Quiz 3

1. a) V is a F- V.S.

Let $S \subseteq V$, $S = \{v_1, \dots, v_n\}$

S is independent iff $\forall v_i \in S, 0 < i \leq n$ no v_i can be written as a linear combination of the other v_i's

2. a) False

$\forall c \in \mathbb{R}, c \in P_3(\mathbb{R}) = a_0 + a_1 x + a_2 x^2 + a_3 x^3, a_i \in \mathbb{R}$

$\forall r_i \in \mathbb{R}, r_1 x + r_2 x^2 + r_3 x^3 \neq c$ (unless $c = 0$), so $sp(x, x^2, x^3) \neq P_3(\mathbb{R})$

3. Prove for $v_1, v_2 \in V$, V is a V.S. over F

$sp(v_1, v_2) = sp(v_1, 2v_1 + 5v_2)$

$sp(v_1, v_2) = r_1 v_1 + r_2 v_2$, $r_i \in F$

$sp(v_1, 2v_1 + 5v_2) = s_1 v_1 + s_2(2v_1 + 5v_2)$, $s_i \in F$

$= s_1 v_1 + s_2 2v_1 + s_2 5v_2$

$= (s_1 + 2s_2)v_1 + (5s_2)v_2$ — $s_1 + 2s_2$ and $5s_2$ can be any elements in F

$= r_1 v_1 + r_2 v_2$ — Let $s_1 + 2s_2 = r_1$, $5s_2 = r_2$

$= sp(v_1, v_2)$ ▨