

This assignment is **due on Aug 21**. All submitted work must be *done individually* without consulting someone else's solutions in accordance with the University's "Academic Dishonesty and Plagiarism" policies.

**Problem 1:** Consider the following variant of the MST problem. We are given an undirected graph  $G = (V, E)$ , edge costs  $c : E \rightarrow \mathbb{R}^+$  and a subset of vertices  $A \subset V$ . Suppose that  $V$  represents a set of computers and  $A$  is the subset of these computers that have a single network port to connect to another computer. We would like to find the cheapest way to design a computer network that will connect all computers.

The abstract problem we are interested in solving is to find a subset  $X \subseteq E$  of edges of minimum cost such that  $(V, X)$  is connected and  $\deg_X(u) = 1$  for all  $u \in A$ .

Your task is to design an algorithm that solves this problem in  $O(m \log n)$  time. Your solution must include:

1. Statement of your algorithm in plain English. (Pseudo-code is optional.)
2. Short proof of correctness.
3. Time complexity analysis.