MATB41 Exam

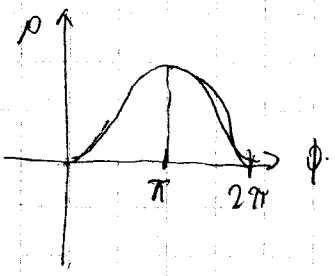
$(x,y,z)=(r\cos\theta, r\sin\theta, z)$

Q1

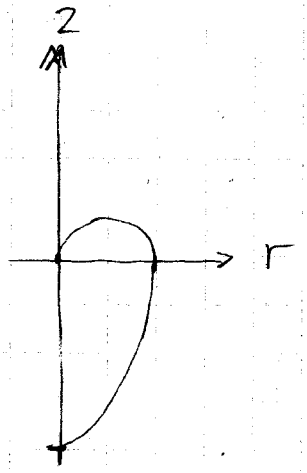
$\sqrt{\rho^2} = 1 - \frac{\rho\cos\phi}{\sqrt{\rho^2}}$

$\rho = 1 - \frac{\rho\cos\phi}{\rho}$

$\rho = 1 - \cos\phi$



1.



$\sqrt{r^2+z^2} = 1 - \frac{z}{\sqrt{r^2+z^2}}$

$\frac{r^2+z^2-z}{\sqrt{r^2+z^2}} = 1$

$r^2+z^2-z = \sqrt{r^2+z^2}$

$r^2+z^2 = \sqrt{r^2+z^2}+z$

2. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi} 1\cdot\rho^2\sin\phi \, d\rho\, d\phi\, d\theta$

$= \int_0^{2\pi}\int_0^{\pi} \sin\phi(1-\cos\phi)\, d\phi\, d\theta$

$u = 1-\cos\phi$

$du = -\sin\phi\, d\phi$

$= \int\int u \, d\phi\, d\theta$

$= \int \left[\frac{(\cos(x)-1)^2}{2}\right]_0^{\pi} d\theta$

$= \int_0^{2\pi} 2 \, d\theta$

$= [2\theta]_0^{2\pi}$

$= 4\pi$