# Experiment-5

**Aim:** To study PDF & CDF functions of different Discrete Random Variables and the effects of Parametric changes.

**Theory:-** A random variable X is a function defined on sample space S of an experiment. Its value are real numbers. For every number 'a' the probability $P(x=a)$, with which X assumes 'a' is defined. Similarly $P(x \in I)$, with which X assumes any value $\in I$ is defined.

## Discrete Random Variable:

By definition, a random variable and its distribution are discrete if X assumes only finitely many or at most countably many values $x_1, x_2, x_3$ - called the possible values of X. Discrete Distribution of X is also determined by the probability function $f(x)$ of $x$; defined by

$$f(x) = \begin{cases} P_j & \text{if } x = x_j \\ 0 & \text{otherwise} \end{cases} \quad (j = 1, 2, ---).$$

From this we get the values of the distribution function.

$$F(x) = \sum_{x_j \leq x} f(x_j) = \sum_{x_j \leq x} P_j$$

## Bernoulli Random Variable:

It is Discrete Probability distribution of a random variable which takes the value 1 with prob. P & value 0, $q = 1-P$

pdf $\rightarrow f_x(x) = \begin{cases} (1-P)\delta(x) + P\delta(x-1) & x = 1 \\ 0 & x = 0 \end{cases}$

cdf $\rightarrow F_x(x) = (1-P)u(x) + P\,u(x-1)$

mean $= P$  Variance $= P(1-P)$

## Binomial Distributions

PDF $f_x(x) = \sum_{k=0}^{n} {}^{n}C_k\, P^k (1-P)^{n-k} \delta(x-k)$

CDF $F_x(x) = \sum_{k=0}^{n} {}^{n}C_k\, P^k (1-P)^{n-k} u(x-k)$

Mean $\bar{x} = nP$  Variance $\sigma_x^2 = nP(1-P)$

The Binomial distribution occurs in game of chance, opinion polls, medicine, & so on.

## Poisson Distribution:

The discrete distribution with infinitely many possible

$$f(x) = \frac{\mu^x}{x!} e^{-\mu}$$

is called Poisson distribution. Special case of Binomial distribution for $n \to \infty,\ p \to 0 \ \| \rightarrow np = \lambda$.

