

Q1. Give a practical example of the application of Uniform random variable, Gaussian Random Variable, and Rayleigh random variable.

→ Uniform RV is used where all the probability of all numbers is same i.e. generation of a Random number say between 0 and 1.

→ Gaussian RV is used when there is white noise in channel.

→ Rayleigh RV is a common RV used in communications theory, where multiple path of dense information exists.

Q2. Discuss Mean, median, & mode values of 3 RVs?

→ Uniform RV

Mean:  $(a+b)/2$ , Median:  $(a+b)/2$

Mode: any value b/w  $a$  &  $b$ .

→ Gaussian RV

Mean = Median = Mode =  $\mu$

→ Rayleigh RV:

Mean:  $\sigma\sqrt{\pi/2}$ , Median:  $\sigma\sqrt{2\ln(2)}$

Mode:  $\sigma$ .

Q3. With the appropriate choice of parameters, the pdf function value can go beyond 1. For each of the random variables, discuss the parametric values that cause this.

→ Uniform: If  $(b-a) < 1$  the Prob  $> 1$

→ Gaussian:  $\sigma^2 < \frac{1}{2\pi}$

→ Rayleigh:  $e^{-1/2} > \sigma^0$

Q4. What is the effect of Parametric changes to the shape of Uniform RV, Gaussian RV & Rayleigh RV?

→ Uniform Random Variable:-

Parameters:  $a, b$

In case of Uniform Random Variable, the peak value of the distribution is dependent on the difference b/w  $b$  &  $a$ .

→ Gaussian Random Variable:-

Parameters:  $\mu, \sigma$

In case of Gaussian Random Variable, the shape of distribution is dependended on variance. For the peak value to go beyond 1, value of  $\sigma^2$  should be less than  $\frac{1}{2\pi}$ .  
 $\mu$  defines the position of peak on x-axis

→ Rayleigh Random Variable:-

In case of Rayleigh RV, the shape depends on the value of  $\sigma$ . If  $\sigma$  is less than  $\sigma^2$ , the peak value will be greater than 1.

Q5. What is physical significance of variance of RV?

→ Variance of random variable represent the AC power present in signal. Variance also defines the broadness of distribution around the mean value in PDF.

It signifies the narrowness or broadness of pdf & slope in PDF.



Q6. The instantaneous slope of CDF function defines the value of PDF function for the same point. Comment.

→ We know CDF is related to PDF by the value that CDF is the running integral of PDF.

$$F_X(x) = \int_{-\infty}^x f_X(z) dz.$$

$$\Rightarrow f_X(x) = \frac{dF_X(x)}{dx}$$

● Also differentiation at any point yields slope. Hence the slope of CDF at any instant defines the value of PDF.

Q7. What is meant by Affine Transformation?

For the variable  $X$

$$E\{X\} = \mu.$$

$$\sigma_x^2 = E\{(X - E\{X\})^2\}.$$

● Using Affine transform on  $X$

$$Y = AX + b.$$

$$E\{Y\} = A\mu + b.$$

$$E\{(Y - E\{Y\})^2\} = A^2 \sigma_x^2.$$

It is used in whitening of RV i.e. zero mean.

$$A = \frac{1}{\sigma_x} \quad \& \quad b = \frac{\mu}{\sigma_x}.$$