(Pages : 2) **031 MAT 011—APRIL—22—171**

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2022

(NEP—DSCC)

Mathematics

ALGEBRA—I AND CALCULUS—I

Time : Two Hours — Maximum : 60 Marks

Answer **all** *questions.*

I. Answer any *five* of the following. Each question carries 2 marks :

1 State Cayley-Hamilton theorem.

2 Find the eigen values of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

3 Find the pedal equation of the curve $r = ae^{\theta \cot \alpha}$.

4 Discuss the continuity of

$$f(x) = \begin{cases} 5x - 4, & \text{when } x \le 1 \\ 4x^3 - 3x, & \text{when } 1 < x < 2, \text{ at a point } x = 1. \end{cases}$$

5 Evaluate : $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

6 When the curve $f(x,y) = 0$ is symmetrical about the x-axis. Give an example.

(5 × 2 = 10 marks)

II. Answer any *four* of the following. Each question carries 5 marks :

7 Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 3 & 4 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing it to normal form.

8 Solve the equations by using Guass elmination method :

$$\begin{aligned} x + 2y - 5z &= -13 \\ 3x - y + 2z &= 1 \\ 2x - 2y + 3z &= 2. \end{aligned}$$

9 With the usual notations prove that $\tan \phi = r.\frac{d\theta}{dr}$, for the curve $r = f(\theta)$.

Turn over

10 If a function f is continuous on $[a, b]$, prove that f is bounded on $[a, b]$.

11 Find the nth derivative of $e^{ax} \cos (bx + c)$.

(4 × 5 = 20 marks)

III. Answer any *three* of the following. Each question carries 10 marks :

12 (a) Prove that multiplication to each element of any row of a matrix by a non-zero constant does not alter the rank.

(b) Verify the Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix}$.

13 (a) Derive the formula for radius of curvature of the curve $y = f(x)$ at any point.

(b) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are connected by the relation $ab = c^2$.

14 (a) State and prove Cauchy's mean value theorem.

(b) Evaluate : $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$.

15 (a) State and prove Leibnitz's theorem for the nth derivative of produt of two functions.

(b) If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + \left(n^2+1\right)y_n = 0$

(3 × 10 = 30 marks)