NARAYANA GROUP OF SCHOOLS

44 YEARS OF EXCELLENCE

CBSE

CLASS-X

MATHEMATICS

CENTUM BOOK

PHASE- II

MATHEMATICS

Code-(041)

Class-X (2023-24)

COURSE STRUCTURE

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
Total		80

QUESTION PAPER DESIGN (Standard)

Time: 3 Hours **Max. Marks: 80**

S.No	Typology of Questions	Total Marks	% Weightage (approx.)
1	Remembering: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. Understanding: Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	43	54
2	Applying: Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24

3	Analysing: Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations Evaluating: Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. Creating: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	80	100

Methods of Questioning-Skills in focus

1. REMEMBERING

Questions where a child just recalls the information learnt.

Direct question where student uses the formula or writes a statement etc.

Eg: Find the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$.

Eg: Evaluate $\sin 30^0 \cos 60^0 - \cos 30^0 \sin 60^0$.

2. UNDERSTANDING

Questions which need interpretation of information given

Questions where we want to test whether a student has understood a concept.

Eg: Prove that "If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio".

Eg: Check whether $(x+2)^3 = x^3 - 4$ is quadratic equation.

3. APPLYING

Questions where application of acquired knowledge in a different situation is needed.

Indirect questions where a child has to facts or rules in a different way.

Eg: Representing the following situation in the form of quadratic equation: The area of a rectangular plot is 528 m2. The length of the plot is 1 metre more than its breadth. We need to find the length and breadth of the plot.

Eg: Find the radius of the largest circle that can be inscribed in the middle space, given that the radius of the outer circle is 10cm.

4. ANALYSING

Questions where a student has to examine and break the given information into parts to make inferences and find evidence to support generalizations.

Eg: Find the two numbers whose sum is 27 and the product is 182.

Eg: Find the two consecutive positive integer's sum of whose squares is 613.

5. EVALUATING:

Questions where a child has to make judgement or opinions after analysing information. Questions where child has to validate the ideas based on a set of criteria.

Eg: It is given that $\Delta ABC \sim \Delta DEF$. Is it true to say that $\frac{CB}{DE} = \frac{BA}{EF}$. Justify your answer.

Eg: Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

6. CREATING

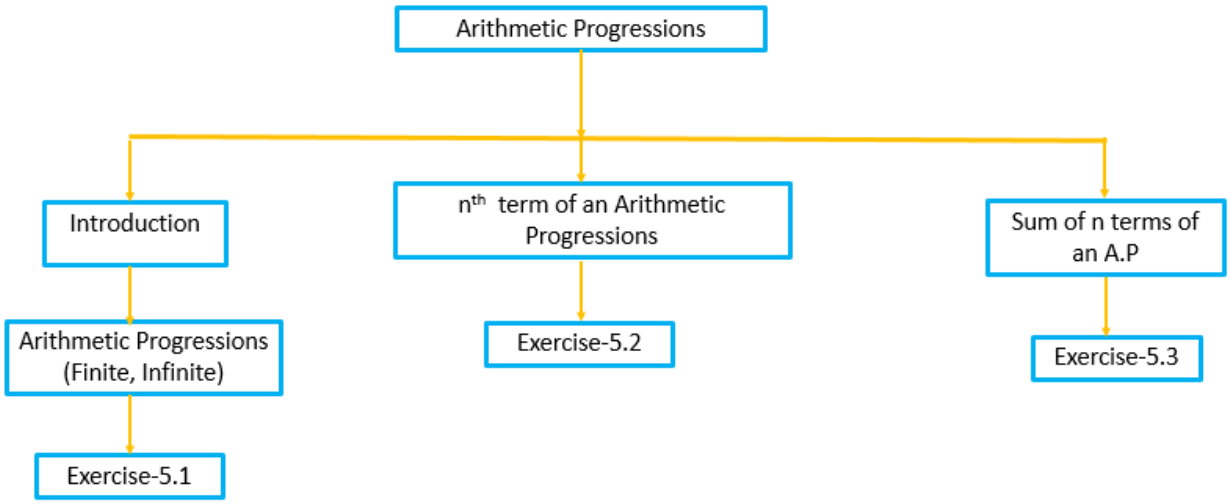
Questions where given information is presented in a different way or a different approach is used to solve the question.

Eg: If the time is taken along X-axis and speed on Y-axis, the coordinates of points O, A, B are (0, 0), (10, 0), (10, 10) respectively, then find the area of triangle OAB. What does the area OAB represent? Justify.

Eg: Solve $x^2 + x - (a-2)(a-3) = 0$.

ARTIHMETIC PROGRESSIONS

MIND MAPPING



BASIC FACTS AND FORMULAE:

1. An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number d to the preceeding term, except the first term.
2. The difference between the two successive terms of an A.P. is called the common difference.
3. **Condition for a list of numbers to be in A.P:** The numbers $a_1,\ a_2,\ a_3, \ldots$ are said to be in A.P., if $a_2 - a_1 = a_3 - a_2 = \ldots\ldots\ldots\ldots$
4. The General Form of an A.P: The general form of an A.P. with the first term a and the common difference d is given by $a,\ a+d,\ a+2d,\ a+3d, \ldots.$
5. **The General or n^{th} term of an A.P:** In an A.P. with first term a and the common difference d, the General term or n^{th} term is given by $a_n = a + (n-1)d$
6. **Selection of Terms in an A.P:**

   i) For three terms, we take $a-d,\ a,\ a+d$

   ii) For four terms, we take: $a-3d, a-d, a+d, a+3d$

   iii) For five terms, we take $a-2d, a-d, a, a+d$ and $a+2d$ and so on.
7. n^{th} term from the end of an A.P. is given by $l-(n-1)$d, where l is last term, d is common difference and n is number of terms.

8. Sum of n terms of an A.P. is given by $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, where '$a$' is the first term, d is the common difference and n is the total number of terms.

9. Sum of n terms of an A.P. is also given by $S_n = \frac{n}{2}\left[a + l\right]$, where a is the first term, and l is the last term.

10. If a, b, c are in A.P, then $b = \frac{a+c}{2}$ and b is called the arithmetic mean of a and c

11. n^{th} **term in terms of** S_n**:** If the sum S_n of n^{th} term of an A.P. is given, then the n^{th} term of the A.P is given by $a_n = S_n - S_{n-1}$, for $n > 1$.

LEVEL – I

MCQ:

1. The 10th term of the sequence $\sqrt{3}, \sqrt{12}, \sqrt{27}$.... is

   a) $\sqrt{243}$ b) $\sqrt{300}$ c) $\sqrt{363}$ d) $\sqrt{432}$

2. The 8th term of an A.P is 17 and its 14th term is 29. The common difference of this AP is:

   a) 3 b) 2 c) 5 d) -2

3. The sum of first 100 even natural numbers is:

   a) 10100 b) 2550 c) 5050 d) 10010

4. The seventh term of an A.P whose first term is 28 and the common difference -4 is:

   a) 0 b) 4 c) 52 d) 56

5. How many terms are there in the A.P. given below?

   14, 19, 24, 29.....,119

   a) 18 b) 14 c) 22 d) 21

6. If -5, x, 3 are three consecutive terms of an A.P., then the value of x is:

   a) -2 b) 2 c) 1 d) -1

7. The common difference of the A.P. whose nth term is given by $a_n = 3n + 7$, is:

   a) 7 b) 3 c) 3n d) 1

8. The 11th term from the end of the A.P.: 10, 7, 4,……., -62 is:

a) 25 b) 16 c) -32 d) 0

9. If p-1, p+1 and 2p+3 are in A.P., then the value of p is:

a) -2 b) 4 c) 0 d) 2

10. Which of the following is an infinite AP?

i) -2, -1, 0, 1, 2, 3, 4, 5, 6, 7…

ii) -14, -12, -10, -8, ___, ____, ____, 0

iii) First 40 even numbers

iv) Odd Numbers

a) i) & ii) b) ii) & iii) c) i) & iv) d) ii) & iv)

2-MARKS:

1. How many two - digit numbers are divisible by 3?
2. In an A.P. it is given that common difference is 5 and sum of its first ten terms is 75. Find the first term of the A.P.
3. Find the sum of first 8 multiples of 3.
4. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots.$ is the first negative term?
5. Which term of the AP: 3, 15, 27, 39 . . . will be 132 more than its 54th term?
6. For what value of n, are the n^{th} terms of two A.P's 63, 65, 67, … and 3, 10, 17, … equal?
7. The 4th term of an $A.P$ is zero, then prove that the 25th term of the A.P. is three times its 11th term.
8. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms.
9. The nth term of an A.P. is $7 - 4n$. Find its common difference.
10. Find the number of terms in the A.P. $18, 15\frac{1}{2}, 13, \ldots\ldots, -47$
11. Check whether -150 is a term of the AP: $11, 8, 5, 2 \ldots.$

12. Find the sum of the odd numbers between 0 and 50.

13. Find the common difference of an A.P, whose first term is $\frac{1}{2}$ and the 8th term is $\frac{17}{6}$. Also write its 4th term.

14. The 7^{th} term of an A.P. is 32 and its 13th term is 62. Find the A.P.

3-MARKS:

1. Find the sum of all 11 terms of an A.P. whose middle most term is 30.

2. Find a, b and c if it is given that the numbers a, 7, b, 23, c are in A.P.

3. The first term of an A.P. is 5, the last term is 45 and sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

4. If the 10th term of an A.P. is 52 and the 17th term is 20 more than the 13th term, find the A.P.

5. How many terms of an A.P. $9, 17, 25, \ldots\ldots$ must be taken to give a sum of 636?

6. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

7. The 14th term of an AP is twice its 8th term. If its 6th term is –8, then find the sum of its first 20 terms.

8. If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, show that its 18th term is zero.

9. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and the nth term.

10. In an A.P. the first term is –4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

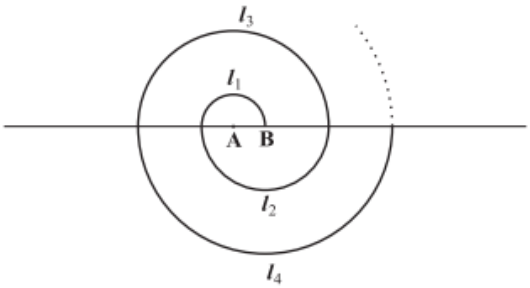
11. If the sum of the first n terms of an AP is $4n - n^2$, what is first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, 10th and the nth terms.

12. If the n^{th} term of an A.P. is $(2n+1)$, find the sum of first n terms of the A.P.
13. How many three digit numbers are divisible by 7?
14. Find whether 55 is a term of the AP 7,10,13,.....or not. If yes, find which term it is.
15. Find the 20^{th} term from the last term of the AP: 3, 8, 13,....., 253.

5-MARKS:

1. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.
2. How many terms of the AP: 24, 21, 18,... must be taken so that their sum is 78?
3. If m times the m^{th} term of an Arithmetic Progression is equal to n times its n^{th} term and $m \neq n$. show that the $(m+n)^{th}$ term of the A.P. is zero.
4. The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first the third term is 5 times the common difference, find the three numbers.
5. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and last term to the product of two middle terms is $7:15$. Find the numbers.
6. If the ratio of the sum of the first n terms of two A.P s is $(7n+1):(4n+27)$, then find the ratio of 9^{th} terms.
7. Find the 60^{th} term of the A.P., 8,10,12, ... if it has a total of 60 terms and hence find the sum 1 last 10 terms.
8. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find: (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years

9. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm . . . as shown in Fig. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$).



10. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

CASE STUDY QUESTIONS

CASE STUDY_1

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6^{th} year and 22600 in 9^{th} year.

Based on the above information answer the following questions:

(i) Find the production during 8^{th} year. 1

(ii) In which year the production is Rs. 29,200? 1

(iii) Find the production during first 3 years. 2

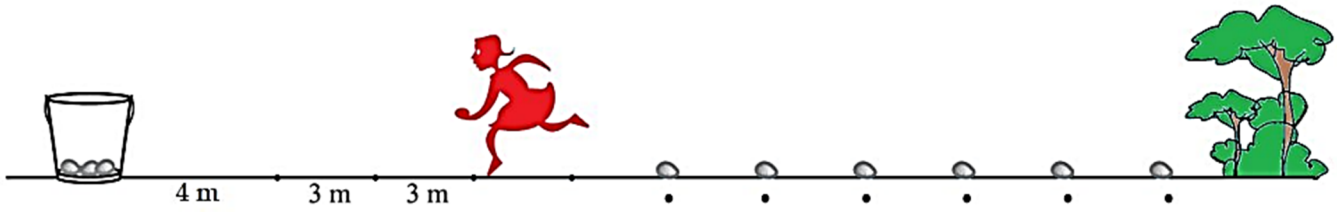
OR

Find the difference of the production during 7^{th} year and 4^{th} year 2

CASE STUDY_2

In a potato race, a bucket is placed at the starting point, which is 8 m from the first potato, and the other potatoes are placed 6m apart in a straight line. There are ten potatoes in the line (see below figure).

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket.



Based on the above information, answer the following questions:

(i) What is the distance covered by the competitor in the first potato? 1

(ii) What is the distance covered by the competitor in second potato? 1

(iii) What is total distance covered by the competitor in first & second potato? 2

OR

If the A.P. 8, 14, 20, ... then find the common difference. 2

CASE STUDY_3

Aditya is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 120 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on...

Based on the above information, answer the following questions:

i) Form an A.P for the above given situation? 1

ii) Find the difference in number of candies placed in 7^{th} and 3^{rd} row. 1

iii) Find the total number of rows of candies. 2

OR

iv) If Aditya decides to make 15 rows, then how many total candies will be placed by him with the same arrangement? 2

LEVEL – II

MCQ:

1. What would be the last term of an A.P with 20 terms whose 4^{th} term is -46 and third term is -70?

   a) -338 b) 338 c) 142 d) -142

2. If the first term of an AP is 2 and the common difference is $\frac{-1}{2}$, what would be the 12th term of the AP?

   a) $2+11\left(-\frac{1}{2}\right)$ b) $2+12\left(-\frac{1}{2}\right)$

   c) $2-11\left(-\frac{1}{2}\right)$ d) $2-12\left(-\frac{1}{2}\right)$

3. Find the n^{th} term of the AP shown below. __, __, -36, __, -44, ... up to n.

   a) 4n-32 b) -40+4n c) 24-4n d) -4n-24

4. Jessie needs ₹1,70,000 for her College admissions in the starting of January 2021.Her mother helped her by creating a fund of ₹12,000 in the end of January 2019. Thereafter she has been collecting ₹5500 in the starting of each month for Jessie's college fund. How much money will be collected in the fund before Jessie's admissions?

a) ₹[12000 + 11(5500)] b) ₹[5500 + 11(12000)]

c) ₹[12000 + 23(5500)] d) ₹[5500 + 23(12000)]

5. Mr. Kapoor is collecting donations for building a new school building in a village. His wife starts by donating ₹125000. His brother also contributes by donating ₹54000. Thereafter every person who contributes in the donation pays ₹5250 more than the previous donor. How much amount has Mr. Kapoor able to collect after 23 donations?

a) ₹164250 b) ₹2400750

c) 2525750 d) ₹2570250

6. What is the sum of all the three-digit numbers divisible by 12?

a) $\left[\frac{73}{2}(108+996)\right]$ b) $\left[\frac{75}{2}(216+996)\right]$

c) $\left[\frac{73}{2}(216+996)\right]$ d) $\left[\frac{75}{2}(108+996)\right]$

7. In an AP, if d = -4, n = 7, a_n = 4, then a is equal to

a) 6 b) 7 c) 20 d) 28

8. In an AP, if a = 3.5, d = 0, n = 101, then a_n will be

a) 0 b) 3.5 c) 103.5 d) 104.5

9. The 11th term of the AP: -5, −5/2, 0, 5/2,...

a) -20 b) 20 c) -30 d) 30

10. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?

a) 30 b) 33 c) 37 d) 38

2-MARKS:

1. Find the number of numbers lying between 146 and 300 which are divisible by both 3 and 5.
2. Find the middle most term of the A.P. -11, -7, -3,.....49.
3. Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7:15.
4. Find the sum of first 30 terms of an A.P. whose second term is 2 and seventh term is 22.
5. If for a given A.P, $a = 7, a_{13} = 35$ find S_{13}.
6. If the sum of first q terms of an AP is $2q + q^2$, find the common difference of the A.P.
7. Which term of the A.P, 120, 116, 112,..... is first negative term?
8. Find the 20^{th} term from the end of the A.P. 23, 18, 13.....-253.
9. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.
10. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?
11. The sum of three numbers in A.P. is -3 and their product is 8. Find the numbers.
12. Justify whether it is true to say that the following are the n^{th} terms of an AP,

    i) $2n - 3$ ii) $3n^2 + 5$ iii) $1 + n + n^2$

12. The n^{th} term of an AP cannot be $n^2 + 1$. Justify your answer.

3-MARKS:

1. If in an A.P. the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m + n)$ terms is $-(m + n)$
2. Find the sum of all natural numbers from 100 to 200 which are divisible by 4.

3. Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by $a_n = 3 + 2n$.

4. Find the sum of all numbers between 250 & 1000 which are exactly divisible by 3.

5. Find the sum of i) the first 1000 positive integers ii) the first n positive integers

6. If the m^{th} term of an A.P. be $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$, then show that its $(mn)^{th}$ term is 1.

7. The sum of 4^{th} and 8^{th} terms of an A.P. is 24 , and sum of 6^{th} and 10^{th} is 44 . Find the first three terms of the A.P.

8. The 8^{th} term of an A.P. is 20 and the 15^{th} term is 12 more than the 11^{th} term. Find the A.P.

9. Determine k so that $k^2+ 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.

10. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

11. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.

12. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer.

13. Determine k so that $k^2 + 4k + 8, 2k^2 + 3k + 6, 3k^2 + 4k + 4$ are three consecutive terms of an AP.

14. Find the sum of all the 11 terms of an AP whose middle most term is 30.

5-MARKS:

1. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7:15. Find the numbers.

2. The sum of first 6 terms of an A.P. is 42. The ratio of its 10^{th} term to 30^{th} term is 1:3. Find the first and the 13^{th} term of the A.P.

3. Solve the equation: $1 + 4 + 7 + 10 + \cdots + x = 287$.

4. Find the sum of 25 terms of an A.P. in which the third term is 7 and 7^{th} term is two more than thrice of its third term.

5. Find the sum of n terms of the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots a + (n-1)d$

6. The 8^{th} term of an A.P. is half the second term and the 11^{th} term exceeds one third of its fourth term by 1. Find the 15^{th} term.

7. If p^{th} , q^{th} and r^{th} terms of an A.P. are a, b, c respectively, then show that

   i) $a(q-r) + b(r-p) + c(p-q) = 0$ ii) $(a-b)r + (b-c)p + (c-a)q = 0$

8. If S_1, S_2, S_3 be the sum of n, $2n$, $3n$ terms respectively on an A.P. Prove that $S_3 = 3(S_2 - S_1)$.

9. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of two middle terms is 7:15. Find the numbers.

10. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

11. The eighth term of an AP is half its second term and the eleventh term exceeds onè third of its fourth term by 1. Find the 15^{th} term.

