1. How many permutations are there of the letters in the word MISSISSIPPI?

$\frac{11!}{4!4!2!} = 34650$ 11 permutations of letters divided by duplicate characters

2. How many arrangements of the letters in the word MISSISSIPPI have no consecutive S's?

$\frac{7!}{4!2!} \times 8C4 = 7350$ 7 permutations of letters (all letters that aren't s) divided by duplicates (that aren't s). It is then multiplied by the possible arrangement of the s's with letters between them so none of the possibilities have consecutive s's. L represents a non-s letter _L_L_L_L_L_L_L_ there are 7 L's meaning there are 8 possible places for the s's to be chosen from.

3. In how many ways can a student answer a 10 question TRUE/FALSE quiz? How many more ways can the student answer the quiz if they are allowed to leave questions blank in order to avoid a penalty for a wrong answer?

$2^{10} = 1024$ possibilities when the options are True or False.

$3^{10} = 1024$ possibilities when the options are True, False, or blank.

4. Your local coffee shop offers 8 different types of pastries and 6 different kinds of muffins. In addition, one can buy a small, medium or large beverage.

a) $14 \times 12 = 168$ 14 bakery items multiplied by 12 medium beverages.

b) $12 \times (4 \times 3) \times 6 \times (6 \times 3) = 18144$ 12 bakery items multiplied by 4 coffees x 3 sizes, multiplied by 6 muffins, multiplied by 6 teas x 3 sizes.

c) $8 \times (6 \times 3) \times 6 \times (1 \times 3) \times 12C2 = 171072$ 6 pastries multiplied by 6 teas x 3 sizes, multiplied by 6 muffins, multiplied by 1 orange juice x 3 sizes, multiplied by 12C2 coffees (assuming the order that you buy the TAs coffee doesn't matter).

5. Tim Hortons sells 20 different kinds of donuts and always has at least a dozen of each type available. How many different ways can someone buy a dozen donuts?

$(20 + 12 - 1)C12 = 31C12$ Assuming the order in which you buy donuts doesn't matter, you can use the combinations with repetition formula where the types of doughnuts are n and the number that you choose is r.

6. Suppose you buy 12 identical donuts and wish to give them to the 5 children you are babysitting. How many different ways can you distribute the donuts if:
a) there are no restrictions

$$(5 + 12 - 1)C12 = 1820$$

b) each child gets at least one donut

$$(5 + (12 - 5) - 1)C(12 - 5) = 330$$

c) each child gets at least one donut and the oldest child gets at least 2 donuts.

$$(5 + (12 - 6) - 1)C(12 - 6) = 120$$

For all the parts in question 6, you can use the combinations with repetition formula, with n being the number of children, and r is the number of donuts being distributed (the amount after subtracting the minimum amount that has to be distributed).

7. A message is made up of 10 different symbols and is transmitted over a communication channel. In addition to the 10 symbols, the transmitter sends 25 blank spaces in total, sending at least 2 blanks between each pair of consecutive symbols. In how many ways can the transmitter send such a message?

$10! \times (11 + (25 - 18) - 1)C7 = 10! \times 17C7$ 10 factorial for all the permutations of the symbols multiplied by the combinations of the blanks after subtracting the required amount of blanks between pairs of symbols:
S__S__S__S__S__S__S__S__S__S (_ represents a blank, S represents a symbol).
The use of the combinations with repetition formula is necessary because all blanks are considered the same and the order in which they are put does not matter. (n in the formula represents the number of symbols and r represents the number of places the blanks can be placed.

8. Determine the number of integer solutions to $x_1 + x_2 + x_3 + x_4 = 32$.

a) $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$

$(4 + 32 - 1)C32 = 35C32 = 6545$ Using the combinations with repetition formula, we can represent the integers as singular objects of r, and the amount of x's as n.

b) $x_1 \geq 5, x_2 \geq 5, x_3 \geq 7, x_4 \geq 7$

$(4 + 8 - 1)C8 = 11C8 = 165$ This is a lot like the previous part, but this time we just subtract the minimum amount of the integers that have to be distributed from the total and use the difference in the same formula as part a).

c) Restate Question 6c) as a linear equation problem with integer solutions.

