CONTROL SYSTEM

VTH SEMESTER

ETEL-307

Department of Electronics and Communication Engineering, BVCOE, New Delhi
Subject: Control System , Instructor: Avinash

UNIT-II

TIME DOMAIN ANALYSIS

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING, BVCOE, NEW DELHI

SUBJECT: CONTROL SYSTEM , INSTRUCTOR: AVINASH

Topics to be covered :-

Time Domain Analysis:

- Time Domain Performance Specifications
- Transient Response Of First & Second Order Systems

Errors:

- Steady State Errors
- Static Error Constants In Unity Feedback Control Systems

Controllers:

- Response With P, PI And PID Controllers

Limitations Of Time Domain Analysis:

Chapter Learning Outcomes:

- Use poles and zeros of transfer functions to determine the time response of a control system
- Describe quantitatively the transient response of first-order systems
- Write the general response of second-order systems given the pole location
- Find the damping ratio and natural frequency of a second-order system
- Find the settling time, peak time, percent overshoot, and rise time for an underdamped second-order system
- Find steady state error depends upon type and input of the system
- Effects of controller to reduce steady state error
- Approximate higher-order systems and systems with zeros as first- or second order Systems

Pole, Zero, System Response:

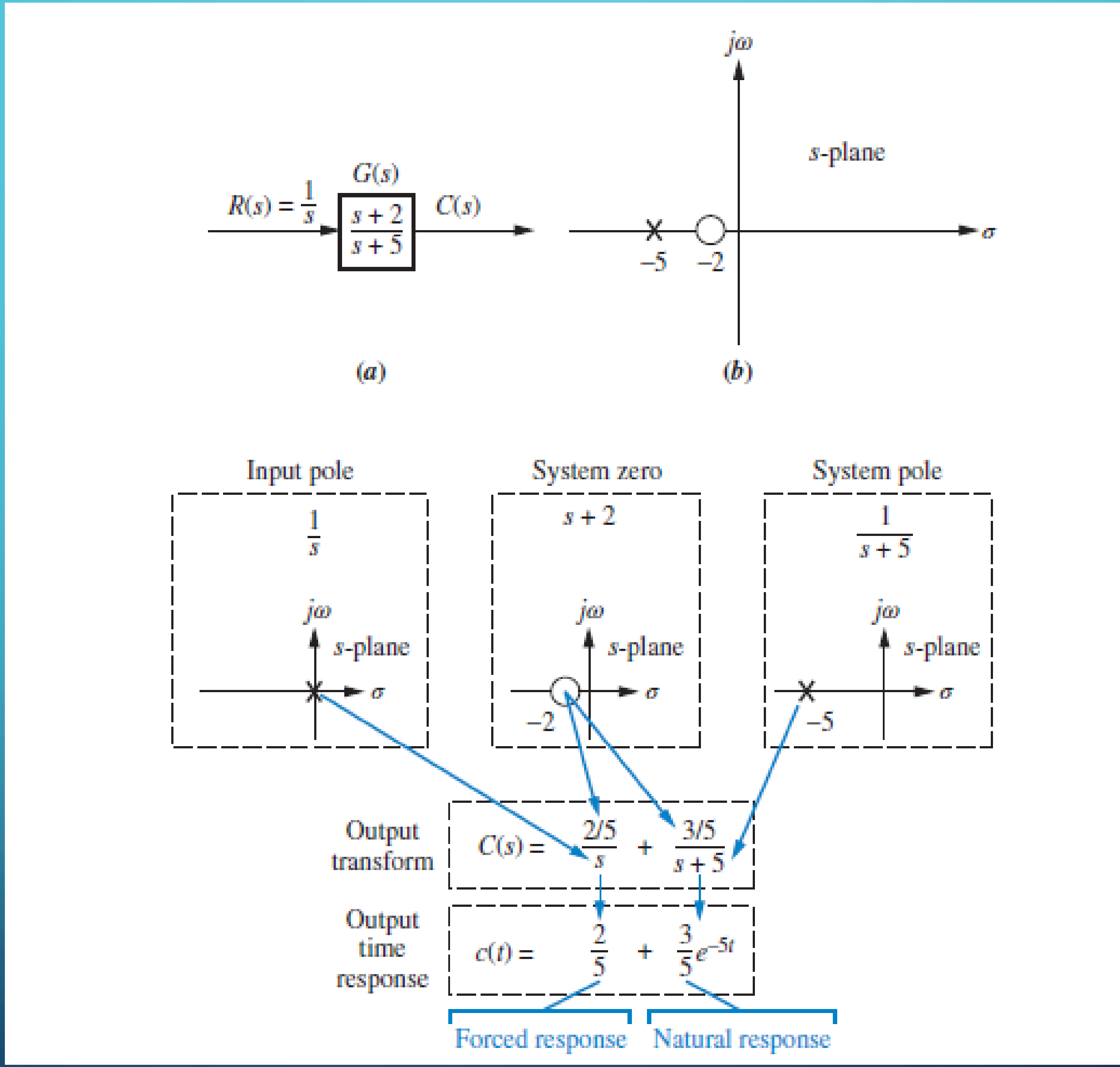
Pole of a Transfer Function: The *poles* of a transfer function are

1) The values of the Laplace transform variable '*s*' that cause the transfer function to become infinite or
2) Any roots of the denominator of the transfer function that are common to roots of the numerator

Zero of a Transfer Function: The *zeros* of a transfer function are

1) The values of the Laplace transform variable, '*s*' that cause the transfer function to become zero, or
2) Any roots of the numerator of the transfer function that are common to roots of the denominator

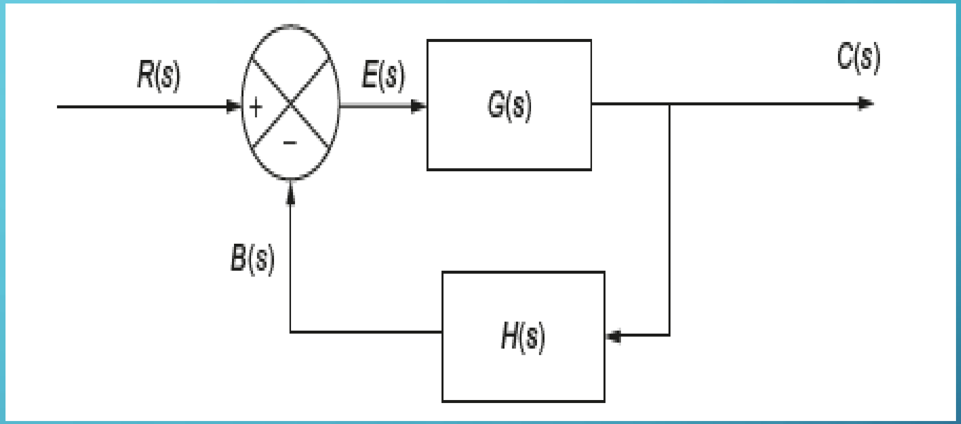
Pole, Zero, System Response:



Type & Order of System:

Every Transfer function representing a control system has certain types & orders

Type of the system:



No of **open loop pole at origin** gives

Type of Control System

$$G(s)H(s) = \frac{K(1+T_a s)}{S^p(1+T_i S)}$$

If P=0, Type =0

If P=1, Type =1

Order of the System: The **highest power** of characteristics equation **(1+ $G(s)H(s)$)** gives order of the system.

