Exam Si Wang 1006090365

1.

a) 2 1's 1 0's

0.95 0.90

$P(\text{No bits flipping}) = (0.95^2)(0.9)$
$= 0.81225$

b) A: (1,1) was the sent message
B: (1,1) was the recieved message

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{(0.95)^2 \cdot \frac{1}{4}}{77/320} \approx 0.9376623$$

$P(A) = \frac{1}{4}$

$P(B|A) = 0.95^2$

$$P(B|A)P(A) + P(B|A^c)P(A^c) = \frac{1}{4}0.95^2 + \frac{3}{4}(0.1^2 + 2(0.1)(0.0$$
$$= \frac{77}{320}$$

0 0 0 0

2.

a) 2^4 - 2 possible variation of q, 4 total q

b)

possible configurations of exams for 2 distinct students is $(2^4)^2 - 2^4$

exams are not distinct if they have same q's

$\frac{2^4}{2^8 - 2^4}$ possible exams / possible exams for 2 people

$= \frac{1}{15}$

c) For any config of exam there are $\binom{4}{2}$ ways to make a exam that differs by 2

$2^4(6)$ is the number of ways to get 2 exams that differ by 2 q's

$\frac{2^4(6)}{2^8 - 2^4} = \frac{2}{5}$

3.

a) $E(W) = 250 \quad V(X) = 20^2$

$$P(W \geq 300) = P(|W-250| \geq 50) \leq \frac{20^2}{50^2}$$

$$= \frac{4}{25} \quad \text{by Chebyshev ineq}$$

b) $\rho = -0.5$ $\quad E\left(\frac{W_1 + W_2}{2}\right) =$

$$-0.5 = \frac{Cov(W_1, W_2)}{(20)^2}$$

$$Cov(W_1, W_2) = -200$$

c) $\bar{X}_{36} = \frac{1}{36} \sum_{i=1}^{36} X_i \quad X_i \sim W$

$$P(36\bar{X}_{36} > 9120) = P\left(\bar{X}_{36} > \frac{760}{3}\right)$$

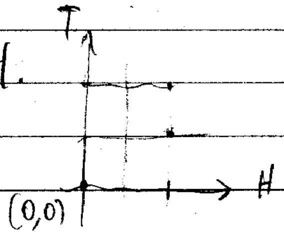
$$= P\left(\frac{\bar{X}_{36} - 250}{20/\sqrt{36}} > 1\right)$$

$$= 1 - P(Z \leq 1)$$

$$= 1 - 0.8413447$$

$$= 0.1586553$$

4.



a) $\dfrac{3}{2^4}$ $\dfrac{\text{tosses that lead to (2,1)}}{\text{possible toss combos}}$

b) $\dfrac{6}{2^4}$ same as a) reasoning

c) Let T be # of tails $T \sim Bin(n, \frac{1}{2})$

$D_4 = T^2 + (n-T)^2 = 2T^2 + 2nT + n^2$

d) $\int$

5. $\sum_{i=0}^{z} p_{Y_1,Y_2}(y_i, z-y_i) = p_X(x)$

$\Rightarrow \sum (1-p)^{y_i} p (1-p)^{z-y_i} p$

$= p^2 \sum (1-p)^2$

$= p^2 \left(\sum 1 - 2p + p^2 \right)$

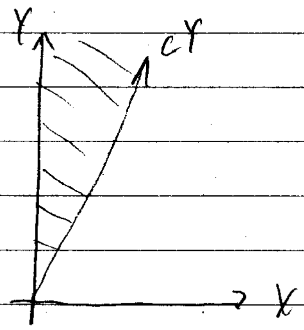
$= p^2 \left(\frac{z(z+1)}{2} - p(z+1)z + \frac{p^2(z+1)z}{2} \right)$

$= \frac{p^2 z(z+1) - 2p^3 z(z+1) + p^4 z(z+1)}{2}$

$= p^2 z(z+1) \cdot \frac{1 - 2p + p^2}{2}$

$= \frac{p^2 z(z+1)(p-1)^2}{2}$

6.



