1. $P((A \cup B) \cap C) = P(A \cup B)P(C)$    $C$ is indep of $A$ and $B$

$$= (P(A) + P(B) - P(A \cap B))P(C)$$
$$= (P(A) + P(B) - P(A)P(B))P(C)$$
$$= \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{2}\cdot\frac{1}{3}\right)\frac{1}{4}$$
$$= \frac{1}{6}$$

2. $* * T * B * * *$

$n$ total people, $k$ inbetween Tom and Ben

Num of ways to organize $k+2$ people in a row of $n$ is $n-(k+2)+1$
$= n-k-1$

There are 2 ways to arrange Tom and Ben (either $*T*B*$
or $*B*T*$)

Num of ways to arrange $n$ people is $n!$

So $P(k$ people between Ben and Tom$) = \frac{2\cdot(n-k-1)}{n!}$

3. $P(C \mid S_1^c \cap S_2) = 1 - P(C^c \mid S_1^c \cap S_2)$    $P(C) = 0.1$

$$= 1 - \frac{P(S_1^c \cap S_2 \mid C^c)P(C^c)}{P(S_1^c \cap S_2)}$$
$$= 1 - \frac{P(S_1^c \mid C^c)P(S_2 \mid C^c)P(C^c)}{P(S_1^c \cap S_2)}$$
$$= 1 - \frac{P(S_1^c \mid C^c)P(S_2 \mid C^c)P(C^c)}{P(S_1^c \cap S_2 \mid C)P(C) + P(S_1^c \cap S_2 \mid C^c)P(C^c)}$$
$$= 1 - \frac{0.95 \times 0.05 \times 0.9}{P(S_1^c \mid C)P(S_2 \mid C)P(C) + P(S_1^c \mid C^c)P(S_2 \mid C^c)P(C^c)}$$
$$= 1 - \frac{0.04275}{0.7 \times 0.7 \times 0.1 + 0.95 \times 0.05 \times 0.95}$$
$$= 1 - \frac{0.04275}{0.094125}$$
$$= \frac{137}{251} \approx 54.58\%$$

TTHH HTHH
THHT

4. $X = \{1, 2, 3, 4\}$

| $x$ | $P(X=x)$ |
| --- | --- |
| 1 | 1/8 |
| 2 | 1/2 |
| 3 | 1/4 |
| 4 | 1/8 |

$\underline{x=2}$

$$P(X=2) = \frac{2\cdot 2 + 2\cdot 2}{2^4} = \frac{1}{2}$$

$\underline{x=3}$

$$P(X=3) = \frac{2\cdot 2}{2^4} = \frac{1}{4}$$

$\underline{x=4}$

$$P(X=4) = \frac{2}{2^4} = \frac{1}{8}$$

$\underline{x=1}$

$$P(X=1) = \frac{2}{2^4} = \frac{1}{8}$$

THTH
HTHT

5.

a) $$\frac{P(X=k+1)}{P(X=k)} = \frac{\binom{n}{k+1} p^{k+1}(1-p)^{n-(k+1)}}{\binom{n}{k} p^k (1-p)^{n-k}}$$

$$= \left( \frac{n!}{(n-(k+1))!(k+1)!} \Big/ \frac{n!}{(n-k)!k!} \right) \frac{p}{1-p}$$

$$= \frac{n-k}{k+1} \cdot \frac{p}{1-p}$$

b) Supp $k < (n+1)p - 1$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(X=k+1) = \binom{n}{k+1} p^{k+1} (1-p)^{n-k-1}$$

$$\frac{n-k}{k+1} \cdot \frac{p}{1-p} > \frac{n-((n+1)p-1)}{(n+1)p-1+1} \cdot \frac{p}{1-p}$$

$$= \frac{n-np-p+1}{(n+1)p} \cdot \frac{p}{1-p}$$

$$= \frac{n+1-(n-1)p}{(n+1)(1-p)}$$

Ratio $\frac{P(X=k+1)}{P(X=k)} > 1$

when $k < (n+1)p - 1$

$\therefore P(X=k+1) > P(X=k)$