Mathematics 12th (Code-041)

General Instructions

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) Question paper is divided into FIVE sections - Section A, B, C, D and E.
(iii) In Section A, Question number 1 to 18 are Multiple Choice Questions (MCQ) and Question number 19 and 20 are Assertion-Reason based questions carrying 1 mark each.
(iv) In Section B, Question number 21 to 25 are Very Short Answer (VSA) type questions carrying 2 marks each.
(v) In Section C, Question number 26 to 31 are Short Answer (SA) type questions carrying 3 marks each.
(vi) In Section D, Question number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Question number 36 to 38 are Case Study Based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculator is NOT allowed.

Time : 3 Hrs.

Max. Marks : 80

Section A Multiple Choice Questions *(Each Que. carries 1 M)*

1. The cartesian equation of the line $\vec{r}=(2\hat{i}+\hat{j})+\lambda(\hat{i}-\hat{j}+4\hat{k})$ is

(a) $\frac{x-2}{1}=\frac{y-1}{1}=\frac{z}{4}$ (b) $\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z}{4}$

(c) $\frac{x+2}{1}=\frac{y+1}{1}=\frac{z}{2}$ (d) $\frac{x-2}{1}=\frac{y+1}{-1}=\frac{z}{4}$

2. If $P(A)=\frac{1}{2}$ and $P(B)=0$, then $P\left(\frac{A}{B}\right)$ is equal to

(a) 1 (b) 0

(c) not defined (d) 0.5

3. If A and B are two independent events, then $P(A\cap\overline{B})$ is equal to

(a) $P(A)-P(A)P(B)$

(b) $P(\overline{A})-P(A)P(B)$

(c) $P(A)-P(\overline{A})P(B)$

(d) $P(A)-P(A)\,P(\overline{B})$

4. The anti-derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equal to

(a) $\frac{2}{3}x^{2/3}+2\sqrt{x}+C$ (b) $\frac{2}{3}x^{3/2}+2\sqrt{x}+C$

(c) $\frac{2}{3}x^{3/2}-2\sqrt{x}+C$ (d) $\frac{3}{2}x^{3/2}+2\sqrt{x}+C$

5. If the sides of an equilateral triangle are increasing at the rate of 8 cm/s, then the rate at which the area increase when side is 9 cm, is

(a) $36\text{ cm}^2/\text{s}$ (b) $36\sqrt{2}\text{ cm}^2/\text{s}$

(c) $36\sqrt{3}\text{ cm}^2/\text{s}$ (d) $12\sqrt{3}\text{ cm}^2/\text{s}$

6. Direction cosines of the vector $-2\hat{i}+\hat{j}-5\hat{k}$ are

(a) $\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}$ (b) $\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$

(c) $\frac{-3}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$ (d) $\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}$

7. The least value of a, such the function f given by $f(x)=x^2+ax+1$ is strictly increasing on (1, 2) is

(a) -1 (b) -2

(c) 0 (d) 1

8. The general solution of
$$\frac{dy}{dx}=1+x^2+y^2+x^2y^2$$
(given that C is the constant of integration)

(a) $\tan^{-1}x=y+\frac{y^3}{3}+C$ (b) $\tan^{-1}y=x+\frac{x^3}{3}+C$

(c) $\tan^{-1}x=\tan^{-1}y+C$ (d) $\tan^{-1}x+\tan^{-1}y=C$

9. If $A=[2\ \ -3\ \ 4]$, $B=\begin{bmatrix}3\\2\\2\end{bmatrix}$, $X=[1\ \ 2\ \ 3]$ and $Y=\begin{bmatrix}2\\3\\4\end{bmatrix}$, then $AB+XY$ is equal to

(a) [28] (b) [24]

(c) 28 (d) 24

10. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is

(a) $\frac{15}{36}$ (b) $\frac{15}{46}$ (c) $\frac{15}{56}$ (d) $\frac{1}{2}$

11. $(\hat{k}\times\hat{j})\cdot\hat{i}+\hat{j}\cdot\hat{k}$ is equal to

(a) -1 (b) 1 (c) 0 (d) -2

12. $\int\tan^2x\cdot dx$ is equal to

(a) $\tan x+x+C$ (b) $\tan x-x+C$

(c) $-\tan x+x+C$ (d) $-\tan x-x+C$

13. If $y=\log(\sin x)$, then $\frac{dy}{dx}$ is equal to

(a) $\tan x$ (b) $\cot x$ (c) $\sec x$ (d) $\tan^2x$

14. If $\begin{vmatrix}x & 9\\6 & 3\end{vmatrix}=\begin{vmatrix}8 & 4\\5 & 2\end{vmatrix}$, then $\frac{x}{2}$ is equal to

