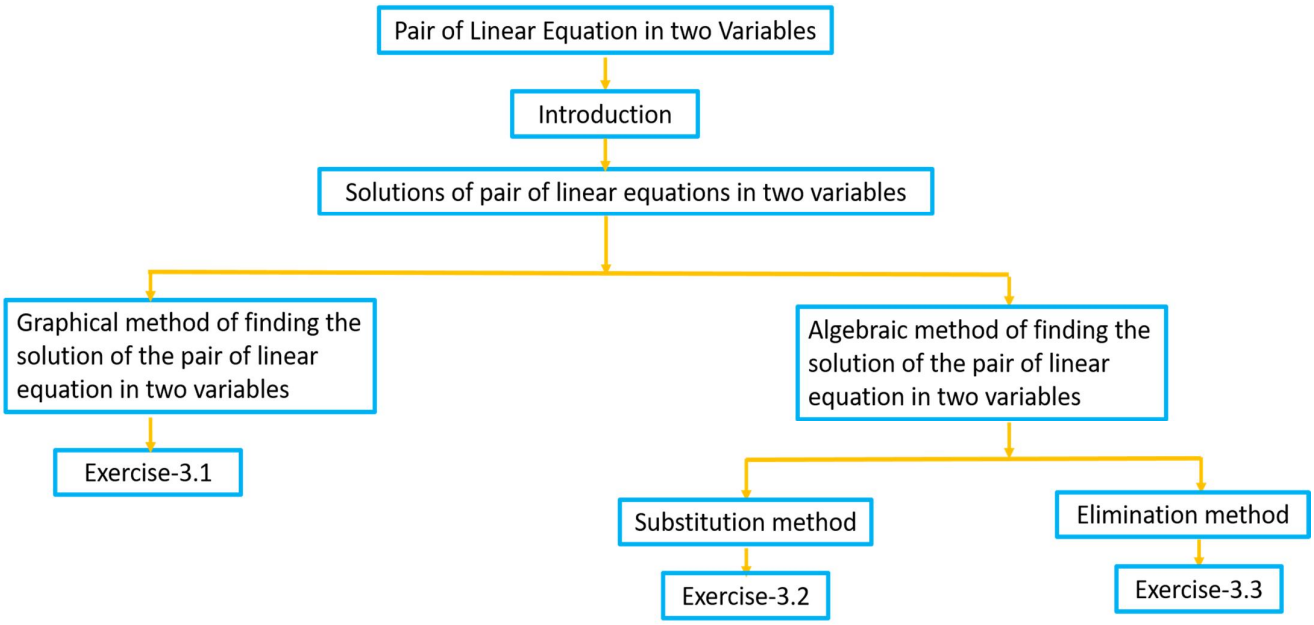
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

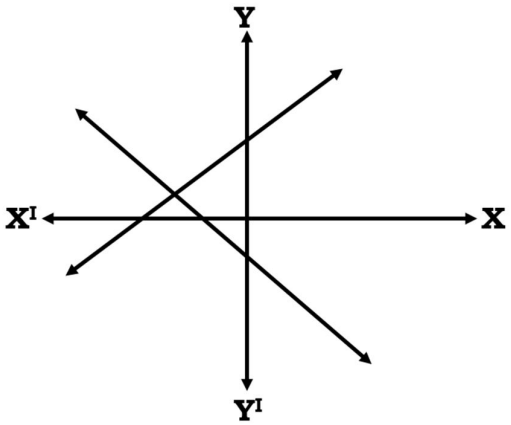
MIND MAPPING



Basic facts and formulae:

