Matching to Produce Information

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Motivation

- In many organizations,
  1. Teams are formed to produce information.
  2. Managers assign workers to teams.
- Are decentralized teams inefficient? If so, when and why?
- We study how complementarities among workers affect **information acquisition** and **team formation**.

Model

- $N \geq 2$ workers.
- Stochastic state θ.
- Common prior: $\theta \sim N(\mu_\theta, \sigma_\theta^2)$.
- Workers can obtain Gaussian signals with mean θ and variance σ^2.
- Signals are correlated:

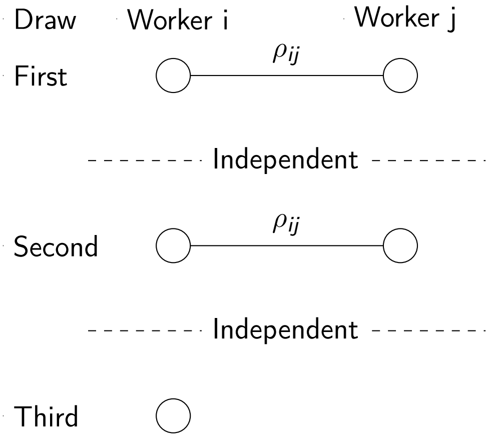
$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} \sim N\left(\begin{pmatrix} \theta \\ \theta \\ \vdots \\ \theta \end{pmatrix}, \sigma^2 \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \cdots & \cdots & 1 \end{bmatrix}\right)$$

Model

Figure: Timing.

- Workers form teams before acquiring signals.
- Each worker can belong only to one team.
- Teams have at most two workers; fixed cost $K > 0$ of forming a pair.
- Within team, worker i chooses integer of draws m_i at cost $c(m_i)$.
- Cost function c satisfies increasing marginal costs and $c(0) = 0$.
- Information generated within team observed only by members.
- Workers choose $a \in \mathbb{R}$ with utility: $u(a, \theta) = -(a - \theta)^2$.

Draw Procedure ($m_i = 3$ and $m_j = 2$)



Solution Concept

We introduce a new solution concept to capture:

1. Cooperative team formation (one-to-one matching).
2. Non-cooperative (after-match) behavior inside teams.

Production Subgame (Inside Each Team)

- Utility in team $S = (i, j)$ given strategy profile (m_i, m_j),

$$v_i^S(m_i, m_j) = -E_{x^S}\left[\max_{a\in\mathbb{R}} E_\theta[(\theta - a)^2 \mid x^S]\right] - c(m_i)$$
$$= \underbrace{-E_{x^S}\left[E_\theta[(\theta - E(\theta \mid x^S))^2 \mid x^S]\right]}_{:=f(m_i, m_j, \rho_{ij})} - c(m_i),$$

  where x^S is the vector of realized draws.

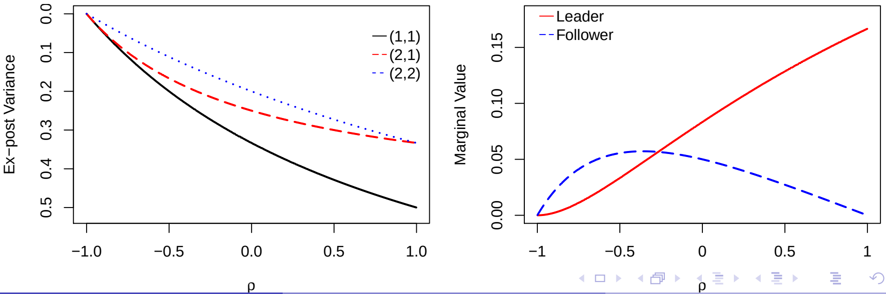
- We require that the chosen vector $m^* = (m_i^*, m_j^*)$ is a pure-strategy **Pareto-Efficient Nash Equilibrium**.

