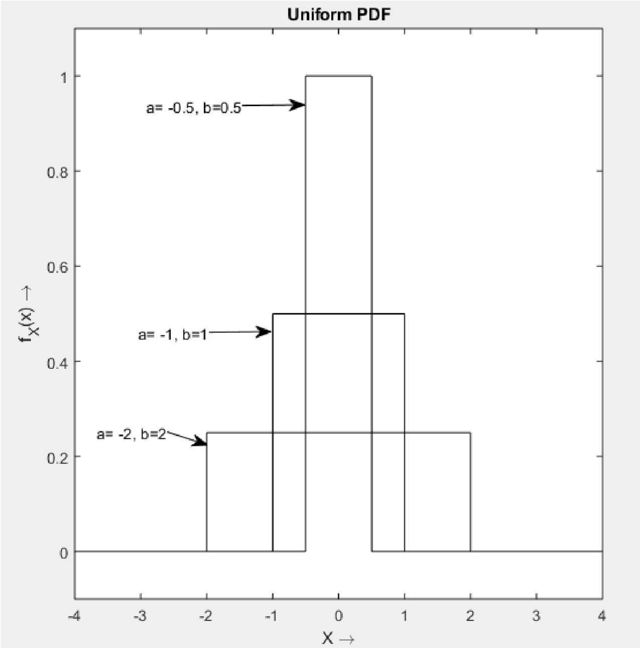
Mean $\bar{x} = \sigma\sqrt{\pi/2}$

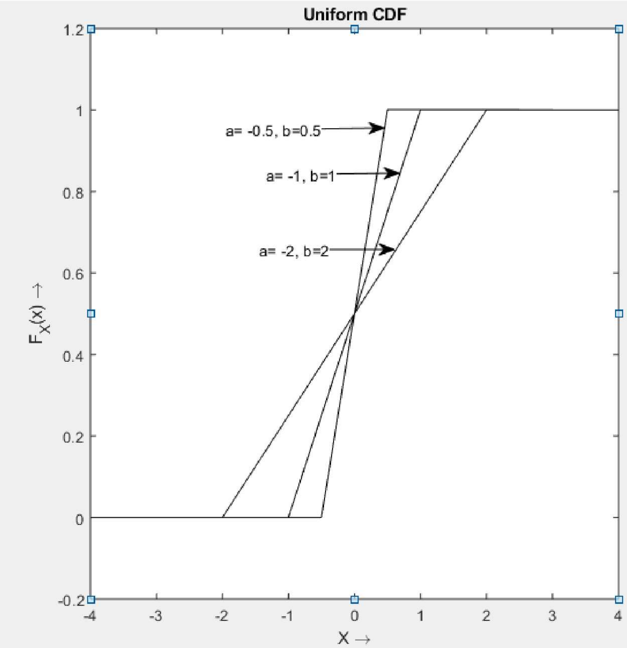
Variance: $\sigma_x^2 = \frac{4-\pi}{2}\sigma^2$

