Experiment-1.

Aim:- To study sampling theorem and simulate the above using Matlab/Octave.

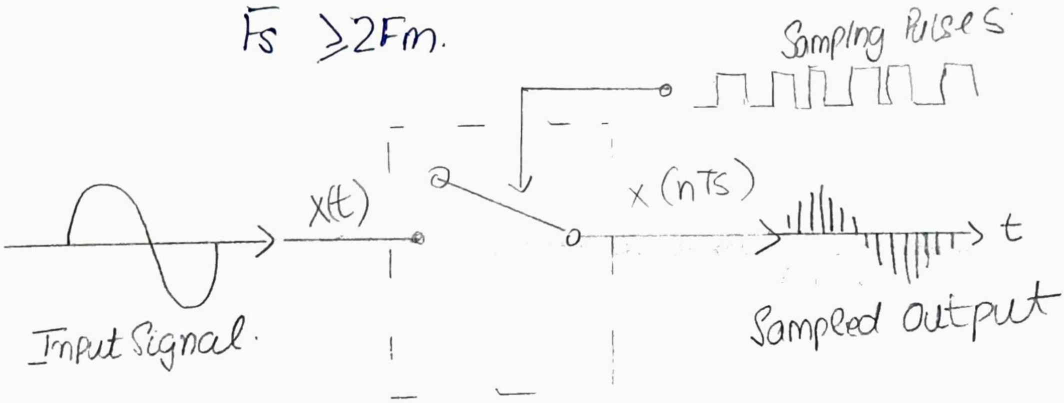
Software Used:- GNU/Octave.

Theory:

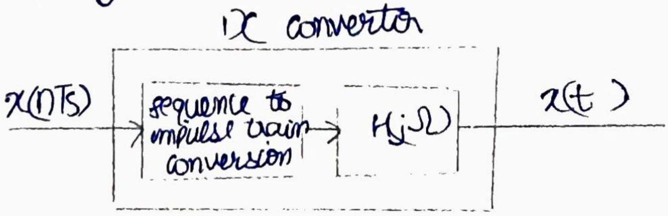
Sampling Theorem: It can be defined as the conversion of an analog signal into a discrete form by taking the sampling frequency as twice the input analog signal frequency. Input signal frequency denoted by F_m & sampling signal frequency denoted by F_S.

Sampling Frequency $F_S = \frac{1}{T_S}$

$F_S \geq 2F_m$.



The process of transforming back the signal (sampled) $x(nT_S)$ to the original input signal $x(t)$ is known as the reconstsuction of the sampling ~~theorem~~ signal.



Experiment - 1

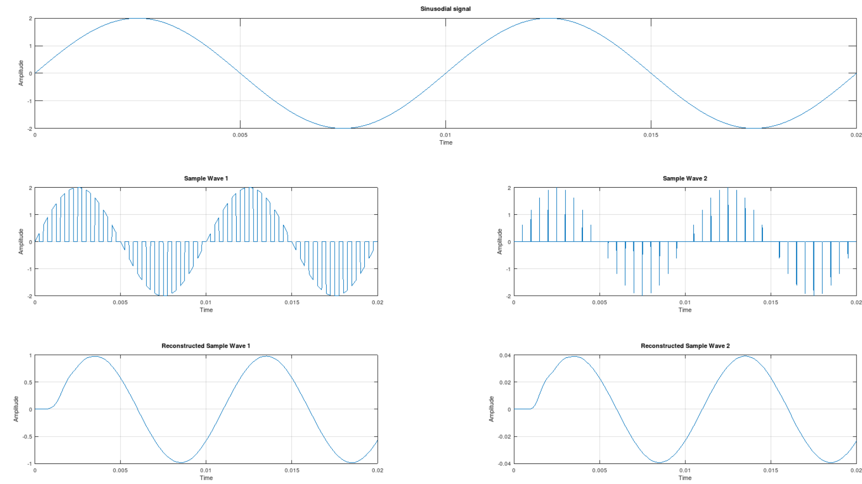
Aim: To Study Sampling Theorem and Simulate the Above using Matlab/Octave.