PEN Characterization

Lemma

The posterior variance in a team acquiring (m, n) signals with pairwise correlation $\rho \in (-1, 1]$ is,

$$f(m, n, \rho) = \left(\left(\min\{n, m\} \frac{2}{1+\rho} + |m - n| \right) \sigma^{-2} + \sigma_\theta^{-2} \right)^{-1}.$$



PEN Characterization

Proposition

If $c(1) < \frac{\min\{\sigma_\theta^2, \sigma^2\}}{1+\gamma}$ *and* $\gamma = \frac{\sigma^2}{\sigma_\theta^2} \geq 1$*, then there exist correlations,*

$$-1 < \rho^* \leq \rho^{***} \leq \rho^{**} < 1$$

for which the following properties hold:

1. *For* $\rho \leq \rho^*$*, there is a unique PEN. It is symmetric.*
2. *For* $\rho \in (\rho^*, \rho^{***}]$*, there is at least one symmetric and one asymmetric PEN.*
3. *For* $\rho > \rho^{***}$*, all PEN are asymmetric.*
4. *For* $\rho > \rho^{**}$*, there is a unique PEN up to the identity of each worker. In it, one worker takes a strictly positive number of draws and the other takes none.*

Coalitional Subgame Perfect Equilibrium (CSPE)

Definition

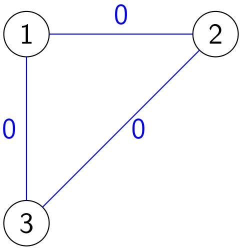
Let Π be a feasible partition of workers and $M^* = \{m^*(S)\}$ be a collection of PEN, one for every feasible team. The tuple (Π, M^*) is a **Coalitional Subgame Perfect Equilibrium (CSPE)** if there does not exist a feasible team S' such that for all $i \in S'$,

$$v_i(m^*(S')) - K * \mathbb{I}_{\{|S'|=2\}} > v_i(m^*(S_\Pi(i))) - K * \mathbb{I}_{\{|S_\Pi(i)|=2\}}.$$

Existence CSPE

Theorem

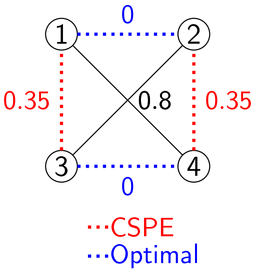
A CSPE exists.

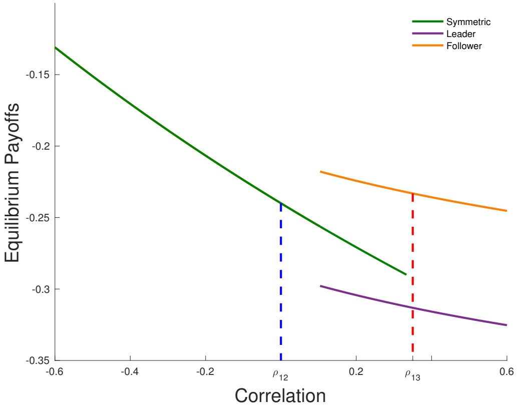


Correlation	Equilibrium	Payoff
0	$(5, 4)$	$(-0.23, -0.21)$
Alone	(7)	(-0.32)

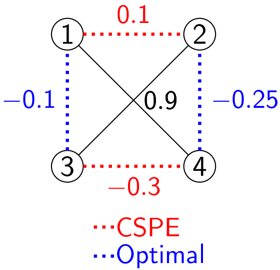
Figure: Roommate Problem.

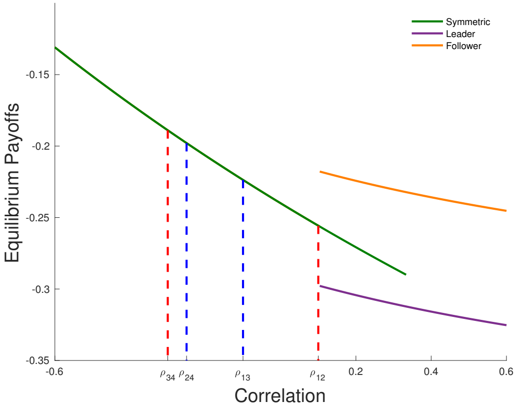
Asymmetric Effort Inefficiency
($\sigma = \sigma_\theta = 1$, $c(m) = 0.01m^2$)