CASE STUDY QUESTIONS

CASE STUDY-1

A road roller (sometimes called a roller- compactor, or just roller) is a compactor - type engineering vehicle used to compact soil, gravel, concrete or asphalt in the construction of roads and foundations. Similar rollers are used also at landfills or in agriculture. Road rollers are frequently referred to as steamrollers, regardless of their method of propulsion.

RCB machine Pvt Ltd started making road roller 10 years ago. Company increased its production uniformly by fixed number every year. The company produces 800 roller in the 6th year and 1130 roller in the 9th year.

Based on the above information, answer the following questions:

i) What was the company's production in first year? 1

ii) What was the company's production in the 8th year? 1

iii) What roller the company's total production of the first 6 years? 2

OR

What was the increase in the company's production every year? 2

LEVEL – III

MCQ:

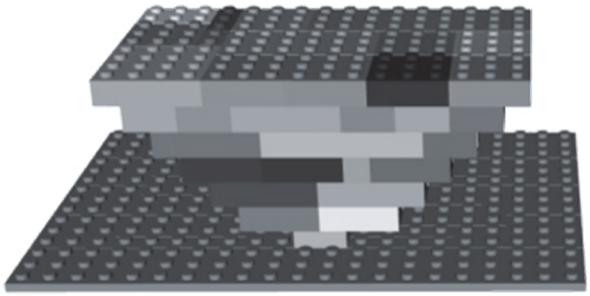
1. Mr. Singh buys a property every year for 12 years. Every year he buys X acres more than the previous year. If in the 8th year he bought 45 acres of land and in the 5th year he bought 30 acres of land, how many acres did he buy in the last year?

   a) [10 + 11(5)] Acres b) [10 + 12(5)] Acres

   c) [5 + 11(10)] Acres d) [5 + 12(10)] Acres

2. The cylindrical bumps on top of Lego blocks are called studs.

   Pragun has built a solid inverted Lego pyramid as shown below. The number of studs in successive floors forms an arithmetic progression. Pragun figures out that the sum of the number of studs used in the first p floors is given by $\left(6p^2-2p\right)$.



(Note: The figure is only for visual representation.)

How many studs are there in the 5th floor?

a) 140 b) 88 c) 64 d) 52

3. If the first, second and last term of an A.P are a, b and 2a respectively, then its sum is:

a) $\frac{ab}{2(b-a)}$ b) $\frac{ab}{b-a}$ c) $\frac{3ab}{2(b-a)}$ d) None

4. In an A.P $a_{m+n}+a_{m-n}$ is equal to:

a) 0 b) 1 c) $2a_m$ d) a_m

5. If pth term of an A.P. $\frac{1}{q}$ and qth term is $\frac{1}{p}$, then the sum of pq terms is:

a) $\frac{p-q}{2}$ b) $\frac{p+q}{2}$ c) $\frac{pq+1}{2}$ d) $\frac{pq-1}{2}$

6. If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers then the value of k will be:

a) 1/n b) (n-1)/n c) (n+1)/2n d) (n+1)/n

7. If in an A.P., $S_n = n^2p$, $S_m = m^2p$ and S_r denotes the sum of r terms of the AP., then S_p = _____.

a) $\frac{1}{2}p^3$ b) mnp c) p^3 d $(m+n)p^2$

8. The number of terms of an A.P. 3, 7, 11, 15... to be taken so that the sum is 406 is

a) 5 b) 10 c) 12 d) 14

9. If p^{th} term of an A.P is $\frac{3p-1}{6}$, then the sum of first n terms of an A.P is:

a) $\frac{n}{12}(3n+1)$ b) $\frac{n}{12}(3n-1)$

c) $\frac{n}{6}(3n+1)$ d) $\frac{n}{6}(3n-1)$

10. Sum of all natural numbers lying between 250 and 1000 which are exactly divisible by 3 is

a) 157365 b) 153657 c) 156375 d) 155637

2-MARKS:

1. The taxi fare after each km, when the fare is Rs 15 for the first km and Rs 8 for each additional km, does not form an AP as the total fare (in Rs) after each km is 15, 8, 8, 8,... Is the statement true? Give reasons.
2. Which term of the AP 3, $10, 17$……. will be 84 more than its 13^{th} term?
3. Find the number of odd integers between 2 and 100 divisible by 3.
4. The 8^{th} term of an A.P. is zero. Prove that its 38^{th} term is triple of its 18^{th} term.
5. Show that the sequence defined by $a_n = 2n^2 + 1$ is not an A.P.
6. Write an A.P. whose first term and common difference are -1.25 and -0.25 respectively.
7. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10^{th} terms is the same as the difference between their 21^{st} terms, which is the same as the difference between any two corresponding terms. Why?
8. In the AP: $10,5,0,-5,\ldots$ the common difference d is equal to 5 . Justify whether the above statement is true or false.
9. Divya deposited ₹1000 at compound interest at the rate of 10% per annum. The amounts at the end of first year, second year, third year,..., form an A.P. Justify your answer.
10. For the AP: $-3,-7,-11,\ldots$, can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20} ? Give reasons for your answer.
11. Match the APs given in column A with suitable common differences given in column B.

	Column A		Column B
(A_1)	2, –2, –6, –10,...	(B_1)	$\frac{2}{3}$
(A_2)	$a=-18, n=10, a_n=0$	(B_2)	–5
(A_3)	$a=0, a_{10}=6$	(B_3)	4
(A_4)	$a_2=13, a_4=3$	(B_4)	–4
		(B_5)	2
		(B_6)	$\frac{1}{2}$
		(B_7)	5

3-MARKS:

1. Find the sum of all two digit natural numbers which when divided by 3 gives 1 as the remainder.
2. Find the sum of all odd integers between 2 and 100 divisible by 3.
3. Find the sum of all three digit numbers which leave the remainder 3, when divided by 5.
4. If the 8^{th} term of the A.P. is 37 and the 15^{th} term is 15 more than the 12^{th} term, find the A.P. Hence find the sum of first 15 terms of the A.P.
5. Consider the sequence defined by $a_n = an^2 + bn + c$. If $a_1 = 1, a_2 = 5$ and $a_3 = 11$ then find the value of a_{10}.
6. Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7:15.
7. Find the sum of first 20 terms of an A.P. in which 3^{rd} term is 7 and 7^{th} term is two more than thrice of its 3^{rd} term.
8. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots\ldots 4\frac{1}{3}$.
9. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.
10. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2n terms of another AP whose first term is -30 and the common difference is 8 . Find n.

11. The 26th, 11th and the last term of an AP are 0, 3 and $-\frac{1}{5}$ respectively. Find the common difference and the number of terms.

12. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.

13. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

14. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

15. Find the sum of last ten terms of the AP: 8, 10, 12,.....126

16. Find the sum of first seven numbers which are multiples of 2 as well as of 9

5-MARKS:

1. The sum of the first 3 terms of an A.P. is 21 and the sum of their squares is 155. Find the AP.

2. The 10th term of an AP is 29 and the sum of its first 20 terms is 610. Find the sum of its first 30 terms.

3. In an A.P. 6th term is half the 4th term, and the 3rd term is 15. How many terms are needed to give a sum that is equal to 66?

4. Find the sum:

   a) $4-\frac{1}{n}+4-\frac{2}{n}+4-\frac{3}{n}.....upto\ n\ terms$

   b) $\frac{a-b}{a+b}+\frac{3a-2b}{a+b}+\frac{5a-3b}{a+b}+.....to 11\ terms$

5. If there are $(2n+1)$ terms in an A.P. then prove that the ratio of the sum of odd terms and the sum of even terms is $(n+1):n$.

6. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

7. Find the sum of the integers between 100 and 200 that are:

   i) divisible by 9 ii) not divisible by 9

8. Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

9. Find the:

   i) Sum of those integers between 1 & 500 which are the multiples of 2 as well as of 5.

   ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

   iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

10. The ratio of the 11^{th} term to the 18^{th} term of an A.P is 2:3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of first 21 terms.

SKILL BASED QUESTIONS

1. The following distribution shows the daily pocket allowance of children of locality. The mean pocket allowance is Rs. 18. Find the missing frequency '*f*'.

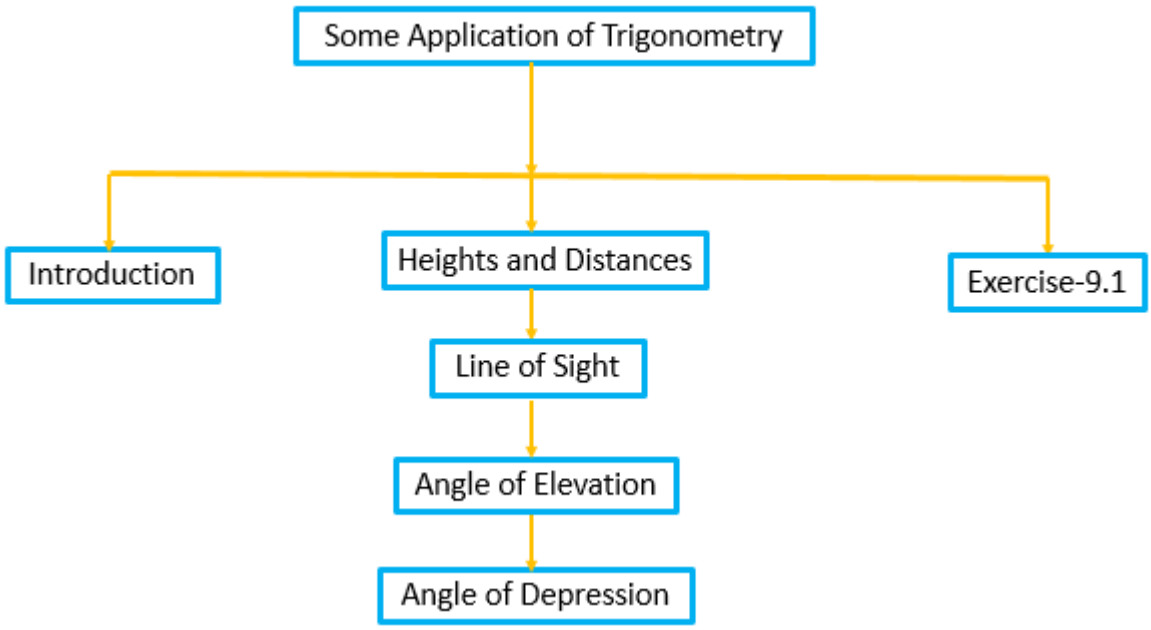
Daily pocket allowance (in Rs)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	*f*	5	4

2. Compare the modal age of two groups A and B of students appearing for an entrance test.

Age in years	**Number of students**	
	Group A	**Group B**
16-18	50	54
18-20	78	89
20-22	46	40
22-24	28	25
24-26	23	17

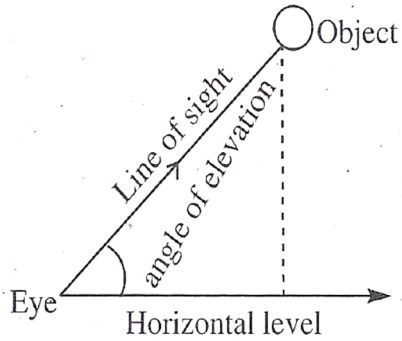
SOME APPLICATIONS OF TRIGONOMETRY

MIND MAPPING:

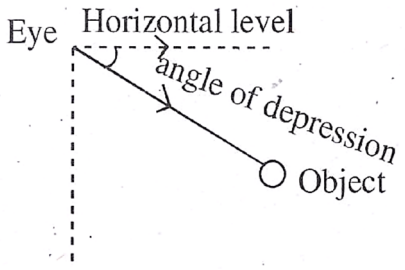


BASIC FACTS AND FORMULAE:

1. **Line of sight:** It is the line drawn from the eye of an observer to the point in the object viewed by the observer.
2. **Angle of elevation:** It is the angle formed by the line of sight with horizontal through the eye of observer when the object is above the horizontal level.



3. **Angle of depression:** It is the angle formed by the line of sight with the horizontal when the object is below the horizontal level.



4. **Trigonometric ratios**:

$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}}$, $\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}}$, $\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A}$

5. **Values of trigonometric ratios:**

θ	30^0	45^0	60^0
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

6. **Trigonometric ratios of complementary angles:**

$\sin(90^\circ - \theta) = \cos\theta$ $\cos(90^\circ - \theta) = \sin\theta$

$\tan(90^\circ - \theta) = \cot\theta$ $\text{cosec}(90^\circ - \theta) = \sec\theta$

$\sec(90^\circ - \theta) = \text{cosec}\,\theta$ $\cot(90^\circ - \theta) = \tan\theta$

LEVEL – I

MCQ

1. If the length of the shadow of a tree is decreasing then the angle of elevation is:

a) Increasing b) Decreasing

c) Remains the same d) None of the above

2. The angle of elevation of the top of a building from a point on the ground, which is 30 m away from the foot of the building, is 30°. The height of the building is:

a) 10 m b) 30/√3 m c) √3/10 m d) 30 m

3. If the height of the building and distance from the building foot's to a point is increased by 20%, then the angle of elevation on the top of the building:

a) Increases b) Decreases

c) Do not change d) None of the above

4. If a tower 6m high casts a shadow of 2√3 m long on the ground, then the sun's elevation is:

a) 60° b) 45° c) 30° d) 90°

5. The angle formed by the line of sight with the horizontal when the point is below the horizontal level is called:

a) Angle of elevation b) Angle of depression

c) No such angle is formed d) None of the above

6. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. The height of the tower (in m) standing straight is:

a) $15\sqrt{3}$ b) $10\sqrt{3}$ c) $12\sqrt{3}$ d) $20\sqrt{3}$

7. The height or length of an object or the distance between two distant objects can be determined with the help of:

a) Trigonometry angles b) Trigonometry ratios

c) Trigonometry identities d) None of the above

8. When the shadow of a pole h metres high is $\sqrt{3}$h metres long, the angle of elevation of the Sun is

a) 30° b) 60° c) 45° d) 15°

9. A ladder makes an angle of 60° with the ground, when placed along a wall. If the foot of ladder is 8 m away from the wall, the length of ladder is

a) 4 m b) 8 m c) $8\sqrt{3}$ m d) 16 m

10. The angle of depression of an object on the ground, from the top of a 25 m high tower is 30°. The distance of the object from the base of tower is

a) $25\sqrt{3}$ m b) $50\sqrt{3}$ m c) $75\sqrt{3}$ m d) 50 m

2-MARKS:

1. At an instant, the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole, thenwhat is the angle of elevation of the sun?

2. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

3. The string of a kite is 100 metres long and it makes an angle of 60° with the horizontal. Find the height of the kite, assuming that there is no slack in the string.
4. If the angles of elevation of the top of a tower from two points at a distance of 4m & 9 m from base of tower and in the same straight line with it are complementary, find the height of the tower.
5. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house.
6. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball.

3 (OR) 5 MARKS:

1. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.
2. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
3. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 30 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.
4. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

5. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal as 45°. Find the height of the pedestal.

6. The shadow of tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find, height of tower. (Given $\sqrt{3}$ = 1.732).

7. As observed from the top of a 100 m high light house from the sea- level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of light house, find distance between the two ships. (Use $\sqrt{3}$ = 1.732).

8. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

9. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° & 60° respectively. Find width of river. [Use $\sqrt{3}$ = 1.732]

10. A man observes a car from the top of a tower, which is moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30° to 45° in 12 minutes, find the time taken by the car now to reach the tower.

11. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30^0. Find the distance of the hill from the ship and the height of the hill.

12. From a point on the ground, the angle of elevation of the top of a tower is observed to be 60^0. From a point 40m vertically above the first point of observation, the angle of elevation of the top of the tower is 30^0. Find the height of the tower and its horizontal distance from the point of observation.

13. The angle of elevation of an aeroplane from a point A on the ground is 60^0. After a flight of 15 seconds, the angle of elevation changes to 30^0. If the aeroplane is flying at a constant height of $1500\sqrt{3}\,m$, find the speed of the plane in km/hr.

14. If the angles of elevation of the top of a tower from two points at the distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary, find the height of the tower.

15. A tree is broken by the wind. The top struck the ground at an angle of 30° and at a distance of 30 m from the root. Find the whole height of the tree.

16. From the top of a building 100 m high, the angles of depression of the top and bottom of a tower are observed to be 45^0 and 60^0 respectively. Find the height of the tower.

CASE STUDY QUESTIONS

CASE STUDY_1

A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

Based on the above information, answer the following questions:

(i) What is the angle of elevation of the sun, if the ratio of length of the rod and its shadow is 1:1? 1

(ii) What is the angle of elevation if they are standing at a distance of 42m away from the monument? 1

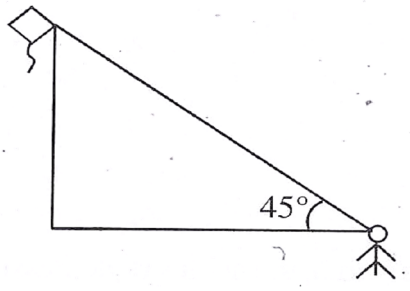
(iii) Find the height of the vertical tower that cast a shadow of length 20m, given that the altitude of the sun is at 60^0. 2

OR

Find the distance from the point of observation to the base of the tower given the angle of elevation is 60^0. 2

CASE STUDY_2

A boy is flying a kite with the length of the string is 100 m with angle of elevation from the ground 45^0 (ignore the height of boy)



i) What is the perpendicular height from the ground? 1

ii) If the angle of elevation become 30° with the same [Height from question i)]. What is the length of string? 1

iii) From the given case study question if the length of string becomes double then find the perpendicular height from ground with the angle of elevation 45^0 ? 2

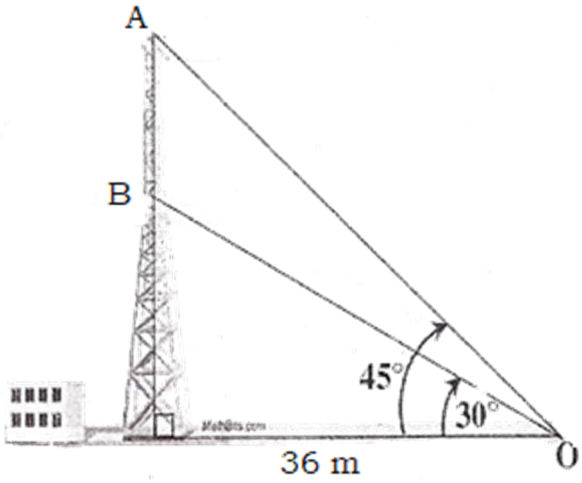
OR

If another boy standing exactly perpendicular to kite with the height 100 m and with the angle of elevation 30°, then find the distance between two boys. 2

CASE STUDY_3

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes.

They are among the tallest human - made structures. There are 2 main types. guyed and self - supporting structures.