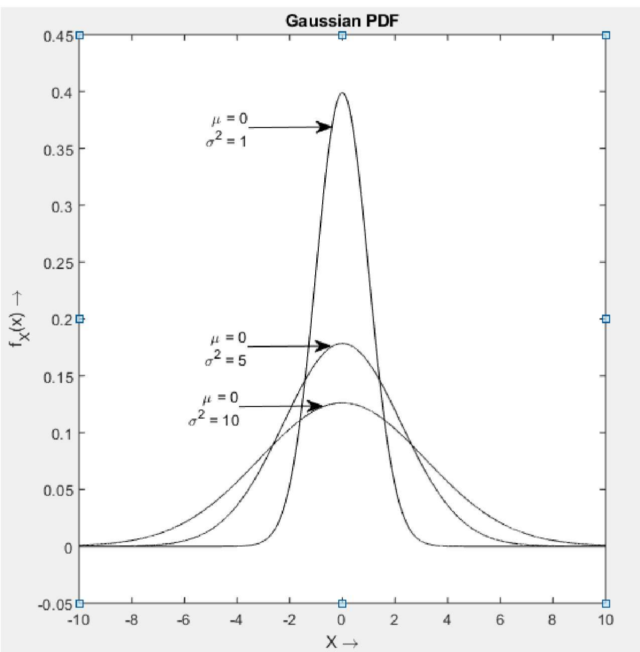
Experiment 6

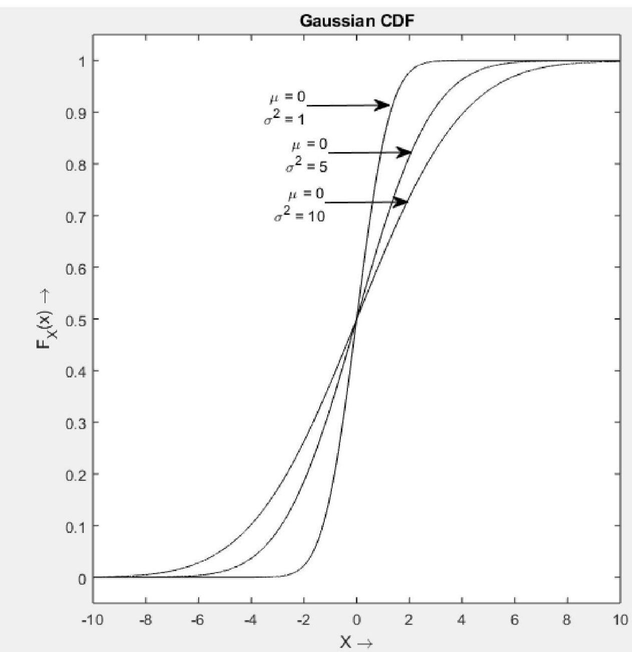
Aim: To study PDF and CDF functions of different Continuous Random Variables and the effect of parametric changes.

