Quiz 3

1. A transformation $T: V \to W$ is a LT iff $T(\vec{v}+\vec{w}) = T(\vec{v}) + T(\vec{w})$, $\forall \vec{v}, \vec{w} \in V$
and $T(r\vec{v}) = rT(\vec{v})$, $\forall \vec{v} \in V$, $r \in F$

2. Let $T: M_{n\times m}(\mathbb{R}) \to \mathbb{R}^{n\times m}$ be a LT

$$\begin{bmatrix} 11 & \cdots & n1 \\ \vdots & \ddots & \vdots \\ 1m & \cdots & nm \end{bmatrix} \to \begin{pmatrix} 11 \\ \vdots \\ n1 \\ \vdots \\ nm \end{pmatrix}$$

*Notate entries in matrix by a_{ij}

$\ker(T) = \{\vec{v} \in M_{n\times m}(\mathbb{R})$ st $T(\vec{v}) = \vec{0}\}$

Since the values are directly transfered, only the 0 matrix will yield the 0 vector in $\mathbb{R}^{n\times m}$ $\therefore$ 1-1

All entries ij, $1 \le i \le n$, $1 \le j \le m$ are real numbers

$\forall \vec{v} \in \mathbb{R}^{n\times m}$, $\vec{v} = (v_{11}, \ldots, v_{n1}, \ldots, v_{nm})$ let $A \in M_{n\times m}$ st all $a_{ij} = v_{ij}$ $\quad 1 \le i \le n$, $1 \le j \le m$

$\therefore T(A) = \vec{v} \Rightarrow$ onto $\Rightarrow$ iso.

3. Let $T': A \mapsto P^{-1}AP$ be a LT

Let $A \in M_{n\times n}$ be arbi.

$$T(T'(A)) = T(P^{-1}AP) = PP^{-1}APP^{-1} = IAI = A$$

$\therefore T \circ T' = id_{M_{n\times n}}$

$$T'(T(A)) = P^{-1}PAP^{-1}P = IAI = A$$

$\therefore T' \circ T = id_{M_{n\times n}}$

We can say T' is the inverse of T so T is invertible

Prove T is a LT for iso.

Let $A, B \in M_{n\times n}$, $r \in F$

$T(A + rB) = P(A + rB)P^{-1}$
$= PAP^{-1} + PrBP^{-1}$ dist of matrices
$= PAP^{-1} + rPBP^{-1}$ scalar multi. of matrices
$= T(A) + rT(B)$

$\therefore T$ is a LT and an iso.