An Open-Source Optimal Power Flow Formulation: Integrating Pyomo & OpenDSS in Python

Valentin Rigoni, Student member, IEEE and Andrew Keane, Senior member, IEEE
University College Dublin
Dublin, Ireland
valentin.rigoni@ucdconnect.ie and andrew.keane@ucd.ie

***Abstract*—It has been widely demonstrated that active network management (ANM) strategies will be required to avoid the violation of network operational limits in distribution networks with rich presence of distributed energy resources (DERs). With the characteristics of different ANM strategies been varied, e.g. centralized or decentralized, a platform where the benefits and drawbacks of multiple approaches can be easily quantified and benchmarked is required. This work introduces a three-phase optimal power flow (OPF) method which can be directly formulated from any network modelled in OpenDSS. Crucially, both OpenDSS and OPF models are integrated into the same Python script, opening a wide range of simulation opportunities. The described simulation platform is publicly available to the academic community and distribution utilities.**

***Index Terms*--distributed energy resources, distribution networks, optimization, Smart Grids, unbalanced power flow.**

I. Introduction

Despite the benefits that distributed energy resources (DERs) can bring to the energy system [1], their wide-scale adoption is expected to present many technical problems for utilities, such as the violation of the operational constraints of the underlying distribution network. These violations have been widely studied by the literature and, in non-dynamic studies, typically encompass the violation of voltage statutory limits and the overloading of the network assets (e.g. distribution transformers and cable sections). The first problem is very common in networks with distributed generation (DG) units, where reverse power flows, at times of high generation but low demand, result in the increment of nodal voltage magnitudes [2]. On the other hand, overloading can result from the increments on current and power flows derived from the connection of, e.g., electric vehicles and electric heat pumps, and from demand response mechanisms [3].

Distribution network operators (DNOs) are accountable for a reliable supply of electricity to end-use customers. Consequently, they must ensure a secure integration of DERs by, firstly, guaranteeing compliance with network operational constraints. In other to achieve this, in addition to circumstantial network reinforcement, DNOs are expected to implement active network management (ANM) strategies [4].

At the transmission system, there exists plenty of well-established tools for operation and planning, with centralized optimal power flow (OPF) algorithms being used for the solution of a large number of problems such as economic dispatch and optimal device allocation. With the inclusion of DERs, various publications have proposed the use of OPF for ANM in distribution systems [5]. Most common strategies include the regulation of power injections from DERs and the operation of OLTC-fitted transformers for voltage regulation. However, the lack of comprehensive telemetry and difficult real-time operation can impede near-term deployment of centralized OPF strategies in distribution grids. Consequently, recent focus has been placed on decentralized ANM solutions [6] (e.g. volt-var curves). These solutions divide the system into operation clusters, managed by independent control units that take decisions without complete network observability [7].

With different types of DERs resulting in different technical problems and requiring different levels of coordination, ANM solutions will vary depending on the technology under consideration, the expected available level of telemetry and the network characteristics. Consequently, there exists a need for a platform where the benefits and drawbacks of multiple approaches can be easily quantified and benchmarked. This work introduces a three-phase OPF method which can be directly formulated from any network modelled in OpenDSS [8]. The proposed platform provides the following benefits:

1. Both OpenDSS and OPF model can be integrated into the same Python [9] script, opening a wide range of simulation opportunities.
2. It can facilitate the benchmark of decentralized solutions, implemented in OpenDSS, with OPF solutions.
3. With many publicly available OpenDSS distribution networks models [10], [11], it can expedite the validation of ANM solutions under multiple network topologies.

The presented platform remains an open-source development in Python and is freely available to download for the academic community and distribution utilities. The integrated models are described in Section II. Section III validates the OPF model under a real LV network with domestic-scale photovoltaics (PVs). Finally, discussion takes place in Section IV and conclusions are drawn in Section IV.

