Why Informationally Diverse Teams Need Not Form, Even When Efficient *

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October 15, 2023

Abstract

In recent decades, research organizations have brought the "market inside the firm" by allowing workers to sort themselves into teams. How do teams form absent a central authority? We introduce a model of team formation in which workers first match and then produce correlated signals about an unknown state. While it is efficient to maximize the number of informationally diverse teams, such teams need not form in equilibrium. Our analysis identifies the two sources of matching inefficiency: (i) workers may form diverse teams that are beneficial to its members, but force excluded workers to form homogeneous teams, and (ii) even when a diverse team is efficient, a worker may prefer to join a homogeneous team if she can exert less effort than her teammate. We completely characterize each inefficiency, discuss contracts that restore efficiency, and relate these contracts to observed management practice.

Keywords: Matching, Teams, Information Acquisition, Correlation

JEL Classification: C78, L23, D83

*We thank Aislinn Bohren, SangMok Lee, George Mailath, Steven Matthews, and Andy Postlewaite for support throughout the project. We also thank participants at the UPenn Micro Theory Lunch, UPenn Micro Theory Seminar, the 28th Jerusalem School in Economic Theory, the 2017 Summer School of the Econometric Society, the 2018 ICGT at Stony Brook, and the 2018 LAMES Conference. A previous draft, "Matching To Produce Information: A Model of Self-Organized Research Teams", studies a discrete version of the signal-acquisition game analyzed in this paper. It appears as the Penn Institute for Economic Research paper 18-002 and has been in circulation since 2018.

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1 Introduction

1.1 Background

Self-organized teams are playing an increasingly important role in economic activity. From 1987 to 1996, the fraction of Fortune 1000 firms with workers in self-managed work teams rose from 27 percent to 78 percent (Lawler, Mohrman and Benson (2001) and Lazear and Shaw (2007)). More recently, a 2016 survey of more than 7,000 executives in over 130 countries indicates that organizations are increasingly operating as a network of teams in which workers engage in self-directed research (Deloitte, 2016). These human resources trends are particularly important in organizations such as Universities (Wuchty, Jones and Uzzi (2007)) and large technology companies, like Google and Amazon, that rely on flexible internal labor markets in order to take advantage of informational complementarities among workers with diverse backgrounds. Yet while the free-ridership problem within teams has garnered considerable theoretical attention (see, for instance, Holmström (1982), Legros and Matthews (1993), and Winter (2004)), less has been devoted to the study of how moral hazard within teams affects matching. Furthermore, little existing work studies this interaction in the context of the production of information.[1]

The case of the Danish hearing-aid manufacturer Oticon illustrates well these broad trends in research and development, as well as the incentive problems that arise when decision making is delegated to productive actors themselves (see Foss (2003) for a comprehensive account). In 1987, Oticon lost almost half of its equity when its competitors began selling cosmetically superior devices. In an attempt to regain its competitive advantage, Oticon re-structured its research department, replacing vertical, hierarchical production with horizontal, project-based team production (Foss (2003) coins this organizational form a *spaghetti organization*). Beyond cosmetic changes to the office spaces — desks were no longer permanent and were located in large open spaces — there was extensive delegation of decision rights. Most notably, employees chose which projects (teams) they would join and had discretion over their compensation.

[1] Subsequent to the first circulated draft of this paper, Kaya and Vereshchagina (2022) study the optimal sorting of workers to teams who differ in their ability to acquire information and engage in team production. As in Chade and Eeckhout (2018), they focus on the sorting of workers of differing expertise into teams, whereas we hold this dimension fixed and consider heterogeneity in the correlation of workers' information. Kambhampati and Segura-Rodriguez (2022) study the optimal allocation of workers to teams in a standard production setting in the presence of both moral hazard and adverse selection. They then identify when decentralized sorting is an optimal organizational structure.

At first, these organizational changes were profitable. Eliminating hierarchies and allowing workers to lead their own teams enabled the firm to take advantage of the existing information dispersed among its workers (Kao, 1996).[2] However, new problems arose. First, some teams were far better than others "in terms of how well the team members worked together and what the outcome of team effort was" (Larsen, 2002). Second, competition meant that "anybody [at a project] could leave at will, if noticing a superior opportunity in the internal job market" (Foss, 2003). These problems eventually led Oticon to introduce a company-wide employee stock option program and selectively intervene in the assignment of roles to workers within teams, designating particular workers as project managers.[3]

1.2 This Paper

We posit a model of moral hazard and matching in the context of information production to better understand the managerial problems faced by firms that decentralize information production and to rationalize management solutions observed within companies like Oticon. In the setting we study, workers form teams (match) in order to forecast the value of a Gaussian state. Each worker then acquires any number of costly Gaussian signals about it. Pairwise correlations between signals in a team can be positive, i.e., the team is *homogeneous*, or negative, i.e., the team is *diverse*. After observing all signals produced within a team, each team guesses the state and each worker receives a payoff proportional to the quadratic distance between her team's forecast and the state realization.

The baseline matching environment features imperfectly transferable utility (Legros and Newman (2007)); one worker cannot compensate another for producing more or less signals than her and a team's profit is divided equally. The literature on partnerships

[2]Oticon's CEO commented that decentralization "improved markedly [Oticon's] ability to invent new ideas, concepts, and make use of what [Oticon] actually [had]" (Kao, 1996). In particular, the firm was able to revive old projects that later turned out to be profitable.

[3]While a prominent example, Oticon is not the only company to have experimented with decentralized research teams and had problems. In 2012, the multibillion-dollar video-game developer Valve publicly released a New Employer Handbook describing the company's non-hierarchical organizational structure. Valve's co-founder adopted this approach in the hope of spurring the company's research and innovation (Keighley, 2020). But, once again, decentralization led to new problems. First, talented workers refused to leave prestigious projects, and it became hard for other projects to recruit them. Second, the flat management model gave workers latitude to "minimize their work" because of the lack of "checks and balances" (Grey, 2013). In 2014, GitHub introduced a middle-management level to supervise its previously unsupervised allocation system of workers to teams (Rusli, 2014). More recently, in 2016, Medium abandoned its use of holocracy, a system "designed to move companies away from rigid corporate structures and toward decentralized management and dynamic composition" (Doyle, 2016).

(see, for instance, Farrell and Scotchmer (1988)) has argued that social convention and social norms might rule out unequal division of surplus even if transfers within teams are permitted. Indeed, we need not look further than our own profession to see that economics scholars receive equal credit for joint work, even if work is not divided equally. As in Farrell and Scotchmer (1988), our interest is in identifying the social-cost that equal-sharing rules play in generating utilitarian welfare losses when sorting is endogenous. Utilitarian welfare is the relevant efficiency notion because, if it is not maximized, then there is a clear way to improve the welfare of all workers even without interfering with the convention of equal-sharing. Specifically, the firm can simply re-assign workers and compensate them directly for the gain and/or loss accrued to them due to their change in teammate. The additional surplus generated by this change can either be distributed among the workers or be pocketed by management.

We identify and completely characterize the two channels leading to matching inefficiency. First, productive, diverse teams composed of workers producing negatively correlated signals may form at the expense of excluded workers who must form homogeneous teams whose workers produce positively correlated information. We call this phenomenon *stratification inefficiency*, which coheres with the observation that teams inside flat organizations tend to be unequal in productivity and with the existing literature on matching with nontransferabilities. Second, diverse teams may not form even when efficient; a worker in such a team may prefer to join a homogeneous team if, in this deviating team, she can exert less effort. We call this phenomenon *asymmetric effort inefficiency*, which rationalizes observations of unequal effort between employees in the same team and which, to our knowledge, has not been systematically studied in the literature. Because both inefficiencies we identify resemble those that led management at Oticon to abandon decentralized sorting, we then seek to rationalize Oticon's organizational responses.

1.3 Overview of Analysis

The formal analysis proceeds as follows. First, we characterize the Nash equilibrium correspondence of the signal-acquisition game played within teams in order to determine each team's payoff frontier. We identify a cutoff value on the (state-conditional) pairwise correlation between workers' signals that orders within-team Nash equilibria in terms of their symmetry. Intuitively, more positively correlated signals contain more redundant information. Thus, the marginal value of producing a signal when one's teammate has already produced one is decreasing in correlation. It follows that, when signals are positively correlated, i.e., the team is homogeneous, there is a unique asymmetric equilibrium up to worker identity. In it, one worker produces all of the team's information, while the other free-rides off her production. Conversely, when signals are negatively correlated, i.e., the team is diverse, there is a unique symmetric equilibrium in which effort is matched (Proposition 1).

We then study the welfare efficiency of self-enforcing matchings and effort levels, i.e., core allocations. We first show that an allocation is efficient if and only if it is *maximally diverse*, i.e., if and only if the matching forms as many diverse teams as possible (Proposition 2). We next formally define stratification inefficiency and asymmetric effort inefficiency and characterize the correlation structures under which each arises. Our main result, Proposition 3, shows that these two inefficiencies are exhaustive and identifies necessary and sufficient conditions under which each arises.

Our characterization has two parts. First, we show that whether stratification inefficiency or asymmetric inefficiency arises depends on the degree to which diverse teams can exploit informational complementarities. Specifically, there is a cutoff correlation below which a worker would rather form a diverse team than free-ride in a homogeneous team, and above which the opposite holds. Below this cutoff, all inefficient core allocations are characterized by stratification inefficiency, while, above it, all inefficient core allocations are characterized by asymmetric effort inefficiency. Second, we provide necessary and sufficient graph-theoretic conditions on the correlation structure among workers that result in each inefficiency. Specifically, we define a simple graph whose vertices equal the set of workers and whose edges link workers whom compose diverse teams. We then identify necessary and sufficient conditions on the graph under which each type of inefficiency arise.

Two interesting applied insights emerge from this analysis. First, we find that it is not enough for this graph to be connected to prevent stratification inefficiency. Instead, there must exist a cycle through all workers in the firm; formally, there must be a *Hamiltonian cycle*. Second, simply adding more diverse connections within an organization need not improve efficiency; the entire matrix of information matters. For instance, when diverse teams are sufficiently unproductive, constructing a Hamiltonian cycle can generate asymmetric effort inefficiency when it did not exist previously. Our results thus suggest a rich interaction between the productivity of diverse teams, on one hand, and the network structure inside of an organization, on the other.

Our final result, Proposition 4, shows that if management can selectively intervene in team formation by controlling how team contracts are written, then asymmetric effort inefficiency can be eliminated. This coheres with Foss (2003)'s account of Oticon's retreat from decentralization; in 1996, Oticon developed a "Competence Center" which dictated wages within teams. Nevertheless, under efficient bilateral contracting, the range of parameters under which endogenous sorting leads to stratification inefficiency increases. Hence, giving workers bonus pay that depends on the performance of other teams is required to restore sorting efficiency. In practice, Oticon's CEO introduced an employee stock option program that might be rationalized as an attempt to have workers internalize the externalities they might generate on residual matches when forming individually profitable teams.

