$K2 \rightarrow \binom{5}{2} (2x)^{5-2} \cdot 3y^2 = 10 \cdot 8x^3 \cdot 9y^2 = 720x^3y^2$

$K3 \rightarrow \binom{5}{3} (2x)^{5-3} \cdot 3y^3 = 10 \cdot 2x^2 \cdot 27y^3 = 1080x^2y^3$

$K4 \rightarrow \binom{5}{4} (2x)^{5-4} \cdot 3y^4 = 5 \cdot 2x \cdot 81y^4 = 810xy^4$

$K5 \rightarrow \binom{5}{5} 2x^{5-5} \cdot 3y^5 = 1 \cdot 1 \cdot 243y^5 = 243y^5$

Problem 19

$$6x^3 + 11x^2 - 31x + 15 \;\big|\; 3x - 2$$

$-(6x^3 + 4x^2)$ $\qquad 2x^2 + 5x + 7$

$15x^2 - 31x + 15$

$-(15x^2 - 10x)$

$21x + 15$

$-(21x + 14)$

1

Problem 20

$f(x) = 2x^4 - 5x^3 + x^2 - 4$

Constant term $= (-4)$ $\quad p = \pm 1, \pm 2, \pm 4$

Leading coefficient $= 2$ $\quad q = \pm 1, \pm 2$

$\frac{p}{q} = \pm\frac{\cancel{1}}{\cancel{1}}; \pm\frac{1}{2} \qquad \frac{p}{q} = \pm\frac{\cancel{2}}{\cancel{1}}; \pm\frac{\cancel{2}}{\cancel{2}} \qquad \frac{p}{q} = \pm\frac{4}{\cancel{1}}; \pm\frac{\cancel{4}}{\cancel{2}}$

$\frac{p}{q} = \pm 1; \pm 2; \pm 4; \pm\frac{1}{2}$

HW2

Problem №1

$S = 5 \overset{4}{+} 9 \overset{4}{+} 13 \dots + 89$ in sigma notation

$a_1 = 5$

$d = 4$

$a_n = a_1 (n-1)d$

$89 = 5(n-1)4$

$89 - 5 = (n-1)4$

$84 = 4(n-1) \quad | : 4$

$21 = n - 1$

$n - 1 = 21$

$n = 21 + 1$

$n = 22$

$$S = \sum_{k=1}^{22} \left(5 + (k-1) \cdot 4\right)$$

HW2

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Problem N2

Rewrite $\sum_{k=3}^{15}(2k+1)$ ~~to~~ to start at $k=1$

$$\sum_{k=3}^{15}(2k+1) = 2\cdot 3+1 = 7$$

$+2$

$2\cdot 4+1 = 9$

$+2$

$2\cdot 5+1 = 11$

$$\sum_{k=1}^{15} = (2(k+2)+1) \rightarrow \text{my solution}$$

$$\sum_{j=1}^{13} = [2(j+2)+1] = \sum_{j=1}^{13} = 2j+5$$ – correct solution

Problem N3

$a_1 = 12$; $a_{10} = 57$; $d = ?$; $a_{25} = ?$

$a_n = a_1 + (n-1)d$

$$d = \frac{a_y - a_x}{y - x}$$

$$d = \frac{57-12}{10-1} = \frac{45}{9} = 5$$

$$a_{25} = 12 + (25-1)5 = 12 + 120 = 132$$

Problem №2

Rewrite $\sum_{k=3}^{15}(2k+1)$ ~~to~~ start at $k=1$

$$\sum_{k=3}^{15}(2k+1) = 2\cdot 3+1 = 7$$

$+2$

$$2\cdot 4+1 = 9$$

$+2$

$$2\cdot 5+1 = 11$$

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$$a_{25} = 12 + (25-1)5 = 12 + 120 = 132$$

Problem 4,

$a_1 = 7 \cdot 15 = 105$

$a_n = 7 \cdot 142 = 994$

$n = \frac{a_n - a_1}{d} + 1 = \frac{994 - 105}{7} + 1 = 128$

$S = \frac{n}{2}(a_1 + a_n) = \frac{128}{2}(105 + 994) = 64 \cdot 1099 = 70336$

Problem N5,

$S = \sum_{k=1}^{n} (3k+2) \quad S = 2650 \quad n = ?$

$S = \frac{n}{2}(a_1 + a_n)$

$a_1 = 3 \cdot 1 + 2 = 5$

$a_n = 3n + 2$

$S = \frac{n}{2}(5 + (3n+2)) = \frac{n}{2}(3n+7)$

$\frac{n}{2}(3n+7) = 2650$

$n(3n+7) = 5300$

$3n^2 + 7n - 5300 = 0$

$n = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 3 \cdot (-5{,}300)}}{2 \cdot 3}$

