Project 2 : Image Compression

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1 Introduction

The goal of this project is to investigate image compression techniques. Mainly, the discrete cosine transform (DCT) and the fast wavelet transform (FWT) based Image Compression techniques will be thoroughly investigated. Both these models will be realized on MATLAB for the purpose of performance evaluation. We will implement the transforms, quantize the transform coefficients, evaluate the quality degradation due to quantization and estimate the bit rate needed for coding of the quantized transform coefficients.

2 DCT-based Image Compression

This problem investigates the rate-distortion performance of a transform image coder using the DCT for 8 X 8 blocks of an image.

2.1 Blockwise 8x8 DCT

The DCT transform used in this project is DCT-II which is a separable orthonormal transform. The MxM DCT-II matrix A is defined as follows : $a_{ik} = \alpha_i cos(\frac{(2k+1)i\pi}{2M})$ for $i, k = 0, 1, ..M-1$ with $\begin{cases} \alpha_0 = \sqrt{\frac{1}{M}} \\ \alpha_i = \sqrt{\frac{2}{M}} \ \forall i > 0 \end{cases}$.

The DCT matrix of size 8x8 is computed and shown in FIG.1. Size 8 is chosen to reduce the computational burden.

$$A = \begin{pmatrix} 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\ 0,4904 & 0,4157 & 0,2778 & 0,0975 & -0,0975 & -0,2778 & -0,4157 & -0,4904 \\ 0,4619 & 0,1913 & -0,1913 & -0,4619 & -0,4619 & -0,1913 & 0,1913 & 0,4619 \\ 0,4157 & -0,0975 & -0,4904 & -0,2778 & 0,2778 & 0,4904 & 0,0975 & -0,4157 \\ 0,3536 & -0,3536 & -0,3536 & 0.3536 & 0.3536 & -0,3536 & -0,3536 & 0,3536 \\ 0,2778 & -0,4904 & 0,0975 & 0,4157 & -0,4157 & -0,0975 & 0,4904 & -0,2778 \\ 0,1913 & -0,4619 & 0,4619 & -0,1913 & -0,1913 & 0,4619 & -0,4619 & 0,1913 \\ 0,0975 & -0,2778 & 0,4157 & -0,4904 & 0,4904 & -0,4157 & 0,2778 & -0,0975 \end{pmatrix}$$

FIGURE 1 – Coefficients of DCT Matrix A

The obtained DCT matrix is applied onto the image cut into 8x8 blocks. The DCT of a block is given by $Y = AXA^T$. Then, the inverse DCT is performed by $X = A^TYA$. DCT operations are shown in FIG.2 for the image "peppers512x512.tif".

