W6 Q2

$u = u(x,y)$

$u_x + 2u_y = 2x - y$ Let $s = x + 2y$ $t = 2x - y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x} \qquad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$

$$= \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(2) \qquad = \frac{\partial u}{\partial s}(2) + \frac{\partial u}{\partial t}(-1)$$

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = \frac{\partial u}{\partial s} + 2\frac{\partial u}{\partial t} + 2\left(\frac{\partial u}{\partial s}(2) - \frac{\partial u}{\partial t}\right)$$

$$= 5\frac{\partial u}{\partial s} = 2x - y$$

$$\frac{\partial u}{\partial s} = \frac{2x-y}{5}$$

$$\int \frac{\partial u}{\partial s}\,ds = \int \frac{2x-y}{5}\,ds$$

$$u = \left(\frac{2x-y}{5}\right)s + f(t) \quad f \text{ is arbi.}$$

$$u(x,y) = \left(\frac{2x-y}{5}\right)(x+2y) + f(2x-y)$$