On a similar concept, a radio station tower was built in two sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is $36m$. From point O, the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45^0.

i) What is the height of the section B? 1

ii) What is the height of the section A ? 1

iii) What is the length of the wire structure from point 0 to the top of section A ? 2

OR

What is the angle of depression from top of tower to point? 2

LEVEL – II

MCQ:

1. The ratio of the height of a tower and the length of its shadow on the ground is √3 : 1. The angle of elevation of the Sun is

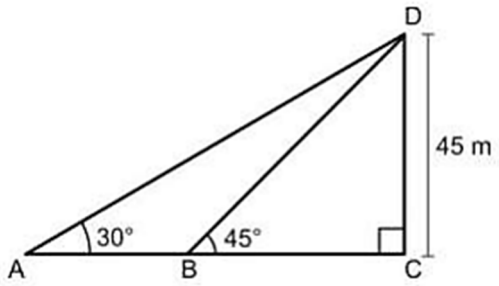
a) 30° b) 45° c) 60° d) 75°

2. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will be

a) Greater than 60° b) Equal to 30°

c) Less than 60° d) Equal to 60°

3. In the given below figure, what is the length of AB?

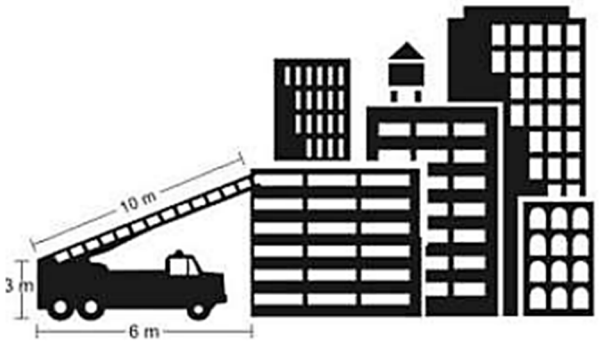


(Note: The figure is not to scale.)

a) $45\sqrt{3}\,m$ b) $\frac{45}{\sqrt{3}}m$

c) $45\left(\sqrt{3}-1\right)m$ d) $45\left(\sqrt{3}+1\right)m$

4. A fire engine, standing near a building, extends its ladder to a length of 10 metres to reach a certain window in the building.



(Note: The figure is not to scale.)

What is the height of the window from the ground?

a) 6 m b) 8 m c) 9 m d) 11 m

5. A person standing 100 m away from a building measures the angle of elevation to the top of the building to be 60°. If the person's eye level is 1.5 m above the ground, how tall is the building?

a) 50 m b) 86.6 m c) 100 m d)173.2 m

6. The angle of elevation of the top of a tower from a point on the ground is 30°. If the distance between the point and the base of the tower is 50 m, what is the height of the tower?

a) 25 m b) 43.3 m c) 50 m d) 86.6 m

7. A flagpole casts a shadow of 12 m when the angle of elevation of the sun is 45°. How tall is the flagpole?

a) 8 m b) 12 m c) 16 m d) 24 m

8. The angle of depression of a car from the top of a cliff is 30°. If the car is 100 m away from the foot of the cliff, how high is the cliff?

a) 50 m b) 86.6 m c) 100 m d) 173.2 m

9. The angle of elevation of a ladder leaning against a wall is 60° and the ladder is 10 m long. How far is the foot of the ladder from the wall?

a) 5 m b) 10 m c) 15 m d) 20 m

10. The angle of depression from the top of a tower to a point on the ground is 45°. If the tower is 50 m high, what is the distance between the tower and the point on the ground?

a) 25 m b) 50 m c) 75 m d) 100 m

2-MARKS:

1. The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle θ with the ground level such that $\tan\theta = \frac{15}{8}$, how high is the kite? Assume that there is no slack in the string.

2. The shadow of a tower, when the angle of elevation of the sun is 45^0, is found to be 10 m longer than when it was 60^0. Find the height of the tower.

3. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^0. Find the length of the string assuming that there is no slack in the string. [Take $\sqrt{3} = 1.732$].

4. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30^0. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45^0. Find the length of the flagstaff and distance of building form point P. [Take $\sqrt{3} = 1.732$]

5. From the top of a 300 m high light - house, the angles of depression of two ships, which are due south of the observer and in a straight line with its base, are 60^0 and 30^0. Find their distance apart.

6. The angle of elevation of the top of a tower from two points P and Q at distances of a and b respectively from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$.

7. The angle of elevation of a kite flying at a height of h m from the ground is θ. If the kite string makes an angle of 60° with the ground, what is the length of the string?

3 (OR) 5 MARKS:

1. From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30^0. A flagstaff is fixed at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of point P from the building. (Take $\sqrt{3} = 1.732$)

2. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of the reflection in the lake is 60^0, find the height of the cloud from the surface of lake.

3. A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

4. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant of 60°. After sometime, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

5. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point A on the ground in 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of the tower.

6. The angle of elevation of a top of a hill at the foot of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the height of the tower is 40 m, then find the height of the hill.

7. A vertical pole 10 m long casts a shadow $10\sqrt{3}$ m long at the same time a tower casts a shadow 90 m long. Determine the height of the tower.

8. A bridge across a river makes an angle of 45° with the river. If the length of the bridge across the river is 100 m, what is the width of the river?

9. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30^0, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60^0. Find the time taken by the car to reach the foot of the tower from this point.

10. A ladder 9 m long reaches a point 9 m below, the top of a vertical flag staff. From the foot of the ladder the angle of elevation of the top of flagstaff is 60^0. Find the height of the flag staff.

11. The angle of elevation of the top of a tower from certain point is 30°.If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

12. The angle of depression from the top of a tower of a point on the ground, which is 30 m from the foot of tower is 30°. Find the height of tower. (Use $\sqrt{3} = 1.732$)

13. A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.

14. A point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?

15. An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from his eyes is 45°. Find the height of the chimney.

16. A boy of height 1.7 m is standing 30 m away from a flagstaff on the same level ground. He observes that the angle of elevation of the top of the flagstaff is 30°. Calculate the height of the flag staff.

17. A 1.6 m tall girl stands at a distance of 3.2 m from a lamp post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post.

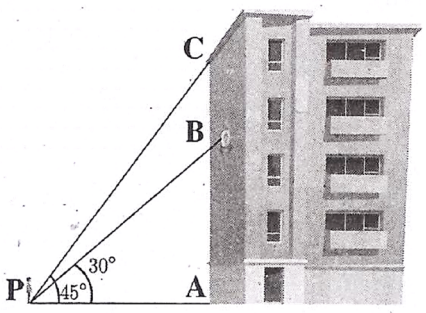
18. The angle of elevation of the top of a tower from a point on the ground is θ. If the person moves 20 m closer to the tower, the angle of elevation becomes 2θ. What is the height of the tower?

CASE STUDY QUESTIONS

CASE STUDY_1

A clinometer is a tool that is used to measure the angle of elevation, or angle from the ground in a right- angled triangle. We can use a clinometer to measure the height of tall things that you can't possibly reach to the top of, flag poles buildings, trees.

Ravish got a clinometer from school lab and started the measuring elevation angle in surrounding. He saw a building on which society logo is painted on wall of building.



From a point P on the ground level, the angle of elevation of the roof of the building is 45°. The angle of elevation of the centre of logo is 30° from same point. The point *P* is at a distance of 24 m from the base of the building.

Based on the above information, answer the following questions:

i) What is the height of the building logo from ground? 1

ii) What is the height of the building from ground? 1

iii) What is the aerial distance of the point P from the top of the building? 2

OR

If the point of observation P is moved 9 m towards the base of the building, then find the angle of elevation θ of the logo on building? 2

LEVEL – III

MCQ

1. In the triangle ABC; $AD \perp BC$, $\tan B = \frac{5}{12}$ $\tan C = \frac{3}{4}$ and BC = 56 cm, then find the value of AD.

a) 10 b) 20 c) 15 d) 13

2. From the top of a temple which is 96 m high, the angles of depression of 2 cars on the road on the same side of the temple are x^0 and y^0 where $\tan x^0 = \frac{3}{4}$ and $\tan y^0 = \frac{1}{3}$. Find the distance between the cars.

a) 160 m b) 260 m c) 200 m d) 80 m

3. If a 1.5-m-tall girl stands at a distance of 3mfrom a lamp-post and casts a shadow of length 4.5mon the ground, then the height of the lamp-post is

a) 1.5 m b) 2 m c) 2.5 m d) 2.8 m

4. From the top of a cliff 20 m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is:

a) 20 m b) 40 m c) 60 m d) 80 m

5. The length of the shadow of a tower standing on level ground is found to be $2x$ metres longer when the sun's elevation is 30° than when it was 45°. The height of the tower is

a) $\left(2\sqrt{3}x\right)m$ b) $\left(3\sqrt{2}x\right)m$

c) $\left(\sqrt{3}-1\right)x\,m$ d) $\left(\sqrt{3}+1\right)x\,m$

6. The lengths of a vertical rod and its shadow are in the ratio $1:\sqrt{3}$. The angle of elevation of the sun is

a) 30° b) 45° c) 60° d) 90°

7. The tops of two towers of heights x and y, standing on a level ground subtend angles of 30° and 60° respectively at the centre of the line joining their feet. Then, $x:y$ is

a) 1 : 2 b) 2 : 1 c) 1 : 3 d) 3 : 1

8. If the angles of elevation of the top of a tower from two points at distances a and b from the base and in the same straight line with it are complementary then the height of the tower is:

a) $\sqrt{\frac{a}{b}}$ b) $\sqrt{ab}$ c) $\sqrt{a+b}$ d) $\sqrt{a-b}$

9. In a rectangle, the angle between a diagonal and a side is 30° and the length of this diagonal is 8 cm. The area of the rectangle is:

a) 16 cm^2 b) $\frac{16}{\sqrt{3}}cm^2$ c) $16\sqrt{3}\,cm^2$ d) $8\sqrt{3}\,cm^2$

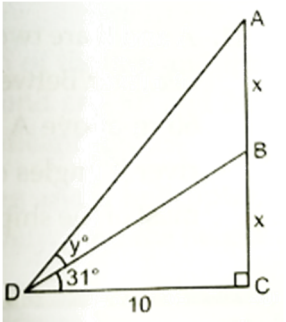
10. From the top of a hill, the angles of depression of two consecutive km stones due east are found to be 30° and 45°. The height of the hill is:

a) $\frac{1}{2}\left(\sqrt{3}-1\right)km$ b) $\frac{1}{2}\left(\sqrt{3}+1\right)km$

c) $\left(\sqrt{3}-1\right)km$ d) $\left(\sqrt{3}+1\right)km$

2-MARKS:

1. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing. Justify your answer.
2. If a man standing on a platform 3 metres above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection. Justify your answer.
3. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled. Justify your answer.
4. If the height of a tower and the distance of the point of observation from its foot, both, are increased by 10%, then the angle of elevation of its top remains unchanged. Justify your answer.
5. Find x and y in the given figure, AB = BC = x and $\angle BDC = 31^0$, $\angle ADB = y^0$ and CD = 10.



6. A boy 1.4 m tall is 20 m away from a tower and observes the angle of elevation of the top of the tower to be 60^0.
   (i) Find the height of the tower to the nearest metres.
   (ii) If he walks 10 metres towards the tower and observes the top of tower, what is the new angle of elevation?

7. At a point on the level ground, the tangent of the angle of elevation of the top of the tower is 5/9. On walking 56 m towards the tower, the tangent of the angle is 3/4. Find the height of the tower.

3 (OR) 5 MARKS:

1. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this, will the car reach the tower? Give your answer to the nearest second.
2. There is a coconut tree on the bank of a river. From a boat 5 m above the water, the angle of elevation of the top of the tree is 45^0 and the angle of depression of reflection of tree top is 60^0. Find the height of the tree.
3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are α and β respectively. Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta-\tan\alpha}$.
4. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$.
5. From the top of a building AB, 60mhigh, the angles of depression of the top and bottom of a vertical lamp-post CD are observed to be 30° and 60° respectively. Find

   (i) The horizontal distance between AB and CD,

   (ii) The height of the lamp-post,

   (iii) The difference between the heights of the building and the lamp-post.

6. From a balloon vertically above a straight road, the angles of depression of two cars at an instant are found to be 45° and 60°. If the cars are 100 m apart, find the height of the balloon.

7. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

8. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30°.

9. The angle of elevation of the top of a vertical tower from a point on the ground is. 60°. From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower.

10. A window of a house is h metres above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β, respectively. Prove that the height of the other house is $h\left(1+\tan\alpha.\cot\beta\right)$ metres.

11. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30° respectively. Find the height of the balloon above the ground.

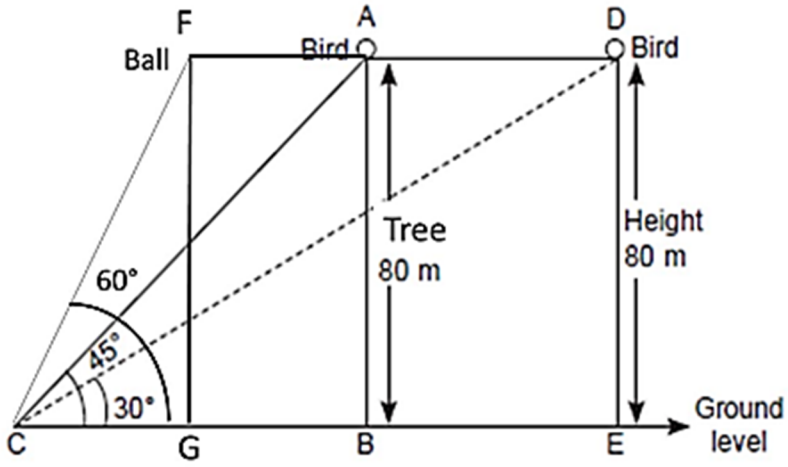
12. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower.

13. A boy standing on a horizontal plane finds a bird flying at a distance of 100m from him at an elevation of 30°. A girl standing on the roof of a 20-m-high building, finds the angle of elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. [Given $\sqrt{2}=1.414$].

CASE STUDY QUESTIONS

CASE STUDY_1

One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80m. He observed a bird on the tree at an angle of elevation of 45°. When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.



(i) At what distance from the foot of the tree was he observing the bird sitting on the tree? 1

(ii) How far did the bird fly in the mentioned time? 2

OR

After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?

(iii) What is the speed of the bird in m/min if it had flown $20\left(\sqrt{3}+1\right)m$? 1

CASE STUDY_2

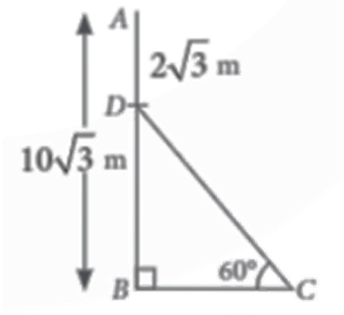
Suppose a straight vertical tree is broken at some point due to storm and the broken part is inclined at a certain distance from the foot of the tree.

Based on the above information, answer the following questions:

(i) If the top of the upper part of the broken tree touches the ground at a distance of 30m (from the foot of the tree) and makes an angle of inclination 30^0, find the height of the remaining part of the tree. 1

(ii) If the top of broken part of tree touches the ground at a point whose distance from the foot of the tree is equal to the height of remaining part, find its angle of inclination. 1

(iii) If AB = $10\sqrt{3}\,m$ and $AD = 2\sqrt{3}\,m$, find the value of CD. 2



OR

If the height of the tree is 6m, which is broken by wind in such a way that its top touches the ground and makes an angle 30^0 with the ground. At what height from the bottom of the tree is broken by the wind?

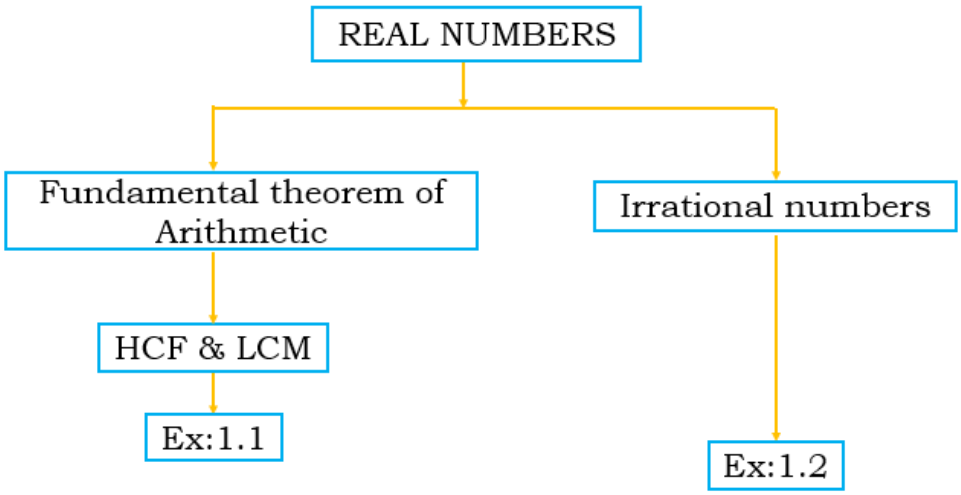
SKILL BASED QUESTIONS

1. Solve the following system of linear equations graphically: $4x - 5y - 20 = 0$ and $3x + 5y - 15 = 0$. Determine the vertices of the triangle formed by the lines representing the above equations and the y-axis.

2. Solve the following system of equations graphically: $3x+2y-11=0$ and $2x-3y+10=0$. Shade the region bounded by these lines and the x-axis.

REAL NUMBERS

MIND MAPPING:



BASIC FACTS AND FORMULAE:

(i) **Rational numbers:** Rational numbers are those numbers which can be written in the form of $\frac{p}{q}$, where both p, q are integers and q is not equal to zero.

(ii) **Irrational numbers:** The numbers which cannot be expressed in the form of $\frac{p}{q}$ are irrational numbers.

(iii) **Real numbers:** The set of rational and irrational numbers together are called real numbers.

(iv) **The Fundamental Theorem of Arithmetic:** Every composite number can be expressed (factorised) as a product of powers of primes, and this factorization is unique, apart from the order in which the prime factors occur.

(OR)

All the natural numbers except ' 1 ' can written as a product of their prime factors.

(v) 1 is neither a prime nor a composite.

(vi) Every composite number can be written as the product of powers of primes.

(vii) For any two positive integers a and b, $HCF(a,b) \times LCM(a,b) = a \times b$

(vii) The greatest number that will divide x, y and z leaving remainders a, b and c respectively is given by $|\text{HCF of } (x-a),(y-b),(z-c)|$

(viii) The least number which when divided by x, y and z leaves the remainders a, b,

and c respectively is given by [LCM of $(x, y, z) - P$], where $P = x - a = y - b = z - c$

(ix) The sum or difference of a rational and an irrational number is irrational.

(x) The product and quotient of a non-zero rational and irrational number is irrational.

(xi) **LCM by Prime Factorisation Method:** Given natural numbers are written as the product of prime factors. The lowest common multiple will be the product of all prime factors with the highest degree (power).

(xii) **HCF by Prime Factorisation Method:** Given natural numbers to be written as the product of prime factors. To obtain the highest common factor multiply all the common prime factors with the lowest degree (power).

