# HomeWork №1

№1a

1. ~~Basi~~ State $P(n)$: The train stops at the all ($n$th) stations.

2. Basis ~~Step~~: We need to show $P(1)$ is true which it is.

3. Inductive Step:

   • IH: Assuming that $P(k)$ is true ~~for~~ all arbitrary positive integers.

   • Goal: We need to ~~prove~~ show that $P(k+1)$ is also true.

   • Prove $P(k+1)$: Problem states „if a train stops at current station, it will stop at the next one" meaning that $P(k+1)$ is true since $P(k)$ is true.

4. Conclusion $P(n)$ is true for all positive integers $n$.

№3a, b, c, d, e and f.

a) $P(1)$ is the problem statement, $1^2+2^2+\ldots n^2 = n(n+1)(2n+1)/6$ for the positive integers $n$.

b) $P(1) \Rightarrow 1^2 \cancel{+1} = 1(1+1)(2\cdot 1+1)/6 \Rightarrow \cancel{2}\,1 = 2\cdot 3/6 \Rightarrow$
$\rightarrow \cancel{2}\,1 = 1$, $P(1)$ is true.

c) We assume that $P(k)$ is true, meaning $1^2+2^2\ldots k^n = k(k+1)(2k+1)/6$

d) We need to prove $P(k+1)$ is also true in inductive step (hypothesis) assuming that $P(k)$ is true.

$1^2+2^2\ldots k^2 = k(k+1)(2k+1)/6 = 1^2+2^2+(k+1)^2 = (k+1)((k+1)+1)((2(k+1)+1))/6. \rightarrow 1^2+1^2+2^2+\ldots \cancel{k}\,k^2+(k+1)^2 =$

$=(k+1)(k+2)(2k+3)/6$

e)

Proof $P(k+1)$:

$P(k+1) \Rightarrow LHS \Rightarrow 1^2+2^2+\dots(k+1)^2 = (1^2+2^2+\dots k^2)+(k+1)^2$

$(k(k+1)(2k+1)/6)+(k+1)^2 = (k+1)(k+2)(2k+3)/6.$

$\cancel{(k+1)\cdot(2k^2+k+6k+6)}\ (k+1)\cdot(k(2k+1)/6+(k+1))6$

$(k+1)\cdot(k(2k+1)/6+6(k+1)/6) = (k+1)(k(2k+1)+6(k+1))/6$

$(k+1)\cdot(2k^2+k+6k+6)/6 = (k+1)\cdot(2k^2+7k+6)/6$

$2k^2+7k+6: (2k+3)(k+2) = (k+1)\cdot((k+2)(2k+3))/6 =$
$=(k+1)(k+2)(2k+3)/6$

f) 1. Bases Step + Inductive step $\to P(1) \Rightarrow P(k) = P(k+1)$

N5

1. State $P(n)$: $1^2+3^2+5^2+\dots(2n+1)^2 = (n+1)(2n+1)(2n+3)/3$ whenever $n$ is a ~~noninteger~~ nonnegative integer.

2. Basis Step: $P(0)$ is true since $n$ is nonnegative integer.

$(2\cdot0+1)^2 = (0+1)(2\cdot0+1)(2\cdot0+3)/3 = 3/3 = 1$

3. Inductive Step:

• IH: Assume that $P(k)$ is true for an arbitorary nonnegative integer $k$.

• Goal: We need to show that $P(k+1)$ is also true.

$\cancel{1^2+3^2+5^2+\dots+(2k+1)^2=(k+1)(2k+1)(2k+3)/3}$

$1^2+3^2+5^2+\dots+(2k+1)^2+(2(k+1)+1)^2=((k+1)+1)(2(k+1)+1)(2(k+1)+3)/3. \to 1^2+3^2+5^2+\dots+(2k+1)^2+(2k+3)^2=$
$=(k+2)(2k+3)(2k+5)/3$

· Proof $P(k+1)$:

$1^2+3^2+5^2+\dots(2k+1)^2+(2k+3)^2=(1^2+3^2+5^2+\dots+(2k+1)^2)+$
$+(2k+3)^2$

$[(k+1)(2k+1)(2k+3)/3]+(2k+3)^2$

$(2k+3)\cdot[(k+1)(2k+1)/3+(2k+3)]$

$(2k+3)\cdot[(k+1)(2k+1)/3+3(2k+3)/3]=(2k+3)\cdot[(k+1)(2k+1)+3(2k+3)]/3.$

$(2k+3)\cdot[(2k^2+k+2k+1)+(6k+9)]/3=(2k+3)\cdot[2k^2+3k+1+6k+9]/3=(2k+3)\cdot[2k^2+9k+10]/3.$

