

Q9

1.

$$\begin{aligned} a) E(\bar{X}_n) &= E\left(\frac{1}{n}(X_1 + \dots + X_n)\right) \\ &= \frac{1}{n}(E(X_1) + \dots + E(X_n)) \text{ by linearity of } E \\ &= \frac{1}{n}(n) \\ &= 1 \end{aligned}$$

$$\begin{aligned} P(\bar{X}_n \geq 2) &\leq \frac{E(\bar{X}_n)}{2} \quad \text{by Markov Ineq} \\ &= \frac{1}{2} \end{aligned}$$

b)  $\text{Cov}(X_i, X_j) = 0$  for  $i \neq j$  bc  $X_i$ 's are indep.

$$\begin{aligned} \text{So } \text{Var}(\bar{X}_n) &= \text{Var}\left(\frac{1}{n}(E(X_1) + \dots + E(X_n))\right) \\ &= \frac{1}{n^2} \text{Var}(E(X_1) + \dots + E(X_n)) \\ &= \frac{1}{n^2}(n) = \frac{1}{n} \end{aligned}$$

$$\begin{aligned} P(\bar{X}_n \geq 2) &= P(\bar{X}_n - 1 \geq 1) \\ &\leq P(|\bar{X}_n - 1| \geq 1) \\ &\leq \frac{1}{n} \quad \text{by Chebyshev's Ineq} \\ &= \frac{1}{n} \end{aligned}$$

$$\begin{aligned} c) V(\bar{X}_n) &= V\left(\frac{1}{n} \sum X_i\right) \\ &= \frac{1}{n^2} V\left(\sum X_i\right) \\ &= \frac{1}{n^2} \left( \sum V(X_i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^n 2\text{Cov}(X_i, X_j) \right) \\ &= \frac{1}{n^2} \left( n + \sum_{i=1}^{n-1} i \right) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{n^2} \left( n + \frac{n(n-1)}{2} \right) \quad P(|\bar{X}_n - \mu| \geq \varepsilon) \leq \frac{V(X)}{\varepsilon^2} \\ &= \frac{1}{n} + \frac{n-1}{2n} = \frac{n+1}{2n\varepsilon^2} \end{aligned}$$

$$= \frac{1}{n} + \frac{n-1}{2n}$$

$$= \frac{n+1}{2n}$$

$$\lim_{n \rightarrow \infty} \frac{n+1}{2n\varepsilon^2} = \frac{1}{2\varepsilon^2} \neq 0$$

$\therefore \bar{X}_n$  doesn't conv to mean