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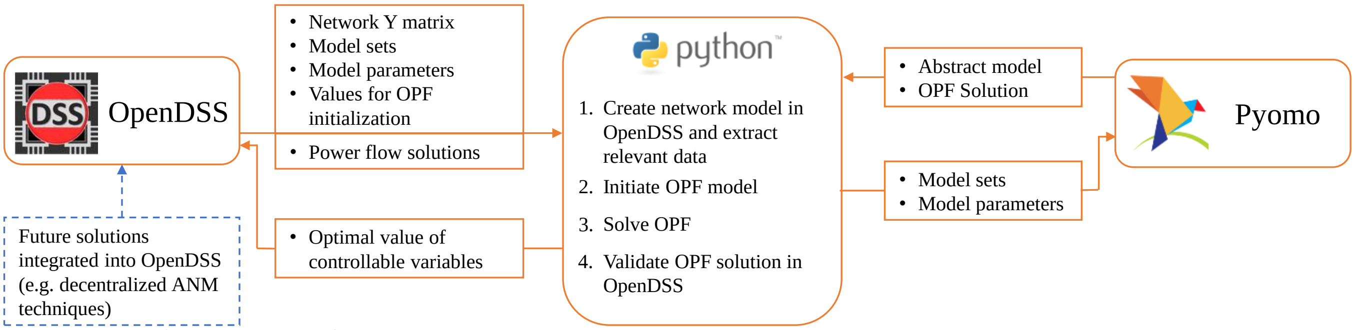

Fig. 1. Overview of the simulation platform

II. Methodology

Power flow simulators are a powerful tool for distribution network analysis and are used in a variety of studies such as quantification of DERs hosting capacity and verification of compliance with thermal and voltage limits under different network configurations. However, they are limited to exploring state variables under predefined nodal power injections, with future distribution network analysis requiring more flexibility in terms of simulation possibilities.

The objective of the presented simulation platform is to enable to easily incorporate power flow optimization into the formulation and validation of future ANM strategies. While there exist other simulation packages for optimization in power systems, they do not typically facilitate integration with external software/algorithms, account for 3-phase unbalanced operation and/or provide a wide variety of realistic network models. Hence, the platform presented in this paper is focused on translating -automatically- to a three-phase OPF problem, any network model available in OpenDSS. Therefore, enabling to retrieve OPF solutions, run power flow simulations and incorporate external algorithms/tools (from volt-var curves to forecasting techniques) into the same Python script; all of this, using open-source software packages.

A. *Summary of the simulation platform*

An overview of the simulation platform is presented in Fig. 1. It incorporates both OpenDSS (i.e. an open-source electrical power system simulation tool) and Pyomo [12] (an open-source Python-based optimization library), as objects of a single Python script.

1) *OpenDSS: network model*

OpenDSS is an open-source electrical power system simulation tool designed primarily for distribution systems. It is characterized by its solving speed and its flexible access and definition of each of the network components (i.e. lines, loads, transformers, etc.) as exemplified in Fig. 2. More importantly, it -and all of its functions- can be imported as an object in any Python instance.

Provided that a network model is available in OpenDSS, the simulation platform uses the latter for:

a) *Exporting the network data*

After compiling the created network, OpenDSS can export the network admittance matrix, the sets of network elements (i.e. loads, lines and buses) and the assets' parameters (e.g. cable ratings and nominal voltages). This data will be used for the definition of the OPF problem and the initialization of its variables.

Example of OpenDSS line definition
```
New Line.LINE1 Bus1=secondary Bus2=F_Bus_1 Linecode=185_SAC Length=0.03 phases=3 Units=km
New Line.LINE2 Bus1=F_Bus_1   Bus2=F_Bus_2 Linecode=95_SAC Length=0.08 phases=3 Units=km
```

Example of OpenDSS load definition
```
New Load.LOAD1 Phases=1 Bus1= F_Bus_2.1 kV=0.230 kW=1 Kvar=0.2 Model=8 ZIPV=[0 0 1 0 0 1 0.8]
New Load.LOAD2 Phases=1 Bus1= F  Bus  3.2 kV=0.230 kW=1 Kvar=0.2 Model=8 ZIPV=[0 0 1 0 0 1 0.8]
```
Fig. 2. Example of OpenDSS line and load definition

b) *Validating OPF model*

To ensure that both OpenDSS and OPF models are comparable, the values of the controllable variables (e.g. DERs' operational setpoints), obtained from the solution of the OPF problem, are incorporated as parameters in OpenDSS. A power flow simulation is solved, and the value of the state variables are compared to ensure a match.

c) *Testing and benchmarking ANM solutions*

In addition to providing the data required for the definition of the OPF model, the OpenDSS circuit can be used as a test network for ANM strategies that do not consist of the solution of a constraint power flow problem. For example, in a system with DG, the magnitude of the voltage at a bus with DG can be used as input for a user-defined volt-var curve. Then, the efficiency of the resulting network operation can be benchmarked against the solution of the OPF problem (which should be considered as the most optimal case).

