~~then~~ the $(n - \sqrt{n}) / \lg n$ term dominates for $n \geq 4$,
and sum is $O(n / \lg n)$

j) $T(n) = \sqrt{n}\, T(\sqrt{n}) + n =$

$\cancel{T(n) =}$ $= \sqrt{n}(\sqrt{n} \lg \lg \sqrt{n}) + n =$

$= n \lg \lg n^{1/2} + n =$

$= n \lg ((1/2) \lg n) + n =$

$= n (\lg (1/2) + \lg \lg n) + n =$

$= -n + n \lg \lg n + n = n \lg \lg n$

$T(n) = \Theta(n \lg \lg n)$

Home work W7

Problem 1

$O(n) \cdot O(\log_2 n) \cdot O(n) \rightarrow O(n^2)$

Problem 2

False. $f(n) = 2^{2n} \rightarrow \neq O(2^n)$

Problem 3

$T(n) = aT(n/b) + f(n) \rightarrow$ compare $f(n)$ with $n^{\log_b a}$

a) $T(n) = 5T(n/3) + n \log n$

$n \log n = n^{\log_3 5}$

$\log_b a = \frac{\log a}{\log b} \Rightarrow \log_3 5 = \frac{\log 5}{\log 3} \approx \frac{0.69897}{0.47712} \approx 1.465$

$n^{\log_3 5} \approx n^{1.465}$

Since $n \log n = O(n^{\log_3 4 - \epsilon})$ for any $0 < \epsilon \leq 0.46$ by case 1 of the master theorem, we have $T(n) = \Theta(\cancel{\log}\, n^{\log_3 5})$

b) $T(n) = 3T(n/3) + n/\lg n$ $\quad g(n) = ?$ $\quad f(n) = ?$ $\quad \frac{f(n)}{g(n)} = ?$

$a = 3,\ b = 3,\ f(n) = n/\lg n$

$n^{\log_3 3} \rightarrow \log_3 3 = 1 \rightarrow n^1 = n \rightarrow g(n)$

$f(n) = \frac{n}{\lg n}$, since $n \rightarrow \infty$ and $\lg n \rightarrow \infty$, $\frac{n}{\lg n} = \infty$

$\frac{f(n)}{g(n)} = \frac{\frac{n}{\lg n}}{n} = \frac{1}{\lg n}$ which is consistent with $T(n) = \Theta(n)$

c) $T(n) = 8T(n/2) + n^3\sqrt{n}$

$a = 8,\ b = 2,\ f(n) = n^3\sqrt{n}$

$n^{\log_b a} \Rightarrow \log_2 8 = 3 \rightarrow n^3$

$f(n) = n^3\sqrt{n} \rightarrow n^3 \cdot n^{1/2} = \frac{3}{1} + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2} \rightarrow n^{7/2}$

$\frac{f(n)}{n^{\log_b a}} = \frac{n^{7/2}}{n^3} > 1 \rightarrow f(n)$ grows faster than $n^{\log_b a}$

which is consistent with ~~case№3~~ case№3 of Master Theorem

$a \cdot f(n/b) \leq c \cdot f(n) \rightarrow T(n) = \Theta(f(n))$

$8\left(\frac{n}{2}\right)^{7/2} = \frac{8n^{7/2}}{2^{7/2}} = \frac{8n^{7/2}}{11.31} \approx 0.706n^{7/2} \leq c\,n^{7/2} \rightarrow 0.99n^{7/2}$

d) $T(n) = 2T(n/2 - 2) + n/2 \rightarrow T(n) = 2T(n/2) + \Theta(n)$

~~$a = 2,\ b = (n/2 - 2),\ f(n) = n/2$~~

~~$n^{\log_b a} = n^{\log_{(n/2-2)} 2}$~~

$a = 2,\ b = 2,\ f(n) = \Theta(n)$

$n^{\log_2 2} = 1 \rightarrow n^1 = n$

$\frac{f(n)}{n^{\log_b a}} = \frac{\Theta(n)}{n} = 1$ which is consistent with case 2 of Master theorem, f(n) grows as fast as $n^{\log_b a}$