(a) $\frac{25}{2}$ (b) $\frac{25}{3}$

(c) $\frac{50}{3}$ (d) All of these

15. Let A and B be the events associated with the sample space S, then the value of $P(A/B)$ lies in the interval

(a) (0, 1) (b) [0, 1] (c) (0, 1] (d) [0, 1)

16. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$

(a) 1 (b) 2 (c) 3 (d) 4

17. Area of the region bounded by the curve $y^2=4x$, Y-axis and the line $y=3$ is

(a) 2 sq units (b) $\frac{9}{4}$ sq units

(c) $\frac{9}{3}$ sq units (d) $\frac{9}{2}$ sq units

18. The direction ratios of the line $\frac{4-x}{2}=\frac{y-2}{6}=\frac{z+5}{-3}$

(a) 2, 6, -3 (b) 2, -6, 3

(c) -2, 6, -3 (d) 6, -2, 3

Assertion-Reason Based Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion** (A) Scalar matrix $A=[a_{ij}]=\begin{cases}k; & i=j\\0; & i\neq j\end{cases}$, where k is a scalar, is an identity matrix when $k=1$.

Reason (R) Every identity matrix is not a scalar matrix.

20. **Assertion** (A) The relation R on the set $N\times N$ defined by $(a,b)\,R\,(c,d)\Leftrightarrow a+d=b+c$, for all $(a,b),(c,d)\in N\times N$ is an equivalence relation.

Reason (R) Any relation is an equivalence relation, if it is reflexive, symmetric and transitive.

Section B Very Short Answer Type Questions (Each Que. carries 2 M)

21. If $AB = BA$ for any two square matrices, then prove by mathematical induction that $(AB)^n = A^n B^n$.

Or If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then show that

$$(A-2I)(A-3I)=O.$$

22. Find the position vector of a point R which divides the line joining the points $P(\hat{i}+2\hat{j}-\hat{k})$ and $Q(-\hat{i}+\hat{j}+\hat{k})$ in the ratio 2 : 1
(i) internally. (ii) externally.

23. Show that the points $(a+5, a-4)$, $(a-2, a+3)$ and (a, a) do not lie on a straight line for any value of a.

Or If (a, b), (a', b') and $(a-a', b-b')$ are collinear, then prove that $ab' = a'b$.

24. Find the direction cosines of the line
$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}.$$

25. The cartesian equation of a line is
$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}.$$
Find the vector equation of the line.

Section C Short Answer Type Questions (Each Que. carries 3 M)

26. Find the inverse of the matrix
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}.$$

27. Find the particular solution of the differential equation $(1+e^{2x})\,dy + (1+y^2)e^x\,dx = 0$, given that $y=1$, when $x=0$.

Or Solve the following differential equation

$$y^2dx + (x^2 - xy + y^2)dy = 0.$$

28. Find the angle between the lines
$$\frac{x}{2}=\frac{y}{2}=\frac{z}{1} \text{ and } \frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}.$$

Or Find the angle between the lines with direction ratios proportional to 4, 5, 2 and 5, 2, 4.

29. A closed right circular cylinder has volume 539 / 2 cu units. Find the radius and the height of the cylinder so that the total surface area is minimum.

30. Find the value of
$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right].$$

31. Find the value of a, for which the function
$$f(x)=\begin{cases} \dfrac{\sqrt{1+ax}-\sqrt{1-ax}}{x}, & \text{if } -1 \le x < 0 \\ \dfrac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$ is
continuous at $x=0$.

Or If $x = a(2t - \sin t)$ and $y = a(1-\cos t)$, then find $\frac{dy}{dx}$, when $\theta = \frac{\pi}{6}$.

Section D Long Answer Type Questions (Each Que. carries 5 M)

32. Using integration, find the area of the region bounded by the curves $y = |x+1|+1$, $x=-3, x=3$ and $y=0$.

33. Evaluate $\displaystyle\int_0^{\pi/4} \frac{\sin x + \cos x}{9+16\sin 2x}dx$

Or Evaluate $\displaystyle\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}dx$

34. Determine graphically the minimum value of the objective function $Z = -50x + 20y$, subject to constraints are $2x - y \ge -5$, $3x + y \ge 3, 2x - 3y \le 12$ and $x \ge 0, y \ge 0$.