PDF $f_x(x) = \sum_{k=0}^{\infty} \frac{e^{-\lambda}}{k!} \lambda^k \delta(x-k)$

Mean $\bar{x} = \lambda$

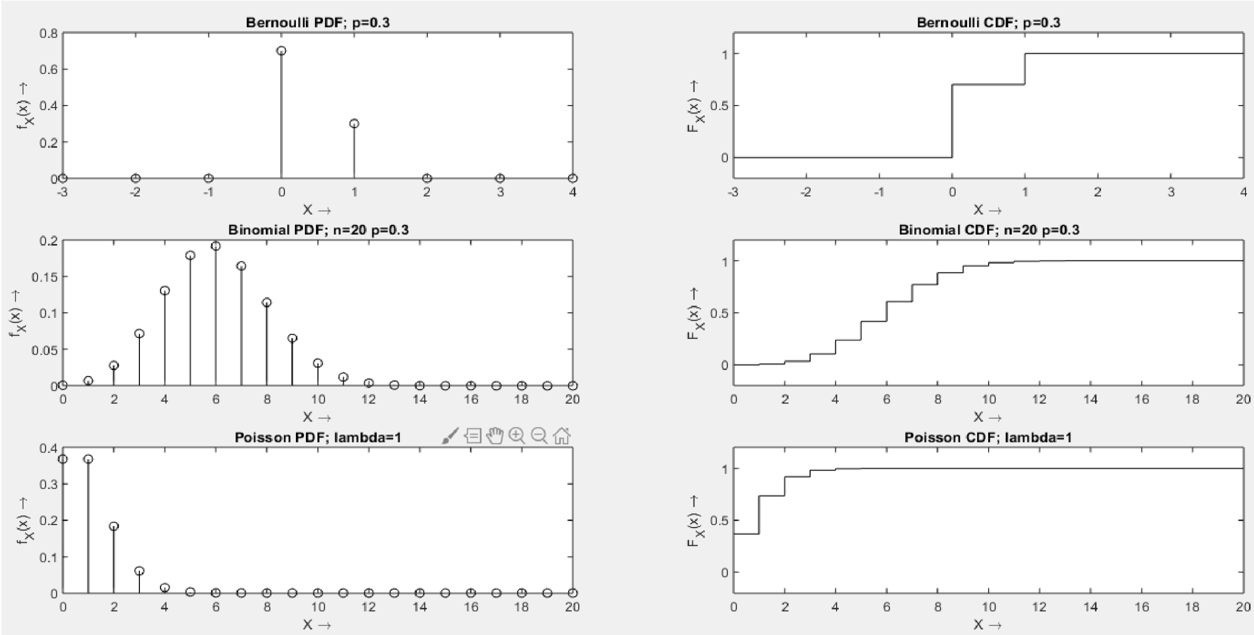
Variance $\sigma_x^2 = \lambda$

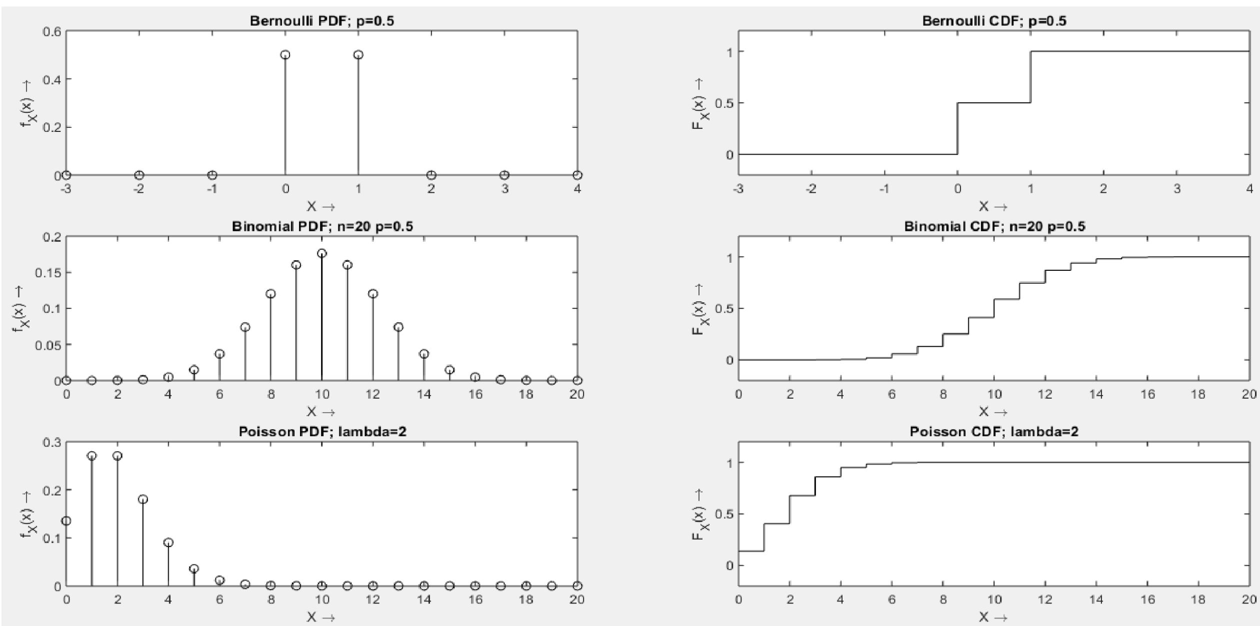
# Experiment 5

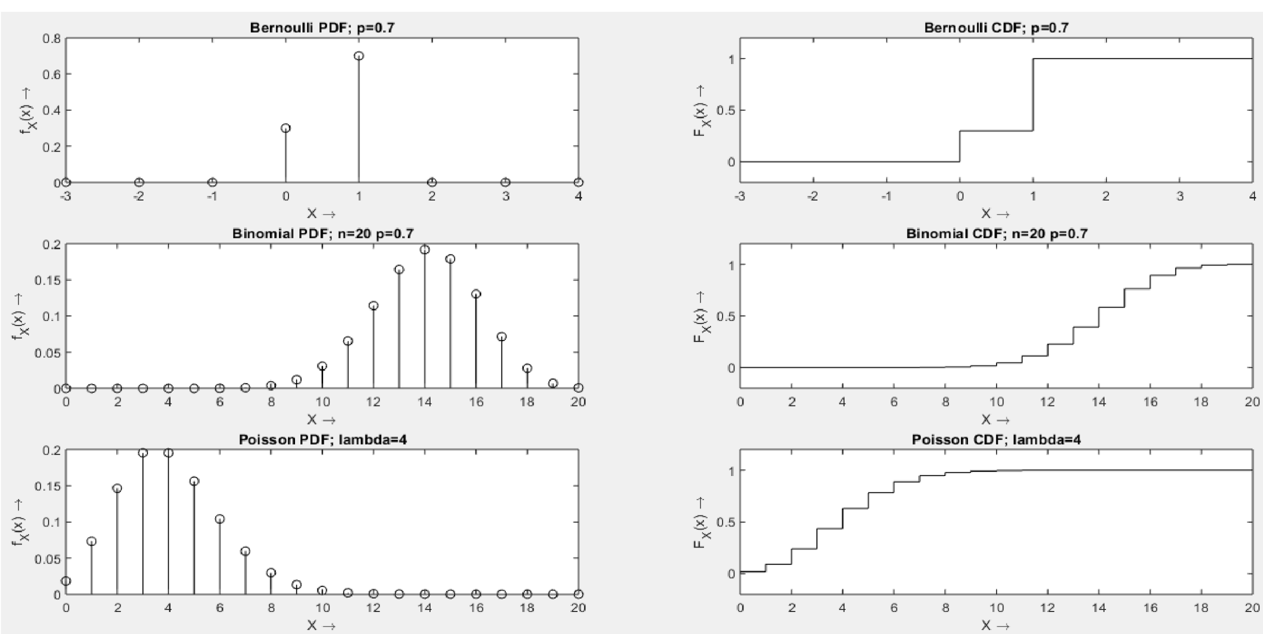
Aim: To study PDF and CDF functions of different Discrete Random Variables and the effect of parametric changes.

Code:

```
clc
clear all

%Bernoulli Random Variable
fprintf('Bernoulli Random Variable\n');
q=input('Success Probability : ');
x_be= -3:1:4;
f_be= zeros(size(x_be));
f_be(x_be==1) = q;
f_be(x_be==0) = 1-q;
F_be = zeros(size(f_be));
for i=1:length(x_be)
    for j=1:i
        F_be(i) = F_be(i)+f_be(j);
    end
end
m_be = sum(x_be.*f_be);
var_be= sum(((x_be-m_be).^2).*f_be);
fprintf('Mean = %3f and Variance =
%3f\n\n',round(m_be,2),round(var_be,2));
figure(1);
subplot(3,2,1)
stem(x_be,f_be,'black');
xlabel('X \rightarrow');
ylabel('f_X(x) \rightarrow');
title(['Bernoulli PDF; p=',num2str(q)]);
subplot(3,2,2);
stairs(x_be,F_be,'black');
xlabel('X \rightarrow');
ylabel('F_X(x) \rightarrow');
title(['Bernoulli CDF; p=',num2str(q)]);
ylim([-0.2 1.2]);

%Binomial Random Variable
fprintf('Binomial Random Variable\n');
n=input('Number of incidents: ');
p=input('Success Probability : ');
x_bi= 0:1:n;
f_bi= zeros(size(x_bi));
for k= 0:n

nCk=factorial(n)/(factorial(k)*factorial
