CBSE

CLASS-X

MATHEMATICS

CENTUM BOOK

PHASE- I

MATHEMATICS

Code-(041)

Class-X (2023-24)

COURSE STRUCTURE

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
Total		80

QUESTION PAPER DESIGN (Standard)

Time: 3 Hours **Max. Marks: 80**

S.No	Typology of Questions	Total Marks	% Weightage (approx.)
1	Remembering: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. Understanding: Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	43	54
2	Applying: Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24

3	Analysing: Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations Evaluating: Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. Creating: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	80	100

Methods of Questioning-Skills in focus

1. REMEMBERING

Questions where a child just recalls the information learnt.

Direct question where student uses the formula or writes a statement etc.

Eg: Find the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$.

Eg: Evaluate $\sin 30^0 \cos 60^0 - \cos 30^0 \sin 60^0$.

2. UNDERSTANDING

Questions which need interpretation of information given

Questions where we want to test whether a student has understood a concept.

Eg: Prove that "If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio".

Eg: Check whether $(x+2)^3 = x^3 - 4$ is quadratic equation.

3. APPLYING

Questions where application of acquired knowledge in a different situation is needed.

Indirect questions where a child has to facts or rules in a different way.

Eg: Representing the following situation in the form of quadratic equation: The area of a rectangular plot is 528 m2. The length of the plot is 1 metre more than its breadth. We need to find the length and breadth of the plot.

Eg: Find the radius of the largest circle that can be inscribed in the middle space, given that the radius of the outer circle is 10cm.

4. ANALYSING

Questions where a student has to examine and break the given information into parts to make inferences and find evidence to support generalizations.

Eg: Find the two numbers whose sum is 27 and the product is 182.

Eg: Find the two consecutive positive integer's sum of whose squares is 613.

5. EVALUATING:

Questions where a child has to make judgement or opinions after analysing information. Questions where child has to validate the ideas based on a set of criteria.

Eg: It is given that $\Delta ABC \sim \Delta DEF$. Is it true to say that $\frac{CB}{DE} = \frac{BA}{EF}$. Justify your answer.

Eg: Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

6. CREATING

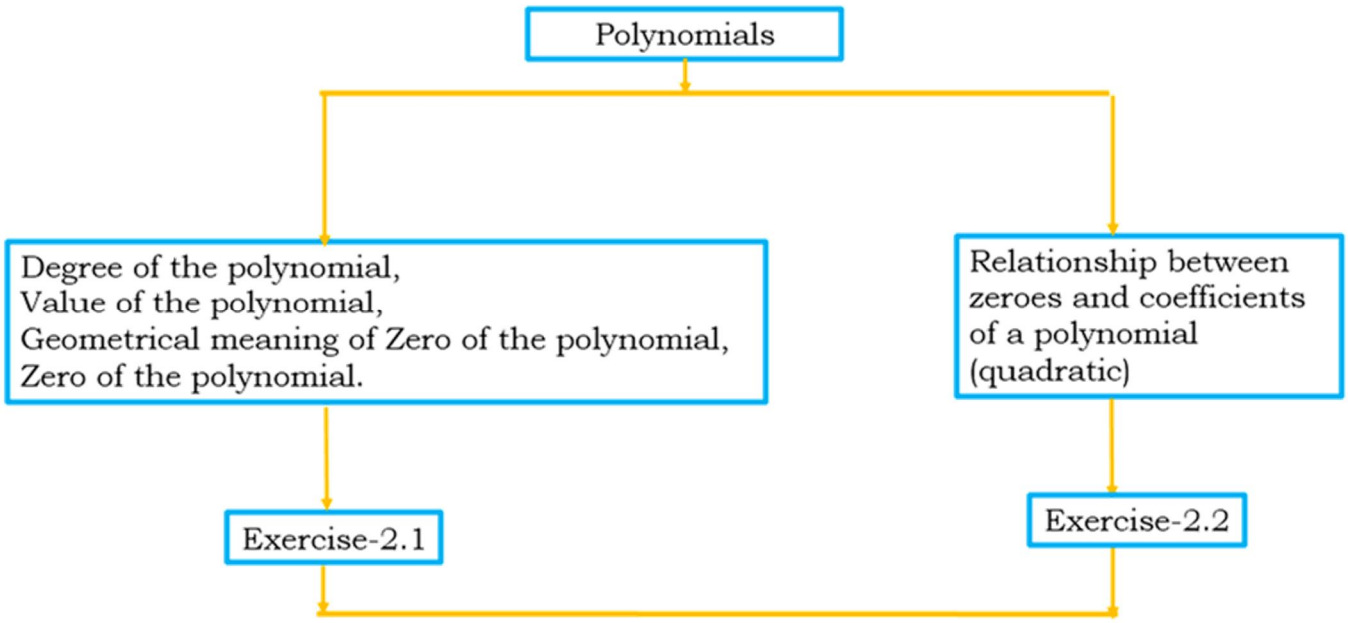
Questions where given information is presented in a different way or a different approach is used to solve the question.

