~~Then~~ the $(n-\sqrt{n})/\lg n$ term dominates for $n \geq 4$, and sum is $O(n/\lg n)$

j) $T(n) = \sqrt{n}\,T(\sqrt{n}) + n =$

$\cancel{T(n)} = \sqrt{n}(\sqrt{n} \lg\lg\sqrt{n}) + n =$

$= n \lg\lg n^{1/2} + n =$

$= n\lg((1/2)\lg n) + n =$

$= n(\lg(1/2) + \lg\lg n) + n =$

$= -n + n\lg\lg n + n = n \lg\lg n$

$T(n) = \Theta(n \lg\lg n)$

HomeWork N8

Problem N1

a) $19/7 = 2(5) \rightarrow 19 = 7\cdot 2 + 5 \; (2 = q;\ 5 = r)$

b) $-111/11 = -10(1) \rightarrow -111 = 11\cdot 11 + 10 \; (-11 = q;\ 10 = r)$

c) $789/23 = 34(7) \rightarrow 789 = 23\cdot 34 + 7 \; (34 = q;\ 7 = r)$

d) $1001/13 = 77 \rightarrow 1001 = 13\cdot 77 + 0 \; (77 = q;\ 0 = r)$

e) $0/19 = 0(0) \rightarrow 0 = 19\cdot 0 + 0 \; (0 = q;\ 0 = r)$

f) $3/5 = 0(3) \rightarrow 3 = 5\cdot 0 + 3 \; (0 = q;\ 3 = r)$

g) $-1/3 = 0(-1) = -1 = 3\cdot(-1) + 2 \; (-1 = q;\ 2 = r)$

h) $4/1 = 4(0) = 4 = 1\cdot 4 + 0 \; (4 = q;\ 0 = r)$

Problem №2

$$f(x) = \begin{cases} x \bmod m & \text{if } x \bmod m \le \lceil m/2 \rceil \\ (x \bmod m) - m & \text{if } x \bmod m > \lceil m/2 \rceil \end{cases}$$

Problem №3

a) $a = 0,\ b = 2,\ c = 2$, mod 4

$0 \cdot 2 = 2 \cdot 2 \pmod 4$

$0 \equiv 2 \pmod 4$

b) $\cancel{a = 0,\ b = 1,\ c = 2,\ d = 1, \text{ mod } 4}$

$\cancel{0 \equiv 1 \pmod 4}$

$\cancel{2 \equiv 1 \pmod 4}$

$\cancel{0^2 \equiv 1 \pmod 4}$

$a = 3,\ b = 3,\ d = 6,\ c = 1$, mod 5

$3 \equiv 3 \pmod 5 \rightarrow 3$

$1 \equiv 6 \pmod 5 \rightarrow 1$

$3^1 \not\equiv 3^3 \pmod 5$

Problem №4

1) Theorem №5

Since $a \equiv b \pmod m$

$a \cdot a \equiv b \cdot b \pmod{m} \longrightarrow a^2 \equiv b^2 \pmod{m} \longrightarrow$

$\longrightarrow a^3 \equiv b^3 \pmod{m} \longrightarrow a^k \equiv b^k \pmod{m}$

2) Theorem N1

$a^k - b^k = (a \cdot b)(a^{k-1} + a^{k-2} b + \ldots + ab^{k-2} + b^{k-1})$

$a \equiv b \pmod{m} \longrightarrow m \mid (a-b) \longrightarrow m \mid (a^k - b^k) \longrightarrow$

$\longrightarrow a^k \equiv b^k \pmod{m}$

Problem 5

$Z_5 = \{0, 1, 2, 3, 4\}$

$a +_4 b = (a+b) \bmod 4$

....

$a +_0 b = (a+b) \bmod 0$

$a \cdot_4 b = (a \cdot b) \bmod 4$

....

