$$
\begin{aligned}
v^T A w &= \langle v, w \rangle \\
&= T(v)\cdot T(w) \quad \text{by a)} \\
&= Bv \cdot Bw \\
&= v^T B^T B w
\end{aligned}
$$

Let $v = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ - ith entry $\quad w = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ - jth entry $\quad 1 \leq i, j \leq n$

Let $A = \begin{bmatrix} a_{11} & \cdots & a_{n1} \\ \vdots & & \\ a_{1n} & \cdots & a_{nn} \end{bmatrix} \quad B^T B = \begin{bmatrix} b_{11} & \cdots & b_{n1} \\ \vdots & & \\ b_{1n} & \cdots & b_{nn} \end{bmatrix}$

$$v^T A w = (0, \ldots, 1, \ldots, 0) \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} = (0, \ldots, 1, \ldots, 0) \begin{bmatrix} a_{j1} \\ \vdots \\ a_{jn} \end{bmatrix} = a_{ji}$$

$v^T B^T B w = b_{ji}$ by similar process as A

So $v^T A w = v^T B^T B w \Rightarrow a_{ji} = b_{ji}$ for any $1 \leq i, j \leq n$

Thus $A = B^T B$

We know B is invertible as its the matrix representation of an iso. and the transpose a an invertible matrix is invertible

Since B and B^T are invertibe, $B^T B$ must be invertble

Choose $C = B^{-1}(B^T)^{-1}$

$B^T B C = B^T B B^{-1} B^T = I_n \therefore \exists C$ st $B^T B C = I_n$ thus $B^T B$ is invertible