Stratification Inefficiency
$(\sigma = \sigma_\theta = 1,\ c(m) = 0.01m^2)$





Inefficiencies are Ubiquitous

- We find conditions (non-zero measure sets of correlation matrices) under which the CSPE is inefficient.
- I focused on welfare inefficiencies, but there are information inefficiencies as well.
  - In a simulation, we find that 21.06% of correlation matrices with a unique CSPE do not maximize welfare and 18.28% do not maximize information production.
- **Economic Takeaway:** If a manager knows the correlation structure of her workers, and workers are sufficiently "heterogeneous", she can profitably intervene.

Thank you!

Existence PEN

Proposition

In every team, there exists a pure strategy PEN of the Production Subgame.

Proof.

NE existence show by exhibiting a Potential Function:

$$\Phi(m, n, \rho) := -f(m, n, \rho) - c(m) - c(n).$$

No mixed strategy NE Pareto-dominates all pure strategy NE. $\square$

Equilibrium Correspondence

Suppose $c(m) = 0.019m$, $\sigma^2 = \frac{1}{4}$ and $\sigma_\theta^2 = 1$.

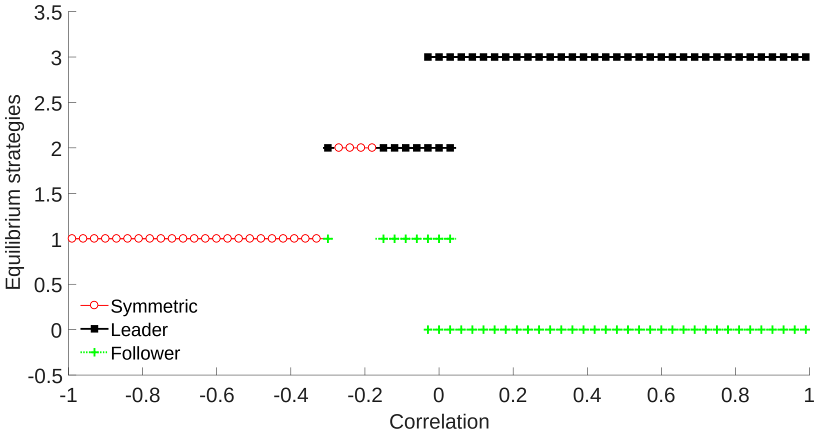


Figure: Equilibrium correspondence when $c(m) = 0.019m$, $\sigma^2 = \frac{1}{4}$ and $\sigma_\theta^2 = 1$.

After-Match Game Matters (Chade and Eeckhout (2018))

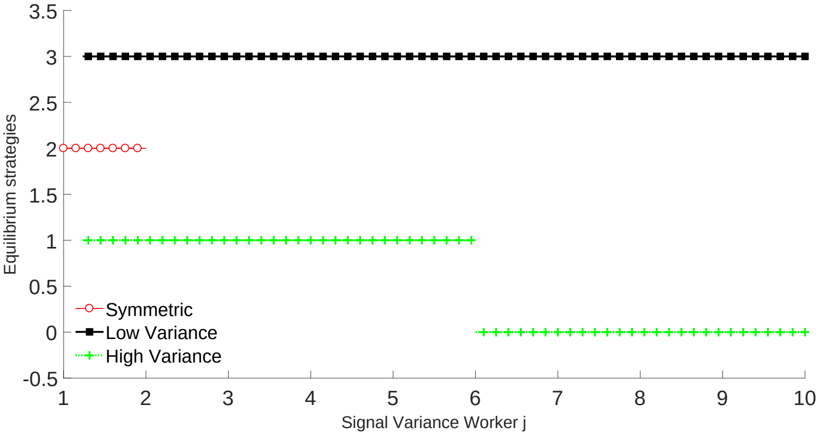


Figure: Equilibrium correspondence when we fix the variance of Player 1 to be 1, $\sigma_\theta = 1$ $d = 0.01$ and $h(m) = m^2$ and allow the variance for the second player to take different values.