

W4 Q2

$$\nabla f = (2x + 2y, 3y^2 + 2x) = (0, 0)$$

$$\begin{cases} 2x + 2y = 0 \end{cases} \rightarrow x = -y$$

$$\begin{cases} 3y^2 + 2x = 0 \end{cases} \Rightarrow$$

$$3y^2 + 2(-y) = 0$$

$$3y^2 - 2y = 0$$

$$y(3y - 2) = 0$$

$$y = 0, \frac{2}{3}$$

Crit points

$$x = 0, -\frac{2}{3}$$

$(0, 0), (\frac{2}{3}, \frac{2}{3})$

$$H = \begin{bmatrix} 2 & 2 \\ 2 & 6y \end{bmatrix}$$

$$H(0, 0) = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix} \quad \det(H) = -4 < 0 \quad (0, 0) \text{ is a saddle}$$

$$H(-\frac{2}{3}, \frac{2}{3}) = \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix} \quad \det(H) = 8 - 4 = 4 > 0 \quad \frac{\partial^2 f}{\partial x \partial x} = 2 > 0 \quad \therefore (-\frac{2}{3}, \frac{2}{3}) \text{ is a local min}$$