$T(n) = \Theta(n^{\log_b a} \cdot \lg n)$

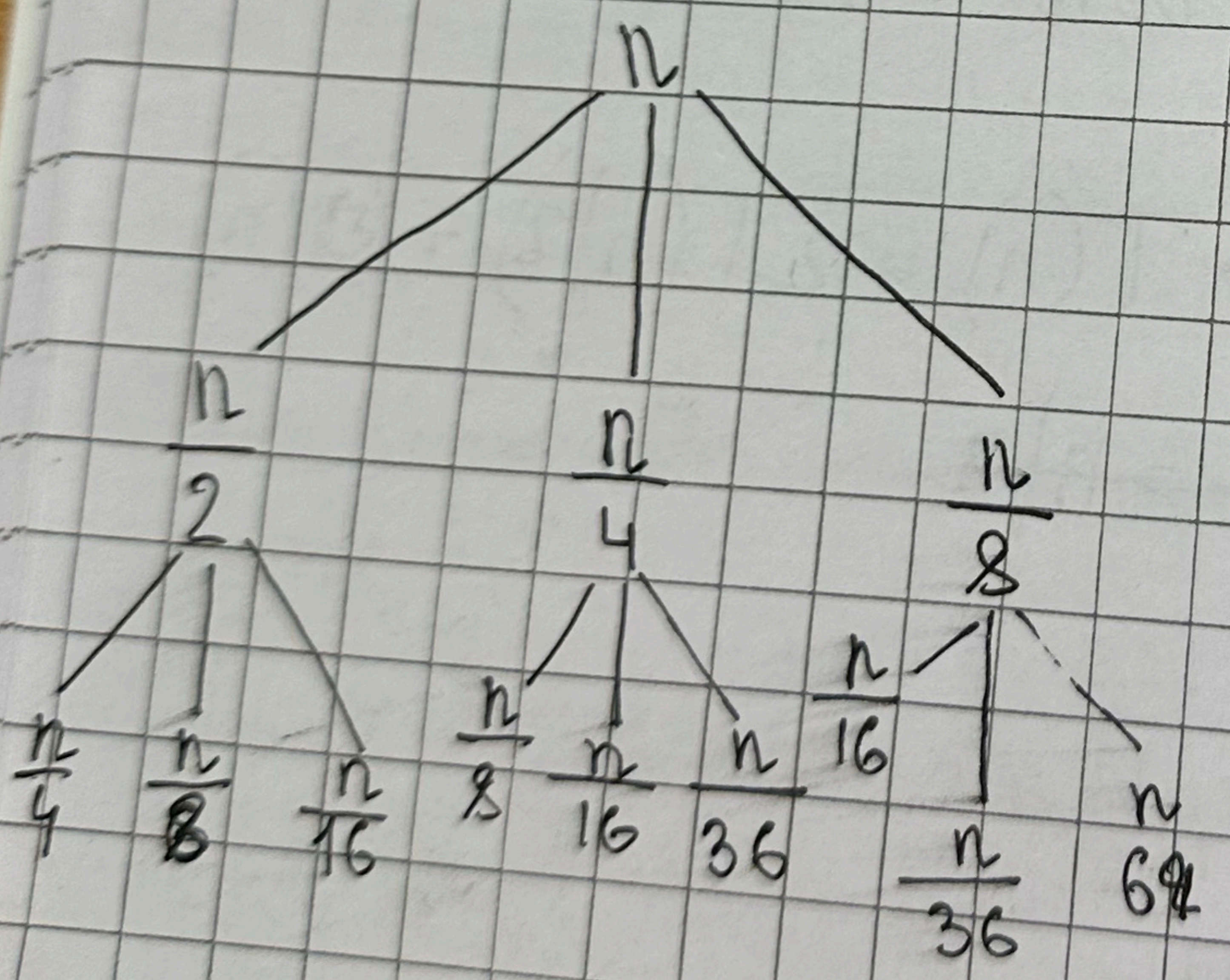
e) $T(n) = 2T(n/2) + n/\lg n$

$a=2,\ b=2,\ f(n) = n/\lg n$

$n^{\log_b a} \Rightarrow \log_2 2 = 1 \rightarrow n^1 = n$

$\frac{f(n)}{n^{\log_b a}} = \frac{n/\lg n}{n} = \frac{1}{\lg n} < 1$, it would be consistent with case n/1 of Master theorem but due the result introduces extra lg factor, so → $T(n) = \Theta(n \cdot \lg \cdot \lg \cdot n)$

