034MAT011 – S – 23 – 2462

FOURTH SEMESTER B.SC. (NEP) DEGREE EXAMINATION, AUG./SEPT. 2023

MATHEMATICS (DSC – 1)

Partial Differential Equations and Integral Transforms

Time : 2 Hours] [Max. Marks : 60

***Instruction :** Answer **all** questions.*

I. Answer **any five** of the following : **(5×2=10)**

1) Form a partial differential equation by eliminating arbitrary constants a and b from $z = ax + by$.

2) Find the general integral for the partial differential equation $z = px + qy - 2\sqrt{pq}$.

3) Classify the partial differential equation $2\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 2$.

4) Find the Laplace transform of sin 2t. cos 3t.

5) Find the Fourier coefficient a_0 for the function $f(x) = x + x^2$ in $[-\pi, \pi]$.

6) Write the half-range Fourier cosine series.

II. Answer **any four** of the following : **(4×5=20)**

7) Solve the Lagrange's partial differential equation $y^2p - xyq = x(z - 2y)$.

8) Solve $(D^2 - 5DD' + 2D'^2)z = 5 \sin(2x + 3y)$.

9) If $L[f(t)] = F(S)$ then prove that $L[f''(t)] = S^2 L[f(t)] - S f(0) - f'(0)$.

10) Solve using Laplace transform $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ given $y(0) = 0 = y'(0)$.

11) Find the Fourier series of $f(x) = |x|$ in $[-\pi, \pi]$, hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

[P.T.O.

III. Answer **any three** of the following : (3×10=30)

12) a) Solve by Charpit's method $px + qy - pq = 0$.

b) Find the complete integral of $p^2 + q^2 = x + y$.

13) a) Solve $4(r - s) + t = 16 \log(x+2y)$.

b) Solve $(D^3 - 4D^2D' + 4DD'^2)z = \cos(2x + 3y)$.

14) a) Prove that the Laplace transform of a periodic function f(t) with period T is given by $L[f(t)] = \frac{1}{1-e^{-ST}}\int_0^T e^{-St} f(t)\, dt$.

b) Evaluate $L^{-1}\left[\log\left(1+\frac{a^2}{S^2}\right)\right]$.

15) a) Find the finite Fourier sine transform for the function $f(x) = ax - x^2$ in $(0, a)$.

b) Find the Half-range cosine series of $f(x) = \sin x$ in $(0, \pi)$.