2) *Pyomo OPF model*

Pyomo is an open-source Python library that supports a diverse set of capabilities for formulating and solving optimization models [12]. One of the characteristics of Pyomo is the possibility of creating abstract models; i.e. models that contain all the expressions and variables related to an optimization problem but with unspecified parameter values.

In the presented simulation package, an abstract model is formulated where the values for the problem sets (i.e. time, loads, buses, lines, etc.) and parameters (i.e. conductance and susceptance admittance matrices, connectivity matrix, etc.) are not defined until the time a solution is to be obtained. Essentially, providing a context for defining and initializing an OPF problem independently of the topology of the network to be studied.

B. *Unbalanced Optimal Power Flow*

This subsection describes the core expressions that define the 3-phase OPF formulation in Pyomo. Equations are derived from the current mismatch method presented in [13] for a 3-

phase network with no explicit modelling of the neutral cable. It is important to highlight that while single-phase network representations are common at the transmission system, the characteristics of distribution systems (e.g. single-phase customers, stochastic load behaviour, etc.) impede the assumption of a balanced system.

1) Definition of nodal current injections

The current mismatch equations are used to relate the nodal voltage phasors with the active and reactive power injections from each load and generation unit in the system. These current mismatches are defined in equations (1)-(5):

$$\Delta I_k^s = I_{calc_k}^s - I_{sp_k}^s \tag{1}$$

$$P_{sp_k}^s = \Re(V_k^s)\,\Re\left(I_{sp_k}^s\right) + \Im(V_k^s)\,\Im\left(I_{sp_k}^s\right) \tag{2}$$

$$Q_{sp_k}^{s,t} = \Im(V_k^s)\,\Re\left(I_{sp_k}^s\right) - \Re(V_k^s)\,\Im\left(I_{sp_k}^s\right) \tag{3}$$

$$\Re\left(I_{calc_k}^s\right) = \sum_{i\in\Omega}\sum_{a\in\sigma_p}\left[G_{ki}^{sa}\,\Re\left(V_i^a\right) - B_{ki}^{sa}\,\Im\left(V_i^a\right)\right] \tag{4}$$

$$\Im\left(I_{calc_k}^s\right) = \sum_{i\in\Omega}\sum_{a\in\sigma_p}\left[G_{ki}^{sa}\,\Im\left(V_i^a\right) + B_{ki}^{sa}\,\Re\left(V_i^a\right)\right] \tag{5}$$

where
$s, a \in \sigma_p$
$k, i \in \Omega$

Ω	set of network buses
σ_p	set of phases $\{a,b,c\}$
$I_{calc_k}^s$	calculated current injections
$I_{sp_k}^s$	specified current injections
V_k^s, V_i^a	phase voltage at bus k
G_{ki}^{sa}, B_{ki}^{sa}	conductance and susceptance from nodal admittance matrix

The specified active power and reactive power injections, $P_{sp_k}^s$ and $Q_{sp_k}^{s,t}$, are defined by equations (6)-(7):

$$P_{sp_k}^s = P_{g_k}^s - \left[P_{Z_k}^s\left(\frac{V_k^s}{V_{nom_k}}\right)^2 + P_{I_k}^s\left(\frac{V_k^s}{V_{nom_k}}\right) + P_{P_k}^s\right] \tag{6}$$

$$Q_{sp_k}^s = Q_{g_k}^s - \left[Q_{Z_k}^s\left(\frac{V_k^s}{V_{nom_k}}\right)^2 + Q_{I_k}^s\left(\frac{V_k^s}{V_{nom_k}}\right) + Q_{P_k}^s\right] \tag{7}$$

where

$P_{g_k}^s, Q_{g_k}^s$	active and reactive power generation at bus k and phase s
$P_{Z_k}^s, P_{I_k}^s, P_{P_k}^s$	ZIP components of active power demand at bus k and phase s
$Q_{Z_k}^s, Q_{I_k}^s, Q_{P_k}^s$	ZIP components of reactive power demand at bus k and phase s
V_{nom_k}	nominal voltage for loads in bus k