Or

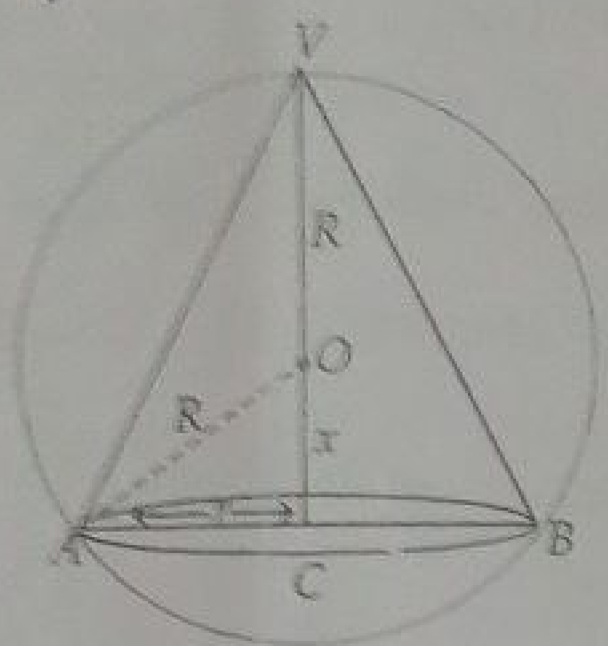
Find graphically the maximum value of $Z = 2x + 5y$, subject to constraints given below

$$2x + 4y \le 8, 3x + y \le 6, x + y \le 4, x \ge 0, y \ge 0.$$

35. If $y = \tan^{-1}(\sec x + \tan x)$, then find $\frac{dy}{dx}$.

Section E Case-Study/Passage-based *(Each Que. carries 4 M)*

36. Let a cone be inscribed in a sphere of radius R. The height and radius of cone are h and r, respectively.



On the basis of above information, answer the following questions.

(i) Write the relation between r and R in terms of x.

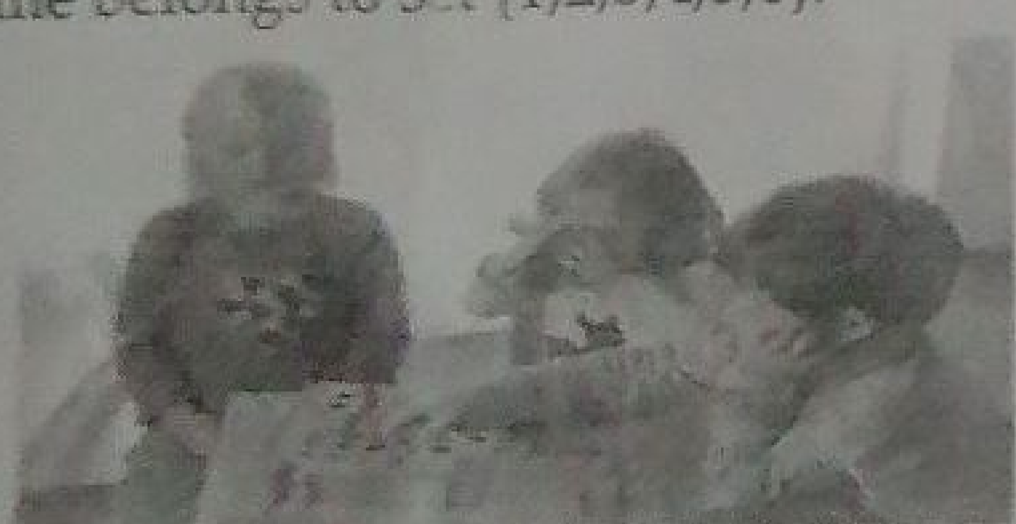
(ii) Write the volume V of the cone in terms of R and x.

(iii) Show that volume V of the cone is maximum, when $x = \frac{R}{3}$.

Or

If volume V of the cone is maximum at $x = \frac{R}{3}$, then find the maximum value of V and find the ratio of volume of cone and volume of sphere, when volume of cone is maximum,

37. Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1,2,3,4,5,6\}$.



Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1,2,3,4,5,6\}$

On the basis of above information, answer the following questions.

(i) Show that the relation R on B, defined by $R = \{x, y : y$ is divisible by $x\}$ is reflexive, transitive but not symmetric.

Or Let R be a relation on B defined by $R = \{(1,2), (2,2), (1,3), (3,4), (3,1), (4,3)$ $(5,5)\}$. Then, show that R is reflexive and transitive but not symmetric.

(ii) Raji wants to know the number of functions from A to B. How many number of functions are possible?

(iii) Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

38. A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots

From this situation answer the following:

(i) Let the target be hit by A, B: the target is hit by B and C: the target is hit by A and C. Then, find the probability that A, B and C all will hit.

(ii) With reference to the events mentioned in (i), what is the probability that 'any two of A, B and C will hit'?