Home Work N3

Problem N1.

$$\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4)$$

$$\log_2\left(\frac{8\sqrt{2}}{16}\right) = \frac{8 \cdot 2^{\frac{1}{2}}}{16_2} = \frac{2^{\frac{1}{2}}}{2} = \frac{1}{2} \cdot 2^{\frac{1}{2}} = 2^{-1} \cdot 2^{\frac{1}{2}} =$$

$$= 2^{-1+\frac{1}{2}} = 2^{-\frac{1}{2}} \longrightarrow -\frac{1}{2}$$

$$\log_2(32) = 5$$

$$2\log_2(4) = 2 \times \log_2(4) = 2 \cdot 2 = 4$$

$$-\frac{1}{2} + 5 - 4 = \frac{1}{2} = 0.5$$

Problem N2.

$$\log_3(x-1) + \log_3(x+1) = 2$$

$$\log_6(A) + \log_6(B) = \log_6(AB)$$

$$\log_3((x-1)(x+1)) = 2$$

$$\log_3(x^2 - 1) = 2$$

$$x^2 - 1 = 3^2 = 9$$

$$x^2 - 1 = 9$$

$$x^2 = 10$$

$$x = \pm\sqrt{10}$$

Problem N3

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$20000 = 10000\left(1 + \frac{0,06}{4}\right)^{4t}$$

$$2 = \left(1 + 0,015\right)^{4t}$$

$$2 = \left(1,015\right)^{4t}$$

$$\ln(2) = \ln\left(\left(1,015\right)^{4t}\right)$$

$$\ln(2) = 4t \ln(1.015)$$

$$t = \frac{\ln(2)}{4\ln(1.015)} \approx \frac{0,6931}{4 \cdot 0.014889} \approx \frac{0,6931}{0.059556} \approx 11.64$$

Problem N4

$$N(t) = N_0 e^{-kt}$$

$$N(t_{1/2}) = \frac{N_0}{2}$$

$$\frac{N_0}{2} = N_0 e^{-kt_{1/2}}$$

$$\frac{1}{2} = e^{-k5}$$

$$\ln\left(\frac{1}{2}\right) = -5k$$

$$k = \frac{-0.6931}{5} \approx 0.1386 \text{ per year.}$$

≈ 11.64

Problem №4

$N(t) = N_0 e^{-kt} \rightarrow N(1/2) = \frac{N_0}{2}$

$\frac{N_0}{2} = N_0 e^{-kt}$

$\frac{1}{2} = e^{-k5}$

$\ln\left(\frac{1}{2}\right) = -5k$

$0.6931 = -5k$

$k = \frac{0.631}{5} \approx 0,1386$

Problem 5

$N_0 = 70$

$t = 3$ hours

$20g \rightarrow ?$

$N(t) = N_0 e^{-kt}$

$N_{(1/2)} = \frac{N_0}{2}$

$70 = 100 e^{-k3} \quad | : 100$

$0,7 = e^{-k3}$

$\ln(0,7) = -3k$

$-0.3567 = -3k$

$k = \frac{+0.3567}{3} \approx 0.1189.$

~~Problem 6.~~

$N(t) = 20g$

$N(t) = N_0 e^{-kt}$

$20 = 100\, e^{-0.1189t}$ /100

$0.2 = e^{-0.1189t}$

$\ln(2) = -0.1189t$

$-1.6094 = -0.1189\,t$

$t = \frac{+1.6094}{0.1189} = 13.54$ hours.

Problem 6.

Find the unit vector $A(1,2,3) \rightarrow B(4,6,9)$

$V = \cancel{A-B}\ B-A$

$V = (4-1, 6-2, 9-3) = (3,4,6)$ - Vector

Magnitude $\rightarrow \|V\| = \sqrt{x^2+y^2+z^2}$

$\|V\| = \sqrt{3^2+4^2+6^2} = \sqrt{9+16+36} = \sqrt{61} = 7.81$

Unit Vector

$\vec{V} = \frac{3}{7.81}$

Problem 7.