(n-k));
    f_bi(k+1)=nCk*p^k*(1-p)^(n-k);
end
F_bi = zeros(size(f_bi));
for i=1:length(x_bi)
    for j=1:i
        F_bi(i) = F_bi(i)+f_bi(j);
    end
end
m_bi = sum(x_bi.*f_bi);
var_bi= sum(((x_bi-m_bi).^2).*f_bi);
fprintf('Mean = %3f and Variance =
%3f\n\n',round(m_bi,2),round(var_bi,2));
figure(1);
subplot(3,2,3)
stem(x_bi,f_bi,'black');
xlabel('X \rightarrow');
ylabel('f_X(x) \rightarrow');
title(['Binomial PDF; n=',num2str(n),'
p=',num2str(p)]);
subplot(3,2,4);
stairs(x_bi,F_bi,'black');
xlabel('X \rightarrow');
ylabel('F_X(x) \rightarrow');
title(['Binomial CDF; n=',num2str(n),'
p=',num2str(p)]);
ylim([-0.2 1.2]);

%Poisson Random Variable
fprintf('Poisson Random Variable\n');
lam = input('Lambda parameter : ');
x_p= 0:1:20;
f_p= zeros(size(x_p));
for k= 0:n

f_p(k+1)=((lam^k)/factorial(k))*exp(-
lam);
end
F_p = zeros(size(f_p));
for i=1:length(x_p)
    for j=1:i
        F_p(i) = F_p(i)+f_p(j);
    end
end
m_p = sum(x_p.*f_p);
var_p= sum(((x_p-m_p).^2).*f_p);
fprintf('Mean = %3f and Variance =
%3f\n\n',round(m_p,2),round(var_p,2));
figure(1);
subplot(3,2,5)
stem(x_p,f_p,'black');
xlabel('X \rightarrow');
ylabel('f_X(x) \rightarrow');
title(['Poisson PDF;
lambda=',num2str(lam)]);
subplot(3,2,6);
stairs(x_p,F_p,'black');
xlabel('X \rightarrow');
ylabel('F_X(x) \rightarrow');
title(['Poisson CDF;
lambda=',num2str(lam)]);
ylim([-0.2 1.2]);
```