$n = \frac{-7 \pm \sqrt{49 + 63600}}{6}$

$$n = \frac{-7 \pm \sqrt{63\,849}}{6}$$

$$\sqrt{63849} = 252.29$$

$$n = \frac{-7 + 252.29}{6} = \frac{245.29}{6} \approx 40.88$$

$$n = 41$$

$$S = \frac{n}{2}(a_1 + a_n) = \frac{41}{2}\left(5 + (3 \cdot 41 + 2)\right) = \frac{41}{2}(5 + 125) =$$

$$= \frac{41}{2} \cdot 130 = 41 \cdot 65 = 2665 \quad \times$$

$$S = \frac{40}{2}\left(5 + (3 \cdot 40 + 2)\right) = 20 \cdot 127 = 2540 \quad \times$$

$$S = \frac{42}{2}\left(5(3 \cdot 42 + 2)\right) = 21 \cdot 133 = 2793 \quad \times$$

$S = 2650$ is not achivable with integer n.

Problem N6

$$n = \frac{-7 \pm \sqrt{63649}}{6}$$

$$n = \frac{-7 + 252.29}{6} = \frac{245.29}{6} \approx 40.88$$

$\sqrt{63649} = 252.29$

$n = 41$

$$S = \frac{n}{2}(a_1 + a_n) = \frac{41}{2}\left(5 + (3 \cdot 41 + 2)\right) = \frac{41}{2}(5 + 125) =$$

$$= \frac{41}{\cancel{2}_{1}} \cdot \cancel{130}_{65} = 41 \cdot 65 = 2665 \quad X$$

$$S = \frac{40}{2}\left(5 + (3 \cdot 40 + 2)\right) = 20 \cdot 127 = 2540 \quad X$$

$$S = \frac{42}{2}\left(5(3 \cdot 42 + 2)\right) = 21 \cdot 133 = 2793 \quad X$$

$S = 2650$ is not achivable with integer n.

Problem N6

$a_5 = 20 \qquad a_{15} = 60 \qquad a_{10} = ?$

$$a_n = \frac{a_x + a_y}{2} *$$

$$a_{10} = \frac{20 + 60}{2} = 40$$

$a_{10} = a_1 + 9d$

$a_{15} = a_1 + 14d$

$5+1.25) =$

$(60-20) = (a_1 + 14d) - (a_1 + \cancel{10}\,4d)$

$40 = 10d \quad | :10$

$4 = d$

$d = 4$

$a_5 \Rightarrow 20 = a_1 + 4 \cdot 4$

$20 = a_1 + 16$

$\cancel{a_1 = 20 + 16}$

$20 - 16 = a_1$

$4 = a_1$

$a_1 = 4$

$a_{10} = a_1 + 9d$

$a_{10} = 4 + 9 \cdot 4$

$a_{10} = 40$

Problem 7

$\cancel{\sum_{k=1}^{20} 5+(k \cdot 0.5)}$ $\quad S_{20} = \frac{20}{2}\left[2 \cdot 5 + (20-1) \cdot 0.5\right]$

$\cancel{\sum_{k=0}^{20} 5+(k \cdot 0.5)}$ $\quad S_{20} = \cancel{20}\,10\left[10 + 9.5\right] = 10 \cdot 19.5 = 195$

$\cancel{S_n = \frac{1+n}{2} = \frac{n(n+1)}{2}}$ $\quad S_n = \frac{a_1 + (a_1 + (n+1)d)}{2} \cdot n$

Problem 8.

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[22 + 3(n-1)\right]$$

$$S_n = \frac{n}{2}\left[22 + 3n - 3\right]$$

$$S_n = \frac{n}{2}\left[19 + 3n\right]$$

$$S_n = \frac{n(3n+19)}{2}$$

$$\frac{n(3n+19)}{2} > 1000 \quad /\cdot 2$$

$$n(3n+19) > 2000$$

$$n(3n+19) - 2000 = 0$$

$$3n^2 + 19n - 2000 = 0$$

$$n = \frac{-19 \pm \sqrt{19^2 - 4\cdot 3\cdot(-2000)}}{2\cdot 3}$$

$$n = \frac{-19 \pm \sqrt{361 + 24000}}{6}$$

$$n = \frac{-19 \pm \sqrt{24361}}{6}$$

$$\sqrt{24361} = 156.03$$

$$n = \frac{-19 + 156.03}{6} \approx \frac{137.03}{6} \approx 22.84$$

$$n = 23$$

$$S_{23} = \frac{23(3\cdot 23 + \ldots}{2}$$

$$= 1012$$

$S_{23} > 1000$, n

Problem 9.

$$\sum_{K=3}^{12} 4\left(\frac{1}{2}\right)\ldots$$

$$\sum_{j=0}^{9} \left(\frac{1}{2}\right)^j \ldots$$

$n = 23$

$S_{23} = \frac{23(3 \cdot 23 + 19)}{2} = \frac{23(69+19)}{2} = \frac{23 \cdot \overset{44}{\cancel{88}}}{\cancel{2}} = 23 \cdot 44 =$