f) $T(n) = T(n/2) + T(n/4) + T(n/8) + n$



n

$n\left(\frac{4+2+1}{8}\right) = \frac{7}{8}n$

$n\left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{2}{36} + \frac{1}{64}\right) =$

$= \frac{16+16+12+4+1}{64} = \frac{49}{64} =$

$= \frac{7}{8}^2 n$

$$\sum_{i=1}^{\log n} \left(\frac{7}{8}\right)^i n = \Theta(n)$$

$T(n) = T(n/2) + T(n/4) + T(n/8) + n$

$\leq cn/2 + cn/4 + cn/8 + n$

$= 7cn/8 + n$

$= (1 + 7c/8)n$

$\leq cn$ if $c \geq 8 \rightarrow T(n) = O(n)$

$T(n) = T(n/2) + T(n/4) + T(n/8) + \geq n \rightarrow T(n) = \Omega(n)$

Since $T(n) = O(n)$ and $T(n) = \Omega(n) \rightarrow T(n) = \Theta(n)$

$T(n) = T(n/2) + T(n/4) + T(n/8) + n$

$= 4n + 2n + 1n + n = 8n$

g) $T(n) = T(n-1) + 1/n$

$T(n) = H_n \rightarrow H_n = 1/1 + 1/2 + 1/3 + 1/4 \ldots\ldots + 1/n$

$T(1) = 1 = H_1$

$T(n-1) = H_{n-1}$ and $T(n) = T(n-1) + 1/n$

$= \cancel{H_{n-1}}\ H_{n-1} + 1/n = H_n$

$H_n = \Theta(\lg n) \longrightarrow T(n) = \Theta(\lg n)$

h) $T(n) = T(n-1) + \lg n$

Guess that $T(n) = \Theta(n \lg n)$

$n = 1 \rightarrow T(n) = \sum_{i=1}^{n} \lg i$

$T(n) = \sum_{i=1}^{n} \lg 1 \le \sum_{i=1}^{n} \lg n = n \lg n \rightarrow$ upper bound

$T(n) = \sum_{i=1}^{n} \lg i \ge \sum_{n=n/2}^{n} \lg i \ge [n/2] \lg [n/2]$

$\ge (n/2 - 1) \lg (n/2) = (n-2-1) \lg n - (n-2-1) = \Omega(n \lg$

Since $T(n) = O(n \lg n)$ and $T(n) = \Omega(n \lg n) \rightarrow$

$T(n) = \Theta(n \lg n)$

i) $T(n) = T(n-2) + 1/\lg n$

$T(n) = T(n-2) + \frac{1}{\lg n} = T(n-4) + \frac{1}{\lg(n-2)} + \frac{1}{\lg n} =$

$= T(n-6) + \frac{1}{\lg(n-4)} + \frac{1}{\lg(n-2)} + \frac{1}{\lg n} = \ldots\ldots$

$= T(2) = \frac{1}{\lg 4} + \ldots + \frac{1}{\lg(n-4)} + \frac{1}{\lg(n-2)} + \frac{1}{\lg n} =$

$= \frac{1}{\lg 2} + \frac{1}{\lg 4} + \ldots + \frac{1}{\lg(n-4)} + \frac{1}{\lg(n-2)} + \frac{1}{\lg n}$

The lower bound $= n/(2 \lg n) = \Omega(n / \lg n)$

The upper bound $= \sqrt{n}/2 + n - \sqrt{n} / \lg n$

~~then~~ the (n -

and sum is

j) $T(n) = \sqrt{n}$

$\cancel{T(n)} =$

$= n \lg$

$= n \lg (($

$= n(\lg$

$= -n +$

$T(n) =$