PRE BOARD EXAMINATION - 2, DECEMBER – 2024-25

(SET 1)

NAME :

DATE :

CLASS & SECTION: XII

SUBJECT: MATHEMATICS(041)

MAX.MARKS: 80 MARKS

TIME: 3 HOURS

General Instructions:

1. This Question Paper has 5 Sections A, B. C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study based questions carrying 4 marks each
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 question in Section D and 3 questions in Section E
8. Use of calculator is not allowed.

I

SECTION A (20 x 1 =20)

1 If A is a square matrix of order 3, $|A^T| = -3$, then $|AA^T| =$

(a) 9 (b) -9 (c) 3 (d) -3

$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ $\begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

2 From the set $\{1, 2, 3, 4, 5, 6\}$, two numbers x and y $(x \neq y)$ are chosen at random. The probability that $\frac{x}{y}$ is an integer is

(a) $\frac{6}{25}$ (b) $\frac{7}{25}$ (c) $\frac{9}{25}$ (d) $\frac{8}{25}$

3 Equation of line passing through the origin and making $30^o, 60^o$ $and 90^o$ with x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$ (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$ (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$ (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

4 Given that A is a square matrix of order 3 and $|A| = -2$, then $|adj(2A)|$ is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

5 $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) -1 (d) 1

6 If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=5$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is

(a) 0 (b) 1 (c) -19 (d) 38

7 $\int e^x\left(\log\sqrt{x}+\frac{1}{2x}\right)dx =$

(a) $e^x \times \log x + C$ (b) $e^x \times \log\sqrt{x} + C$ (c) $e^x \times \frac{1}{2x} + C$ (d) $\frac{e^x}{\log\sqrt{x}} + C$

8 The vector in the direction of the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 9 is

(a) $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (b) $\frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$

(c) $3(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ (d) $9(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

9 Let $f(x) = \begin{cases} ax^2+1, & x>1 \\ x+\frac{1}{2}, & x\le 1 \end{cases}$ then f is differentiable at x = 1 if a =

(a) 2 (b) −2 (c) 1 (d) −1

10 If $A^2 - A + I = 0$ then the inverse of A is

(a) A + I (b) I − A (c) A − I (d) None of these

11 If $\int e^{-2\log x}\,dx = f(x) + C$, then $f(x)$ is

(a) $\frac{e^{-2\log x+1}}{-2\log x+1}$ (b) $\frac{1}{x^2}$ (c) $-\frac{1}{x}$ (d) $-2\log x\, e^{-2\log x+1}$

12 If $\hat{\imath},\ \hat{\jmath},\ \hat{k}$ are unit vectors then

(a) $\hat{\imath}.\hat{\jmath} = 1$ (b) $\hat{\imath}.\hat{\imath} = 1$ (c) $\hat{\imath} \times \hat{\jmath} = 1$ (d) $\hat{\imath} \times (\hat{\jmath} \times \hat{k}) = 1$

13 A family has 2 children and the elder child is a girl. The probability that both children are girls is: GB, GG

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

14 If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric matrix and Q is skew symmetric matrix then Q is equal to

(a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

15 If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{y}{x}$ (b) $\frac{y}{x}$ (c) $sec^2\frac{y}{x}$ (d) $-sec^2\frac{y}{x}$

16 Which of the following points satisfies both the inequalities $2x + y \le 10$ and $x + 2y \ge 8$?

(a) (−2, 4) (b) (3, 2) (c) (−5, 6) (d) (4, 2)

17 The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If Z = 30x + 24y is the objective function, then (maximum value of Z – minimum value of Z) is equal to

(a) 40 (b) 96 (c) 144 (d) 136

18 The order of the matrix A such that $\begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 3 & 4 \end{bmatrix}_{3\times 2} A = \begin{bmatrix} 1 & -8 \\ 1 & -2 \\ 9 & 2 \end{bmatrix}_{3\times 2}$, is

3x2 2x2 2

(a) 2 x 3 (b) 3 x 2 (c) 2 x 2 (d) 1 x 2

ASSERTION-REASON BASED QUESTIONS

In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19 **Assertion (A)** :The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin\left(\frac{dy}{dx}\right) + 1 = 0$ is 3

Reason (R) : The highest power of the highest order derivative involved in a differential equation, when it is written as a polynomial in derivatives, is called its degree. d)

20 **Assertion (A) :** Let $X = \{1, 2, 3\}$ and $S: X \to X$ such that $S = \{(1,1), (2,2), (3,3), (1,2)\}$. Then, the relation S is a reflexive relation on X.

Reason (R) : A relation S defined in a set X is called symmetric relation. if $(a, b) \subset S$ implies $(b, a) \in S$ for all $a, b \in X$. b)

SECTION B

(5 x 2 =10)

21 A box contains 10 tickets, 2 of which carry a prize of ₹ 6 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

OR

10% of the bulbs produced by the factory are of red colour and 2% are of red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

22 The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

23 Find the value of $\cos^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \operatorname{cosec}^{-1}(2)$

OR

Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Also, write range of f(x).