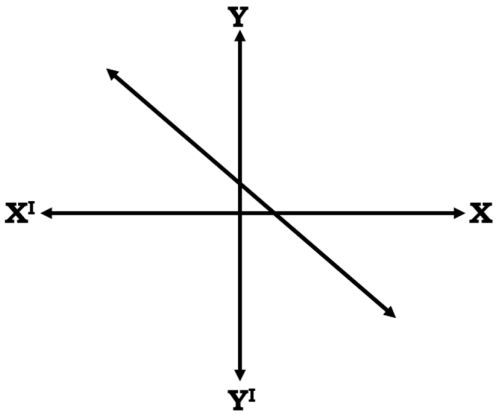
1. A linear equation is an equation of straight line in the form of ax + by + c = 0 where a ≠ 0, b ≠ 0 and a, b, c are real numbers. In this equation $a \& b$ are coefficients and c is a constant.
2. Two linear equations having two same variables are called pair of linear equation in two variables. The general form of pair of linear equations is $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$. Where $a_1, a_2, b_1, b_2, c_1 \, and \, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0; \; a_2^2 + b_2^2 \neq 0$.
3. Every solution of the equation is a point on the line representing it.
4. Each solution (x, y) of linear equation in two variables, ax +by+ c=0 corresponds to a point on the line representing the equation, and vice versa.
5. A system of linear equations is said to be consistent if it has at least one solution.
6. A system of linear equations is said to be inconsistent if it has no solution.
7. Graphical method of pair of linear equation in two variables.

   a) If the two lines intersect each other at one particular point then that point will be the only solution of that pair of linear equations. It is said to be consistent pair of equations.



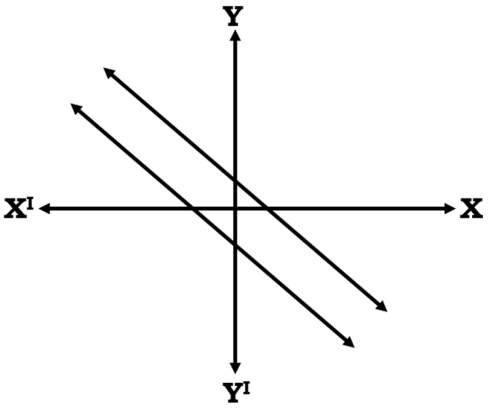
One Solution

b) If two lines coincide with each other, then there will be infinite solutions as all the points on the line will be the solution for the pair of linear equations. It is said to be dependent or consistent pair of equations.



Infinitely Many Solution

c) If the two lines are parallel then there will be no solution as the lines are not intersecting at any point. It is said to be inconsistent pair of equations.



No Solution

8. **Interpretation of the pair of equations:**

Pair of Equations	**Ratio Comparison**	**Graphical Representation**	**Algebraic Interpretation**
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$	Intersecting lines	Only one solution
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$	Coincident lines	Infinite solutions
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$	Parallel lines	No solution

9. **Algebraic methods of solving a pair of linear equations:**

a) **Substitution method:**

If we have a pair of Linear Equations with two variables x and y, then we have to follow these steps to solve them with the substitution method-

Step 1: We have to choose any one equation and find the value of one variable in terms of other variable i.e. y in terms of x.

Step 2: Then substitute the calculated value of y in terms of x in the other equation.

Step 3: Now solve this Linear Equation in terms of x as it is in one variable only i.e. x.

Step 4: Substitute the calculate value of x in the given equations and find the value of y.

b) **Elimination method:**

In this method, we solve the equations by eliminating any one of the variables.

Step 1: Multiply both the equations by such a number so that the coefficient of any one variable becomes equal.

Step 2: Now add or subtract the equations so that the one variable will get eliminated as the coefficients of one variable are same.

Step 3: Solve the equation in that leftover variable to find its value.

Step 4: Substitute the calculated value of variable in the given equations to find the value of the other variable.

