HomeWork Week 5

Problem N1

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \cdots \infty$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

Let's assume that e is rational $e = \frac{p}{q}$

$$x = q!\left(e - \sum_{n=0}^{q} \frac{1}{n!}\right)$$

$$x = q!\left(\frac{p}{q} - \sum_{n=0}^{q} \frac{1}{n!}\right) = q!\left(\frac{p}{q}\right) - \sum_{n=0}^{q} \frac{q!}{n!} = a(b-1)! - \sum_{n=0}^{q} \frac{q!}{n!}$$

$$x = q!\left(\sum_{n=0}^{\infty} \frac{1}{n!} - \sum_{n=0}^{q} \frac{1}{n!}\right) = \sum_{n=q+1}^{\infty} \frac{q!}{n!}$$

$$\frac{q!}{n!} = \frac{q \cdot (q-1) \cdot (q-2) \cdots}{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots (q+1) \cdot q \cdot (q-1) \cdots} =$$

$$= \frac{1}{n(n-1)(n-2) \cdots (q+1)} \le \frac{1}{(q+1)^{n-q}}$$

$$0 < x = \sum_{n=q+1}^{\infty} \frac{q!}{n!} < \sum_{k=1}^{\infty} \frac{1}{(q+1)^k} = \frac{\frac{1}{q+1}}{1 - \frac{1}{q+1}} = \frac{1}{q} < \underline{1}$$

Problem №2

$a^2 - b^2 = (a-b)(a+b)$

$n = 11 \rightarrow a = 6; \ b = 5$

$n = 2k + 1$

$a = k + 1; \ b = k$

$n = 2k + 1 \rightarrow (k+1)^2 - k^2 = k^2 + 2k + 1 - k^2 = 2k + 1 = n$

Problem №3

r - rational num   i - irrational num

$s = r + i \rightarrow s =$ rational

If $s = a/b$ and $r = c/d$ where a, b, c and d = integers

$b \neq 0, \ d \neq 0 \rightarrow s + (-r) = (ad - bc)/(bd)$

So $s + (-r)$ is a rational number.

But $s + (-r) = r + i - r = i$ forcing us to conclusion that i is rational. This contradicts our hypothesis that i is irrational. Therefore the assumption that s was rational was incorrect, and we conclude, that s is irrational.

Problem №4

$\sqrt{2}$ - irrational number $\rightarrow$ 2 is rational.

This counterexample refutes the proposition.

Problem №5

$q + r \geq p$

$q = x \geq 1 \quad \rightarrow \quad q = x < 1 \quad \rightarrow q - r \leq 2$

$r = y \geq 1 \quad \rightarrow r = y < 1$

$p \geq 2$

Problem №6

a) if n is odd $\rightarrow n^3 + 5 =$ even

Then $n = 2k+1 \rightarrow n^3 + 5 = (2k+1)^3 + 5 = 8k^3 + 12k^2 + 6k$

$+6 = 2(4k^3 + 6k^2 + 3k + 3)$. Thus n^3+5 is two times integer, so it's even.

b) n^3+5 is odd ; $n =$ is odd

$(n^3+5) - n^3 =$ even $\neq 5$

Problem №7

$U =$ all men in community

$S = U \rightarrow \{T, F\}$

$\forall x \in U: (\neg S(x)) \leftrightarrow x$ is shaved by barber

$\forall x \in U: (\neg S(x)) \leftrightarrow B(x)$

$B(b) \leftrightarrow S(b)$

$S(b) \to B(b) \leftrightarrow (\neg S(b))$

$\neg((p \leftrightarrow q) \wedge (q \leftrightarrow r)) \to (p \leftrightarrow r)$

$S(b) \leftrightarrow (\neg S(b))$

Proplem №8

$x \neq$ even $y \neq$ even

$x = 2m+1$

$xy =$ odd $x+y =$ odd

i) $y =$ even. $y = 2n$, so $x+y = (2m+1) + 2n = 2(m+n)+1 =$ odd

ii) $y =$ odd. $y = 2n+1$. so $xy = (2m-1)(2n+1) = 4mn+2m+2n+1 =$
$= 2(2mn+m+n)+1 =$ odd

$2k^2 + 6k +$

times