1.4 Related Literature

Team Theory. Chade and Eeckhout (2018) study the optimal assignment of workers to teams in the same (canonical) Gaussian environment that we consider, but with two important differences: (i) each worker produces exactly one signal within a team and (ii) utility is transferable. In our environment, in contrast to (i), workers can acquire any number of signals and, in contrast to (ii), utility is non-transferable. This first difference means that the utility of workers in a team are affected not only by their pairwise correlation, but also the number of signals each worker (endogenously) acquires (see (1), which subsumes the formula for the value of a team in Chade and Eeckhout (2018) when each worker acquires a single signal). The second difference allows us to study the impact of moral hazard on sorting, a "relevant open problem with several economic applications" (Chade and Eeckhout, 2018). Our analysis, consequently, focuses on the *efficiency* of equilibrium teams as opposed to their assortativity, as is the focus of Chade and Eeckhout (2018).[4]

An additional difference between our setup and that of Chade and Eeckhout (2018) is that they assume that signals between workers possess a *common* correlation parameter, but differ in variance, whereas we assume the opposite. We make this assumption to capture research settings in which workers are identical in their level of "expertise", but may come from different backgrounds. Our work, therefore, contributes to the literature on

[4]As the latter question is of independent interest, however, we study this question in a previous working paper Kambhampati, Segura-Rodriguez and Shao (2021). Fixing the signal structure of Chade and Eeckhout (2018), we show that, once effort choice is endogenous, optimal matching must simultaneously diversify, while incentivizing effort.

diversity in teams, i.e., Prat (2002), Hong and Page (2001), and Hong and Page (2004).[5] In particular, asymmetric effort inefficient core allocations are characterized by excessive homogeneity, i.e., high correlation, within teams. Our results thus illustrate a new channel through which moral hazard can cause homogenous teams to form even diverse teams are efficient.

Sorting and Bilateral Moral Hazard. Legros and Newman (2007) consider general two-sided matching environments in which, for each matched pair, there is an exogenously specified utility possibility frontier.[6] Our paper joins a small literature that considers matching settings in which the utility possibility frontier of each matched pair is affected by the presence of bilateral moral hazard.[7] Kaya and Vereshchagina (2015) study one-sided matching between partners who, after matching, play a repeated game with imperfect monitoring (due to moral hazard) and transfers. While moral hazard limits the achievable joint surplus attainable by a matched pair, transfers ensure that the Pareto-frontier is linear, i.e. payoffs are transferable. Hence, stable matchings exist and (constrained) efficiency is ensured by standard arguments, in contrast to our setting.[8]

Vereshchagina (2019) studies two-sided matching between financially-constrained entrepreneurs in the presence of bilateral moral hazard and incomplete contracts; entrepreneurs can only sign contracts under which the realized revenue is split between the partners according to an equity-sharing rule.[9] Non-transferability of output gives rise to inefficient positive sorting through the following channel: wealthy entrepreneurs, whom con-

[5] Prat (2002) finds conditions under which a team should be composed of homogenous information structures when these information structures are priced according to market forces. Hong and Page (2004) and Hong and Page (2001) consider the performance of heterogeneous non-Bayesian problem solvers. In contrast, we consider the *endogenous* formation of teams by Bayesian workers within a firm with a fixed information structure.

[6] A well-known application of this framework is to risk-sharing within households. Legros and Newman (2007) and Chiappori and Reny (2016) show that if couples share risk efficiently, then all stable matchings are negative assortative. Gierlinger and Laczó (2018) show that if the assumption of perfect risk-sharing is relaxed, then positive assortative matching can occur. Schulhofer-Wohl (2006) finds necessary and sufficient conditions for preferences under which risk-sharing problems admit a transferable utility representation.

[7] Wright (2004), Serfes (2005), Serfes (2007), and Sperisen and Wiseman (2016) study the assortativity of stable matchings in the presence of one-sided moral hazard, i.e. principals matching agents. For more recent contributions to this literature, see Section 5.2 of Chade and Swinkels (2020) and Chade and Eeckhout (2022).

[8] Kaya and Vereshchagina (2014) study a special case of their model in which workers form partnerships that may involve "money burning" to provide incentives. They then ask whether workers would prefer to work for an entrepreneur, i.e. hire a budget-breaker, as in Franco, Mitchell and Vereshchagina (2011) to avoid this problem. Chakraborty and Citanna (2005) consider a model similar to that of Kaya and Vereshchagina (2015) in which partners play asymmetric roles.

[9] Two-sidedness again ensures that a stable matching exists, in the sense of Legros and Newman (2007), unlike in our setting.

tribute more resources to joint production, are willing to form partnerships with poor entrepreneurs only if they receive a high equity share. But, joint surplus maximizing equity shares may be constant across all partnerships. Hence, wealthy entrepreneurs prefer to match even if the overall benefit of re-matching with poor entrepreneurs is large. The logic behind inefficiency thus resembles that of Stratification Inefficiency.[10]

Finally, Kräkel (2017) considers a very different channel through which moral hazard leads to inefficient endogenous sorting. He studies an environment in which a firm posts an initial contract that determines both wages and a sorting protocol (workers either endogenously sort into teams or are randomly assigned to teams). The firm then receives interim information about the efficiency of the matches formed and can re-negotiate the initial contract. Under endogenous sorting, workers may form inefficient teams in order to force the firm to re-negotiate the initial contract.

Correlation and Information Acquisition. More broadly, our analysis of the information acquisition game played within teams is related to recent work defining notions of complementary and substitutable information. In the environment we consider, lower correlation implies higher complementarity in terms of the value of information. Börgers, Hernando-Veciana and Krähmer (2013) define signals as complements or substitutes in terms of their value across *all* decision problems, therefore requiring stronger conditions. Liang and Mu (2020) adapt the definition of Börgers, Hernando-Veciana and Krähmer (2013) to a multivariate Gaussian environment and use it characterize the learning outcomes of a sequence of myopic players.

2 Model

There are four workers, indexed by the set $\mathcal{N} := \{1, 2, 3, 4\}$, who cooperatively form teams of two. Each team completes a project. This project involves guessing the value of a state θ, which has a standard Gaussian distribution. The final assignment of workers to teams is described by a (matching) function $\mu : \mathcal{N} \to \mathcal{N}$ with the property that the teammate of worker i's teammate, j, is i — that is, if $j = \mu(i)$, then $\mu(j) = i$. Let $\mathcal{M}$ denote the set of all such functions.

[10]We note, however, that there is no analog to Asymmetric Effort Inefficiency in her model. A related, earlier contribution is that of Sherstyuk (1998), who shows that equal-sharing equity rules may preclude efficient heterogeneous partnerships.

2.1 Production Subgame

We first define a game, parameterized by K, in which each worker acquires a discrete number of signals. We then define a continuous limit game, obtained as $K \to \infty$. Our analysis focuses on the limit game due to its superior tractability. In a previous working paper Kambhampati, Segura-Rodriguez and Shao (2021), we analyze the properties of K-discrete games and characterize their (qualitatively similar) equilibria.

In each game parameterized by K, workers produce unbiased, conditionally independent Gaussian signals with variance $\frac{\sigma^2}{K}$. However, the signals of workers in the same team are correlated. In our baseline analysis, we assume that, for any distinct workers i and j, $\rho_{ij} \in \{\rho_\ell, \rho_h\}$ is the state-conditional correlation coefficient between worker i's and worker j's signals. In addition, we assume that (i) $-1 < \rho_\ell < 0 < \rho_h < 1$ and (ii) for each $\rho \in \{\rho_\ell, \rho_h\}$ there exist workers i and j with $\rho_{ij} = \rho$. We call a team (i,j) **diverse** if $\rho_{ij} = \rho_\ell$, as workers in such a team produce negatively correlated signals, and **homogeneous** if $\rho_{ij} = \rho_h$, as workers in such a team produce positively correlated signals.[11] Each signal costs $0 < c < \min\{\sigma^{-2}, \sigma^2\}$ to produce[12] and the number of signals worker i produces, n_i, belongs to a grid $\{0, 1, 2, ..., KM\}$, where $M > \sqrt{\sigma^2}/\sqrt{c}$.[13]

The correlation structure captures the economics of a situation in which matched effort is affected by complementarities, while unilateral effort is not. In particular, if $n_i \geq n_j > 0$, then workers i and j produce n_j conditionally correlated signals and worker i produces $n_i - n_j$ signals, each of which is uncorrelated with all other signals.[14] After observing the signal realizations of every team member, team (i,j) takes an action $a^* \in \mathbb{R}$ to minimize the expected value of a quadratic loss function. Formally,

$$a^* \in \arg\max_{a \in \mathbb{R}} E_\theta \left[1 - (a - \theta)^2 \mid x^S \right],$$

where x^S denotes the concatenation of signals observed in the team. When a team's payoff

[11] The terms "diverse" and "homogeneous" should thus be interpreted in relative, rather than absolute, terms. A team with a correlation coefficient that is positive, but close to zero, might be considered a "diverse" team relative to teams outside of the firm.

[12] The condition on the cost function ensures that at least one worker has an incentive to acquire a strictly positive number of signals and that acquiring negatively correlated signals is sufficiently valuable.

[13] The choice of M ensures that the upper bound never binds.

[14] Diminishing marginal returns and complementarity are separate forces shaping workers' information acquisition strategies in our model. While acquiring more signals reduces the marginal productivity of acquiring one's own signals, correlation between signals across workers captures the degree of complementarity of information. These are plausible forces in information-producing teams that, nevertheless, are not easy to analyze separately in reduced-form production models. Our specification allows us to analyze how moral hazard is shaped by complementarities, holding fixed individual marginal returns.

is shared equally, worker i's payoff in team (i,j) given (n_i, n_j) is thus

$$u_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - E_x\left[\min_{a\in\mathbb{R}} E_\theta\left[(a-\theta)^2 \mid x^S\right]\right]\right) - \frac{c}{2K} n_i.$$

By simplifying the posterior variance, $E_x\left[\min_{a\in\mathbb{R}} E_\theta\left[(a-\theta)^2 \mid x^S\right]\right]$, Appendix A.1 establishes that worker i's payoff function is equal to

$$u_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - \frac{\sigma^2}{K}\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \frac{\sigma^2}{K}\right)^{-1}\right) - \frac{c}{2K} n_i, \tag{1}$$

where $\underline{n}_{ij} = \min\{n_i, n_j\}$ and $\bar{n}_{ij} = \max\{n_i, n_j\}$. Notice that if n_i is interpreted as producing $\frac{n_i}{K}$ signals, then we can re-write (1) as

$$v_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - \sigma^2\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \sigma^2\right)^{-1}\right) - \frac{1}{2} c n_i. \tag{2}$$

Put differently, each K-discrete game is strategically equivalent to one in which each worker chooses a fraction of signals $n_i \in \{0, \frac{1}{K}, \ldots, M\}$ and payoffs are defined by (2).