$\vec{V} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$

$\|\vec{V}\| =$

Problem

$\vec{a}$

3 c

$\vec{b}$

Unit Vector $\longrightarrow \hat{v} = \frac{v}{\|v\|}$

$\hat{v} = \frac{3}{7.81} ; \frac{4}{7.81} ; \frac{6}{7.81}$

Problem 7.

$$\vec{v} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$$

$$\|\vec{v}\| = \sqrt{7^2 + (-2)^2 + 4^2)} = \sqrt{49 + 4 + 16} = \sqrt{69} = 8.31$$

Problem 8.

$\vec{a} = (2, -1, 3)$ $\vec{b} = (1, 4, 2)$ $3\vec{a} - 2\vec{b} = ?$

$3\vec{a} = (2 \cdot 3 ; -1 \cdot 3 ; 3 \cdot 3) = (6 ; -3 ; 9)$

$2\vec{b} = (1 \cdot 2 ; 4 \cdot 2 ; 2 \cdot 2) = (2 ; 8 ; 4)$

$3\vec{a} - 2\vec{b} = (6 - (-2) ; -3 - 8 ; 9 - 4) = (8 ; -11 ; 5)$

Problem 9.

$\vec{p} = (1 ; 2 ; 3)$

$\vec{q} = (4, -5, 6)$

~~$\vec{p} + \vec{q} = (1 + 4 ; 2 + (-5) ; 3 + 6) = (5 ; -3 ; 9)$ → dot prod~~

$\vec{p} \cdot \vec{q} = \|\vec{p}\| \cdot \|\vec{q}\| \cdot \cos\theta = (1 \cdot 4) + (2 \cdot (-5)) + (3 \cdot 6) =$

$= 4 + (-10) + 18 = \blacksquare\ 12$

$$\cos\theta = \frac{A \cdot B}{\|A\| \cdot \|B\|}$$

~~$\|A\| =$~~

$$\|p\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14} = 3.74$$

$$\|q\| = \sqrt{4^2 + (-5)^2 + 6^2} = \sqrt{16 + 25 + 36} = \sqrt{77} = 8.$$

$$\cos\theta = \frac{12}{3.74 \cdot 8.77} \approx 0.3658$$

$$\theta = \arccos(0.3658) \approx 68.85^\circ$$

Product 10

$\vec{u} = (2; -1; 4)$

$\vec{v} = (-8; 4; -16)$

$\vec{u} \cdot \vec{v} = (2 \cdot -8) + (-1 \cdot 4) + (4 \cdot (-16)) = (-16) + (-4) + (-64)$

$= -84$

$\vec{v} \cdot \vec{v} \neq 0$

Ppodlect 11

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$$

$2A - 3B = ?$

$2A = \begin{bmatrix} 2 \cdot 2 & 2 \cdot \\ 2 \cdot 0 & \end{bmatrix}$

$3B = \begin{bmatrix} 3 \cdot 4 & 3 \cdot \\ 3 \cdot (-2) & 3 \cdot \end{bmatrix}$

$A - B = \begin{bmatrix} 4 - \\ 0 - \end{bmatrix}$

Problem 12

$C = \begin{bmatrix} 1 & 2 \\ 3 & \end{bmatrix}$

$E = \begin{bmatrix} 5 \\ (1 \cdot \\ (3 \end{bmatrix}$

Proble

$\begin{cases} x \\ 2 \\ - \end{cases}$

$$2A - 3B = ?$$

$$2A = \begin{bmatrix} 2\cdot 2 & 2\cdot -1 \\ 2\cdot 0 & 2\cdot 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix}$$

$$3B = \begin{bmatrix} 3\cdot 4 & 3\cdot 5 \\ 3\cdot(-2) & 3\cdot 1 \end{bmatrix} = \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 4-12 & -2-15 \\ 0-(-6) & 6-3 \end{bmatrix} = \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$$

Problem 12

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \qquad E = CD$$

$$E = \begin{bmatrix} \overset{5}{(1\cdot 5)} + \overset{14}{(2\cdot 7)} & \overset{6}{(1\cdot 6)} + \overset{16}{(2\cdot 8)} \\ \overset{15}{(3\cdot 5)} + \overset{28}{(4\cdot 7)} & \overset{18}{(3\cdot 6)} + \overset{32}{(4\cdot 8)} \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

