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Optimal solution :

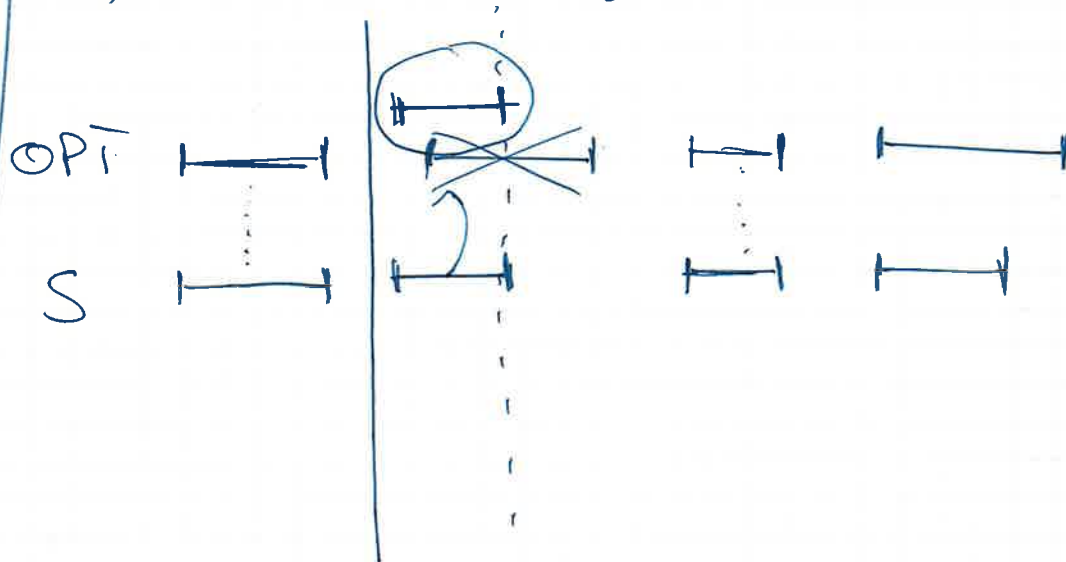
 $\{4, 6, 7\}$  ? No $\{10, 11, 6, 7\}$  ? Yes $\{1, 11, 5, 7\}$  ? Yes

1) Increasing length



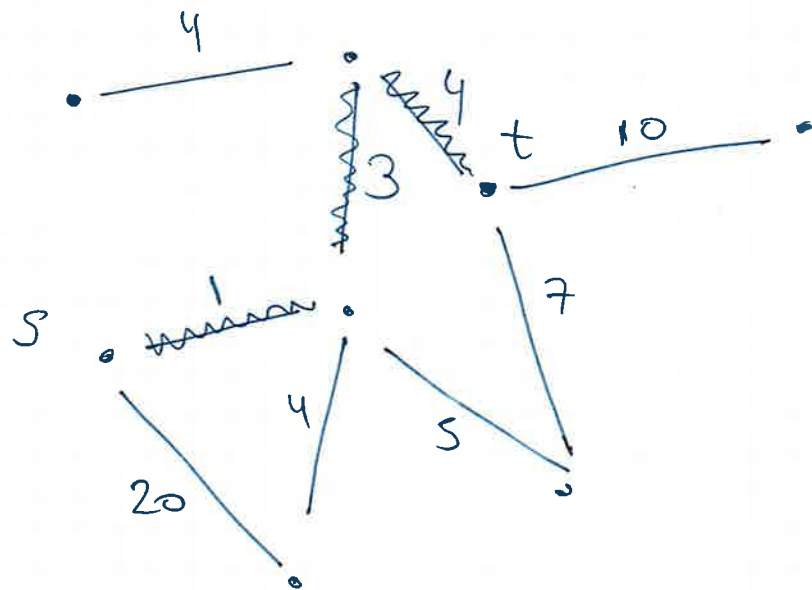
Not optimal

3) Increasing starting time

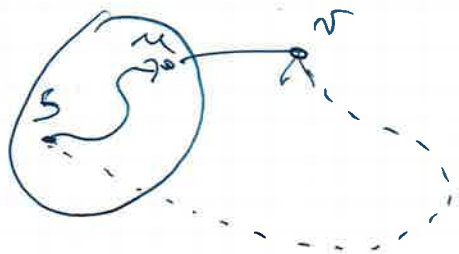
4) ~~Feas~~ Increasing f-time $\Rightarrow$  New OPT is feasible

&  
S and OPT agree on  
one more interval

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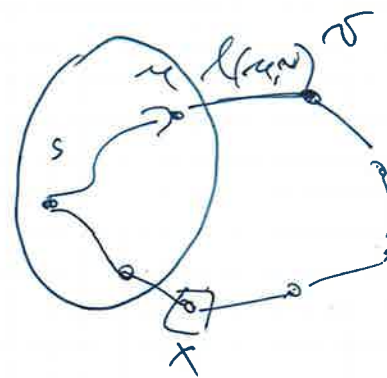