We study the limit game of the K-discrete games as K approaches infinity. Specifically, within each team, we assume that, for each worker i, the strategy space is $[0, M]$ and the payoff function is as defined in Equation 2. This normal-form game is called the **Production Subgame**. In Appendix A.2, we provide justification for the study of this limit game; we show that, as $K \to \infty$, the strategy space for each worker in the re-parameterized K-discrete game, $\{0, \frac{1}{K}, \frac{2}{K}, \ldots, M\}$, converges to the real interval $[0, M]$. In addition, we show that the set of Nash equilibria converges to the set of Nash equilibria in the continuous game in the Hausdorff metric, i.e., the equilibrium correspondence is both upper- and lower- hemicontinuous in K. Hence, strategic behavior in the limit game is appropriately interpreted as an approximation of strategic behavior in nearby, discrete signal-acquisition games.

2.2 Solution Concept

A signal-acquisition strategy for worker i is a function mapping teammate identity to a non-negative real number of signals, $n_i : \mathcal{N}\setminus\{i\} \to [0, M]$. Denote the profile of signals chosen within team (i,j) by $n(i,j) := (n_i(j), n_j(i))$. In each team (i,j), we require that the strategy profile $n^*(i,j)$ is a Nash equilibrium of the corresponding Production Subgame. For the two-stage game, we use the core as our solution concept, the standard solution concept in the literature on matching with imperfectly transferable utility.[15] In these settings, an **allocation** is a pair (μ, N^*), where $\mu \in \mathcal{M}$ is a matching and $N^* = \{n^*(i,j)\}_{i,j=\mu(i)}$ is a collection of within-team Nash equilibria. An allocation is in the core if no pair can match and play a Nash equilibrium that makes both strictly better off than under the initial allocation. A formal definition follows below. To ease notation, we denote $n_i(j)$ and $n_j(i)$ by n_i and n_j and drop the dependence of v_i in Equation 2 on ρ_{ij} whenever there is no confusion that j is i's teammate.

Definition 1. *An allocation (μ, N^*) is in the* **core** *if there does not exist a matching, $\hat{\mu} \in M$, a worker i with match $j = \hat{\mu}(i)$, and a Nash equilibrium $(\hat{n}_i, \hat{n}_j)$ for which*

$$v_i(\hat{n}_i, \hat{n}_j) > v_i(n_i^*, n_{\mu(i)}^*) \quad \textit{and} \quad v_j(\hat{n}_i, \hat{n}_j) > v_j(n_i^*, n_{\mu(j)}^*).$$

3 Production Subgame Analysis

We begin our analysis by characterizing Nash equilibria played within a team (i,j) with arbitrary correlation $\rho \in (-1,1)$.

3.1 Marginal Value of Information

To develop intuition, we first define and analyze the **marginal value of information** generated by worker i given a signal profile $n(i,j)$. Recall that agent i's ex-post variance in team (i,j) is

$$\frac{1}{2}\left(1 - \sigma^2\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \sigma^2\right)^{-1}\right).$$

So, the marginal value of information is

$$MV(n_i; n_j, \rho) := \frac{1}{2}\sigma^2\left(\frac{\frac{\partial}{\partial n_i}\left(\underline{n}_{ij}\left(\frac{1-\rho}{1+\rho}\right) + \bar{n}_{ij}\right)}{\left(\underline{n}_{ij}\left(\frac{1-\rho}{1+\rho}\right) + \bar{n}_{ij} + \sigma^2\right)^2}\right) > 0,$$

where the bracketed term is the marginal reduction in ex-post variance. If $n_i \geq n_j$, we call worker i a **high producer**. In this case, additional information produced by worker i is novel; it is uncorrelated with existing information. Consequently, the marginal value of information is affected by the correlation coefficient between the two workers only

[15] See Legros and Newman (2007) for a general definition in two-sided environments and Kaya and Vereshchagina (2015) for a definition in a one-sided environment.

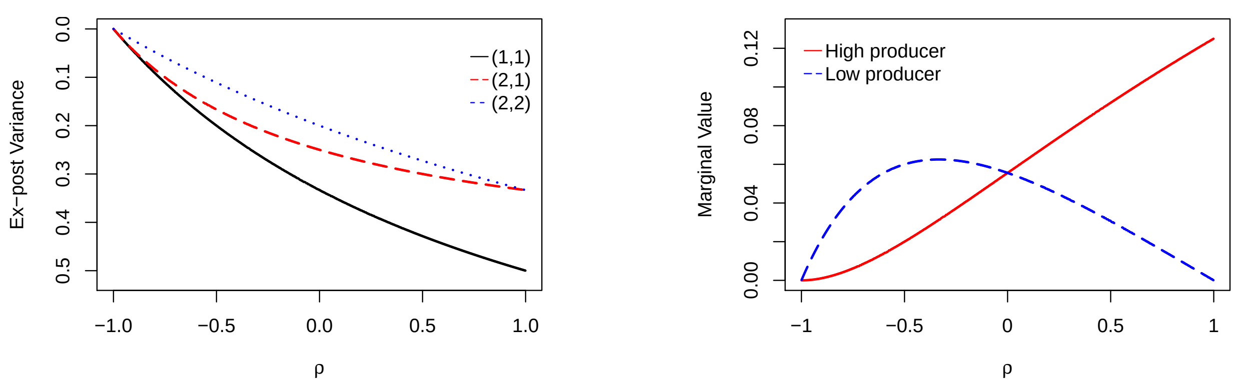
through the value of existing information:

$$MV(n_i; n_j, \rho) = \frac{1}{2}\sigma^2 \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^{-2}.$$

Conversely, if $n_i > n_j$, we call worker j a **low producer**. Consequently, the marginal value of information for j is affected by the correlation coefficient between the two workers both through the value of existing information and the value of matching worker i's information:

$$MV(n_j; n_i, \rho) = \frac{1}{2}\sigma^2 \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^{-2} \left(\frac{1-\rho}{1+\rho} \right).$$

Figure 1 illustrates comparative statics on the marginal value of information with respect to the correlation coefficient ρ, fixing $\sigma^2 = 1$. In Figure 1a, we plot ex-post variance under various strategy profiles (n_i, n_j). We make a number of observations. First, for a fixed number of draws, ex-post variance is increasing in ρ, i.e., higher values of ρ correspond to lower complementarities in information. Second, as ρ approaches -1, if both workers acquire a positive amount of information, then ex-post variance shrinks to zero. Third, when $\rho = 1$, a low producer generates completely redundant information and ex-post variance does not decrease at all.



(a) Ex-post Variance.

(b) Marginal Value of Second Signal.

Figure 1: Ex-post Variance and Marginal Values.

We further explore these intuitions in Figure 1b. The solid red line represents a high producer's marginal value at the strategy profile $(1,1)$; formally, it is the right-derivative of the marginal value of information at $(1,1)$. The dashed blue line represents the low producer's marginal value; formally, it is the left-derivative of the marginal value of information at $(1,1)$.

We, again, make three observations about Figure 1b, which generalize beyond the parameterization we consider, and which we exploit in the analysis. First, the marginal value of information to the high producer is strictly increasing in ρ. This happens because the value of the information obtained from working together with the low producer *decreases* in ρ. By concavity of the information production function, the marginal value of information left to learn increases.

Second, the marginal value of information to a low producer is non-monotonic in ρ. Indeed, we see that the blue line in Figure 1b is hump-shaped. The marginal value of the low producer is increasing in an initial region for the same reason the high producer's marginal value is increasing; when ρ increases, the value of work done together decreases and so the marginal value of information left to learn increases. However, there is another effect to consider. When ρ increases, the value of additional matched work *decreases* — information is less complementary. After an interior cutoff value, the second effect dominates and the marginal value of information to the low producer decreases.

Third, the marginal value of information produced by a high producer is higher than the marginal value of information produced by a low producer for positive correlations and lower for negative correlations. For negative values of ρ, complementarities imply that it is more valuable for the low producer to match the high producer's information than for the high producer to produce novel information. Alternatively, for positive values of ρ, the low producer's new information is, at least, partially redundant, which makes her marginal value always lower than the high producer's marginal value of information.

3.2 Nash Equilibria and Payoff Frontiers

Building upon the preceding intuitions, we solve for the Nash equilibrium correspondence within an arbitrary team (i, j) as we vary the team's correlation coefficient ρ.

Proposition 1. *Fix a team (i, j) with correlation coefficient $\rho \in (-1, 1)$. Then, the following properties hold:*

1. *If $\rho > 0$, i.e., (i, j) is homogeneous, then there are two Nash equilibria. In one,*

$$n_i = 0 \quad \textit{and} \quad n_j = \sqrt{\frac{\sigma^2}{c}} - \sigma^2.$$

In the other, $n_j = 0$ and $n_i = \sqrt{\frac{\sigma^2}{c}} - \sigma^2$. The following payoff vectors are therefore feasible:

$$\gamma_1 := \left(\frac{1}{2}(1 + c\sigma^2 - 2\sqrt{c\sigma^2}), \frac{1}{2}(1 - \sqrt{c\sigma^2})\right) \quad \textit{and} \quad \gamma_2 := \left(\frac{1}{2}(1 - \sqrt{c\sigma^2}), \frac{1}{2}(1 + c\sigma^2 - 2\sqrt{c\sigma^2})\right).$$

2. *If $\rho = 0$, then a profile $n(i,j)$ is a Nash equilibrium if and only if*

$$n_i + n_j = \sqrt{\frac{\sigma^2}{c}} - \sigma^2.$$

The following payoff vectors are therefore feasible:

$$\left\{\alpha \cdot \gamma_1 + (1-\alpha) \cdot \gamma_2, \quad \alpha \in [0,1]\right\}.$$

3. *If $\rho < 0$, i.e., (i,j) is diverse, then the unique Nash equilibrium has*

$$n_i = n_j = \left(\frac{1+\rho_{ij}}{2}\right)\left(\sqrt{\frac{1-\rho_{ij}}{1+\rho_{ij}}}\sqrt{\frac{\sigma^2}{c}} - \sigma^2\right).$$

The unique feasible payoff vector has each worker achieve a payoff of

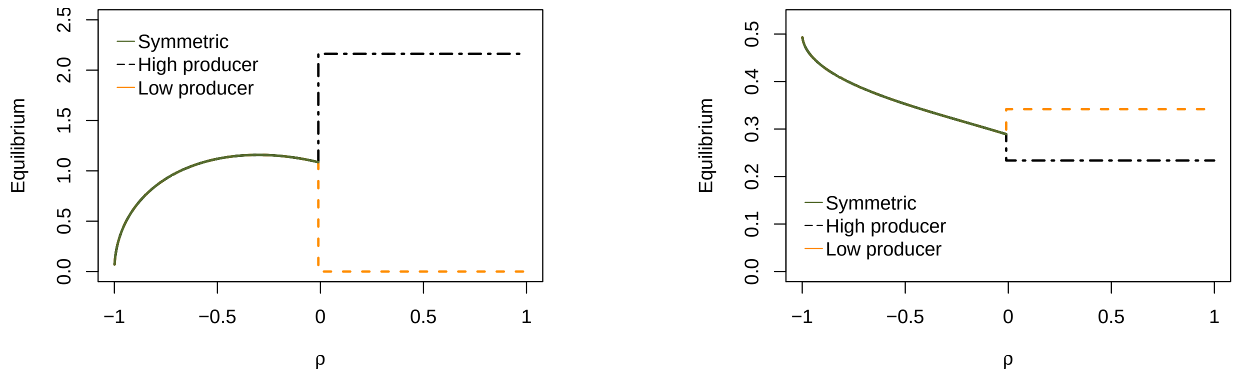
$$\frac{1}{2}\left(1 - \left(\sqrt{\frac{1+\rho_{ij}}{1-\rho_{ij}}}\right)\sqrt{c\sigma^2} - c\left(\frac{1+\rho_{ij}}{2}\right)\left(\sqrt{\frac{1-\rho_{ij}}{1+\rho_{ij}}}\sqrt{\frac{\sigma^2}{c}} - \sigma^2\right)\right).$$

Proof. See Appendix A.3. □

From Proposition 1, we see that, except when $\rho = 0$, there exists a unique Nash equilibrium (up to player identity). Interestingly, for negative correlation coefficients, this equilibrium is symmetric, while for positive coefficients it is asymmetric.[16] The intuition is simple: When the correlation coefficient is negative, complementarities make it more valuable for a low producer to increase his effort up to the level of a high producer than for a high producer to increase his amount of effort. The opposite intuition holds for positive correlation coefficients: Independently of how many signals each worker produces, the high producer's marginal value of information is always higher than the low producer's marginal value of information.