Code

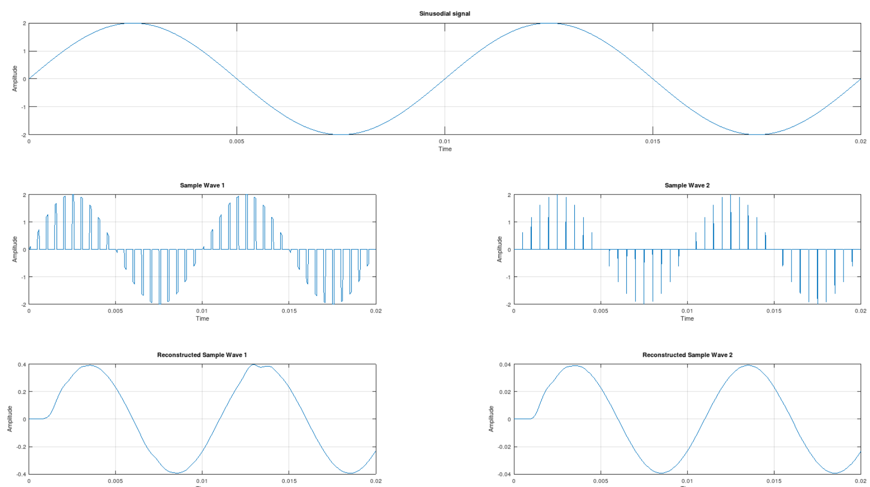
```
% octave pkg to load signal based utils
pkg load signal

clc;
clear alll;
close all;


%Inputs
a = input('Enter the Amplitude: ')
fm = input('Enter the Frequency: ')

fs = 20*fm;
t = 0:1/(1000*fm):2/fm;
s = a*sin(2*pi*fm*t);

% p = (1 + square(2*pi*fs*t, 50))/2;
p = square(2*pi*fs*t, 50);
p(p<0) = 0;

p1 = (1 + square(2*pi*fs*t, 0.1))/2;

sam1 = s.*p;
sam2 = s.*p1;


% Plotting

subplot(3, 1, 1);
plot(t, s);
grid on;
title('Sinusodial signal');
xlabel('Time');
ylabel('Amplitude');

subplot(3, 2, 3);
plot(t, sam1);
grid on;
title('Sample Wave 1');
xlabel('Time');
ylabel('Amplitude');


subplot(3, 2, 4);
plot(t, sam2);
grid on;
title('Sample Wave 2');
xlabel('Time');
ylabel('Amplitude');

% Reconstruction

[n, d] = butter(10, 1/50);
y = filter(n, d, sam1);     %low Pass filtering
y1 = filter(n, d, sam2);

%Plotting

subplot(3, 2, 5);
plot(t, y);
grid on;
title('Reconstructed Sample Wave 1');
xlabel('Time');
ylabel('Amplitude');

subplot(3, 2, 6);
plot(t, y1);
grid on;
title('Reconstructed Sample Wave 2');
xlabel('Time');
ylabel('Amplitude');

%pause in octave
pause
```

