University of Toronto at Scarborough
Department of Computer & Mathematical Sciences

MIDTERM - MATA37 Calculus II for Mathematical Sciences

Examiner: K. Smith

Date: June 28, 2019
Duration: 110 Minutes

FAMILY NAME: ____________________

GIVEN NAMES: ____________________

STUDENT NUMBER: ____________________

CIRCLE your Tutorial/TA:

TUT0006 Mon. 10:00am Paolo Labuguen
TUT0008 Mon. 2:00pm Pourya Memarpanahi
TUT0001 Tues. 2:00pm Qin Deng
TUT0002 Wed. 3:00pm Yuan Bian
TUT0003 Fri. 12:00pm Kunihiro Ito
TUT0004 Fri. 10:00am Kunihiro Ito

SIGNATURE: ____________________

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

NOTES:

- There are 10 numbered pages the exam. It is your responsibility to ensure that, at the start of the exam, this booklet has all its pages. The last page is intentionally left blank.
- Answer all questions. Explain and justify your answers.
- **Show all your work**. Credit will not be given for numerical answers if the work is not shown. If you need more space use the back of the page.
- No cell phones and any type of e-mail or instant messaging devices are allowed to be brought to the exam. Be sure that if you have any, that they are OFF and in your backpack away from you.
- The use of a calculator is **NOT** permitted.

FOR MARKERS ONLY	
Question	Marks
1	/20
2	/4
3	/ 13
4	/ 15
5	/ 8
6	/ 5
7	/ 26
8	/ 4
9	/ 5
BONUS Cover Page Info	/2
TOTAL	**/100**

1. **[20 marks]** **TRUE OR FALSE** Carefully read each statement. If the assertation must be true, the **circle T** (for true). Otherwise, **circle F** (for false). Justification is neither required nor rewarded, but a small workplace is given for your rough work. Each correct answer earns 2 points and each incorrect or blank answer earns 0 points.

(a) Let $a, b \in \mathbb{R},\ a < b$. If $|\ f\ |$ is integrable on $[a, b]$ then so is the product $|\ f\ |\ f$.

T or F

(b) Every continuous function has at least 3 antiderivatives.

T or F

(c) Let $n \in \mathbb{N}$. Then $\displaystyle\sum_{k=0}^{n} \frac{1}{k+1} + \sum_{k=1}^{n} k^2 = \sum_{k=0}^{n} \frac{1+k^3+k^2}{k+1}$.

T or F

(d) $\displaystyle\int_{-2}^{1} 8x^{-4} dx = -3$.

T or F

(e) Let $a, b \in \mathbb{R},\ a < b$. If f is positive, continuous and concave up on $[a, b]$, then any left Riemann sum approximation for f on $[a, b]$ will be an underestimate of $\displaystyle\int_a^b f(x)dx$.

T or F

(f) Every improper rational function can be expressed as the sum of a polynomial and a proper rational function.

T or F

(g) If $\int_0^{10} f(x)dx = 17$ and $\int_0^{8} f(t)dt = 12$, then $\int_8^{10} -f(x)dx = -5$.

T or F

(h) Suppose that f is continuous, positive and decreasing on $\mathbb{R}$. Then $A(x) = \int_x^{-6} f(t)dt$ is increasing for all $x \leq -6$.

T or F

(i) The partial fraction decomposition of $\frac{e^{2x}+1}{e^{2x}(e^x-3)}$ is of the form $\frac{A}{e^x} + \frac{B}{e^{2x}} + \frac{C}{e^x-3}$, where A, B, C are unknown real constants.

T or F

(j) $$\int_{-\pi}^{\pi} \left(\frac{e^{3\cos(x^2+1)}\sin(x^2+1)}{x} + x - 6x^9 \right) dx = 0.$$

T or F

2. **[4 marks]** Let f be a continuous everywhere real-valued function. Suppose that the graph of f passes through the point $(2, 5)$ and that the slope of its tangent line at $(x, f(x))$ is $3 - 4x$. Find $f(1)$.

3. (a) **[5 marks]** Let $a, b \in \mathbb{R}$, $a < b$. Let f be a real-valued function such that $[a, b] \subset \text{dom}(f)$. Provide a complete and accurate statement of the Fundamental Theorem of Calculus Part II with respect to f on $[a, b]$.

(b) **[8 marks]** Prove that the function $F(x) = \displaystyle\int_0^{3x} \frac{x}{t^{20}+1}\,dt$ is increasing for all $x \geq 0$. Do not evaluate the integral. Make sure to fully justify your work.

4. Let $a, b \in \mathbb{R}$, $a < b$.

   (a) **[5 marks]** Suppose that f is bounded on $[a, b]$. Provide a complete and accurate statement of what it means for the function f to be integrable on $[a, b]$ by the Darboux definition.

   (b) **[10 marks]** Let function $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 1 & \text{, if } x \in \mathbb{Q} \\ e & \text{, if } x \notin \mathbb{Q} \end{cases}$

   Prove that the function $\ln(f(x))$ is <u>not</u> integrable on $[a, b]$ by :

   i. the Darboux definition.

   ii. the Integrability ϵ- Reformulation. (You may re-use the values of any sums that were computed in part *i.*, just cite "from *i.*" where appropriate.)

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5. **[8 marks]** Let $a, b \in \mathbb{R},\ a < b.$. Suppose that f and g are continuous on $[a, b]$. Use only the Riemann sum definition of the definite integral to prove :

   If f is positive on $[a, b]$ and $f(x) \geq g(x)$ on $[a, b]$, then $\int_a^b 2f(x)dx \geq \int_a^b g(x)dx$.

   Do not use any of your integral properties.

6. **[5 marks]** Express $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\sqrt{1+\frac{3i}{n}}$ as a definite integral with lower integration limit $a=\frac{1}{3}$. Make sure to justify your solution.

7. Evaluate the following integrals. Make sure to appropriately justify your solutions.

   (a) **[3 marks]** $\displaystyle\int_0^5 f(x)dx$ where $f(x)=\begin{cases}2 & \text{, if } x<3\\ x & \text{, if } x\geq 3\end{cases}$.

   (b) **[5 marks]** $\displaystyle\int \sqrt{x}\ln(x)dx$.

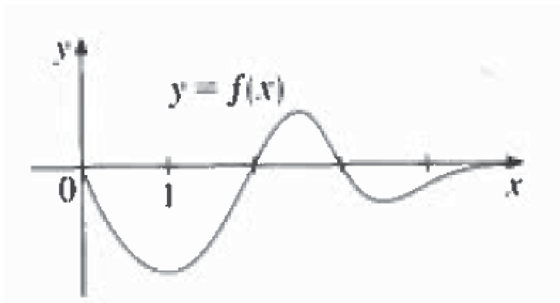
(c) **[8 marks]** $\displaystyle\int \frac{\ln(x)}{x\left(\ln(x)+2\right)\left(\left(\ln(x)\right)^2-4\right)}dx.$

(d) **[3 marks]** $\displaystyle\int \left(\tan(x)+x\sec^2(x)\right)dx.$

(e) **[6 marks]** $\displaystyle\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

8. **[4 marks]** Let $c \in \mathbb{R}$. Suppose that f is an odd function and that f is integrable on $[-c, c]$. Prove that $\displaystyle\int_{-c}^{0} f(x)dx = -\int_0^c f(x)dx$.

9. **[5 marks]** The graph of a function $y = f(x)$ is shown in the figure below. Make a rough sketch of an antiderivative, F, of f given that $F(0) = 1$. Make sure to justify your solution.



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