LEVEL_1

MCQ:

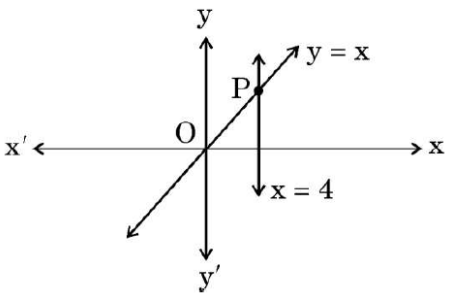
1. If the lines represented by equations $3x+2my=2$ and $2x+5y+1=0$ are parallel, then the value of m is:

   a) $\frac{2}{5}$ b) $-\frac{5}{4}$ c) $\frac{3}{2}$ d) $\frac{15}{4}$

2. The pair of linear equations $x+2y-5=0$ and $2x-4y+6=0$:

   a) is inconsistent
   b) is consistent with many solutions
   c) is consistent with a unique solution
   d) is consistent with two solutions

3. The lines represented by the linear equations $y=x$ and $x=4$ intersect at P. The coordinates of the point P are:



   a) $(4,0)$ b) $(4,4)$ c) $(0,4)$ d) $(-4,4)$

4. The value of 'k' for which the system of equations $kx+2y=5$ and $3x+4y=1$ have no solution, is:

   a) $k=\frac{3}{2}$ b) $k\neq\frac{3}{2}$ c) $k\neq\frac{2}{3}$ d) $k=5$

5. The pair of linear equations $2x=5y+6$ and $15y=6x-18$ represents two lines which are:

   a) intersecting
   b) parallel
   c) coincident
   d) either intersecting or parallel

6. Graphically pair of equations $6x-3y+10=0$ and $2x-y+9=0$ represents two lines which are:

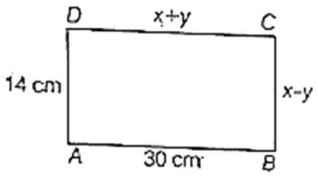
   a) intersecting at exactly one point
   b) intersecting exactly at two points
   c) coincident
   d) parallel

7. The point of intersection of the lines $y=3x$ and $x=3y$ is:

   a) $(0,0)$ b) $(0,3)$ c) $(3,0)$ d) $(3,3)$

2-Marks:

1. Solve the following pair of linear equations: $3x-5y=4$ and $2y+7=9x$.
2. In the fig. ABCD is a rectangle. Find the values of x and y.



3. Equation $2x=5y+4$ is given. Write another linear equation, so that the lines represented by the pair are: (i) intersecting (ii) coincident (iii) parallel
4. On comparing the ratios, $\frac{a_1}{a_2},\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the following pair of equations are consistent or inconsistent:
   (i) $3x+2y=5;\ 2x-3y=7$ (ii) $5x-3y=11;-10x+6y=-22$
5. For which values of p does the pair of equations given below has unique solutions $2x+2y+2=0$ and $4x+py+8=0$.
6. Find a, if the line $3x+ay=8$ passes through the intersection of lines represented by the equations $3x-2y=10$ and $5x+y=8$.
7. Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.
8. For what values of k, will the following equations represent coincident lines? $2x+3y=4$ and $4x+(k+4)y=3k+2$.
9. Solve the system of equations by substitution method: $0.2x+0.3y=1.3$ and $0.4x+0.5y=2.3$.
10. Solve: $99x+101y=499$; $101x+99y=501$.
11. Is the pair of equations $x+2y-3=0$ and $6y+3x-9=0$ consistent? Justify your answer.
12. The angles of a triangle are x, y and 40^0. The difference between two angles x and y is 30^0. Find x and y.

3-Marks:

1. Draw the graph of the equations $x-y+1=0$ and $3x+2y-12=0$. Using this graph find the values of x and y which satisfy both the equations.
2. Two numbers are in the ratio of 5 : 6. If 7 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

3. Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
   (i) intersecting lines (ii) parallel lines (iii) coincident lines.
4. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.
5. For what values of a and b does the following pair of linear equations have an infinite number of solutions? $2x+3y=7$ and $(a-b)x+(a+b)y=3a+b-2$.
6. The sum of the digits of a two digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the fraction.