Figure 2a illustrates our results. The team's correlation parameter, ρ, is on the x-axis, while Nash equilibrium strategies are on the y-axis. The solid, green line indicates the strategy of each worker in a symmetric Nash equilibrium; the symmetrically-spaced, dashed, orange line indicates the strategy of a low producer in an asymmetric Nash equilibrium; and the asymmetrically-spaced, dashed, black line indicates the strategy of a high producer in an asymmetric Nash equilibrium.

[16] We remark that while the Nash equilibrium in homogeneous teams involves one worker acquiring zero signals, this property need not hold in related, discrete signal-acquisition games (see Kambhampati, Segura-Rodriguez and Shao (2021)). We interpret the lower bound on effort of zero as a "minimum" amount of effort that a worker can exert.



(a) Equilibrium Correspondence.

(b) Payoff Correspondence.

Figure 2: Equilibrium Correspondence with $\sigma^2 = 1$ and $c(n) = 0.1n$.

Figure 2b presents corresponding Nash equilibrium payoffs. From Proposition 1, we see that, for any value of $\rho \in (-1, 1)$, a team is associated with a unique *total* surplus value. However, due to nontransferable effort costs, workers cannot freely divide this surplus. When choosing a teammate, each worker thus takes into account not only the total surplus the team generates, but how much of it she can achieve. The following corollary states a number of useful properties of feasible payoff vectors, necessarily satisfied in Figure 2a, that will be useful when we analyze equilibrium sorting.

Corollary 1.

1. *In any diverse team, the unique feasible payoff vector is strictly decreasing (in both components) in the team's correlation coefficient.*

2. *In any homogeneous team, the set of feasible payoff vectors is constant in the team's correlation and, in any Nash equilibrium in the team, the low producer obtains a strictly higher utility than the high producer.*

3. *A high producer in a homogeneous team always obtains a strictly lower utility than he would in a diverse team.*

4. *There exists a unique value $\rho^* \in (-1, 0)$ such that, in any diverse team with correlation coefficient $\rho_\ell \geq \rho^*$, each worker would obtain a higher payoff as a low producer in a homogeneous team.*

The proof follows from simple algebraic manipulation and is therefore omitted. Property 4 of Corollary 1 will be particularly important when characterizing equilibrium sorting patterns: A worker may sometimes prefer to join a homogeneous team, which produces less valuable information than a diverse team, if she can save on effort costs.

4 Efficient Allocations

Before proceeding to our equilibrium sorting analysis, we characterize which allocations are (constrained) efficient. Informally, an allocation is efficient if there does not exist another matching and collection of equilibria that strictly increases utilitarian welfare. A formal definition follows below.

Definition 2 (Efficient Allocations)**.** *An allocation (μ, N^*) is **efficient** if there does not exist a matching $\hat{\mu} \in \mathcal{M}$ and a collection of Nash equilibria $\hat{N} = \{\hat{n}(i,j)\}_{i,j=\mu(i)}$ for which*

$$\sum_{\ell \in \mathcal{N}} v_\ell(n^*(\ell, \mu(\ell))) < \sum_{\ell \in \mathcal{N}} v_\ell(\hat{n}(\ell, \hat{\mu}(\ell))).$$

*It is **inefficient** otherwise.*

We say that an allocation is maximally diverse if it contains a matching that forms as many diverse teams as possible.

Definition 3 (Maximally Diverse Allocations)**.** *An allocation (μ, N^*) is **maximally diverse** if there does not exist another matching $\hat{\mu} \in \mathcal{M}$ for which*

$$|\{i \in \mathcal{N} : \rho_{i\hat{\mu}(i)} = \rho_\ell\}| > |\{i \in \mathcal{N} : \rho_{i\mu(i)} = \rho_\ell\}|.$$

We observe that the set of efficient allocations is equivalent to the set of maximally diverse allocations. Moreover, there always exists an efficient, i.e., maximally diverse, allocation.

Proposition 2 (Characterization of Efficient Allocations)**.**

1. *An allocation (μ, N^*) is efficient if and only if μ is maximally diverse.*
2. *There always exists an efficient matching and every efficient matching is in the core.*

Proof. See Appendix A.4. □

The result in Proposition 2 can be related to the literature on assortative matching. Specifically, for each worker i, let i's type be define as $t_i := \min_j \rho_{ij}$, where if $t_i = \rho_\ell$, then i is a "high" type and if $t_i = \rho_h$, then i is a "low" type. Under these definitions, Proposition

2 equivalently asserts that any *positive assortative matching*, i.e., any matching that pairs highs as often as possible, is efficient and only such matchings are efficient. Since our setting features imperfectly transferable utility, however, the entire correlation matrix among workers will matter for *equilibrium* sorting. Our theoretical contribution is to characterize precisely when core allocations are inefficient.

5 Core Allocations

Our analysis of the Production Subgame yields two important insights. First, fixing a strategy profile within teams, reducing correlation increases the value of information the team generates. Hence, there is a tendency for workers with a low pairwise correlation to match, ignoring effort costs. Second, increasing correlation decreases the symmetry of equilibria; as signals become more substitutable, the marginal value of matching a high producer's signal decreases. Hence, the existence of nontransferable effort costs may tempt workers with a low pairwise correlation to join less productive teams if they can free-ride on their teammate's effort. We now show how these two within-team properties influence the efficiency of sorting of workers into teams.

5.1 Stratification Inefficiency

We first exposit an inefficiency that arises when a diverse team forms, but causes excluded workers to form an inefficient, homogeneous team. It will be useful to define $G_\ell = (V, E)$ to be the (simple) graph with vertices equal to the set of workers, $V := \mathcal{N}$, and edges linking workers whom compose diverse teams,

$$E := \{(i,j) | \ i,j \in V, \ \ i < j, \text{ and } \rho_{ij} = \rho_\ell\}.$$

Figure 3a represents one possible specification of G_ℓ.

In the graph depicted in Figure 3a, we argue that, if $\rho_\ell \leq \rho^*$ as defined in Corollary 1, then there is an inefficient core allocation, (μ, N^*), in which worker 1 and worker 4 form a diverse team, and worker 2 and worker 3 form a homogeneous team. To see why such an allocation can be in the core, suppose that $\mu(1) = 4$ and $\mu(2) = 3$. Then, worker 1 and worker 4 play a symmetric Nash equilibrium, $n^*(1,4)$, while worker 2 and worker 3 play an asymmetric Nash equilibrium, $n^*(2,3)$, in which one worker does not exert any effort and the other exerts a strictly positive amount. By the assumption that $\rho_\ell \leq \rho^*$, however, worker 1 and worker 4 each obtains a weakly higher payoff together than they can achieve

(a) Correlation Graph G_ℓ.

(b) Sorting when $\rho_\ell \leq \rho^*$.

Figure 3: Stratification Inefficiency.

in any other Nash equilibrium in any other team. So, neither has a (strict) incentive to form a deviating team and (μ, N^*) must be a core allocation.[17] Nevertheless, μ is *not* maximally diverse. In particular, the matching $\hat{\mu} \in \mathcal{M}$ satisfying $\hat{\mu}(1) = 3$ and $\hat{\mu}(2) = 4$ forms more diverse teams than μ. Thus, by Proposition 2, the original core allocation could not have been efficient. See Figure 3b.

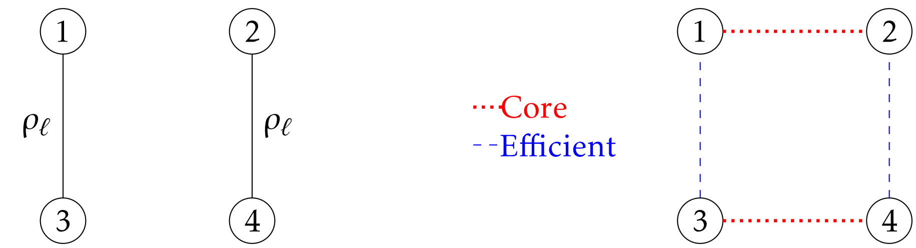
We now formalize the previous logic and define our first notion of inefficiency.

Definition 4 (Stratification Inefficiency)**.** *An allocation (μ, N^*) is* ***stratification inefficient*** *if it is an inefficient allocation and there exist two workers $i, j \in \mathcal{N}$, $i \neq j$, for which $\mu(i) = j$ and $v_k(n^*(i,j))$ is the highest payoff any worker $k \in \{i,j\}$ can obtain in any Nash equilibrium in any team.*

In a stratification inefficient allocation, a pair of teammates are each as well off as in *any* other feasible team playing *any* other Nash equilibrium, e.g., worker 1 and worker 4. In addition, there exists another matching, e.g., $\hat{\mu}$, such that $\hat{\mu}(1) = 3$ and $\hat{\mu}(2) = 4$, and a collection of Nash equilibria that increases utilitarian welfare. Stratification inefficiency therefore arises when a diverse team forms, but does not internalize the "externality" it generates on the productivity of the residual match. This is consistent with existing findings in the literature on matching with nontransferabilities, e.g., Legros and Newman (2007).

Our contribution here is to answer to the following question: Under what conditions does a stratification inefficient core allocation exist? We borrow some terminology from graph theory to state our main characterization result (see Gross, Yellen and Anderson (2018) for all definitions we use). Define a **walk** to be a sequence of edges which joins a sequence of vertices and a **trail** to be a walk in which all edges are distinct. A **path**

[17]This example, of course, generalizes to the case in which worker 2 and worker 3 can work alone. We considered this case in an earlier version of this paper.



(a) Correlation Graph G_ℓ.

(b) Sorting when $\rho_\ell \geq \rho^*$.

