WS 8.1

$f(x,y) = 3y^2 - 2x^2 + 2x$

Find max/min on $x^2+y^2=1$

$g(x,y) = x^2+y^2-1$

$\nabla f = (-4x+2, 6y) \quad \nabla g = (2x, 2y)$

$\nabla f = \lambda \nabla g$

$$\begin{cases} -4x+2 = \lambda 2x & (1) \\ 6y = 2\lambda y & (2) \\ x^2+y^2 = 1 & (3) \end{cases}$$

Case 1, $y=0$

$x^2+0=1, \quad x=1,-1$

(1) $-4(1)+2 = \lambda 2(1)$

$-2 = 2\lambda$

$\lambda = -1$

$-4(-1)+2 = \lambda 2(-1)$

$6 = -2\lambda$

$\lambda = -3$

Possible pts: $(1,0), (-1,0)$

Case 2, $y \neq 0$

(2) $6y = 2\lambda y$

$3 = \lambda$

(1) $-4x+2 = (3)2x$

$2 = 6x+4x$

$\frac{1}{5} = x$

(3) $\left(\frac{1}{5}\right)^2 + y^2 = 1$

$y^2 = 1 - \frac{1}{25}$

$y^2 = \frac{24}{25}$

$y = \pm \frac{24}{25}$

pts: $\left(\frac{1}{5}, \frac{24}{25}\right), \left(\frac{1}{5}, -\frac{24}{25}\right)$