2) Equality constraints

a) Current mismatch: One of the bases for this OPF formulation is to force the current mismatches from Equation (1) to be equal to zero in Constraint (8):

$$\Delta I_k^s = 0 \tag{8}$$

b) Slack bus: The voltage at the slack bus, V_{slack}^s, is constrained to be equal to the specified value $V_{sp_{slack}}^s$ by Constraint (9):

$$V_{slack}^s = V_{sp_{slack}}^s \tag{9}$$

3) Network operational limits: inequality constraints

a) Voltage limits: the magnitude of the steady-state voltage at each node must comply with the voltage statutory limits defined in Constraint (10); where the upper and lower voltage statutory limits for node k, i.e. V_{min_k} and V_{max_k}, are those of the corresponding distribution code.

$$V_{min_k} \le V_k^s \le V_{max_k} \tag{10}$$

b) Line thermal limits: the current flow at each phase s of every line l, I_l^s, must not exceed the rated current, $I_{l_{max}}$, delimited by the cable manufacturer as per Constraint (11):

$$I_l^s \le I_{l_{max}} \tag{11}$$

c) Transformer rating limits: the total apparent power flow across each transformer must not exceed its rating, $S_{transformer_{max}}$, as per Constraint (12).

$$\sum_{s\in\Omega} S_{transformer}^s \le S_{transformer_{max}} \tag{12}$$

III. Test Case

The proposed platform is validated on a British low voltage (LV) network. The network consists of residential customers that have adopted domestic-scale PVs. Here, the amount of lines in the original model has been greatly reduced, without compromising results, to facilitate the solution of the OPF problem. All mentioned data is provided with the model.

A. Network model

The studied network is obtained from a set of 25 real networks provided by ENWL [11]. It supplies power to a total of 330 single-phase residential customers distributed among 3 radial feeders and is depicted in Fig. 3. Nominal voltages of 0.4 kV and 10 kV are adopted for the LV and MV levels with upper and lower statutory limits of ±10% according to the EN 50160 standard [14]. The distribution transformer has a rating of 800 kVA. Cable data was obtained from manufacturers' datasheets.

The models from [11] were created using GIS data, with the cable paths from one point to another consisting of a series of very small linear segments. Here, adjacent line segments having the same cable type and connecting buses with no customers (unnecessarily increasing the number of state variables in the OPF) were merged. This modification enabled to reduce the number of lines in approximately 90% as shown in Table I.

B. Profiles for residential customers

A pool of 100 residential loads profiles is obtained using the model from [15]. This model creates realistic daily time-series profiles for residential loads accounting for house occupancy, activity, electrical appliances and thermal operation. The provided profiles have a resolution of 5 minutes and represent load composition in terms of constant impedance, constant current and constant power, i.e. variating ZIP model, for either active or reactive power as exemplified in Fig. 4.

C. PV models

1) Generation profiles

PV profiles are allocated from the pool of 100 profiles available from [11]. Given the limited geographical extension of LV feeders, it is assumed that solar irradiance is uniform across all customers. Every house is considered to have a PV (the worst-case scenario in terms of voltage impacts). A power factor limit of 0.95 is considered for the PV inverters, that have a rating, $S_{inverter_{pv}}$, that is 10% greater than the PV active power rating.

2) Controllable variables

PV inverters are capable of absorbing reactive power. This capability, together with active power curtailment, is used to operate them via the two controllable variables $P_{control_{pv}}$ and $Tan\varphi_{control_{pv}}$ as described in equations (13)-(17):

$$P_{pv} = P_{sp_{pv}} P_{control_{pv}} \tag{13}$$

$$Q_{pv} = P_{pv} Tan\varphi_{control_{pv}} \tag{14}$$

$$0 \leq P_{control_{pv}} \leq 1 \tag{15}$$

$$-\tan(\text{acos}(\varphi_{min})) \leq Tan\varphi_{control_{pv}} \leq 0 \tag{16}$$

$$\sqrt{P_{pv}{}^{2} + Q_{pv}{}^{2}} \leq S_{inverter_{pv}} \tag{17}$$

where P_{pv} and Q_{pv} are the injected active and reactive power for each PV, pv, and $P_{sp_{pv}}$ the value from the PV profile in [11].

