

that the $(n - \sqrt{n}) / \log n$ term dominates for $n \geq 4$,

and sum is $\Theta(n/\log n)$

and $\sum \lceil \sqrt{n} \rceil + n =$

$$\rightarrow T(n) = \sum \lceil \sqrt{n} \rceil \log(\lceil \sqrt{n} \rceil) + n =$$

upper bound

$$\begin{aligned} &= n \log(\lceil \sqrt{n} \rceil) + n = \\ &= n \log((1/2)\log n) + n = \\ &= n(\log(1/2) + \log \log n) + n = \\ &= -n + n \log \log n + n = n \log \log n \\ &\rightarrow T(n) = \Theta(n \log \log n) \end{aligned}$$

Homework 18

problem 1

- 1) $10/7 = 2(5) \rightarrow 10 = 7 \cdot 2 + 5 (2 = q_1, 5 = r_1)$
- 2) $-11/1 = -10/1 \rightarrow -11 = 11 + 10 (-11 = q_1, 10 = r_1)$
- 3) $489/23 = 34(7) \rightarrow 489 = 23 \cdot 34 + 7 (34 = q_1, 7 = r_1)$
- 4) $1001/13 = 77 \rightarrow 1001 = 13 \cdot 77 + 0 (77 = q_1, 0 = r_1)$
- 5) $0/10 = 0(0) \rightarrow 0 = 10 \cdot 0 + 0 (0 = q_1, 0 = r_1)$
- 6) $3/5 = 0(3) \rightarrow 3 = 5 \cdot 0 + 3 (0 = q_1, 3 = r_1)$
- 7) $-1/3 = 0(-1) = -1 = 3 \cdot (-1) + 2 (-1 = q_1, 2 = r_1)$
- 8) $4/1 = 4(0) = 4 = 1 \cdot 4 + 0 (4 = q_1, 0 = r_1)$

Problem N2

$$f(x) = \begin{cases} x \bmod m & \text{if } x \bmod m \leq \lfloor m/2 \rfloor \\ (x \bmod m) - m & \text{if } x \bmod m > \lfloor m/2 \rfloor \end{cases}$$

$$\begin{aligned} a \cdot a &\equiv b \\ a \cdot a^2 &\equiv b^2 \end{aligned}$$

2) Theorem

$$a = b \pmod{r}$$

$$a^2 = b^2 \pmod{r}$$

problem

$$2^5 = a + b$$

$$2^6 = a - b$$

$$a + b = 6$$

$$a - b = 0$$

$$a = 3$$

$$b = 3$$

$$a \cdot b = 9$$

$$p_n = 9$$

$$Q = 1$$

Problem N3

a) $a = 0, b = 2, c = 2 \pmod{4}$

~~$0 \cdot 2 = 2 \cdot 2 \pmod{4}$~~

~~$0 \equiv 2 \pmod{4}$~~

b) ~~$a = 0, b = 1, c = 2, d = 1 \pmod{4}$~~

~~$0 \equiv 1 \pmod{4}$~~

~~$2 \equiv 1 \pmod{4}$~~

~~$0^2 \equiv 1 \pmod{4}$~~

~~$a = 3, b = 3, c = 1, d = 1 \pmod{5}$~~

~~$3 \equiv 3 \pmod{5} \rightarrow 3$~~

~~$1 \equiv 6 \pmod{5} \rightarrow 1$~~

~~$3 \not\equiv 3 \pmod{5}$~~

Problem N4

i) Theorem

Since $a \equiv b \pmod{m}$

$$a \equiv b \pmod{m} \rightarrow a^k \equiv b^k \pmod{m} \rightarrow$$

$$a^k \equiv b^k \pmod{m} \rightarrow a^k \equiv b^k \pmod{m}$$

Theorem 11.1

$$\begin{aligned} a^k - b^k &= (a - b)(a^{k-1} + a^{k-2}b + \dots + ab^{k-2} + b^{k-1}) \\ a^k - b^k &\pmod{m} \rightarrow m \mid (a - b)(a^{k-1} - b^{k-1}) \rightarrow \\ a^k - b^k &\pmod{m} \rightarrow a^k \equiv b^k \pmod{m} \end{aligned}$$

