

## Quiz 4

1.

a) A change of coords matrix  $C_{A \rightarrow B}$

$$\forall \vec{v} \in V, C_{A \rightarrow B} [\vec{v}]_A = [\vec{v}]_B \quad A, B \text{ are bases of } V$$

A matrix  $C_{A \rightarrow B}$  transforms vectors in coordinate A to vectors in coordinate B

$$\text{Let } A = (a_1, \dots, a_n) \text{ be a Basis of } A \quad C_{A \rightarrow B} = \begin{bmatrix} [a_1]_B & \dots & [a_n]_B \\ \vdots & & \vdots \end{bmatrix}$$

2.

$$a) \text{ True, } C^{-1} = C_{E \rightarrow B} = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \text{ where } B = (b_1, \dots, b_n)$$

$$\text{Since } C_{B \rightarrow E} = I \quad C_{E \rightarrow B} = I^{-1} = I$$

Thus all  $b_i$ 's are  $\begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}$  -  $i$ th row so  $B$  is the standard basis  $\Rightarrow B = E$

3.

$$a) V = P_2$$

$$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & 0 & -1 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & 0 & 0 & 1 \end{bmatrix}$$

$$T: V \rightarrow F^3 \\ \vec{v} \mapsto [\vec{v}]_B$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 1 & 1 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b-c \\ c \\ a-c \end{pmatrix}$$

$$\sim \begin{bmatrix} I & | & 1 & 1 & -1 \\ & & 0 & 0 & 1 \\ & & 1 & 0 & -1 \end{bmatrix}$$

$$\text{So } T(a+bt+ct^2) = [a+b-c, c, a-c]$$