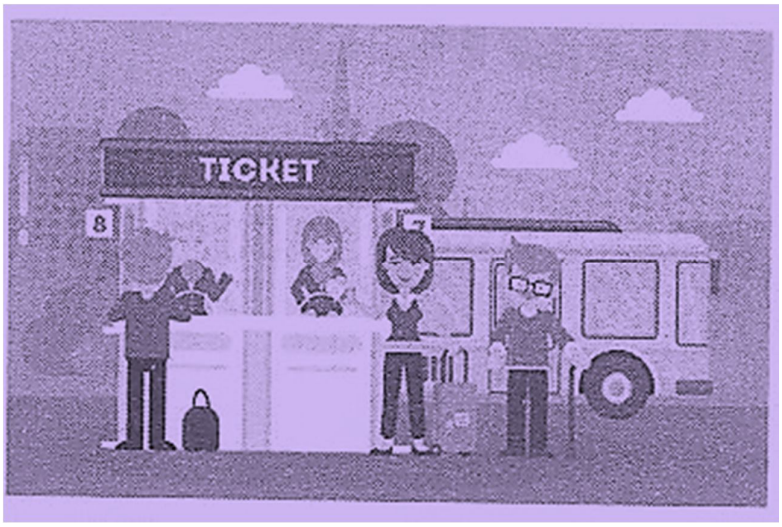
5-Marks:

1. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save Rs. 2000 per month, find their monthly incomes.
2. The area of the rectangle gets reduced by 9 square units, if its length reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 3 units, then the area increases by 67 sq.units. Find the dimensions of the rectangle.
3. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
4. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid Rs. 27 for a book kept for seven days, while Rekha paid Rs. 21 for the book she kept for five days. Find the fixed charge and the additional charge paid by them?
5. Solve for $x\&y$: $mx-ny=m^2+n^2$, $x-y=2n$.
6. Angles of cyclic quadrilateral ABCD are $\angle A=(6x+10)^0$, $\angle B=(5x)^0$, $\angle C=(x+y)^0$, $\angle D=(3y-10)^0$. Find x and y, also the values of the four angles.

CASE STUDY QUESTIONS

CASE STUDY_1

From Bengaluru bus stand, if Riddhima buys 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, then total cost is Rs. 46, but if she buys 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur, then the total cost is Rs. 74.



Consider the fares from Bengaluru to Malleswaram and that to Yeshwanthpur as Rs x and Rs y respectively and answer the following questions.

(i) Represent 1st situation algebraically. 1

(ii) Represent 2nd situation algebraically. 1

(iii) What is the fare from Bengaluru to Malleswaram? 2

OR

What is the fare from Bengaluru to Yeshwanthpur?

CASE STUDY_2

Raman usually go to a dry fruit shop with his mother. He observes the following two situations.

On 1st day: The cost of 2 kg of almonds and 1 kg of cashew was Rs 1600.
On 2nd day: The cost of 4 kg of almonds and 2 kg of cashew was Rs 3000.
Denoting the cost of 1 kg almonds by Rs x and cost of 1 kg cashew by Rs y, answer the following questions.

(i) Represent algebraically the situation of day-I. 1

(ii) Represent algebraically the situation of day- II. 1

(iii) At what point the linear equation represented by day-I, intersect the x axis. 2

OR

At what point the linear equation represented by day-II, intersect the y-axis

CASE STUDY_3:

A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is Rs 9000 and from batch II is Rs 26,000. Assume that each poor child pays Rs. x per month and each rich child pays Rs. y per month.

Based on the above information, answer the following questions:

(i) Represent the information given above in terms of x and y. 1

(ii) Find the monthly fee paid by a poor child. 2

OR

Find the difference in the monthly fee paid by a poor child and a rich child. 2

(iii) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of the fees from batch II? 1

LEVEL_2

MCQ:

1. The pair of equations $y = 0$ and $y = -7$ has

   a) one solution
   b) two solutions
   c) infinitely many solutions
   d) no solution

2. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

   a) parallel
   b) intersecting at (b, a)
   c) coincident
   d) intersecting at (a, b)

