Experiment-5

Aim:- To find time response specification of second order system

Software Used:- MATLAB 2016.

Theory: The order of a control system is determined by the power of s in the denominator of its transfer function higher order systems are based on second order system.

The general expression for a second order system is:-

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where ξ & ω_n are damping ratio & natural frequency of the system response.

Rise Time: It is the time required for the response to rise from 10% to 90%, 5% to 95% of its final value.

$$t_r = \frac{\pi - \phi}{\omega_d} \; ; \text{ where } \omega_d = \omega_n\sqrt{1-\xi^2} \;\&$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

Peak Time: Time required for the response to reach the first peak of the overshoot

$$t_p = \frac{\pi}{\omega_d}$$

Settling Time: Time required for the response curve to reach & stay within a range of about the final value of size specified by absolute percentage of the final value

$$t_s = \frac{4}{\xi \omega_n}$$

Classification of Second Order System:

Nature of System	Damping ratio	Roots of char. Eq^n	Nature of Roots
Undamped	0	$\pm j\omega_n$	Purely Img
Underdamped	<1	$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$	Complex
Critically Damped	1	$-\omega_n$	Real & Unique
Overdamped	>1	$-\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$	Real & Unequal

Time response:

Case 1 $t = 0$: $G(S) = \dfrac{\omega_n^2}{\omega_n^2 + S^2}$

$$c(t) = \omega_n^2 + e^{-\omega_n t}$$

Case 2 $\xi > 1$: $S_1 S_2 = -\bar{v} + j\omega_d$

$$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} + e^{-\xi n t} \sin \omega_d t$$

Case 3 $\xi = 1$ $G(S) = \dfrac{\omega_n^2}{(S+\omega_n)^2}$ $c(t) = \omega_n^2 + e^{-\omega_n t}$

Case 4 $\xi > 1$: $S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$

$$c(t) = \frac{\omega_n}{2\sqrt{\xi^2-1}} \left(e^{-\xi t} - e^{-\xi\omega_n t}\right)$$