Figure 4: Asymmetric Effort Inefficiency.

is a trail in which all vertices are distinct and a **cycle** is a non-empty trail in which the first and last vertices are equal. A **Hamiltonian path (cycle)** is a path (cycle) that visits each vertex exactly once. To illustrate these definitions, observe that Figure 6a contains a Hamiltonian path starting at vertex 3 and ending at vertex 2, but does not contain a Hamiltonian cycle.

The following result establishes that a stratification inefficient allocation is in the core if and only if diverse teams are sufficiently productive and G_ℓ contains a Hamiltonian path, but does not contain a Hamiltonian cycle.

Lemma 1. *There exists a stratification inefficient allocation in the core if and only if (i) $\rho_\ell \leq \rho^*$ and (ii) G_ℓ contains a Hamiltonian path, but does not contain a Hamiltonian cycle.*

Proof. See Appendix A.5. □

Lemma 1 demonstrates that, to prevent inefficiency, it is not enough for all workers in an organization to be connected through diverse paths. In fact, this type of connectivity can lead to the "wrong" connections being made; in a stratification inefficient allocation, an efficiency-minded manager can better exploit the entire correlation matrix among workers. The more nuanced sufficient condition for decentralized sorting to be efficient is that, instead, it is possible to identify a diverse cycle through all workers in the organization.

5.2 Asymmetric Effort Inefficiency

We now study a sorting inefficiency that arises due to asymmetric effort provision within teams.

Consider the correlation graph depicted in Figure 4a and suppose that $\rho_\ell \geq \rho^*$, where ρ^* is defined in Corollary 1. We claim that there is an inefficient core allocation in which only homogeneous teams form, even though any efficient matching forms only diverse teams. To see why such a matching can be in the core, suppose that $\mu(1) = 2$ and $\mu(3) = 4$. Then, the unique equilibrium in the homogeneous teams $(1,2)$ and $(3,4)$ involves one worker producing a strictly positive number of signals and the other producing zero signals. Let $n_1^*(2) = 0 < n_2^*(1)$ and $n_4^*(3) = 0 < n_3^*(4)$, so that worker 1 and worker 4 produce zero signals. If $\rho_\ell \geq \rho^*$, worker 1 is unwilling to from a diverse team with worker 3 and worker 4 is unwilling to form a diverse team with worker 2 by property 4 of Corollary 1. Moreover, neither can do better in any other homogeneous team. Finally, worker 2 and worker 3 cannot form a team and strictly increase their payoffs; since $\rho_{23} = \rho_h$, one of them would, again, be forced to produce all signals in the team. It follows that the constructed allocation is in the core.

We again formalize the logic just described and define our second notion of inefficiency.

Definition 5 (Asymmetric Effort Inefficiency)**.** *An allocation (μ, N^*) is **asymmetric effort inefficient** if there exist two workers $i, j \in \mathcal{N}$, $i \neq j$, for which $\mu(i) = j$ and the following properties hold:*

1. *There is a Nash equilibrium $\hat{n}(i,i')$, $i' \neq i$, satisfying*

$$n_i^*(j)\hat{n}_{i'}(i) < \hat{n}_i(i')n_j^*(i)$$

and for which $v_i(n^*(i,j))$ is the highest payoff worker i can obtain in any Nash equilibrium in any team;

2. *There exists a matching $\hat{\mu} \in \mathcal{M}$ satisfying $\hat{\mu}(i) = i'$ and a collection of Nash equilibria $\hat{N} = \{\hat{n}(i,j)\}_{i,j=\mu(i)}$, including $\hat{n}(i,i')$, such that*

$$\sum_{\ell \in \mathcal{N}} v_\ell(n^*(\ell, \mu(\ell))) < \sum_{\ell \in \mathcal{N}} v_\ell(\hat{n}(\ell, \mu'(\ell))).$$

To understand the definition, consider again the example. Let $(i,j) = (1,2)$ and $(i',j') = (3,4)$. The manager prefers to match worker 1 with worker 3 because there is a symmetric Nash equilibrium inside the team, $\hat{n}(1,3)$, in which worker 1 exerts relatively more effort than her partner compared to the "on-path" Nash equilibrium in which $n_1^*(3) = 0$. In particular, $\frac{n_1^*(3)}{n_3^*(1)} = 0 < \frac{\hat{n}_1(2)}{\hat{n}_2(1)}$ so that, by cross-multiplying, we see that the first inequality of the definition is satisfied. The second part of the definition ensures that, upon re-matching worker 1 and worker 2 and fixing $\hat{n}(1,2)$, the manager can select a Nash equilibrium in the residual match that increases utilitarian welfare. To our knowledge, a formalization of this type of inefficiency is missing in the literature on matching with nontransferabilities.

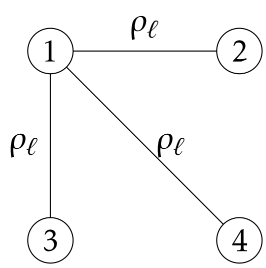


Figure 5: Star (Claw) Graph.

Under what conditions does an asymmetric effort inefficient core allocation exist? Say that $G = (V, E)$ is a **star** if one node has degree $|V| - 1$ and all other nodes have degree one (see Figure 5 for an example).[18] The following lemma states that there exists an asymmetric effort inefficient allocation in the core if diverse teams are not too productive, G_ℓ is not a star, and G_ℓ does not contain a Hamiltonian cycle, as in Figure 4. However, an additional qualification is required to obtain a *necessary* and sufficient condition. Say that two vertices are **connected** if there is a path containing both. Say that a graph is **almost connected** if all, but one, pair of vertices is connected.

We obtain the following result.

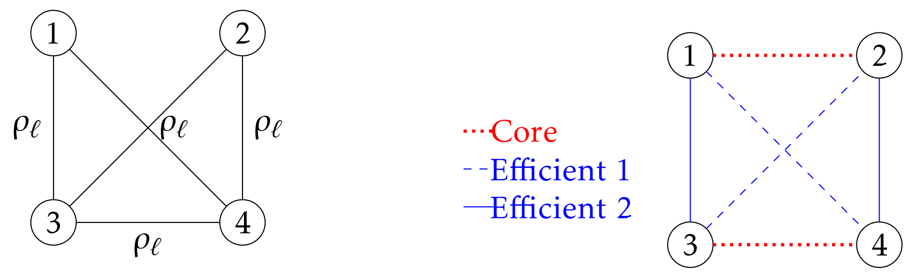
Lemma 2. *There exists an asymmetric effort inefficient allocation in the core if and only if $\rho_\ell \geq \rho^*$, G_ℓ is not a star, and either G_ℓ does not contain a Hamiltonian cycle or is almost connected.*

Proof. See Appendix A.6. □

A star graph does not contain a Hamiltonian path. But, there are many non-star graphs that also do not contain Hamiltonian paths, such as the graph in Figure 4a. So the condition that G_ℓ is not a star and does not contain a Hamiltonian cycle, which is sufficient for asymmetric effort inefficiency, is weaker than the necessary and sufficient condition for stratification inefficiency. In addition, asymmetric effort inefficiency also arises when G_ℓ is almost connected, as illustrated in Figure 6. In this case, there is an asymmetric effort inefficient core allocation in which the *only* feasible homogeneous team forms.

We remark that, counter-intuitively, efficiency can be restored in this example by *removing* the edge $(3, 4)$, thereby constructing a graph that contains a Hamiltonian cycle, but which is not almost connected. Hence, merely adding more diverse connections within an organization need *not* improve efficiency; the entire matrix of informational interdependence matters.

[18] When $|V| = 4$, this graph is sometimes referred to as a "claw".



(a) Correlation Graph G_ℓ.

(b) Sorting when $\rho_\ell \geq \rho^*$.

Figure 6: Asymmetric Effort Inefficiency in an Almost Connected Graph.

5.3 Complete Inefficiency Characterization

We are ready to state our portmanteau result. The following proposition states that, generically, any inefficient core allocation is stratification inefficient or asymmetric effort inefficient. Hence, the necessary and sufficient conditions identified in Lemma 1 and Lemma 2 are necessary and sufficient for the existence of an inefficient core allocation.

Proposition 3. *Suppose $\rho_\ell \neq \rho^*$. Then, any inefficient core allocation is either stratification inefficient or asymmetric effort inefficient, where the "or" is exclusive. Thus, there exists an inefficient core allocation if and only if either (a) $\rho_\ell < \rho^*$ and G_ℓ contains a Hamiltonian path, but does not contain a Hamiltonian cycle; or (b) $\rho_\ell > \rho^*$, G_ℓ is not a star, and either does not contain Hamiltonian cycle or is almost connected.*

Proof. See Appendix A.7. □

Notice that, for any value of ρ_ℓ, there exists a graph G_ℓ that leads to an inefficient core allocation. Whether the core allocation is a stratification inefficient or asymmetric effort inefficient depends on the productivity of the diverse teams in the organization. If these teams are sufficiently productive, then diverse teams may form at the expense of other workers. If these teams are sufficiently unproductive, then workers have opportunities to form homogeneous teams in order to minimize their effort. Both types of inefficiencies cohere with the informal accounts of inefficiencies inside of decentralized organizations described in the Introduction. The formal analysis adds value by providing a taxonomy for these inefficiencies, identifying the precise conditions under which each arises, and pointing out that it is important that the "right" diverse connections are formed inside of an organization.

6 Management Implications

In the baseline model, we assumed that workers in a team shared their profits equally, as might be the case in an unregulated decentralized organization. We then identified and completely characterized the two sources of sorting inefficiency. We now consider solutions that relax the equal-sharing rule within teams and relate these solutions to management practice at Oticon.

6.1 Efficient Bilateral, Linear Contracts

We first relax the assumption that profits within a team must be shared equally. In particular, suppose that ex-post profits can be divided linearly among workers in a team. That is, in a team (i,j), worker i receives a share α_i and worker j receives a share α_j, and the budget-balance condition, $\alpha_i + \alpha_j = 1$, is satisfied. The following Proposition characterizes shares that maximize total surplus.

Proposition 4 (Efficient Bilateral Linear Contracts)**.**

1. *In a diverse team (i,j), the surplus-maximizing division of team profits, (α_i^*, α_j^*), satisfies $\alpha_i^* = \alpha_j^* = \frac{1}{2}$.*
2. *In a homogeneous team (i,j), the surplus-maximizing division of team profits, (α_i^*, α_j^*), either satisfies $\alpha_i^* = 0$ and $\alpha_j^* = 1$, or $\alpha_i^* = 1$ and $\alpha_j^* = 0$.*

Proof. See Appendix A.8. □

The first property of the Proposition states that the equal-sharing rule imposed in the baseline model is actually *efficient* in diverse teams. Hence, the analysis of within-team profits and payoffs is unchanged. The second property states, however, that in a homogeneous team, it is optimal to make one worker in a team the sole residual claimant of profits. An immediate implication of Proposition 4 is that no core allocation can be asymmetric effort inefficient; no worker in a diverse team is tempted to join a homogeneous team in order to exert less effort.