$(2k+5)(k+2)=(2k+3)[(k+2)(2k+5)]/3=(k+2)(2k+3)(2k+5)/3.$

· Conclusion: $P(k)$ is true $\to$ $P(k+1)$ is also true

## №6

1. $P(n) = 1\cdot 1! + 2\cdot 2! + \ldots + n\cdot n! = (n+1)! - 1$ for pos

2. $P(1) = 1\cdot 1! = 1\cdot 1 = 1 \rightarrow (1\cdot 1)! - 1 = 2! - 1 = 2 - 1 = 1$

3. $\cdot$ IH: $P(k) \rightarrow 1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k! = (k+1)! - 1$

$\cdot$ Goal: $P(k+1)$ is true $\rightarrow$ $1\cdot 1! + 2\cdot 2! + \ldots + (k+1)\cdot(k+1)! = ((k+1)+1)! - 1 \rightarrow$

$1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k! + (k+1)(k+1)! = (k+2)! + 1$

$1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k! +$

$\cdot$ Proof of $P(k+1)$.

$1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k! + (k+1)(k+1)! = (1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k!) +$

$+ (k+1)(k+1)!$.

$(k+1)! + (k+1)\cdot(k+1)! - 1 = (k+1)!\cdot[1 + (k+1)] - 1 =$

$= (k+1)!\cdot(k+2) - 1 = (k+1)!(k+2) = (k+2)! = (k+2)! - 1$.

## №8.

1. $P(n) = 2\cdot 2\cdot 7 + 2\cdot 7^2 - \ldots + 2\cdot(-7)^n = (1-(-7)^{n+1})/4$

2. $P(0) = 2\cdot(-7)^0 = 2\cdot 1 = 2. = (1-(-7)^{0+1})/4$. $P(0)$ is true.

3.

$\cdot$ IH: $P(k) = 2\cdot 2\cdot 7 + 2\cdot 7^2 - \ldots + 2(-7)^k = (1-(-7)^{k+1})/4$

$\cdot$ Goal: $P(k+1) = 2 - 2\cdot 7 + 2\cdot 7^2 - \ldots + 2\cdot(-7)^k +$

$+ 2\cdot(-7)^{k+1} = (1-(-7)^{k+2})/4$

Proof of $P(k+1)$. $2-2\cdot 7+2\cdot 7^2-\ldots+2\cdot(-7)^k+2\cdot(-7)^{k+1}=$

$=(2\cdot 2\cdot 7+2-7^2-\ldots+2(-7)^k)+2(-7)^{k+1}=[(1-(-7)^{k+1})$

$/4]+2(-7)^{k+1}=[1-(-7)^{k+1}]/4+[8\cdot(-7)^{k+1}]/4=$

$=[1-(-7)^{k+1}+8(-7)^{k+1}]/4=\cancel{(-7)^{k+1}}\ -1\cdot(-7)^{k+1}+$

$+8(-7)^{k+1}=7(-7)^{k+1}=-(-7)\cdot(-7)^{k+1}=-(-7)^{(k+1)+1}=$

$=-(-7)^{k+2}=[1+(-7)(-7)^{k+1}]/4=[1-(-7)^{(k+1)+1}(-7)]/4$

$=[(1-(-7)^{k+1})/4]+2(-7)^{k+1}=(1/4)-(1-4)\cdot(-7)^{k+1}+$

$+2(-7)^{k+1}=(1/4)+(-7)^{k+1}\cdot[2-(1/4)]=(1/4)+$

$+(-7)^{k+1}\cdot[8/4-1/4]=(1/4)+(-7)^{k+1}\cdot(7/4)=(1/4)\cdot$

$[1+7\cdot(-7)^{k+1}]=(1/4)\cdot[1-(-7)\cdot(-7)^{k+1}]=(1/4)\cdot$

$[1-(-7)^{(k+1)+1}]=(1-(-7)^{k+2})/4.$

N18.

a) $P(2)=2!<2^2$

b) $P(2)=|2!=2\cdot 1=2|=|2^2=4|\rightarrow 2<4$ is true

c) $P(k)\rightarrow k!<k^k,\ k>1$

d) $P(k+1)\rightarrow(k+1)!<(k+1)^{(k+1)}$

e) proof. $(k+1)!=(k+1)\cdot k!<(k+1)\cdot k^k=(k+1)^k(k+1)^1=$

$=(k+1)^k\cdot(k+1).$

f) Basis Step → Inductive Step.