$\int_0^\infty \int_0^{cy} f_{X,Y}(x,y)\,dx\,dy$

$= \iint f_X(x) f_Y(y)\,dx\,dy$ independant

$= \iint 3e^{-3x} e^{-x}\,dx\,dy$

$\int_0^\infty \int_{\frac{x}{c}}^\infty 3e^{-4x}\,dy\,dx$ $\qquad = \int_0^\infty \left[\frac{-3e^{-4x}}{4} \right]_0^{cy} dy$

$\int \left[\frac{-3e^{-4x}}{4} \right]_{\frac{x}{c}}^\infty dx$ $\qquad = \int \frac{-3e^{-4cy}}{4} + \frac{3}{4}\,dy$

$\int_0^\infty \frac{3e^{-\frac{4x}{c}}}{4}\,dx$ $\qquad = \left[\frac{3e^{-4cy}}{16} + \overset{\to \infty}{\frac{3y}{4}} \right]_0^\infty$

$= \frac{3}{4} \left[-\frac{ce^{-\frac{4x}{c}}}{4} \right]_0^\infty$ $\leftarrow$ try other integoal

$= \frac{3}{4}\left(\frac{c}{4}\right) = \frac{3c}{16}$

7.

a) $M_{X_i}(t) = E(e^{tX_i})$ $\qquad X_i = 1 \qquad X_i = -1$

$= \frac{1}{2}e^{t} + \frac{1}{2}e^{-t}$ $\qquad P(X_i = 1) = \frac{1}{2}$ similar w -1

$= \frac{1}{2}(e^{t} + e^{-t})$

b) $E(Z_n) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} E(X_i)$ $\qquad E(X_i) = \frac{1}{2}(1) + \frac{1}{2}(-1)$

$= 0$ $\qquad\qquad = 0$

Show $P(Z_n \geq z) \leq e^{-tz + \frac{t^2}{2}}$

$P(e^{tZ_n} \geq z) \leq \frac{E(e^{tZ_n})}{z}$ $\qquad P(tZ_n \geq \ln(z)) = P\left(Z_n \geq \frac{\ln(z)}{t}\right)$

$= \frac{m_{Z_n}(t)}{z}$

$= \frac{\prod m_{X_i}(\frac{1}{\sqrt{n}} t)}{z}$

$= \frac{\prod \left(\frac{1}{2}(e^{\frac{t}{\sqrt{n}}} + e^{-t/\sqrt{n}})\right)}{z}$

c) $\frac{d}{dt} e^{-tz + \frac{t^2}{2}} = e^{-tz + \frac{t^2}{2}}(-z + t) = 0$

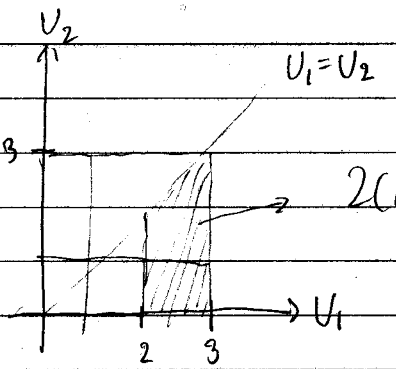
$\Rightarrow t = z$

$\Rightarrow P(Z_n \geq z) = e^{-z^2 + \frac{z^2}{2}}$

$= e^{-\frac{z^2}{2}}$

8.

a)



$2(1) + \frac{1}{2}(1)(1) = 2.5$

$P(U_1 > 2 \mid U_1 \geq U_2) = 2.5\, p_{U_1}(u_1)$
$= \frac{5}{6}$

b) $f_{U_1,U_2}(u_1,u_2) = \frac{1}{u_1 u_2} \quad 0 \leq u_1, u_2 \leq 0$

c) $\int_0^3 \int_0^{u_1} (u_1 - u_2) \frac{1}{u_1 u_2}\, du_2\, du_1$

$= \int_0^3 \left[\ln(|u_2|) - \frac{u_2}{u_1} \right]_0^{u_1} du_1$

$= \int_0^3 \ln(u_1) - 1 \; du_1$