University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 Winter 2020

Assignment # 6

You are expected to work on this assignment prior to your tutorial during the week of February 24th. You may ask questions about this assignment in that tutorial.

STUDY: Chapter 5: Section 5.3, Section 5.6 (OMIT Thm 5.23 & Example 4 – these are to appear on the next assignment. Do NOT memorize or cite/use Thm's 5.21 & 5.22, read and understand these theorems.)

HOMEWORK:

At the beginning of your TUTORIAL during the week of March 2nd you may be asked to either submit the following "Homework" problems or write a quiz based on this assignment and/or related material from the lectures and textbook readings. This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

Reminder : There are no UTSC classes Mon. Feb 18 - Fri. Feb. 21 due to Reading Week.

1. Write the partial fraction decomposition, form only, for the following :
   (a) $f(x) = \frac{x^2-1}{(x+3)(x-2)^2}$
   (b) $f(x) = \frac{3+3x}{x^3(x^3-1)}$
2. Textbook Section 5.3 - # 28, 34, 40, 42, 50, 54.
3. Which of the following integrals are improper and which are not? Explain.

(a) $\displaystyle\int_{-2}^{0} \frac{2x}{2x+3}dx$

(b) $\displaystyle\int_{1}^{7} \frac{x+1}{\sqrt{x^4-1}}dx$

(c) $\displaystyle\int_{-\infty}^{-6} x^{-2}dx$

4. Textbook Section 5.6 - # 18, 20, 32, 34, 38, 40, 44, 50, 74, 76.

EXERCISES: You do not need to submit solutions to the following problems but you should make sure that you are able to answer them.

1. Textbook Section 5.3 - # 1(a)-(g), 17-55 (ODD numbered questions) — You get better at integrating by practicing!

2. Textbook Section 5.6 - # 1(a)-(f), 2, 3, 4, 7, 21-53 (ODD numbered questions), 75, 77 — You get better at these computations by practicing!

3. Which of the following integrals are improper? Why?

   (a) $\displaystyle\int_{1}^{2} \frac{x}{x-1}dx$

   (b) $\displaystyle\int_{0}^{\infty} \frac{1}{1+x^3}dx$

   (c) $\displaystyle\int_{-\infty}^{\infty} x^2e^{-x^2}dx$

   (d) $\displaystyle\int_{0}^{\frac{\pi}{4}} \cot(x)dx$

4. Determine whether each integral is convergent or divergent :

   (a) $\displaystyle\int_{2}^{\infty} \frac{dv}{v^2+2v-3}$

   (b) $\displaystyle\int_{1}^{\infty} \frac{dx}{\sqrt{x}+x\sqrt{x}}$

   (c) $\displaystyle\int_{-1}^{2} \frac{x}{(x+1)^2}dx$

   (d) $\displaystyle\int_{1}^{\infty} \frac{1}{(w-1)^2}dw$

(e) $\displaystyle\int_{-\infty}^{\infty} x^3 e^{-x^4} dx$

(f) $\displaystyle\int_0^1 x \ln(x) dx$

(g) $\displaystyle\int_0^{\pi} \sec^2(x) dx$

5. Determine whether the following statements are true or false. If it is true, then explain why. If it is false, provide a counterexample (an example to show the statement is false).

   (a) If f is a continuous, decreasing function on $[1, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_1^{\infty} f(x)dx$ is convergent.

   (b) If $\int_7^{\infty} f(x)dx$ converges and let $c \in \mathbb{R}$, then $\int_7^{\infty} cf(x)dx$ also converges.

   (c) Let $a \in \mathbb{R}$. If $\int_a^{\infty} f(x)dx$ and $\int_a^{\infty} g(x)dx$ are both convergent, then $\int_a^{\infty}[f(x) + g(x)]dx$ is convergent.

   (d) Let $a \in \mathbb{R}$. If $\int_0^1 f(x)dx$ is a convergent and $\int_0^1 g(x)dx$ is a divergent improper integral, then $\int_0^1 f(x)g(x)dx$ is also divergent.

Do. Or do not. There is no try. — *Yoda*
