

W7 Q2

$$u = u(x, t)$$

$$u_t - k u_{xx} + b u_x = 0 \quad v = x - b t \quad w = t$$

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial u}{\partial v} \frac{\partial v}{\partial t} + \frac{\partial u}{\partial w} \frac{\partial w}{\partial t} & \frac{\partial u}{\partial x} &= \frac{\partial u}{\partial v} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial w} \frac{\partial w}{\partial x} \\ &= \frac{\partial u}{\partial v} (-b) + \frac{\partial u}{\partial w} (1) & &= \frac{\partial u}{\partial v} (1) + \frac{\partial u}{\partial w} (0) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right) &= \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial v} \right) \\ &= \frac{\partial}{\partial v} \left(\frac{\partial u}{\partial v} \right) \frac{\partial v}{\partial x} + \frac{\partial}{\partial w} \left(\frac{\partial u}{\partial v} \right) \frac{\partial w}{\partial x} \\ &= \frac{\partial^2 u}{\partial v^2} (1) + \frac{\partial^2 u}{\partial v \partial w} (0) \\ &= u_{vv} \end{aligned}$$

$$\begin{aligned} u_t - k u_{xx} + b u_x &= -b u_v + u_w - k u_{vv} + b u_v \\ &= u_w - k u_{vv} \end{aligned}$$

Simplifies to $u_w - k u_{vv} = 0$