FIFTH SEMESTER B.SC. (NEP) DEGREE EXAMINATION, FEBRUARY 2024

MATHEMATICS

Paper – I : Real Analysis – II and Complex Analysis (DSC – 1)

Time : 2 Hours] [Max. Marks : 60

Instruction : *Answer **all** questions.*

I. Answer **any five** of the following : **(5×2=10)**

1) Define upper and lower Darboux Sums.

2) If $f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ -1 & \text{when } x \text{ is irrational,} \end{cases}$ then prove that f is not R-integrable.

3) State Abel's test for the convergence of an improper integral of product of two functions.

4) Define harmonic function and prove that u = sinx coshy is harmonic.

5) Find the fixed points and normal form of $w = \dfrac{z}{2-z}$ by bilinear transformation.

6) Evaluate $\int_C \dfrac{e^z}{z-2}$ where C is the circle |z| = 3.

II. Answer **any four** of the following : **(4×5=20)**

7) If f(x) and g(x) are R-integrable in [a, b], then prove that f(x) + g(x) is also R-integrable in [a, b].

8) State and prove Dirichlet's test for the convergence of the product of two functions.

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9) Show that $u = e^x \cos y$ is a harmonic function and find its harmonic conjugate.

10) If f(z) is a regular function $z = x + iy$, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

11) Show that the transformation $w = \frac{1}{z}$ transforms a line not passing through the origin in z-plane to a circle passing through the origin in w-plane.

III. Answer **any three** of the following : **(3×10=30)**

12) a) Prove that every continuous function is R-integrable in [a, b].

b) If $f(x) = x^2$ defined in [0, 1], show that f(x) is R-integrable in [0, 1] and hence find $\int_0^1 f(x)\, dx$.

13) a) State and prove fundamental theorem of integral calculus.

b) Examine the convergence of :

i) $\int_1^\infty \frac{dx}{x^{1/3}(1+x)^{1/2}}$

ii) $\int_1^\infty \frac{dx}{(1+x)\sqrt{x}}$

14) a) State and prove necessary condition for f(z) to be analytic function.

b) Show that $u = x^3 - 3xy^2$ is harmonic, find its harmonic conjugate and corresponding analytic function.

15) a) Discuss the transformation for $w = z^2$ and show that it transforms lines parallel to y-axis in z-plane into parabolas in w-plane.

b) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$ and $w_3 = -1$.