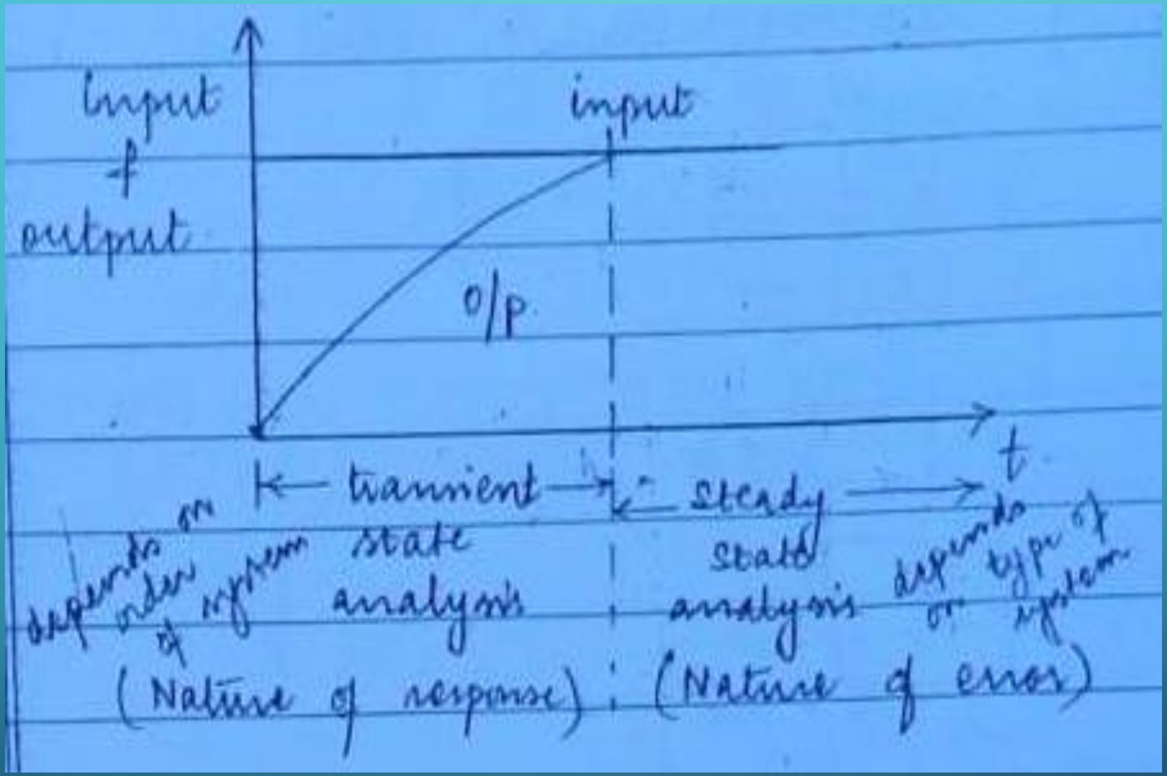
Standard Test Signals:

Type of the signal	$r(t)$ in time domain	$R(s)$ in frequency-domain
Impulse	$\delta(t)$	1
Step	$Au(t)$	$\frac{A}{s}$
Ramp	$A\,t$	$\frac{A}{s^2}$
Parabolic	$\frac{At^2}{2}$	$\frac{A}{s^3}$

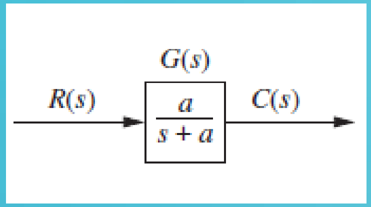
- Sudden Shocks – Impulse Signal – Stability Analysis
- Sudden Change Type Input – Step Signal – Time Domain Analysis
- Velocity Type Input – Ramp Signal- Time Domain Analysis
- Acceleration Type Input- Parabolic Signal – Time Domain Analysis

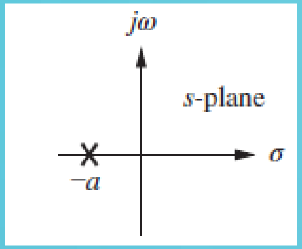
Time Domain Analysis:

Total time response of a system = Transient Response + Steady State Response



Time Domain Performance Specifications First Order System:





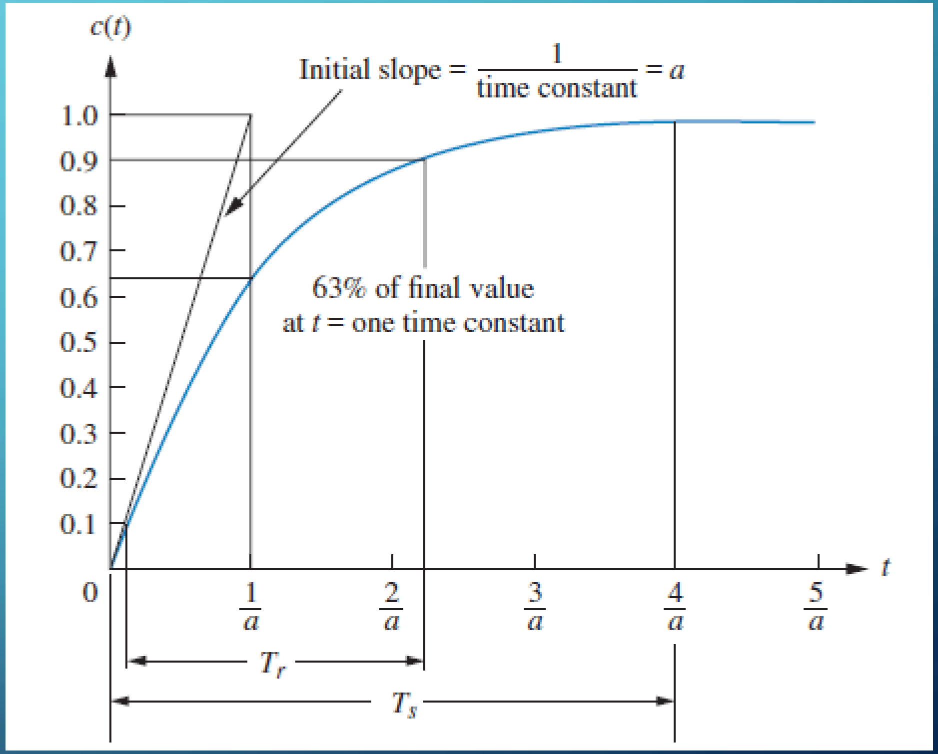
$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

$$C(t) = C_f(t) + C_n(t) = 1 - e^{-at}$$

For, $t = \frac{1}{a}, C(t) = 1 - e^{-at} = 0.63$

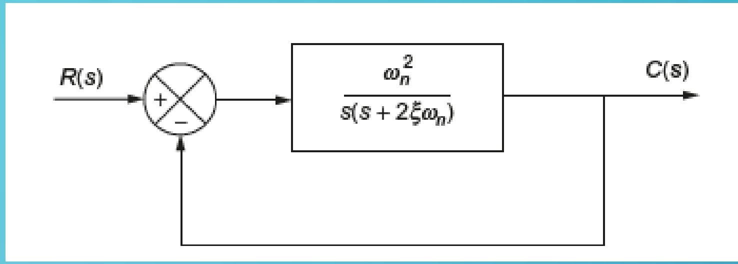
$$Rise\ Time = T_r = \frac{2.2}{a}$$

$$Setteling\ Time = T_s = \frac{4}{a}$$



Time Domain Performance Specifications Second Order System:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)}$$

