Quiz 1

(1) The identity of a binary operation is an element which applied to any other element in the set in the operation yields itself.
Let S be a non-empty set
Let $v \in S$ be arbi. $+$ be the operation
If $v+0=0+v=v$, 0 is the identity element of $+$ over S

(2) This doesn't exist, let the binary operation be $+$
For an element to be invertible (v)
There exists v' such that $v+v'=v'+v=0$, where 0 is the identity element.
A element cannot be invertible if there is no identity by the definition of invertability

(3) Non-empty
Let $f(x)=0$
$\forall x \in \mathbb{R}, f(x)=0$.
$\therefore f(x)=0$ is in the set

Addition
Let $f, g \in F(\mathbb{R},\mathbb{R})$
st $f(x)=f(-x)$, $g(x)=g(-x)$
$(f+g)(x) = f(x)+g(x)$ by func. addition
$= f(-x)+g(-x)$ by def. of f and g
$= (f+g)(-x)$
$\Rightarrow (f+g)(x)=(f+g)(-x)$
$\therefore$ closed under addition

S. Multiplication
Let $r \in \mathbb{R}$, $f \in F(\mathbb{R},\mathbb{R})$ st $f(x)=f(-x)$
$(rf)(x) = rf(x)$ by func. multiplication
$= rf(-x)$ by def of f
$=(rf)(-x)$
$\Rightarrow (rf)(x)=(rf)(-x)$
$\therefore$ closed under scalar multiplication

By subspace test, $\{f(x)=f(-x) \mid f \in F(\mathbb{R},\mathbb{R})\}$ is a subspace of $F(\mathbb{R},\mathbb{R})$