Test 1   CSCB36

1. $D = \{ n \in \mathbb{N} : \text{for some } j, k \in \mathbb{N}, n = 5j + 12k \}$
   Prove $n \in D$, $\forall n \geq 44$

   <u>Basis</u>

   $n = 44$
   Let $k = 2, j = 4$
   $5(4) + 12(2) = 44 = n$

   $n = 45$
   Let $k = 0, j = 9$
   $5(9) + 12(0) = 45 = n$

   $n = 46$
   Let $k = 3, j = 2$
   $5(2) + 12(3) = 46 = n$

   $n = 47$
   Let $k = 1, j = 7$
   $5(7) + 12(1) = 47 = n$

   $n = 48$
   Let $k = 4, j = 0$
   $5(0) + 12(4) = 48 = n$

   <u>Induction</u>
   Let $n > 48$
   Supp. $k \in D, k \in \mathbb{N}, 44 \leq k < n$ [IH]
   Prove $n \in D$

   We know $44 \leq n - 5 < n$ by our supposition
   This means $\exists j, k \in \mathbb{N}, \text{ st } n - 5 = 5j + 12k$, bc $n - 5 < n$
   Let $j' = j + 1, k' = k, j', k' \in \mathbb{N}$
   $5j' + 12k' = 5(j+1) + 12k$ by def of j', k'
   $\qquad\qquad = 5j + 12k + 5$ by basic arithmetic
   $\qquad\qquad = n - 5 + 5$ by IH
   $\qquad\qquad = n$
   $\blacksquare$ as wanted