TT2 STAB52

1.

a) $P(Y=1) = P(Y=1|X=0)P(X=0)$
$+ P(Y=1|X=1)P(Y=1)$
$= \frac{1}{3}p + \frac{1}{2}(1-p)$
$= \frac{1}{2} - \frac{1}{6}p$

$P(Y=0) = P(Y=0|X=1)P(X=1) + P(Y=0|X=0)P(X=0)$
$= \frac{1}{2}(1-p) + \frac{2}{3}(p)$
$= \frac{1}{2} - \frac{1}{2}p + \frac{2}{3}p$
$= \frac{1}{2} + \frac{1}{6}p$

$$p_Y(y) = \begin{cases} \frac{1}{2} + \frac{1}{6}p, & y = 0 \\ \frac{1}{2} - \frac{1}{6}p, & y = 1 \end{cases}$$

$$p_X(x) = \begin{cases} p, & x = 0 \\ (1-p), & \cancel{x} = 1 \end{cases}$$

$\frac{1}{2} + \frac{1}{6}p = p$
$\frac{1}{2} = \frac{5}{6}p$
$p = \frac{3}{5}$

$\frac{1}{2} - \frac{1}{6}p = 1 - p$
$-\frac{1}{2} = -\frac{5}{6}p$
$p = \frac{3}{5}$

$\therefore p = \frac{3}{5}$ makes them equal

b) $p_{X|Y}(x|y) = \frac{P(X=x, Y=y)}{P(Y=y)}$

2.
a) $p_X(x) = \sum_y P(X=x, Y=y)$

$p_X(1) = P(X=1, Y=1)$
$= \frac{1}{3}$

$p_X(0) = P(X=0, Y=1) + P(X=0, Y=0)$
$= \frac{2}{3}$

$p_X(x) = \begin{cases} \frac{1}{3}, & x=1 \\ \frac{2}{3}, & x=0 \end{cases}$ $\quad p_Y(y) = \begin{cases} \frac{2}{3}, & y=1 \\ \frac{1}{3}, & y=0 \end{cases}$

b)

$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$
$= -\frac{5}{36}$

$E(X) = (1)\frac{1}{3} + (0)\frac{2}{3}$
$= \frac{1}{3}$

$V(X) = \frac{1}{3}^2 + \frac{1}{3}^2$
$= \frac{2}{9}$

$E(Y) = (1)\frac{2}{3} + (0)\frac{1}{3}$
$= \frac{2}{3}$

$V(Y) = \frac{2}{3}^2 + \frac{2}{3}^2$
$= \frac{8}{9}$

$Cov(X,Y) = E((\frac{1}{3} - \frac{2}{9})(\frac{1}{3} - \frac{8}{9}))$
$= -\frac{5}{81}$

3. RV $X \sim Exp(1)$ $Y \sim Exp(2)$

$$P(X > Y) = 1 - e^{-2x}$$

$$= 1 - e^{-2(e^{-x})}$$

4. a) $Y \in [0,1]$ as $X \in [0,1]$

b). $P(Y \leq y) = P(1 - X^2 \leq y)$
$= P(1 - y \leq X^2)$
$= P(\sqrt{1-y} \leq X)$
$= 1 - P(X \leq \sqrt{1-y})$
$= 1 - \frac{\sqrt{1-y} - 0}{1 - 0}$
$= 1 - \sqrt{1-y}$

c) $\frac{d}{dy} P(Y \leq y) = \frac{1}{2\sqrt{1-y}}$