Problem 5

$$Z_5 = \{0, 1, 2, 3, 4\}$$

$$0+4 \not\equiv (0+0) \pmod{4}$$

$$0+0 \equiv (0+0) \pmod{0}$$

$$0+0 \equiv (0+0) \pmod{1}$$

Problem 6

$$0+0 \equiv (0+0) \pmod{0}$$

$$0+1 = 1$$

$$1+1 = 2$$

$$4+4 = 8$$

$$4+4 = 3$$

$$1+0 = 1$$

$$1+1 = 0$$

$$0 \cdot 0 = 0 \quad 0 \cdot 1 = 0 \quad \dots \quad 0 \cdot 4 = 0$$

$$1 \cdot 0 = 0 \quad 1 \cdot 1 = 1 \quad \dots \quad 1 \cdot 4 = 4$$

$$\dots \quad 4 \cdot 0 = 4 \quad 4 \cdot 1 = 4 \quad \dots \quad 4 \cdot 4 = 1$$

if $a=1$ then $f(a)=a$ and $g(a)=0$

$f \rightarrow$ one-to-one and onto because $f(0) = f(1) = 0$
 $g \rightarrow$ not into, because it ranges in $\{0, 1, \dots, 4\}$
and it's not one-to-one because $g(0) = g(1) = 0$

Problem N7

a)

$$\begin{array}{r} 238 \\ - 230 \end{array} \left| \begin{array}{c} 2 \\ 15 \\ \hline 2 \end{array} \right.$$

①

$$\begin{array}{r} 57 \\ - 56 \end{array} \left| \begin{array}{c} 2 \\ 14 \\ \hline 2 \end{array} \right.$$

②

$$\begin{array}{r} 1 \\ - 28 \end{array} \left| \begin{array}{c} 2 \\ 12 \\ \hline 2 \end{array} \right.$$

③

$$\begin{array}{r} 0 \\ - 28 \end{array} \left| \begin{array}{c} 2 \\ 12 \\ \hline 2 \end{array} \right.$$

④

⊕ ⊕ ⊕ ⊕

9764

976

$$238 = 2 \cdot 1100111$$

The image shows a series of handwritten numbers and symbols arranged vertically on a grid background. The sequence begins with '282' on the left, followed by a minus sign, and a circle containing a minus sign. This pattern repeats several times: '153' (minus, circle minus), '215' (minus, circle plus), '100' (minus, circle plus), '150' (minus, circle plus), and ends with a large circle containing a plus sign at the bottom.

$1000 = 11101000$

A child's drawing on lined paper. The drawing consists of a vertical oval shape, possibly representing a cylinder or a tree trunk, with a horizontal line extending from its left side. A small, branching black line extends from the top of this horizontal line. Below the main vertical shape, the word "cat" is written in a cursive script. The entire drawing is done in black ink on white paper with horizontal ruling lines.

10 10 10
13 13 13
10 10 10
1 1 1
5 5 5
0 0 0
3 3 3
1 1 1
0 0 0

$\log_{10} 1000 = 3$

A hand-drawn diagram on grid paper featuring a series of numbered circles (1 through 10) connected by lines. The circles are arranged in two main horizontal rows. The top row contains circles 1 through 6, and the bottom row contains circles 7 through 10. Each circle is a simple black outline. A horizontal line connects circle 1 to circle 2. From circle 2, a line goes up to circle 3 and another line goes right to circle 4. From circle 4, a line goes up to circle 5 and another line goes right to circle 6. From circle 6, a line goes down to circle 7. From circle 7, a line goes right to circle 8. From circle 8, a line goes up to circle 9. From circle 9, a line goes right to circle 10. There is also a small arrow pointing downwards from the bottom of circle 6 towards circle 7.

