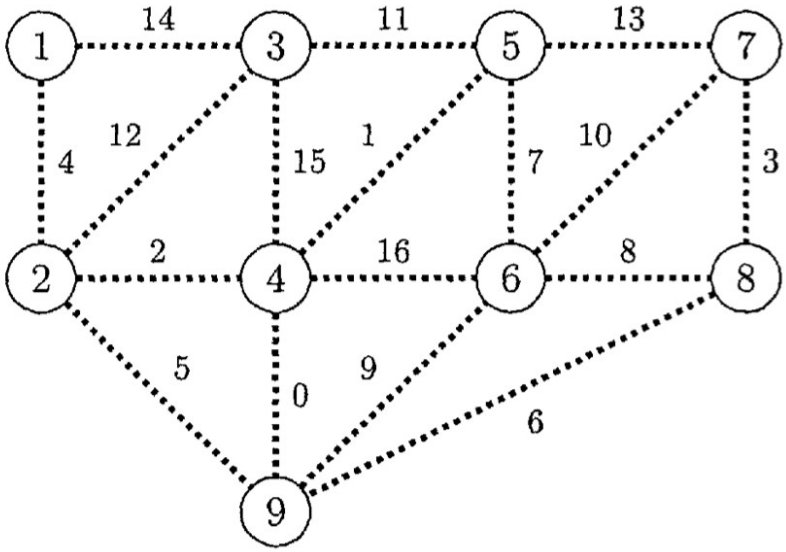
Q1 Kruskals

4 Points

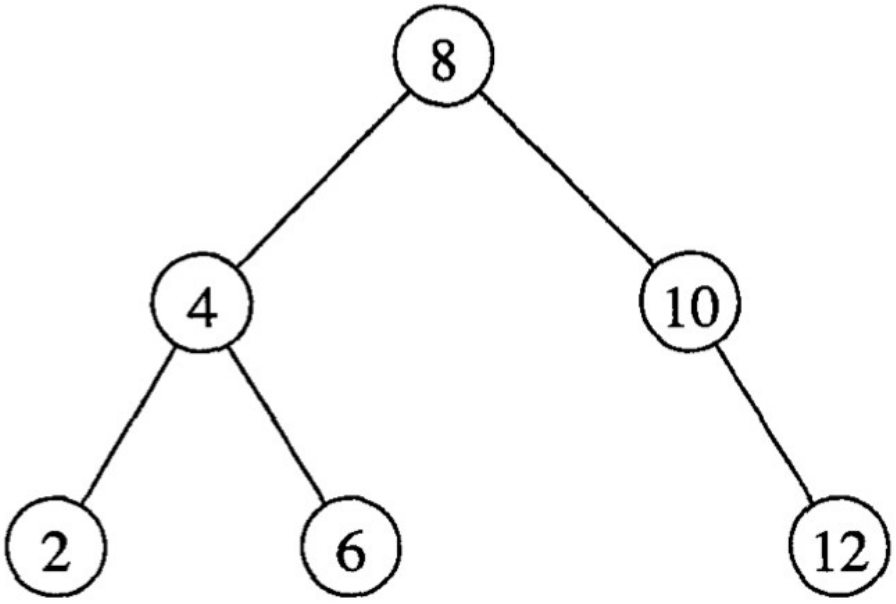


Perform Kruskal's algorithm on this graph and select the edge from the list below that is chosen last.

- ○ 6
- ○ 7
- ○ 8
- ○ 9
- ○ 10
- ◉ 11
- ○ 12
- ○ 13
- ○ 14
- ○ 15
- ○ 16

Q2

4 Points



This AVL tree T is balanced. Suppose we insert into T a key chosen uniformly at random from the set

{1, 3, 5, 7, 9, 11, 13, 14, 15, 16}

What is the exact numeric value of the expected height. Express your answer as a fraction in reduced terms. Ie, if your answer is 3, you would write

Numerator: 3, Denominator 1

If your answer is 2.5, you would write

Numerator 5, Denominator 2

Your answer:

Numerator

- ○ 2
- ○ 3
- ○ 4
- ○ 5
- ○ 6
- ○ 7
- ○ 8
- ○ 9
- ○ 10
- ○ 11
- ◉ 12
- ○ 13
- ○ 14

Denominator

- ○ 1
- ○ 2
- ○ 3
- ○ 4
- ◉ 5
- ○ 6

Q3

8 Points

In the next two questions we consider hashing with chaining in a hash table $T[0...99]$ with 100 slots. A slot $T[i]$ is even if i is even and odd if i is odd. The hash function for this table does not have simple uniform hashing. Starting from an empty hash table T, we enter 6000 keys such that:

- a key is 3 times as likely to hash to an even slot than to an odd slot.
- keys that hash to an even slot are equally likely to hash to any even slot
- keys that hash to an odd slot are equally likely to hash to any odd slot

Q3.1
4 Points

What is the expected length of the chain for $T[33]$?

