University of Toronto at Scarborough
CSCC73 - Algorithm Design and Analysis

Assignment #2: Greedy Algorithms

Due: January 28, 2023 at 11.59pm This exercise is worth 5% of your final grade.

Warning: Your electronic submission on Gradescope affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSCC73. Late assignments will not be accepted. If you are working with a partner your partners' name must be listed on your assignment and you must sign up as a "group" on Gradescope. Recall you must not consult **any outside sources except your partner, textbook, TAs and instructor**.

1. (10 marks) Consider a communications graph. Each edge of the connected graph $G = (V, E)$ represents a communication link between sites (represented as nodes). Each edge e has a bandwidth b_e.

   Each pair of nodes $u, v \in V$ needs to be able to communicate. For any $u, v - path$ P the *bottleneck transmission rate* $b(P)$ of P is the minimum bandwidth of any edge it contains. In other words, $b(P) = min_{e \in P} b_e$. The *best achievable bottleneck rate* for a pair $u, v \in V$ is the maximum, over all $u - v$ paths P in G, of the value $b(P)$. Our goal is to determine a set of $u - v$ paths for each pair $u, v \in V$ with best achievable bottleneck rate.

   Fortunately, we can construct a spanning tree T of G such that for every pair of nodes $u, v \in V$ the unique $u - v$ path in the tree T actually achieves the best achievable bottleneck rate for u, v in G.

   Give an efficient algorithm to construct such a spanning tree. Your algorithm should construct a spanning tree T in which, for each $u, v \in V$, the bottleneck rate of the $u - v$ path in T is equal to the best achievable bottleneck rate for the pair u, v in G. Prove the correctness of your algorithm and give it's complexity. If you have forgotten about minimum spanning trees please feel free to take a look at https://mathlab.utsc.utoronto.ca/bretscher/b63/lectures/w5/mst_dijkstra_idea.pdf.

2. (10 marks) Given a set P of n people. Suppose the i^{th} person claims to know d_i other people in P. Determine in polynomial time if P is a feasible set by constructing a greedy algorithm. By feasible, we mean that it's possible for each person to know the number of people they claim to know. Prove that your algorithm correctly determines if such a set up is possible and give the complexity of your algorithm. HINT: Think of the i^{th} person being represented by a vertex v_i and the number of people that the person knows as the degree d_i of v_i.