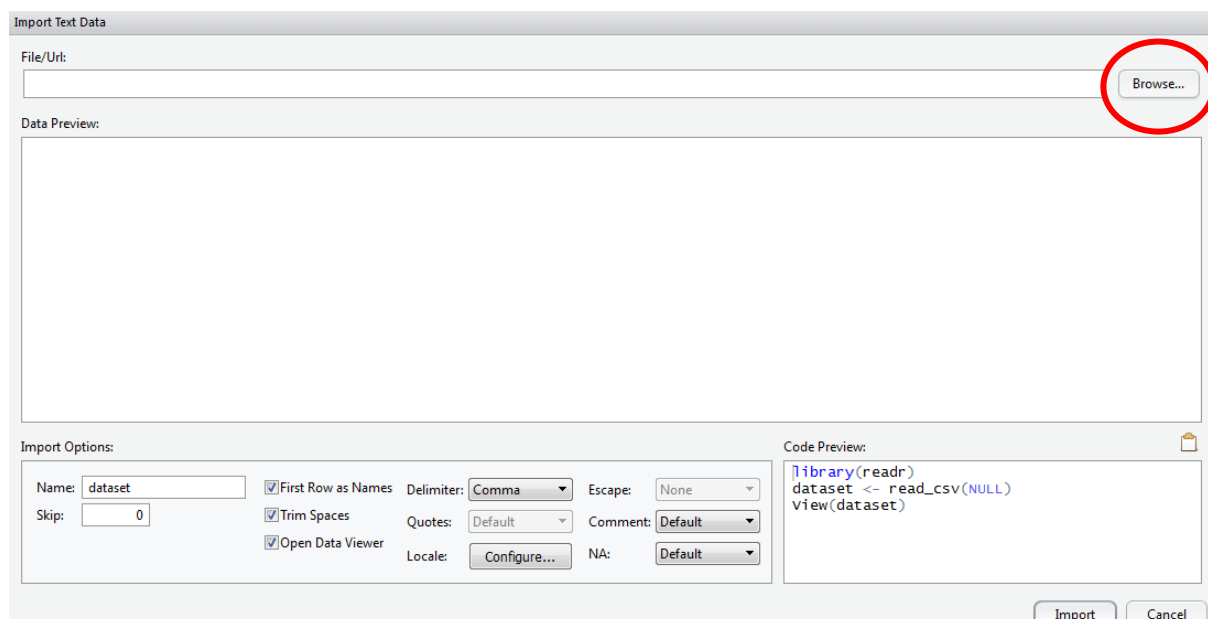
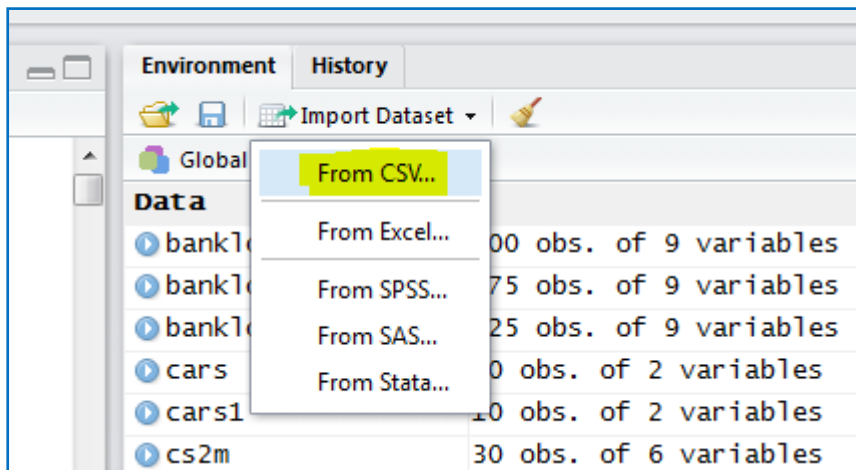
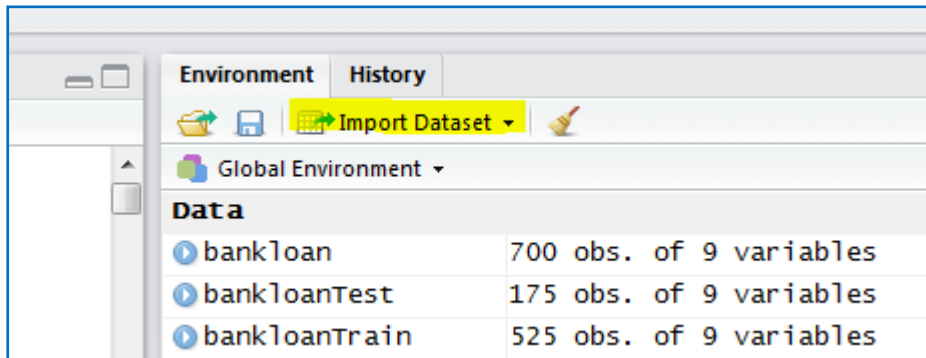


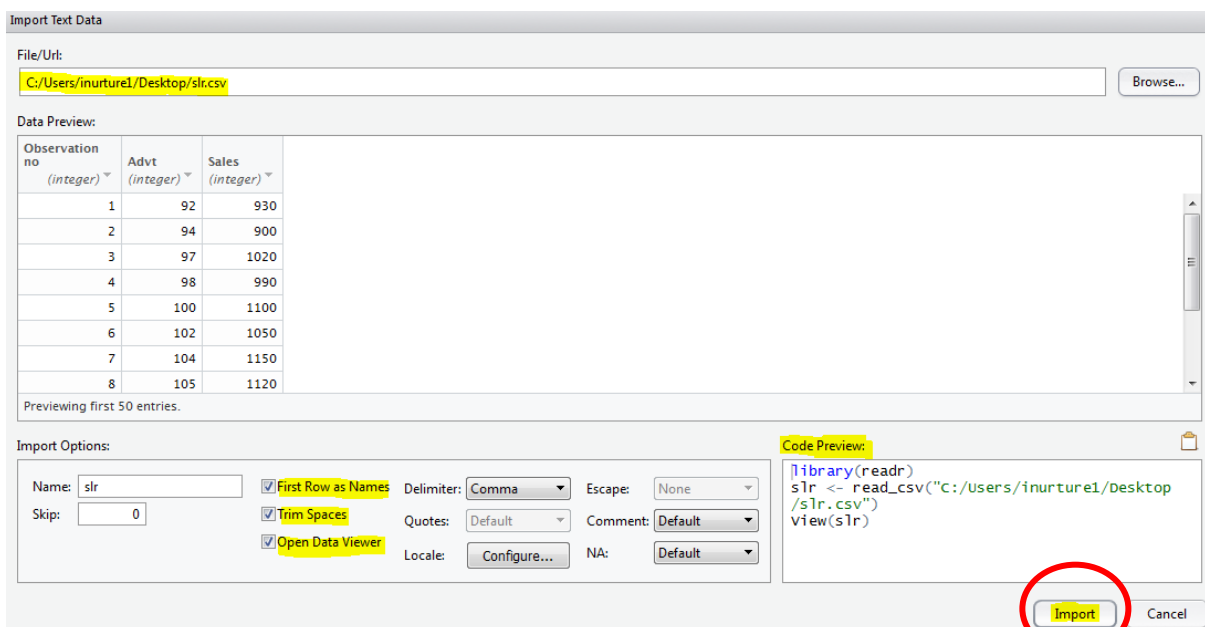
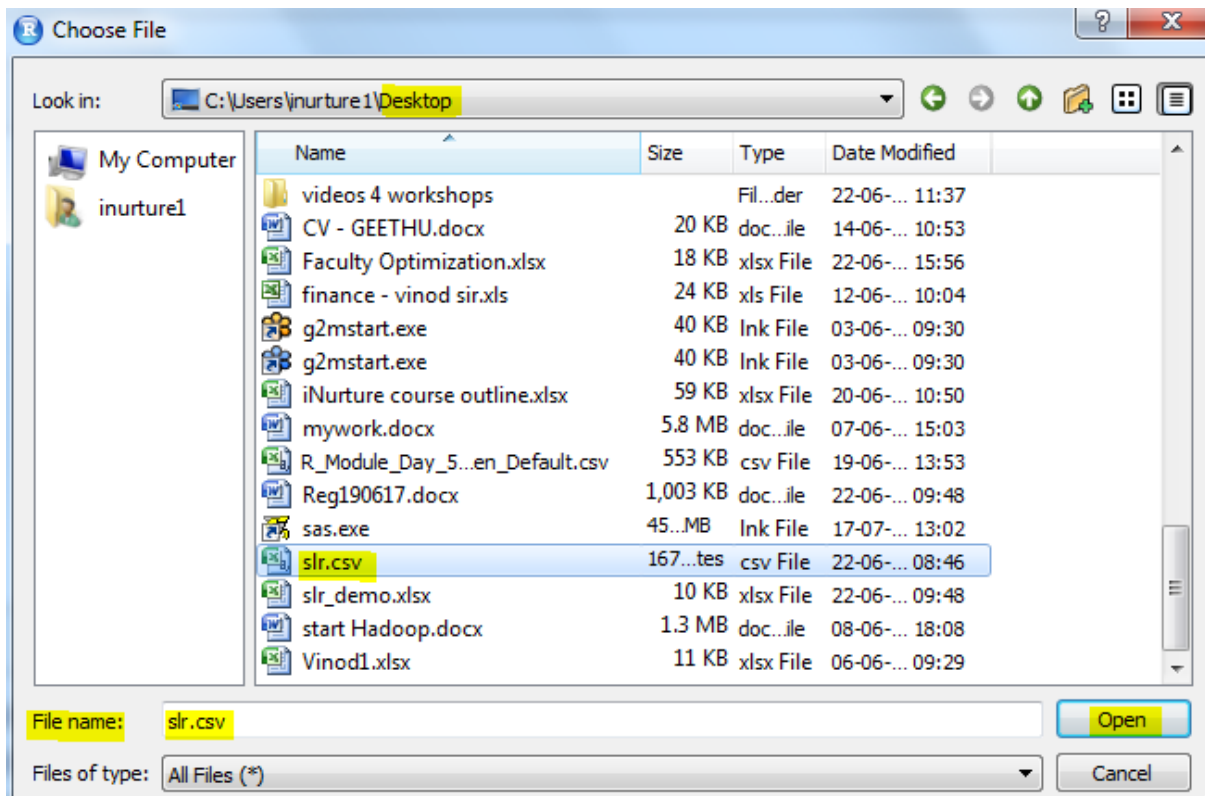
Simple Linear Regression

