

W3 Q2

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\begin{aligned} a) D_{\vec{a}} f(\vec{a}) &= \lim_{h \rightarrow 0} \frac{f(\vec{a} + h\vec{a}) - f(\vec{a})}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(ha, hb) - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{(ha)(hb)^2}{(ha)^2 + (hb)^2} \\ &= \lim_{h \rightarrow 0} \frac{h^2 ab^2}{h^2(a^2 + b^2)} \\ &= \frac{ab^2}{a^2 + b^2} \\ &= ab^2 \end{aligned}$$

$$\begin{aligned} b) \lim_{\vec{h} \rightarrow \vec{0}} \frac{f(\vec{a} + \vec{h}) - f(\vec{a}) - \nabla f(\vec{a}) \cdot \vec{h}}{|\vec{h}|} \\ = \lim_{\vec{h} \rightarrow \vec{0}} \frac{f(\vec{a} + \vec{h}) - 0 - (0,0) \cdot \vec{h}}{|\vec{h}|} \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial x}, \vec{a} = (1,0) & \quad (1)(0)^2 = 0 \\ \frac{\partial f}{\partial y}, \vec{a} = (0,1) & \quad (0)(1)^2 = 0 \end{aligned}$$

$$\nabla f(\vec{a}) = (0,0)$$

$$\begin{aligned} &= \lim_{(x,y) \rightarrow (0,0)} \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{xy^2}{x^2+y^2} \\ &= \lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{(x^2+y^2)^{3/2}} \end{aligned}$$

$$\vec{h} = (x,y)$$

$$\begin{aligned} \text{Let } y=0 \\ \frac{x(0)^2}{(x^2+0^2)^{3/2}} &= \frac{0}{x^3} = 0 \end{aligned}$$

$$\text{Let } y=x$$

$$\frac{x(x^2)}{(x^2+x^2)^{3/2}} = \frac{x^3}{(2x^2)^{3/2}} = \frac{x^3}{2x^3} = \frac{1}{2} \quad \text{limits don't equal}$$

$$\therefore \lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{(x^2+y^2)^{3/2}} \text{ DNE}$$

$\Rightarrow f$  is not diff at  $(0,0)$