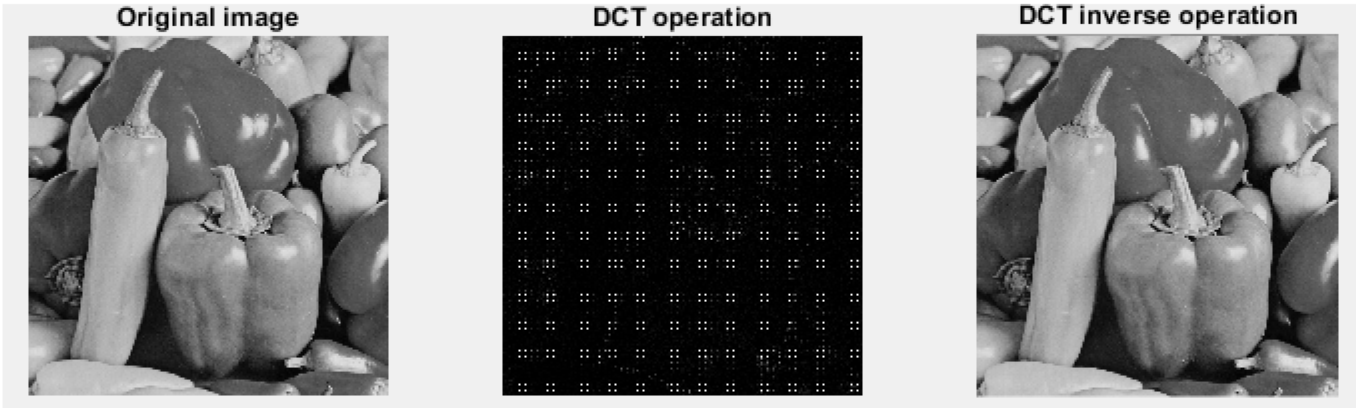

FIGURE 2 – DCT Operations on the peppers512x512.tif image

A natural image is mostly represented in low-frequencies, the DCT operation will compact the energy for each block. We can see from the FIG.2 (DCT Operation) that the energy of each block is concentrated in the upper left of the blocks which represent the DC component and the low-frequencies. With the same analysis, we can see that the high-frequencies of each block are represented in the lower right of the blocks.

2.2 Uniform Quantizer

DCT transformed image shall be quantized in order to encode the coefficients. The quantizer will reduce the number of discrete symbols thus the image is more compressible.