LEVEL - I

MCQ:

1. 385 in the form of product of prime factors is written as:

   a) $5^1 \times 7^1 \times 11^1$ b) $5^2 \times 7^2 \times 11^1$

   c) $5^1 \times 7^2 \times 11^2$ d) $7^1 \times 11^1$

2. Two positive numbers have their HCF as 12 and their product as 6336. How many number of pairs possible for the numbers?

   a) 2 b) 3 c) 4 d) 5

3. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then, the least prime factor of $(a + b)$ is:

   a) 2 b) 3 c) 5 d) 8

4. Given that $HCF(135, 225) = 45$, then the $LCM(135, 225) =$

   a) 765 b) 675 c) 576 d) 867

5. Write one irrational number between 0.15 and 0.21.

   a) 0.17243561902..... b) 0.15

   c) 0.21 d) 0.17171717....

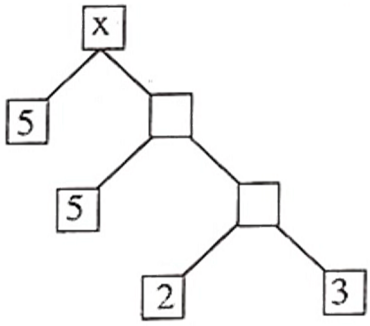
6. The HCF of 12, 18 and 30 is:

a) 5 b) 4 c) 3 d) 6

7. What is the exponent of 2 in the prime factorisation of 144?

a) 4 b) 5 c) 6 d) 3

8. Find the value of x in the factor tree.



a) 150 b) 200 c) 250 d) 450

9. If p is a prime number then find the LCM of p, p^2 *and* p^3 is:

a) p b) p^2 c) p^3 d) $2p^2$

10. What is the HCF of the smallest composite number and the smallest prime number?

a) 2 b) 4 c) 3 d) 6

11. Prime factorisation of 156 is:

a) $2\times3\times13$ b) $2\times3\times13^2$ c) $2^2\times3\times13$ d) $2^2\times3^2\times13$

2-MARKS:

1. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

2. Check whether 12^n can end with the digit 0 for any natural number n.

3. Given that $\sqrt{2}$ is irrational. Prove that $5+3\sqrt{2}$ is an irrational number.

4. Using fundamental theorem of arithmetic, find the H.C.F of 26, 51 and 91.

5. Explain why: i) $7\times11\times13+13$ ii) $7\times6\times5\times4\times3\times2\times1+5$ are composite numbers.

6. The HCF and LCM of two numbers are 9 and 90 respectively. If one number is 18, find the other.

7. Show that the numbers 143 and 187 are not co-prime.

3-MARKS:

1. Prove that $\sqrt{5}$ is an irrational number.

2. Assuming that $\sqrt{3}$ is an irrational number, prove that $5\sqrt{3}-7$ is an irrational number.

3. Prove that $\sqrt{3}$ is an irrational number.

4. Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.

5. Find the L.C.M and H.C.F of 404 and 96 and verify that L.C.M × H.C.F = Product of the two numbers.

6. Find the LCM and HCF of 12, 72 and 120 using prime factorisation. Also show that HCF × LCM ≠ Product of three given numbers.

7. Prove the following are irrational numbers:

   i) $13+25\sqrt{2}$ ii) $3-\sqrt{5}$ iii) $\frac{1}{2+\sqrt{3}}$ iv) $\sqrt{6}+\sqrt{2}$

8. Three bells ring regularly at intervals of 4, 7 and 14 minutes. If all three bells rang together at 6 a.m, when will they ring together again?

5-MARKS:

1. There is a circular path around a sports field. Sonia takes 18 minutes for one round of the field and Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time. After how many minutes will they meet again at the starting point?

2. Find HCF of 44, 96 and 404 by prime factorization method. Hence find their LCM.

3. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance and complete steps?

4. Find HCF of 378,180 and 420 by prime factorization method. Is $HCF \times LCM$ of three number equal to the product of the three numbers?

5. Find the largest positive integer that will divide 122, 150 and 115 leaving remainders 5, 7, 11 respectively.

6. Find the smallest number when dividing by 28 and 32 leaving remainders 8 and 12 respectively.

CASE STUDY QUESTIONS

CASE STUDY_1

To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

Based on the above information answer the following questions:

(i) Express 36 as product of prime factors. 1

(ii) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then find the LCM (p, q). 1

(iii) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B? 2

OR

If the product of two positive integers is equal to the product of their HCF and LCM is true then, find the HCF (32, 36).

CASE STUDY_2

A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

Based on the above information, answer the following questions:

(i) Express 108 as the product of prime factors. 1

(ii) In each room the same number of participants are to be seated and all of them being in the same subject, hence find the maximum number participants that can accommodated in each room. 2

OR

What is the minimum number of rooms required during the event? 2

(iii) Find the product of HCF and LCM of 60, 84 and 108. 1

CASE STUDY_3

Aditya works as a librarian in Bright Children International School in Indore. He ordered for books on English, Hindi and Mathematics. He received 96 English books, 240 Hindi books and 336 Maths books. He wishes to arrange these books

in stacks such that each stack consists of the books on only one subject and number of books in each stack is the same. He also wishes to keep the number of stacks minimum.

Based on the above information, answer the following questions:

(i) Find the number of books in each stack. 2

OR

If the thickness of each English book is 3cm, then find the height of each stack of English books.

(ii) How many stacks of Mathematics books will be formed. 1

(iii) Find the total number of stacks formed. 1

LEVEL – II

MCQ:

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then the result obtained by dividing the product of the positive integers by the LCM (a, b) is

a) xy b) xy^2 c) x^3y^3 d) x^2y^2

2. Which of the following is a pair of co-primes?

a) (14, 35) b) (18, 25) c) (31, 93) d) (32, 62)

3. If $a = \left(2^2 \times 3^3 \times 5^4\right)$ and $b = \left(2^3 \times 3^2 \times 5\right)$ then HCF(a, b) = ?

   a) 90 b) 180 c) 360 d) 540

4. Total number of factors of a prime number is:

   a) 4 b) 1 c) 3 d) 2

5. Product of a non-zero number and an irrational number is an:

   a) Always rational b) always irrational

   c) Rational or irrational d) none of these

6. If x and y are co-primes then, HCF of x and y is:

   a) xy b) 1 c) 0 d) 2

7. The largest number which divides 245 and 1037, leaving remainder 5 in each case is:

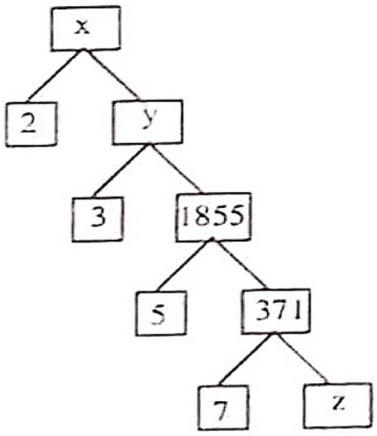
   a) 42 b) 48 c) 12 d) 24

8. Which of the following is an irrational number?

   a) $\frac{\sqrt{2}}{\sqrt{8}}$ b) $\frac{\sqrt{3}}{3\sqrt{5}}$ c) $\frac{\sqrt{5}}{\sqrt{20}}$ d) $\frac{\sqrt{63}}{\sqrt{7}}$

2-MARKS:

1. Determine the values of 'm' and so that the prime factorisation of 10500 is expressible as $2^m \times 3 \times 5^n \times 7$.
2. Show that 8^n cannot end with the digit zero for any natural number n.
3. Find the LCM of 72, 80 and 120 using the fundamental theorem of arithmetic.
4. Prove that the sum of rational number and an irrational number is always irrational.
5. Find LCM of 92 and 510. Also find their HCF by using LCM.
6. Prove that $3\sqrt{7}$ is an irrational number.
7. Complete the following factor tree and find the composite number x.



8. What is the largest number that divides 245 & 1029, leaving remainder 5 in each?

3-MARKS:

1. Write 32875 as a product of prime factors. Is this factorisation unique?
2. Find the HCF and LCM of 84 and 402 by the prime factorisation method. Hence, prove that the product of terms is equal to the product of their HCF and LCM.
3. In a school, the duration of a period in junior section is 40 minutes and senior section is 1 hour. If the first bell for each section rings at 9.00 a.m, when will the two bells ring together again?
4. There are 156, 208 and 260 students in Groups A, B, C respectively. Buses are to be hired to take them for a field trip. Find the minimum number of buses to be hired if the same number of students should be accommodated in each bus.
5. Examine whether the following numbers are rational or irrational.

   (i) $\left(3+\sqrt{2}\right)^2$ ii) $\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)$ iii) $\frac{6}{2\sqrt{3}}$

6. Two tankers contain 620 litres and 840 litres of diesel respectively. Find the maximum capacity of a container which can measure the diesel of both the tankers in exact number of times.
7. Find the largest number that will divide 398, 436 and 542, leaving remainders 7, 11 and 15 respectively.

5-MARKS:

1. Prove that $\sqrt{3}$ is an irrational number. Hence prove that $\frac{3}{2\sqrt{3}}$ is an irrational number.
2. The product of two numbers x and y is 217728. Find the LCM and HCF of x and y if it is given that $LCM(x,y) = 42\ HCF(x,y)$
3. Find the greatest number of six digits exactly divisible by 18, 24 and 36.
4. Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.
5. If p is a prime number then prove that $\sqrt{p}$ is an irrational.
6. Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p and q are primes.

CASE STUDY QUESTIONS

CASE STUDY_1

Shalvi is a tuition teacher and teaches mathematics to some kids at her home. She is very innovative and always plan new games to make her students learn concepts.

Today, she has planned a prime number game. She announce the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to second student. Second student also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number the

last student got 173250. He told this number to Shalvi in class. Now she asked some questions to the students as given below.

i) How many students are in the class? 1

ii) What is the highest prime number used by student? 1

iii) Which prime numbers has been used maximum times and minimum times? 2

OR

Is it possible to have two numbers whose HCF is 18 & LCM is 760? Give reason.

LEVEL - III

MCQ:

1. Three bulbs red, green and yellow flash at intervals of 80 seconds, 90 seconds and 110 seconds. All three flash together at 8:00 am. At what time will the three bulbs flash altogether again?

   a) 9:00 am b) 9:12 am c) 10:00 am d) 10:12 am

2. Rahul has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?

   a) 4 cm as it is the LCM of 40 and 84 b) 4 cm as it is the HCF of 40 and 84

   c) 8 cm as it is the LCM of 40 and 84 d) 8 cm as it is the HCF of 40 and 84

3. If two positive integers p & q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers then, the $LCM(p,q)$ is:

   a) a^3b^2 b) a^2b^3 c) a^3b^3 d) a^2b^2

4. If $p^2 = \frac{32}{50}$, then p is/an

   a) Whole number b) integer

   c) rational number d) irrational number

5. Write the sum of the exponents of prime factors in the prime factorisation of 98.

   a) 2 b) 3 c) 4 d) 5

6. If the prime factorization of a natural number n is $2^3 \times 3^2 \times 5^2 \times 7$, write the number of consecutive zeroes in n.

a) 5 b) 1 c) 3 d) 2

7. The LCM and HCF of two numbers are equal, then the numbers must be ____.

a) prime b) co-prime c) composite d) equal

8. The sum of LCM and HCF of two numbers is 1260. If their LCM is 900 more than their HCF, find the product of two numbers.

a) 203400 b) 194400 c) 198400 d) 205400

9. The HCF of the polynomials $\left(x^2-4x+4\right)(x+3)$ and $\left(x^2+2x-3\right)(x-2)$ is:

a) $x+3$ b) $x-2$ c) $(x+3)(x-2)$ d) $(x+3)(x-2)^2$

10. If HCF (72, q) = 12 then how many values can q take? (Assume q be a product of 2 and a power of 3 only)

a) 1 b) 2 c) 3 d) 4

2-MARKS:

1. Explain why $3 \times 5 \times 7 + 7$ is a composite number.
2. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.
3. Show that 12^n cannot end with digits 0 or 5 for any natural number, n.
4. Can the given number 6^n, n being a natural number, end with the digit 5? Give reasons?
5. Find two numbers which on multiplication with $\sqrt{180}$ gives a rational number. Are these numbers rational or irrational?
6. Prove that $\frac{3\sqrt{2}}{4}$ is an irrational number.
7. In a school there are two sections - section A and section B of class X. There are 32 students in section A and 36 students in section B. Determine the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.

3-MARKS:

1. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer

2. Prove that $\frac{1}{2}-\frac{\sqrt{5}}{3}$ is irrational.

3. A fruit vendor has 732 apples and 942 oranges. He distributes these fruits among the students of an orphanage, such that each of them gets either only apples or only oranges of equal number. Find the least possible number of students.

4. Aloukya and Manoghna run around a circular track and they take 180 seconds and 150 seconds respectively to complete one revolution. If they start together at 9 am form the same point, how long it would take for them to meet again for the first time at the starting point.

5. Is square root of every non-square number always irrational? Find the smallest natural number which divides 2205 to make its square root a rational number.

6. If two positive integers $x \& y$ are expressed in terms of primes as $x = p^2q^3$, $y = p^3q$ what can you say about their LCM & HCF. Is LCM a multiple of HCF? Explain.

7. State fundamental theorem of arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively? Justify your answer.

5-MARKS:

1. Prove that $\left(3+2\sqrt{5}\right)^2$ is irrational.

2. If x is rational and $\sqrt{y}$ is irrational, then prove that $\left(x+\sqrt{y}\right)$ is irrational.

3. State whether the given statement is true or false. Justify.

   (i) The sum of two rationals is always rational.

   (ii) The product of two rationals is always rational.

(iii) The sum of two irrationals is always an irrational.

(iv) The product of two irrationals is always an irrational.

(v) The sum of a rational and an irrational is irrational.

4. Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

5. Write the HCF and LCM of the smallest odd composite number and the smallest odd prime number. If an odd number p divides q^2, then will it divide q^3 also? Explain.

6. If P is a prime number, then prove that $\sqrt{p}$ is irrational.

7. (a) If a is a non-zero rational number and $\sqrt{b}$ is irrational then show that $a\sqrt{b}$ is irrational.

(b) If a is a rational and $\sqrt{b}$ is irrational then prove that $\left(a+\sqrt{b}\right)$ is irrational.

CASE STUDY QUESTIONS

CASE STUDY_1

On 71st republic day Parade on 26/01/2021 in Delhi, Captain RS Meel is planning for parade of following two groups:

(a) First group of Army contingent of 624 members behind an army band of 32 members.

(b) Second group of CRPF troops with 468 soldiers behind the 228 members of bikers. These two groups are to march in the same number of columns. This sequence of soldiers is followed by different states of Jhanki which are showing the culture of the respective states.

i) What is the maximum number of columns in which, army troop can march? 1

ii) What is the maximum number of columns in which CRPF troop can march? 1

iii) What should be subtracted with the numbers of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop? 2

OR

What should be added with the numbers of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

CASE STUDY_2

Three friends Virat, Rohit, and Sachin used to ride their bicycles around their Society's park every day in the evening. The shape of the society's park is a Rectangle.

One day, they decide to play a game. All three have a ringing bell on their bicycle. They ring the bell at an interval of 9, 12, 15 minutes respectively means after every 9 mins Virat rings the bell, after every 12 mins Rohit rings the bell and after every 15 mins, Sachin rings the bell.

Based on the above information, answer the following questions.

i) If they start ringing together a time, after what time will they ring together next time? 1

ii) If only Sachin and Virat play this game, then after what time will they next ring together? 1

iii) If only Rohit and Sachin play this game, then after what time will they next ring together? 2

OR

If one more friend, Rishab also join them from the beginning. Now Virat, Rohit, Sachin and Rishab ring at an interval of 9,12, 15 and 20 minutes then after what time will they next ring together.

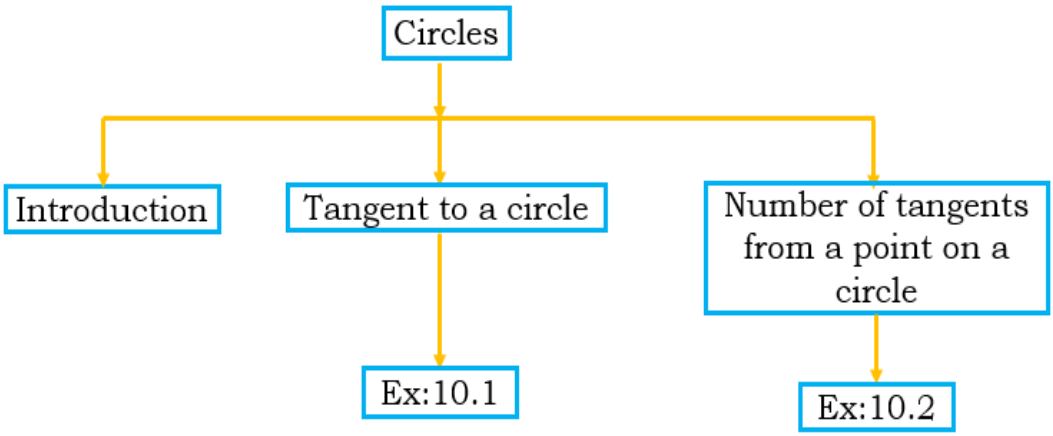
SKILL BASED QUESTIONS

Graph: Pair of linear equations in two variables

1. Romila went to a stationery shop and purchased 2 pencils and 3 erasers for Rs. 9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for Rs.18. Represent this situation algebraically and graphically.

2. Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$. Represent this situation graphically.

CIRCLES



BASIC FACTS AND FORMULAE:

1. A collection of all the points in a plane which are equidistant from a fixed point is called a circle. Fixed point is known as its centre and constant distance its radius. If centre of circle is 0 and radius r, then circle is denoted by $C(0,r)$
2. Circles having the same centre are said to be concentric circles.

   Eg: Circles $C(O,r_1)$, $C(O,r_2)$, $C(0,r_3)$ are concentric.
3. A line which intersects circle in two points is called a secant line.
4. A line which touches circle at one point or two coincident points is called a tangent line, to the circle and the point where it meets the circle is called a point of contact.
5. i) If a point lies inside the circle, then no tangent can be drawn to the circle.

   ii) If a point lies on the circle, then one tangent can be drawn to the circle through that point.

   iii) If a point lies outside the circle, then two tangents can be drawn to the circle
6. Tangent-Radius theorem: The tangent at any point of a circle is perpendicular to the radius through the point of contact.
7. Equal tangent - Lengths theorem: The lengths of the two tangents from an external point to a circle are equal.

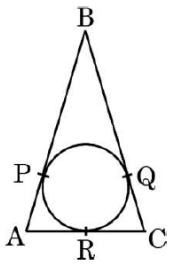
8. In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

9. If two tangents TP and TQ are drawn to a circle with centre O from an external point T, then $\angle PTQ = 2\angle OPQ$

LEVEL – I

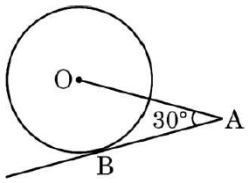
MCQ:

1. In the given figure, AB = BC = 10 cm. If AC = 7 cm, then the length of BP is:



a) 3·5 cm b) 7 cm c) 6·5 cm d) 5 cm

2. In the given figure, AB is a tangent to the circle centered at O. If OA = 6 cm and $\angle OAB = 30^0$, then the radius of the circle is:

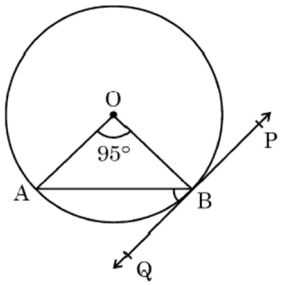


a) 3 cm b) 3 3 cm c) 2 cm d) 3 cm.

