

$$1. \frac{\partial f}{\partial x} = \begin{cases} \frac{y^3(x^2+y^2) - (2x)(xy^3)}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\frac{\partial f}{\partial y} = \begin{cases} \frac{3xy^2(x^2+y^2) - 2y(xy^3)}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\frac{\partial}{\partial y} f(x,y) = \lim_{h \rightarrow 0} \frac{g(\vec{a} + h\vec{u}) - g(\vec{a})}{h} \quad \vec{a} = (0,1)$$

$$= \lim_{h \rightarrow 0} \frac{g(0,h) - 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^3(0+h^2) - 2(0)(0h^3)}{(0^2+h^2)^2} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^5}{h^4} \cdot \frac{1}{h}$$

$$= 1$$

$$\frac{\partial}{\partial x} k(x,y) = \lim_{h \rightarrow 0} \frac{k(\vec{a} + h\vec{u}) - k(\vec{a})}{h} \quad \vec{a} = (1,0)$$

$$= \lim_{h \rightarrow 0} \frac{k(h,0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h(0)(h^2+0) - 2(0)(h(0))}{(h^2+0)^2} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0}{h^3}$$

$$= 0$$

Not equal

2. Mixed partials not equal \therefore not C^2 by contrap.