3. For what values of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ has infinitely many solutions.

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

4. If the lines given by $3x + 2ky = 2$ and $2x + 5y = 1$ are parallel, then the value of k is:

   a) $-\frac{5}{4}$ b) $\frac{2}{5}$ c) $\frac{15}{4}$ d) $\frac{3}{2}$

5. One equation of a pair of dependent linear equation is $-5x + 7y - 2 = 0$. The second equation can be:

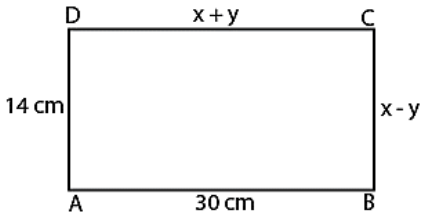
   a) $10x + 14y + 4 = 0$
   b) $-10x - 14y + 4 = 0$
   c) $-10x + 14y + 4 = 0$
   d) $10x - 14y + 4 = 0$

6. A pair of linear equation which has a unique solution $x = 2$ and $y = -3$ is:

   a) $x + y = 1$ & $2x - 3y = -5$
   b) $2x + 5y = -11$ & $4x + 10y = -22$
   c) $2x - y = 1$ & $3x + 2y = 0$
   d) $x - 4y - 14 = 0$ & $5x - y - 13 = 0$

2-MARKS

1. Determine the values of a and b for which the pair of linear equations has infinitely many solutions: $3x-(a+1)y=2b-1$ and $5x+(1-2a)y=3b$
2. Two straight paths are represented by the lines $7x-5y=3$ and $14x-10y=5$. Check whether the paths cross each other.
3. Solve for x and y: $\frac{x}{a}+\frac{y}{b}=2,\ ax-by=a^2-b^2$.
4. For what value of k will the following pair of linear equations have no solution? $3x+y=1;\ (2k-1)x+(k-1)y=2k+1.$
5. In the figure ABCD is a rectangle. Find the values of x and y.



6. Find whether the lines representing the following pair of linear equations intersect at a point or parallel or coincident. $\frac{3}{2}x+\frac{5}{3}y=7$ and $\frac{3}{2}x+\frac{3}{2}y=6$
7. Solve the following equations by elimination method: $\frac{x}{7}+\frac{y}{3}=5;\ \frac{x}{7}-\frac{y}{9}=1$
8. Is it true to say that the pair of equations $-x+2y+2=0$ and $\frac{1}{2}x-\frac{1}{4}y-1=0$ has a unique solution? Justify your answer.
9. For the pair of equations $\lambda x+3y=-7$ & $2x+6y=14$ to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.
10. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
11. The sum of two numbers is 120 and one of numbers is 3 times the other. Find the numbers.

3-MARKS

1. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and denominator. If 3 is added to both the numerator and denominator, fraction becomes $\frac{3}{4}$. Find the fraction.
2. Seven times a two digit number is equal to four times the number obtained by reversing the order of the digits. If the sum of both the digits is 9, find the number.

3. The students of a class are made to stand in rows. If three students are extra in each row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

4. Solve: $\frac{4}{x}+3y=14; \frac{3}{x}-4y=23$

5. Write an equation of a line passing through the point representing solutions of the pair of linear equations $x+y=2$ and $2x-y=1$. How many such lines can be found?

6. The age of father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

5-MARKS

1. A two digit number is obtained by either multiplying the sum of digit by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the numbers.

2. Anuj decided to donate some money for books for the children living in an orphanage. If there are children less, everyone will get Rs. 20 more. If there are 7 children more everyone will get Rs.10 less. What is the number of children and how much does each get? What is the total amount distributed? Why Anuj decided to distribute money for books?

3. Raghav scored 70 marks in a test, getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been rewarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test?

4. Determine graphically, the vertices of the triangle formed by the lines $y=x$, $3y=x$ and $x+y=8$.

5. Solve graphically, the pair of equations $2x+y=6$ and $2x-y+2=0$. Find the ratio of the areas of the two triangle formed by the lines representing these equations with X-axis and the lines with Y-axis.

6. Draw the graph of the lines $x=-2$ and $y=3$. Write the vertices of the figure formed by these lines, the $x-axis$ and $y-axis$. Also find the area of the figure.

