

Q2

Def set G of formulas that uoc in $\{\neg, \vee\}$

Let G be the smallest set st:

Basis: if x is a var $x \in G$

Ind: If $F, F_2 \in G$ then $\neg F, F \vee F_2 \in G$

Prove $\forall F \in G \exists \text{Formula } F' \text{ st } F' \text{ uoc } \{\llcorner, \lrcorner\} \text{ and } F \text{ LEQV } F'$

Basis

Let $F = x$, x is a var

Consider $F' = x$, F' uoc in $\{\llcorner, \lrcorner\}$ (trivial)

$F' = F \therefore F' \text{ LEQV } F$

Ind

Supp $F, F_2 \in G$ and \exists formulas F', F_2' that uoc in $\{\llcorner, \lrcorner\}$ and $F \text{ LEQV } F'$ and $F_2 \text{ LEQV } F_2'$

Case 1

$F = \neg F_1$ Consider $F' = F_1' \llcorner \lrcorner F_1'$

F_1	$\neg F_1$	$F_1 \llcorner \lrcorner F_1$	F' uoc in $\{\llcorner, \lrcorner\}$ [IH]
0	1	0	$F' \text{ LEQV } F_1 \llcorner \lrcorner F_1$ [IH]
1	0	1	$\text{LEQV } \neg F_1$ by truth table
			$\text{LEQV } F$ as wanted

Case 2 * Using \lrcorner in place of \llcorner

$F = F_1 \vee F_2$ Consider $F' = (F_2' \lrcorner (F_1' \lrcorner \lrcorner F_1')) \lrcorner \lrcorner F_1'$

F_1	F_2	$F_1 \vee F_2$	$(F_2' \lrcorner (F_1' \lrcorner \lrcorner F_1')) \lrcorner \lrcorner F_1'$
0	0	0	0
1	0	1	1
0	1	1	1
1	1	1	1

F' uoc in $\{\llcorner, \lrcorner\}$ [IH], $F' \text{ LEQV } (F_2' \lrcorner (F_1' \lrcorner \lrcorner F_1')) \lrcorner \lrcorner F_1'$ [IH]

$\text{LEQV } F_1 \vee F_2$ by table

$\text{LEQV } F$ as wanted

Since $\{\neg, \vee\}$ is complete
So is $\{\llcorner, \lrcorner\}$ \square