University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 | Winter 2020

# Assignment # 7

You are expected to work on this assignment prior to your tutorial during the week of March 2nd. You may ask questions about this assignment in that tutorial.

**STUDY:** Chapter 5: Section 5.4, Section 5.6 & corresponding lecture material (CT for type II improper integrals). Note that we will NOT memorize or cite/use Thm's 5.21 & 5.22, read and understand these theorems.)

**HOMEWORK:**

At the beginning of your TUTORIAL during the week of March 9th you may be asked to either submit the following "Homework" problems or write a quiz based on this assignment and/or related material from the lectures and textbook readings. This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

1. Determine whether each integral is convergent or divergent. Make sure to fully justify your answers.
   (a) $\displaystyle\int_1^{\infty} \frac{\sqrt{x} - 4\sin^2(x)}{x^3} dx$
   (b) $\displaystyle\int_{-\infty}^{0} \frac{xe^x}{x^3+1} dx$
   (c) $\displaystyle\int_5^{6} \frac{1}{(x-3)\sqrt{x-5}} dx$
   (d) $\displaystyle\int_1^{\infty} \frac{x^3+2x+1}{\ln(x)} dx$
   (e) $\displaystyle\int_{-1}^{1} \frac{3x+5}{x^2+2x+1} dx$

2. Let $a, b \in \mathbb{R}$ with $a < b$. Suppose that $f$ and $g$ are continuous on $(a, b]$. Suppose that both $f$ and $g$ have a vertical asymptote at $x = a$. Prove : If $0 \leq g(x) \leq f(x)$ on $[a, b]$ and $\int_a^b f(x)dx$ converges then $\int_a^b g(x)dx$ also converges.

3. Evaluate the following:

   (a) $\int_0^{\frac{\pi}{2}} \cos^8(x)\sin^3(x)dx$

   (b) $\int \sin^3(5x)\cos^5(5x)dx$

   (c) $\int_0^{\frac{\pi}{4}} \tan^5(x)dx$

   (d) $\int \sin^3(x)\tan^2(x)(x)dx$

   (e) $\int \sec^4(x)\tan^7(x)dx$

   (f) $\int \sin^4(t)\cos^2(t)dt$

   (g) $\int \tan^2(x)\sec^3(x)dx$

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**EXERCISES:** You do not need to submit solutions to the following problems but you should make sure that you are able to answer them.

1. Textbook Section 5.4 - # 1(a)-(g), 2, 21-76 (ODD numbered questions) — You get better at integrating by practicing!

2. Textbook Section 5.6 - # 57 - 64 (ODD numbered questions) — You get better at these computations by practicing!

3. Prove the divergence case for CT for type I improper integrals over $[a, \infty)$ for any $a \in \mathbb{R}$.

4. Find the value of the constant $C$ for which the following integral

$$\int_0^{\infty} \left( \frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2} \right) dx$$

   converges. Evaluate the integral for that value of $C$. Make sure to fully justify your answer.

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$$\lim_{\text{time}\to\infty} \text{everything} = \text{OK}$$