035MAT011 - F - 24 - 6765



FIFTH SEMESTER B.SC. (NEP) DEGREE EXAMINATION, FEBRUARY 2024 MATHEMATICS

Paper – I: Real Analysis – II and Complex Analysis (DSC – 1)

Time: 2 Hours]

[Max. Marks: 60

Instruction : Answer all questions.

I. Answer any five of the following:

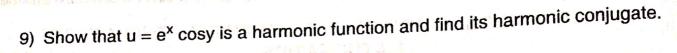
 $(5 \times 2 = 10)$

- 1) Define upper and lower Darboux Sums.
- 2) If $f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ -1 & \text{when } x \text{ is irrational,} \end{cases}$ then prove that f is not R-integrable.
- 3) State Abel's test for the convergence of an improper integral of product of two functions.
- 4) Define harmonic function and prove that $u = \sin x \cosh y$ is harmonic.
- 5) Find the fixed points and normal form of $w = \frac{z}{2-z}$ by bilinear transformation.
- 6) Evaluate $\int_C \frac{e^z}{z-2}$ where C is the circle |z| = 3.
- II. Answer any four of the following:

 $(4 \times 5 = 20)$

- 7) If f(x) and g(x) are R-integrable in [a, b], then prove that f(x) + g(x) is also R-integrable in [a, b].
- 8) State and prove Dirichlet's test for the convergence of the product of two functions.

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10) If f(z) is a regular function z = x + iy, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \left| f(z) \right|^2 = 4 \left| f'(z) \right|^2.$$

11) Show that the transformation $w = \frac{1}{z}$ transforms a line not passing through the origin in z-plane to a circle passing through the origin in w-plane.

III. Answer any three of the following:

 $(3 \times 10 = 30)$

- 12) a) Prove that every continuous function is R-integrable in [a, b].
 - b) If $f(x) = x^2$ defined in [0, 1], show that f(x) is R-integrable in [0, 1] and hence find $\int_0^1 f(x) dx$.
- 13) a) State and prove fundamental theorem of integral calculus.
 - b) Examine the convergence of :

i)
$$\int_{1}^{\infty} \frac{dx}{x^{\frac{1}{3}} (1+x)^{\frac{1}{2}}}$$

E) Evaluate
$$\int_{0}^{\infty} \frac{xb}{(x+1)} \int_{1}^{\infty} \left(ii \frac{xb}{x^2 - 2} \right)^{\infty} dx$$
 (ii)

- 14) a) State and prove necessary condition for f(z) to be analytic function.
 - b) Show that $u = x^3 3xy^2$ is harmonic, find its harmonic conjugate and corresponding analytic function.
- 15) a) Discuss the transformation for $w = z^2$ and show that it transforms lines parallel to y-axis in z-plane into parabolas in w-plane.
 - b) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$ and $w_3 = -1$.