Import File in R

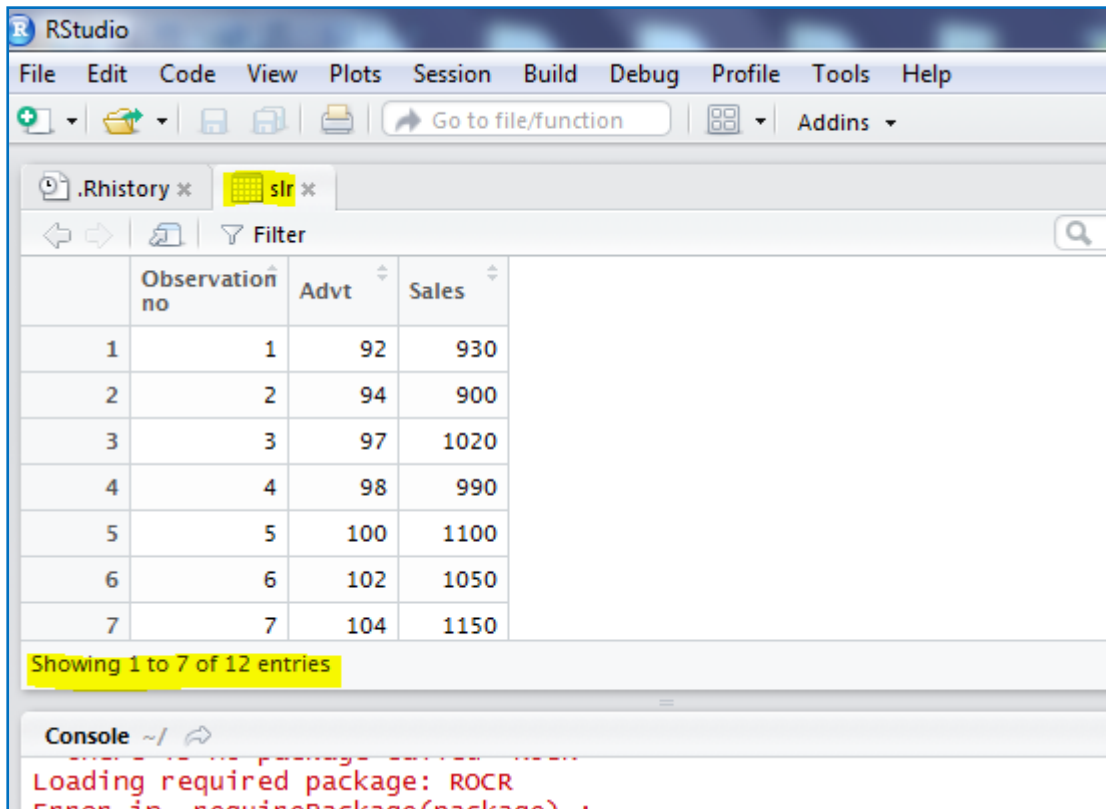
File name **slr.csv**



Simple Linear Regression



Simple Linear Regression

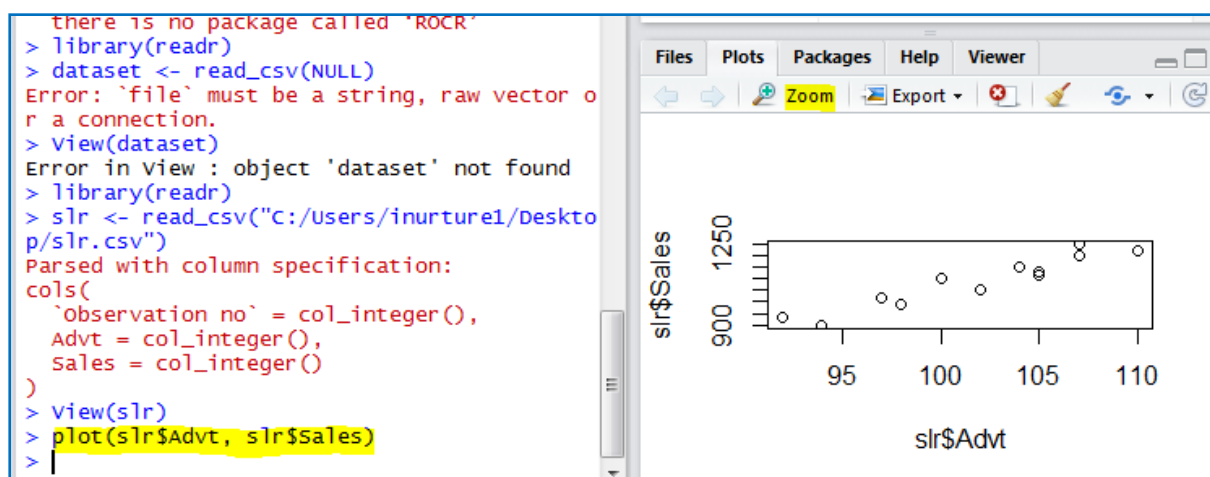


	Observation no	Advt	Sales
1	1	92	930
2	2	94	900
3	3	97	1020
4	4	98	990
5	5	100	1100
6	6	102	1050
7	7	104	1150

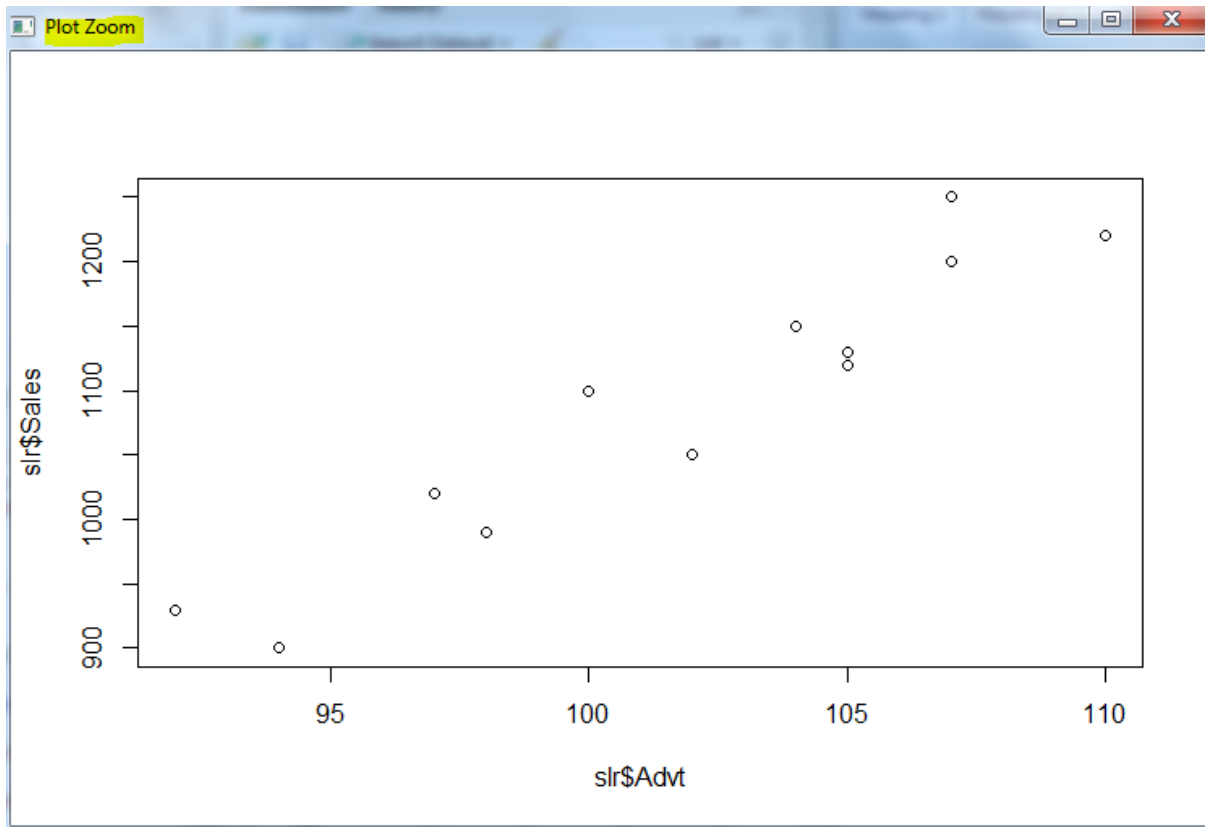
Showing 1 to 7 of 12 entries

Console ~/
Loading required package: ROCR
Error in requirePackage(package) :