D. Objective function

The objective of the OPF problem, Equation (18), is to minimise PV active power curtailment and reactive power consumption (the first one is prioritized).

$$\min \sum_{t \in T} \sum_{pv \in PVs} -P_{control_{pv}} + \frac{1}{\alpha} Tan\varphi_{control_{pv}} \tag{18}$$

where T is the simulation time horizon and PVs the set of photovoltaics. T includes the 24 hours of the day with a 10-minute time step. Note that $\alpha \gg 1$.

E. Problem formulation

- Objective function: (18)
- Constraints and expressions: (1)-(18)
- Control variables: $P_{control_{pv}}$ and $Tan\varphi_{control_{pv}}$

IV. Results

The network from Section III is incorporated into the simulation platform from Section II. All the data required for the OPF problem is extracted from the OpenDSS circuit and used to create an instance for the Pyomo abstract model. This instance is solved using the NLP solver Knitro [16].

A. OPF model validation

The solution of the OPF model in the Pyomo instance provides with the optimal active and reactive power injections for all PVs (characterised by $P_{control_{pv}}$ and $Tan\varphi_{control_{pv}}$). To validate the OPF model, these injections are incorporated as parameters into OpenDSS and the circuit is solved for each of the time steps in T. Then, results are compared with the OPF variables and expressions to ensure a satisfactory match; this is exemplified in Fig. 5 and Fig. 6, where it can be observed that voltages at the end of Feeder3 and current flows at Feeder3's main cable section coincide between both models.

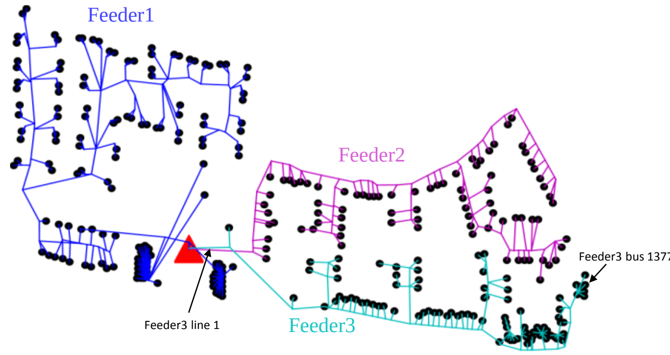

Fig. 3. ENWL Network 12 (simplified model). Dots represent the residential customers and the red triangle, the distribution transformer

TABLE I
ENWL Network Model 12

Feeder	no. of customers	no. of lines (original model)	no. of lines (simplified model)
Feeder1	157	3,154	240
Feeder2	83	1,817	157
Feeder3	90	1,376	149

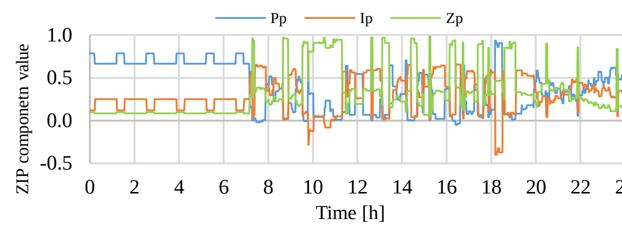

Fig. 4. Active power ZIP model components - profile for a single house

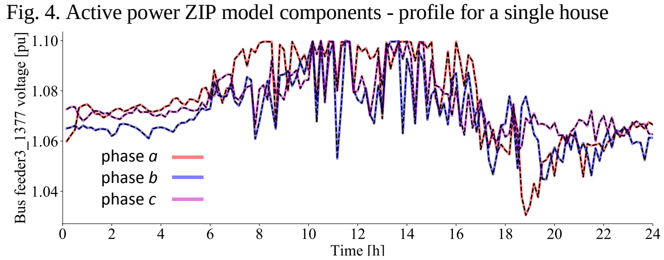

Fig. 5. Feeder3 bus 1377 voltages. OpenDSS (dashed line) vs OPF (solid line)

