Q11

1. MGF of Poisson: $m(t) = \exp(\lambda(e^t - 1))$

of Binomial: $m(t) = (1 - p + pe^t)^n$

Let $p = \frac{\lambda}{n}$ in the Bin MGF

$$\lim_{n\to\infty} m(t) = \lim_{n\to\infty} \left(1 - \frac{\lambda}{n} + \frac{\lambda}{n} e^t\right)^n$$

$$= \lim_{n\to\infty} \left(1 + \frac{\lambda}{n}(e^t - 1)\right)^n \qquad \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$= e^{\lambda(e^t - 1)} \Rightarrow$ MGF of Poisson

$\therefore$ Poisson$(\lambda) \sim$ Bin$(n, \frac{\lambda}{n})$ as $n \to \infty$

2.

a) Let $X_1, \ldots, X_{36} \overset{iid}{\sim} \text{Exp}(1)$ $\quad E(X_i) = \frac{1}{1} = 1$, $V(X_i) = \frac{1}{1^2} = 1$

$\bar{X}_{36} = \sum_{i=1}^{36} X_i$ $\quad E(\bar{X}_{36}) = \sum E(X_i) = 36$, $V(\bar{X}_{36}) = \sum V(X_i) = 36$

$\bar{X}_{36} \overset{approx}{\sim} N(36, \frac{36}{36}) = N(36, 1)$

$$P(\bar{X}_{36} > 45) = P\left(\frac{\bar{X}_{36} - \mu}{36/\sqrt{36}} > \frac{45 - 36}{36/\sqrt{36}}\right) \approx P(Z > 4.5)$$

$$= 0.0668072$$

b) $\bar{X}_{36} \sim \text{Gamma}(36, 1)$ as X_i are iid $\sim \text{Exp}(1)$

$P(\bar{X}_{36} > 45) = 0.0742175$ by wolfram alpha