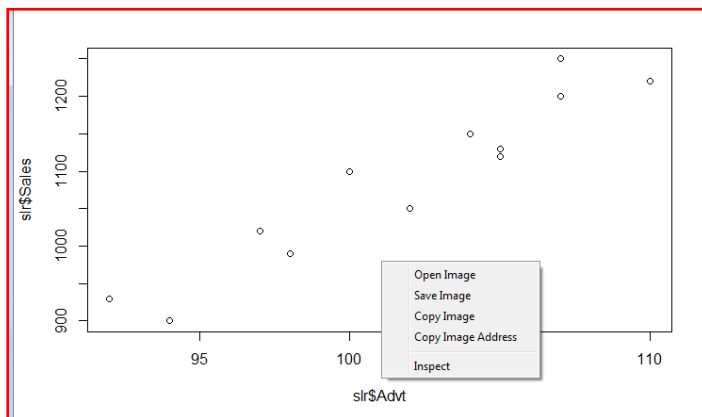
Always first plot scatter plot and see relationship

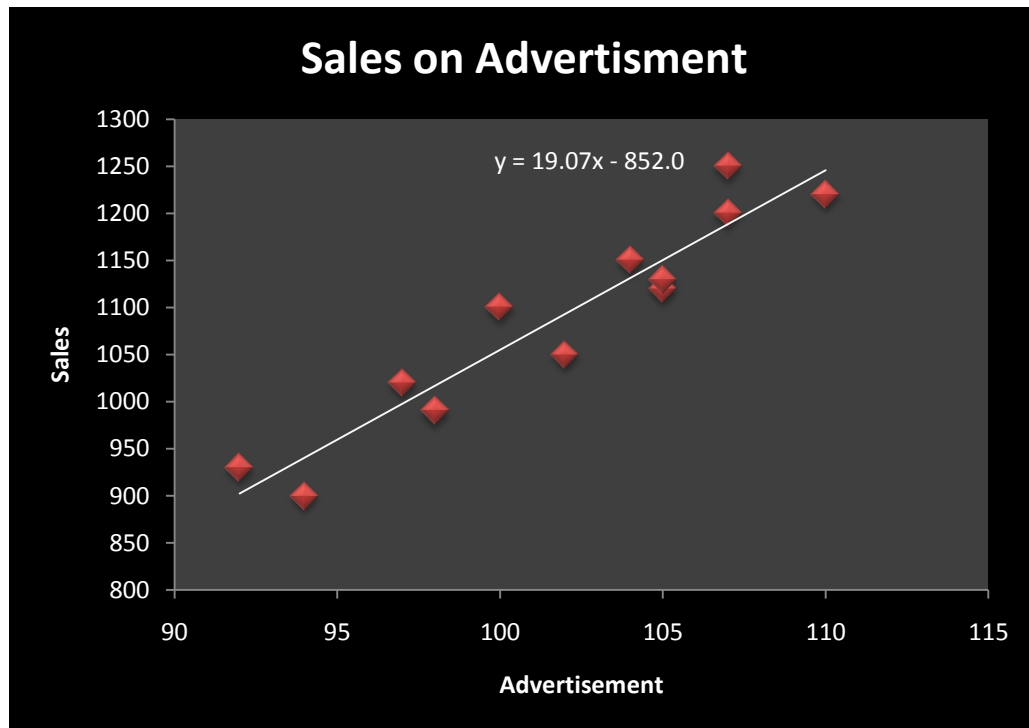


Simple Linear Regression



Copy plot and paste at your desired destination





Type equation here.

Build Linear Regression Model

```
> mod<-lm(slr$Sales~slr$Advt)
> mod

call:
lm(formula = slr$Sales ~ slr$Advt)

Coefficients:
(Intercept)      slr$Advt
    -852.08         19.07
```

Simple Linear Regression

```
> summary(mod)

Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78  -4.181   0.00188 **
slr$Advt       19.07       2.00    9.535  2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF,  p-value: 2.454e-06
```

	A	B	C	D
1				
2	Observat ion no	Advt(x)	Sales(y)	$x^2(xx)$
3	1	92	930	=B3*B3
4	2	94	900	
5	3	97	1020	
6	4	98	990	
7	5	100	1100	
8	6	102	1050	
9	7	104	1150	
10	8	105	1120	
11	9	105	1130	
12	10	107	1200	
13	11	107	1250	
14	12	110	1220	
15				

Leave Row 1 as
blank

Simple Linear Regression

	A	B	C	D
1				
2	Observat ion no	Advt(x)	Sales(y)	$x^2(xx)$
3	1	92	930	8464
4	2	94	900	
5	3	97	1020	
6	4	98	990	
7	5	100	1100	
8	6	102	1050	
9	7	104	1150	
10	8	105	1120	
11	9	105	1130	
12	10	107	1200	
13	11	107	1250	
14	12	110	1220	
15				

D3		fx =B3*B3		
	A	B	C	D
1				
2	Observat ion no	Advt(x)	Sales(y)	$x^2(xx)$
3	1	92	930	8464
4	2	94	900	
5	3	97	1020	
6	4	98	990	
7	5	100	1100	
8	6	102	1050	
9	7	104	1150	
10	8	105	1120	
11	9	105	1130	
12	10	107	1200	
13	11	107	1250	
14	12	110	1220	
15				

This is called
formula bar

There is a +
sign...**hold it** and
drag till D14 and
leave cursor

Simple Linear Regression

	A	B	C	D
1				
2	Observat ion no	Advt(x)	Sales(y)	$x^2(xx)$
3	1	92	930	8464
4	2	94	900	8836
5	3	97	1020	9409
6	4	98	990	9604
7	5	100	1100	10000
8	6	102	1050	10404
9	7	104	1150	10816
10	8	105	1120	11025
11	9	105	1130	11025
12	10	107	1200	11449
13	11	107	1250	11449
14	12	110	1220	12100
15				

	A	B	C	D	E
1					
2	Observat ion no	Advt(x)	Sales(y)	$x^2(xx)$	xy
3	1	92	930	8464	=B3*C3
4	2	94	900	8836	
5	3	97	1020	9409	
6	4	98	990	9604	
7	5	100	1100	10000	
8	6	102	1050	10404	
9	7	104	1150	10816	
10	8	105	1120	11025	
11	9	105	1130	11025	
12	10	107	1200	11449	
13	11	107	1250	11449	
14	12	110	1220	12100	
15					

Press ENTER
 → 85560 will
 appear → There
 is a + sign...**hold**
it and drag till
 E14 and leave
 cursor

Simple Linear Regression

	A	B	C	D	E
1					
2	Observat	Advt(x)	Sales(y)	$x^2(xx)$	xy
3	ion no				
4	1	92	930	8464	85560
5	2	94	900	8836	84600
6	3	97	1020	9409	98940
7	4	98	990	9604	97020
8	5	100	1100	10000	110000
9	6	102	1050	10404	107100
10	7	104	1150	10816	119600
11	8	105	1120	11025	117600
12	9	105	1130	11025	118650
13	10	107	1200	11449	128400
14	11	107	1250	11449	133750
15	12	110	1220	12100	134200
16					

Select B3 to B14 → Go to **Sum** and click it. You will have sum at B15

Sum (Alt+=)

Click here to display the result of a simple calculation, such as Average or Maximum Value, after the selected cells.

15 SUM =

You will get sum here in B15

Simple Linear Regression

	A	B	C	D	E
1					
2	Observat ion no	Advt(x)	Sales(y)	$x^2(xx)$	xy
3	1	92	930	8464	85560
4	2	94	900	8836	84600
5	3	97	1020	9409	98940
6	4	98	990	9604	97020
7	5	100	1100	10000	110000
8	6	102	1050	10404	107100
9	7	104	1150	10816	119600
10	8	105	1120	11025	117600
11	9	105	1130	11025	118650
12	10	107	1200	11449	128400
13	11	107	1250	11449	133750
14	12	110	1220	12100	134200
15	SUM =	1221			
16					

There is a + sign...**hold it**
and drag till E15 and
leave cursor

Simple Linear Regression

	A	B	C	D	E
1					
2	Observat	Advt(x)	Sales(y)	x^2(xx)	xy
3	ion no				
4	1	92	930	8464	85560
5	2	94	900	8836	84600
6	3	97	1020	9409	98940
7	4	98	990	9604	97020
8	5	100	1100	10000	110000
9	6	102	1050	10404	107100
10	7	104	1150	10816	119600
11	8	105	1120	11025	117600
12	9	105	1130	11025	118650
13	10	107	1200	11449	128400
14	11	107	1250	11449	133750
15	12	110	1220	12100	134200
16	SUM =	1221	13060	124581	1335420

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$SS_{xy} = 1335420 - \frac{1221 \times 13060}{12} = 6565$$

$$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 124581 - \frac{(1221)^2}{12} = 344.25$$

This is called
Regression
Coefficient

$$b_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{6565}{344.25} = 19.07$$

This is called
Intercept

$$b_0 = \frac{\sum y}{n} - b_1 \frac{\sum x}{n} = \frac{13060}{12} - 19.07 \frac{1221}{12} = -852.08$$