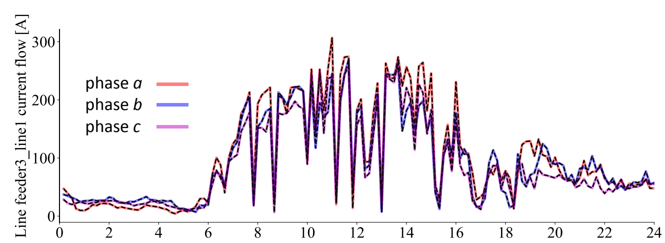

Fig. 6. Feeder3 line 1 current flows. OpenDSS (dashed line) vs OPF (solid line)

B. Results from the OPF solution

Results from the OPF solution are shown in Fig. 7 to Fig. 9. In Fig. 7, the voltage magnitude across all nodes (3 phases per bus) are plotted for the time horizon T. It can be observed that voltages never exceed the upper limit from Constraint(10). On

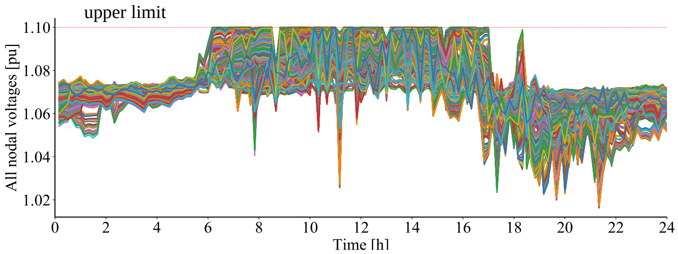

Fig. 7. Voltage magnitudes across all network nodes

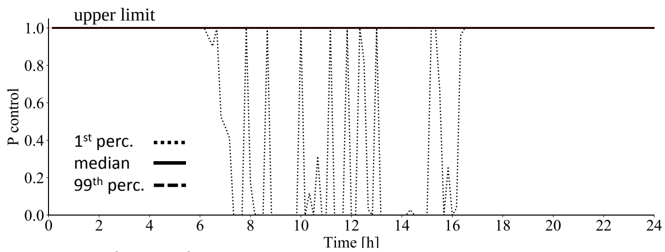

Fig. 8. 1st, 50th and 99th percentiles for all PVs $P_{control_{pv}}$

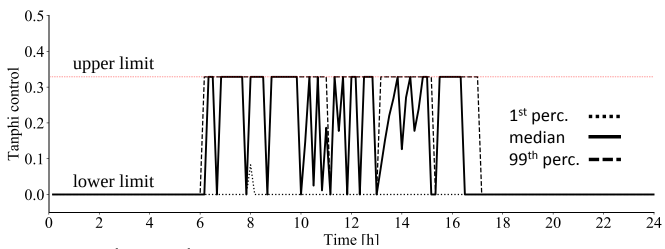

Fig. 9. 1st, 50th and 99th percentiles for all PVs $Tan\varphi_{control_{pv}}$

the other hand, Fig. 8 and Fig. 9 show the obtained value for the controllable variables $P_{control_{pv}}$ and $Tan\varphi_{control_{pv}}$, which regulate the active and reactive power injected by each PV inverter. Results are presented in terms of median, and 1st and 99th percentiles. It is shown that in order to comply with constraints (10)-(12), reactive power consumption and, less commonly, active power curtailment take place during generation hours. It must be mentioned that the resulting operational setpoints for each PV depend largely on its location. PVs that have a greater impedance path to the head of the feeder will tend to operate with lower power factors and higher curtailment; this is because their actions will have a greater repercussion on the feeder voltages.

V. Discussion

A. Representation of neutral conductor

The presented OPF uses a 3-wire network representation. In networks with a neutral conductor), Kron's reduction is required; this assumes the neutral to be perfectly connected to the ground.