Obs: At any point in time  
 $\text{dist}(s, v) \geq \text{dist}(s, v)$   
 for all  $v \in V \setminus S$



If  $v$  minimizes  $\text{dist}'(s, v)$   
 then  $\text{dist}'(s, v) = \text{dist}(s, v)$

Assume otherwise



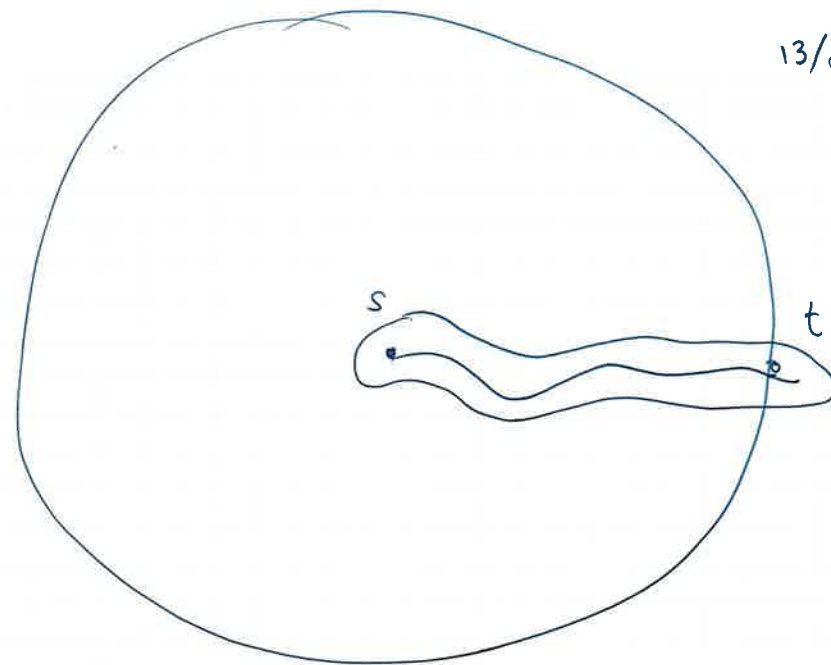
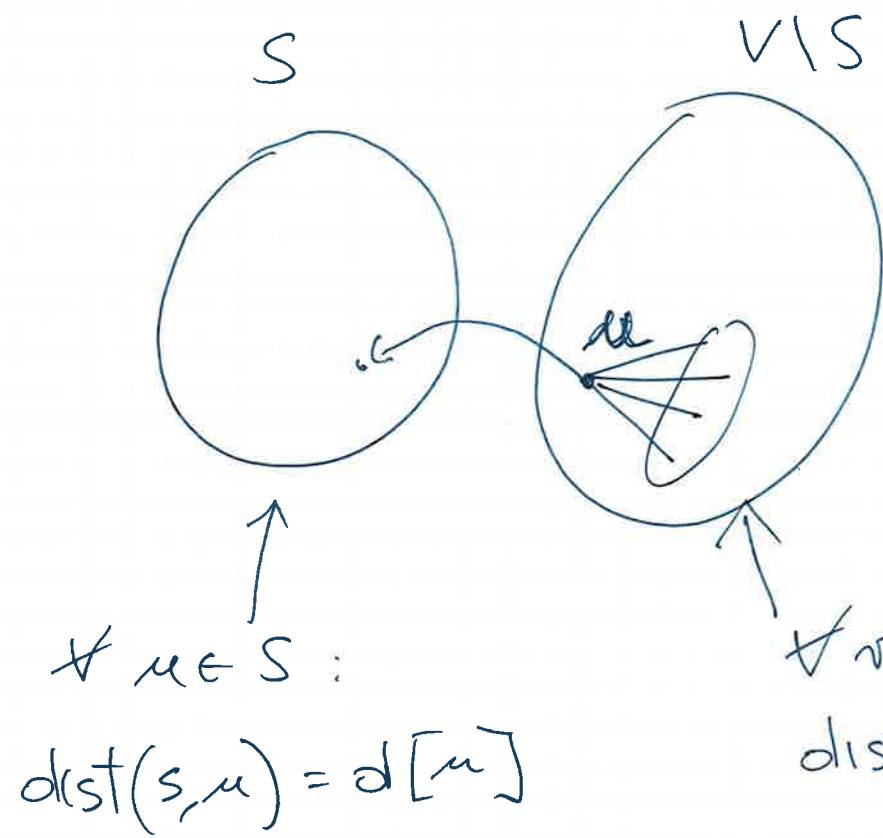
$$\text{dist}(s, v) < \text{dist}'(s, v)$$