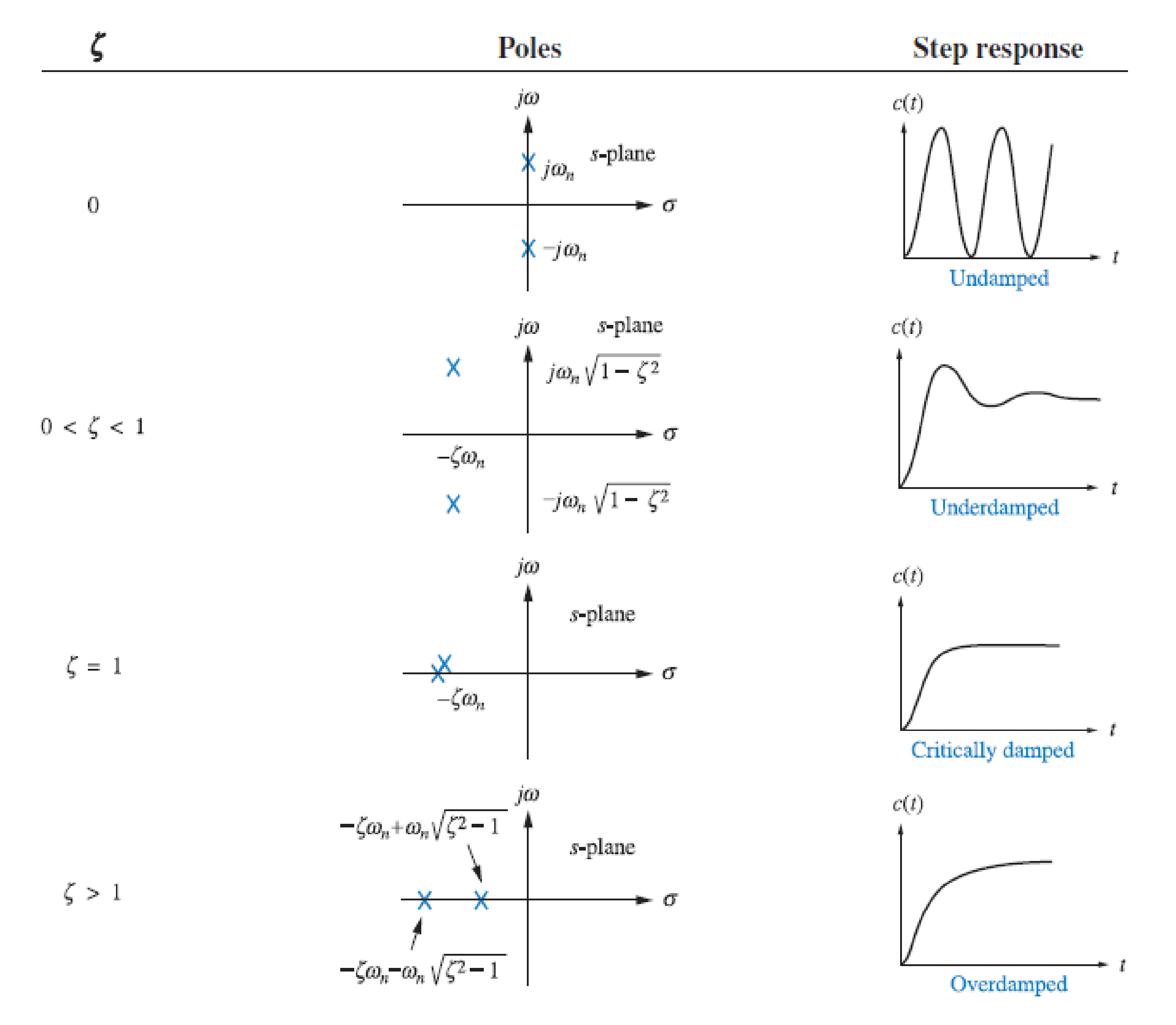


$Where, \omega_n = Undamped\ Natural\ frequency\ \&\ \zeta = \ Damping\ Ratio$

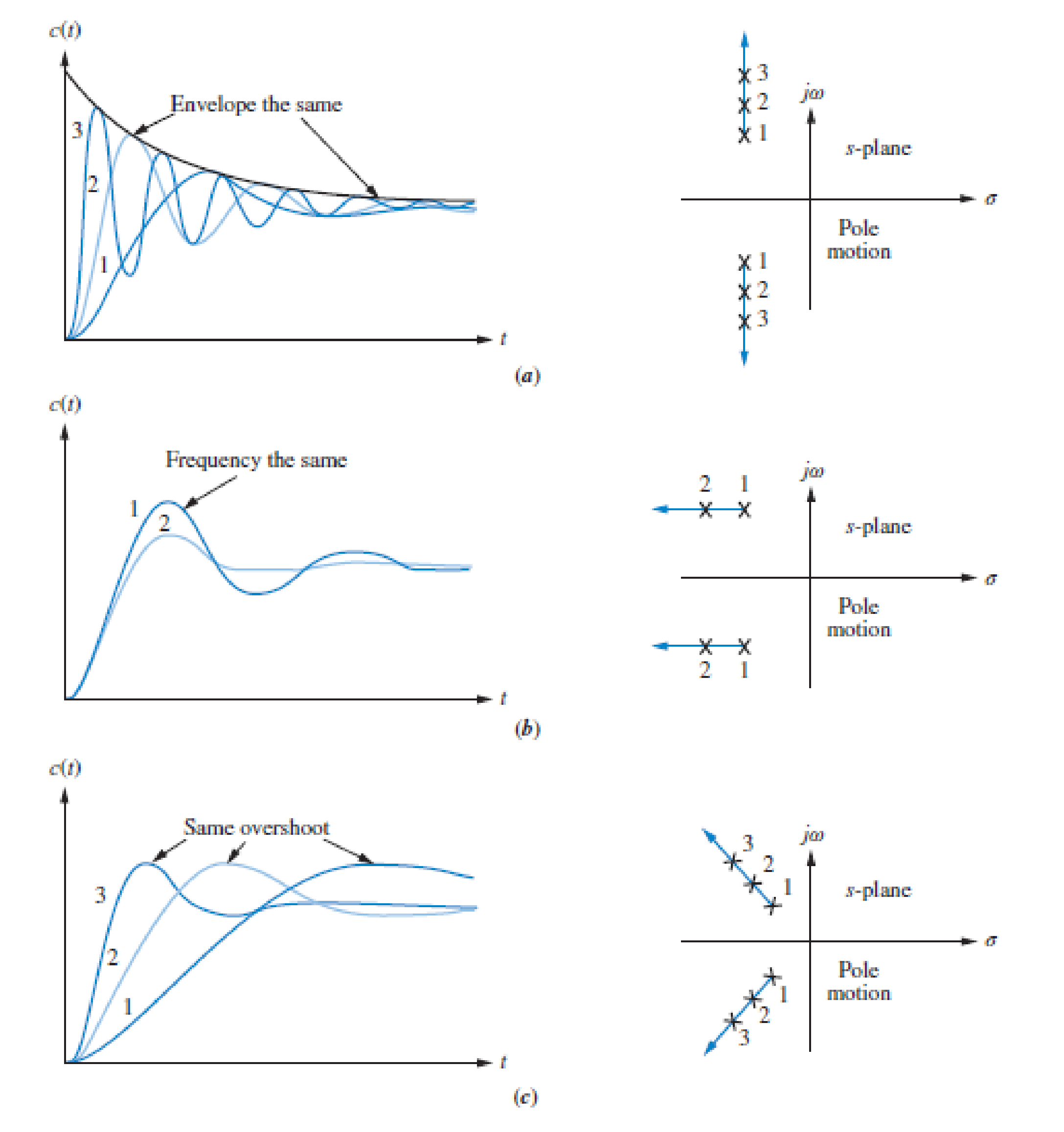
Classification of second order system based on damping ratio:

Nature of the system	Damping ratio ξ	Roots of the characteristic equation s_1, s_2	Nature of roots
Undamped or oscillating system	0	$\pm j\omega_n$	Purely imaginary
Underdamped	<1	$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ or $-\sigma \pm j\omega_d$	Complex
Critically damped	1	$-\omega_n$	Real and equal
Overdamped	>1	$-\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$	Real and unequal

Time Domain Performance Specifications Second Order System:



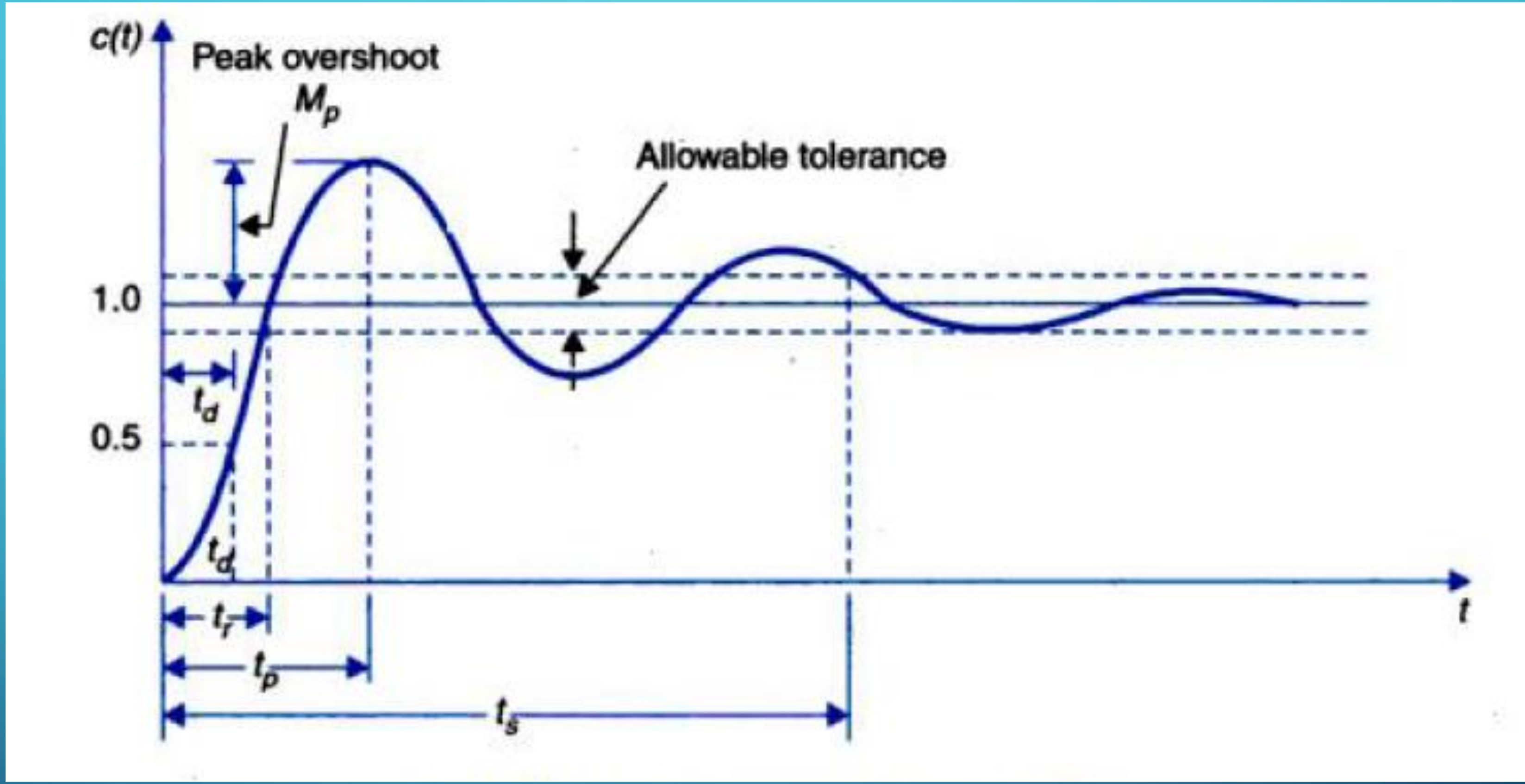
Time Domain Performance Specifications Second Order System:



Time Domain Performance Specifications Second Order System:

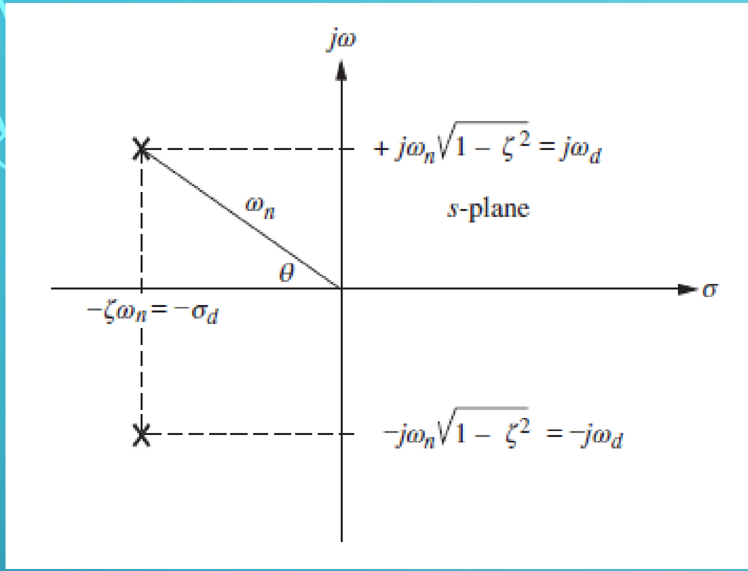
- Delay Time:- The time taken by the response of a system to reach 50 % of the final value
- Rise Time:- The time taken by the response of a system to reach 90 % of the final value from 10 %
- Peak Time:- The time taken by the response of a system to reach the first peak overshoot is known as the peak time
- Maximum Peak Overshoot:- The maximum peak overshoot is obtained at the peak time of the response of a system
- Settling Time:- The time taken by the response of a system to reach 98% of the final value
- Steady State Error:-It is the measurement of the difference existing between the standard Input signal *r(t)* applied and the output response obtained from the system *c(t)* when such standard signals are applied

Time Domain Performance Specifications Second Order System:



Time Domain Performance Specifications Second Order System:

Relation of Rise Time, Peak Time, % Overshoot, Settling Time with Location of Poles:



Radial distance from the origin to the pole is the natural frequency ω_n

$$Cos\ (\theta) = \zeta$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d},\ \omega_d = \text{Damped Frequency}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{\pi}{\sigma_d},\qquad \sigma_d = Exponential\ damping\ frequency$$

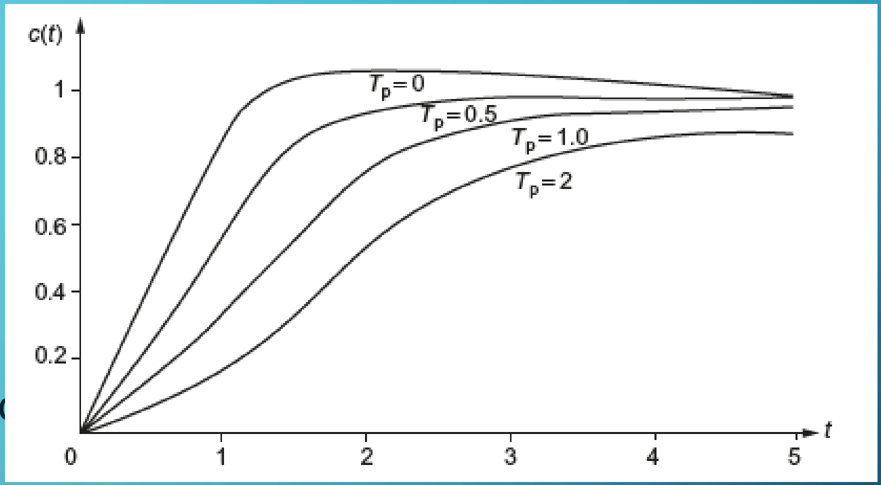
Tp is inversely proportional to the imaginary part of the pole. Since horizontal lines on the *s*-plane are lines of constant imaginary value, they are also lines of constant peak time

Settling time is inversely proportional to the real part of the pole. Since vertical lines on the *s*-plane are lines of constant real value, they are also lines of constant settling time

Since ζ . cos ϑ, radial lines are lines of constant ζ. Since percent overshoot is only a function of ζ, radial lines are thus lines of constant percent overshoot, *%OS*.

Effect of Adding Pole & Zeros On Time Domain Performance Specifications

Second Order System:



Effect of Adding Poles:

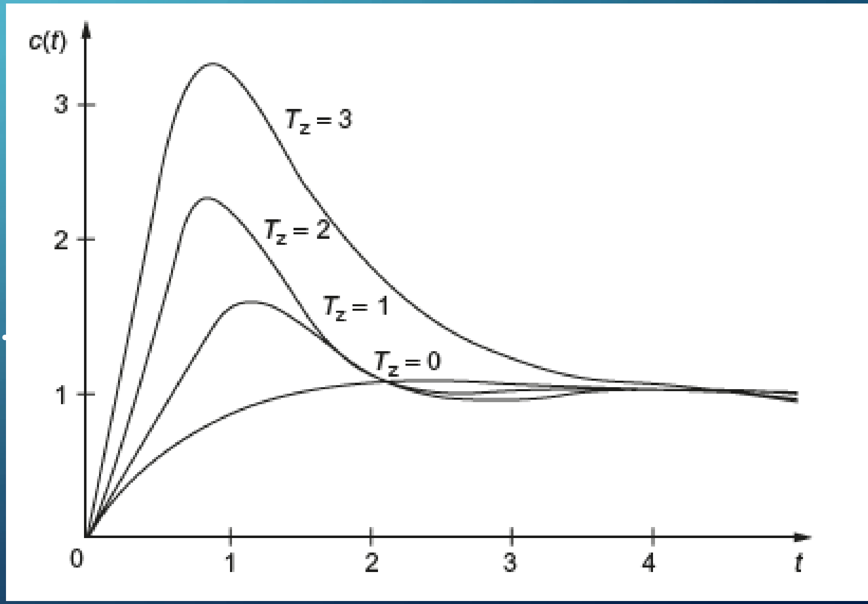
$$\frac{C_{(s)}}{R_{(s)}} = \frac{{\omega_n}^2}{(s^2+2\zeta\omega_n s + {\omega_n}^2)(1+sT_p)}$$