Regression Equation:

$$y = a + bx$$

$$y = b_0 + b_1 x$$

Sales = intercept + regression coefficient × advertisement

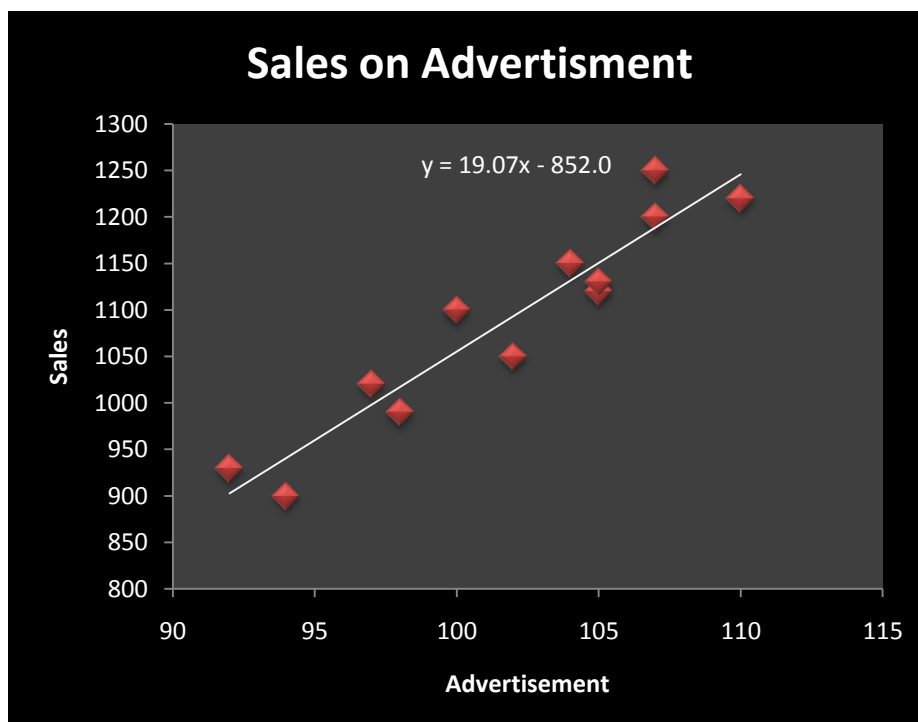
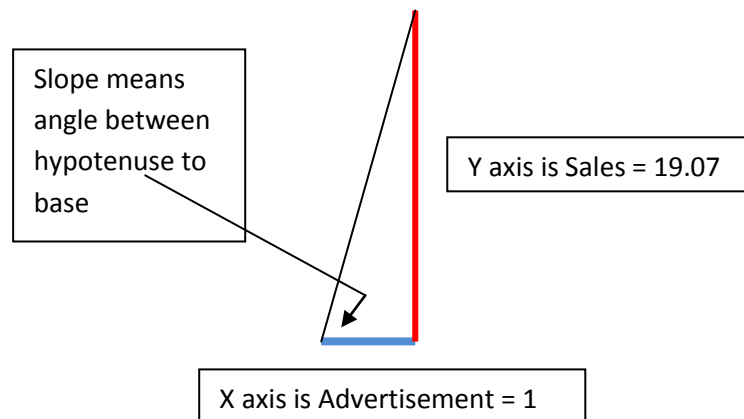
$$\text{Sales} = -852.08 + 19.07 \times \text{advertisement}$$

Simple Linear Regression

Interpretation of b_1

One unit change in **Advertisement** will result in 19.07 times change in **Sales**

Regression coefficient is also called Slope (or $\tan \theta = \text{Perpendicular/Base} = \text{Sales/Advertisement}$)



Standard Error of Mean

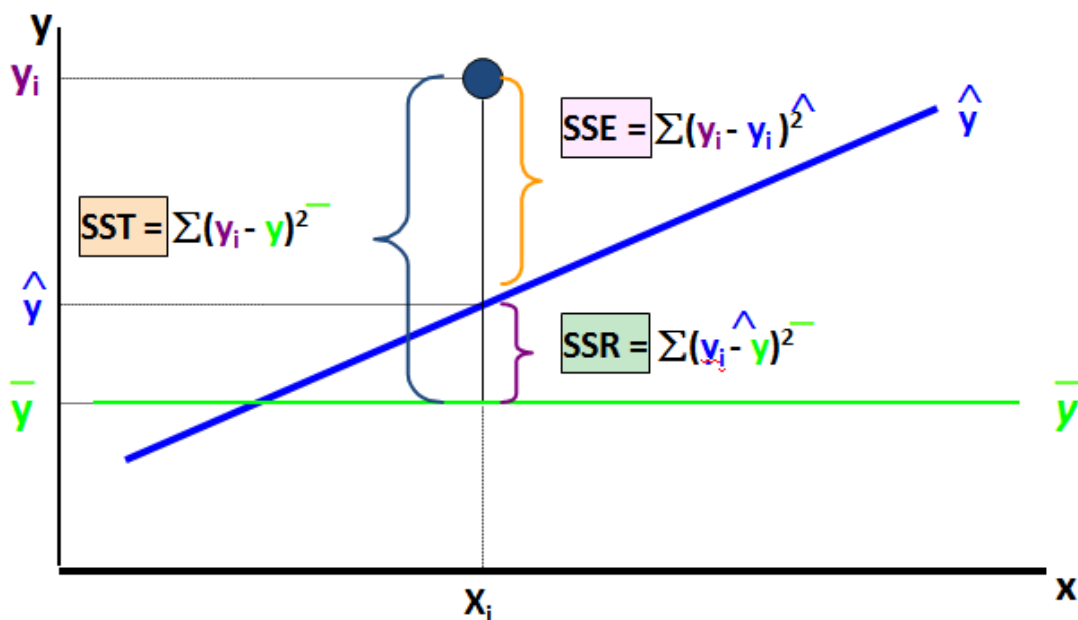
```
> summary(mod)

Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78  -4.181   0.00188 **
slr$Advt       19.07       2.00   9.535 2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF,  p-value: 2.454e-06
```



Simple Linear Regression

ZTEST						=SE\$1+\$F\$1*B3
	A	B	C	D	E	F
1					-852.08	19.07
2	Observation no	Advt(x)	Sales(y)	x^2(xx)	xy	Pred= yhat = a + bx = -852.08+ 19.07x
3	1	92	930	8464		=SE\$1+\$F\$1*B3
4	2	94	900	8836	84600	
5	3	97	1020	9409	98940	
6	4	98	990	9604	97020	
7	5	100	1100	10000	110000	
8	6	102	1050	10404	107100	
9	7	104	1150	10816	119600	
10	8	105	1120	11025	117600	
11	9	105	1130	11025	118650	
12	10	107	1200	11449	128400	
13	11	107	1250	11449	133750	
14	12	110	1220	12100	134200	
15	SUM =	1221	13060	124581	1335420	
16						

\$ sign is used for fixing the same calculation for all cells

Press enter → hold the + sign at the right bottom corner of cell F3 → drag till F15 → LEAVE the cursor → you will have calculations in all cells (F4:F15)

	A	B	C	D	E	F
1					-852.08	19.07
2	Observation no	Advt(x)	Sales(y)	x^2(xx)	xy	Pred= yhat = a + bx = -852.08+ 19.07x
3	1	92	930	8464	85560	902.36
4	2	94	900	8836	84600	940.5
5	3	97	1020	9409	98940	997.71
6	4	98	990	9604	97020	1016.78
7	5	100	1100	10000	110000	1054.92
8	6	102	1050	10404	107100	1093.06
9	7	104	1150	10816	119600	1131.2
10	8	105	1120	11025	117600	1150.27
11	9	105	1130	11025	118650	1150.27
12	10	107	1200	11449	128400	1188.41
13	11	107	1250	11449	133750	1188.41
14	12	110	1220	12100	134200	1245.62
15	SUM =	1221	13060	124581	1335420	22432.39
16						

Simple Linear Regression

```
> Pred<-predict(lm(slr$Sales~slr$Advt))
```

```
> Pred
      1      2      3      4      5      6      7
902.3965 940.5374 997.7487 1016.8192 1054.9601 1093.1009 1131.2418
      8      9     10     11     12
1150.3123 1150.3123 1188.4532 1188.4532 1245.6645
```

	A	B	C	D	E	F	G
1					-852.08	19.07	
2	Observation no	Advt(x)	Sales(y)	x^2(xx)	xy	Pred= yhat = a + bx = -852.08+ 19.07x	Residuals
3	1	92	930	8464	85560	902.36	=C3-F3
4	2	94	900	8836	84600	940.5	
5	3	97	1020	9409	98940	997.71	
6	4	98	990	9604	97020	1016.78	
7	5	100	1100	10000	110000	1054.92	
8	6	102	1050	10404	107100	1093.06	
9	7	104	1150	10816	119600	1131.2	
10	8	105	1120	11025	117600	1150.27	
11	9	105	1130	11025	118650	1150.27	
12	10	107	1200	11449	128400	1188.41	
13	11	107	1250	11449	133750	1188.41	
14	12	110	1220	12100	134200	1245.62	
15	SUM =	1221	13060	124581	1335420	22432.39	
16							

Press enter →
hold the + sign
at the right
bottom corner
of cell G3 →
drag till G15 →
LEAVE the
cursor → you
will have
calculations in
all cells
(G4:G15)

```
> error<- residuals(lm(slr$Sales~slr$Advt))
> error
      1      2      3      4      5      6      7
27.60349 -40.53740 22.25127 -26.81917 45.03994 -43.10094 18.75817
      8      9     10     11     12
-30.31227 -20.31227 11.54684 61.54684 -25.66449
> summary(error)
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-43.100 -27.690  -4.383   0.000  23.590  61.550
```

Simple Linear Regression

	A	B	C	D	E	F	G
1					-852.08	19.07	
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy	Pred= yhat = $a + bx =$ -852.08+ 19.07x	Residuals
3	1	92	930	8464	85560	902.36	27.64
4	2	94	900	8836	84600	940.5	-40.5
5	3	97	1020	9409	98940	997.71	22.29
6	4	98	990	9604	97020	1016.78	-26.78
7	5	100	1100	10000	110000	1054.92	45.08
8	6	102	1050	10404	107100	1093.06	-43.06
9	7	104	1150	10816	119600	1131.2	18.8
10	8	105	1120	11025	117600	1150.27	-30.27
11	9	105	1130	11025	118650	1150.27	-20.27
12	10	107	1200	11449	128400	1188.41	11.59
13	11	107	1250	11449	133750	1188.41	61.59
14	12	110	1220	12100	134200	1245.62	-25.62
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39
16							