Problem 13.

$$\begin{cases} x + y + z = 6 \\ 2x - y + 3z = 14 \\ -3x + 2y - 2z = -10 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{array}\right]$$

$R_2 = R_2 - 2R_1$

$2 - 2(1) = 0$

$-1 - 2(1) = -3$

$3 - 2(1) = 1$

$14 - 2(6) = 2$

$R_3 = R_3 + 2R_1$

$-3 + 2(1) = -5 \quad 0$

$2 + 2(1) = 0 \quad 5$

$-2 + 2(1) = -4 \quad 1$

$-10 + 2(6) = -12 \quad 8$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 5 & 1 & 8 \end{array}\right]$$

$R_3 = R_3 +$

$R_3(2) \quad 5 +$

$R_3(3) =$

$R_3(4) =$

$z = \frac{14}{3}$

$-3y +$

$-3y =$

$-3y$

y

$-x +$

x

x

x

p

$$R_3 = R_3 + \left(\frac{5}{-3} R_2\right)$$

$$R_3(2) \quad 5 + \left(\frac{5}{-3} \cdot -3\right) = 10$$

$$R_3(3) = 1 + \left(\frac{5}{-3} \cdot 1\right) = 1 - \frac{5}{3} = \frac{3}{3} - \frac{5}{3} = -\frac{2}{3}$$

$$R_3(4) = 8 + \left(\frac{5}{-3} \cdot 2\right) = 8 + \frac{10}{-3} = \frac{24}{3} - \frac{10}{3} = \frac{14}{3}$$

$$z = \frac{14}{3} \cdot \left(-\frac{2}{3}\right) = \frac{14}{3} \cdot \left(-\frac{3}{2}\right) = -7$$

$$-3y + 1(-7) = 2$$

$$-3y - 7 = 2$$

$$-3y = 9 \mid \cdot(-3)$$

$$y = -3$$

$$x + (-3) + (-7) = 6$$

$$x - 10 = 6$$

$$x = 16$$

$$x = 16, \; y = -3, \; z = -7$$

Problem 14

$$B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

1) $R_1 = R_1 + R_3 \cdot 1$

$R_1 = -1 + (1) \cdot 1 = 0$

$0 + 1(-1) = 0 - 1 = -1$

$R_1[4] = R_1[4] + 1 \cdot R_3[4]$

$0 + 1(-1) = -1$

$R_1 = [1, 2, 0, -1]$

2) $R_2 = R_2 - 3R_3$

$R_2 = 3 - \cdot 3 \cdot (1) = 0$

$5 - 3(-1) = 8$

$R_2 = [0, 1, 0, 8]$

3) $R_1 = [1, 0, 0, -17]$

$$\begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Problem 15

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \qquad A^{-1} = ?$$

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$a_{11} = 1$

$R_1 = R_1 :$

$R_1 = [1$

$R_2 = R$

$5 - 5$

$3 - 5$

$0 - 5$

$1 -$

R_2

a_2

R

$$A=\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]=I$$

$a_{11}=1$

$R_1=R_1:2$

$$R_1=\left[1;\ \frac{1}{2}\ \middle|\ \frac{1}{2},0\right]$$

$R_2=R_2-5R_1$

$5-5(1)=0$

$3-5\left(\frac{1}{2}\right)=3-\frac{5}{2}=\frac{1}{2}$

$0-5\left(\frac{1}{2}\right)=0-\frac{5}{2}=-\frac{5}{2}$

$1-5(0)=1$

$$R_2=\left[0,\ \frac{1}{2}\ \middle|\ -\frac{5}{2},1\right]$$

$a_{22}=1$

$R_2=R_2\cdot 2$

$$R_2=\left[0;\ 1\ \middle|\ -5,2\right]$$

$R_1=R_1-\left(\frac{1}{2}R_2\right)$

$1-\frac{1}{2}(0)=1$

$\frac{1}{2}-\frac{1}{2}(1)=0$

$$\frac{1}{2} - \frac{1}{2}(-5) = \frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$$

$$0 - \frac{1}{2}(2) = 0 - 1 = -1$$

$$R_1 = [1, 0 \mid 3, -1]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$