- Peak overshoot decreases as T_p decreases.
- Number of oscillations existing in the system becomes less as T_p de
- Increase in rise time.
- System becomes more sluggish as T_p decreases

Effect of Adding Zeros:

$$\frac{C_{(s)}}{R_{(s)}} = \frac{{\omega_n}^2(1+sT_z)}{(s^2+2\zeta\omega_n s + {\omega_n}^2)}$$

- Increases rise time as T_z increases.
- Increases peak overshoot and number of oscillations as T_z increases.



Steady State Response Analysis:

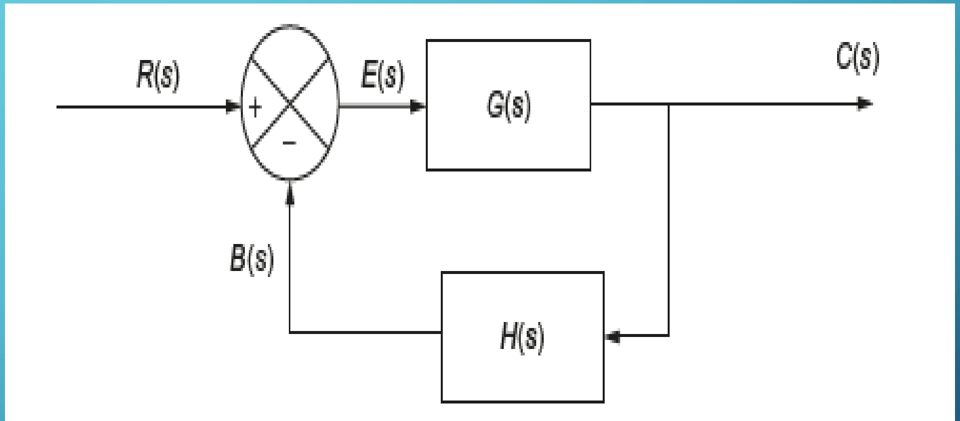
It deals with the estimation of magnitude of steady state error between input and output and depends on type of control system.

$$E(s) = R(s) - \mathrm{B}(s)$$

$$= R(s) - C(s)H(s)$$

$$= R(s) - E(s)G(s)H(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$



Steady State Response Analysis:

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

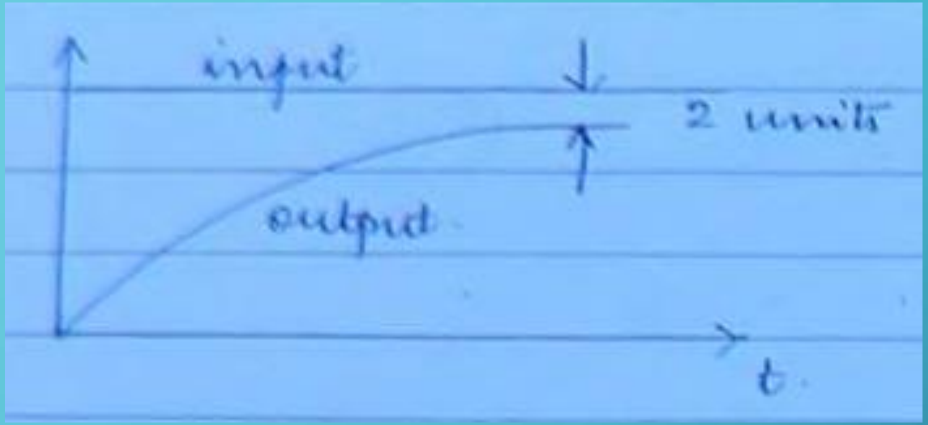
$$\lim_{t\to\infty} e_{(t)} = e_{ss}$$

Applying Final Value Theorem,

$$e_{ss} = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} \frac{s\,R(s)}{1 + G(s)H(s)}$$

$$= \frac{\lim_{s\to 0}\{s\,R(s)\}}{1 + \lim_{s\to 0} G(s)H(s)}$$



Steady State Response Analysis:

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)H(s)} = \frac{\lim_{s\to 0}\{s\,R(s)\}}{1+\lim_{s\to 0} G(s)H(s)}$$

For $R(s) = \frac{A}{s}, e_{ss} = \frac{\lim_{s\to 0}\left\{s*\frac{A}{s}\right\}}{1+\lim_{s\to 0} G(s)H(s)} = \frac{A}{1+\lim_{s\to 0} G(s)H(s)} = \frac{A}{1+K_p}, K_p = Position\ Error\ Constant$

For $R(s) = \frac{A}{s^2}, e_{ss} = \frac{\lim_{s\to 0}\left\{s*\frac{A}{s^2}\right\}}{1+\lim_{s\to 0} G(s)H(s)} = \frac{A}{\lim_{s\to 0} s+\lim_{s\to 0} sG(s)H(s)} = \frac{A}{K_v}, K_v = Velocity\ \ Error\ Constant$

For $R(s) = \frac{A}{s^3}, e_{ss} = \frac{\lim_{s\to 0}\left\{s*\frac{A}{s^3}\right\}}{1+\lim_{s\to 0} G(s)H(s)} = \frac{A}{\lim_{s\to 0} s^2+\lim_{s\to 0} s^2G(s)H(s)} = \frac{A}{K_a}, K_a = Acceleration\ \ Error\ Constant$

	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$

Controllers:

The controllers are used to maintain time domain specifications (maximum overshoot, steady state error) at a specific value instead of zero or infinity.

The output signal of the controller or the input signal of the system Is known as actuating signal.

Type of Electronic Controllers:

- Proportional Controller
- Integral Controller
- Derivative Controller
- Proportional Integral Controller
- Proportional Derivative Controller
- Proportional Integral Derivative Controller

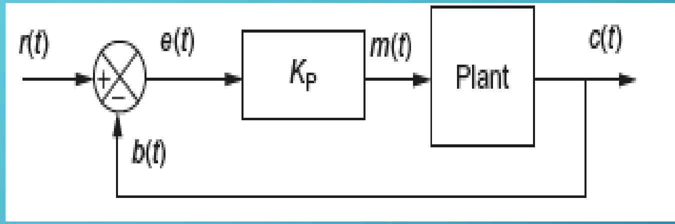
Proportional Controllers:

In proportional controller, the error signal is amplified to generate the control (output) signal. The output signal of the proportional controller *m*(*t*), is proportional to the error signal *e* (*t*).

In P controller, $m_t \propto e_t$

$$m_t = K_p e_t, K_p = Proportional\ Gain$$

$$M_s = K_p E_s$$



Advantages of P-controller:

- It amplifies the error signal by the gain value K_p
- It increases the loop gain by
- It improves the steady-state accuracy, disturbance signal rejection and relative stability.
- The use of controller makes the system less sensitive to parameter variations

Disadvantages of P-controller

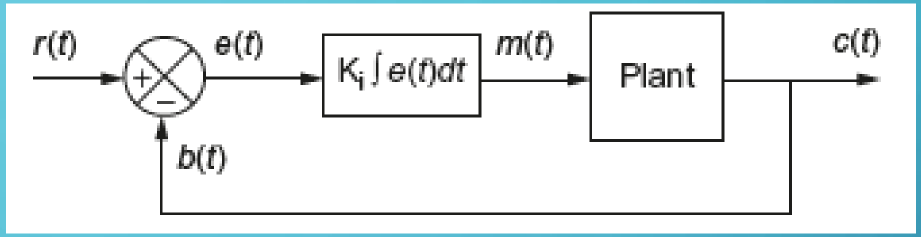
- System becomes unstable if the gain of the controller increases by large value.
- P-controller leads to a constant steady-state error.