Find average in cell B16

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1					-852.08	19.07								
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy	Pred= yhat = $a + bx =$ -852.08+ 19.07x	Residuals							
3	1	92	930	8464	85560	902.36	27.64							
4	2	94	900	8836	84600	940.5	-40.5							
5	3	97	1020	9409	98940	997.71	22.29							
6	4	98	990	9604	97020	1016.78	-26.78							
7	5	100	1100	10000	110000	1054.92	45.08							
8	6	102	1050	10404	107100	1093.06	-43.06							
9	7	104	1150	10816	119600	1131.2	18.8							
10	8	105	1120	11025	117600	1150.27	-30.27							
11	9	105	1130	11025	118650	1150.27	-20.27							
12	10	107	1200	11449	128400	1188.41	11.59							
13	11	107	1250	11449	133750	1188.41	61.59							
14	12	110	1220	12100	134200	1245.62	-25.62							
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39							
16	Average =	=av												
17														
18														
19														
20														

Type =av in cell B16 → You will have a drop down box → select AVERAGE → press tab key → AVERAGE will appear in the cell B16 (see next picture)

Simple Linear Regression

	A	B	C	D	E
1					-852.08
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy
3	1	92	930	8464	85560
4	2	94	900	8836	84600
5	3	97	1020	9409	98940
6	4	98	990	9604	97020
7	5	100	1100	10000	110000
8	6	102	1050	10404	107100
9	7	104	1150	10816	119600
10	8	105	1120	11025	117600
11	9	105	1130	11025	118650
12	10	107	1200	11449	128400
13	11	107	1250	11449	133750
14	12	110	1220	12100	134200
15	SUM =	1221	13060	124581	1335420
16	Average =	=AVERAGE(
17		AVERAGE(number1, [number2], ...)			
18					

Simple Linear Regression

	A	B	C	D	E	F	G
1					-852.08	19.07	
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy	Pred= yhat = $a + bx =$ $-852.08 +$ $19.07x$	Residuals
3	1	92	930	8464	85560	902.36	27.64
4	2	94	900	8836	84600	940.5	-40.5
5	3	97	1020			97.71	22.29
6	4	98	990			916.78	-26.78
7	5	100	1100			954.92	45.08
8	6	102	1050			993.06	-43.06
9	7	104	1150			1031.2	18.8
10	8	105	1120	11025	117600	1150.27	-30.27
11	9	105	1130	11025	118650	1150.27	-20.27
12	10	107	1200	11449	128400	1188.41	11.59
13	11	107	1250	11449	133750	1188.41	61.59
14	12	110	1220	12100	134200	1245.62	-25.62
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39
16	Average =	=AVERAGE(B3:B14)					
17		AVERAGE(number1, [number2], ...)					
18							

Simple Linear Regression

	A	B	C	D	E	F	G
1					-852.08	19.07	
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy	Pred= yhat = $a + bx =$ -852.08+ 19.07x	Residuals
3	1	92	930	8464	85560	902.36	27.64
4	2	94	900	8836	84600	940.5	-40.5
5	3	97	1020	9409	98940	997.71	22.29
6	4	98	990	9604	97020	1016.78	-26.78
7	5	100	1100	10000	110000	1054.92	45.08
8	6	102	1050	10404	107100	1093.06	-43.06
9	7	104	1150	10816	119600	1131.2	18.8
10	8	105	1120	11025	117600	1150.27	-30.27
11	9	105	1130	11025	118650	1150.27	-20.27
12	10	107	1200	11449	128400	1188.41	11.59
13	11	107	1250	11449	133750	1188.41	61.59
14	12	110	1220	12100	134200	1245.62	-25.62
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39
16	Average =	101.75					
17							

There is a **+** sign...**hold it**
and drag till G16 and
leave cursor

Simple Linear Regression

	A	B	C	D	E	F	G
1					-852.08	19.07	
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy	Pred= yhat = $a + bx =$ -852.08+ 19.07x	Residuals
3	1	92	930	8464	85560	902.36	27.64
4	2	94	900	8836	84600	940.5	-40.5
5	3	97	1020	9409	98940	997.71	22.29
6	4	98	990	9604	97020	1016.78	-26.78
7	5	100	1100	10000	110000	1054.92	45.08
8	6	102	1050	10404	107100	1093.06	-43.06
9	7	104	1150	10816	119600	1131.2	18.8
10	8	105	1120	11025	117600	1150.27	-30.27
11	9	105	1130	11025	118650	1150.27	-20.27
12	10	107	1200	11449	128400	1188.41	11.59
13	11	107	1250	11449	133750	1188.41	61.59
14	12	110	1220	12100	134200	1245.62	-25.62
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39
16	Average =	101.75	1088.33	10381.8	111285	1088.2925	0.040833

Congratulations! Good work done so far! Now we are set to crack the mystery of complicated terms (almost scary!) with the help of above numbers. We just need to calculate three more terms that is SST, SSR \$ SSE

[This time am not showing each excel step and believing that you will be able to create in your excel sheet and find SUM in row 15, HOWEVER, showing that how you should use the \$ sign in formulas]

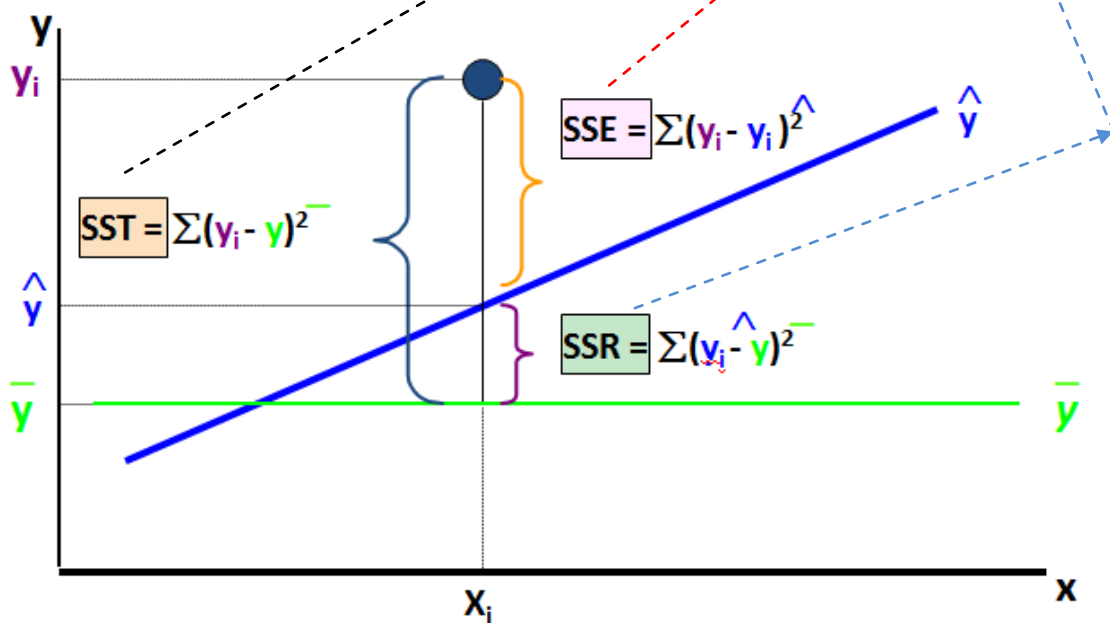
Simple Linear Regression

	A	B	C	D	E	F	G	H	I	J
1					-852.08	19.07				
2	Observation no	Advt(x)	Sales(y)	$x^2(xx)$	xy	Pred= yhat = a + bx = -852.08+ 19.07x	Residuals	SST = $(y-AVGy)^2$	SSR = $(\text{yhat}-AVG\text{yhat})^2$	SSE = $(y-\text{yhat})^2$
3	1	92	930	8464	85560	902.36	27.64	$=(C3-\$C\$16)^2$	$=(F3-\$F\$16)^2$	$=(C3-F3)^2$
4	2	94	900	8836	84600	940.5	-40.5			
5	3	97	1020	9409	98940	997.71	22.29			
6	4	98	990	9604	97020	1016.78	-26.78			
7	5	100	1100	10000	110000	1054.92	45.08			
8	6	102	1050	10404	107100	1093.06	-43.06			
9	7	104	1150	10816	119600	1131.2	18.8			
10	8	105	1120	11025	117600	1150.27	-30.27			
11	9	105	1130	11025	118650	1150.27	-20.27			
12	10	107	1200	11449	128400	1188.41	11.59			
13	11	107	1250	11449	133750	1188.41	61.59			
14	12	110	1220	12100	134200	1245.62	-25.62			
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39			
16	Average =	101.75	1088.33	10381.8	111285	1088.2925	0.040833			
17										