24 Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.

25 If $\vec{p} = 5\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$ and $\vec{q} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then find the value of λ, so that $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ are perpendicular vectors.

SECTION C

(6 x 3 =18)

26 Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

OR

Find $\frac{dy}{dx}$, if $y = (\cot x)^x + \sin^{-1}\sqrt{x}$

27 If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three sides of a triangle, then show that

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

28 Evaluate $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}\, dx$

OR

Evaluate $\int_0^{\pi} \frac{1}{1+e^{\sin x}}\, dx$

29 Solve the following linear programming problem graphically.

Maximise Z = 3x + 9y

Subject to the constraints

$x + y \geq 10$

$x + 3y \leq 60$

$x \leq y$

$x \geq 0, y \geq 0$

30 Find the particular solution of the differential equation $2y\, e^{\frac{x}{y}}\, dx + \left(y - 2x\, e^{\frac{x}{y}}\right) dy = 0$, given that $x = 0$ when $y = 1$

OR

Find the general solution of the differential equation $(x-1)\frac{dy}{dx} = 2x^3 y$.

31 Find : $\int \frac{2x+1}{\sqrt{3+2x-x^2}}\, dx$.

SECTION D

(4 x 5 =20)

32 Find the image of the point (2, -1, 5) in the line $\frac{11-x}{-10} = \frac{y+2}{-4} = \frac{z+8}{-11}$

OR

Vertices Q and R of ΔPQR lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of ΔPQR given that point P has coordinates $(1, -1, 2)$ and the line segment QR has length of 5 units.

33 Find the points of local maxima and local minima, of the function,

$f(x) = \sin x - \cos x$, $0 < x < 2\pi$. Also find the local maximum and local minimum values.

34 If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, Find A^{-1}. Hence, solve the system of equations

$3x + 3y + 2z = 1$

$x + 2y = 4$

$2x - 3y - z = 5$

OR

If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$; $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$

35 Using integraton, find the area of the smaller region bounded by the curves $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $\frac{x}{5} + \frac{y}{3} = 1$.

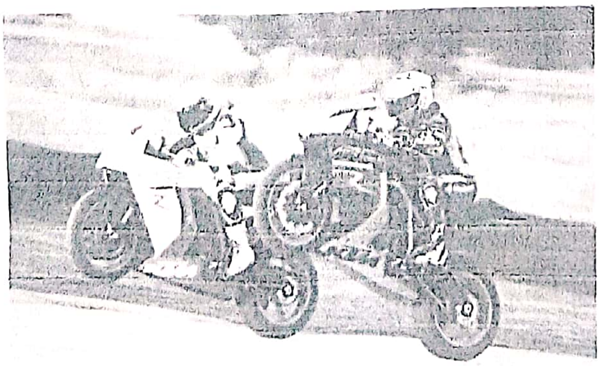
SECTION E

(3 x4 =12)

36 **CASE STUDY 1 : Read the following passage and answer the questions given below.**

An organisation conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(i) How many relations are possible from B to G ? b 1

(ii) Among all the possible relations from B to G, how many functions can be formed from B to G? 2 1

(iii) (a) Let R: B → B be a relation from B to B as $R_1 = \{(b_1, b_2), (b_2, b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes 2

(I) reflexive but not symmetric (b_1, b_3)

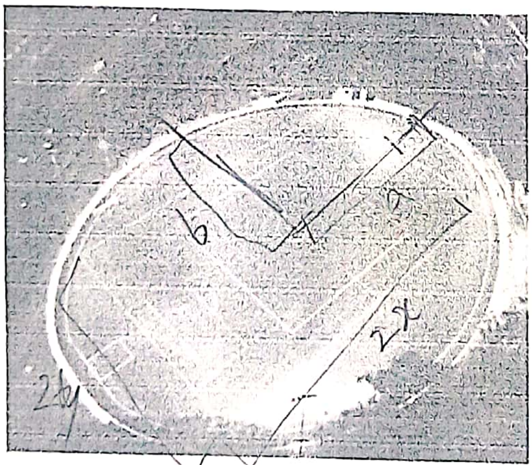
(II) reflexive and symmetric but not transitive. $(b_1,b_1), (b_2,b_2), (b_3,b_3)$ $(b_1,b_2), (b_2,b_1), (b_1,b_3), (b_3,b_1)$ $(b_2,b_3), (b_3,b_2)$

OR

(b) If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; (where $0 \leq x \leq 20\sqrt{2}$ *and* $0 \leq y \leq 200$), then state whether the track represents a one-one and onto function or not. (Justify).

37 **Case Study 2 :** Read the following passage and answer the questions given below

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



Based on the above information, answer the following questions :

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x. 1

(ii) Find the critical point of the function. 1

(iii) (a) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area 2

OR

(b) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area

38 **Case Study 3 :** A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80

Let E_1: represent the event when many workers were not present for the job ;

E_2: represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job? 1

(ii) What is the probability that construction will be completed on time? 1

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time? 2

OR

(b) What is the probability that all workers were present given that the construction job was completed on time?