Corollary 2. *Suppose within-team contracts are efficient linear contracts. Then, there cannot exist an asymmetric effort inefficient allocation in the core.*

Foss (2003)'s account of Oticon's retreat from decentralization indicates a management intervention consistent with our efficiency analysis. In particular, in 1996, Oticon developed a "Competence Center" which approved projects and their financing. This Center "carefully examined [projects] with respect to technical feasibility and commercial implications" in a desire to "avoid harming motivation (i.e., diluting incentives) by overruling ongoing projects". To achieve these goals, the Competence Center appointed project leaders and constrained project leaders' decision rights over salaries. In the context of our stylized model, one such intervention that would seem attractive would be to allow diverse teams to have a "flat" internal structure, with parties receiving equal compensation, while forcing homogeneous teams to be structured hierarchically, with project leaders receiving most of the compensation arising from a project's success. However, as will next be shown, such interventions may actually increase inequality in productivity across teams when sorting is endogenous.

6.2 Joint Performance Evaluation

Unfortunately, requiring that output is shared efficiently does not restore the efficiency of endogenous sorting. Effort remains non-contractible. In particular, suppose that management restricts within-team contracts to be bilaterally efficient linear contracts. Then, the range of parameters under which a stratification inefficient core allocations exists *increases*. Specifically, for any value of ρ_ℓ, as long as G_ℓ contains a Hamiltonian path, but does not contain a Hamiltonian cycle, there exists a stratification inefficient core allocation.

Corollary 3. *Suppose within-team contracts are efficient linear contracts. If G_ℓ contains a Hamiltonian path, but does not contain a Hamiltonian cycle, then there exists a stratification inefficient core allocation.*

One way to eliminate this inefficiency, while respecting within team budget-balance and allowing for endogenous sorting, is to construct a transfer scheme involving *joint performance evaluation*, i.e., a payment scheme in which workers are compensated based on output produced in other teams. For instance, returning to the example presented in Figure 3, suppose that worker 1 receives a bonus based on the productivity of the team they are *not* a part of, e.g., the team composed of 2 and 3. If this bonus is sufficiently large, then he will internalize the externality generated on the residual match and always sort efficiently, i.e., always match with worker 3.

In the case of Oticon, joint performance evaluation schemes, in the form of stock options, appeared to be particularly important in alleviating stratification inefficiency (whose existence is suggested by the quotation of Larsen (2002) in the Introduction). In Foss (2003)'s account of the company's history, the author notes that Oticon's CEO, Lars Kolind, introduced an employee stock program in which

> "shop floor employees were invited to invest up to 6.000 Dkr (roughly 800 USD) and managers could invest up to 50.000 Dkr (roughly 7.500 USD). Although these investments may seem relatively small, in Kolind's view they were sufficiently large to significantly matter for the financial affairs of individual employees; therefore, they would have beneficial incentive effects. More than half of the employees made these investments."

Our analysis rationalizes these compensation schemes as a way of resolving stratification inefficiency, which we have argued is exacerbated by selective management intervention to prevent asymmetric effort inefficiency.

7 Discussion

Our paper is a first step towards understanding how research teams form absent a central authority and in the absence of transfers. We shed light on how workers' incentives for effort within teams are affected by their informational complementaries and therefore impact equilibrium sorting. Our analysis uncovers two plausible forces leading to inefficient sorting. First, workers producing complementary information may match and force excluded workers to form highly unproductive teams composed of workers producing substitutable information. Hence, there is too much inequality in productivity *across* teams. Second, even when it is efficient for a team composed of workers producing complementary information to form, such a team may not arise in equilibrium if one of its members has an opportunity to form a less productive team in which she exerts relatively less effort. Hence, there is too much inequality in effort *within* teams. Our theoretical results link the productivity of diverse teams and the network structure of an organization, on one hand, to the efficiency of endogenous sorting, on the other. In addition, we point out that management interventions that eliminate asymmetric effort inefficiency can exacerbate stratification inefficiency.

Our paper makes several simplifying assumptions. Most starkly, (i) the signal-acquisition game played by workers within a team corresponds to the limit of a sequence of games in which workers acquire a discrete number of signals and (ii) the entries of the correlation matrix among workers can only take on two values. These assumptions enable us to provide the sharpest possible characterization of the sources of sorting inefficiency. Nevertheless, a previous working paper Kambhampati, Segura-Rodriguez and Shao (2021) shows that the nature of within-team equilibria and the sources of sorting inefficiency are qualitatively similar when both assumptions are relaxed.

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A Proofs

A.1 Posterior Variance Simplification

For any measurable function $g : X \rightarrow \mathbb{R}$, where X is the set of possible realizations of signals,

$$-\mathbb{E}_{x,\theta}\Big[(g(x)-\theta)^2\Big] \leq -\mathbb{E}_x\Big[(\mathbb{E}(\theta \mid x)-\theta)^2\Big] = -\mathbb{E}_x\Big[\mathbb{E}_\theta\Big[(\mathbb{E}(\theta \mid x)-\theta)^2 \mid x\Big]\Big] = -Var(\theta \mid x).$$

The inequality follows because $\mathbb{E}\Big[(b-\theta)^2|x\Big]$ is minimized by setting $b = \mathbb{E}[\theta|x]$. The first equality follows from the Law of Iterated Expectations. The second equality follows from the definition of conditional variance.

Let Σ be the correlation matrix of joint signals x, and 1_N be a N-column vector of ones. The likelihood function of the signals is

$$p(x|\theta) = \det(2\pi K\sigma^{-2}\Sigma)^{-\frac{1}{2}} \exp\Bigg(-\frac{1}{2}\Big[(\theta \cdot 1_N - x)' K\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\Bigg)$$

and the prior density is

$$p(\theta) = (2\pi)^{-\frac{1}{2}} \exp\Bigg(-\frac{1}{2}\Big[\theta^2\Big]\Bigg),$$

because θ follows a standard normal distribution. By Bayes rule, the posterior distribution of $\theta|x$ is proportional to,

$$\begin{aligned} p(x|\theta)p(\theta) &\propto \exp\left(-\frac{1}{2}\Big[\theta^2 + (\theta\cdot 1_N - x)'K\sigma^{-2}\Sigma^{-1}(\theta\cdot 1_N - x)\Big]\right) \\ &\propto \exp\left(-\frac{1}{2}\Big[\theta^2(1+K\sigma^{-2}1_N'\Sigma^{-1}1_N) - \theta K\sigma^{-2}(x'\Sigma^{-1}1_N + 1_N'\Sigma^{-1}x)\Big]\right) \\ &\propto \exp\left(-\frac{1}{2}\Big[\theta - A\Big]'B\Big[\theta - A\Big]\right), \end{aligned}$$

where $B = (1 + K\sigma^{-2}1_N'\Sigma^{-1}1_N)$, $A = B^{-1}K\sigma^{-2}1_N'\Sigma^{-1}x$, and the proportionality operator eliminates positive constants. Because the derived expression is the kernel of a normal distribution, $Var(\theta \mid x) = B^{-1}$.

We construct B^{-1} when workers take $n_j \geq n_i$ draws. The prior covariance matrix, Σ^{-1}, is block diagonal with n_i blocks of the form

$$\Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

and $n_j - n_i$ scalar blocks each equal to 1. The inverse of a block diagonal matrix is equal to the block diagonal matrix formed by inverting each block. Then, $1_N'\Sigma^{-1}1_N$ is equal to $n_i 1_2'\Sigma_0^{-1}1_2 + (n_j - n_i)$. Because

$$\Sigma_0^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix},$$

we have, $1_2'\Sigma_0^{-1}1_2 = \frac{2}{1+\rho}$. Hence,

$$\begin{aligned} Var(\theta \mid n_i, n_j) &= B^{-1} \\ &= \left(K\sigma^{-2}1_N'\Sigma^{-1}1_N + 1\right)^{-1} \\ &= \left(K\sigma^{-2}\left(\frac{2n_i}{1+\rho} + (n_j - n_i)\right) + 1\right)^{-1} \\ &= \frac{\sigma^2}{K}\left(\frac{n_i(1-\rho)}{1+\rho} + n_j + \frac{\sigma^2}{K}\right)^{-1}. \end{aligned}$$

Define $\underline{n}_{ij} = \min\{n_i, n_j\}$ and $\bar{n}_{ij} = \max\{n_i, n_j\}$. Then, worker i's utility function becomes

$$u_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - \frac{\sigma^2}{K}\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \frac{\sigma^2}{K}\right)^{-1}\right) - \frac{c}{2K}n_i.$$

A.2 Microfoundation for Continuous Signal-Acquisition Game

We first show that, as K grows large, the action space for each worker in the K-discrete game converges to their action space in the continuous game. We then show that the equilibrium strategy profiles converge to the corresponding strategy profiles in the continuous game.

In what follows, let

$$d(X,Y) := \max\{\sup_x \inf_y d(x,y), \sup_y \inf_x d(x,y)\}$$

be the Hausdorff metric. The following Lemma demonstrates the convergence of action sets.

Lemma 3. *Let $S_K := \{0, \frac{1}{K}, \frac{2}{K}, \ldots, M\}$. Then, $d(S_K, [0,M]) \to 0$ as $K \to \infty$.*

Proof. Since $S_K \subset [0,M]$, then $\inf_{m\in[0,M]} d(s,m) = 0$ for any $s \in S_K$, and, consequently, $\sup_{s\in S_K} \inf_{m\in[0,M]} d(s,m) = 0$. Then $d^*(S_K,[0,M]) = \sup_{m\in[0,M] s\in S_K} \inf_{s\in S_K} d(s,m)$.

Now we want to show $[0,M] \subset \overline{\cup_{K=1} S_K}$. Let $x \in [0,M]$. We can partition $[0,M]$ into the non-overlapping intervals $[0,\frac{1}{K}) \cup [\frac{1}{K},\frac{2}{K}) \cup ... \cup [\frac{M-1}{K}, M]$. Therefore, x is in one of these intervals. Denote by x_K the initial point of the interval containing x. Then, by construction, $x_K \to x$ as $K \to \infty$. Therefore, $\{x_K\}_{K=1}^{\infty} \subset \overline{\cup_{K=1} S_K}$. Since $\inf_{s\in S_K} d(s,m)$ is a continuous function in m, the extreme value theorem implies there exists a $m_K^* \in [0,M]$, such that $\sup_{m\in[0,M] s\in S_K} \inf_{s\in S_K} d(s,m) = \inf_{s\in S_K} d(s,m_K^*)$. And since $[0,M] \subset \overline{\cup_{K=1} S_K}$, then $d^*(S_K,[0,M]) = \inf_{s\in S_K} d(s,m_K^*) \to 0$ as $K \to \infty$. □

The following Lemma demonstrates convergence of equilibrium strategy profiles.