$a \cdot_0 b = (a \cdot b) \bmod 0$

Problem 6

$0+0=0 \quad 0+1=1 \quad \ldots\ldots \quad 0+4=4$

$1+0=1 \quad 1+1=2 \quad \ldots\ldots \quad 1+4=5$

......

$4+0=4 \quad 4+1=0 \quad \ldots\ldots \quad 4+4=3$

$0 \cdot 0 = 0$ $\quad 0 \cdot 1 = 0 \ \dots \quad 0 \cdot 4 = 0$

$1 \cdot 0 = 0$ $\quad 1 \cdot 1 = \underline{1} \ \dots \quad 1 \cdot 4 = 4$

$\dots$

$4 \cdot 0 = 4$ $\quad 4 \cdot 1 = 4 \ \dots \quad 4 \cdot 4 = \underline{1}$

if $d = 1$ then $f(a) = a$ and $g(a) = 0$

$f \rightarrow$ one-to-one and into because $f(0) = f(1) = 0$

$g \rightarrow$ not into, because it ranges is $\{0, 1, \dots, d-1\}$

and it's not one-to-one because $g(0) = g(d) = 0$

Problem N7

a)

$$
\begin{array}{l}
231 : 2 = 115, \quad 231 - 230 = \boxed{1} \\
115 : 2 = 57, \quad 115 - 114 = \boxed{1} \\
57 : 2 = 28, \quad 57 - 56 = \boxed{1} \\
28 : 2 = 14, \quad 28 - 28 = \boxed{0} \\
14 : 2 = 7, \quad 14 - 14 = \boxed{0} \\
7 : 2 = 3, \quad 7 - 6 = \boxed{1} \\
3 : 2 = 1, \quad 3 - 2 = \boxed{1} \\
1 : 2 = 0, \quad 1 - 0 = \boxed{1}
\end{array}
$$

$231 = 11100111$

b) 4532 | 2
−4532 → 2266, remainder 0
2266 | 2
−2266 → 1133, remainder 0
1133 | 2
−1132 → 566, remainder 1
566 | 2
−566 → 283, remainder 0
283 | 2
−282 → 141, remainder 1
141 | 2
−140 → 70, remainder 1
70 | 2
−70 → 35, remainder 0
35 | 2
−34 → 17, remainder 1
17 | 2
−16 → 8, remainder 1
8 | 2
−8 → 4, remainder 0
4 | 2
−4 → 2, remainder 0
2 | 2
−2 → 1, remainder 0
1 | 2
0, remainder 1

4532 = 1000111101000

c) 97644 | 2
−97644 → 48822, remainder 0
48822 | 2
−48822 → 24411, remainder 0
24411 | 2
−24410 → 12205, remainder 1
12205 | 2
−12204 → 6102, remainder 1
6102 | 2
−6102 → 3051, remainder 0
3051 | 2
−3050 → 1525, remainder 0
1525 | 2
−1524 → 762, remainder 1

1 0 1 0 1 1 1 0 1 0 1

97644 = 10111110101001100

Problem №8

a) $(11111)_2 \rightarrow 1\cdot2^4 + 1\cdot2^3 + 1\cdot2^2 + 1\cdot2^1 + 1\cdot2^0 =$
$= 16 + 8 + 4 + 2 + 1 = 31$

c) $(1\,0101\,0101)_2 \rightarrow 1\cdot2^8 + 0\cdot2^7 + 1\cdot2^6 + 0\cdot2^5 + 1\cdot2^4 +$
$+ 0\cdot2^3 + 1\cdot2^2 + 0\cdot2^1 + 1\cdot2^0 = 256 + \overset{64}{\cancel{128}} + 16 + 4 + 1 =$
$= 341$

