4. <u>Basis</u>

Let $e = x$

$vr(e) + cn(e) = 1 + 0 = 1 \qquad pr(e) + ag(e) = 0 + 0 = 0$

$\Rightarrow vr(e) + cn(e) = pr(e) + ag(e) + 1$

Calculations are the same for $e = y$ and $e = z$

<u>Induction</u>

Let $e_1, e_2 \in \mathcal{E}$ be arbi.

Supp. $vr(e_1) + cn(e_1) = pr(e_1) + ag(e_1) + 1$ [IH]

$vr(e_2) + cn(e_2) = pr(e_2) + ag(e_2) + 1$

Prove $\forall e \in \mathcal{E}$, $vr(e) + cn(e) = pr(e) + ag(e) + 1$

Case 1: $e = (e_1 \wedge e_2)$

$vr(e) + cn(e) = vr((e_1 \wedge e_2)) + cn((e_1 \wedge e_2))$ by def of e

$= vr(e_1) + vr(e_2) + cn(e_1) + cn(e_2) + 1$ by def of vr, cn

$= pr(e_1) + ag(e_1) + 1 + pr(e_2) + ag(e_2) + 1 + 1$ by IH

$= pr((e_1 \wedge e_2)) + ag((e_1 \wedge e_2)) + 1$ by def of pr, ag

$= pr(e) + ag(e) + 1$ as wanted

Case 2: $e = (e_1 \vee e_2)$

Same as case 1, replacing $\wedge$ with $\vee$

Case 3: $e = \langle e_1 \wedge e_2 \rangle$

$vr(e) + cn(e) = vr(\langle e_1 \wedge e_2 \rangle) + cn(\langle e_1 \wedge e_2 \rangle)$ by def of e

$= vr(e_1) + vr(e_2) + cn(e_1) + cn(e_2) + 1$ by def of vr, cn

$= pr(e_1) + ag(e_1) + 1 + pr(e_2) + ag(e_2) + 1 + 1$ by IH

$= pr(\langle e_1 \wedge e_2 \rangle) + ag(\langle e_1 \wedge e_2 \rangle) + 1$ by def of pr, ag

$= pr(e) + ag(e) + 1$

as wanted

This differs from case 1, as in this case, 2 is being added to ag rather than pr.

Case 4: $e = \langle e_1 \vee e_2 \rangle$

Same as case 3, replacing $\wedge$ with $\vee$

$\therefore \forall e \in \mathcal{E}$, $vr(e) + cn(e) = pr(e) + ag(e) + 1$ ▨