In this project we will use a uniform mid-tread quantizer that is realized by the formula :

```
y = delta*floor(x/delta + 0.5);
% delta is the fixed step size.
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It can be observed from the formula that no threshold bars are used. A plot of the quantizer is shown in FIG.3. The step of the quantizer will determine the amount of information about the image we will lose (.ie the amount of distortion wanted).

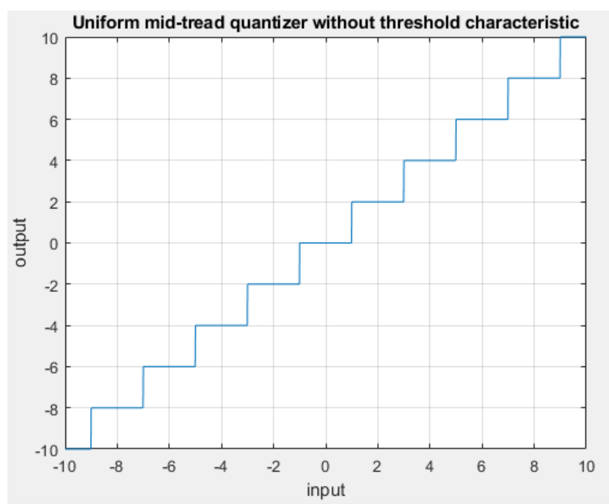

FIGURE 3 – Uniform quantizer function with delta=2

2.3 Distortion and Bit-Rate Estimation

By quantizing the image, we introduce distortion in the reconstructed image. We have lossy compression and we want the analyze the performance of the compression for different quantization steps.

The quality of the reconstructed will measured by the PSNR, while the compression ratio will be represented by the bit-rate.

- The PSNR is defined : $PSNR = 10log_{10}(\frac{255^2}{d})$ with d the average distortion in the image (d will be approximated by the mean squared error between the original and the reconstructed image).

  As the DCT-II is an orthonormal transform, computing the mean square error between the original image and the reconstructed image is the same as computing the mean squared error between the original and the quantized DCT transformed images. Indeed, if we represent the images as vector we have $y = Ax$ and $x = A^T y \Rightarrow d = \frac{1}{M^2}(\hat{x}-x)^T(\hat{x}-x) = \frac{1}{M^2}(\hat{y}-y)^T AA^T(\hat{y}-y) = \frac{1}{M^2}(\hat{y}-y)^T(\hat{y}-y)$.

- In order to estimate the bit-rate required to encode our image DCT quantized coefficients, we employ variable length coding (VLC). We will assume that the ideal VLC is considered (.ie we have no redundancy in the code-words), therefore the bit-rate R is equal to the entropy. The entropy is calculated as follows : $H = -\sum p_i log_2(p_i)$.

- For a better representation we can compute the compression ratio is computed as : $Tx = 1 - \frac{R}{8}$.

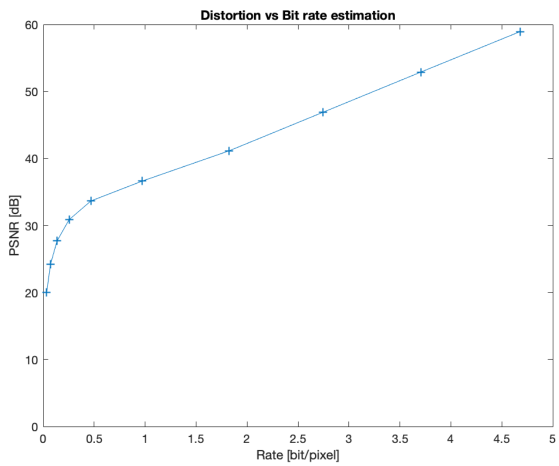

FIGURE 4 – PSNR vs Bit-Rate for "peppers512x512.tif"

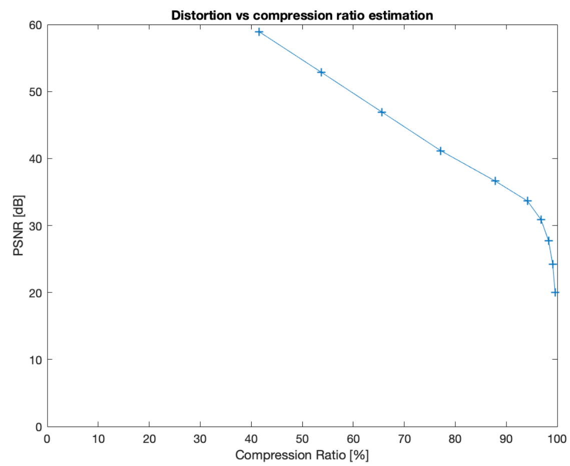

FIGURE 5 – PSNR vs Compression ratio for "peppers512x512.tif"

Graph of PSNR vs Bit-rate is shown in FIG.4. It can be observed that with the increase in quantization step size, the bit rate is reduced (this is what is expected because we have eliminated information about the image) and the distortion in the image increase. We can see that we have a trade-off between the distortion in the image (represented by the PSNR) and the bit-rate. The graph of PSNR vs compression ratio is shown in FIG.5 is a complement to understand that the more we compress the image, the more distortion.

3 FWT-based Image Compression

This problem investigates the rate-distortion performance of a transform image coder using the Fast Wavelet Transform (FWT).

3.1 The Two-Band Filter Bank

