**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** Normal Distribution with = 45 and 8.

Z-Scores at X=50;

Z = (X - µ) / σ

Z=(50-45)/8

Z = 0.625

Probability p(X>50) =1-stats.norm.cdf(abs(z\_score))

1-stats.norm.cdf(abs(0.625))

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| =0.2676 |
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1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans: More employees at the processing center are older than 44 than between 38 and 44.**

p(X>44); Employees older than 44 yrs of age

1-stats.norm.cdf(44,loc=38,scale=6)

= 0.1586

Employees between 38 to 44 yrs of age

# p(38<X<44);

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

= 0.3413= 34.13%

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

**A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Employees under 30 yrs of age

# P(X<30);

= 0.0912

No. of employees attending training program from 400 nos. is N\*P(X<30)

400\*stats.norm.cdf(30,38,6)

=36.484 Therefore statement B also True.

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1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:** As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore

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| 2X1~ N(2 u,4 σ^2) and | |
| X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) | |
| 2X1-(X1+X2) = N( 4µ,6 σ^2) | |
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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:** stats.norm.interval(0.99,100,20)

a = 48.48 and b = 151.52

Option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:** Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

Mean Profit is Rs 540 Million

Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD = (3^2)+(4^2)

print('Standard Deviation is Rs', SD\*45, 'Million')

Standard Deviation is Rs 315 Million

**A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

print('Range is Rs',(stats.norm.interval(0.95,540,315)),'in Millions')

Range is Rs (-77.38865513011706, 1157.388655130117) in Millions

**B. Specify the 5th percentile of profit (in Rupees) for the company**

X=μ + Zσ; wherein from z table, 5 percentile = -1.64

X= 540+(-1.64)\*(315)

print('5th percentile of profit (in Million Rupees) is',np.round(X,2))

5th percentile of profit (in Million Rupees) is 23.4

**C.Which of the two divisions has a larger probability of making a loss in a given year?**

Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

Ans: 0.0477

Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

Ans: 0.040