1 Useful Formulas

1.1 General

- Arithmetic Series: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- Geometric Series: $\begin{cases} \sum_{i=0}^{n} a^i = \frac{1-a^{n+1}}{1-a}, & \forall a \in \mathbb{R} \\ \sum_{i=0}^{\infty} a^i = \frac{1}{1-a}, & \forall |a| < 1 \end{cases}$
- Binomial Thm: $\begin{cases} (x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i} \\ \forall\, x, y \in \mathbb{R},\ n \in \mathbb{N} \end{cases}$
- Exponential Fun: $\sum_{i=0}^{\infty} \frac{a^i}{i!} = e^a, \quad \forall a \in \mathbb{R}$
- Gamma Fun: $\begin{cases} \Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx,\ a > 0 \\ \Gamma(n) = (n-1)!,\ n \in \mathbb{N} \end{cases}$

1.2 Probability

- Law of Total Probability: $\forall$ partition $\{A_i\}_{i=1}^n$

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

- Bayes Rule: $\forall$ partition $\{A_i\}_{i=1}^n$

$$P(A_k|B) = \frac{P(B|A_k)P(A_k)}{\sum_{i=1}^n P(B|A_i)P(A_i)}$$

- Multiplication Rule: $P\left(\bigcap_{i=1}^n A_i\right) =$

$$P(A_1)P(A_2|A_1) \times \cdots \times P(A_n|A_1, \ldots, A_{n-1})$$

- Inclusion-Exclusion Principle:

$$\begin{aligned} P\left(\bigcup_{i=1}^n A_i\right) &= \sum_{i=1}^{n} P(A_i) - \sum_{1 \le i < j \le n} P(A_i \cap A_j) + \\ &+ \sum_{1 \le i < j < k \le n} P(A_i \cap A_j \cap A_j) \\ &\vdots \\ &+ (-1)^{n-1} P\left(A_1 \cap \cdots \cap A_n\right) \end{aligned}$$

1.3 Calculus

- Fundamental Theorem of Calculus:

$$F'(x) = f(x) \Rightarrow \int_a^b f(x)dx = F(b) - F(a)$$

$$F(x) = \int_a^x f(u)du \Rightarrow F'(x) = f(x)$$

- Integration by Parts:

$$\int_a^b f(x)g'(x)dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$$

- Change of Variables:

$$\int_a^b f(x)dx \overset{(x=h(u))}{=} \int_{h^{-1}(a)}^{h^{-1}(b)} f(h(u))h'(u)du$$

1.4 Important Distributions

Name	PMF/PDF	CDF	Mean	Variance
Discrete Distributions				
Binomial(n, p)	$\binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, \ldots, n$		np	$np(1-p)$
Geometric(p)	$p(1-p)^{x-1}, \quad x = 1, \ldots, \infty$	$1 - (1-p)^x$	$1/p$	$(1-p)/p^2$
NegBinom(r, p)	$\binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad x = r, \ldots, \infty$		r/p	$r(1-p)/p^2$
HyperGeom(N, M, n)	$\binom{M}{x}\binom{N-M}{n-x}/\binom{N}{n}$, $\max(0, n+M-N) \le x \le \min(n, M)$		nM/N	$n\frac{M}{N}\frac{(N-M)}{N}\frac{N-n}{N-1}$
Poisson(λ)	$e^{-\lambda}\lambda^x/x!, \quad x = 0, \ldots, \infty$		λ	λ
Continuous Distributions				
Uniform(l, u)	$1/(u-l), \quad x \in [l, u]$	$(x-l)/(u-l)$	$(u+l)/2$	$(u-l)^2/12$
Exponential(λ)	$\lambda e^{-\lambda x}, \quad x \ge 0$	$1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$
Normal(μ, σ^2)	$\frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \quad x \in \mathbb{R}$		μ	σ^2
Gamma(a, λ)	$\lambda^a x^{a-1} e^{-\lambda x}/\Gamma(a), \quad x \ge 0$		a/λ	a/λ^2