Quiz 8

1) $A \in M_n(\mathbb{C})$ is normal iff $A^*A = AA^*$

2) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = A \qquad AA^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad A^*A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad AA^* = A^*A$

A is normal $\Rightarrow$ A is unitarily dia.

$A \neq A^*$

3) Supp T is self-adj, $\langle T(v), w\rangle = \langle v, T(w)\rangle$
and positive $\langle v, v\rangle \geq 0 \ \forall v \in V$

Let v be a eigenvector w. eigenvalue λ

$\langle T(v), v\rangle = \langle \lambda v, v\rangle \Rightarrow \lambda\langle v, v\rangle > 0$
$= \langle v, \lambda v\rangle \qquad \bar{\lambda}\langle v, v\rangle$

$\lambda\langle v, v\rangle > 0$ and $\langle v, v\rangle > 0$
$\therefore \lambda > 0$

$\lambda\langle v, v\rangle = \bar{\lambda}\langle v, v\rangle$
$\lambda = \bar{\lambda}$
$\therefore \lambda$ is real