3. In the given below figure, AC and AB are tangents to a circle centered at O. If $\angle COD = 20^0$, then $\angle BAO$ is equal to:

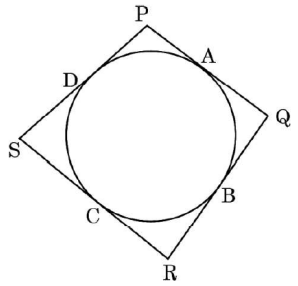
a) 30^0 b) 60^0 c) 45^0 d) 90^0

4. In the given figure, PQ is tangent to the circle centred at O. If $\angle AOB = 95^0$, then the measure $\angle ABQ$ is called.



a) 47.5^0 b) 42.5^0 c) 85^0 d) 95^0

5. In the given below figure, the quadrilateral PQRS circumscribes a circle. Here $PA + CS$ is equal to:

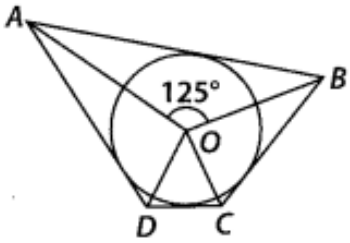


a) QR b) PR c) PS d) PQ

6. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is

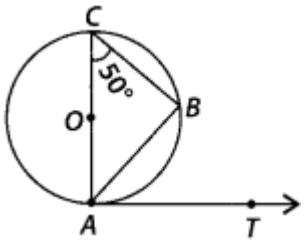
a) 3 cm b) 6 cm c) 9 cm d) 1 cm

7. In figure, if ∠AOB = 125°, then ∠COD is equal to



a) 62.5° b) 45° c) 35° d) 55°

8. In figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^0$. If AT is the tangent to the circle at the point A, then ∠BAT is equal to



a) 65° b) 50° c) 60° d) 40°

9. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is:

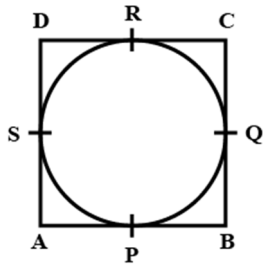
a) 60 cm^2 b) 65 cm^2 c) 30 cm^2 d) 32.5 cm^2

10. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XT and at a distance 8 cm from A is:

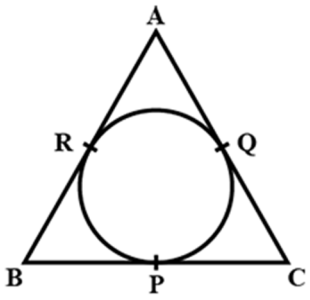
a) 4 cm b) 5 cm c) 6 cm d) 8 cm

2-MARKS:

1. In figure a circle touches all four sides of quadrilateral ABCD in which AB = 6cm BC = 7cm and CD = 4cm. Find AD.



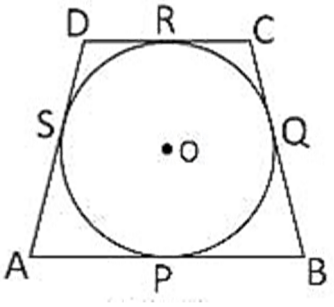
2. An isosceles triangle ABC, with $AB = AC$, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.



3. The length of a tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. Find the diameter of the circle.

4. Prove that the parallelogram circumscribing a circle is a rhombus.

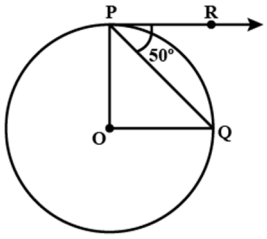
5. A quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.



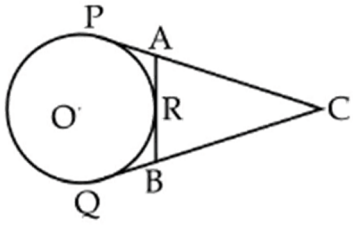
6. Prove that the tangents at the extremities of any chord make equal angles with the chord.

8. Prove that the tangents drawn at the end points of a diameter of a circle are parallel.

9. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of $50°$ with PQ, then find $\angle POQ$.

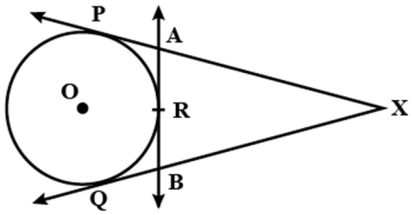


10. In the given figure, CP and CQ are tangents to a circle with centre D and line segment AB touches the circle at R with CP = 11cm, AR = 3cm, BC = 7cm then find BR.

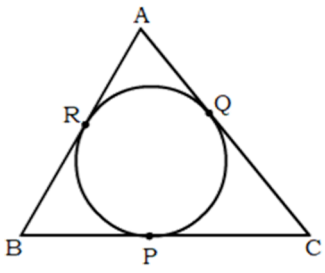


3-MARKS:

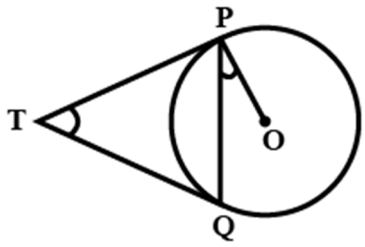
1. In figure XP and XQ are tangents from X to the circle with centre O. R is a point on the circle and AB is tangeat at R. Prove that: $XA + AR = XB + BR$.



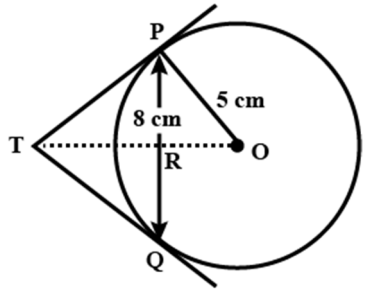
2. In figure a circle is inscribed in a $\triangle ABC$ touching BC, CA and AB at P, Q and R respectively: If $AB = 10$ cm, $AQ = 7$ cm, $CQ = 5$ cm find the length of BC.



3. In figure two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



4. In the given below figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.



5. Prove that opposite sides of quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

6. Prove that the lengths of tangents drawn from an external point to a circle are equal.

7. AB is a diameter and AC is a cbord of a circle such that $\angle BAC = 30^2$. If the tangent at C intersects AB produced at D, prove that BC = BD.

8. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

9. ABCD is a quadrilateral such that $\angle D = 90°$. A circle $C(O, r)$ touches the sides AB, BC, CD and DA as P, Q, R and S respectively. If $BC = 38$ cm, $CD = 25$ cm and $BP = 27$ cm, then find r.

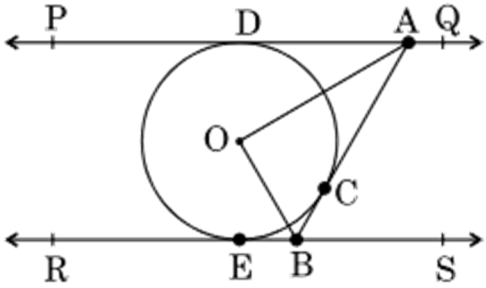
10. Prove that the perpendicular bisector of a chord of a circle always passes through the centre.

11. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

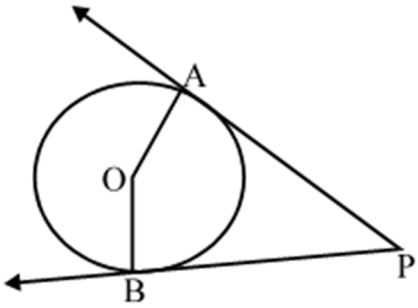
12. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

5-MARKS:

1. Prove that the lengths of the tangents drawn from an external point to a circle are equal.
2. In the figure, PQ and RS are two parallel tangents to a circie with centre 0 and another tangent AB, with point of contact C intersects PQ at A and RS at B. Prove that $\angle AOB = 90°$.

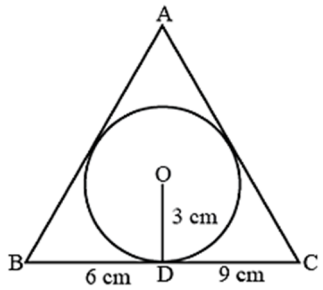


3. Prove that the tangent to a circle is perpendicular to the radius through the point of contact.
4. It the figure, O is the centre of the circle. PA and PB are tangent segments. Show that the quadrilateral AOBP is cyclic.



5. A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR. inito which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If area of $\triangle PQR$ is 336 cm^2, find the sides PQ and PR.
6. Two circles with centres A and B of radii 3 cm and 4 cm respectively intersect at two points C and D sach that AC and BC are tangents to the two circles. Find the length of the common chord CD.

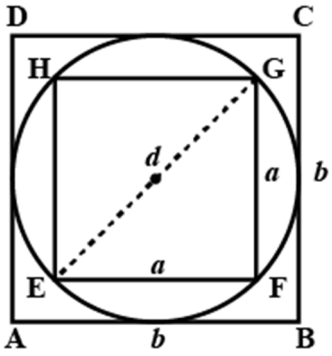
7. A triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If area of $\triangle ABC$ is 54 cm^2, then find the lengths of sides AB and AC.



CASE STUDY QUESTIONS

CASE STUDY_1

In the figure, a square is inscribed in a circle of diameter d and another square is circumscribing the circle.



i) Find the length of the diagonal of inner square EFGH? 1

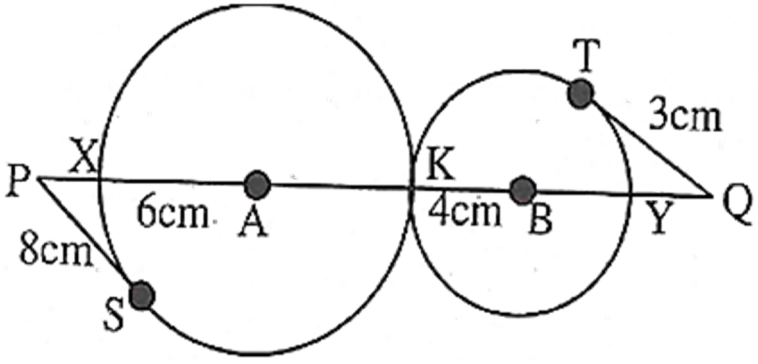
ii) Find the area of outer square PQRS? 1

iii) Find the ratio of area of outer square PQRS to area of inner square ABCD? 2

OR

Find the ratio of perimeter of outer square PQRS to perimeter of inner square ABCD? 2

CASE STUDY_2



Based on the above information, answer the following questions.

(i) Find the value of PA? 1

(ii) Find the value of BQ ? 1

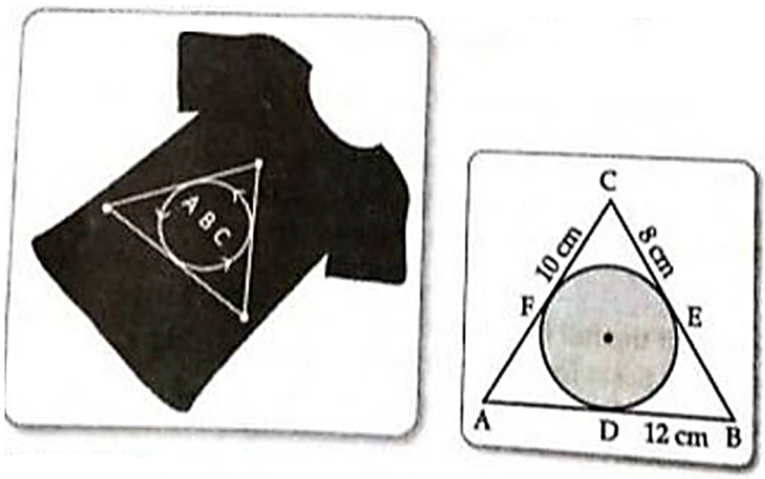
(iii) Find the value of PX and QY? 2

OR

Calculate the length of the tangent which is drawn from a point at a distance of 13cm from the centre of circle and the largest chord of that circle is 10cm. 2

CASE STUDY_3

Varun has been selected by has School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12cm, 8cm and 10cm respectively.



i) Find the length of AD and BE? 1

ii) If radius of the circle is 4 cm, Find the area of ΔOAB ? 1

iii) Calculate the perimeter of ΔABC 2

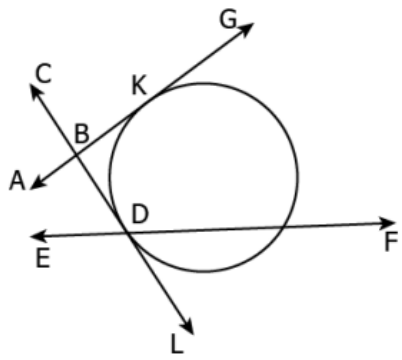
OR

Find the area of ΔABC ? 2

LEVEL – II

MCQ:

1. A figure is shown below.



Which of the following is true?

a) Lines AG and CL are the tangents and line EF is a secant to the circle.

b) Lines AG and CL are the secants and line EF is a tangent to the circle.

c) Line AG is a tangent and lines EF and CL are the secants to the circle.

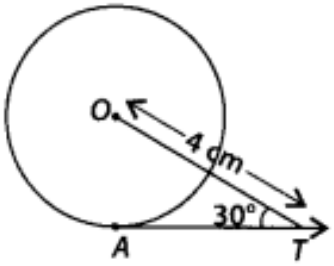
d) Line AG is a secant and lines EF and CL are the tangents to the circle.

2. A circle passes through point P. How many tangents and secants to the circle are possible that pass through P?

a) Tangent: 1; Secant: 1 b) Tangent: Infinite; Secant: 1

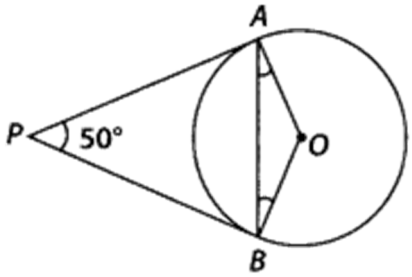
c) Tangent: 1; Secant: Infinite d) Tangent: Infinite; Secant: Infinite

3. In figure, AT is a tangent to the circle with centre O such that 0T = 4 cm and ∠OTA = 30°. Then AT is equal to:



a) 4 cm b) 2 cm c) $2\sqrt{3}$ cm d) $4\sqrt{3}$ cm

4. In figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^0$, then ∠OAB is equal to:

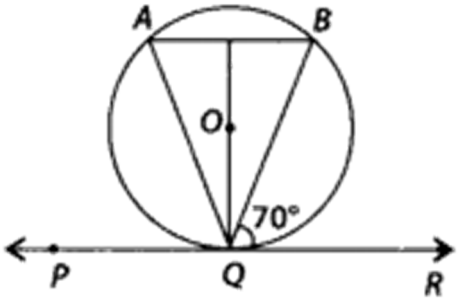


a) 25° b) 30° c) 40° d) 50°

5. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to:

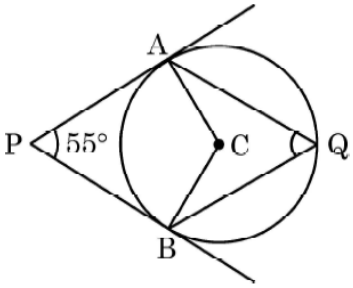
a) $\frac{3}{2}\sqrt{3}\,cm$ b) 6 cm c) 3 cm d) $3\sqrt{3}\,cm$

6. In figure, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and ∠BQR = 70°, then ∠AQB is equal to:



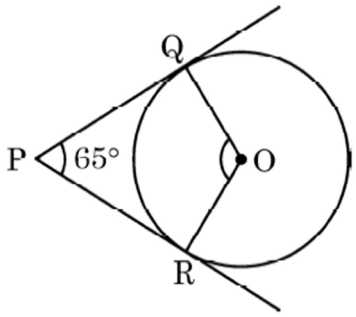
a) 20° b) 40° c) 35° d) 45°

7. In the given figure, PA and PB are the tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of $\angle AQB$ is:



a) $62\frac{1}{2}^0$ b) 125^0 c) 55^0 d) 90^0

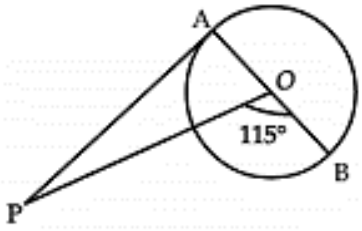
8. In the given figure, PQ and PR are tangents drawn from P to the circle with centre O such that $\angle QPR = 65^0$. The measure of $\angle QOR$ is:



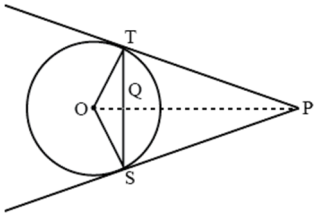
a) 65^0 b) 125^0 c) 115^0 d) 90^0

2-MARKS:

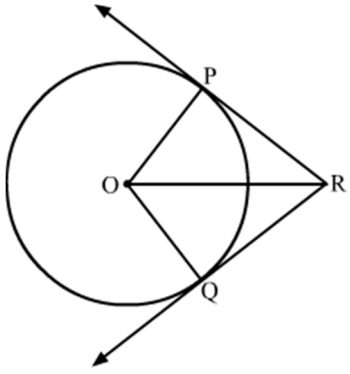
1. In the figure PA is a taggent from an extemal point P to a circle with centre 0. If $\angle POB = 115^0$, find $\angle APO$.



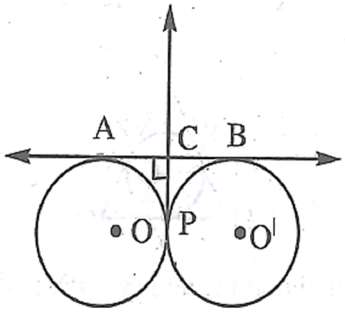
2. From an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If $OP = 2r$, show that $\angle OTS = \angle OST = 30^*$.



3. Two tangents RQ and RP are drawn from an extémal point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that $OR = PR + RQ$.

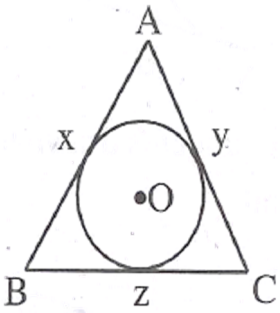


4. In the given figure If BC = 4.5 cm, then find the length of AB.



6. Two tangents TP and TQ are drawn from an external point T to a circle with centre O inclined to each other at an angle of 70^0. What is the value of $\angle POQ$?

7. ABC is an isosceles triangle in which AB = AC which is circumscribed about a circle as shown in the figure. Show that BC is bisected at the point of contact.



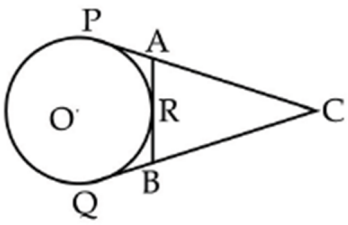
8. The length of tangent from an external point to a circle is always greater than the radius of the circle. Justify your answer.

