(Pages : 2) **032 MAT 011—Sep. 2022—5115**

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, SEPTEMBER 2022

(NEP—DSCC)

Mathematics

ALGEBRA—II AND CALCULUS—II

Time : Two Hours Maximum : 60 Marks

Answer **all** *questions.*

I. Answer any *five* of the following :

1 State Bolzano Weierstrass theorem.

2 Prove that in a group G inverse of an element is unique.

3 Find the partial derivatives of $z = x^2 - xy + y^2$.

4 If $x = r\cos\theta$ $y = r\sin\theta$ then show that $\dfrac{\partial(x,y)}{\partial(r\theta)} = r$.

5 Evaluate $\int_C x^2 dx + xy dy$ taken along the quarter circle $x = \cos t$ $y = \sin t$ joining the points (1, 0) to (0, 1).

6 Evaluate $\int_0^1 \int_0^2 (x+y)\, dx\, dy$.

(5 × 2 = 10 marks)

II. Answer any *four* of the following :

7 Prove that the set of rational numbers is denumerable.

8 Prove that a non-empty subset H of a group (G, •) is a subgroup of G if and only if it satisfies :

i) $\forall a, b \in H \Rightarrow a \, . \, b \in H$

ii) $\forall a \in H \Rightarrow a^{-1} \in H$.

9 If $Z = \dfrac{x^2 + y^2}{x + y}$ prove that $\left[\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right]^2 = 4\left[1 - \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right]$.

Turn over

10 Expand $e^x \cos y$ in powers of x, y as far as the terms of third degree using Maclaurin's theorem.

11 Find the area of a circle $x^2 + y^2 = a^2$ by using double integrations.

(4 × 5 = 20 marks)

III. Answer any *three* of the following :

12 (a) State and prove Archimedean property of real numbers.

(b) Prove that set $N \times N$ is countable.

13 (a) State and prove Lagrange's theorem.

(b) Prove that if H be a subgroup of group G then two right cosets of H in G are either identical or disjoint.

14 (a) State and prove Euler's theorem on homogenous functions of two variables.

(b) Find the maximum and minimum values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

15 (a) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $x + y + z = 1$.

(b) Shoe that if $-1 < a < 1$ and $\frac{-\pi}{2} < \sin^{-1} a < \frac{\pi}{2}$

$$\int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x} dx = \pi \sin^{-1} a.$$

(3 × 10 = 30 marks)