7. Determine, algebraically the vertices of the triangle formed by the lines. $5x-y=5$, $x+2y=1$ and $6x+y=17$.

CASE STUDY QUESTIONS

CASE STUDY_1:

A test consists of "True or False" questions. One mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

Type of Question	Marks given for correct answer	Marks deducted for wrong answer
True/False	1	0.25

(i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly? 1

(ii) How many questions did he guess? 1

(iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got? 2

OR

If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks? 2

LEVEL-3

MCQ:

1. Two numbers are in the ratio of 3:4. If 5 is subtracted from each other, then the ratio will be 2:3. What will be the smallest number?

   a) 15 b) 18 c) 20 d) 24

2. If $\frac{4x-3y}{7x-6y}=\frac{4}{13}$, then the value of $\frac{x}{y}$ is:

   a) $\frac{5}{8}$ b) $\frac{8}{5}$ c) $-\frac{5}{8}$ d) $-\frac{8}{5}$

3. In the equations shown below, a and b are unknown constants. $ax+2y=14$ & $2x+by=14$. If (-3, 4) is the solution of the given equations, what are the values of a and b?

   a) a = 2, b = 5 b) a = -2, b = 5 c) a = 5, b = 2 d) a = 5, b = -2

4. Consider the equations as shown: $(x-a)(y-b)=(x-2a)\left(y-\frac{b}{2}\right)$ and $x\left(x+\frac{1}{2b}\right)+y\left(y+\frac{a}{2}\right)-2xy=5+(x-y)^2$. On comparing coefficients, a student says these pair of equations is consistent. Is he/she correct? Which of these explains why?

a) Yes, because they are intersecting lines

b) Yes, because they are parallel lines

c) No, because they are parallel lines

d) No, because they are intersecting lines.

5. The solutions of the equation $\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$ is given by:

a) $x=2, y=1$ b) $x=1, y=1$ c) $x=-1, y=-1$ d) $x=1, y=2$

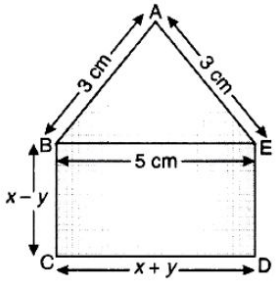
6. A farmer divides his herd of x cows among his 4 son's such that first son gets one-half of the herd, the second son gets one fourth, the third son gets one-fifth and the fourth son gets 7 cows, then the value of x is:

a) 100 b) 140 c) 160 d) 180

2-MARKS

1. Father's age is 3 times sum of the ages of his two children. After 5 years, his age will be twice the sum of the ages of two children. Find the age of the father.

2. In the adjacent figure ABCDE is a pentagon with $BE \parallel CD$ and $BC \parallel ED$. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm. Find the value of x & y.



3. Find the values of a and b for which the following system of linear equations has infinite solutions: $2x-(a-4)y=2b+1$ and $4x-(a-1)y=5b-1$.

4. Cars are parked in a parking place at a particular point of time in rows. If 3 cars are extra in a row, there would be one row less. If 3 cars are less in a row there would be 2 rows more. Find the number of cars in the parking place at that particular point of time.

5. At a certain time in a deer park, the number of heads and the number of legs of deer and human visitors were counted and it was found there were 39 heads and 132 legs. Find the number of deer and human visitors in the park.
6. Eight times a two digit number is equal to three times the number obtained by reversing the order if its digits. If the difference between the digit is 5, find the number.
7. The larger of two supplementary angles exceeds the smaller by 18^0, find them.
8. For what values of a and b will the following system of linear equations has infinitely many solutions? $2x-3y=7;\ (a+b)x-(9a+b-3)=4a+b$

3-MARKS

1. Find the value of p and q for which the system of equations represent coincident lines. $2x+3y=7$, and $(p+q+1)x+(p+2q+2)y=4(p+q)+1$
2. Solve the following system of linear equations by elimination method:
$2(ax-by)+(a+4b)=0$, $2(bc+ay)+(b-4a)=0$.
3. For what values of p and q will the following pair of linear equations has infinitely many solutions? $4x+5y=2;\ (2p+7q)x+(p+8q)y=2q-p+1$
4. Students are made to stand in rows. If one student is extra in a row there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.
5. For which value (s) of λ, do the pair of equations $\lambda x+y=\lambda^2$ and $x+\lambda y=1$ have (i) No solution? (ii) Infinitely many solutions? (iii) a unique solution?
6. Draw the graph of the pair of equations $2x+y=4$ and $2x-y=4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