Eg: If the time is taken along X-axis and speed on Y-axis, the coordinates of points O, A, B are (0, 0), (10, 0), (10, 10) respectively, then find the area of triangle OAB. What does the area OAB represent? Justify.

Eg: Solve $x^2 + x - (a-2)(a-3) = 0$.

POLYNOMIALS

MIND MAPPING:



Basic facts and formulae:

1) The general form of a polynomial of nth degree is given by $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n + a_0$; where $a_0, a_1, a_2, ...a_n$ are real coefficients and $a_0 \neq 0$.

2) If $p(x)$ is a polynomial in x , then the highest power of x in the $p(x)$will be the degree of polynomial $p(x)$.

3) If all the coefficients of the polynomial is equal to zero, then it is called as zero polynomial. The degree of zero polynomial is not defined.

4) Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

5. The value of x for which the given polynomial becomes zero, is called zero of the polynomial.

6) The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of y = p(x) intersects the x –axis.

7) A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

8) The zero of a linear polynomial, $ax + b$ is $\frac{-b}{a}$, i.e.... $x = \frac{-(\text{Constant term})}{\text{Coefficent of } x}$

9) Let α, β are the zeroes of the quadratic polynomial $ax^2+bx+c, a\neq 0$, then

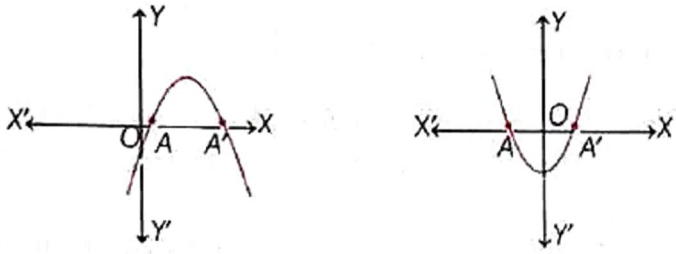
Sum of the zeroes $(\alpha+\beta)=\dfrac{-(Coefficent\ of\ x)}{Coefficient\ of\ x^2}=-\dfrac{b}{a}$ and

Product of the zeroes $(\alpha\beta)=\dfrac{\text{Constant term}}{Coefficient\ of\ x^2}=\dfrac{c}{a}$.

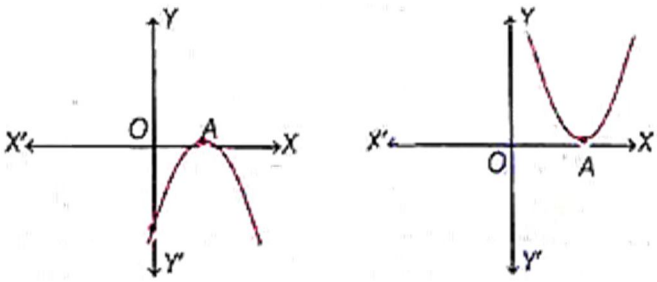
10) If a > 0 then the shape of the parabola open upwards.

11) If a < 0 then the shape of the parabola open downwards.