Let x represent the amount of donuts a child has, and let x_n represent the amount for the oldest child, where n = 5.

$x_1 + x_2 + x_3 + x_4 + x_5 = 12$

$x_1 \geq 1, x_2 \geq 1, x_3 \geq 1, x_4 \geq 1, x_5 \geq 2$

$(5 + 6 - 1)C6 = 11C8 = 210$ This is solved the same way as part b).

9. You are having a dinner party with your extended family. You have a square table that seats 8 people (two people per side). We say that two "seatings" are unique if rotating the table by 90, 180 or 270 degrees cannot make the seatings the same.

a) How many unique seatings are there for 8 people named Ann, Bob, Cal, Dan, Ed, Fran, Gina, Hal?

$\frac{8!}{4} = 10080$ Treating it like a circle, the total amount of arrangements is 8! But we divided by 4 because every other rotation of the table is considered the same arrangement. This is similar to a circle, with the only difference being that every rotation of the arrangement is considered the same while only every other rotation is considered the same in this question.

b) If two of the people, Ann and Bob do not wish to sit on the same side together, how many unique seatings are there?

$\frac{8 \times 6 \times 6!}{4} = 8640$ First we choose any seat for Ann (or Bob, either one), then because they cannot sit on the same side, there are only 6 other seats for Bob (or

Ann depending on the choice), to sit at. After Ann and Bob's seats are decided we can multiply that by 6! for the rest of the family. The total amount of seatings is still divided by 4 for the rotation duplicates.

c) If Ann and Bob really do not like each other, and are not even willing to sit on the same side or across from each other, how many unique seatings are there?

$\frac{8\times4\times6!}{4} = 5760$ Similar to part b), but instead of 6 seats for the Bob (if Ann gets first choice), there are only 4, as they cannot sit on the same or opposite side of each other. All the rest of the equation is the same as part b).

10. Suppose you have an x − y grid with origin (0, 0). You have a robot that can travel one unit at a time along the grid lines from (x, y) to either (x + 1, y) or to (x, y + 1). We will use R to denote a move increasing x by 1 and U to denote a move increasing y by 1. How many different paths can the robot take from (0, 0) to (5, 5)? How many paths are there from (2, 7) to (9, 14)? In the following picture we show two possible paths from (0, 0) to (5, 5).

Using this formula: $\frac{(x+y)!}{x!y!}$ (where x and y are the dimensions of the grid), you can calculate the number of paths from one corner to the other in any regular grid. This can also be written as $(x+y)Cx$ *or* $(x+y)Cy$ (these are the same because of the symmetry in Pascal's triangle). This formula works because of how the number of possible paths follow Pascal's triangle. If you were going from corner A to corner B on a regular grid (where A and B are on opposite sides), you need to make x + y moves to get from A to B (assuming you can only move towards the destination). Meaning if A is start of Pascal's triangle, B is the xth(or yth) term on the (x+y)th row (which is why it can be represented as (x+y)C(x or y) in two dimensions).

$$\frac{(7+7)!}{7!7!} = \frac{14!}{7!7!} = 14C7 = 3432$$

11. We now consider the same concept as in Question 10 but in three dimensions. In three dimensions we represent a point as (x, y, z). Suppose your robot is to travel from (x, y, z) to (x + m, y + n, z + p). How many possible paths does your robot have?

We can use the same formula as question 10, just adding a dimension z.
If the dimensions of the grid is $m \times n \times p$ then the number of paths from (x, y, z) to (x + m, y + n, z + p) is:
$\frac{(m+n+p)!}{m!n!p!}$ (Note: in 3 dimensions you cannot write this in combinations form).