B. Publicly available OpenDSS networks

There exists a large set of networks modelled in OpenDSS that are available to the public. For example, in addition to the 25 LV networks from [11] (a total of 128 LV feeders), IEEE Test Cases and EPRI Test Circuits are available from [10].

C. Problem size and right solver

As in any optimization problem, not every solver will be able to find a satisfactory solution; this will greatly depend on the problem definition. While no issues were encountered with Knitro, a different solver may not be able to converge to an optimal solution with the presented network and time resolution. Note that the provided platform enables to model the complete network or just one feeder at the time. The larger the network and the time set, the more difficult it will be for a solver to converge to an optimal solution.

I. Conclusions

This paper introduces a three-phase OPF method which can be directly formulated from any network modelled in OpenDSS. Using a real network with PVs, the presented method shows to be effective at coordinating, according to an objective function, the operation of DERs with the network operational constraints.

Both OpenDSS and OPF models, which share the same network admittance matrix, are integrated into the same Python script, opening a wide range of simulation opportunities and facilitating the validation of future ANM solutions. The presented platform remains an open-source development in Python and is freely available to download for the academic community and distribution utilities

References

[1] P. Siano, "Demand response and smart grids - A survey," *Renew. Sustain. Energy Rev.*, vol. 30, pp. 461–478, 2014.

[2] A. T. Procopiou and L. F. Ochoa, "Voltage Control in PV-Rich LV Networks Without Remote Monitoring," *IEEE Transactions on Power Systems*, vol. 32, no. 2. pp. 1224–1236, 2017.

[3] M. Nijhuis, M. Gibescu, and J. F. G. Cobben, "Assessment of the impacts of the renewable energy and ICT driven energy transition on distribution networks" *Ren. Sust. Energy Rev.*, v 52, p 1003-1014, 2015.

[4] A. Alarcon-Rodriguez, E. Haesen, G. Ault, J. Driesen, and R. Belmans, "Multi-objective planning framework for stochastic and controllable distributed energy resources," *IET Renew. Power Gener.*, vol. 3, no. 2, pp. 227–238, 2009.

[5] G. Valverde and T. Van Cutsem, "Model predictive control of voltages in active distribution networks," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2152–2161, 2013.

[6] P. N. Vovos, A. E. Kiprakis, A. R. Wallace, and G. P. Harrison, "Centralized and distributed voltage control: Impact on distributed generation penetration," *IEEE Trans. Power Syst.*, v 22, p. 476–483, 2007.

[7] J. W. Smith, W. Sunderman, R. Dugan, and B. Seal, "Smart inverter volt/var control functions for high penetration of PV on distribution systems," in *2011 IEEE/PES PSCE*, 2011.

[8] EPRI, "OpenDSS - Open Distribution System Simulator," *smartgrid.epri.com*, 2014.

[9] Python Core Team, "Python: A dynamic, open source programming language," *Python Software Foundation*, 2015. .

[10] Electric Power Research Institute - EPRI, "Smart Grid Resource Center" [Online]. Available: http://smartgrid.epri.com/SimulationTool.aspx.

[11] ENWL, "LV network models," *Low Voltage Network Solutions*, 2014. [Online]. Available: http://www.enwl.co.uk/about-us/the-future/nia-lcnf-tier-1/low-voltage-network-solutions. [Accessed: 01-Jul-2016].

[12] W. E. Hart, C. Laird, J.-P. Watson, and D. L. Woodruff, "Pyomo – Optimization Modeling in Python," *Adv. Model. Agric. Syst.*, 2012.

[13] P. A. N. Garcia, J. L. R. Pereira, S. Carneiro, and V. M. Da Costa, "Three-phase power flow calculations using the current injection method," *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 508–514, 2000.

[14] CENELEC, "Voltage Disturbances Standard EN 50160 Voltage Characteristics in Public Distribution Systems," 2004.

[15] K. McKenna and A. Keane, "Residential Load Modeling of Price-Based Demand Response for Network Impact Studies," *IEEE Transactions on Smart Grid*, vol. 7, no. 5. pp. 2285–2294, 2016.

[16] R. H. Byrd, J. Nocedal, and R. A. Waltz, "Knitro: An Integrated Package for Nonlinear Optimization," *Energy*, vol. 83, pp. 35–59, 2006.