Supp. $h(L)$ is regular for $L = \mathcal{L}(01^*)$

Let n be as in PL.

Let $y = 01^n$, $y \in h(L)$, $|y| = n+1 \geq n$

By PL we know:

$y = uvw$, $v \neq \varepsilon$, $|uv| \leq n$, $uv^kw \in h(L), \forall k \in \mathbb{N}$

Case 1: $u = 0$

Then $v = 1^j$ for $1 \leq j \leq n-1$

and $w = 1^i$ for $n-1 = j+i$

If n is odd:

$|y|$ is even, but if 01^n is even, it cannot be in $h(L)$

This is bc if $|01^n|$ is even, then $s = 01^k$, $t = 1^{n-k}$ for $k = \frac{n-1}{2}$

So even numbers in $h(L)$ are in the form $1^{n-k}01^k$

PL must hold for all $y \in h(L)$ with $|y| \geq n$, thus a contradiction.

Case 2: $u = \varepsilon$

Then $v = 01^j$ for $1 \leq j \leq n-1$

and $w = 1^i$ for $n-1 = j+i$

$uv^2w = 01^j01^j1^i \notin h(L)$

$\therefore$ Thus a contradiction.

By contradiction, $h(L)$ is not regular for $L = \mathcal{L}(01^*)$ and thus h does not preserve regular lang.