032MAT011 – S – 23 – 2277

SECOND SEMESTER B.A./B.SC. (NEP) DEGREE
EXAMINATION, AUGUST/SEPTEMBER 2023

MATHEMATICS (DSC – 1)

Algebra – II and Calculus – II

Time : 2 Hours] [Max. Marks : 60

Instruction : *Answer **all** questions.*

I. Answer **any five** of the following : **(5×2=10)**

1) State Archimedian properties of real numbers.

2) Prove that the identity element in a group is unique.

3) If $u = e^x \sin y$ and $v = x + y$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

4) If $f = xy^2 + 2y$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

5) Evaluate $\int_C y\,dx - x\,dy$ taken along the curve $y = x^2$ from (0, 0) to (1, 1).

6) Evaluate $\int_0^1 \int_0^1 (x^2 + y^2)\,dx\,dy$.

II. Answer **any four** of the following : **(4×5=20)**

7) Prove that every subset of countable set is countable.

8) Define subgroup of a group. Prove that intersection of two subgroups of G is a subgroup of G.

9) If u and v are functions of r and s, and r and s are functions of x and y, then prove that $\frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$.

10) If $u = \log\left(\frac{x^4 + y^4}{x - y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

11) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using double integration.

[P.T.O.

III. Answer **any three** of the following : **(3×10=30)**

12) a) Define closed set. Prove that union of two closed sets is a closed set.

b) Prove that unit interval [0, 1] is not countable.

13) a) Prove that every subgroup H of a cyclic group G is also cyclic.

b) If H be a subgroup of G, then all right cosets of H in G have the same number of elements.

14) a) If $u = f(x, y)$ is homogeneous function of degree n, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

b) Expand sinx siny in powers of x and y as far as the terms of third degree using Maclaurin's series.

15) a) If $f(x, \alpha)$ and $\frac{\partial}{\partial \alpha}(f(x, \alpha))$ be continuous function of x and α and

$F(\alpha) = \int_a^b f(x, \alpha)$, then prove that $\frac{d}{d\alpha}(F(\alpha)) = \int_a^b \frac{\partial}{\partial \alpha} f(x, \alpha)\,dx$.

b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration.

B