Q11

1. MGF of Poisson: $m(t) = \exp(\lambda(e^t - 1))$
   of Binomial: $m(t) = (1-p+pe^t)^n$

Let $p = \frac{\lambda}{n}$ in the Bin MGF

$$\lim_{n\to\infty} m(t) = \lim_{n\to\infty} \left(1 - \frac{\lambda}{n} + \frac{\lambda}{n} e^t\right)^n \qquad \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$$= \lim_{n\to\infty} \left(1 + \frac{\lambda}{n}(e^t - 1)\right)^n$$

$$= e^{\lambda(e^t-1)} \Rightarrow \text{MGF of Poisson}$$

$\therefore$ Poisson$(\lambda) \sim$ Bin$(n, \frac{\lambda}{n})$ as $n \to \infty$

2.

a) Let $X_1, \dots, X_{36} \overset{iid}{\sim} Exp(1)$ $\quad E(X_i) = \frac{1}{\lambda} = 1$

$V(X_i) = \frac{1}{1^2} = 1$

$\overline{X_{36}} = \sum_{i=1}^{36} X_i \quad E(\overline{X_{36}}) = \sum E(X_i) = 36$

$V(\overline{X_{36}}) = \sum V(X_i) = 36$

$\overline{X_{36}} \overset{aprx}{\sim} N(36, 36)$

$$P(\overline{X_{36}} > 45) = P\left(\frac{\overline{X_{36}} - \mu}{\sigma} > \frac{45-36}{\sqrt{36}}\right) \approx P(Z > 1.5)$$

$= 0.0668072$

b) $\overline{X_{36}} \sim$ Gamma$(36, 1)$ as X_i are iid $\sim Exp(1)$

$P(\overline{X_{36}} > 45) = 0.0742175$ by wolfram alpha