Peer Review 1 Idea 3 Recog

September 25, 2022 2:22 PM

$L = \{\langle M_1, M_2, n\rangle \mid M_1, M_2$ are TM's, $n \in \mathbb{N}$, and $|L(M_1) \cap L(M_2)| \geq n\}$

We claim L is neither recognizable nor co-recognizable.

We prove that $HALT \leq_m L$, hence L is not co-recognizable

Consider the following TM F and the reduction it computes:

F = "On input $\langle M, w\rangle$

1. Construct TM M_1 as follows:

   M_1 = "On input x

   1.
   2. run M on w
   3. accept "

2. Construct TM M_2 as follows:

   M_2 = "On input x

   1. if $x = w$ then accept else reject
   2.
   3. "

3. return $\langle M_1, M_2, 1\rangle$"

We argue that $\langle M, w\rangle \in HALT \iff \langle M_1, M_2\rangle \in L$

($\Rightarrow$) $\langle M, w\rangle \in HALT$

$\Rightarrow$ M halts	[def of HALT]
$\Rightarrow$ M_1 accepts all strings	[desc. of M_1]
$\Rightarrow$ M_2 accepts w	[desc. of M_2]
$\Rightarrow L(M_1) = \Sigma^*$	[M_1 acc. all str]
$\Rightarrow L(M_2) = \{w\}$	[M_2 acc w]
$\Rightarrow L(M_1) \cap L(M_2) = \{w\}$	[w is in Σ^*]
$\Rightarrow |L(M_1) \cap L(M_2)| = 1 \geq 1$	[$|\{w\}| = 1$]
$\Rightarrow \langle M_1, M_2, 1\rangle \in L$	[def of L]

($\Leftarrow$) $\langle M, w\rangle \notin HALT$

$\Rightarrow$ M loops	[def of HALT]
$\Rightarrow$ M_1 accepts no strings	[desc of M_1]
$\Rightarrow$ M_2 accepts w	[desc of M_2]
$\Rightarrow L(M_1) = \emptyset$	[M_1 acc no str]

$\therefore L$ is not co-recognizable ☑

$\Rightarrow M_2$ accepts w [desc of M_2]

$\Rightarrow L(M_1) = \emptyset$ [M_1 acc no str]

$\Rightarrow L(M_2) = \{w\}$ [M_2 acc w]

$\Rightarrow L(M_1) \cap L(M_2) = \emptyset$ [$\emptyset \cap x = \emptyset$]

$\Rightarrow |L(M_1) \cap L(M_2)| = 0 < 1$ [$|\emptyset| = 0$]

$\Rightarrow \langle M_1, M_2, 1 \rangle \notin L$ [def of L]