Coefficients and Identities
precisely the number of ways
a set of n items. $\binom{n}{k}$
$C(n,k)$
binomial coefficients
expansion $(x+y)^3$
$j = 0$ till $j = n$.
angle

Homework 27

N1b. $(x+y)^5$

$$(x+y)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4y + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}xy^4 + \binom{5}{5}y^5 = 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$$

N4. $x^5y^8 = (x+y)^{13}$

$$\binom{n}{j}x^{n-j}y^j \rightarrow \binom{13}{8} = \frac{13!}{(13-8)!} = \frac{13!}{8!\,5!} = \frac{13\times 12\times 11\times 10\times 9}{5\times 4\times 3\times 2\times 1} =$$

$$= 13 * 11 * 9 = 1287.$$

N6. x^7 in $(1+x)^{11}$

$$\sum_{j=0}^{11}\binom{11}{j}x^j \rightarrow \binom{11}{7} = \frac{11!}{7!(11-7)!} = \frac{11!}{7!\,4!} =$$

$$= \frac{11\times 10\times 9\times 8}{4\times 3\times 2\times 1} = 11\times 10\times 3 = 330$$

N12. $\binom{10}{k}$, $0 \le k \le 10$

1 10 45 120 210 252 210 120 45 10 1.

1 11 56 165 330 462 462 330 55 11 1.

N15 $\binom{n}{k} \le 2^n$, $0 \le k \le n$

$$\binom{n}{k} \le \sum_{i=0}^{n}\binom{n}{i} = 2^n$$

N21

Method 1 (LHS): $k \times \binom{n}{k}$

Method 2 (RHS): $n \times \binom{n-1}{k-1}$

Conclusion: $k\binom{n}{k} = n\binom{n-1}{k-1}$