Direct implementation method is employed in MATLAB for the implementation of one-dimensional two-band analysis filter with the signal and prototype scaling vectors as arguments. QMF filters will be used in order to have a perfect signal reconstruction. The QMF filters bank is constructed as follows :

— h_0 = prototype scaling vectors of length N

— $h_1[n] = (-1)^n h_0[N-1-n]$

— $g_0[n] = h_0[N-1-n]$

— $g_1[n] = h_1[N-1-n]$

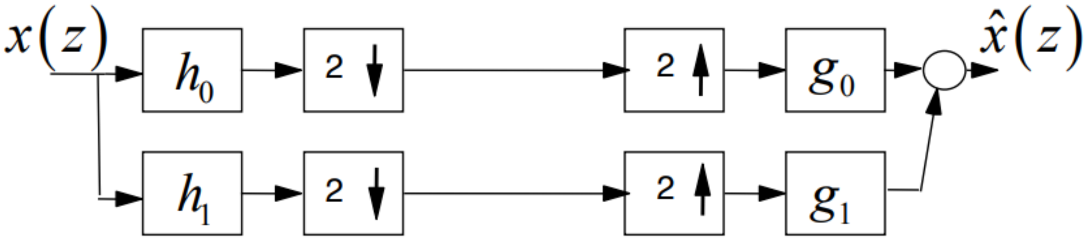

FIGURE 6 – The Two-Band Filter Bank for a 1D-signal

The direct implementation involves the computation of a convolution which extends the signal. Wishing keeping the same length between the output and the input signal during the analysis and synthesis, signal sequence data is extended in a periodic fashion in order to conserve the energy of the input signal in the output signal with the same length.

3.2 FWT

To implement the two-dimensional FWT (and inverse FWT), we recursively apply the 1-dimension analysis and synthesis filter banks on the rows, then on the columns of the image. Daubechies 8-taps filter is used for a scale of 4. FIG.7 shows the results, comparing the original image, intermediate signal and reconstructed image.

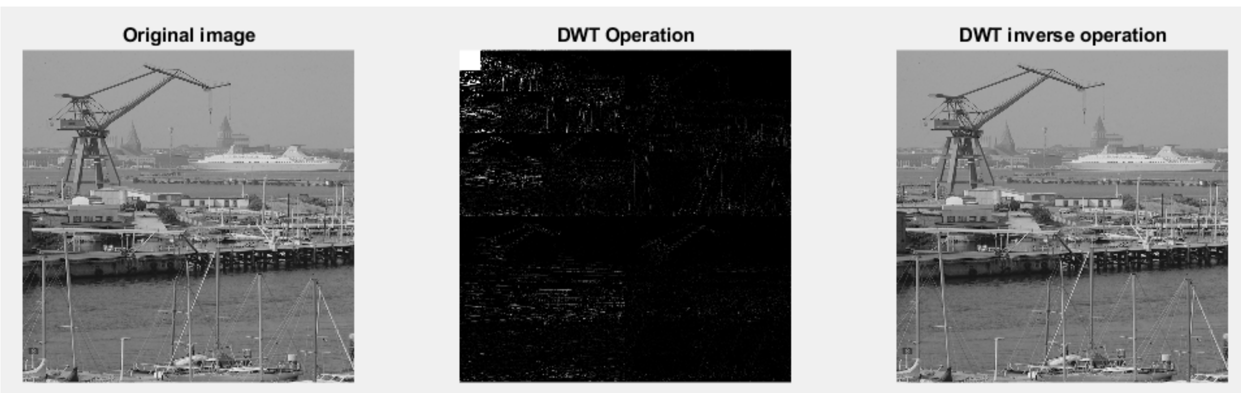

FIGURE 7 – Wavelet coefficients for db4 scale 4 of the image harbour.

Here we can see that we were successful in reconstructing the image using our 1-dimensional two-bank filter banks for our two-dimensional FWT. The biorthogonal filters allow us to reconstruct the image perfectly.

3.3 Uniform Quantizer

Uniform Quantization is applied for the wavelet coefficients obtained after the recursive analysis filter banks. We will use the same quantization function which is shown in FIG.3.

3.4 Distortion & Bit-Rate Estimation

Like in the DCT part we will analyze the performance of the compression for different quantization steps.

Power to Signal Noise Ratio (PSNR) is used to measure the quality of the reconstructed image where d represents the mean square error between the original image and the reconstructed image. Bit-rate required to encode the wavelet coefficients are assumed to be ideal code-word length of VLC that encodes each coefficient individually. However, each sub-band is encoded individually to exploit the mathematical characteristics within each sub-bands and thus, the entropy of each sub-band is normalized based on its contribution in the overall wavelet coefficients.

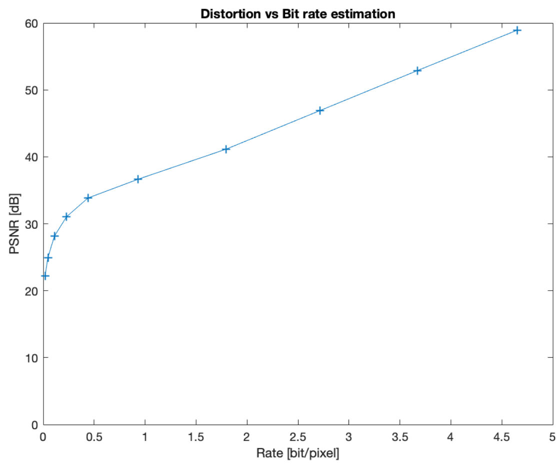

FIGURE 8 – PSNR vs Bit-Rate for "peppers512x512.tif"

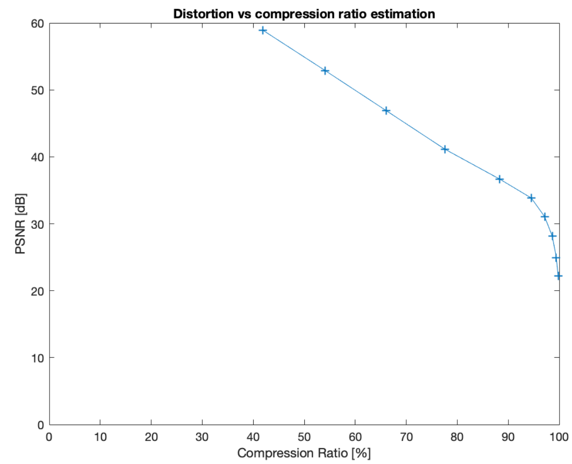

FIGURE 9 – PSNR vs Compression ratio for "peppers512x512.tif"

FIG.8 depicts the relationship between measured PSNR and encoded bit-rate. It was observed that the more the distorted the image is, the less bit-rate is required to store the compressed image. This is because the details in the image are lost during the compression process, and hence we need fewer bits to store the information.

FIG.9 helps to understand the trade-off between the distortion and the compression ratio.

4 Comparison between DCT and FWT

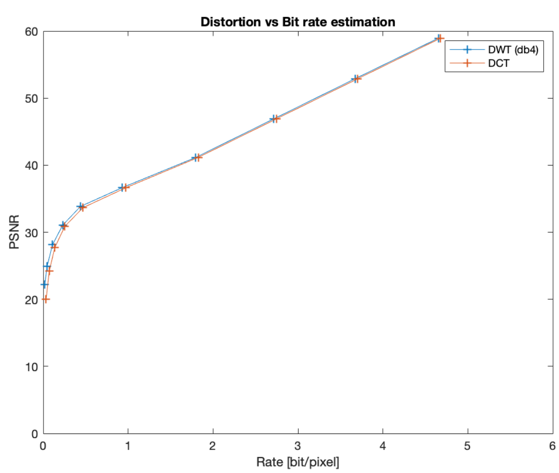

FIGURE 10 – PSNR vs Bit-Rate curve for DCT & DWT for "peppers512x512.tif"

