

Experiment-8

Aim: To study & plot bode plot of any higher order system.

Software Used: Matlab 2016 b.

Theory: Bode Plot is a graphical representation of the transfer function for determining the stability of the control system. Bode plot consists of two separate plots one is a plot of the log of magnitude of a sinusoidal transfer function the other is a plot of phase angle. The curves are drawn on semilog graph paper using the log scale for frequency and linear scale for magnitude (in db) or phase angle (in degrees)

1. $20 \log_{10} |G(j\omega)|$ vs $\log \omega$
2. Phase Shift vs $\log \omega$

Gain Margin: It is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability.
Mathematically gain margin is defined as the reciprocal of the magnitude gain margin of $G(j\omega) + j\omega$ & Plots over frequency.

$$GM = \frac{1}{|G(j\omega) + j\omega|_{\omega_c}} ; \omega_c = \text{Phase cross over Frequency.}$$

This frequency is the point where phase curve cross the 180° line.

Phase Margin: For gain the additional phase lag can be defined as the amount of additional phase lag which can be introduced in the system till system reaches on the verge of instability.

$$PM = |\angle G(j\omega)H(j\omega)|_{\omega_c} - (-180^\circ)$$

$$PM = 180 + \angle G(j\omega)H(j\omega)|_{\omega_c}$$

Where ω_c = gain crossover frequency

The point at which the magnitude curve crosses the 0dB line is the gain crossover frequency.

Practical Solution: Let the transfer function be:

$$G(s)H(s) = \frac{5(s+2)}{(s+1)(s+4)}$$

$$\therefore \angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{4} + \tan^{-1}\frac{\omega}{2}$$

Acc to definition:

$$-180^\circ = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{4} + \tan^{-1}\frac{\omega}{2}$$

$$\tan^{-1}\omega + \tan^{-1}\frac{\omega}{4} - \tan^{-1}\frac{\omega}{2} = 90^\circ$$

taking tangent on both sides,

$$\tan\left(\tan^{-1}\omega + \tan^{-1}\frac{\omega}{4} - \tan^{-1}\frac{\omega}{2}\right) = \tan 90^\circ$$

∴ Phase Crossover $\omega = 5.87 \text{ rad/s}$

$$\text{Now } G(j\omega)U(j\omega) = \frac{5(j\omega+2)}{(j\omega+1)(j\omega+4)}$$

For $\omega = 5.87 \text{ rad/s}$

$$G(j\omega)U(j\omega) = \frac{5(j \times 5.87 + 2)}{(j \times 5.87 + 1)(j \times 5.87 + 4)}$$

∴ Gain Crossover Frequency = 6.68 rad/s

Result :- Successfully coded this on MATLAB.

Code

```
pkg load control

num = [100]
den = [1 5 6 3]
sys = tf(num, den)

bode(sys)
[mag, phase] = bode(sys)

pause
```

Output

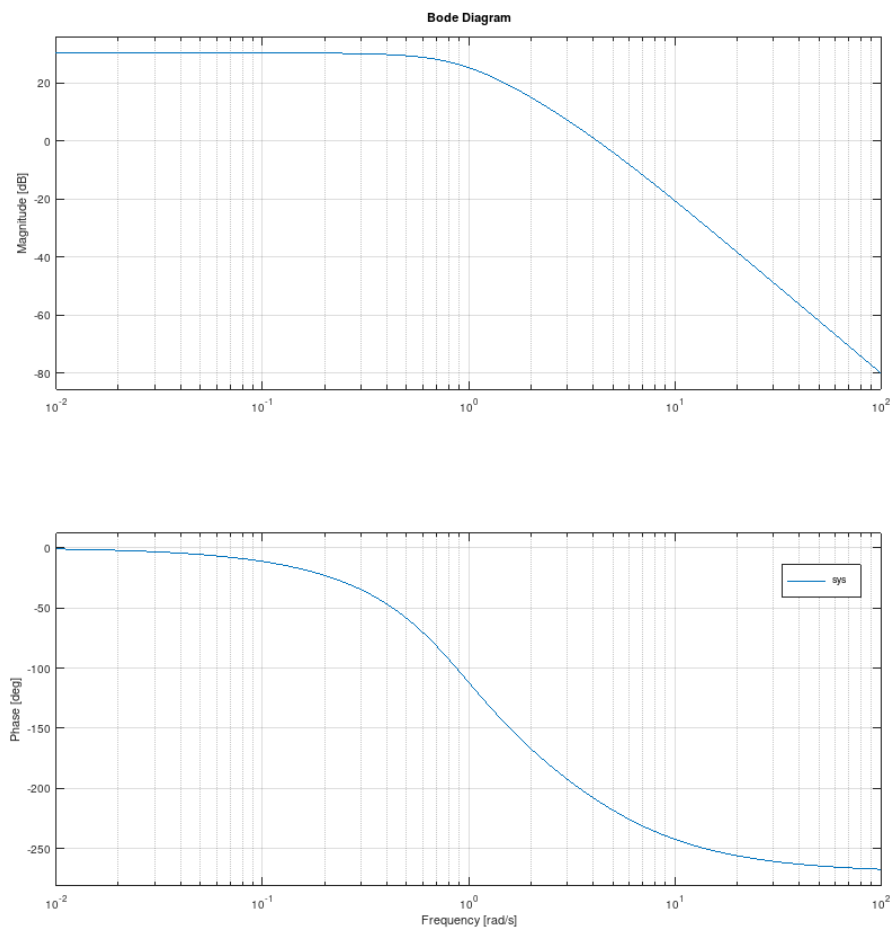


Figure 1: Bode Plot of Higher Order System