Outputs

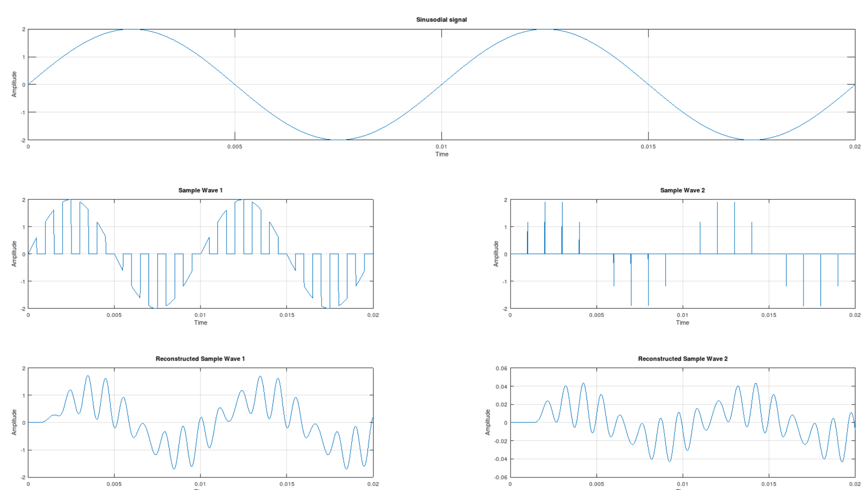
Case 1: Sampling With 50% duty Cycle (No aliasing)



Case 2: Sampling With 20% duty Cycle (Noise in Recovery)



Case 3: Sampling With 50% duty Cycle (Aliasing)



Q. Describe the need of Antialiasing filters.

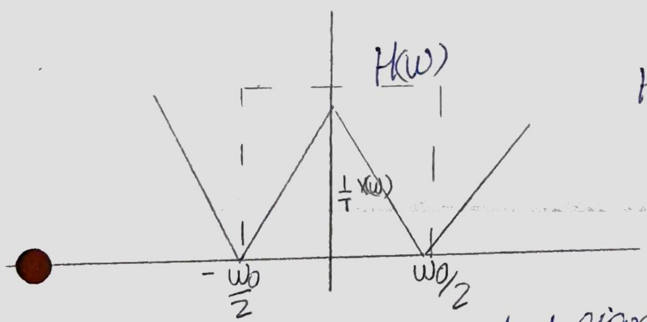
→ During recovery, while designing lowpass filter, it is essential to keep in mind of 2 points:

→ In practise real signal have infinite band width and it is necessary to make it into band limited signal. The real signal is filtered to get power rich signal.

→ Following nyquist rate, the sampling frequency is high, which could seep higher frequency noise components from channel introducing aliasing, the low pass filter removes such high frequency terms.

Q. For the example used in the experiment, explain how low pass filters recovers the message signal. Use spectrum diagram.

→ For the reconstruction of signal, we use a low pass filter to sampled signal.



The filter has a frequency response H(ω) and impulse respons as

$$h(t) = \text{sinc}\left(\frac{t}{T}\right)$$

Thus the reconstructed signal can be given as.

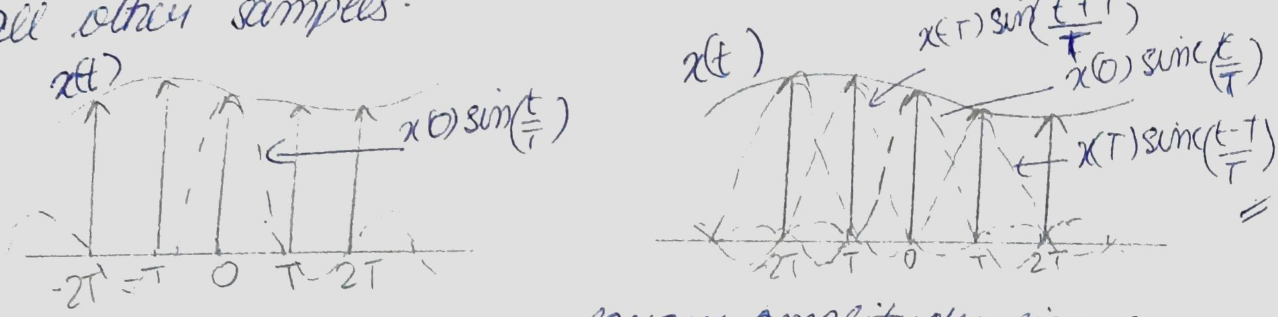
$$\hat{x}(t) = \underbrace{x(t)}_{\text{sampled signal}} * h(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT) * h(t)$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\ \text{sinc}\left(\frac{t-nT}{T}\right)$$

$$x[n] = x(t)\Big|_{t=nT} \rightarrow \boxed{\text{sinc pulse generator}} \rightarrow \hat{x}(t) = \sum_{n=-\infty}^{\infty} x[n]\, h(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} x[n]\ \text{sinc}\left(\frac{t-nT}{T}\right)$$

