MATB24

Quiz 1

(1) Let $s_1, s_2 \in S$ be arbi., $\otimes$ be the binary operation

Commutative binary operation: $s_1 \otimes s_2 = s_2 \otimes s_1$

(2) Identity element: $e \otimes v = v = v \otimes e \quad \forall v \in (\mathbb{Q} \setminus \{-1\})$

Let $e = 0 \in (\mathbb{Q} \setminus \{-1\}) \; v \in (\mathbb{Q} \setminus \{-1\})$

$e \otimes v = ev + e + v$
$= 0v + 0 + v$
$= v$

$v \otimes e = ve + v + e$
$= v(0) + v + 0$
$= v$

(3) Prove $\forall v \in \mathbb{R}^2, \exists u \in \mathbb{R}^2$ st $v + u = \vec{0}$

Let $v, \in \mathbb{R}^2$ be arbi. $v = (v_1, v_2)$
Let $u, \in \mathbb{R}^2$, $u = -v$ $u = (-v_1, -v_2)$

$v + u = (v_1, v_2) + (-v_1, -v_2)$
$= (v_1 - v_1, v_2 - v_2)$ by standard vector addition
$= (0, 0)$
$= \vec{0}$ zero element in $\mathbb{R}^2$ ∎