$= 1012$

$S_{23} > 1000, \; n = 23$

Problem 9

$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j = \sum_{j=0}^{9} \left(\frac{1}{2}\right)^j \frac{1}{2}$

$\sum_{j=0}^{9} \left(\frac{1}{2}\right)^j \cdot \frac{1}{2} = \sum_{j=0}^{9} \left(\frac{1}{2}\right)^{j+1}$

$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^j = \sum_{j=0}^{9} \left(\frac{1}{2}\right)^j \cdot \frac{1}{2}$

$S = \sum_{k=0}^{9} \left(\frac{1}{2}\right)^k \cdot \frac{1}{2}$

Problem 10

$a_2 = -6$

$a_5 = 48$

$a_{10} = ?$

$a_n = a_1 r^{n-1}$

$a_2 \longrightarrow -6 = a_1 r^1$

$a_5 \longrightarrow 48 = a_1 r^4$

$\frac{48}{-6} = \frac{a_1 r^4}{a_1 r^1}$

$-8 = r^3$

$r^3 = -8$

$r = -2$

$a_1 \longrightarrow -6 = a_1(-2)$

$a_1 = \frac{-6}{-2}$

$a_1 = 3$

$a_5 \longrightarrow 48 = a_1(-2)^4$

$48 = 3 \cdot 16$

$a_{10} \longrightarrow a_{10} = a_1 r^9$

$a_{10} = 3 \cdot (-2)^9$

$a_{10} = 3 \cdot (-512)$

$a_{10} = -1536$

Problem 11.

$a_4 = 54$

$a_7 = 1458$

$r = ?$

$a_n = a_1 r^{(n-1)}$

$a_4 \rightarrow 54 = a_1 r^3$

$a_7 \rightarrow 1458 = a_1 r^6$

$$\frac{1458}{54} = \frac{a_1 r^6}{a_1 r^3}$$

$27 = r^3$

$r^3 = 27$

$r = 3$

Problem 12.

$a_1 = 8$

$r = \frac{3}{4}$

$a_{15} = ?$

$\cancel{a_{15} = a_1 r^{(15-1)}}$

$\cancel{a_{15} = 8 \left(\frac{3}{4}\right)^{14}}$

$\cancel{a_{15}} = S_n = a_1 \frac{1-r^n}{1-r}$

$r^{15} = \left(\frac{3}{4}\right)^{15}$

$S_{15} = 8\frac{1-\left(\frac{3}{4}\right)^{15}}{1-\frac{3}{4}} = 8 \cdot \frac{1-r^{15}}{\frac{1}{4}} = 32(1-r^{15})$

$S_{15} \approx 32 \cdot 1 = 32$

Problem 13

$P(x) = x^5 - 4x^3 + x^2 - 7$

degree: 5

Number of terms: 4

Problem 14

$(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7)$

$2x^4 - 2x^3 + 5x + 2 - 2x^2 \longrightarrow 2x^4 - 2x^3 - 2x^2 + 5x + 2$

Problem 15

$(x^2 - x + 2)(x^2 + x + 1)$

$x^2(x^2 + x + 1) + (-x)(x^2 + x + 1) + 2(x^2 + x + 1) = x^4 \cancel{+ x^3} \cancel{+ x^2} -$

$\cancel{- x^3} \cancel{- x^2} - x + 2x^2 + 2x + 2 = x^4 + x^2 + \cancel{x}\, x + 2$

Problem 16

$24x^3y^2z^5$

GCD

LCM

Problem N

$x^4 - 13x^2 + \ldots$

Problem 16.

$24x^3y^2z^5$ $\qquad\qquad$ $36x^5y^3z^2$

$\cancel{GCD} = 2\cdot 12 \cdot xxx \cdot yy\,zzzzz \qquad 3\cdot 12\; xxxxx\; yyy\; zz$

$\cancel{LCM}\quad GCD = 12x^3y^2z^2$

$$LCM = \underset{2\cdot 3\cdot 12}{\cancel{2\cdot 3\cdot 12}}\cdot x^5y^3z^5 = \underset{72}{\cancel{216}}\,x^5y^3z^5$$

Problem N17.

$x^4 - 13x^2 + 36$

$u^2 - 13u + 36 = 0$

$$\begin{array}{ccc} u & & 4 \\ & \times & \\ u & & 9 \end{array}$$

$(u-9)(u-4) = 0$

$(x^2-9)(x^2-4) = 0$

$(x-3)(x+3)(x-2)(x+2) = 0$

Problem 18.

$(2x+3y)^5$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}\cdot b^k$$

$k=0 \rightarrow \binom{5}{0}(2x)^{5-0}\cdot 3y^0 = 1\cdot 32x^5\cdot 1 = 32x^5$

$k=1 \rightarrow \binom{5}{1}(2x)^{4-1}\cdot 3y^1 = 5\cdot 16x^4\cdot 3y = 240x^4y$