9. Can the angle between two tangents to a circle may be 0°. Justify your answer.

10. If angle between two tangents drawn from a point P to a circle of radius 'a' and centre 'O' is 90^0, then $OP = a\sqrt{2}$, Justify your answer.

11. If angle between two tangents drawn from a point P to a circle of radius ' a ' and centre 'O' is 60^0, then $OP = a\sqrt{3}$. Justify your answer.

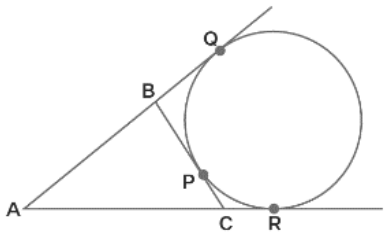
12. CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11cm, BC = 7cm, then find the length of BR.



3-MARKS:

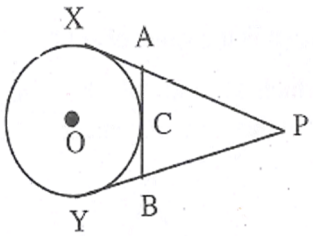
1. Two concentric circles are of radii 5cm and 3cm. Find the length of chord of the larger circle which touches the smaller circle.

2. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$.

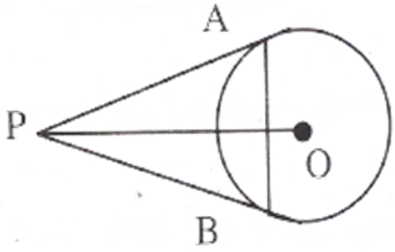


3. In the given figure a circle touches the sides PQ, QR and PR of $\triangle PQR$ at points X, Y & Z respectively. Show that $PX + QY + RZ = XQ + YR + ZP = \frac{1}{2}$ (Perimeter of $\triangle PQR$).

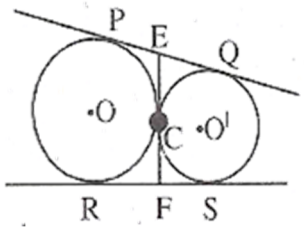
4. In the given figure, from an external point P, tangents PX and PY are drawn to a circle with centre 0. If AB is another tangent to the circle at C and PX = 14cm, then find the perimeter of ΔPAB.



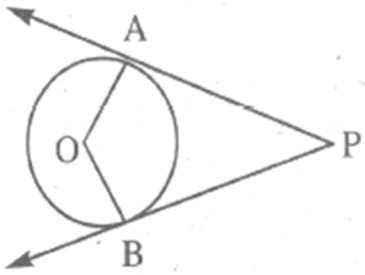
5. In the given figure, PA and PB are two tangents drawn to a circle with centre 0 and radius r. If OP = 2r, then show that $\triangle$ APB is equilateral.



6. In the given figure, the two circles touch each other externally at C. Prove that the common tangent it C bisects the other two tangents.



7. In the given figure, *O* is the centre of the circle and PA and PB are tangent segments, Show that quadrilateral ADBP is cyclic.



8. In two concentric circles, prove that all chords of the outer circle which touches the inner circle are of equal lengths.

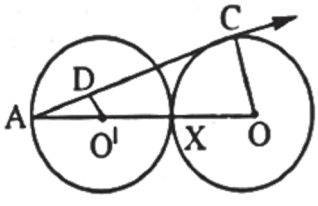
9. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6cm and 8cm. Determine the other two sides of the triangle.

10. Two tangents BC and BD are drawn to a circle with centre 0 from a point B, such that $\angle CBD = 120^0$. Prove that $OB = 2BC$.

11. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

5-MARKS:

1. In the figure two equal circles, with centres O and O^I, touch each other at X. O^{I}X produced meets the circle with centre O at A. AC is tangent to the circle with centre O, at the point C. O^{I}D is perpendicular to AC. Find the value of $\frac{DO^I}{CO}$.

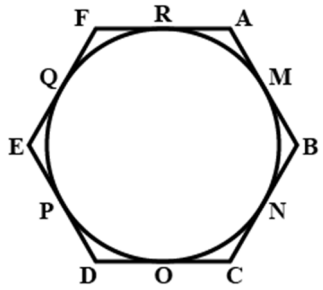


2. The radius of the inner circle of triangle is 4 cm and the segment into which one side is divided by the point of contact are 6 cm and 8 cm determine the other two sides of the triangle.

3. Two circle with centers O and O^{I} of radii 3cm and 4cm respectively intersect at two points P and Q such that OP and O^{I}P are tangents to the two circles. Find the length of the common chord PQ.

4. If an isosceles triangle ABC in which $AB = AC = 6\,cm$ is inscribed in a circle of radius $9\,cm$, then find the area of the triangle.

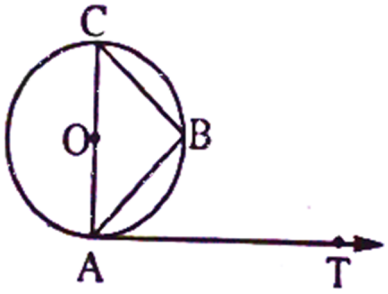
5. If a hexagon $ABCDEF$ circumscribe a circle, prove that $AB + CD + EF = BC + DE + FA$



7. Let 's' denote the semi - perimeter of a triangle ABC in which BC = a, CA = b and AB = c. If a circle touches the sides BC, CA and AB at D, E, F respectively. Prove that BD = s - b.

8. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10cm, find the perimeter of the triangle PCD.

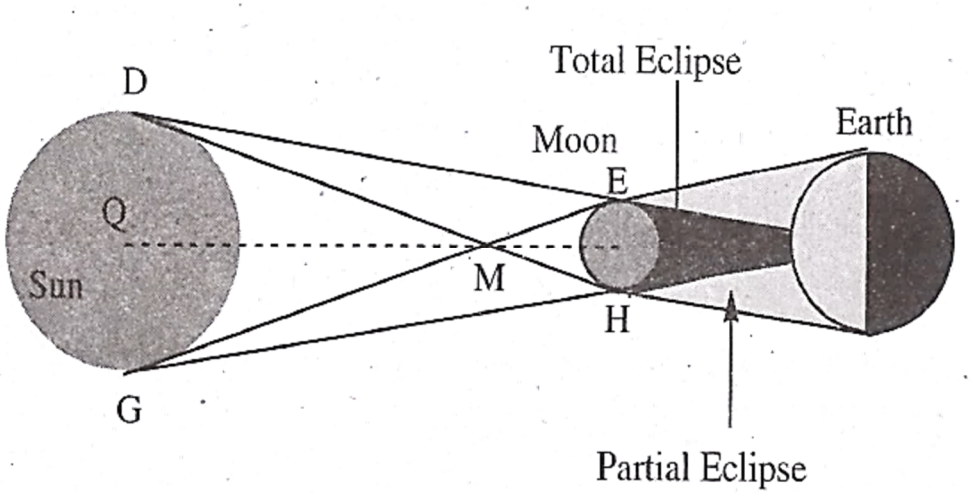
9. If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in the figure. Prove that $\angle BAT = \angle ACB$.



CASE STUDY QUESTIONS

CASE STUDY_1

A tangent line to a circle is a line that intersects the circle at exactly one point. (It appears to brush the edge of a circle). A point of tangency is the point where a tangent line intersects with a circle. Common external tangents do not intersect the segment that has its endpoints on the centers of the two circles. Common internal tangents intersect the segment that has its endpoints on the centers of the two circles. The dotted line represent the line segment that has its endpoints on the centres of the sun and the moon.



i) The drawing above shows how the sun, moon and earth are aligned for a solar eclipse. Identify the tangents lines which partition an area on the earth that experiences a total solar eclipse? 1

ii) From the above drawing, identify the common external tangents between the sun and the moon? 1

iii) What is the length of DM in the given picture, if EM = y units, HM = 10 units, $GM = (y + 10)$ units? 2

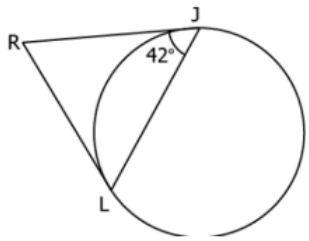
OR

What is the measure of ∠DOG in the given picture, if $\angle DMG = x^0$? 2

LEVEL-III

MCQ:

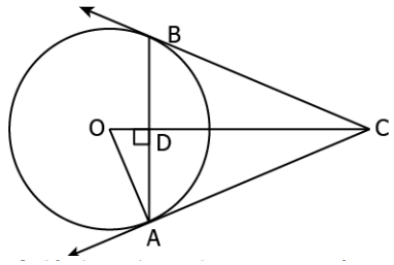
1. In the figure shown, RJ and RL are tangents to the circle.



What are the measure of $\angle JRL$?

a) 90° − 42° b) 90° − 84° c) 180° − 42° d) 180° − 84°

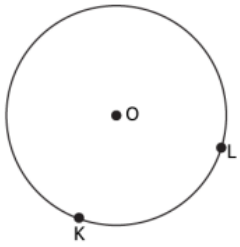
2. In figure is shown, AC and BC are two tangents to the circle O with radius 5 cm.



If OC = 13 cm and OD : DC = 3 : 10, then what is the perimeter of triangle ABC?

a) 24 cm b) 30 cm c) 32 cm d) 34 cm

3. A circle is shown below.



Statement I: There is only one line passing through point K which makes an angle of 90° with OK.

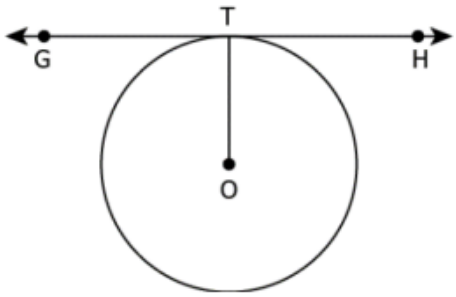
Statement II: The shortest distance of a tangent passing through point L from the centre O is equal to the radius of the circle, OL.

Statement III: One tangent can pass through two points K and L of a circle. Which statement(s) is/are correct?

a) Statement I and II b) Statement I and III

c) Statement II and III d) Statement I, II and III

4. A circle with centre O is shown below.



Which of the following statements is true?

a) There can be only one line passing through point T such that it is parallel to OT.

b) There can be only one line passing through point T such that it is perpendicular to OT.

c) There can be any number of lines passing through point T such that they are parallel to OT.

d) There can be any number of lines passing through point T such that they are perpendicular to OT.

5. A circle is drawn. Two points are marked outside the circle such that only 3 tangents can be drawn to the circle using these two points.

Which of the following is true based on the above information?

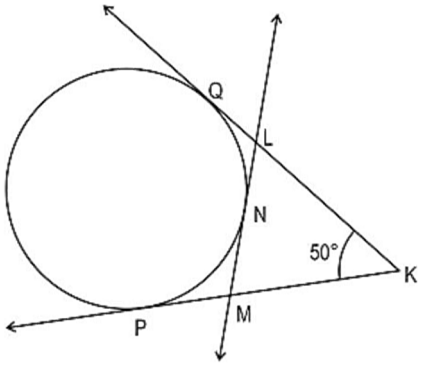
a) All 3 tangents are equal in length.

b) Both the points lie on one of the tangents.

c) The tangents and the circle have two common points in total.

d) (Such a situation is not possible as with 2 points, there will be 4 tangents to the circle).

6. Shown below is a circle with 3 tangents KQ, KP and LM. QL = 2 cm and KL = 6 cm. PM = 1/2 KL.



(Note: The figure is not to scale.)

What is the measure of $\angle LMK$?

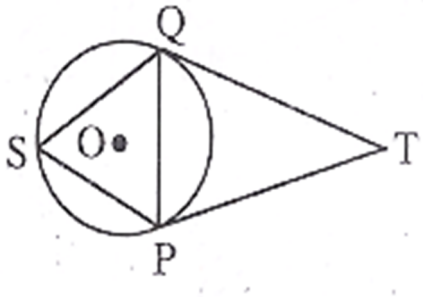
a) 50^0 b) 65^0

c) 80^0

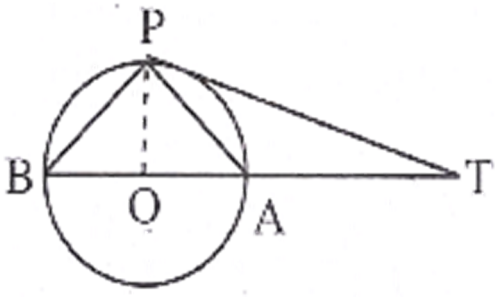
d) (Cannot be uniquely determined with the given information)

2-MARKS:

1. PT and QT are two tangents to the circle, if ∠PTQ = 50°, then find ∠PSQ and ∠OPQ



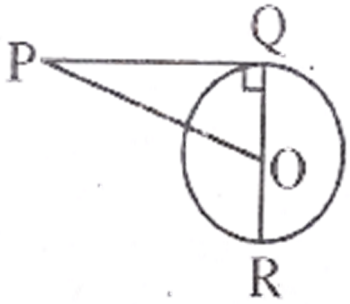
2. In the figure, BOA is a diameter of the circle with centre O and the tangent at a point P meets BA extended at T. If ∠ABP = 40°, then find ∠PTA.



3. Two circles with centres X and Y touch externally at P. If tangents AT and BT meet the common tangent at T, then prove that AT = BT.

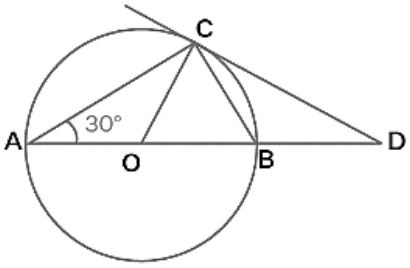
4. The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm and 7 cm respectively. Determine the length of the fourth side of the quadrilateral.

5. In the figure $OQ:PQ = 3:4$ and perimeter of $\triangle POQ = 60cm$. Determine PQ, QR and OP.



6. If a number of circles touch a given line segment PQ at a point A, then their centres lie on the perpendicular bisector of PQ. Write 'True' or 'False' and justify your answer.

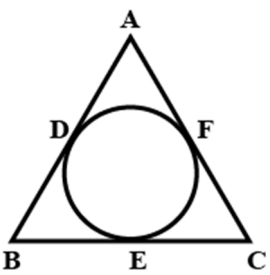
7. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD. Write 'True' or 'False' and justify your answer



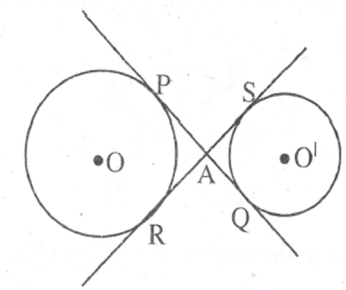
8. In the given figure, if AB = AC, prove that BE = EC.

OR

ABC is an isosceles triangle in which AB = AC, circumscribed about a circle, as shown in the given Figure. Prove that the base is bisected by the point of contact.



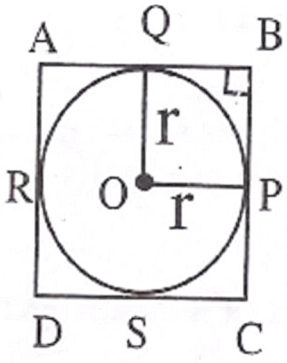
9. In the figure common tangents PQ asd RS are two circles intersect at A. Prove that $PQ = RS$.



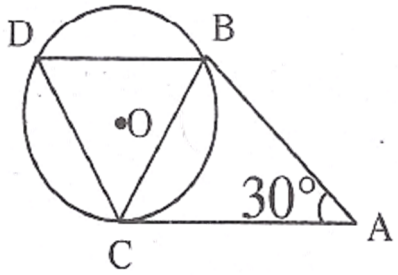
10. In the figure, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^0$. Prove that OA = 2AB.

3-MARKS:

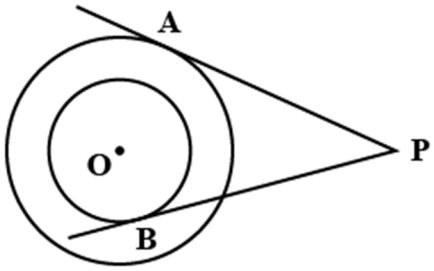
1. In the given below figure a circle is inscribed in quadriateral ABCD in which $\angle B = 90°$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, then find radius (r) of the circle.



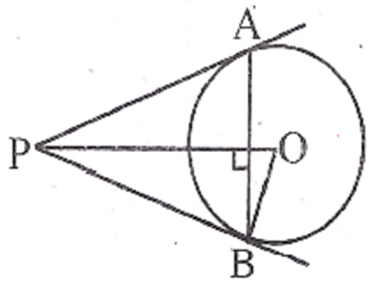
2. In the given figure, tangents AC and AB are trawn to a circle from a point A such that $\angle BAC = 30^0$, a chord BD is drawn parallel to the tangent AC. Find $\angle DBC$,



3. In the figure there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA & PB are drawn to these circles. If AP = 12 cm, then find the length of BP.



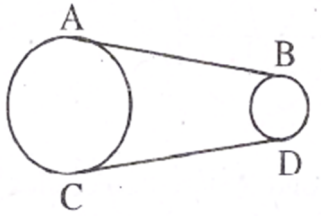
4. In AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.



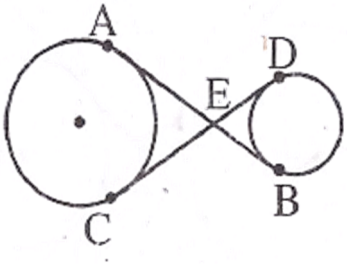
5. If d_1, d_2 $(d_2 > d_1)$ be the diam of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.

6. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

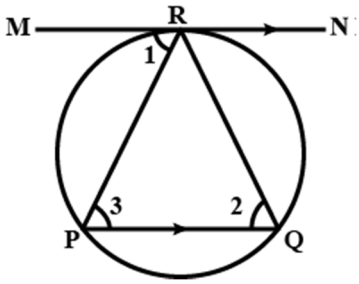
7. In figure, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



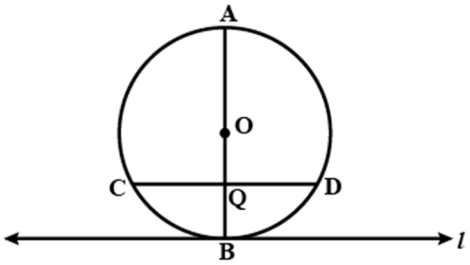
8. In figure, common tangents AB and CD to two circles intersect at E. Prove that $AB = CD$.



9. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

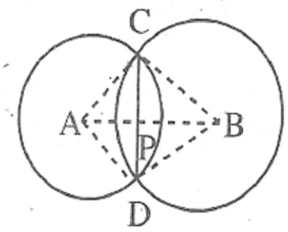


10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

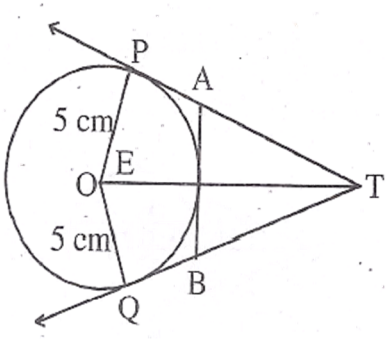


5-MARKS:

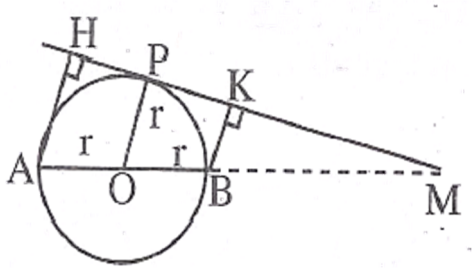
1. Two circles with centres A and B of radili 3cm and 4cm respectively intersect at two points C and D such that AC and BC are tangents to the two circles. Find the length of the common chord CD.



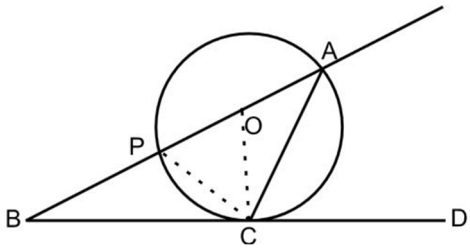
2. O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle it E. If AB is the tangent to the circle at E, then find length of AB.



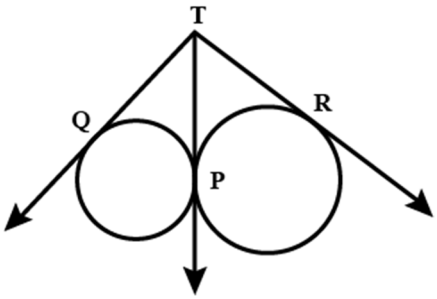
3. AB is a diameter of a circle, P is a point on the semi-circle APB. AH and BK are perpendiculars from A & B respectively to tangent at P. Prove that AH + BK = AB.



4. In the figure, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle CAD = 90^0$.

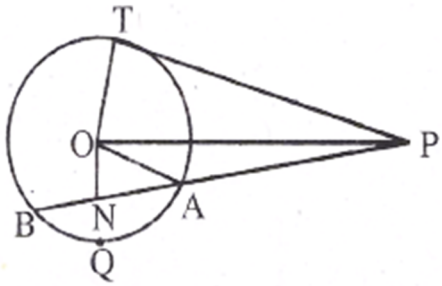


7. Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that TQ = TR.



8. In the figure, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that.

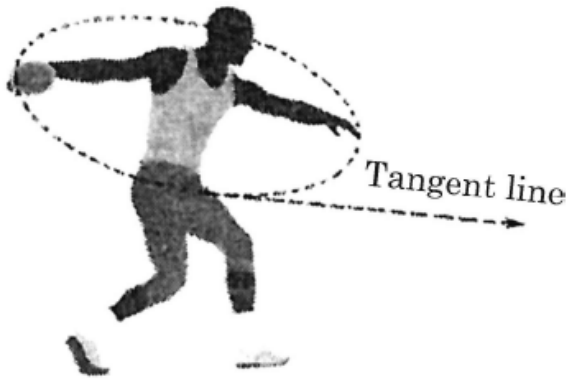
i) $PA.PB = PN^2 - AN^2$ ii) $PN^2 - AN^2 = OP^2 - OT^2$ iii) $PA.PB = PT^2$



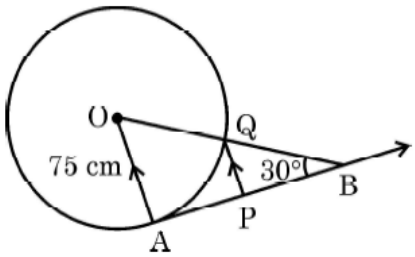
CASE STUDY QUESTIONS

CASE STUDY_1

The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75cm. Point O is centre of the circle and $\angle ABO = 30^0$. PQ is parallel to OA.



Based on the above information:

i) Find the length of AB. 1

ii) Find the length of OB. 1

iii) Find the length of AP. 2

OR

Find the length of PQ.

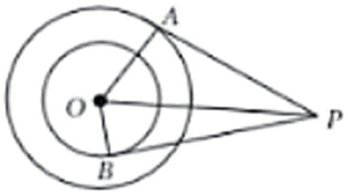
CASE STUDY_2

If a tangent is drawn to a circle from an external point, then the radius at the point of contact is perpendicular to the tangent. Answer the following questions using the above condition.

i) Two concentric circles are of radii 5cm and 3cm. Find the length of the chord of the larger circle which touches the smaller circle. 1

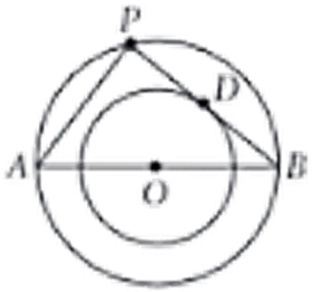
ii) Two concentric circles are such that the difference between their radii is 4cm and the length of the chord of the larger circle which touches the smaller circle is 24cm. Find the radius of the smaller circle. 1

iii) In the given figure, O is the centre of two concentric circles of radii 5cm and 3cm. From an external point P tangents PA and PB are drawn to these circles. If PA = 12cm then find the value of PB. 2



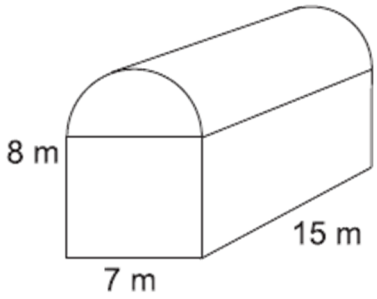
OR

The diameter of two concentric circles are 10cm and 6cm. AB is a diameter of the bigger circle and BD is the tangent to the smaller circle touching it at P on producing. Find the length of BP.



SKILL BASED QUESTIONS

1. Deepa runs an industry in a shed which is in the shape of a cuboid surmounted by a half-cylinder. The base of the shed is of dimensions 7 m × 15 m and the height of the cuboidal portion is 8 m.

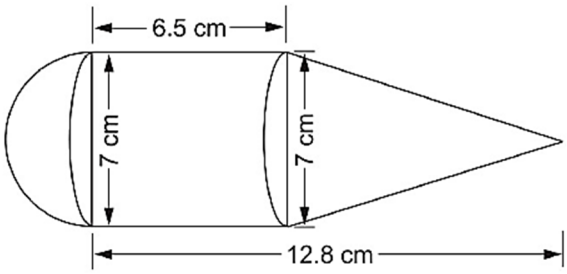


(i) Find the volume of air that the shed can hold.

(ii) Suppose the machinery in the shed occupies a total space of 300 m3 and there are 20 workers, each of whom occupy about 0.08 m3 space on an average. Then, how much air is in the shed?

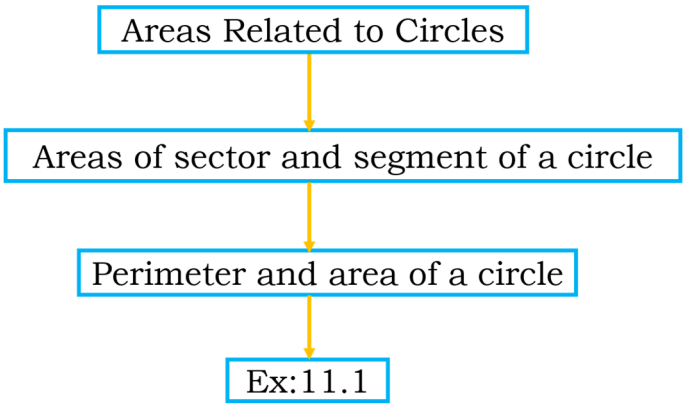
(iii) Find the internal surface area of the shed, excluding the floor.

2. The given figure represents a solid consisting of a cylinder surmounted by a cone at one end and a hemisphere at the other. Find the volume of the solid.



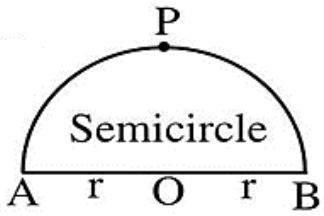
AREAS RELATED TO CIRCLES

MIND MAPPING:



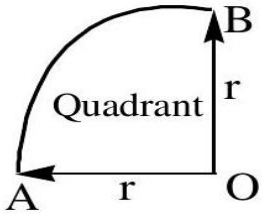
BASIC FACTS AND FORMULAE:

1. A circle is a collection of all points in a plane which arc at a constant distance from a fixed point in the same plane.
2. Circumference (Perimeter) of a circle = πd or $2\pi r$, where r is the radius of the circle.
3. Area of a circle = πr^2
4. Semicircle: Diameter of a circle divides it into two equal parts, each being called a semicircle, (i.e., half of the circle). As shown in figure, APB is a semi circle (Semi ⇒ half)



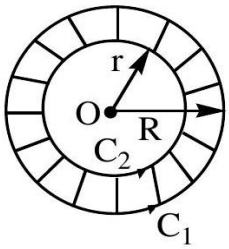
i) Perimeter of a semicircle = $(\pi r + 2r)$ ii) Area of the semi-circle = $\frac{1}{2}\pi r^2$

5. **Quadrant of a circle:** It is one - fourth of a circle. As shown in the below fig.



Perimeter of a quadrant of a circle = $\frac{1}{4}(2\pi r) + 2r = \frac{\pi r}{2} + 2r$

6. Area of a ring: It is the area enclosed between two concentric circles. Let $C_1(0,R)$ and $C_2(0,r)$ be two concentric circles as shown in fig.



Area of ring = (Area of the outer circle, C_1) − (Area of the inner circle, C_2)

$= \pi R^2 - \pi r^2 = \pi(R+r)(R-r)\,unit^2$

7. **Circular wheel in Motion:** Suppose a circular wheel of radius r is in motion and it completes n number of revolutions in 1 minute. Then

Distance travelled by rotating wheel in 1 revolution, D = Perimeter of the wheel

No of revolutions in 1 minute, $n = \frac{\text{distance travelled by the wheel in 1 minute}}{\text{Perimeter/Circumference of the wheel}}$

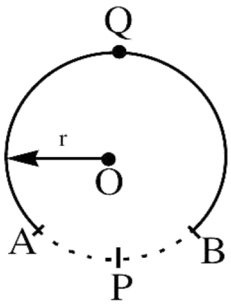
$\Rightarrow n = \frac{D}{2\pi r}$

8. **Arc of a circle:** It is a continuous part of perimeter/ circumference of a circle and its length is denoted by 'l '. It is of two types.

i) **Minor arc:** An arc whose length is less than half of the perimeter of a circle is called a minor - arc.

ii) **Major arc:** An arc whose length is more than half of the perimeter of a circle is called a major - arc. As shown in figure in circle $C(0,r)$.

APB is the minor arc and BQA is the major arc.

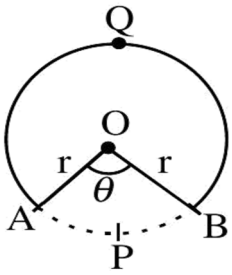


9. **Sector of a circle:** It is a part of the circular region enclosed by two radii and the corresponding arc of the circle. It is of two types.

i) **Minor sector:** A sector is called a ' minor sector', if the bounding arc is a minor arc of the circle.

ii) **Major sector:** A sector is called a 'major sector', if the bounding arc is the major arc of the circle.

The region OAPB is a minor sector, (or simply a sector) denoted by sector OAB. The region OAQB is the major sector, denoted by sector OAQB.



10. a) Area of the sector = $\frac{\theta}{360}\times\pi r^2$. where, r is the radius of the circle and θ is angle of the sector in degrees.

b) Area of major sector OAQB = [Area of circle $C(O,r)$] – [Area of minor sector]

Area of the major sector OAQB = $\pi r^2 - \frac{\pi r^2}{360}\times\theta$

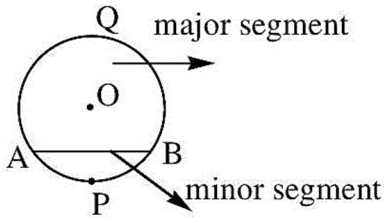
11. Length of an arc of sector $l = \frac{\theta}{360}\times 2\pi r$

12. Area of a sector in terms of arc length, l and r is $\frac{lr}{2} sq.units$

13. **Segment of a circle:** It is a part or portion of the circular region enclosed between a chord and corresponding arc of the circle. It is also of two types.

i) **Minor segment:** A segment is called a minor segment, if the bounding arc of the circle is a minor arc.

ii) **Major segment:** A segment is called a major segment if the bounding arc of the circle is a major arc. As shown in fig in circle $C(O,r)$.



The region APB is a minor segment, (or simply a segment)

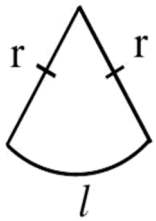
The region AQB is the major segment.

14. Area of major segment = Area of the circle − Area of minor segment
$= \pi r^2 -$ Area of minor segment.

15. If a chord subtend 90^0 at centre, then the area of corresponding segment $= \left(\frac{\pi}{4} - \frac{1}{2}\right) r^2$

16. If a chord subtend 60° at centre, then area of corresponding segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) r^2$

17. Perimeter of sector = $\frac{\pi r\theta}{180^0} + 2r$ (OR) perimeter $l + 2r$



18. Angle described by minute hand in 60 minutes $= 360^0$

Angle described by minute hand in one minute $= \left(\frac{360}{60}\right)^0 = 6^0$

Thus, minute hand rotates through an angle of 6^0 in one minute.

19. Angle described by hour hand in 12 hours $= 360^0$

Angle described by hour hand in one hour $= \left(\frac{360}{12}\right)^0 = 30^0$

Angled described by hour hand in one minute = $\left(\frac{30}{60}\right)^0 = \frac{1}{2}^0$

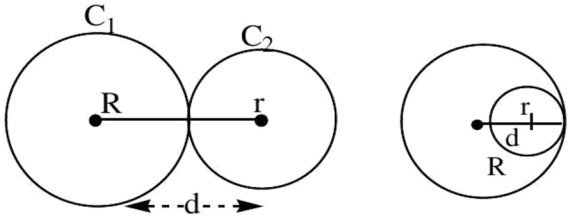
Thus, hour hand rotates through $\frac{1}{2}^0$ in one minute.

20. Touching circles: Two circles can touch each either externally or internally as shown below.

Case-(I): External touching: distance between the centres = Sum of radii of circles.

$d = R + r$, where d is the distance between the centres.

Case-(II): Internal touching: distance between centres = Difference of radii of circle d = R − r where d is the distance between the centres.



LEVEL - I

MCQ:

1. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

   a) $R_1 + R_2 = R$ b) $R_1^2 + R_2^2 = R^2$ c) $R_1 + R_2 = R$ d) $R_1^2 + R_2^2 < R^2$

2. The area of a sector of a circle with radius 6 cm if angle of the sector is 60^0 is:

   a) $\frac{123}{7}cm^2$ b) $\frac{132}{7}cm^2$ c) $\frac{144}{7}cm^2$ d) $\frac{156}{7}cm^2$

3. In a circle of diameter $42cm$, if an arc subtends an angle of 60^0 at the centre where $\pi = \frac{22}{7}$ then what is the length of arc?

   a) 33 cm b) 11 cm c) 15 cm d) 22 cm

4. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.

   a) 18 cm b) 24 cm c) 36 cm d) 33 cm

5. The length of an arc of a circle of radius 12 cm is 10πcm, then the central angle of this arc is:

   a) 150^0 b) 130^0 c) 120^0 d) 140^0

6. Perimeter of the sector with radius 10.5 cm and sector angle 60° is:

   a) 32 cm b) 23 cm c) 64 cm d) 48 cm

7. Areas of two circles are in the ratio of $9:4$, then ratio of their circumferences is:

   a) 3 : 2 b) 2 : 3 c) 5 : 4 d) 4 : 5

8. The circumference of a circle is 39.6 cm, then its area is:

   a) 124.74 cm^2 b) 156.73 cm^2 c) 136.74 cm^2 d) 179.62 cm^2

2-MARKS:

1. Find the perimeter of a square circumscribing a circle of radius 3.5 cm.
2. The radius of a circle is 17.5 cm. Find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

3. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
4. The radii of two circles are 8cm and 6cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
5. If the perimeter and area of a circle are numerically equal, then the radius of the circle is 2 units. Justify your answer
6. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of $16.5\,km$. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)
7. If the perimeter of a protractor is 72 cm, calculate its area. (Take $\pi = \frac{22}{7}$)
8. Find the area of the quadrant of a circle whose circumference is 22 cm.
9. Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use $\pi = 3.14$)
10. Find the area swept by 7 cm long minute hand of a clock in 10 minutes?
11. A wire can be bent in the form of a circle of radius 56cm. If the same wire is bent in the form of a square, then find the area of the square.

3-MARKS:

1. A chord of a circle of radius 15 cm subtends an angle of 60^0 at the centre. Find the areas of corresponding minor segment of the circle. (Use $\pi = \frac{22}{7}$ & $\sqrt{3} = 1.73$)
2. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60^0. Also find the area of the corresponding major segment.(Use $\pi = \frac{22}{7}$)
3. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

4. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115^0. Find the total area cleaned at each sweep of the blades.

5. The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs.5280. Find the area of the field. (Use $\pi = \frac{22}{7}$).

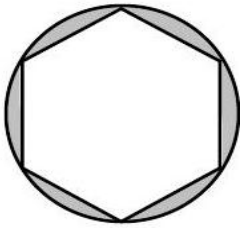
6. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: i) minor segment ii) major sector (Use $\pi = 3.14$)

7. A wire when bent in the form of a square enclose an area 121 cm^2. If wire was bent in the form of a circle, then find the area enclosed by circle. (Use $\pi = \frac{22}{7}$).

5-MARKS:

1. In a circle of radius 21cm, an arc subtends an angle of 60^0 at the centre. Find:
   i) the length of the arc
   ii) area of the sector formed by the arc
   iii) area of the segment formed by the corresponding chord

2. A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 (Use $\sqrt{3} = 1.732$).



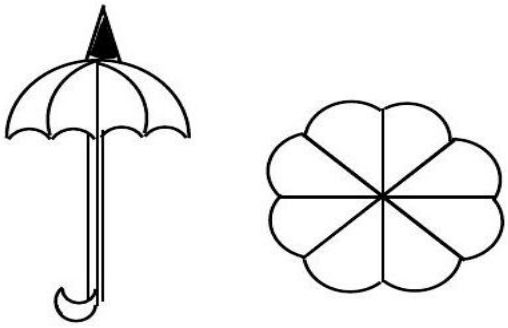
3. A brooch is made with silver wire in the form of a circle with diameter 35 mm for the inauguration of a Social club in a school to keep environment clear in its neighbourhood. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find
   i) Total length of the silver wire required.
   ii) The area of each sector of the brooch.

4. There is a circular park in front of a shopping mall whose radius is 20 m. There is a gravel path of uniform width of 2 m around the park.

   i) A person takes five rounds of the circular park. What distance he covers?

   ii) What is the area of travelled path?