Simple Linear Regression

	A	B	C	D	E	F	G	H	I	J
1					-852.08	19.07				
2	Observation no	Advt(x)	Sales(y)	x^2(xx)	xy	Pred= yhat = a + bx = -852.08+ 19.07x	Residuals	SST = (y-AVGy) ²	SSR = (yhat-AVGyhat) ²	SSE = (y-yhat) ²
3	1	92	930	8464	85560	902.36	27.64	25069.44444	34570.89456	763.9696
4	2	94	900	8836	84600	940.5	-40.5	35469.44444	21842.62306	1640.25
5	3	97	1020	9409	98940	997.71	22.29	4669.444444	8205.189306	496.8441
6	4	98	990	9604	97020	1016.78	-26.78	9669.444444	5114.037656	717.1684
7	5	100	1100	10000	110000	1054.92	45.08	136.1111111	1113.723756	2032.2064
8	6	102	1050	10404	107100	1093.06	-43.06	1469.444444	22.72905625	1854.1636
9	7	104	1150	10816	119600	1131.2	18.8	3802.777778	1841.053556	353.44
10	8	105	1120	11025	117600	1150.27	-30.27	1002.777778	3841.210506	916.2729
11	9	105	1130	11025	118650	1150.27	-20.27	1736.111111	3841.210506	410.8729
12	10	107	1200	11449	128400	1188.41	11.59	12469.44444	10023.51381	134.3281
13	11	107	1250	11449	133750	1188.41	61.59	26136.11111	10023.51381	3793.3281
14	12	110	1220	12100	134200	1245.62	-25.62	17336.11111	24751.94226	656.3844
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39	138966.6667	125191.6418	13769.2285
16	Average =	101.75	1088.33	10381.8	111285	1088.2925	0.040833			
17										

Explained and Unexplained Variation



Now the moment has to come to understand the calculation Residual Standard Error (37.11)

```
> summary(mod)

Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78   -4.181  0.00188 **
slr$Advt       19.07       2.00    9.535 2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF,  p-value: 2.454e-06
```

$$Standard Error_{residuals} = \sqrt{\frac{SSE}{n - k - 1}} = \sqrt{\frac{13769.2285}{12 - 1 - 1}} = 37.11$$

Simple Linear Regression

What is the practical use of 37.11 (Standard Error Residual)?

Say, you want to predict sales for a given advertisement budget of 95.

$$Sales_{@95} = b_0 + b_1x = -852.08 + 19.07 \times 95 = 959.57$$

It is never a good idea to express prediction in POINT ESTIMATE....rather; we should present CONFIDENCE INTERVALS as shown below:

$$Upper Limit_{sales} = \hat{y} + \text{Critical value of } Z \text{ at } 95\% CL \times SE_{residuals}$$

$$Upper Limit_{sales} = 959.57 + 1.96 \times 37.11 = \mathbf{1032.306}$$

$$Lower Limit_{sales} = \hat{y} - \text{Critical value of } Z \text{ at } 95\% CL \times SE_{residuals}$$

$$Lower Limit_{sales} = 959.57 - 1.96 \times 37.11 = \mathbf{886.83}$$

So, you will say that you are 95% confident that the sales correspond to 95 budget of advertisement will lie between 886.83 and 1032.306

Now let's understand t test for slope

```
> summary(mod)

Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78  -4.181  0.00188 **
slr$Advt       19.07       2.00    9.535 2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF,  p-value: 2.454e-06
```

These 3 terms, *t*-statistics (9.535), Std Error of regression coefficient (2.00) and Probability (of committing Type I Error) is 2.45e-06 [2.45/1000000 = 0.00000245 = almost 0] are associated with a *t* test which tests following Null Hypothesis:

Ho: The slope of advertisement with sales is **not** significant

Ha: The slope of advertisement with sales is significant

Simple Linear Regression

In mathematical symbols:

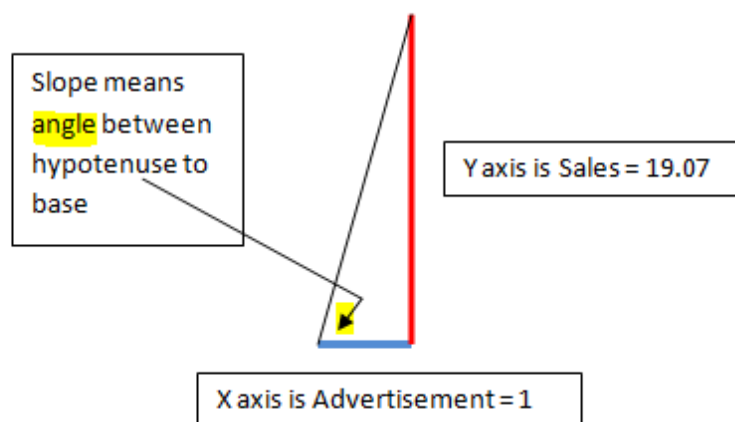
$$H_0: \beta_1 = 0 (\text{there is **no** linear relationship})$$

$$H_a: \beta_1 \neq 0 (\text{there is linear relationship})$$

Interpretation of b_1

One unit change in Advertisement will result in 19.07 times change in Sales

Regression coefficient is also called Slope (or $\tan \theta = \text{Perpendicular/Base} = \text{Sales/Advertisement}$)



NO SLOPE will look like this:



See the angle....as a matter of fact there is NO angle....or

$$\beta_1 = 0 (\text{there is **no** linear relationship})$$

So, first we will calculate the test statistics.

$$t = \frac{b_1 - \beta_1}{S_b}$$

$$S_b = \frac{\text{Standard Error of residuals}}{\sqrt{SS_{xx}}}$$

Simple Linear Regression

$$S_b = \frac{\sqrt{SS_{xx}}}{\sqrt{344.25}} = 2.00$$

S_b is called Standard Error of Regression Coefficient

$$\text{Standard Error}_{\text{residuals}} = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{13769.2285}{12-1-1}} = 37.11$$

	A	B	C	D	E
1					
2	Observat ion no	Advt(x)	Sales(y)	x^2(xx)	xy
3	1	92	930	8464	85560
4	2	94	900	8836	84600
5	3	97	1020	9409	98940
6	4	98	990	9604	97020
7	5	100	1100	10000	110000
8	6	102	1050	10404	107100
9	7	104	1150	10816	119600
10	8	105	1120	11025	117600
11	9	105	1130	11025	118650
12	10	107	1200	11449	128400
13	11	107	1250	11449	133750
14	12	110	1220	12100	134200
15	SUM =	1221	13060	124581	1335420
16					

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$SS_{xy} = 1335420 - \frac{1221 \times 13060}{12} = 6565$$

$$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 124581 - \frac{(1221)^2}{12} = 344.25$$

```
> summary(mod)
```

```
Call:
```

```
lm(formula = slr$Sales ~ slr$Advt)
```

```
Residuals:
```

```
      Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547
```

```
Coefficients:
```

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78  -4.181  0.00188 **
slr$Advt       19.07       2.00   9.535 2.45e-06 ***
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 37.11 on 10 degrees of freedom
```

```
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
```

```
F-statistic: 90.93 on 1 and 10 DF,  p-value: 2.454e-06
```

S_b is called Standard Error of Regression Coefficient

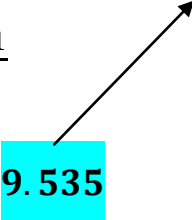
```
> summary(mod)

Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78   -4.181  0.00188 **
slr$Advt       19.07       2.00    9.535 2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF,  p-value: 2.454e-06
```

$$t = \frac{b_1 - \beta_1}{S_b}$$
$$t = \frac{19.07 - 0}{2} = 9.535$$


Now let's discuss about $\text{Pr}(>|t|)$ or Probability value (Significance value)

Here the rule goes like this:

If, p value is ≤ 0.05 (for 5% Level of Significance) \rightarrow **REJECT** H_0

If, p value is > 0.05 (for 5% Level of Significance) \rightarrow **ACCEPT** H_0

*As p value is 0.000 which is less than 0.05, **REJECT** the H_0 (and **ACCEPT** H_a) and conclude that "Slope is significant"*

Simple Linear Regression

```
> summary(mod)

Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101  -27.692   -4.383   23.589   61.547

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78  -4.181  0.00188 **
slr$Advt       19.07       2.00   9.535 2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF, p-value: 2.454e-06
```

Verify residuals with following R commands

```
> error<- residuals(lm(slr$Sales~slr$Advt))
> error
      1      2      3      4      5      6      7
27.60349 -40.53740 22.25127 -26.81917 45.03994 -43.10094 18.75817
      8      9     10     11     12
-30.31227 -20.31227 11.54684 61.54684 -25.66449
> summary(error)
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-43.100 -27.690  -4.383   0.000  23.590   61.550
```

Multiple R- squared: 0.9009

90% of the variance in Sales can be explained by Advertisement [Remaining 10% is unexplained variance....due to factors outside the model]

Simple Linear Regression

	A	B	C	D	E	F	G	H	I	J	K	L
1					-852.08	19.07						
2	Observation no	Advt(x)	Sales(y)	x^2(xx)	xy	Pred=yhat = a + bx = -852.08+ 19.07x	Residuals	SST = (y-AVGy) ²	SSR = (yhat-AVGyhat) ²	SSE = (y-yhat) ²		
3	1	92	930	8464	85560	902.36	27.64	25069.44444	34570.89456	763.9696		
4	2	94	900	8836	84600	940.5	-40.5	35469.44444	21842.62306	1640.25		
5	3	97	1020	9409	98940	997.71	22.29	4669.444444	8205.189306	496.8441		
6	4	98	990	9604	97020	1016.78	-26.78	9669.444444	5114.037656	717.1684		
7	5	100	1100	10000	110000	1054.92	45.08	136.1111111	1113.723756	2032.2064		
8	6	102	1050	10404	107100	1093.06	-43.06	1469.444444	22.72905625	1854.1636		
9	7	104	1150	10816	119600	1131.2	18.8	3802.777778	1841.053556	353.44		
10	8	105	1120	11025	117600	1150.27	-30.27	1002.777778	3841.210506	916.2729		
11	9	105	1130	11025	118650	1150.27	-20.27	1736.111111	3841.210506	410.8729		
12	10	107	1200	11449	128400	1188.41	11.59	12469.44444	10023.51381	134.3281		
13	11	107	1250	11449	133750	1188.41	61.59	26136.11111	10023.51381	3793.3281		
14	12	110	1220	12100	134200	1245.62	-25.62	17336.11111	24751.94226	656.3844		
15	SUM =	1221	13060	124581	1335420	22432.39	-9372.39	138966.6667	125191.6418	1376.8895		
16	Average =	101.75	1088.33	10381.8	111285	1088.2925	0.040833		0.900875331			
17								R Square = SSR/SST		Dr. Vinod: R Square = SSR/SST = 125191.6418/138966.66 67		
18												
19												

