035MAT013 – F – 24 – 6766

FIFTH SEMESTER B.SC. (NEP) DEGREE EXAMINATION, FEBRUARY 2024

MATHEMATICS (DSC – 2)

Vector Calculus and Analytical Geometry

Time : 2 Hours] [Max. Marks : 60

***Instruction** : Answer **all** questions.*

I. Answer **any five** of the following. **(5×2=10)**

1) Find the unit normal vector to the surface $x^2 - y^2 + z = 3$ at (1, 0, 2).

2) Show that the vector
$\vec{f} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.

3) If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}.d\vec{r}$, where C is the path along the parabola $y = 2x^2$ from (0, 0) to (1,1).

4) Find the equations of the straight line through the point (2, 1, –2) and equally inclined the axes.

5) Define ruled surfaces.

6) Show that the plane $x + 2y + 3z = 2$ touches the conicoid $x^2 - 2y^2 + 3z^2 = 2$.

II. Answer **any four** of the following. **(4×5=20)**

7) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at (1, –2, –1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.

8) Evaluate $\iint_S \left[(x+z)\hat{i} + (y+z)\hat{j} + (x+y)\hat{k}\right].n \, ds$, where s is the surface of the sphere $x^2 + y^2 + z^2 = 4$ by using Gauss' divergence theorem.

9) Show that the equation $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of planes and find angle between them.

10) Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z - 7 = 0$ which intersect in the line $6x - 3y - 2z = 0 = 3z + 2$.

11) Find the point of intersection of the line $\frac{x+5}{-3} = \frac{y+4}{1} = \frac{z-11}{7}$ with the conicoid $12x^2 - 17y^2 + 7z^2 = 7$.

[P.T.O.

III. Answer **any three** of the following. **(3×10=30)**

12) a) Prove that the necessary and sufficient condition for the vector function $\vec{f}(t)$ to have a constant magnitude is $\vec{f}\cdot\frac{d\vec{f}}{dt}=0$.

b) If $\vec{f}=(y^2+z^2-x^2)\hat{i}+(z^2+x^2-y^2)\hat{j}+(x^2+y^2-z^2)\hat{k}$, find div $\vec{f}$ and curl $\vec{f}$.

13) a) State and prove Stokes theorem.

b) Verify Green's theorem in the plane for $\int_C (3x^2-8y^2)\,dx+(4y-6xy)dy$ where C is the closed curve bounded by $y=x^2$ and $x=y^2$.

14) a) Prove that the equation $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ represents a sphere. Find its centre and radius.

b) Find the equation of the plane which bisects the acute angle between the planes $3x+6y-2z+5=0$ and $4x-12y+3z-3=0$.

15) a) Find the equation of the surface generated by the lines which passes through a fixed point (α, β, γ) and intersect the curve $ax^2+by^2=1$, $z=0$.

b) Find the equations to the tangent planes to conicoid $7x^2-3y^2-z^2+21=0$ which passes through the line $7x-y+9=0$, $z=3$.