W7 Q2

$u = u(x,t)$

$u_t - k u_{xx} + b u_x = 0 \qquad v = x - bt \quad w = t$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial t} + \frac{\partial u}{\partial w}\frac{\partial w}{\partial t} \qquad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial w}\frac{\partial w}{\partial x}$$

$$= \frac{\partial u}{\partial v}(-b) + \frac{\partial u}{\partial w}(1) \qquad = \frac{\partial u}{\partial v}(1) + \frac{\partial u}{\partial w}(0)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial v}\right)$$

$$= \frac{\partial}{\partial v}\left(\frac{\partial u}{\partial v}\right)\frac{\partial v}{\partial x} + \frac{\partial}{\partial w}\left(\frac{\partial u}{\partial v}\right)\frac{\partial w}{\partial x}$$

$$= \frac{\partial^2 u}{\partial v^2}(1) + \frac{\partial^2 u}{\partial w \partial v}(0)$$

$$= u_{vv}$$

$$u_t - k u_{xx} + b u_x = -b u_v + u_w - k u_{vv} + b u_v$$

$$= u_w - k u_{vv}$$

Simplifies to $u_w - k u_{vv} = 0$