Q11

1\. MGF of Poisson: $m(t) = \exp(\lambda(e^t - 1))$

of Binomial: $m(t) = (1-p+pe^t)^n$

Let $p = \frac{\lambda}{n}$ in the Bin MGF

$$\lim_{n\to\infty} m(t) = \lim_{n\to\infty} \left(1 - \frac{\lambda}{n} + \frac{\lambda}{n} e^t\right)^n \qquad \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$

$$= \lim_{n\to\infty}\left(1 + \frac{\lambda}{n}(e^t - 1)\right)^n$$

$$= e^{\lambda(e^t-1)} \Rightarrow \text{MGF of Poisson}$$

$\therefore$ Poisson$(\lambda) \sim$ Bin$(n, \frac{\lambda}{n})$ as $n \to \infty$

2\.

a) Let $X_1, \dots, X_{36} \overset{iid}{\sim} Exp(1)$ $\quad E(X_i) = \frac{1}{1} = 1$

$V(X_i) = \frac{1}{1^2} = 1$

$\bar{X}_{36} = \frac{1}{36}\sum_{i=1}^{36} X_i \qquad \bar{X}_{36}$ = avg service time

$\bar{X}_{36} \overset{aprx}{\sim} N(1, \frac{1}{36})$

$$P\left(\bar{X}_{36} > \frac{45}{36}\right) = P\left(\frac{\bar{X}_{36} - \mu}{1/\sqrt{36}} > \frac{\frac{45}{36} - 1}{1/\sqrt{36}}\right) \approx P(Z > 1.5)$$

$$= 1 - P(Z \leq 1.5)$$

$$= 0.0668072$$

b) $\bar{X}_{36} \sim$ Gamma$(36, 1)$ as X_i are iid $\sim Exp(1)$

$P(\bar{X}_{36} > 45) = 0.0742175$ by wolfram alpha