5. An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



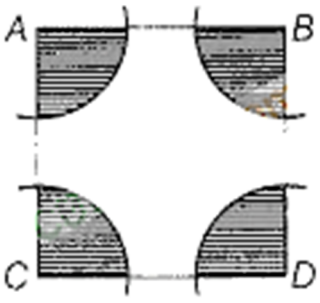
6. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope.

   i) The area of that part of the field in which the horse can graze ($Use\ \pi = 3.14$)

   ii) Increase in grazing area if rope were 10 m long instead of 5m. ($Use\ \pi = 3.14$)

7. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

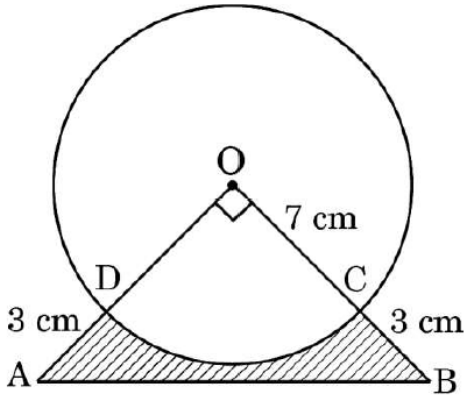
8. In figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.



CASE STUDY QUESTIONS

CASE STUDY_1

In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is 20 per cm^2.



Based on the above information, answer the following questions:

i) What is the area of the quadrant ODCO? 1

ii) Find the area of triangle AOB. 1

iii) What is the total cost of silver plating the shaded part ABCD? 2

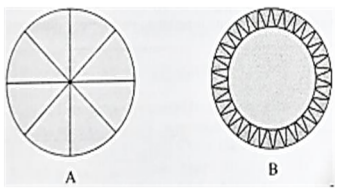
OR

What is the length of the arc CD?

CASE STUDY_2

A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat.

Designs of some brooch are shown below. Observe them carefully.



Design A: Brooch A is made with silver wire in the form of a circle with diameter 28 mm. The wire used for making 4 diam which divide the circle into 8 equal parts.

Design B: Brooch b is made in two colours Gold and silver. Outer part is made with Gold. The circumference of silver part is 44 mm and the gold part is 3 mm wide everywhere.

i) With reference to the design A, what is the total length of silver wire required? 1

ii) With reference to the design A, find the area of each sector of the brooch. 1

iii) With reference to design B, find the difference of areas of golden & silver parts. 2

OR

A boy is playing with brooch B. He makes revolution with it along its edge. How many complete revolutions must it take to cover 80πmm ? 2

CASE STUDY_3:

Can you hit the bullseye? Bullseye or archery is the sport of hunting or shooting at targets using a bow and arrows. Archery target is formed with concentric circles. Look at the figure shown here. The figure depicts an archery target marked with its five scoring areas from the centre outwards as gold, Red, Blue Black and white. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide.

i) What is the area of the region representing the Gold scoring area? 1

ii) What is the radius of the region representing Gold and Red scoring areas? 1

iii) What is the area of region representing Red scoring area? 2

OR

What is the radius of the region representing Gold, Red, Black and Blue scoring areas?

LEVEL - II

MCQ:

1. The difference between circumference and radius of a circle is 111 cm. Find the area of the circle.

   a) 23 cm b) 20 cm c) 21 cm d) 25 cm

2. A pendulum swings through an angle of 30^0 and describes an arc 8.8 cm in length. Find the length of the pendulum.

   a) 16.8 cm b) 24 cm c) 12.3 cm d) 8.4 cm

3. If the radius of a circle is doubled, then its area is increased by:

   a) 100 % b) 200 % c) 300% d) 400%

4. If the perimeter of a semi-circular protractor is 72 cm where $\pi = \frac{22}{7}$, then find the diameter of protractor.

   a) 14 cm b) 33 cm c) 28 cm d) 42 cm

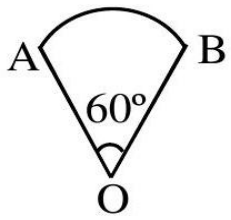
5. The hour-hand of a clock is 6cm long. The angle swept by it between 7:20 a.m. and 7:55 a.m. is:

   a) $\left(\frac{35}{4}\right)^0$ b) $\left(\frac{35}{2}\right)^0$ c) 35^0 d) 70^0

6. The perimeter of a sector of a circle of radius 14 cm is 68 cm. Find the area of the sector.

   a) 420 cm^2 b) 280 cm^2 c) 320 cm^2 d) 165 cm^2

7. In figure is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. (Take $\pi = \frac{22}{7}$).



   a) 15 cm b) 21.5 cm c) 11 cm d) 10.5 cm

8. If the circumference of a circle is tripled, then area becomes

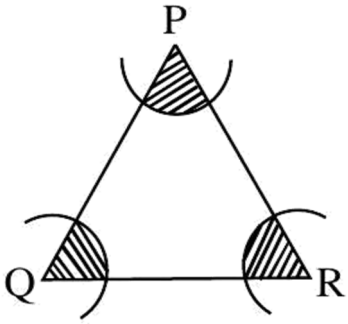
a) 9 times b) 3 times c) 6 times d) 30 times

2-MARKS:

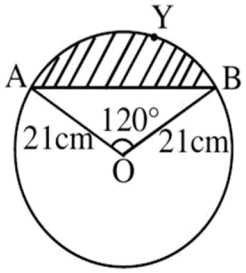
1. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.
2. Find the diameter of a circle whose circumference is equal to the sum of the circumference of the two circles of diam 36 m and 20 m.
4. The perimeter of a sector of a circle is 66 cm and the radius of circle is 12 cm. Find the corresponding length of the arc.
5. Find the area of a right angled triangle, if the radius of its circumcircle is 3 cm and altitude drawn to the hypotenuse is 2 cm.
6. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.
7. In a circle with centre 0 and radius 5 cm, AB is a chord of length $5\sqrt{3}\,cm$. Find the area of sector AOB.
8. Is the following statement true? Give reasons for your answer. Area of a segment of a circle = area of the corresponding sector - area of the corresponding triangle.
9. A road which is $7\,m$ wide surrounds a circular park whose circumference is $88\,m$. Find the area of the road.

3-MARKS:

1. A horse is tethered to one corner of a rectangular field of dimensions 70×52 m, by a rope of length 21 m. How much area of the field can it graze?
2. In figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



3. Find the area of the segment AYB shown in figure, if radius of the circle is 21 cm and $\angle AOB = 120°$. (Use $\pi = \frac{22}{7}$).



4. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimension 20 m × 16 m. Find the area of the field in which the cow can graze.

5. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, circles are drawn with radius equal to half the length of side of triangle. Find the area of that part of triangle which is not included in circles. (Use $\sqrt{3} = 1.73, \pi = \frac{22}{7}$)

6. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour which the boy is cycling.

7. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120^0. Find the total area cleaned at each sweep of the two blades.

8. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of Rs. 50 per metre.

9. Is it true to say that area of a segment of a circle is less than the area of its corresponding sector? Why?

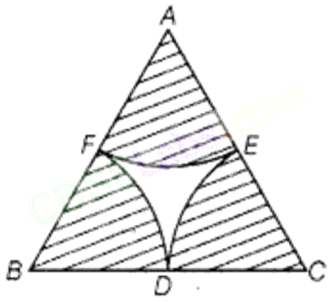
5-MARKS:

1. In a circle of radius 6 cm, a chord of length 10 cm makes an angle of 110° at the centre of the circle. Find:

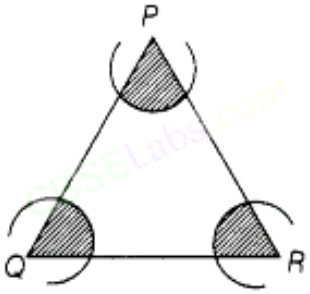
   i) the circumference of the circle ii) the area of the circle

   iii) the length of the arc AB iv) the area of the sector 0AB

2. In figure arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, To intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region, (use π = 3.14).



3. In figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



4. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am and 6:40 am.

5. Two circles touch each other internally. The sum of their areas is 116π cm^2 and distance between their centres is 6 cm. Find the radii of the circles.

6. A truck has front wheels of radius 0.7 m and its rear wheels have radius 1.4 m. If the rear wheel takes 500 revolutions to travel a certain distance, how many

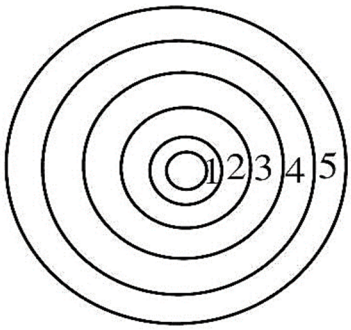
revolutions must the front wheel had taken to cover the same distance? Also, find the difference in their areas. (Use $\pi = \frac{22}{7}$).

7. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use = 3·14)

CASE STUDY QUESTIONS

CASE STUDY_1

Five persons are walking on the ground 1^{st} person is 14 m away from the centre of the ground. The distance between any two consecutive persons is $\frac{1}{2}m$.



i) How much distance travelled by 1^{st} person? 1

ii) What is the Sum of distances travelled by 1^{st} person and 5^{th} person? 1

iii) What is the difference between the 2^{nd} and 3^{rd} person's distances? 2

OR

What is the difference between the 3^{rd} and 4^{th} person's distances?

LEVEL - III

MCQ:

1. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then what is the relation among $R, R_1 \text{ and } R_2$?

a) $R_1 + R_2 = R$ b) $R_1^2 + R_2^2 = R^2$ c) $R_1 + R_2 < R$ d) $R_1^2 + R_2^2 < R^2$

2. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then what is the relation among R, R_1 and R_2 ?

a) $R_1 + R_2 = R$ b) $R_1^2 + R_2^2 = R^2$ c) $R_1 + R_2 < R$ d) $R_1^2 + R_2^2 < R^2$

3. On increasing the diameter of a circle by 40%, its area will be increased by

a) 40% b) 80% c) 96% d) 82%

4. On decreasing the radius of a circle by 30%, its area is decreased by

a) 30% b) 60% c) 45% d) none of these

5. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diam 16 m and 12 m in a locality. Find the radius of the new park?

a) 10m b) 15m c) 20m d) 25m

6. Find the radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diam 36 cm and 20 cm ?

a) 16 cm b) 56 cm c) 28 cm d) 42 cm

7. Find diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm?

a) 25 cm b) 50 cm c) 75 cm d) 12.5 cm

2-MARKS:

1. In covering a distance s metres, a circular wheel of radius r metres makes $\frac{s}{2\pi r}$ revolutions. Is this statement true? Why?

2. Is it true that the distance travelled by a circular wheel of diameter d cm in one revolution is $2\pi d$ cm ? Why?

3. Circumferences of two circles are equal. Is it necessary that their areas will also be equal? Why?

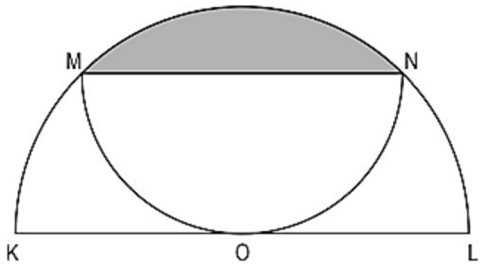
4. Areas of two circles are equal. Is it necessary that their circumferences will also be equal? Why?

5. A bicycle wheel of radius 35 cm is making 25 revolutions in 10 seconds. At what speed in km/hr is the cycle moving? A bicycle wheel makes 5000 revolutions in moving 22 km. Find the diameter of the wheel. (Use $\pi = \frac{22}{7}$).

6. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form a circle, then what is the circumference of the circle?

7. The sum of the radii of two circles is 140 cm and the difference of their circumferences is 88 cm. Find the diameter of the circles.

8. A semi-circle MON is inscribed in another semi-circle. Radius OL of the larger semicircle is 6cm.



(Note: The figure is not to scale.)

Find the area of the shaded segment in terms of π. Draw a rough figure and show your steps.

3-MARKS:

1. Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

2. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending an angle 90° at the centre.

3. The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120^0 and 40^0. Find the areas of the two sectors as well as the lengths of the corresponding arcs. What do you observe?

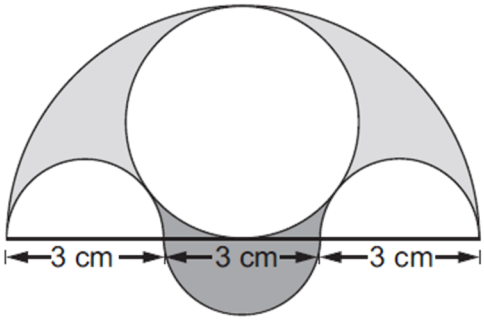
4. Area of a sector of central angle 200° of a circle is 770 cm^2. Find the length of the corresponding arc of this sector.

5. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of $66 km/hr$?

6. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road.

7. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

8. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semi-circle of radius 4.5 cm are drawn in the given figure. Find the area of shaded region.



5-MARKS:

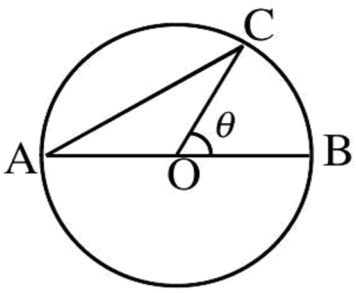
1. Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles.

2. The diameter of front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

3. Sides of a triangular field are 15 m, 16 m, and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which can be grazed by the three animals.

4. A chord of a circle of radius 14 cm subtends an angle of 60^0 at the centre. Find the area of the corresponding minor segment of the circle. Also find the major segment of the circle.

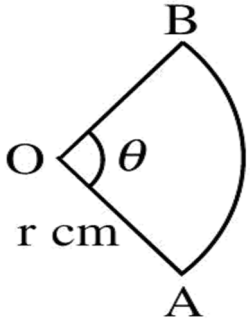
5. AB is the diameter of a circle, with centre O. C is a point on the circumference such that $\angle COB = \theta$. The area of the minor segment cut off by AC is equal to twice the area of the sector BOC. Prove that $\sin\frac{\theta}{2}.\cos\frac{\theta}{2} = n\left(\frac{1}{2} - \frac{\theta}{120}\right)$.



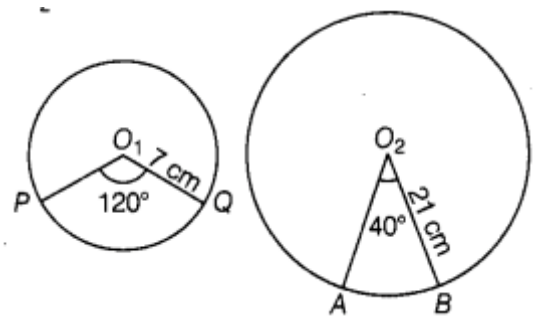
6. A chord of a circle subtends an angle of θ at the centre of the circle. The area of the minor segment cut off by the chord is one eighth of the area of the circle. Prove that $8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$.

7. The diagram shows a sector of a circle of radius rcm containing an angle θ. The area of the sector is Acm^2 and perimeter of the sector is 50 cm. Prove that

i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$ ii) $A = 25r - r^2$



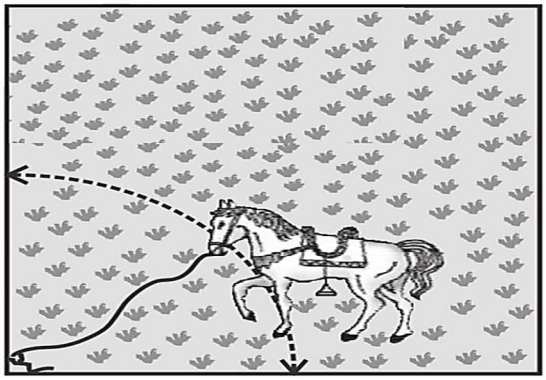
8. The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the lengths of the corresponding arcs. What do you observe?



CASE STUDY QUESTIONS

CASE STUDY_1

A horse is tied to peg at one corner of a square shaped grass field of sides 15 m by means of a 5 m long rope (see the given figure) (Use $\pi = 3.14$).



i) What is the area of the grass field? 1

ii) What is the area of that part of the field in which the horse can graze? 1

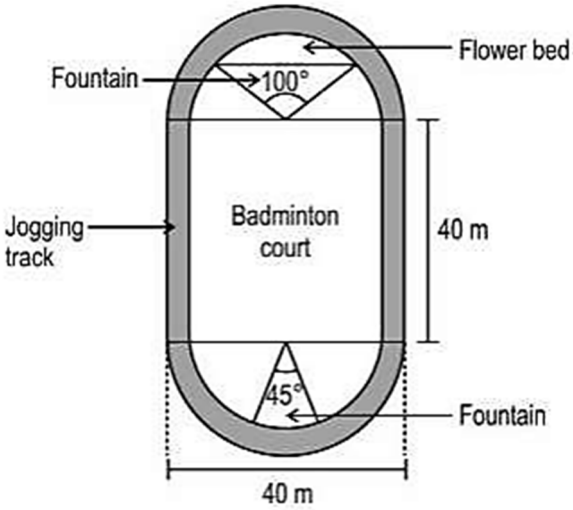
iii) What is the grazing area if the rope were 10 m long instead of 5 m? 2

OR

What is the area of the part of the field in which horse cannot be grazed?

CASE STUDY_2

Shown below is the top view of a stadium. There is a badminton court at the centre. The stadium is surrounded by a jogging track. The track is semi-circular in shape at the top and the bottom of the court. The fountains converge at the centre of the respective semicircles. The jogging track has a uniform width of 2 m.



(Note: The figure is not to scale.)
(Note: Use π = 3.14)

Based on the above information, answer the following questions:

i) What is the area of the jogging track? 1

ii) What is the area occupied by the fountain shown below the court in figure? 1

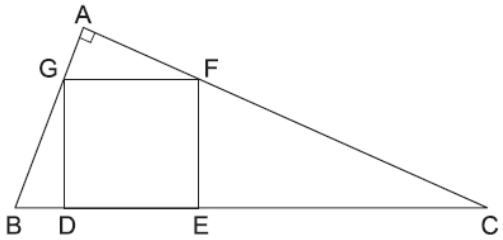
iii) The cost of gardening is Rs 300/m^2 and the area of the fountain next to the flower bed is 150 m^2. What is the cost of gardening the flower bed? 2

OR

If the rate of fencing is Rs 150/m, what is the cost of fencing the flower bed ONLY on the curved portion of its boundary?

SKILL BASED QUESTIONS

1. In the given figure, DEFG is a square and $\angle BAC = 90^0$.



Prove that:

(i) $\Delta AGF \sim \Delta DBG$

(ii) $\Delta AGF \sim \Delta EFC$

(iii) $\Delta DBG \sim \Delta EFC$

(iv) $DE^2 = BD \times EC$

2. In the given figure, BM and EN are respectively the medians of ΔABC and ΔDEF. If $\Delta ABC \sim \Delta DEF$, prove that:

(a) $\Delta AMB \sim \Delta DNE$ (b) $\Delta CMB \sim \Delta FNE$ (c) $\frac{BM}{EN} = \frac{AC}{DF}$