University of Toronto at Scarborough
CSCC73 - Algorithm Design and Analysis

Assignment #7 Linear Programming

Due: March 26, 2023 at 11.59pm This exercise is worth 5% of your final grade.

Warning: Your electronic submission on Gradescope affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSCC73. Late assignments will not be accepted. If you are working with a partner your partners' name must be listed on your assignment and you must sign up as a "group" on Gradescope. Recall you must not consult **any outside sources except your partner, textbook, TAs and instructor**.

1. (15 marks) Your favourite natural food company sells two types of trail mix each of which are made from a blend of dried fruits and nuts. Trail mix A contains $1lb$ of dried fruits and $1.5lbs$ of nuts and retails for \$7. A package of trail mix B contains $2lbs$ of dried fruit and $1lb$ of nuts and retails for \$6. Dried fruits when bought in bulk cost $\$1/lb$ and bulk nuts cost $\$2/lb$. The packaging for trail mix A is a nice metal tin and costs $\$1.40$ to package whereas type B tail mix is packaged in a resealable bag and costs $\$0.60$ to for the packaging. A total of 240,000lbs of dried fruits and 180,000lbs of nuts are available each month. Due to the nature of the packaging, the bottleneck in the production is for type A in that the factory can only produce 110,000 tins of trail mix A per month.

   (a) Formulate the problem as a linear program in two variables where the objective function maximizes profit.

   (b) Graph the feasible region, give the coordinates of the vertices and state the vertex maximizing the profit and the value of the maximum profit.

   (c) Confirm the maximizing vertex by applying the Simplex method to the problem.

2. **(0 marks - will not be graded, just for practice, does not need to be handed in)** Consider the following LP problem:

$$\max 18x_1 + 12.5x_2$$

   subject to

$$\begin{aligned} &x_1 + x2 \leq 20 \quad (1) \\ &x_1 \leq 12 \quad (2) \\ &x_2 \leq 16 \quad (3) \\ &x_1, x_2 \geq 0 \quad (4) \end{aligned}$$

   (a) Solve the program using the SIMPLEX method.

   (b) Write the dual LP and show that your solution in part (a) is optimal.

3. The standard form of an LP problem is to solve for vector $x \in R^n$ defined by: `minimize` $c^t x$ subject to $x \geq 0$ and $Ax \geq b$ where A is an $m \times n$ matrix and $c \in R^n$ and $b \in R^m$ are vectors.

   When we consider an ILP (integer linear program), the added constraint that x is an integer valued vector is added. Consider the following objective function and set of constraints.

$$Minimize : h(x_1) + h(x_2) + \ldots + h(x_n)$$

   where

$$h(x_i) = \begin{cases} c_i x_i + f_i & x_i > 0 \\ 0 & x_i = 0 \end{cases}$$

   subject to:

$$\begin{aligned} x_1 + x_2 + \ldots + x_n \geq d & \\ x_i \geq 0, & \quad 1 \leq i \leq n \\ x_i \leq b_i, & \quad 1 \leq i \leq n \\ x_i \in \mathbb{Z}, & \quad 1 \leq i \leq n \end{aligned}$$

   Notice that the objective function is *not* a linear equation. Introduce new variables and constraints to convert this optimization problem into an ILP problem.

   HINT: Try using indicator variables for your new variables.