SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, SEPTEMBER 2022

(NEP—DSCC)

Mathematics

ALGEBRA—II AND CALCULUS—II

Answer all questions.

I. Answer any five of the following:

Time: Two Hours

- 1 State Bolzano Weierstrass theorem.
- 2 Prove that in a group G inverse of an element is unique.
- 3 Find the partial derivatives of $z = x^2 xy + y^2$.
- 4 If $x = r \cos \theta$ $y = r \sin \theta$ then show that $\frac{\partial (x,y)}{\partial (r\theta)} = r$.
- 5 Evaluate $\int_{C} x^2 dx + xy dy$ taken along the quarter circle $x = \cos t$ $y = \sin t$ joining the points (1, 0) to (0, 1).
- 6 Evaluate $\int_{0}^{1} \int_{0}^{2} (x+y) dx dy.$

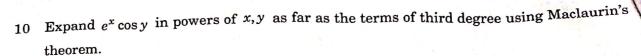
 $(5 \times 2 = 10 \text{ marks})$

Maximum: 60 Marks

- II. Answer any four of the following:
 - 7 Prove that the set of rational numbers is denumerable.
 - 8 Prove that a non-empty subset H of a group (G, •) is a subgroup of G if and only if it satisfies:
 - i) $\forall a, b \in H \Rightarrow a \cdot b \in H$
 - ii) $\forall a \in \mathbf{H} \Rightarrow a^{-1} \in \mathbf{H}$

9 If
$$Z = \frac{x^2 + y^2}{x + y}$$
 prove that $\left[\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]^2 = 4\left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$.

Turn over



Find the area of a circle $x^2 + y^2 = a^2$ by using double integrations.

 $(4 \times 5 = 20 \text{ marks})$

III. Answer any three of the following:

12 (a) State and prove Archimedean property of real numbers.

(b) Prove that set $N \times N$ is countable.

13 (a) State and prove Lagrange's theorem.

(b) Prove that if H be a subgroup of group G then two right cosets of H in G are either identical or disjoint.

14 (a) State and prove Euler's theorem on homogenous functions of two variables.

(b) Find the maximum and minimum values of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

15 (a) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane x + y + z = 1.

(b) Shoe that if -1 < a < 1 and $\frac{-\pi}{2} < \sin^{-1} a < \frac{\pi}{2}$

$$\int_0^\pi \frac{\log(1 + a\cos x)}{\cos x} dx = \pi \sin^{-1} a.$$

 $(3 \times 10 = 30 \text{ marks})$