в) $(10\,0000\,0001)_2 \rightarrow 1\cdot2^9 + 0\cdot2^8 \ldots + 0\cdot2^1 + 1\cdot2^0 =$
$= 512 + 1 = 513$

d) $(110\,1001\,0001\,0000)_2 = 1\cdot2^{14} + 1\cdot2^{13} + 0\cdot2^{12} +$
$+ 1\cdot2^{11} + \ldots 1 + 2^8 + \ldots 1\cdot2^4 + \ldots = 16384 + 8192 +$
$+ 2048 + 256 + 16 = 26896$

Problem №9

a) $(100\,0111)_2 \rightarrow 71$, $(111\,0111)_2 \Rightarrow 119$
$100\,0111 + 111\,0111 = 1011\,1110 \rightarrow 190$
$100\,0111 \cdot 111\,0111 = 10\,0001\,0000\,0001 \rightarrow 8449$

в) $(1110\,1111)_2 \rightarrow 239$, $(1011\,1101)_2 \rightarrow 189$
$1110\,1111 + 1011\,1101 = 1\,1010\,1100\ (428)$
$1110\,1111 \cdot 1011\,1101 = 1011\,0000\,0111\,0011\ (45\,171)$

c) $(10\,1010\,1010)_2 \rightarrow 682$, $(1\,1111\,0000)_2 \rightarrow 496$

$10\,1010\,1010 + 1\,1111\,0000 = 100\,1001\,1010$ (1178)

$10\,1010\,1010 \cdot 1\,1111\,0000 = 101\,0010\,1001\,0110\,0000$ (338 272)

d) $(10\,0000\,0001)_2 \rightarrow 513$, $(11\,1111\,1111)_2 \rightarrow 1023$

$10\,0000\,0001 + 11\,1111\,1111 = 110\,0000\,0000$ (1536)

$10\,0000\,0001 \cdot 11\,1111\,1111 = 1000\,0000\,0001\,1111\,1111$ (524 799)

Problem IV10

a) $88 \rightarrow 2^3 \cdot 11$

b) $126 \rightarrow 2 \cdot 63 \rightarrow 2 \cdot 3 \cdot 21 \rightarrow 2 \cdot 3 \cdot 3 \cdot 7 \rightarrow$

$\rightarrow 2 \cdot 3^2 \cdot 7$

c) $729 \rightarrow 3 \cdot 243 \rightarrow 3 \cdot 3 \cdot 81 \rightarrow 3 \cdot 3 \cdot 3 \cdot 27 \rightarrow 3 \cdot 3 \cdot 3 \cdot 3 \cdot 9 \rightarrow$

$\rightarrow 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \rightarrow 3^6$

d) $1001 \rightarrow 7 \cdot 143 \rightarrow 7 \cdot 11 \cdot 13$

Problem IV11

Number of zeroes $= \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] + \left[\frac{100}{5^3}\right] + \ldots$

$\left[\frac{100}{5}\right] = 20$

$\left[\frac{100}{25}\right] = 4$

$\left[\frac{100}{125}\right] = 0$

$20+4=24$

Problem № 12

a) $\gcd(12,18) \to \gcd(12,6) \to \gcd(6,0) = 6$

b) $\gcd(111,201) \to \gcd(111,90) \to \gcd(90,21) \to \gcd(6,3) \to$
$\to \gcd(3,0) = 3$

c) $\gcd(1001,1331) \to \gcd(1001,330) \to \gcd(330,11) \to$
$\to \gcd(11,0) = 11$

d) $\gcd(12345,54321) \to \gcd(12345,4941) \to \gcd(4941,2463) \to$
$\to \gcd(2463,15) \to \gcd(15,3) \to \gcd(3,0) = 3$

e) $\gcd(1000,5040) \to \cancel{1000,} \gcd(1000,40) \to \gcd(40,0) \to$
$= 40$

f) $\gcd(9886,6060) \to \gcd(6060,3828) \to \gcd(3828,2232) \to$
$\to \gcd(2232,1596) \to \gcd(1596,636) \to \gcd(636,324) \to$
$\to \gcd(324,312) \to \gcd(312,12) \to \gcd(12,0) = 12$