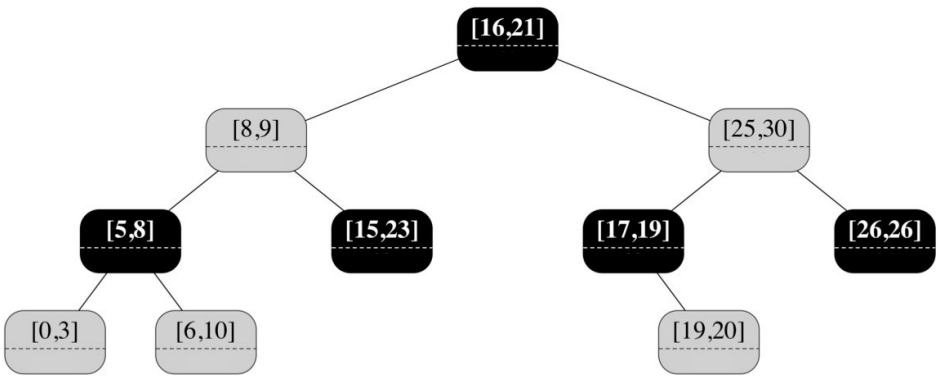
- ○ 20
- ◉ 30
- ○ 40
- ○ 50
- ○ 60
- ○ 70
- ○ 80
- ○ 90
- ○ None of the above

Q3.2
4 Points

After inserting the 6000 keys into our table, what is the expected number of key comparisons when searching for a key that is not in the table.

- ○ 30
- ○ 35
- ○ 40
- ○ 45
- ○ 50
- ○ 55
- ◉ 60
- ○ 65
- ○ 70
- ○ 75
- ○ 80
- ○ 85
- ○ 90
- ○ None of the above

Q4
4 Points



State the interval that search(root, hi, lo) would return for search(root, 25,22) using the interval tree search algorithm from class.

- ○ $[15, 23]$
- ◉ $[25, 30]$
- ○ $[26, 26]$
- ○ None of the above

Q5 DFS Cycles

4 Points

Pick the best answer. DFS can be used to detect a cycle in a graph in

- ○ $\Theta(m)$ time
- ○ $\Theta(n)$ time
- ◉ $\Theta(n+m)$ time
- ○ $\Theta(\log n)$ time
- ○ $\Theta(\log m)$ time
- ○ $\Theta(\log(n+m))$ time
- ○ $\Theta(m \log n)$ time
- ○ None of the above

Q6 MST, Dijkstra's

4 Points

Select all that are true for a weighted graph $G = (V, E)$. Let T be a minimum spanning tree of G.

- [x] Dividing each edge weight in two will not change the set of edges included in the distance tree constructed by Dijkstra's algorithm.
- [x] Adding a constant to every edge's weight may mean T is no longer a minimum spanning tree.
- [x] T is still an MST after squaring the edge weights of G.
- [x] Dijkstra's algorithm works when the edge weights are negative.
- [ ] If the edge weights are not unique then there must be more than one minimum spanning tree.
- [ ] None of them are true.

Q7
12 Points

Consider the Randomized Quicksort sorting algorithm that we described in class for the next three questions. For these questions,

- Make sure your answers are **exact numbers**, do not use asymptotic notation.
- You do not need to justify your answers.

Q7.1
4 Points

What is the worst-case total number of comparisons (between the elements of the sequence) when we execute Randomized Quicksort on a sequence with 7 distinct integers? Do not write any spaces with your answer.

27

Q7.2
4 Points

What is the best-case total number of comparisons (between the elements of the sequence) when we execute Randomized Quicksort on a sequence of 7 distinct integers. Do not write any spaces with your answer.

19

Q7.3
4 Points

Let E_n be the $expected$ total number of comparisons between the elements of the sequence when we execute Randomized Quicksort on a sequence of n distinct integers.

What is E_4? Select the best answer.

- ○ 3
- ○ 3.5
- ○ 4
- ○ 4.25
- ○ 4.5
- ○ 4.75
- ○ 5
- ○ 5.25
- ○ 5.5
- ○ 5.75
- ○ 6
- ◉ None of the above.

Q8 Build Heap

4 Points

Select the best answer. An array of $\log n$ nodes can be converted into a min heap:

- ◉ in $\Theta((\log n)\log(\log n))$ time and $\log n$ space.
- ○ in $\Theta((\log n)\log(\log n))$ time and $2\log n$ space.
- ○ in $\Theta(n \log n)$ time and $\log n$ space.
- ○ in $\Theta(n \log n)$ time and $2\log n$ space.
- ○ in $\Theta(\log n)$ time and $\log n$ space.
- ○ in $\Theta(\log n)$ time and $2\log n$ space.
- ○ in $\Theta(n)$ time and $\log n$ space.
- ○ in $\Theta(n)$ time and $2\log n$ space.
- ○ None of the above.

Q9 Complexity

4 Points

If $f() \in \mathcal{O}(g())$ then ...

Select all that are true:

- [x] there is a positive real number c such that for a natural number n_0, $n \geq n_0 \rightarrow f(n) \leq cg(n)$ for all $n \in \mathbb{N}$.
- [x] there is a positive real number c such that for a natural number n_0, $n \geq n_0 \rightarrow g(n) \geq cf(n)$ for all $n \in \mathbb{N}$.
- [ ] $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty$.
- [ ] $\lim_{n\to\infty} \frac{g(n)}{f(n)}$ exists and is finite.
- [ ] $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.
- [x] $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ exists and is finite.
- [x] $g() \in \Omega(f())$.
- [ ] None of them are true.

Final Part A_13678

UNGRADED

STUDENT

Stephen Guo