Lemma 4. *Let $G_K(\rho) := \Big\{(a^K,b^K) \in \mathbb{Q}^2 |\ (a^K,b^K)$ is a Nash Equilibrium in the K-th discrete game$\Big\}$ and $G(\rho) := \Big\{(a,b) \in \mathbb{R}^2 |\ (a,b)$ is a Nash Equilibrium in the continuous game$\Big\}$. Then, for any $\rho \in [-1,1]$, $d(G(\rho), G_K(\rho)) \to 0$ as $K \to \infty$.*

Proof. First, we show that for every convergent sequence $\{(a^K,b^K)\}_{K=1}^{\infty}$ such that $(a^K,b^K) \in G_K(\rho)$, there exists $(a,b) \in G(\rho)$ such that $(a^K,b^K) \to (a,b)$ as $K \to \infty$. For the sake of contradiction, suppose there exists a convergent sequence $\{(a^K,b^K)\}_{K=1}^{\infty}$ with $(a^K,b^K) \in G_K(\rho)$ that does not converge to any $(a,b) \in G(\rho)$. Since $(a^K,b^K) \in [0,M]^2$ the limit of the sequence $\{(a^K,b^K)\}_{K=1}^{\infty}$ exists and we denote it by (a',b'). By our contradiction assumption, we have that (a',b') it is not a Nash equilibrium. Hence, without loss of generality, in the continuous game, there must exist a profitable deviation $a'' \in [0,M]$ for some player i, i.e., $v_i^{i,j}(a',b') < v_i^{i,j}(a'',b')$. Moreover, it is always possible to find a sequence $(a''^K)_{K=1}^{\infty}$

with $a''^K \in S_K$ and such that $a''^K \to a''$. As v_i is continuous, there exists large enough K such that $v_i^{i,j}(a^K, b^K) < v_i^{i,j}(a''^K, b^K)$, that is, for large K, in the K-th game, player i has a profitable deviation. This contradicts (a^K, b^K) being in $G_K(\rho)$, which concludes the argument. Now we apply this first result to prove,

$$\sup_{\mathbf{x} \in G_K(\rho)} \inf_{\mathbf{y} \in G(\rho)} d(\mathbf{x}, \mathbf{y}) \to 0,$$

as $K \to \infty$. Suppose $\sup_{\mathbf{x} \in G_K(\rho)} \inf_{\mathbf{y} \in G(\rho)} d(\mathbf{x}, \mathbf{y}) > c > 0$. Then there must exist a sequence, $\mathbf{x}_K \in G_K(\rho)$ such that $\inf_{\mathbf{y} \in G(\rho)} d(\mathbf{x}_K, \mathbf{y}) > c$. By construction of S_K, the sequence $\mathbf{x}_K$ is bounded. Therefore, it must have a convergent subsequence, $\mathbf{x}_{K_k}$. But our first result implies that, the subsequence has a limit in $G(\rho)$. Therefore $\inf_{\mathbf{y} \in G(\rho)} d(\mathbf{x}_{K_k}, \mathbf{y}) \to 0$, contradicting $\inf_{\mathbf{y} \in G(\rho)} d(\mathbf{x}_{K_k}, \mathbf{y}) > c > 0$.

Second, we show that, for every $(a, b) \in G(\rho)$, there exists a convergent sequence $\{(a^K, b^K)\}_{K=1}^{\infty}$ such that $(a^K, b^K) \in G_K(\rho)$ and $(a^K, b^K) \to (a, b)$ as $K \to \infty$. First, observe that $G_K(\rho)$ is non-empty because each discrete game is a potential game (see Kambhampati, Segura-Rodriguez and Shao (2021) for details). Because the strategy spaces are compact, we can always construct a convergent sequence $\{(a^K, b^K)\}_{K=1}^{\infty}$ such that $(a^K, b^K) \in G_K(\rho)$. In addition, from Lemma 1, we know that if either $\rho > 0$ or $\rho < 0$, $G(\rho)$ is a singleton set consisting of a point (a, b). Hence, by the first part of the proof, the sequence $\{(a^K, b^K)\}_{K=1}^{\infty}$ converges to (a, b).

Now, we consider the more difficult case in which $\rho = 0$. Select some Nash equilibrium $(a, b) \in G(0)$. Under the assumption that $0 < c < \min\{\sigma^{-2}, \sigma^2\}$, a necessary condition for (a, b) to be an equilibrium is that the marginal utility for both players at (a, b) is equal to the marginal cost of acquiring additional information:

$$\frac{\sigma^2}{(a + b + \sigma^2)^2} = \frac{c}{2}.$$

Denote by $\underline{x}^K$ the largest element in S_K that is less than or equal to the real number x. We claim that either $(\underline{a}^K, \underline{b}^K)$, $(\underline{a}^K + \frac{1}{K}, \underline{b}^K)$, or $(\underline{a}^K + \frac{2}{K}, \underline{b}^K)$ is a Nash equilibrium of the K-th discrete game, where the "or" is exclusive. Hence, there exists a sequence of equilibria $\{(a^K, b^K)\}_{K=1}^{\infty}$ that converges to (a, b).

To prove the claim, first consider the strategy profile $(\underline{a}^K, \underline{b}^K)$. Because $\underline{a}^K < a$ and $\underline{b}^K < b$, no worker is better off deviating to a lower number of signals. Worker 1 is not better off choosing $\underline{a}^K + \frac{1}{K}$ if the difference between her payoff with the strategy $(\underline{a}^K + \frac{1}{K}, \underline{a}^K)$

and the strategy $(\underline{a}^K, \underline{b}^K)$ is negative. That is, if and only if

$$\frac{1/K\sigma^2}{(\underline{a}^K + \underline{b}^K + \sigma^2)(\underline{a}^K + 1/K + \underline{b}^K + \sigma^2)} < \frac{c}{2K} \Leftrightarrow$$
$$(a + b + \sigma^2)^2 < (\underline{a}^K + \underline{b}^K + \sigma^2)(\underline{a}^K + 1/K + \underline{b}^K + \sigma^2).$$

Because the utility function for information is concave and worker 2 faces the same incentives as worker 1, if this inequality holds, then the profile $(\underline{a}^K, \underline{b}^K)$ is a Nash equilibrium.

If, instead,

$$(a + b + \sigma^2)^2 > (\underline{a}^K + \underline{b}^K + \sigma^2)(\underline{a}^K + 1/K + \underline{b}^K + \sigma^2),$$

then worker 1 always has an incentive to produce at least $\underline{a}^K + 1/K$ signals when worker 2 produces $\underline{b}^K$ signals. The profile $(\underline{a}^K + 1/K, \underline{b}^K)$ is a Nash equilibrium if worker 1 does not strictly prefer to choose $\underline{a}^K + \frac{2}{K}$. This happens if and only if the difference of her payoff when choosing $(\underline{a}^K + \frac{2}{K}, \underline{a}^K)$ and $(\underline{a}^K + \frac{1}{K}, \underline{b}^K)$ is negative. That is, if and only if,

$$(a + b + \sigma^2)^2 < (\underline{a}^K + 1/K + \underline{b}^K + \sigma^2)(\underline{a}^K + 2/K + \underline{b}^K + \sigma^2).$$

Therefore, if this inequality holds, the profile $(\underline{a}^K + 1/K, \underline{b}^K)$ is a Nash equilibrium. Here, it is important to observe that worker 2 does not have an incentive to deviate either: the payoff difference for worker 2 of producing $\underline{b}^K$ signals instead of $\underline{b}^K - 1/K$ when worker 1 produces $\underline{a}^K + 1/K$ signals is the same as the difference in payoffs for worker 1 producing $\underline{a}^K + 1/K$ signals instead of $\underline{a}^K$ when worker 2 produces $\underline{b}^K$ signals. In addition, the payoff difference for player 2 of producing $\underline{b}^K + 1/K$ signals rather than $\underline{b}^K$ signals when worker 1 produces $\underline{a}^K + 1/K$ signals is the same as the the difference in payoffs for worker 1 of producing $\underline{a}^K + 2/K$ signals rather than $\underline{a}^K + 1/K$ when worker 2 produces $\underline{b}^K$ signals.

Finally, if the inequality

$$(a + b + \sigma^2)^2 < (\underline{a}^K + 1/K + \underline{b}^K + \sigma^2)(\underline{a}^K + 2/K + \underline{b}^K + \sigma^2)$$

is not satisfied then worker 1 has an incentive to produce $\underline{a}^K + 2/K$ signals when worker 2 produces $\underline{b}^K$ signals. Worker 1 never wants to deviate to a larger number of signals because the difference of her payoff with the strategy $(\underline{a}^K + \frac{2}{K}, \underline{b}^K)$ and under $(\underline{a}^K + \frac{3}{K}, \underline{b}^K)$ is always negative; the inequality

$$(a + b + \sigma^2)^2 < (\underline{a}^K + 2/K + \underline{b}^K + \sigma^2)(\underline{a}^K + 3/K + \underline{b}^K + \sigma^2)$$

is always satisfied. In an analogous way to the previous, it can be argued that worker 2 has no incentive to deviate from $\underline{b}^K$. We have thus proved the desired claim that either

$(\underline{a}^K, \underline{b}^K)$, $(\underline{a}^K + \frac{1}{K}, \underline{b}^K)$, or $(\underline{a}^K + \frac{2}{K}, \underline{b}^K)$ is a Nash equilibrium of the K-th discrete game.

Now we apply this second result to prove,

$$\sup_{\mathbf{x}\in G(\rho)} \inf_{\mathbf{y}\in G_K(\rho)} d(\mathbf{x},\mathbf{y}) \to 0,$$

as $K \to \infty$. Again, $\inf_{\mathbf{y}\in G_K(\rho)} d(\mathbf{x},\mathbf{y})$ is continuous in $\mathbf{x}$ and $G(\rho)$ is bounded by construction. By the extreme value theorem, there exists a $\mathbf{x}_K^* \in G(\rho)$ such that $\sup_{\mathbf{x}\in G(\rho)} \inf_{\mathbf{y}\in G_K(\rho)} d(\mathbf{x},\mathbf{y}) = \inf_{\mathbf{y}\in G_K(\rho)} d(\mathbf{x}_K^*,\mathbf{y})$. But our second result says, $\inf_{\mathbf{y}\in G_K(\rho)} d(\mathbf{x}_K^*,\mathbf{y}) \to 0$ as $K \to \infty$. □

A.3 Proof of Proposition 1

1. Suppose first that $\rho > 0$ and consider the profile $(\sqrt{\frac{\sigma^2}{c}} - \sigma^2, 0)$. In this profile, worker i's marginal value of information is

   $$MV(n_i; n_j, \rho) = \frac{1}{2}c,$$

   which is the marginal cost of acquiring additional information. Hence, worker i is best-responding to n_j. Since $\frac{1-\rho}{1+\rho} \in (0,1)$ when $\rho > 0$, $MV(n_j; n_i, \rho) < MV(n_i; n_j, \rho)$. Hence, the marginal value of information for worker j is strictly less than its cost. So, worker j is best-responding to n_i. It follows that $(\sqrt{\frac{\sigma^2}{c}} - \sigma^2, 0)$ is a Nash equilibrium. By symmetry of the game, $(0, \sqrt{\frac{\sigma^2}{c}} - \sigma^2)$ must also be a Nash equilibrium. Substituting these strategy profiles into the payoff functions for each player yields the payoff vectors in the statement of the Proposition.