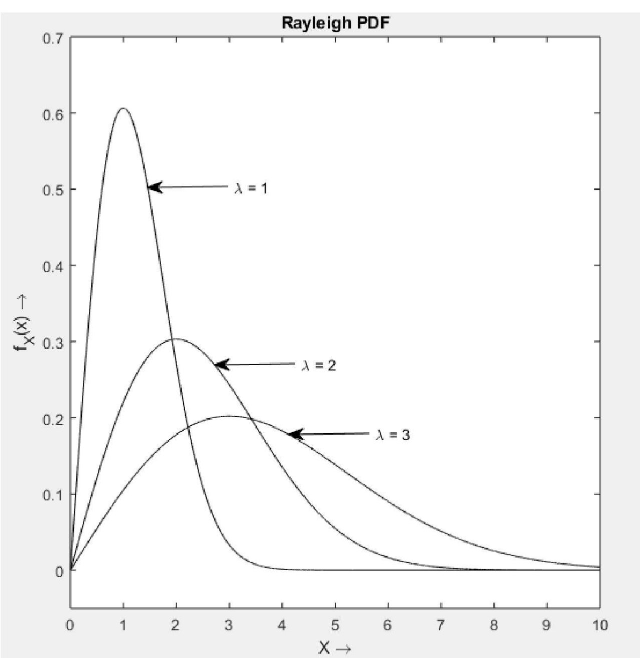
```
clc
clear all
% Uniform Random Variable
fprintf('Uniform Random Variable\n');
a=input('Starting point: ');
b=input('Ending point  : ');
x_u=a-2:0.001:b+2;
inc = x_u(2)-x_u(1);
f_u=zeros(size(x_u));
for i=1:length(x_u)
    if x_u(i)>=a && x_u(i)<=b
        f_u(i)=1/(b-a);
    end
end

F_u=zeros(size(f_u));
for i=1:length(f_u)
    for j=1:i
        F_u(i)=F_u(i)+f_u(j)*inc;
    end
end
m_u=sum(x_u.*f_u)*inc;
var_u=sum(((x_u-m_u).^2).*f_u)*inc;
fprintf('Mean = %3f and Variance =
%3f\n\n',round(m_u,2),round(var_u,2));
figure(2)
subplot(1,2,1)
plot(x_u,f_u,'black')
xlabel('X \rightarrow');
ylabel('f_X(x) \rightarrow');
title('Uniform PDF');
hold on
subplot(1,2,2)
plot(x_u,F_u,'black')
xlabel('X \rightarrow');
ylabel('F_X(x) \rightarrow');
title('Uniform CDF');
ylim([-0.2 1.2]);
hold on

%Gaussian Random Variable
fprintf('Gaussian Random Variable\n');
u= input('Mean of X : ');
var= input('Variance of X : ');
x_g= -10:0.001:10;
inc=x_g(2)-x_g(1);
f_g=(1/sqrt(2*pi*var))*exp(-((x_g-
u).^2)/(2*var));
F_g=zeros(size(f_g));
for i=1:length(f_g)
    for j=1:i
        F_g(i)=F_g(i)+f_g(j)*inc;
    end
end
m_g=sum(x_g.*f_g)*inc;
var_g=sum(((x_g-m_g).^2).*f_g)*inc;
fprintf('Mean = %3f and Variance =
%3f\n\n',round(m_g,2),round(var_g,2));
figure(3)
subplot(1,2,1)
plot(x_g,f_g,'black')
xlabel('X \rightarrow');
ylabel('f_X(x) \rightarrow');
title('Gaussian PDF');
hold on
subplot(1,2,2)
plot(x_g,F_g,'black')
xlabel('X \rightarrow');
ylabel('F_X(x) \rightarrow');
title('Gaussian CDF');
ylim([-0.2 1.2]);
hold on

%Rayleigh Distribution
fprintf('Rayleigh Random Variable\n');
sig=input('Parameter Sigma:');
x_r=0:0.001:10;
inc=x_r(2)-x_r(1);
f_r=(x_r/sig^2).*exp(-
(x_r.^2)/(2*sig^2));
F_r=zeros(size(f_r));
for i=1:length(f_r)
    for j=1:i
        F_r(i)=F_r(i)+f_r(j)*inc;
    end
end

m_r=sum(x_r.*f_r)*inc;
var_r=sum(((x_r-m_r).^2).*f_r)*inc;
fprintf('Mean = %3f and Variance =
%3f\n\n',round(m_r,2),round(var_r,2));
figure(4)
subplot(1,2,1)
plot(x_r,f_r,'black')
xlabel('X \rightarrow');
ylabel('f_X(x) \rightarrow');
title('Rayleigh PDF');
hold on
subplot(1,2,2)
plot(x_r,F_r,'black')
xlabel('X \rightarrow');
ylabel('F_X(x) \rightarrow');
title('Rayleigh CDF');
ylim([-0.2 1.2]);
hold on
```

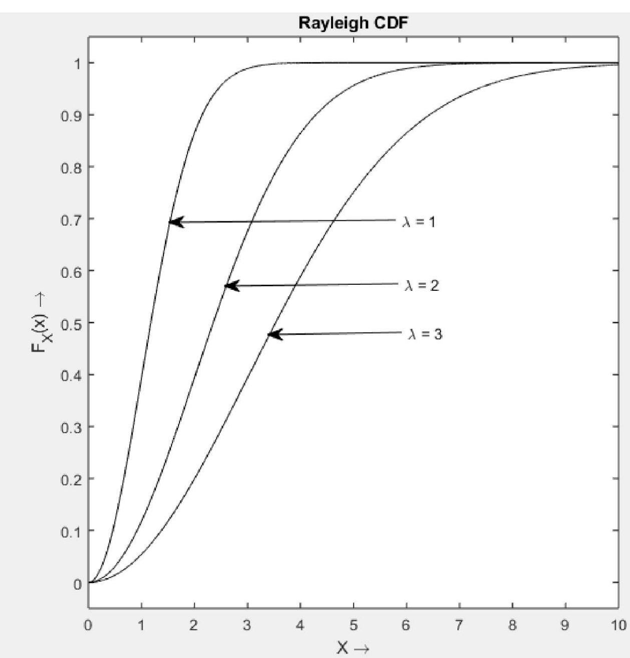

Uniform PDF
a= -0.5, b=0.5
a= -1, b=1
a= -2, b=2
$f_X(x) \rightarrow$
$X \rightarrow$