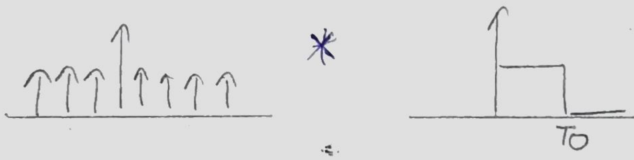
The reconstructed signal is a train of sinc pulses scaled by samples $x[n]$. The interpolated signal is a sum of shifted sincs weighted by the sample $x(nT)$. The sinc function shifted to nT is equal to one at nT & zero at all other samples.



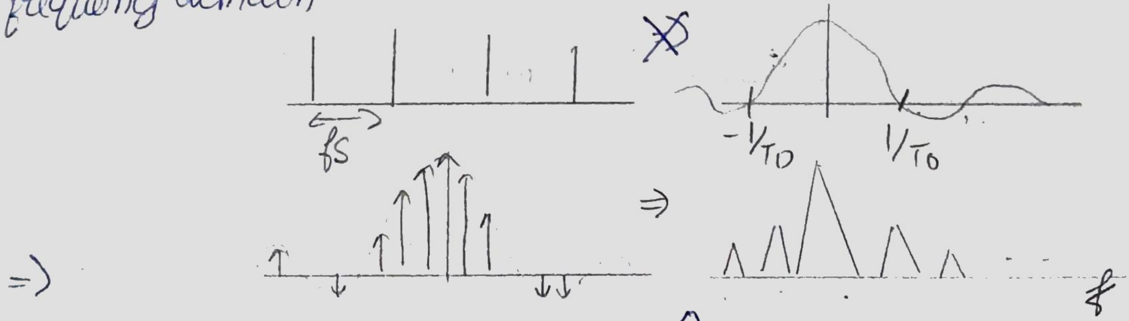
Q3. Explain how we recover larger amplitudes signals from a wave sampled with a higher duty cycle.

→ A wave sampled with a higher duty cycle implies a higher pulse width. Since this pulse is convoluted with the impulse train, it will be multiplied in the frequency domain i.e sinc function.

In time domain:



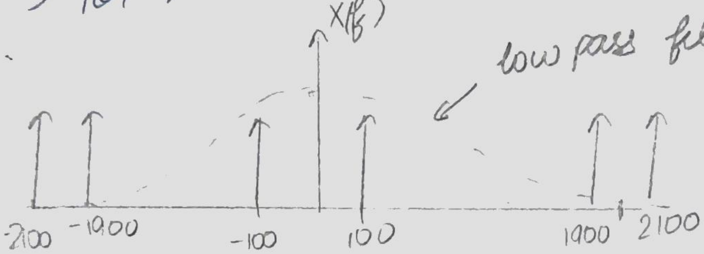
In frequency domain.



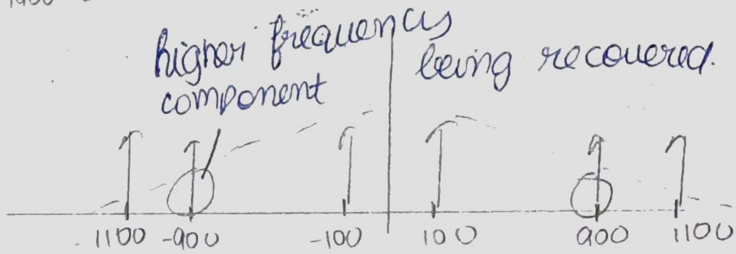
Now as 'T_0' increases, Duty cycle ↑, the sinc waves which has a factor of T_0, starts to achieve a higher central value & rate at which it dies out increases. Therefore the amp. of spectrum after sinc is convolved with the signal ↑. Also recovery power of original signal is increased. This is how larger amplitudes signals are recovered.

