

Q3

$$\begin{aligned} 1. a) P(P_1) &= P(P_1|C)P(C) + P(P_1|C^c)P(C^c) \\ &= 0.9 \cdot 0.1 + 0.2 \cdot 0.9 \\ &= 0.27 \end{aligned}$$

$$\begin{aligned} b) P(P_1 \cap P_2) &= P(P_1 \cap P_2|C)P(C) + P(P_1 \cap P_2|C^c)P(C^c) \\ &= P(P_1|C)P(P_2|C)P(C) + P(P_1|C^c)P(P_2|C^c)P(C^c) \\ &= 0.9 \cdot 0.9 \cdot 0.1 + 0.2 \cdot 0.2 \cdot 0.9 \\ &= 0.117 \end{aligned}$$

$$\begin{aligned} P(P_1)P(P_2) &= (P(P_1|C)P(C) + P(P_1|C^c)P(C^c))(P(P_2|C)P(C) + P(P_2|C^c)P(C^c)) \\ &= 0.27 \cdot 0.27 \\ &= 0.0729 \end{aligned}$$

$\therefore P_1$ and P_2 are indep.

$$\begin{aligned} c) P(P_2|P_1 \cap C) &= \frac{P(P_1 \cap C|P_2)P(P_2)}{P(P_1 \cap C)} \\ &= \frac{P(P_1 \cap C \cap P_2)}{P(P_1 \cap C)} \\ &= \frac{P(P_1 \cap P_2|C)P(C)}{P(P_1|C)P(C)} \\ &= \frac{P(P_1 \cap P_2|C)}{P(P_1|C)} \\ &= \frac{P(P_2|C)}{P(P_1|C)} \\ &= P(P_2|C) \end{aligned}$$

$$\begin{aligned} d) P(C|P_1 \cap P_2^c) &= \frac{P(P_1 \cap P_2^c|C)P(C)}{P(P_1 \cap P_2^c)} \\ &= \frac{P(P_1|C)P(P_2^c|C)P(C)}{P(P_1 \cap P_2)} \\ &= \frac{0.9 \cdot 0.1 \cdot 0.1}{0.117} \\ &= \frac{1}{13} \end{aligned}$$