HomeWork N 22

N1.

1) Basis Step: $P(1)$ and $P(2)$ are true. (Given).

2) Inductive Step: Let's assume ~~that~~ all steps between 1. till k are true $\rightarrow 1 \leq j \leq k$ where $k \geq 2$. We want to prove that $P(k+1)$ is true. To do that we can use $k-1$ since it's still in range so $j = (k-1)+2 = \underline{k+1}$. This ~~me~~ means that $P(k+1)$ is true.

3) By strong induction, $P(n)$ is true for all integers $n \geq 1$

N3.

a) Basis Step: $P(3)$ and $P(5)$ are true. Given.

~~Inductive Step: We need to prove that $P(n)$ is true when $n \geq 8$, $3 \leq j \leq k$ where $k \geq 8$.~~

$P(8)$: $(1 \cdot 3 + 1 \cdot 5) \rightarrow$ True

$P(9)$: $(3 \cdot 3) \rightarrow$ True

$P(10)$: $(2 \cdot 5) \rightarrow$ True.

b) Inductive Step: we need to prove that $P(n)$ is true for all integers j meaning $8 \leq j \leq k$ where $k \geq 10$.

c) That any ~~$k+j$~~ 1 can be formed using only 3 and 5 cent stamps. $3x + 5x = j \leq k$. wher $k \geq 10$.

d) we know that $k-2 \geq 8$ when $k \geq 10$. which means $P(k-2)$ is true. That means that we can $(k-2)+3 = k+1$ which proves that $P(k+1)$ is true.

e) Since $P(n)$ is true for $n = 8, 9, 10$. And we also have that all j between 8 and $k \geq 10$ is true we have a ground to prove $P(k+1)$ true like $(k-2)+3 = k+1$.

N7. $\$2$ and $\$5$.

~~$\leq j \leq k$ where $k \geq 7$.~~

1: No

2: Yes (1×2)

3: No

4: Yes (2×2)

5: Yes (1×5)

6: Yes $(2 \cdot 3)$

7: Yes $(1 \cdot 2 + 1 \cdot 5)$

$\therefore$ all amounts $n \geq 4$

· Basis Step: $P(4)$ is true because $\overset{4}{(2 \cdot 2)}$ and $P(5)$ is true because $(1 \cdot 5) = 5$

· Inductive Step: We want to prove that $P(k+1)$ is also true. $4 \leq j \leq k$ where $k \geq 5$. $(k-1)+2 = k+1$ which proves that $P(k+1)$ is true.

N10

1) Basis Step: $P(1)$ is true since $1-1=0$ which means we broke entire chocolate.

2) Inductive Step: We need to prove that $P(k+1)$

is true. $1 \le j \le k$ where $k \ge 1$. Which means we must have two rectangles each time we breake j_1 and j_2 $\to j_1 + j_2 = k+1$.

Total Breaks: $1 + (j_1 - 1) + (j_2 - 1) = 1 + j_1 + 1 + j_2 - 1 = (j_1 + j_2) - 1 \to k+1$.

N12.

When n is even.

1) Basis Step: $P(1)$ is true because $2^0 = 1$.

2) Inductive Step: We need to prove $\overset{k+1=2m}{\cancel{P(k+1)}/2}$ is true for all positive even integers.

Since $k \ge 1$, $k+1 \ge 2$, so $m \ge 1$. Also $m = (k+1)/2 \le$ $\le (k+k)/2 = k$ (for $k \ge 1$). So $m = \sum_{i \in S} 2^i \to$ $\to k+1 = 2m = 2\left(\sum_{i \in S} 2^i\right) = \sum_{i \in S} 2^{i+1}$. Let's $S' = \{i+1 \mid i \in S\}$

Since $i \ge 0 \Rightarrow i+1 \ge 1$ which makes $P(k+1)$ true.

When n is odd.

2) Inductive Step: $k+1 = 2m+1$ for some integer m.

Since $k+1 \ge 1 \to 2m \ge 0$, so $m \ge 0$.

if $m = 0 \to k+1 = 1 \to 2^0 = 1$

if $m \ge 1 \to k/2 \to m \le k/2 \le k \to 1 \le m \le k$.

$m = \sum_{i \in S} 2^i \to k+1 = 2m+1 = 2\left(\sum_{i \in S} 2^i\right) + 2^0 = \left(\sum_{i \in S} 2^{i+1}\right) + 2^0$.

Since $2^0 \notin 2^{i+1}$ for $i \in S'$, we have $i \ge 0 \Rightarrow$

$\Rightarrow i+1 \ge 1 \to 2^{i+1} \ge 2$.

N37

Suppose $a = dq_1 + r_1$ with $0 \le r_1 < d$ and

$a = dq_2 + r_2$ with $0 \le r_2 < d$. Then $q_1 = q_2$ and

$r_1 = r_2$.

$(dq_1 + r_1) - (dq_2 + r_2) = 0 = d(q_1 - q_2) + (r_1 + r_2)$.

$r_2 - r_1 = d(q_1 - q_2) = -d + 0 < r_2 - r_1 < 0 + d$ or

~~$d($~~ $-d < r_2 - r_1 < d$. So $r_2 - r_1$ is betwen $-d$ and d

$r_2 - r_1 = 0$ ~~and~~ which means $r_2 = r_1$. which also

implies that $d \neq 0$ since it's positive integer.

$q_1 - q_2 = 0$ which $q_1 = q_2 \Rightarrow q$ and r are unique.