Integral Controller:

The integral (I) controller relates the present error value and the past error value to determine the controller output. The output signal of the integral controller $m(t)$ is proportional to integral of the input error signal $e\,(t)$.

In I controller,

$$m_{(t)} \propto \int e_{(t)}\,dt$$



$$M_s = K_i \frac{E_s}{S}$$

Advantages of I-controller

- Reduces the steady-state error without the help of manual reset. Hence, the controller is also called as automatic reset
- Eliminates the error value
- Eliminates the steady-state error

Disadvantages of I-controller

- Action of this controller leads to oscillatory response with increased or decreased Amplitude, which is undesirable and the system becomes unstable.
- It involves integral saturation or wind-up effect.
- There is poor transient response.

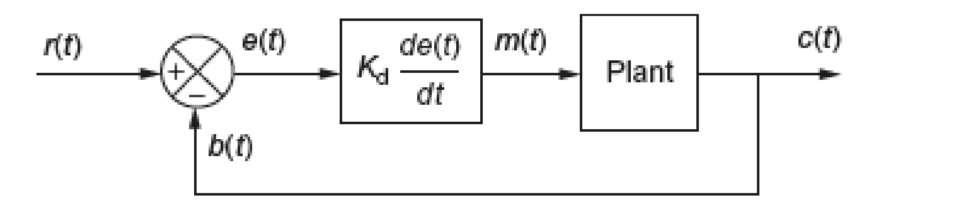
Derivative Controllers:

In Derivative Controller,

$$m_t \propto \frac{de_t}{dt}$$

$$m_t = K_d \frac{de_t}{dt}$$

$$M_s = K_d S E_s$$



Advantages of D-controller:

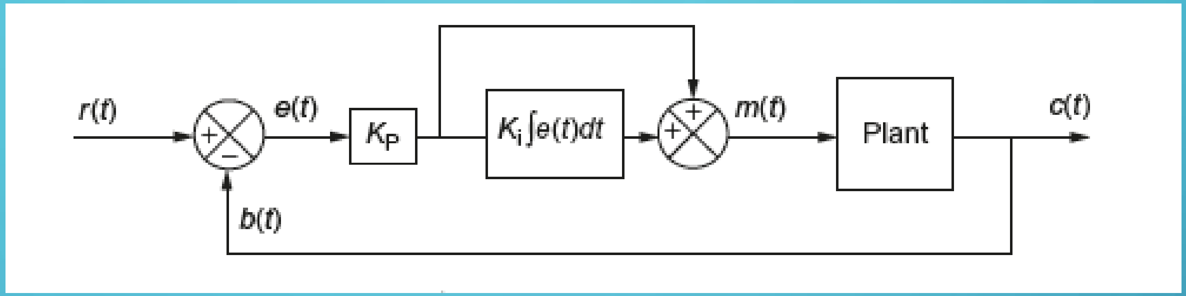
- Resists the change in the system
- Has faster response
- Anticipates the error and initiates an early corrective action that increases the stability Of the system
- Effective during transient period

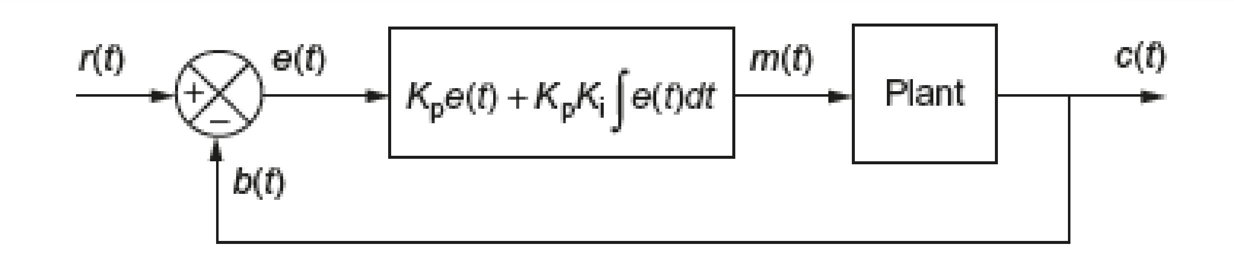
Disadvantages of D-controller:

- steady-state error is not recognized by the controller even when the error is too large.
- This controller cannot be used separately in the system.

Proportional Integral Controller:

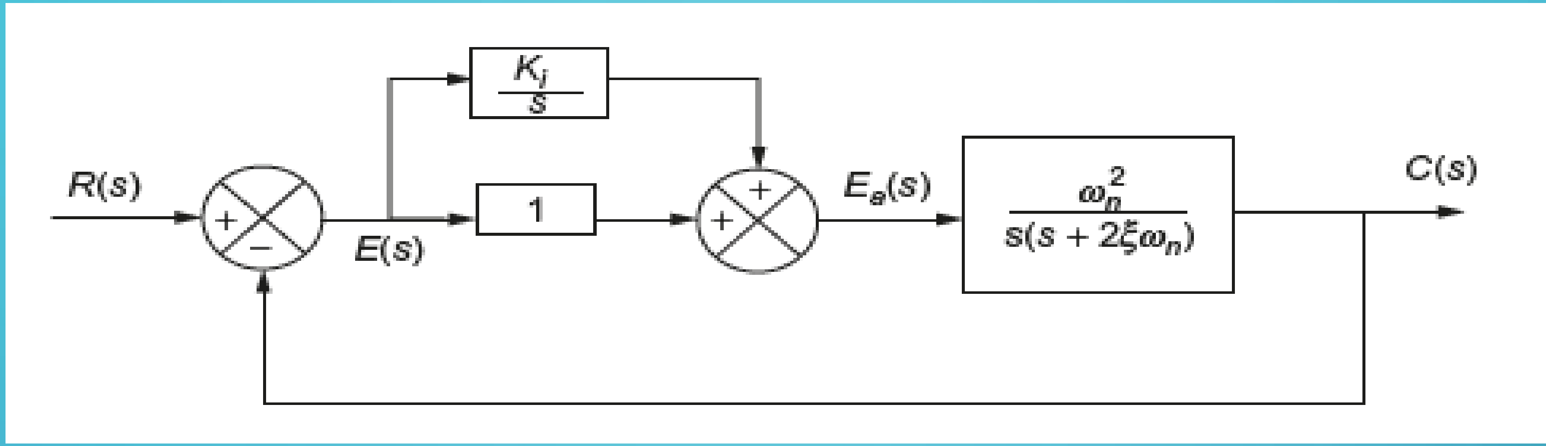
The individual advantages of proportional and integral controller can be used by combining the both in parallel, which results in the pi-controller. The output of the proportional Integral controller (PI-controller) consists of two terms: one proportional to error signal and the other proportional to the integral of error signal.





$$M_s = K_p E_s + K_p K_i \frac{E_s}{S}$$

Response of Second Order System With Proportional Integral Controller:



$$\frac{C_{(s)}}{R_{(s)}} = \frac{(s+K_i)\omega_n{}^2}{s^3+2\zeta\omega_n s^2+\omega_n{}^2 s+K_i\omega_n{}^2}$$

Order of the system has been increased

$$\frac{E_{(s)}}{R_{(s)}} = \frac{s^2(s+2\zeta\omega_n)}{s^3+2\zeta\omega_n s^2+\omega_n{}^2 s+K_i\omega_n{}^2}$$

For Unit Ramp Input, $e_{ss} = 0$

For Unit Parabolic Input, $e_{ss} = \frac{2\zeta}{K_i\omega_n}$

Proportional Integral Controller:

Advantages of PI-controller:

- Eliminates the offset present in the proportional controller
- Provides faster response than the integral controller due to the presence of proportional Controller also.
- Fluctuation of the system around the set point is minimum
- Has zero steady state error
- Form of a feedback control
- Increases the loop gain