```
> summary(mod)
```

Call:

```
lm(formula = slr$Sales ~ slr$Advt)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547
```

Coefficients:

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -852.08      203.78  -4.181  0.00188 **
slr$Advt      19.07        2.00   9.535  2.45e-06 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom

Multiple R-squared: 0.9009, Adjusted R-squared: 0.891

F-statistic: 90.93 on 1 and 10 DF, p-value: 2.454e-06

```
> cor(slr$Advt,slr$Sales)
```

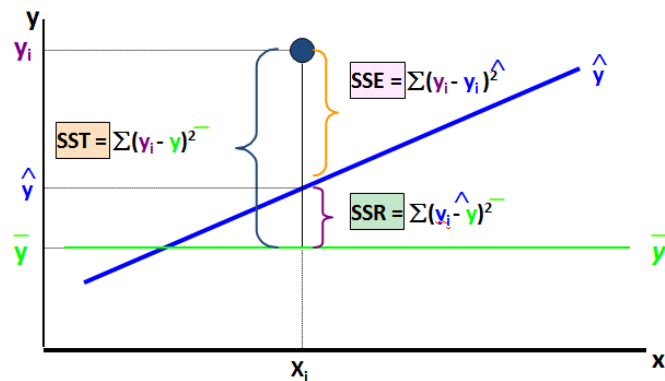
```
[1] 0.9491666
```

```
> 0.949*0.949
```

```
[1] 0.900601
```

```
> 0.949^2
```

```
[1] 0.900601
```



Simple Linear Regression

Adjusted R – squared: 0.891

$$Adj\ R\ Square = 1 - \frac{SSE/(n - k - 1)}{SST/(n - 1)}$$

$n = \text{nos of observations} = 12, \quad k = \text{nos of predictors} = 1$

H	I	J
SST = (y-AVGy)²	SSR = (yhat-AVGyhat)²	SSE = (y-yhat)²
25069.44444	34570.89456	763.9696
35469.44444	21842.62306	1640.25
4669.444444	8205.189306	496.8441
9669.444444	5114.037656	717.1684
136.1111111	1113.723756	2032.2064
1469.444444	22.72905625	1854.1636
3802.777778	1841.053556	353.44
1002.777778	3841.210506	916.2729
1736.111111	3841.210506	410.8729
12469.44444	10023.51381	134.3281
26136.11111	10023.51381	3793.3281
17336.11111	24751.94226	656.3844
138966.6667	125191.6418	13769.2285

$$Adj\ R\ Square = 1 - \frac{SSE/(n - k - 1)}{SST/(n - 1)}$$

Adj R Square

$$= 1 - \frac{13769.2285/(12 - 1 - 1)}{138966.6667/(12 - 1)}$$

> 1-(13769.2285/10)/(138966.6667/11)
[1] 0.8910087

Simple Linear Regression

Now we need to understand the last line: F-statistics 90.93 on 1 and 10 DF, p-value: 2.45e-06

```
> summary(mod)

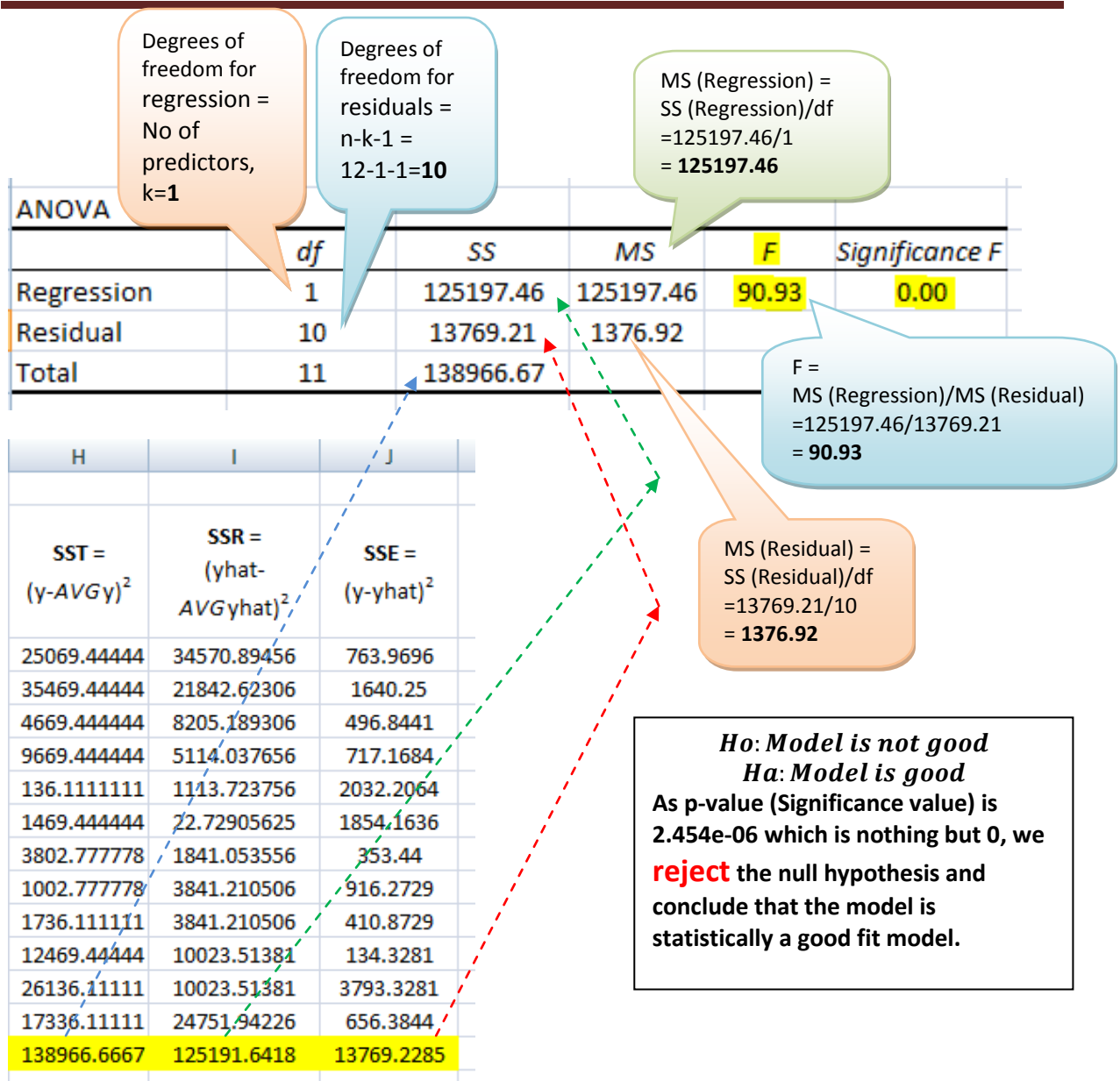
Call:
lm(formula = slr$Sales ~ slr$Advt)

Residuals:
    Min       1Q   Median       3Q      Max
-43.101 -27.692  -4.383   23.589   61.547

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -852.08     203.78   -4.181  0.00188 **
slr$Advt       19.07       2.00    9.535 2.45e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.11 on 10 degrees of freedom
Multiple R-squared:  0.9009,    Adjusted R-squared:  0.891
F-statistic: 90.93 on 1 and 10 DF, p-value: 2.454e-06
```

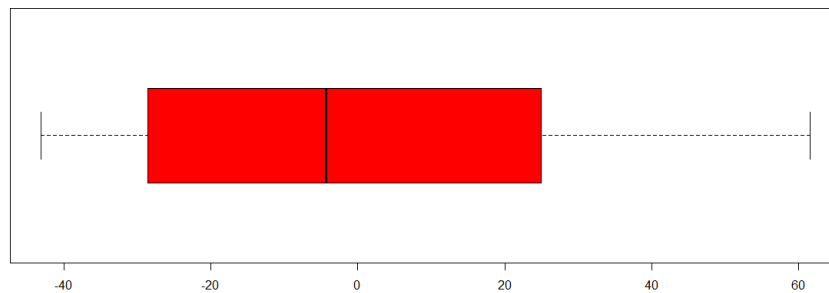
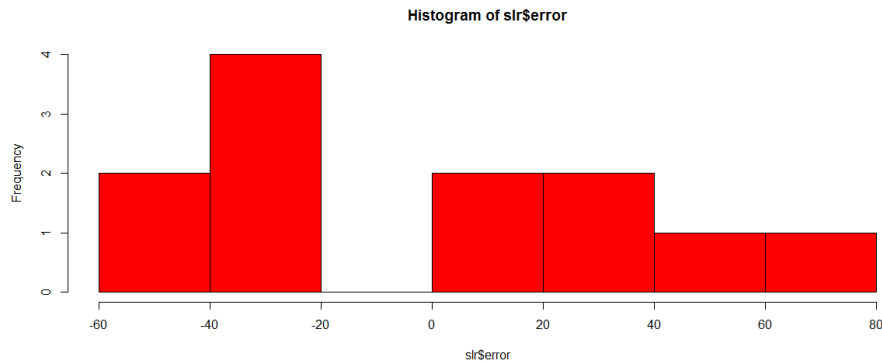
Simple Linear Regression



Now we have understood all involved calculations and their interpretations! Kudos to all!

Assumption 1: Normality of error

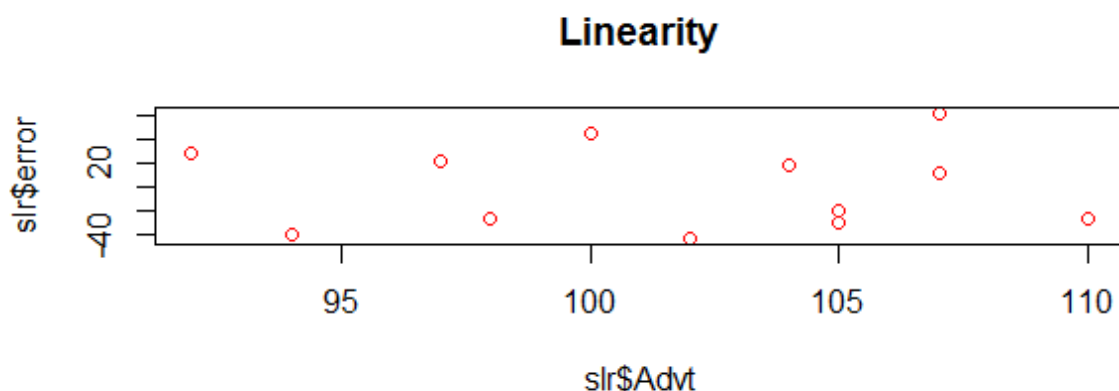
```
> hist(slr$error, col = "red")  
> boxplot(slr$error, col = "red", horizontal = T)
```



Assumption 2: Linearity

X axis: advertisement; Y axis: error

