Quiz 4

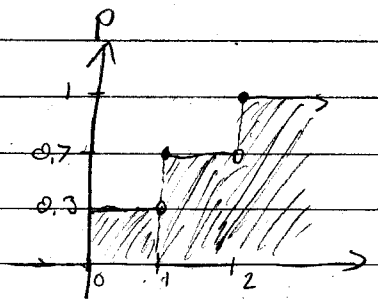
1. a) $P(X=0) = 0.5((1-p_A)(1-p_B)) + 0.5((1-p_B)(1-p_B))$
$= 0.5((1-0.6)(1-0.4)) + 0.5((1-0.4)^2)$
$= 0.3$

$P(X=1) = 0.5(p_A(1-p_A) + (1-p_A)p_B) + 0.5(p_B(1-p_A) + (1-p_B)p_B)$
$= 0.5(0.6(1-0.6) + (1-0.6)(0.4)) + 0.5(0.4(1-0.6) + (1-0.4)(0.4))$
$= 0.4$

$P(X=2) = 0.5(p_A^2) + 0.5(p_B p_A)$
$= 0.5(0.6^2) + 0.5(0.4(0.6))$
$= 0.3$

$$P(X=x) = \begin{cases} 0.3 & x=0 \\ 0.4 & x=1 \\ 0.3 & x=2 \end{cases}$$

$$P(X \le x) \begin{cases} 0 & x<0 \\ 0.3 & 0 \le x < 1 \\ 0.7 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$



b) Coin B is used if the first flip is tails
$P(B \text{ in 2nd}) = 0.5(1-p_A) + 0.5(1-p_B)$
$= 0.5(1-0.6) + 0.5(1-0.4)$
$= 0.5$

c) PMF of $Bin(2, 0.5)$: $P(X=x) = \binom{2}{x}(0.5)^x(0.5)^{2-x}$
$P(X=0) = \binom{2}{0}(0.5)^0(0.5)^2 = 0.25 \neq 0.3$
$P(X=1) = \binom{2}{1}(0.5)^1(0.5)^1 = 0.5 \neq 0.4$
$P(X=2) = \binom{2}{2}(0.5)^2(0.5)^0 = 0.25 \neq 0.3$
$X \not\sim Bin(2, 0.5)$

d) $P(C_1) = 0.5\,p_B + 0.5\,p_A$
$= 0.5$

$P(C_2) = 0.5(p_B p_A + (1-p_B)p_B) + 0.5(p_A p_A + (1-p_A)p_B)$
$= 0.5(0.4(0.6) + (1-0.4)(0.4)) + 0.5(0.6^2 + (1-0.6)(0.4))$
$= 0.5$

$P(C_1 \cap C_2) = 0.5\,p_A p_A + 0.5\,p_B p_A$
$= 0.5(0.6)^2 + 0.5(0.4)(0.6)$
$= 0.3$

$P(C_1)P(C_2) = 0.5^2$
$= 0.25$

$P(C_1 \cap C_2) \neq P(C_1)P(C_2)$
$\therefore C_1$ and C_2 are not indep.