Bernoulli PDF; p=0.3
Bernoulli CDF; p=0.3
Binomial PDF; n=20 p=0.3
Binomial CDF; n=20 p=0.3
Poisson PDF; lambda=1
Poisson CDF; lambda=1
f_X(x) →
F_X(x) →
X →


Bernoulli PDF; p=0.5
Bernoulli CDF; p=0.5
Binomial PDF; n=20 p=0.5
Binomial CDF; n=20 p=0.5
Poisson PDF; lambda=2
Poisson CDF; lambda=2
f_X(x) →
F_X(x) →
X →


Bernoulli PDF; p=0.7
Bernoulli CDF; p=0.7
Binomial PDF; n=20 p=0.7
Binomial CDF; n=20 p=0.7
Poisson PDF; lambda=4
Poisson CDF; lambda=4
f_X(x) →
F_X(x) →
X →

Q1. Give a practical example of the application of Bernoulli random variable, binomial random variable & Poisson random variable.

⇒ When there is only one bit required for transmission of message signal, the Bernoulli RV is used to ~~master~~ understand noise;
It is used for binary data generators and identifying bit error pattern.

→ In case of sequence of 'n' bit transmission, binomial RV are considered, since binomial distribution is combination of 'n' mutually independent yet identically distributed Bernoulli distribution.
It can be used to model total number of bits received from data sequence of length n.

→ Poisson is special case of binomial to identify noise occurence when 'n' is very large.

Q2. For a Bernoulli RV, the mean value exists between 0 & 1, but at that point there is no probability assigned. What does the mean value signify?

→ In case of Bernoulli random variable, the mean is the expectation value that determines whether the incoming bit is zero or binary one. Probability assignment isn't required at every point to define mean. Even in case of 0 probability distribution, the mean can be the weighted average of data based upon modelling of probability. It used to determine the threshold value.

Q3. What is the meaning of Median and mode of Random variable?

For Probability Density Function of a RV,

-> Median is the point on X axis of PDF at which it divides it into 2 halves.

-> Mode is the point on X axis of PDF having it's highest value.

-> Median is not defined for discrete RV.

Q4. Is there an established relationship b/w Bernoulli RV, Binomial RV & Poisson RV?

-> Binomial RV is a combination of mutually independent yet identically distributed bernoulli distribution, and can be modelled after that

-> Poisson is a special case of Binomial where the no. of cases 'n' is very large & the success rate 'p' is very low.

Q5. What is the effect of RV parametric change in the Binomial RV's PDF shape?

-> Changing the 'n' is inversely proportional to the peak value of PDF.

-> Changing the success rate 'p' shifts the peak value of PDF of X axis

Q6. What is the typical value 'λ' in the case of a Poisson RV? What is the shape of the PDF of this value?

-> $\lambda \leq 0.5$ for Poisson RV, PDF is shaped like exponential decay function.