University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA37 Winter 2020

Assignment # 5

You are expected to work on this assignment prior to your tutorial during the week of February 10th. You may ask questions about this assignment in that tutorial.

STUDY: Chapter 5: Section 5.1 (OMIT Thm 5.4 - we never mix variables; *if we perform a u-subst. to a definite integral then our u integrand* ***must*** *have corresponding u integration limits if we keep our integral in definite form*; Section 5.2.

HOMEWORK:

On the week of February 24th **everybody** must SUBMIT the 'Homework" problems from this assignment into the appropriately labeled MATA37 tutorial drop box (4th floor IC) by no later than 5pm Tuesday February 25th. Make sure that your assignment is stapled, has both your name and student number clearly written upon it and, that you submit it in your tutorial's drop box. (Failure to follow these instructions correctly will result in a grade of zero). This part of your assignment will count towards the 20% of your final mark, which is based on weekly assignments / quizzes.

1. (Do not memorize these 'integration formulae' - it is not worth your while.) Prove each of the following :
   (a) $\int \arcsin(x) = x\arcsin(x) + \sqrt{1-x^2} + C$.
   (b) Suppose that $f,\ f',\ f''$ are continuous everywhere. Let $n \in \mathbb{R} - \{-1\}$. Prove that $\int (f(x))^n f'(x)dx = \frac{1}{n+1}(f(x))^{n+1} + C$
   (c) $\int \ln(x)dx = x\ln(x) - x + C$.

2. Evaluate the following :

   (a) $\displaystyle\int (z+1)\sqrt{3z+2}\; dz.$

   (b) $\displaystyle\int_0^2 x^3\left(7-x^2\right)^{\frac{1}{2}}\; dx.$

   (c) $\displaystyle\int \sin^3(x)\cos^4(x)dx.$

   (d) $\displaystyle\int_0^1 2^{\sqrt{x}}dx.$

   (e) $\displaystyle\int_0^1 (x^2+1)e^{-x}dx.$

   (f) $\displaystyle\int \frac{x}{(x+4)^{\frac{1}{3}}}dx.$

   (g) $\displaystyle\int (\ln(x))^2\; dx.$

   (h) $\displaystyle\int_1^e \sqrt{x}\ln(x)dx.$

   (i) $\displaystyle\int \sin(x)\ln\left(\cos(x)\right)dx.$

   (j) $\displaystyle\int \frac{\sqrt{u}}{u+1}du.$

   (k) $\displaystyle\int (\arctan(x))^3\left(\sec(x)\right)^4\; dx.$

   (l) $\displaystyle\int e^x\sin(2x)dx.$

3. Consider the integral $\displaystyle\int_1^9 \sqrt{3-\sqrt{x}}\; dx.$

   (a) Provide three different (and non-identity, i.e., not $u=x$) valid u-substitutions that can be used to evaluate this integral. Make sure to show that your substitution is valid. That is, perform your change of variable but do not evaluate the resulting integral.

   (b) Complete the evaluation of this integral for one of your u-substitutions from part (a).

EXERCISES: You do not need to submit solutions to the following problems but you should make sure that you are able to answer them.

1. Textbook Section 5.1 - # 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 71, 73, 75, 77, 89, 90 — **You get better at integrating by practicing!**

2. Textbook Section 5.2 - # 1(a)-(h), 7, 27-77 (ODD numbered questions excluding any questions with hyperbolic trig functions), 89, 90 — **You get better at integrating by practicing!**

3. If $a, b \in \mathbb{R}^+$, then prove that $\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx$.

4. Suppose that $g''(x)$ is continuous everywhere and that

$$\int_0^{2\pi} g(x)\sin(x)dx + \int_0^{2\pi} g''(x)\sin(x)dx = 2.$$

Given that $g(2\pi) = 1$, prove that $g(0) = 3$.

5. Evaluate the following :

   (a) $\displaystyle\int x\sec^2(x)dx$.

   (b) $\displaystyle\int 2x^2\sin(x^3+1)dx$.

   (c) $\displaystyle\int_0^{3\pi} \sin^5(x)dx$.

   (d) $\displaystyle\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)\, dx$.

   (e) $\displaystyle\int (\arcsin(x))^2\, dx$.

   (f) $\displaystyle\int_0^1 \frac{1}{(1+\sqrt{x})^4}dx$.

   (g) $\displaystyle\int x^3\sqrt{x^2+1}\, dx$.

   (h) $\displaystyle\int x5^{-x}dx$.

   (i) $\displaystyle\int_0^1 \sqrt[3]{1+7x}\, dx$.

   (j) $\displaystyle\int \frac{x^7}{(1+x^4)^{\frac{3}{2}}}dx$.

   (k) $\displaystyle\int \sin(\ln(x))dx$.

(l) $\displaystyle\int \frac{\ln(\ln(x))}{x}dx.$

(m) $\displaystyle\int x\arctan(x)dx.$

(n) $\displaystyle\int \frac{dx}{1+e^x}.$

(o) $\displaystyle\int \sec^3(x)\tan^3(x)dx.$

(p) $\displaystyle\int \frac{\cos(x)\ln\left(\cos(x)\right)}{\csc(x)}dx.$

(q) $\displaystyle\int_0^4 \frac{\sqrt{t}}{1+\sqrt{t}}dt$

(r) $\displaystyle\int e^{\sqrt{x}}dx.$

(s) $\displaystyle\int \sec^3(x)dx.$

6. Let $n \in \mathbb{Z}$, $n > 2$. Prove:

$$\int \sin^n(x)dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx.$$

Hint : Re-write the LHS integrand as an appropriate product and then apply an appropriate integration technique. (**P.S. Do not memorize this formula.)**

7. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$ and f'' is continuous. Find the value of $\displaystyle\int_1^4 xf''(x)dx$.

Difficulties strengthen the mind, as labor does the body. — *Seneca*