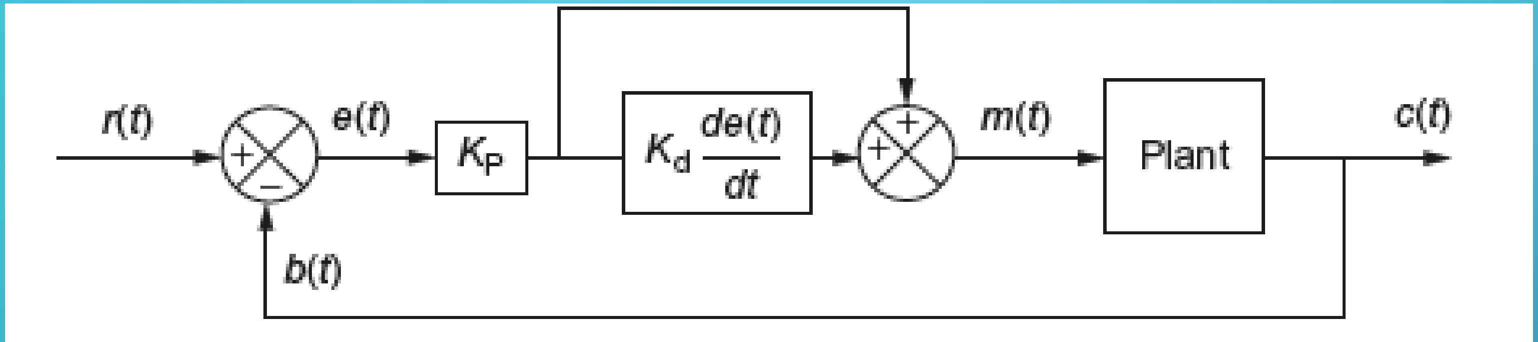
Disadvantages of PI-controller:

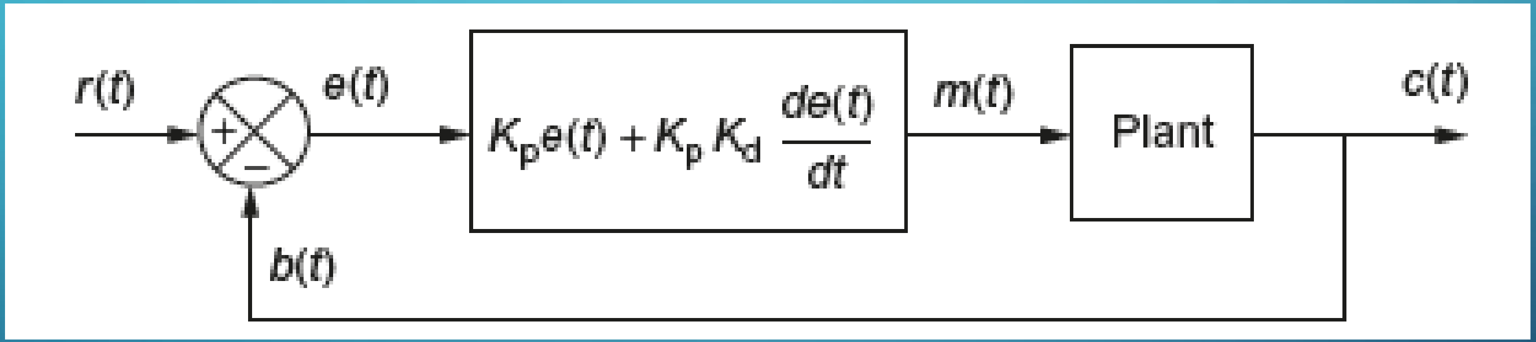
- It has maximum overshoot.
- Settling time is more.

PI controllers Improves Steady State Response

Proportional Derivative Controller:

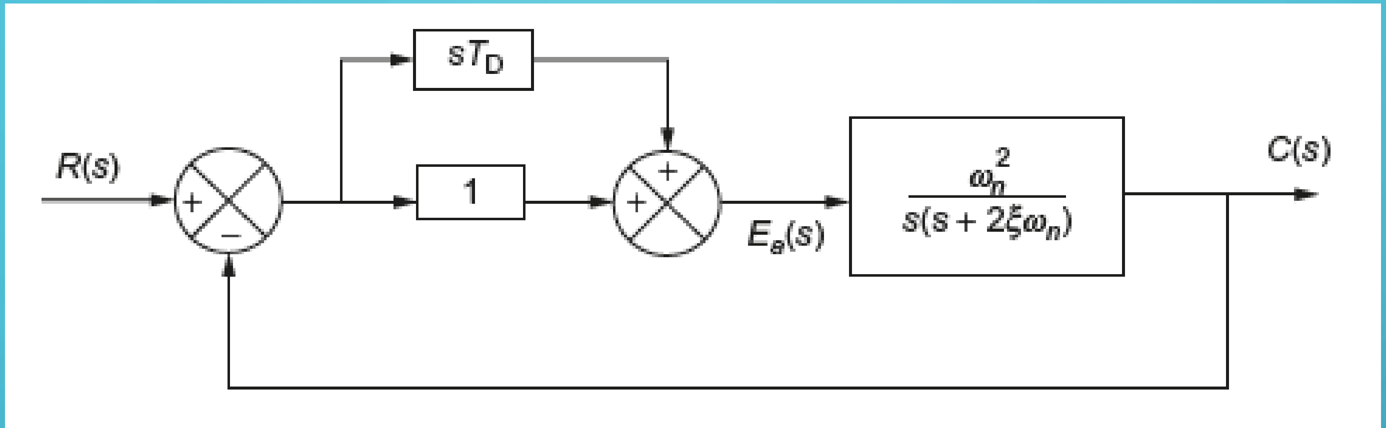
The proportional derivative (PD) controller is used in the system to have faster response from the controller. It combines the proportional and derivative controller in parallel. The output of the proportional derivative controller consists of two terms: one proportional to error signal and the other proportional to the derivative of error signal.





$$M_s = K_pE_s + SK_pK_dE_s$$
$$\frac{M_s}{E_s} = K_p(1 + K_dS)$$

Response of Second Order System With Proportional Derivative Controller:



$$\frac{C_{(s)}}{R_{(s)}} = \frac{(1 + sT_D)\omega_n{}^2}{s^2 + (2\zeta\omega_n + \omega_n{}^2 T_D)s + \omega_n{}^2}$$

$$\zeta' = \zeta + \frac{\omega_n T_D}{2}$$

Effective damping ratio has been increased by using the controller, thereby reducing the maximum overshoot

$$\frac{E_{(s)}}{R_{(s)}} = \frac{s(s + 2\zeta\omega_n)}{s^2 + (2\zeta\omega_n + \omega_n{}^2 T_D)s + \omega_n{}^2}$$

Response of Second Order System With Proportional Derivative Controller:

$$\frac{C_{(s)}}{R_{(s)}} = \frac{(1 + sT_D)\omega_n{}^2}{s^2 + (2\zeta\omega_n + \omega_n{}^2 T_D)s + \omega_n{}^2}$$

$$\zeta' = \zeta + \frac{\omega_n T_D}{2}$$

Effective damping ratio has been increased by using the controller, thereby reducing the maximum overshoot

$$\frac{E_{(s)}}{R_{(s)}} = \frac{s(s + 2\zeta\omega_n)}{s^2 + (2\zeta\omega_n + \omega_n{}^2 T_D)s + \omega_n{}^2}$$

For, $R_{(s)}$ = Unit Ramp

$$E_s = \frac{1}{s^2}\frac{s(s + 2\zeta\omega_n)}{[s^2 + (2\zeta\omega_n + \omega_n{}^2 T_D)s + \omega_n{}^2]}$$

$$e_{ss} = \frac{2\zeta}{\omega_n}$$

Effect of PD:-

1. Damping Ratio - Increased
2. Maximum Overshoot - Decreased
3. Settling Time- Decreased
4. Steady State Error- ?

Proportional Derivative Controller:

Advantages of PD-controller:

- It has smaller maximum overshoot due to the faster derivative action.
- It eliminates excessive oscillations.
- Damping is increased.
- Rise time in the transient response of the system is lower