   To prove that these are the only equilibria, consider any profile (n_i, n_j) with $n_i \geq n_j > 0$. If this profile were to be a Nash equilibrium, then the marginal value of information generated by worker i must be equal to its cost. But then, by $\rho > 0$ and the observation that both workers share the same marginal cost of effort, worker j could reduce n_j and strictly increase her payoff.

2. If $\rho = 0$, then the marginal value of information is identical for both workers given any signal profile (n_i, n_j). When $n_i + n_j = \sqrt{\frac{\sigma^2}{c}} - \sigma^2$, the marginal value of information equals its marginal cost. So, any such profile constitutes a Nash equilibrium and no other profile can be a Nash equilibrium. The line segment joining the feasible payoff vectors when $\rho > 0$ is thus the set of feasible payoff vectors.

3. If $\rho < 0$, the any profile in which $n_i > n_j$ cannot be a Nash equilibrium. If this profile were to be a Nash equilibrium, then the marginal value of information generated

by worker i must be equal to its cost. But, since $\frac{1-\rho}{1+\rho} > 1$ when $\rho < 0$, $MV(n_j; n_i, \rho) > MV(n_i; n_j, \rho)$. So, since both workers share the same marginal cost of effort, worker j would have a strict incentive to acquire more information. It suffices to consider symmetric profiles (n_i, n_j) with $n_i = n_j = n$. The unique value at which the marginal value of information equals the marginal cost, and its corresponding payoff vector, is stated in the Proposition.

A.4 Proof of Proposition 2

1. It suffices to show that any diverse team generates strictly larger total surplus than any homogeneous team. That is, for any $\rho_h > 0 > \rho_\ell$, the sum of payoffs in team (i, j) is strictly larger if $\rho_{ij} = \rho_\ell$ than if $\rho_{ij} = \rho_h$. From the first property of Proposition 1, the sum of payoffs in a homogeneous team is always

$$1 + \frac{1}{2}c\sigma^2 - \frac{3}{2}\sqrt{c\sigma^2}.$$

   From the third property of Proposition 1 and the first property of Corollary 1, the sum of payoffs in a diverse team is strictly larger than

$$1 - \sqrt{c\sigma^2} - \frac{c}{2}\left(\sqrt{\frac{\sigma^2}{c}} - \sigma^2\right) = 1 + \frac{1}{2}c\sigma^2 - \frac{3}{2}\sqrt{c\sigma^2}.$$

   The result follows.

2. Because $|\mathcal{M}|$ is finite, there exists a maximally diverse matching and, hence, an efficient matching. To see that any such matching must be in the core, observe that, by Corollary 1 part 3, any worker in a diverse team obtains a strictly lower utility as a high producer in a homogeneous team and that any worker in a diverse team can do no better in another diverse team. In addition, any worker who may potentially want to form a deviating homogeneous team with a worker in a diverse team would only have a strict incentive to do so if he was guaranteed to be the low producer. It follows that no two workers can match and play a Nash equilibrium that makes both strictly better off than under the maximally diverse matching.

A.5 Proof of Lemma 1

For the"if" direction, suppose that $\rho_\ell \leq \rho^*$ and that G_ℓ contains a Hamiltonian path, but does not contain a Hamiltonian cycle. Without loss of generality, suppose 1 and 4 are connected by a Hamiltonian path consisting of edges $(1,2)$, $(2,3)$, and $(3,4)$. If there is no Hamiltonian cycle, then it must be that $\rho_{14} = \rho_h$. Now, consider the matching $\mu \in \mathcal{M}$ with $\mu(2) = 3$ and $\mu(1) = 4$. Select any Nash equilibria in the two teams, $n^*(1,4)$ and $n^*(2,3)$, and define N^* as the collection of these equilibria. Since $\rho_{23} = \rho_\ell \leq \rho^*$, neither 2 nor 3 can do strictly better in any other team. It follows that (μ, N^*) is in the core. In addition, it is stratification inefficient because μ is not maximally diverse; the matching $\mu' \in \mathcal{M}$ with $\mu'(1) = 2$ and $\mu'(3) = 4$ forms two diverse teams instead of just one.

For the "only if" direction, suppose that either $\rho_\ell > \rho^*$ or that G_ℓ is not a path graph. If $\rho_\ell > \rho^*$, then there is no team in which both workers obtain the highest payoff they can in any Nash equilibrium in any team; a worker in a diverse team can achieve a strictly higher payoff as a low producer in a homogeneous team and a high producer in a homogeneous team can achieve a strictly higher payoff in a diverse team. Hence, a necessary condition for stratification inefficiency is that $\rho_\ell \leq \rho^*$.

To show that it is a necessary condition for stratification inefficiency that G_ℓ contains a Hamiltonian path, but not a Hamiltonian cycle, suppose that there is a stratification inefficient core allocation, (μ, N^*). Then, because $\rho_\ell \leq \rho^*$, μ must form exactly one diverse team and one homogeneous team and the maximally diverse matching must form two diverse teams. Without loss of generality, let $\mu(1) = 2$, with $\rho_{12} = \rho_\ell$, and $\mu(3) = 4$, with $\rho_{34} = \rho_h$. In order for there to exist a maximally diverse matching with two diverse teams, it must be that $\rho_{13} = \rho_\ell$ or $\rho_{14} = \rho_\ell$. Without loss of generality, let $\rho_{13} = \rho_\ell$. If this is the case, then it must be that $\rho_{24} = \rho_\ell$ in order for it to be feasible to form two diverse teams. Hence, there is a Hamiltonian path in G_ℓ joining the edges $(1,3)$, $(1,2)$, and $(2,4)$. Finally, there cannot be a Hamiltonian cycle in G_ℓ because $\rho_{34} = \rho_h$.

A.6 Proof of Lemma 2

For the "if" direction, suppose $\rho_\ell \geq \rho^*$, G_ℓ is not a star, and G_ℓ does not contain a Hamiltonian cycle. Suppose first that there exists a path with at least two edges that is not a Hamiltonian cycle. Without loss of generality, suppose it begins with worker 1 and ends with worker 4. Consider the feasible matching $\mu \in \mathcal{M}$ with $\mu(1) = 4$ and $\mu(2) = 3$. Since $\rho_{14} = \rho_h$, we can choose $n^*(1,4)$ so that $n_1^*(4) = 0$. By $\rho_\ell \geq \rho^*$, worker 1 obtains a higher payoff than he can in any Nash equilibrium in any team. If the original path contained three edges, then it must be that $\rho_{23} = \rho_\ell$, so that $n^*(2,3)$ is a symmetric Nash equilibrium. Hence, (μ, N^*) is asymmetric effort inefficient because it is possible to form a diverse team with worker 1 and either worker 2 or worker 3 and have the residual team also be diverse.

Now, suppose the original path contained two edges. Without loss of generality, let these edges be $(1,2)$ and $(2,4)$. If these are the only edges in G_ℓ, then the resulting matching is asymmetric effort inefficient (an improvement can be made by re-matching worker 1 and worker 2). If the only other edge is $(2,3)$, then the graph is a star. Last, if there is either an edge $(1,3)$ or $(2,4)$, then it is possible to construct two diverse teams and increase the sum of payoffs. Finally, if G_ℓ only contains paths with one edge, then an inefficient core allocation can be constructed in a manner analogous to the construction described in the main text.

For the "if" direction in the case in which G_ℓ is almost connected, simply construct an allocation in which the two disconnected workers match. By $\rho_\ell \geq \rho^*$, one worker in this team obtains a higher payoff than they can in any Nash equilibrium in any other team. In addition, the residual match is diverse and therefore no worker has a strict incentive to form a diverse team with the worker producing zero signals in the constructed homogeneous team. It follows that the allocation is in the core, but is not maximally diverse.

For the "only if" direction, suppose that G_ℓ is a star. Then, any feasible allocation forms exactly one diverse and one homogeneous team. Therefore, there cannot be an inefficient allocation in the core. Now, suppose that G_ℓ contains a Hamiltonian cycle, but is not almost connected. We claim that any core matching must form two diverse teams and, hence, the core allocation cannot be inefficient. Suppose, without loss of generality and towards contradiction, that $\rho_{12} = \rho_h$ and $\mu(1) = 2$ in a core allocation with a homogeneous team. Then, $\rho_{34} = \rho_h$ and both 1 and 2 are connected to both 3 and 4 (if any of these conditions is violated, then the graph would either be almost connected or would not contain a Hamiltonian cycle). Since any Nash equilibrium in both $(1,2)$ and $(3,4)$ involves one worker producing all signals and because this worker obtains a strictly lower utility than he would in a diverse team (property 3 of Corollary 1), there must exist a pair that can re-match and play a Nash equilibrium making each strictly better off. It follows that the original allocation could not have be a core allocation.

A.7 Proof of Proposition 3

Consider first the case in which $\rho_\ell < \rho^*$. If a core allocation is inefficient, then it is not maximally diverse. Since both workers in a diverse team obtain a higher utility than they can in any homogeneous team, any core allocation must consist of at least one diverse team (otherwise, two workers would have an incentive to deviate and form a diverse team). So, any inefficient core allocation must form exactly one diverse team and one homogeneous team. The two workers in the diverse team obtain a higher utility than they can in any other team. Hence, the allocation must be stratification inefficient. It cannot be asymmetric effort inefficient because both the high and low producers in the homogeneous team obtain a strictly lower payoff than if they were in a diverse team.

Now, suppose $\rho_\ell > \rho^*$. Then, only a low producer in a homogeneous team can obtain the highest payoff in any Nash equilibrium in any team. So, no core allocation can be stratification inefficient. In addition, if a core allocation is inefficient, then it contains a matching that is not maximally diverse. So, any welfare improvement must involve moving a low producer in a homogeneous team to a diverse team, i.e., it must increase her effort contribution relative to her teammate. Hence, any inefficient core allocation is asymmetric effort inefficient.

A.8 Proof of Proposition 4

1. Immediate from concavity of the value of information and the observation that, for any profile (n_i, n_j) with $n_i + n_j = n$ for some fixed $n > 0$,
$$\left(\underline{n}_{ij}\left(\frac{1-\rho}{1+\rho}\right) + \bar{n}_{ij} + \sigma^2\right)$$
is maximized with $n_i = n_j = \frac{n}{2}$ if $\rho < 0$.

2. Ignoring the requirement that ex-post profits must be divided between teammates, an efficient signal acquisition profile, (n_i^*, n_j^*), satisfies
$$(n_i^*, n_j^*) \in \underset{(n_i,n_j)}{\arg\max} \quad \left(1 - \sigma^2\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \sigma^2\right)^{-1}\right) - \frac{1}{2}cn_i - \frac{1}{2}cn_j.$$
When $\rho > 0$, it must be that $n_i^* > n_j^* = 0$ or $n_j^* > n_i^* = 0$. The former profile can be implemented in a Nash equilibrium with the budget-balanced shares $\alpha_i^* = 1$ and $\alpha_j^* = 0$. Similarly, the latter profile can be implemented in a Nash equilibrium with $\alpha_i^* = 0$ and $\alpha_j^* = 1$.