CMSC320 Introduction to Data Science

September 30, 2016

Finding Maxima/Minima using Derivatives

The value(s) at which a function attains its maximum value is called a *maximum* of the function. Similarly, the value(s) at which a function attains its *minimum* value is called a minimum of the function.

In a smoothly changing function maxima or minima are found where the function flattens (slope becomes 0). The first derivative of the function tells us where the slope is 0. This is the *first derivate test*.

The derivate of the slope (the second derivative of the original function) can be useful to know if the value we found from first derivate test is a maxima or minima. When a function's slope is zero at x, and the second derivative at x is:

- less than 0, it is a local maximum
- greater than 0, it is a local minimum
- equal to 0, then the test fails (there may be other ways of finding out though)

This is called the *second derivate test*.

Steps to find Maxima/Minima of function $f(x)$

1. Find the value(s) at which $f(x) = 0$ (First derivative test). i.e., Find x's such that $f^{'}(x) = 0$.

2. Find the value of the second derivative for each of the x's found in step 1 (Second derivative test).

3. If the value of the second derivative at x is:

   - less than 0, it is a local maximum
   - greater than 0, it is a local minimum
   - equal to 0, then the test fails (no minima or maxima)

Notes on Finding Derivatives

Sum Rule

The derivative of the sum of two functions is the sum of the derivatives of the two functions:

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$$

Similarly, the derivative of the difference of two functions is the difference of the derivatives of the two functions.

Power Rule

If we have a function f(x) of the form $f(x) = x^n$ for any integer n,

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}(x^n) = nx^{n-1}$$

Chain Rule

If we have two functions of the form $f(x)$ and $g(x)$, the chain rule can be stated as follows:

$$\frac{d}{dx}(f(g(x)) = f^{'}(g(x))g^{'}(x)$$

Eg. Differentiate $y = (3x+1)^2$ with respect to x.
Solution. Applying the above equation, we have the following:

$$\frac{d}{dx}((3x+1)^2) = 2(3x+1)^{2-1}\frac{d}{dx}((3x+1)) = 2(3x+1)(3) = 6(3x+1)$$

Product Rule

If we have two functions f(x) and g(x),

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x)) = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule

If we have two functions f(x) and g(x) ($g(x) \neq 0$),

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \left(\frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{g(x)^2}\right)$$

A useful useful calculus cheat sheet and a discussion on finding maxima/minima can be found in the embedded links.