FIGURE 11 – DCT and FWT for the step size 512 for "peppers512x512.tif"

By using the same image *peppers512x512.tif*, we compare the performance of the compression using the DCT operation and the FWT operation.

From FIG.10, we can see that the DCT and FWT operation have nearly the same performance concerning the rate-distortion trade-off. The difference between the two operations is the type of distortion in the image. The quantization mainly eliminates the high frequency in a natural image and depending on the applied operation we have different distortions.

The DCT is block-based so the distortion is located on the block, this is why on FIG.11 we can see blocks of uniform colour for the DCT transformed image. Regarding the FWT operation, this transform is sub-band based and is applied to the overall image. The quantization eliminates the high-frequencies of the image, this is why we have a feeling of a blurred image in the FWT transformed image.

5 Conclusion

To summarize, we carried out DCT and FWT based image compression for the set of test images. In each case, we analyzed the relationship between the distortion and the estimated bit-rate. In order to compare both these techniques, a test image is considered, DCT and FWT methods were applied separately and estimated Bit-rate vs PSNR curves are superposed. Figure 8 shows the curves. From the graph, we can infer that DCT and FWT differ marginally. DCT image is more widely adopted compression technique because of its compatibility with most of the software in the world, i.e. JPEG vs JPEG 2000.

Besides, we experimented out DCT and FWT for the same test image but with the quantization step size of 512. Then, the images are reconstructed and shown in FIG.11. From observation, DCT reconstructed image can be observed to be a low contrast image while FWT can be observed as a blurred image. This is evident as DCT is spatial operation while in FWT, we separate low and high frequencies.

In conclusion, the goal to demonstrate the two different technique to compress image namely DCT and FWT has been accomplished. Successfully image transformation was performed with the respective technique, we applied quantizer before attempting to reconstruct the image.

6 Appendix

6.1 Who Did What

- Yann Debain : Analyzed the FWT-based image compression and successfully implemented 1D Analysis filter bank, Synthesis filter bank and 2D FWT was realized on MATLAB, including bit-rate and PSNR curve. Contributed to the report.

- Harsha HN : Analyzed the DCT-based image compression and successfully implemented DCT transformation and reconstruction of the image including bit-rate and PSNR curve. Further contributed to the report.

- Tan Tian Fu : Analyzed DCT and Synthesis filter bank contributed to its implementations, even fixed errors and contributed to the report.

6.2 Matlab code

Enclosed.

6.3 References

[1] R. C. Gonzales and R.E. Woods, Digital Image Processing, Prentice Hall, Upper Saddle River, New Jersey, third edition, 2008.

[2] Digital image processing using MATLAB / Rafael C. Gonzalez, Richard E. Woods, Steven L. Eddins