Home Work Week 14

1.1

(13) Let p and q be the propositions

a) $\neg p$ b) $p \land \neg q$ c) $p \to q$

d) $\neg p \to \neg q$ e) $p \to q$ f) $q \land \neg p$

g) $q \to p$

(33) Construct a truth ~~tab~~ table

a)

p	q	$p \lor q$	$p \oplus q$	$(p \lor q) \to (p \oplus q)$
T	T	T	F	F
F	T	T	T	T
T	F	T	T	T
F	F	F	F	T

b)

p	q	$p \oplus q$	$p \land q$	$(p \oplus q) \to (p \land q)$
T	T	F	T	T
F	T	T	F	F
T	F	T	F	F
F	F	F	F	T

c)

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
F	T	T	F	T
T	F	T	F	T
F	F	F	F	F

d)

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
F	T	T	F	T	T
T	F	F	F	T	T
F	F	T	T	F	T

e)

p	q	r	$\neg p$	$\neg r$	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg r$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	F	F	T	T	F
F	T	T	T	F	F	F	F
T	F	T	F	F	F	T	T
T	T	F	F	T	T	F	T
F	F	T	T	F	~~F~~ T	F	T
F	T	F	T	T	F	T	T
T	F	F	F	T	F	F	F
F	F	F	T	T	T	T	F

e)

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
T	T	F	F	T	T
F	T	F	T	F	F
T	F	T	T	F	F
F	F	T	F	T	T

35. Construct a truth table

a)

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
F	T	F	T
T	F	T	T
F	F	T	T

b)

p	q	$\neg p$	$\neg p \leftrightarrow q$
T	T	F	F
F	T	T	T
T	F	F	T
F	F	T	F

c)

p	q
T	T
F	T
T	F
F	F

d)

p	q
T	T
F	T
T	F
F	F

e)

p
T
F
T
F

f)

p
T
F
T

c)

p	q	$\neg p$	$p \to q$	$\neg p \to q$	$(p \to q) \vee (\neg p \to q)$
T	T	F	T	T	T
F	T	T	T	T	T
T	F	F	F	T	T
F	F	T	T	F	T

d)

p	q	$\neg p$	$p \to q$	$\neg p \to q$	$(p \to q) \wedge (\neg p \to q)$
T	T	F	T	T	T
F	T	T	T	T	T
T	F	F	F	T	F
F	F	T	T	F	F

e)

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
T	T	F	T	F	T
F	T	T	F	T	T
T	F	F	F	T	T
F	F	T	T	F	T

f)

p	q	$\neg p \leftrightarrow \neg q$	$p \leftrightarrow q$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	T	T	T	T
F	T	F	F	T
T	F	F	F	T
F	F	T	T	T

(41) $(p \lor q \lor r) \rightarrow$ it will be T since there's $\lor$ (or) and it requires at least one of the variables ~~F~~ is T.

$(\neg p \lor \neg q \lor \neg r) \rightarrow$ same goes here.

1.3

(15) Is it a tautology? → Yes, it's a tautology.

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \land (p \rightarrow q)$	$(\neg q \land (p \rightarrow q)) \rightarrow \neg p$
T	T	F	F	T	F	T
F	T	T	F	T	F	T
T	F	F	T	F	F	T
F	F	T	T	T	T	T

(42) ?

(43) $\neg, \land, \lor$ ~~are~~ form a fuctionally complette collection of logical operators.

(45)

$p_1 \land p_2 \land \ldots \land p_n$, by removing $\land$ with De Morgan's law, we have the equivalent proposition $\neg(\neg p_1 \lor \neg p_2 \ldots \lor \ldots \lor \neg p_n)$.

(51) $p \rightarrow q \equiv \neg p \lor q \equiv ((p \downarrow q) \lor q) \equiv (((p \downarrow p) \downarrow p) \downarrow \downarrow ((p \downarrow p) \downarrow q))$.

with constant F

1.4

(37)

a) $\forall x((P(x,$

b) $\forall x(((M($

c) $M \rightarrow ((H($

d) $\exists x((T(x$

(53)

a) True

(59)

a) $\neg \forall x(P($

b) $\forall x(Q$

c) $\neg \forall x(P($

d) No, it

(61)

a) $\forall x($

b) $\forall x($

c) $\forall x(\neg$

d) $\forall x($

with constant F $\rightarrow F \downarrow ((F \downarrow p) \downarrow q))$.

1.4

37

a) $\forall x ((P(x, 25000)) \lor S(x, 25))$

b) $\forall x (((M(x) \land T(x, 3)) \lor (F(x) \land T(x, 3.5))) \rightarrow Q(x))$

c) $M \rightarrow ((H(60) \lor (H(45) \land T) \land \forall y\, G(B, y))$

d) $\exists x ((T(x, 21) \land G(x, 4.0))$

53

a) True b) False c) True

59

a) $\neg \forall x (P(x) \rightarrow Q(x))$ or $\forall x (P(x) \rightarrow \neg Q(x))$

b) $\forall x (Q(x) \rightarrow R(x))$

c) $\neg \forall x (P(x) \rightarrow R(x))$ or $\forall x ((P(x) \rightarrow \neg R(x))$

d) No, it doesn't follow.

61

a) $\forall x (P(x) \rightarrow \neg Q(x))$

b) $\forall x (R(x) \rightarrow \neg S(x))$

c) $\forall x (\neg Q(x) \rightarrow S(x))$

d) $\forall x (P(x) \rightarrow \neg R(x))$.

e) Yes, it follows.

1.5

(19)

a) $\forall x \forall y(((x<0) \wedge (y<0)) \rightarrow (x+y<0))$

b) $\neg \forall x \forall y(((x>0) \wedge (y>0)) \rightarrow (x-y>0))$

c) $\forall x \forall y(x^2+y^2 \geq (x+y)^2)$

d) $\forall x \forall y(|xy| = |x||y|)$

(21)

$\forall x \exists a \exists b \exists c \exists d((x>0) \rightarrow x = a^2+b^2+c^2+d^2)$

(31)

a) $\neg(\forall x \exists y \forall z T(x,y,z)) \equiv \exists x \neg \exists y \forall z T(x,y,z)$

b) $\neg(\forall x \exists y P(x,y) \vee \forall x \exists y Q(x,y)) \equiv \neg \forall x \exists y P(x,y) \wedge$
$\wedge \neg \forall x \exists y Q(x,y)$

c) $\neg(\forall x \exists y P(x,y) \wedge \exists z R(x,y,z)) \equiv \exists x \neg \exists y (P(x,y) \wedge$
$\wedge \exists z R(x,y,z))$

d) $\neg(\forall x \exists y (P(x,y) \rightarrow Q(x,y))) \equiv \exists x \neg \exists y (P(x,y)) \rightarrow$
$\rightarrow Q(x,y))$

(49)

a) if $\forall x P(x)$ is true and $\exists x Q(x)$ is true, which they are we can have $\forall x \exists y(P(x) \wedge Q(y))$.

b) Same here, ...

$\equiv \forall x \exists y(P(x) ...$

b) Same here, $\forall x P(x)$ ~~and~~ $\vee \exists x Q(x) \equiv$

$\equiv \forall x \exists y (P(x) \vee Q(y))$.