Problem N8

a) $(111)_2 \rightarrow 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 =$

$= 16 + 8 + 4 + 2 + 1 = 34$

b) $(1010101)_2 \rightarrow 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 +$

$(1010101)_2 = 1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 =$

$+ 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 256 + 16 + 4 + 1 =$

$= 273$

c) $(1000000001)_2 \rightarrow 1 \cdot 2^9 + 0 \cdot 2^8 + \dots + 0 \cdot 2^1 + 1 \cdot 2^0 =$

$= 512 + 1 = 513$

d) $(110100010000)_2 = 1 \cdot 2^{14} + 1 \cdot 2^{13} + 0 \cdot 2^{12} +$

$+ 1 \cdot 2^{11} + \dots + 1 \cdot 2^8 + \dots + 1 \cdot 2^4 + \dots +$

$+ 2048 + 256 + 16 = 26896$

e) $(101010101)_2$

f) $(101010101)_2$

g) $(101010101)_2$

h) $(101010101)_2$

i) $(101010101)_2$

j) $(101010101)_2$

k) $(101010101)_2$

l) $(101010101)_2$

m) $(101010101)_2$

n) $(101010101)_2$

o) $(101010101)_2$

p) $(101010101)_2$

q) $(101010101)_2$

r) $(101010101)_2$

s) $(101010101)_2$

t) $(101010101)_2$

u) $(101010101)_2$

v) $(101010101)_2$

w) $(101010101)_2$

x) $(101010101)_2$

y) $(101010101)_2$

z) $(101010101)_2$

aa) $(101010101)_2$

ab) $(101010101)_2$

ac) $(101010101)_2$

ad) $(101010101)_2$

ae) $(101010101)_2$

af) $(101010101)_2$

ag) $(101010101)_2$

ah) $(101010101)_2$

ai) $(101010101)_2$

aj) $(101010101)_2$

ak) $(101010101)_2$

al) $(101010101)_2$

am) $(101010101)_2$

an) $(101010101)_2$

ao) $(101010101)_2$

ap) $(101010101)_2$

aq) $(101010101)_2$

ar) $(101010101)_2$

as) $(101010101)_2$

at) $(101010101)_2$

au) $(101010101)_2$

av) $(101010101)_2$

aw) $(101010101)_2$

ax) $(101010101)_2$

ay) $(101010101)_2$

az) $(101010101)_2$

ba) $(101010101)_2$

ca) $(101010101)_2$

da) $(101010101)_2$

ea) $(101010101)_2$

fa) $(101010101)_2$

ga) $(101010101)_2$

ha) $(101010101)_2$

ia) $(101010101)_2$

ja) $(101010101)_2$

ka) $(101010101)_2$

la) $(101010101)_2$

ma) $(101010101)_2$

na) $(101010101)_2$

oa) $(101010101)_2$

pa) $(101010101)_2$

qa) $(101010101)_2$

ra) $(101010101)_2$

sa) $(101010101)_2$

ta) $(101010101)_2$

ua) $(101010101)_2$

va) $(101010101)_2$

wa) $(101010101)_2$

xa) $(101010101)_2$

ya) $(101010101)_2$

za) $(101010101)_2$

ba) $(101010101)_2$

ca) $(101010101)_2$

da) $(101010101)_2$

ea) $(101010101)_2$

fa) $(101010101)_2$

ga) $(101010101)_2$

ha) $(101010101)_2$

ia) $(101010101)_2$

ja) $(101010101)_2$

ka) $(101010101)_2$

la) $(101010101)_2$

ma) $(101010101)_2$

na) $(101010101)_2$

oa) $(101010101)_2$

pa) $(101010101)_2$

qa) $(101010101)_2$

ra) $(101010101)_2$

sa) $(101010101)_2$

ta) $(101010101)_2$

ua) $(101010101)_2$

va) $(101010101)_2$

wa) $(101010101)_2$

xa) $(101010101)_2$

ya) $(101010101)_2$

za) $(101010101)_2$

ba) $(101010101)_2$

ca) $(101010101)_2$

da) $(101010101)_2$

ea) $(101010101)_2$

fa) $(101010101)_2$

ga) $(101010101)_2$

ha) $(101010101)_2$