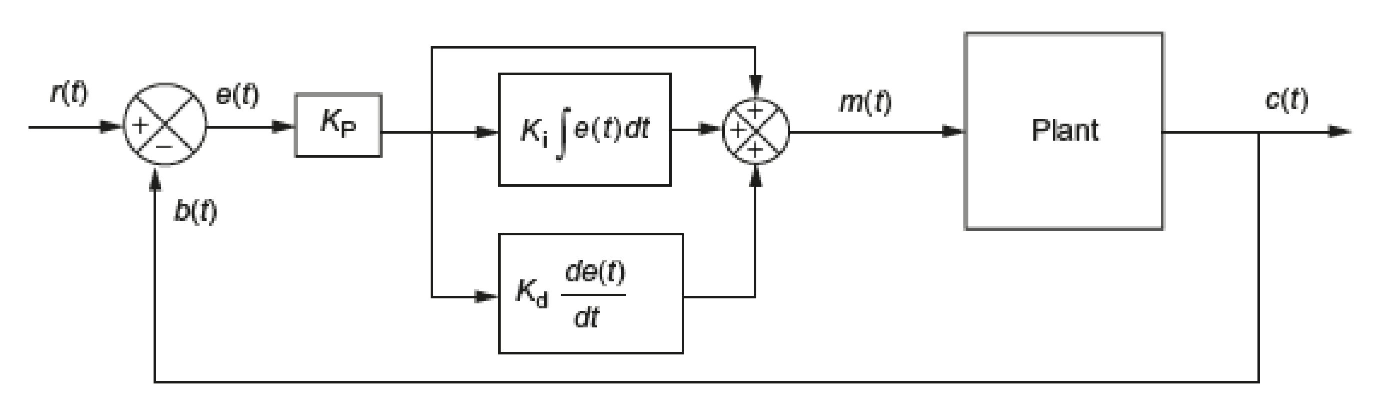
Disadvantages of PD-controller:

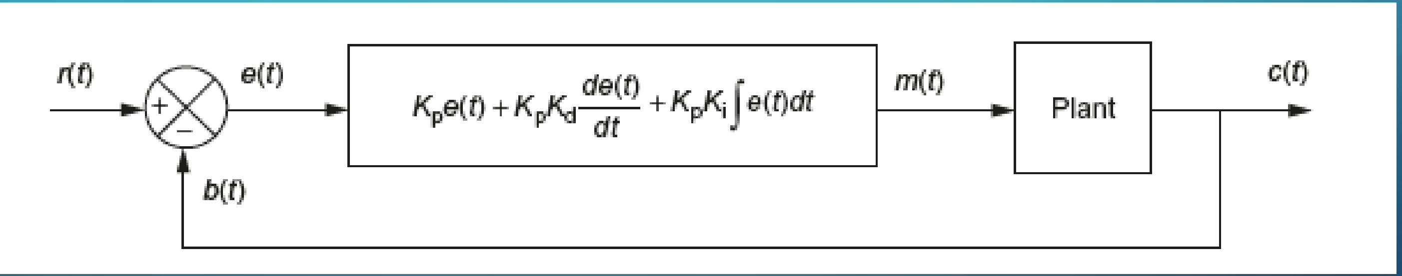
- It does not eliminate the offset.
- It is used in slow systems

PD Controller Improves Transient Response

Proportional Integral Derivative Controller:

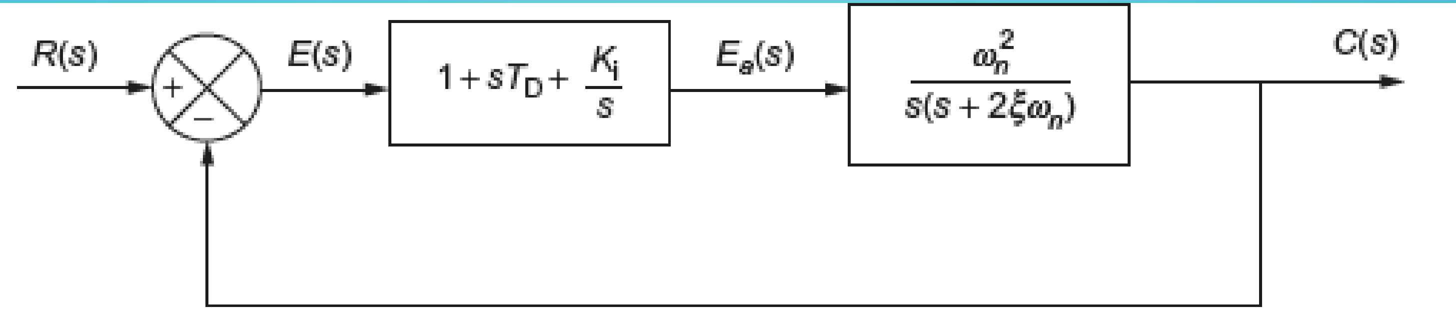
The universally used controller in the control system is the proportional integral derivative (PID) controller. It is the Parallel combination of P I and D-controllers. By tuning the parameters in the PID-Controller, the control action for specific process could be obtained. The output of the PID-controller consists of three terms: first one proportional to error signal, second one proportional to the integral of error signal and the third one proportional to the derivative of error signal





$$M_s = K_p E_s + K_p K_i \frac{E_s}{S} + K_p S E_s$$

Proportional Integral Derivative Controller:



Proportional Integral Derivative Controller:

Advantages of PID-controller

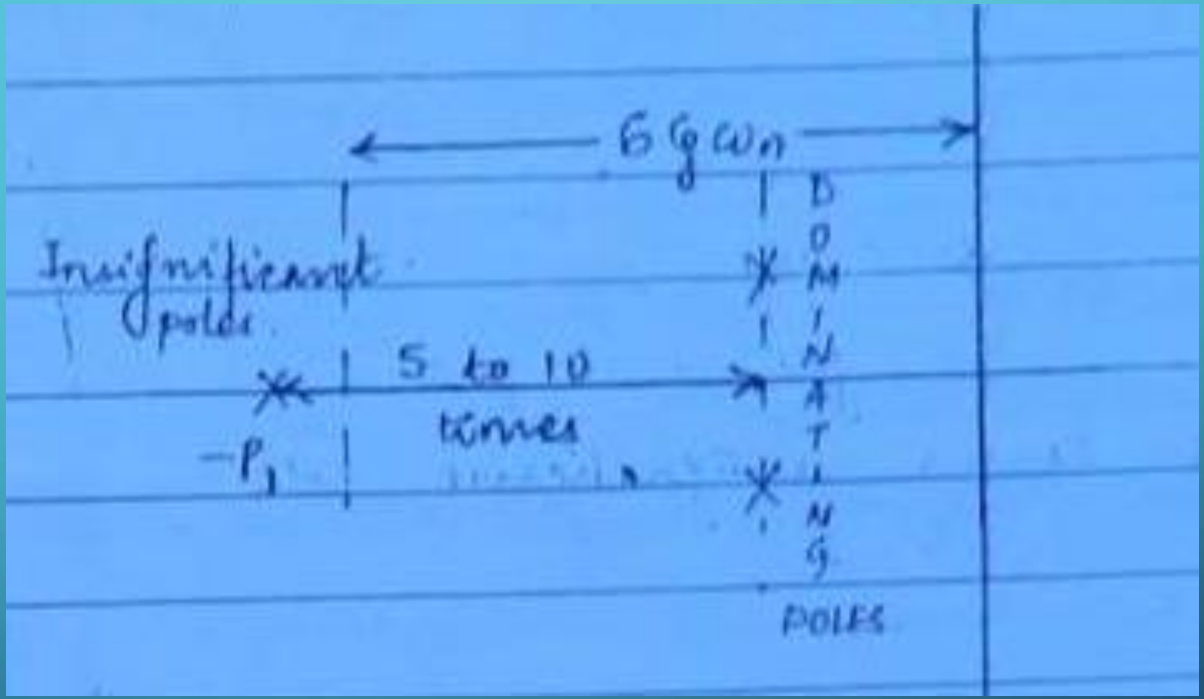
- It reduces maximum overshoot
- Steady-state error is zero.
- It increases the stability of the system.
- It improves the transient response of the system.
- It is possible to tune the parameters in the controllers.

Disadvantages of PID-controller

- It is difficult to use in non-linear systems.
- It is difficult to implement in large industries where complex calculations are required.

PID Improves Transient Response as well as Steady State Response

Analysis of Higher Order System with Approximation:



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