```
> plot(slr$Advt, slr$error, main = "Linearity", col = "red")
```



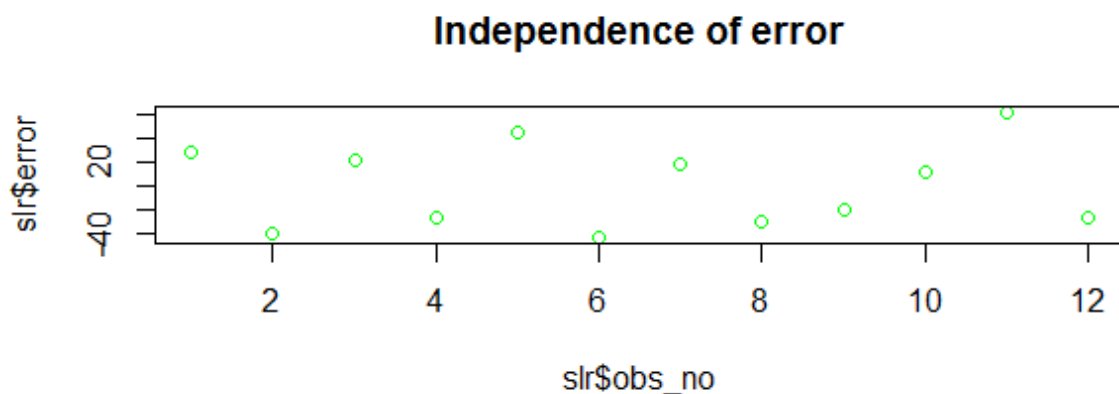
Assumption 3: Independence of error

X axis: observation number; Y axis: error

```
> obs_no<- c(1:12)
> slr$obs_no<- NULL
> slr$obs_no<- obs_no
> slr
```

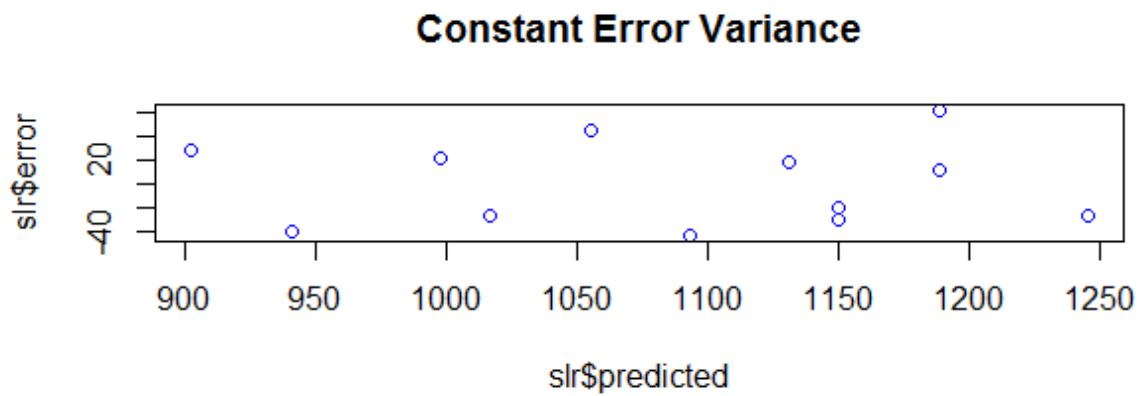
	Advt	Sales	predicted	error	obs_no
1	92	930	902.3965	27.60349	1
2	94	900	940.5374	-40.53740	2
3	97	1020	997.7487	22.25127	3
4	98	990	1016.8192	-26.81917	4
5	100	1100	1054.9601	45.03994	5
6	102	1050	1093.1009	-43.10094	6
7	104	1150	1131.2418	18.75817	7
8	105	1120	1150.3123	-30.31227	8
9	105	1130	1150.3123	-20.31227	9
10	107	1200	1188.4532	11.54684	10
11	107	1250	1188.4532	61.54684	11
12	110	1220	1245.6645	-25.66449	12

```
> plot(slr$obs_no, slr$error, main = "Independence of error", col = "green")
```



Assumption 4: Independence of error

X axis: predicted values; Y axis: error



Multiple Regression

File: **mtcars.csv**

```
> fit<- lm(mpg~ disp+hp+wt+drat, data= mtcars)
> fit

Call:
lm(formula = mpg ~ disp + hp + wt + drat, data = mtcars)

Coefficients:
(Intercept)      disp          hp          wt          drat
  29.148738    0.003815   -0.034784   -3.479668    1.768049

> ki<- lm(mtcars$mpg~ mtcars$disp+mtcars$hp+mtcars$wt+mtcars$drat)
> ki

Call:
lm(formula = mtcars$mpg ~ mtcars$disp + mtcars$hp + mtcars$wt +
    mtcars$drat)

Coefficients:
(Intercept) mtcars$disp  mtcars$hp  mtcars$wt  mtcars$drat
  29.148738    0.003815   -0.034784   -3.479668    1.768049

> summary(fit)

Call:
lm(formula = mpg ~ disp + hp + wt + drat, data = mtcars)

Residuals:
    Min       1Q   Median       3Q      Max
-3.5077 -1.9052 -0.5057  0.9821  5.6883

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  29.148738   6.293588   4.631   8.2e-05 ***
disp         0.003815   0.010805   0.353   0.72675
hp          -0.034784   0.011597  -2.999   0.00576 **
wt          -3.479668   1.078371  -3.227   0.00327 **
drat         1.768049   1.319779   1.340   0.19153
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.602 on 27 degrees of freedom
Multiple R-squared:  0.8376,    Adjusted R-squared:  0.8136
F-statistic: 34.82 on 4 and 27 DF,  p-value: 2.704e-10
```

Multicollinearity

```
> vif(fit)
      disp      hp      wt      drat
8.209402 2.894373 5.096601 2.279547

> vif(fit)>5
      disp      hp      wt      drat
      TRUE FALSE  TRUE FALSE
```

$$VIF_i = \frac{1}{1 - R_i^2}$$

Experiments

```
> summary(ti)

Call:
lm(formula = mtcars$mpg ~ mtcars$hp + mtcars$wt)

Residuals:
    Min       1Q   Median       3Q      Max
-3.941 -1.600 -0.182  1.050  5.854

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  37.22727    1.59879   23.285  < 2e-16 ***
mtcars$hp    -0.03177    0.00903   -3.519  0.00145 **
mtcars$wt    -3.87783    0.63273   -6.129  1.12e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.593 on 29 degrees of freedom
Multiple R-squared:  0.8268,    Adjusted R-squared:  0.8148
F-statistic: 69.21 on 2 and 29 DF,  p-value: 9.109e-12
```

Simple Linear Regression

```
> summary(hi)

Call:
lm(formula = mtcars$mpg ~ mtcars$hp)

Residuals:
    Min       1Q   Median       3Q      Max
-5.7121 -2.1122 -0.8854  1.5819  8.2360

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  30.09886    1.63392   18.421  < 2e-16 ***
mtcars$hp    -0.06823    0.01012   -6.742  1.79e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.863 on 30 degrees of freedom
Multiple R-squared:  0.6024,    Adjusted R-squared:  0.5892
F-statistic: 45.46 on 1 and 30 DF, p-value: 1.788e-07
```

```
> summary(fi)

Call:
lm(formula = mtcars$mpg ~ mtcars$wt)

Residuals:
    Min       1Q   Median       3Q      Max
-4.5432 -2.3647 -0.1252  1.4096  6.8727

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  37.2851    1.8776   19.858  < 2e-16 ***
mtcars$wt    -5.3445    0.5591   -9.559  1.29e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.046 on 30 degrees of freedom
Multiple R-squared:  0.7528,    Adjusted R-squared:  0.7446
F-statistic: 91.38 on 1 and 30 DF, p-value: 1.294e-10
```

Model	Predictors	R Square
1. fit	disp, hp, wt, drat	0.8376
2. ti	hp, wt	0.8268
3. hi	hp	0.6024
4. fi	wt	0.7528

Assumptions of model: fit