Uniform CDF
a= -0.5, b=0.5
a= -1, b=1
a= -2, b=2
$F_X(x) \rightarrow$
$X \rightarrow$


Gaussian PDF
$\mu = 0$ $\sigma^2 = 1$
$\mu = 0$ $\sigma^2 = 5$
$\mu = 0$ $\sigma^2 = 10$
$f_X(x) \rightarrow$
$X \rightarrow$


Gaussian CDF
$\mu = 0$ $\sigma^2 = 1$
$\mu = 0$ $\sigma^2 = 5$
$\mu = 0$ $\sigma^2 = 10$
$F_X(x) \rightarrow$
$X \rightarrow$


Rayleigh PDF
$\lambda = 1$
$\lambda = 2$
$\lambda = 3$
$f_X(x) \rightarrow$
$X \rightarrow$


Rayleigh CDF
$\lambda = 1$
$\lambda = 2$
$\lambda = 3$
$F_X(x) \rightarrow$
$X \rightarrow$

Q1. Give a practical example of the application of Uniform random variable, Gaussian Random Variable, and Rayleigh random variable.

→ Uniform RV is used where all the probability of all numbers is same i.e generation of a Random number say between 0 and 1.

→ Gaussian RV is used when there is white noise in channel.

→ Rayleigh RV is a common RV used in communication theory, where multiple path of dense information exists.

Q2. Discuss Mean, median, & mode values of 3 RVs?

→ Uniform RV

Mean: $(a+b)/2$, Median: $(a+b)/2$

Mode: any value b/w a & b.

→ Gaussian RV

Mean = Median = Mode = μ

→ Rayleigh RV :-

Mean: $\sigma\sqrt{\pi/2}$, Median: $\sigma\sqrt{2\ln(2)}$

Mode: σ.

Q3. With the appropriate choice of parameters, the pdf function value can go beyond 1. For each of the random variables, discuss the parametric values that cause this.

→ Uniform: If $(b-a) < 1$ the Prob > 1

→ Gaussian: $\sigma^2 < \frac{1}{2\pi}$

→ Rayleigh: $e^{-1/2} > \sigma$.

Q4. What is the effect of parametric changes to the shape of Uniform RV, Gaussian RV & Rayleigh RV?

→ Uniform Random Variable:-

Parameters : a, b

In case of Uniform Random Variable, the peak value of the distribution is dependent on the difference b/w b & a.

→ Gaussian Random Variable:-

Parameters : μ, σ_x

In case of Gaussian Random variable, the shape of distribution is dependended on variance. For the peak value to go beyond 1, value of σ^2 should be less than $\frac{1}{2\pi}$.

μ defines the position of peak on x-axis

→ Rayleigh Random Variable:-

In case of Rayleigh RV, the shape depends on the value of σ. If σ is less than $e^{-1/2}$, the peak value will be greater than 1.

Q5. What is physical significance of variance of RV?

→ Variance of random variable represent the AC power present in signal. Variance also defines the broadness of distribution around the mean value in PDF.

It signifies the narrowness or broadness of pdf & slope in CDF.

Q6. The instaneous slope of CDF function defines the value of PDF function for the same point. Comment.

-> We know CDF is related to PDF by the value that CDF is the running integral of PDF.

$$F_X(x) = \int_{-\infty}^{x} f_X(z)\,dz.$$

$$\Rightarrow f_X(x) = \frac{dF_X(x)}{dx}$$

- Also differenciation at any point yeilds slope. Hence the the slope of CDF at any instant defines the value of PDF.

Q7 What is meant by Affine Transformation?

For the variable X

$$E\{X\} = \mu.$$

$$\sigma_X^2 = E\{(X - E\{X\})^2\}.$$

- Using Affine transform on X

$$Y = AX + b.$$

$$E\{Y\} = A\mu + b.$$

$$E\{(Y - E\{Y\})^2\} = a^2 \sigma_X^2.$$

It is used in whitening of RV ie zero mean.

$$a = \frac{1}{\sigma_X} \quad \& \quad b = \frac{\mu}{\sigma_X}.$$