5-MARKS

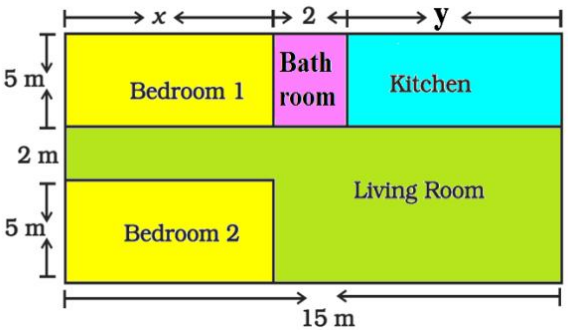
1. The length of the sides of a triangle are $2x+\frac{y}{2}, \frac{5x}{3}+y+\frac{1}{2}$ *and* $\frac{2}{3}x+2y+\frac{5}{2}$. If the triangle is equilateral. Find its perimeter.
2. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the coordinates of vertices of the triangle formed by these lines and x-axis.
3. Determine graphically, the coordinates of the vertices of a triangle whose equations are $2x+2y+6=0$, $2x+3y-18=0$, $y-2=0$. Also find the area of this triangle.

4. Check graphically, whether the pair of equations $x+3y=6$ and $2x-3y=12$ is consistent. If so, solve them graphically.
5. Ram decided to distribute some amount to poor students for their books. If there are 8 students less, everyone will get Rs. 10 more. If there are 16 students more everyone will get Rs. 10 less. What is the number of students and how much does each get? What is the total amount distributed? What is the reason that motivated Ram to distribute money for books?
6. The incomes of two persons A and B are in the ratio 8 : 7 and the ratio of their expenditures is 19 : 16. If their savings Rs. 2550 per month. Find their monthly income.
7. Jamila sold a table and a chair for Rs. 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit 25% on the table and 10% on the chair she would have got Rs. 1065. Find the cost price of each.
8. In an election contested between A and B, A obtained votes equal to twice the number of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 18000 persons on the electoral roll. How many voted for B.
9. When the son will be as old as the father today their ages will add upto 126 years. When the father was old as the son is today, their ages add upto 38 years. Find their present ages.
10. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of Rs.2 of 3 bananas and the second lot at the rate of Rs. 1 per banana and got a total of Rs. 400. If he had sold the first lot at the rate of Rs. 1 per banana, and the second lot at the rate of Rs. 4 for 5 bananas his total collection would have been Rs. 460. Find the total number of bananas he had.

CASE STUDY QUESTIONS

CASE STUDY_1:

In the below given layout, the design and measurements has been made such that area of two bedrooms and Kitchen together is 95 sq. m



(i) Find the area of two bedrooms and kitchen. 1

(ii) Find the length of the outer boundary of the layout. 1

(iii) Find the area of living room in the layout. 2

OR

Find the cost of laying tiles in Kitchen at the rate of Rs. 50 per sq.m 2

CASE STUDY_2:

Piyush sells a saree at 8% profit and a sweater at 10% discount, thereby getting a sum of Rs. 1008. If he had sold the saree at 10% profit and the sweater, he would have got Rs. 1028.

Denote the cost price of the saree and the list price (price before discount) of the sweater by Rs. x and Rs. y respectively and answer the following questions:

(i) Represent the 1st situation can be represented algebraically. 1

(ii) Represent the 2nd situation can be represented algebraically. 1

(iii) Name the point where the linear equation represented by 1st situation intersect the x-axis. 2

OR

Name the point where the linear equation represented by 2nd situation intersect the y-axis. 2

SKILL BASED QUESTIONS:

1. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

2. Draw the graphs of equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and Y-axis.