

4. a) Supp there are DFSA's  $M, M'$  st  $L = \mathcal{L}(M), L' = \mathcal{L}(M')$

$$M = (Q, \Sigma, \delta, s, F) \quad M' = (Q', \Sigma', \delta', s', F')$$

Let  $M_1$  be a DFSA st:

$$Q_1 = Q \times Q' \times \{\text{odd}, \text{even}\}$$

$$\delta_1(q_1, q_2, \text{odd}) = (\delta(q_1, a), q_2, \text{even})$$

$$\delta_1(q_1, q_2, \text{even}) = (q_1', \delta'(q_2, a), \text{odd}) \quad q_1 \in Q, q_2 \in Q', a \in \Sigma$$

$$s_1 = (s, s', \text{odd})$$

$$F_1 = \{(q_1, q_2, \text{odd}), q_1 \in F, q_2 \in F'\}$$

Final state is odd index as the last processed state should be a even number bc  $\lambda(x, y) = a_1 b_1 \dots a_n b_n$  so  $\text{len}(\lambda(x, y)) = 2n$