ia) $(101010101)_2$

ja) $(101010101)_2$

ka) $(101010101)_2$

la) $(101010101)_2$

ma) $(101010101)_2$

na) $(101010101)_2$

oa) $(101010101)_2$

pa) $(101010101)_2$

qa) $(101010101)_2$

ra) $(101010101)_2$

sa) $(101010101)_2$

ta) $(101010101)_2$

ua) $(101010101)_2$

va) $(101010101)_2$

wa) $(101010101)_2$

xa) $(101010101)_2$

ya) $(101010101)_2$

za) $(101010101)_2$

ba) $(101010101)_2$

ca) $(101010101)_2$

da) $(101010101)_2$

ea) $(101010101)_2$

fa) $(101010101)_2$

ga) $(101010101)_2$

ha) $(101010101)_2$

ia) $(101010101)_2$

ja) $(101010101)_2$

ka) $(101010101)_2$

la) $(101010101)_2$

ma) $(101010101)_2$

na) $(101010101)_2$

oa) $(101010101)_2$

pa) $(101010101)_2$

$$\begin{aligned}
 & 2^0 = 1010101010 \rightarrow 682, (11110000)_2 \rightarrow 496 \\
 & 2^1 = 10101010111110000 = 1001000110101010 (1148) \\
 & 2^2 = 10101010111110000000 \rightarrow 513, (11111111)_2 \rightarrow 1023 \\
 & 2^3 = 10101010111111111 = 1100000000 (1586) \\
 & 2^4 = 101010100001 + 111111111 = 10000000011111111 (5244983)
 \end{aligned}$$

$$\begin{aligned}
 & 1 + 1 \cdot 2^0 = \text{problem 1110} \\
 & 2 + 2 \cdot 2^1 = 2 \cdot 11 \rightarrow 2^3 \cdot 11 \rightarrow 6 \\
 & 2 + 2 \cdot 2^2 = 2 \cdot 3^2 \cdot 7 \rightarrow 3 \cdot 243 \Rightarrow 3 \cdot 3 \cdot 81 \rightarrow 3 \cdot 3 \cdot 27 \rightarrow 3 \cdot 3 \cdot 9 \rightarrow \\
 & 2 + 2 \cdot 2^3 = 2 \cdot 3^3 \cdot 13 \rightarrow 3 \cdot 3 \cdot 3 \cdot 3 \rightarrow 3^6 \\
 & 2 + 2 \cdot 2^4 = 2 \cdot 3^4 \cdot 13 \rightarrow 4 \cdot 143 \rightarrow 4 \cdot 11 \cdot 13
 \end{aligned}$$

Problem 1111

$$\begin{aligned}
 \text{Number of zeroes} &= \left[\frac{100}{5} \right] + \left[\frac{100}{5^2} \right] + \left[\frac{100}{5^3} \right] + \dots \\
 &= \left[\frac{100}{25} \right] + \left[\frac{100}{125} \right] + \dots
 \end{aligned}$$

$$\begin{aligned}
 \left[\frac{100}{5} \right] &= 20 \\
 \left[\frac{100}{25} \right] &= 4 \\
 \left[\frac{100}{125} \right] &= 0
 \end{aligned}$$

$$20+4 = \textcircled{24}$$

Problem N 12

- a) $\gcd(12, 18) \rightarrow \gcd(12, 6) \rightarrow \gcd(6, 0) = 6$
- b) $\gcd(111, 201) \rightarrow \gcd(111, 90) \rightarrow \gcd(90, 21) \rightarrow \gcd(6, 3) \rightarrow \gcd(3, 0) = 3$
- c) $\gcd(1001, 1331) \rightarrow \gcd(1001, 330) \rightarrow \gcd(330, 11) \rightarrow \gcd(11, 0) = 11$
- d) $\gcd(12345, 54321) \rightarrow \gcd(12345, 4841) \rightarrow \gcd(4841, 2463) \rightarrow \gcd(2463, 15) \rightarrow \gcd(15, 3) \rightarrow \gcd(3, 0) = 3$
- e) $\gcd(1000, 5000) \rightarrow \cancel{1000}, \rightarrow \gcd(1000, 40) \rightarrow \gcd(40, 0) = 40$
- f) $\gcd(9886, 6060) \rightarrow \gcd(6060, 3828) \rightarrow \gcd(3828, 2232) \rightarrow \gcd(2232, 1596) \rightarrow \gcd(1596, 636) \rightarrow \gcd(636, 324) \rightarrow \gcd(324, 312) \rightarrow \gcd(312, 12) \rightarrow \gcd(12, 0) = 12$