Question 2. [15 marks]

Let

$$f(x,y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2}$$

a) [3 marks] Using Polar Coordinates, integrate $f(x,y)$ over B, the solid unit circle at the origin.

b) We will try to do part a) using a different change of variables.
Consider

$$u = x^2 + y^2 \qquad v = \frac{x}{\sqrt{x^2+y^2}}$$

In other words, we have defined

$$h(x,y) = (x^2 + y^2, \frac{x}{\sqrt{x^2+y^2}}) = (u,v)$$

1. [1 marks] It is clear that $x = v\sqrt{u}$. Find an expression for $|y|$ in terms of u and v.
2. [3 marks] Find D_h. Find $DetD_h$ in terms of u and v.
3. [5 marks] Using the change of variable formula presented in class, integrate $f(x,y)$ over B by integrating the corresponding region A in the uv-plane.
   Strategy: To find A, first understand what u and v represent in the xy-plane.
   Also, it may be useful to know that $\int \frac{1}{\sqrt{1-x^2}} = \arcsin x$
4. [3 marks] Did you get the same answer as part a)? Comment.

Congratulations – you've made it to the end of your assessment for this course! We hope that you feel proud of the work that you did here because you know that it was your own and no one elses. Please know that all suspected cases of academic dishonesty will be investigated following the procedures outlined in the Code of Behaviour on Academic Matters. If you have violated that Code, admitting it now will significantly reduce any penalty you incur if its discovered by your instructor later. Admitting your mistakes is as much a matter of pride as never making them from the beginning. Thus, please check the appropriate statement below:

_____I confirm that the work I've done here is my own and no one elses, in line with the principles of scholarship and the University of Torontos Code of Behaviour

_____I regret that I violated the Code of Behaviour on this assessment and would like to admit that now so that I can take responsibility for my mistake

I, (full name here)_____ ,

University of Toronto student number _____,
confirm that my response here is an accurate and true representation of my behaviour, knowing that by signing this declaration untruthfully I will incur an even greater penalty if it is later discovered that I have cheated or behaved dishonestly on this assessment.