031MAT011 – F – 24 – 6334

FIRST SEMESTER B.SC. (NEP) DEGREE EXAMINATION, FEBRUARY 2024

MATHEMATICS (DSC – 1)

Algebra – I and Calculus – I

Time : 2 Hours]

[Max. Marks : 60

Instruction : *Answer **all** questions.*

I. Answer **any five** of the following. **(5×2=10)**

1) Define rank of a matrix.

2) Find eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

3) Find ϕ for the curve $r^2 = a^2\cos 2\theta$ at $\theta = \pi/6$.

4) Discuss the continuity of the function

$$f(x) = \begin{cases} \dfrac{2}{5-x} & \text{for} \quad x < 3 \\ 5-x & \text{for} \quad x \geq 3 \end{cases} \text{ at } x = 3.$$

5) Evaluate $\lim\limits_{x \to 0} \dfrac{e^{x} - e^{-x}}{\sin x}$.

6) When the curve $f(x, y) = 0$ is symmetrical about the

i) X-axis

ii) Y-axis.

II. Answer **any four** of the following. **(4×5=20)**

7) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{bmatrix}$ to row reduced echelon form and hence find the rank of the matrix A.

[P.T.O.

8) Show that the equation of circle of curvature for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$ is $\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}$.

9) Find the evolute for the parabola $y^2 = 4ax$.

10) State and prove Lagrange's Mean Value theorem.

11) Prove that the n^{th} derivative of $e^{ax} \sin(bx + c)$ is $(a^2 + b^2)^{n/2} e^{ax} \sin\left(bx + c + n\tan^{-1} b/a\right)$.

III. Answer **any three** of the following. **(3×10=30)**

12) a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix}$ using Cayley-Hamilton theorem.

b) Solve the system of equations using Gauss elimination method.

$$x - y - z = 1$$
$$2x + y + z = 2$$
$$x - 2y + z = 4$$

13) a) Derive the formula for radius of curvature for the curve $y = f(x)$ at any point (x, y) on it.

b) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are connected by the relation $a + b = c$.

14) a) State and prove intermediate value theorem.

b) Evaluate $\lim\limits_{x \to 0} \frac{\log \sin x}{\log \sin 2x}$.

15) a) State and prove Leibnitz's theorem for the n^{th} derivative of product of two functions.

b) If $y = \sin(m\sin^{-1}x)$, then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.