Q4. For the recovery system used in the experiment, changing the sampling frequency from 20 fm to 10 fm leads to the extra frequency signal in the recovered signal. Can this phenomenom be called aliasing? Explain.

→ For the initial case of $f_s = 20 \times f_m = 2000 Hz$ ($f_m = 100 Hz$)



If $f_s = 10 \times f_m$.



Since the sampling frequency follows the Nyquist criteria i.e $f_s \geq 2f_m$, this not an case of aliasing. The problem in this case is of use of poor low pass filter. This not as case of improper sampling hence it is not aliasing.

Q5: How can we fix the problem caused in the question above?

→ The poor choice of low pass filter in the above question cause the problem. Using a better performance low pass filter can be used to resolve the problem.

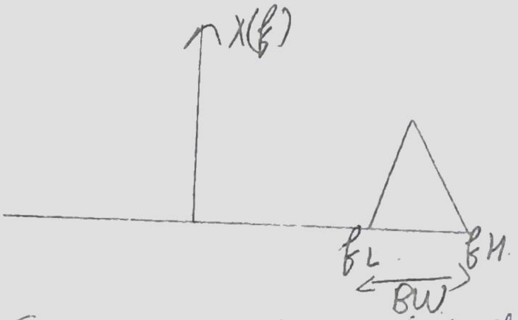
Q6. In practise, what is the sampling frequency used for audio & for speech transmission.

→ Audio transmission :- 44.1 KHz.

→ Speech transmission :- 8KHz.

Q7. Briefly describe the concept of Band pass sampling.

→ A band pass signal is a signal containing a band of frequencies that are not adjacent to (or not centered at) zero frequency i.e lowest frequency in signal. $f_L > 0Hz$



Here, the Bandwidth B(W), lowest frequency (f_L) & highest freq (f_H)

$\Rightarrow \boxed{BW = f_H - f_L}$

For sampling of such signal, the sampling rate (f_s):

$$\boxed{\frac{2f_H}{k} \leq f_s \leq \frac{2f_L}{k-1}}$$; where $k = 1, 2, 3 \ldots N$ and

$N =$ Index of $\left[\frac{f_H}{BW}\right]$

* This type of sampling allows the signal to be sampled at at much lower rate than it is permitted if the Nyquist condition is used:

Eg: A band pass signal with f_L & f_H as 4KHz & 6KHz respectively can be sampled at a rate of 4KHz effectively in contrast to 12 KHz rate required as per Nyquist condition.

Q8. Briefly compare the ideal, flat top & natural sampling.

Ans. Ideal sampling: It is also known as instantaneous or impulse sampling. Train of impulse is used as a carrier. The sampling function is a train of impulses & principle used is multiplication principle

Spectrum of ideally sampled signal is given as:

$$G(f) = f_s\left[\Sigma X(f - nf_s)\right].$$

Flat top Sampling:- This sampling is practical in nature and is easily obtained. The tip of the samples remain constant and is equal to the instantaneous value of the message signal $x(t)$ at the start of the sampling process. Sample & hold circuit are used. Spectrum is given as:

$$G(f) = f_s \cdot \left[\sum X(f - nf_s) \cdot H(f) \right]$$

Natural Sampling: It is also a practical method with pulses having finite equal width T. Sampling is done in accordance with carrier signal (digital in nature).

Spectrum is given as:

$$G(f) = \frac{A\tau}{T_s} \sum \sin((nf_s T) \; X(f - nf_s))]$$

Sampled signal is multiplication of