University of Toronto Scarborough

Department of Computer & Mathematical Sciences

STAB52H3 Introduction to Probability

Term Test 2
November 16, 2020

Duration: 60 minutes

Examination aids allowed: Open notes-books, scientific calculator

Instructions:

- Read the questions carefully and answer only what is being asked.
- Answer all questions directly on the examination paper; use the last pages if you need more space, and provide clear pointers to your work.
- Show your intermediate work, and write clearly and legibly.

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1. (20 points) Consider two random variables X and Y that are binary valued, i.e., the set of possible values is $\{0, 1\}$. Let the marginal distribution of X be

$$\mathbb{P}(X = 0) = p, \quad \mathbb{P}(X = 1) = 1 - p.$$

It is given that the conditional distribution of Y given X is

$$\mathbb{P}(Y = 0|X = 0) = \frac{2}{3}, \quad \mathbb{P}(Y = 1|X = 0) = \frac{1}{3},$$
$$\mathbb{P}(Y = 1|X = 1) = \frac{1}{2}, \quad \mathbb{P}(Y = 0|X = 1) = \frac{1}{2}.$$

   (a) (10 points) Find the value of $p \in [0, 1]$ such that the marginal distributions of X and Y are the same.

   (b) (10 points) For the value of p given in (a), find the conditional distribution of X given Y.

2. (20 points) Let (X, Y) be a random vector whose distribution is given by

$$\mathbb{P}((X, Y) = (0, 0)) = \mathbb{P}((X, Y) = (0, 1)) = \mathbb{P}((X, Y) = (1, 1)) = \frac{1}{3}.$$

   (a) (4 points) Find the marginal distributions of X and Y, and give their name and parameters.

   (b) (16 points) Find the correlation coefficient $\mathrm{Cor}(X, Y) = \rho_{XY}$ between X and Y.

3. (20 points) Consider two *independent* Exponential RVs: $X \sim$ Exponential(1) and $Y \sim$ Exponential(2). Find the probability $\mathbb{P}(X > Y)$.

4. Let the RV X follow Uniform$(0, 1)$ distribution, and define the new RV $Y = 1 - X^2$.

   (a) (3 points) What is the range of possible values of Y?

   (b) (10 points) Find the CDF of Y.

   (c) (7 points) Find the PDF of Y.

5. Consider two RVs X, Y where X has marginal PDF $f_X(x) = \begin{cases} x/2, & x \in (0, 2) \\ 0, & \text{otherwise} \end{cases}$ and Y has conditional PDF $f_{Y|X}(y|x) = \begin{cases} 1/x, & y \in (0, x) \\ 0, & \text{otherwise} \end{cases}$ (i.e. Y given X is uniformly distributed in $(0, X)$.)

   (a) (10 points) Find the marginal PDF of Y; are X and Y independent? (justify your answer).

   (b) (10 points) Find the conditional PDF of X given Y, and identify its distribution (i.e. give its name and parameters).