THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, FEBRUARY 2023

(NEP-DSCC)

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS AND REAL ANALYSIS—I

Time : Two Hours

Maximum : 60 Marks

Answer **all** *questions.*

I. Answer any *five* of the following :

1 Verify the exactness of the equation $\left(x^3+3xy^2\right)dx+\left(3x^2y+y^3\right)dy=0$ and hence solve it.

2 Solve : $p^2-7p+12=0$ where $p=\dfrac{dy}{dx}$.

3 Find the particular integral of $\dfrac{d^2y}{dx^2}+9y=\cos 4x$.

4 Define bounded sequence and give an example.

5 Test the convergence of the series :

$$1+\frac{1}{4^{2/3}}+\frac{1}{9^{2/3}}+\frac{1}{16^{2/3}}+\ldots.$$

6 Test the convergence of the series :

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots.$$

(5 × 2 = 10 marks)

II. Answer any *four* of the following :

7 Solve the equation $y=2px+x^2p^4$.

8 Solve : $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+3y=e^{-3x}$.

9 Solve $\left(x^2D^2-3xD+4\right)y=2x^2$ where $D=\dfrac{d}{dx}$.

Turn over

10 Prove that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is (i) monotonically increasing ; (ii) bounded ; and (iii) tends to limit $\frac{2}{3}$.

11 Discuss the convergence of the series $\sum \frac{3^n n!}{n^n}$.

(4 × 5 = 20 marks)

III. Answer any *three* of the following :

12 (a) Prove that the necessary and sufficient condition for the equation $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(b) Find the orthogonal trajectories of the family of parabolas $y = ax^2$ where a is a parameter.

13 (a) With usual notations, prove that $\frac{1}{f(D^2)} \sin ax = \frac{1}{f(-a^2)} \sin ax$ provided $f(-a^2) \neq 0$.

(b) Solve the simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = e^t ; \frac{dy}{dt} - x + 6y = e^{2t}$.

14 (a) If the sequence $\{a_n\}$ converges to 'l' then prove that $\lim_{n \to \infty} \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} = l$.

(b) Prove that the sequence $\{x_n\}$ defined by $x_1 = 1, x_n = \sqrt{2 + x_{n-1}}\ \forall\ n \geq 2$ is convergent and converges to 2.

15 (a) State and prove Cauchy's root test for a series of positive terms.

(b) Test the convergence of the series $\sum ne^{-n^2}$.

(3 × 10 = 30 marks)