$$\text{dist}'(s, x) < \text{dist}(s, v)$$

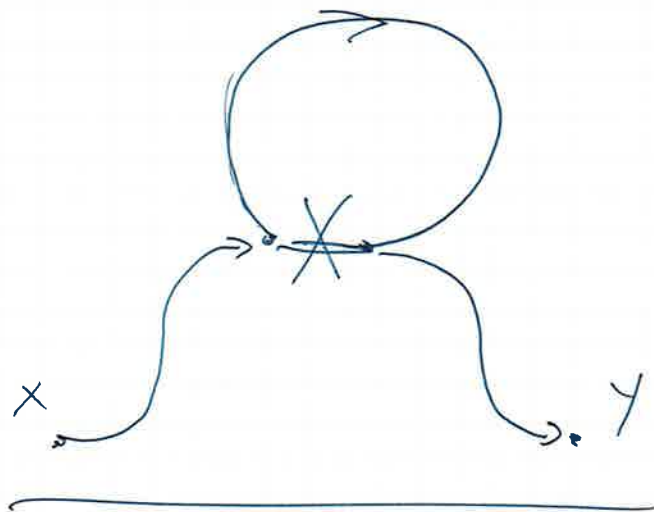
$$\Rightarrow \text{dist}(s, x) < \text{dist}'(s, v)$$

A contradiction

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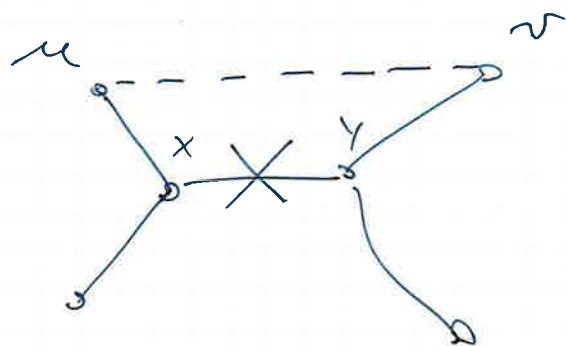


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Let  $(u, v)$  be edge with smallest cost

$\Rightarrow (u, v) \in \text{OPT}$



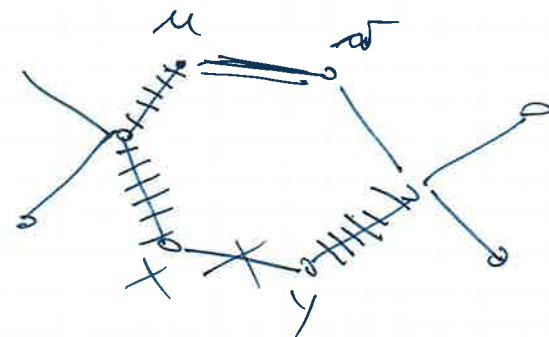
$\Rightarrow$  New tree has lower cost

$\text{OPT} \neq X$

agree up to just before  $(u, v)$

$(u, v) \in X \neq (u, v) \notin \text{OPT}$

$\text{OPT} \rightarrow$



$\text{thick line} \in \text{OPT} \cap X$

When  $(u, v)$  was added to  $X$

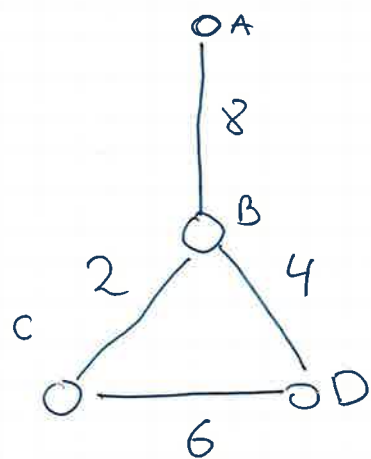
want to avoid

$c(x, y) < c(u, v)$

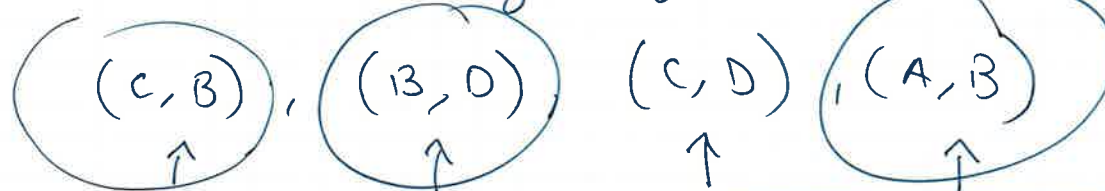
take old tree

add  $(u, v)$

delete  $(x, y)$



Sorted in increasing weight



make-set ( ['A', 'B', 'C', 'D'] )

find('C')  $\neq$  find('B')

union('C', 'B')

find('B')  $\neq$  find('D')

union('B', 'D')

find('C') = find('D')

find('A')  $\neq$  find('B')

union('A', 'B')

{A} {B} {C} {D}

{A} {B, C} {D}

{A} {B, C, D}

{A, B, C, D}

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$$V = \{0, 1, \dots, n-1\}$$

$$id = [0, 1, 2, 3, \dots, n-1]$$

Initially  $id[v] == v$

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update id of first set

union(0, 1)

union(0, 2)

union(0, 3)

⋮

union(0, n-1)

union(0, 1)

union(2, 3)

union(2, 4)

union(0, 3)

[0, 1, 2, 3, 4]

[1, 1, 2, 3, 4]

[1, 1, 3, 3, 4]

[3, 3, 3, 4, 4]

[4, 4, 4, 4, 4]