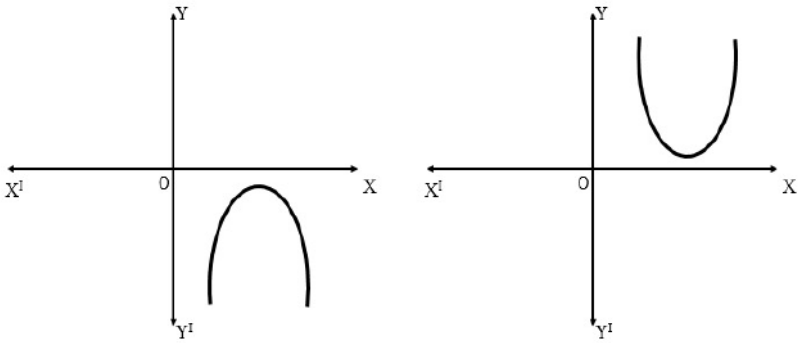
12) The graph of cuts the X-axis at two distinct points A and A^I, then the X-coordinate of A and A^I are the two zeroes of the quadratic polynomial ax^2+bx+c.



13) The graph of $y=ax^2+bx+c$ touches the X-axis at exactly one point A, then the X-coordinate of A is the only zero of the quadratic polynomial ax^2+bx+c.



14) The graph of $y=ax^2+bx+c$ is either completely above the X-axis or completely below the X-axis. So the quadratic polynomial ax^2+bx+c has no real zeroes.



15) In general, a polynomial p(x) of degree 'n' intersects the X-axis atmost 'n' points. Hence a polynomial p(x) of degree 'n' has atmost 'n' zeroes.

16) If $(x-\alpha),(x-\beta)$ are the factors of a quadratic polynomial $f(x)$, then $f(x)=k\left[x^2-(\alpha+\beta)x+\alpha\beta\right]$, where $k\neq 0, k\in R$.

LEVEL_1

MCQ:

1. The quadratic polynomial sum of whose zeroes is 4 and product of the zeroes is 1 is given by:

   a) x^2+4x-1 b) x^2-4x+1 c) x^2+4x+1 d) x^2-4x-1

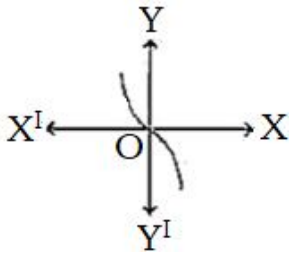
2. The degree of the polynomial $\frac{-t^9+4t^6+7t^5}{t^5}$ is:

   a) 4 b) 3 c) 1 d) 0

3. The value of k, for which -2 is a zero of the polynomial $3x^2+4x+2k$ is:

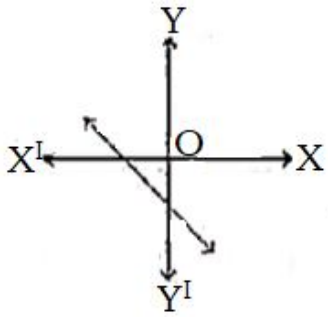
   a) -2 b) 2 c) 3 d) -4

4. Number of zeroes of the polynomial from the below given graph are:



   a) 0 b) 1 c) 2 d) 3

5. Number of zeroes of the polynomial from the below given graph are:

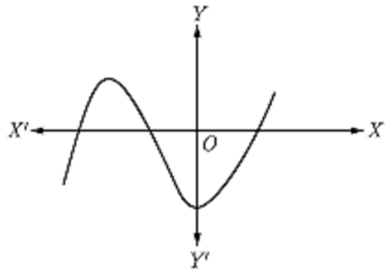


   a) 0 b) 1 c) 2 d) 3

6. If one of the zero of the polynomial $2x^2-3x+k$ is reciprocal to the other, then the value of k is:

   a) 1 b) -1 c) 3 d) 2

7. The graph of a polynomial is shown in figure, then the number of its zeroes are:

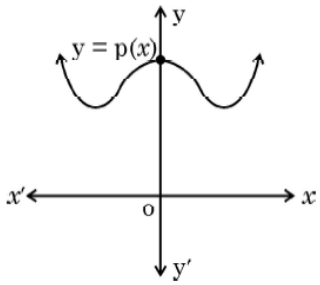


a) 0 b) 1 c) 2 d) 3

8. If -1 is the zero of the polynomial $f(x)=x^2-7x-8$, then the other zero is:

a) 7 b) 8 c) 9 d) 3

9. The graph of $y=p(x)$ is shown in the figure for some polynomial $p(x)$. The number of zeroes of $p(x)$ are:



a) 0 b) 1 c) 2 d) 3

10. The number of polynomials having zeroes as -3 and 5 are:

a) only one b) infinite c) exactly two d) at most two

2-Marks:

1. Write whether the following expressions are polynomials or not.

(i) $x^3+\frac{1}{x^2}+\frac{1}{x}+1$ (ii) x^2+x+3 (iii) $y^{\frac{-1}{2}}-3y+2$ (iv) $\sqrt{5}z^3+\sqrt{7}z$

2. Find the zeroes of the following polynomial $4\sqrt{3}x^2+5x-2\sqrt{3}$.

3. If one of the zeroes of the polynomial $\left(a^2+9\right)x^2+13x+6a$ is reciprocal of other, then find the value of a.

4. If one of the zeroes of the polynomial $(k-1)x^2+kx+1$ is -3 then find the value of k.

5. Find the quadratic polynomial whose sum and the product of the zeroes are $\frac{21}{8}$ *and* $\frac{5}{16}$ respectively.

6. If α, β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$ then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

7. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of other, find the value of k.

8. If one of the zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and also other zero.

9. Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

10. Find the quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively: $\sqrt{2}, \frac{1}{3}$.

3-Marks:

1. Find the zeroes of the quadratic polynomial $4s^2 - 4s + 1$ and verify the relation between zeroes and its coefficients.

2. Find the zeroes of the quadratic polynomial $x^2 - 3$ and verify the relationship between zeroes and its coefficients.

3. If the sum of the zeroes of the polynomial $p(x) = (a+1)x^2 + (2a+3)x + (3a+4)$ is -1, then find the product of zeroes.

4. If α, β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$.

5. Show that $\frac{1}{2}$ & $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relation between zeroes and coefficients of the polynomial.

6. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. (i) $\frac{-8}{3}, \frac{4}{3}$ (ii) $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

7. If m, n are the zeroes of the polynomial $3x^2 - x - 2$, find the values of the following without factorising the polynomial.

(i) $\frac{1}{m} + \frac{1}{n}$ (ii) $m^2 + n^2$. Show your steps.

5-Marks:

1. Find a quadratic polynomial each with the given numbers as the sum and the product of its zeroes respectively.

   (i) $\frac{-1}{4}, \frac{1}{4}$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$

2. Find the zeroes of the quadratic polynomial $f(x) = 6x^2 - 3$ and verify the relationship between zeroes and the coefficients of the polynomial.

3. If α, β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then find the value of:

   (i) $\alpha^3 + \beta^3$ (ii) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (iii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

4. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

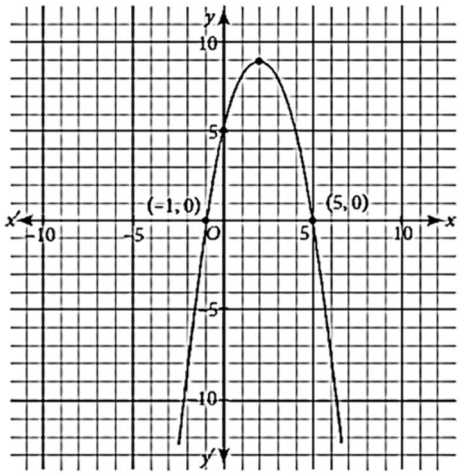
5. If α, β are the zeroes of the polynomial $f(x) = x^2 - 3x - 2$, find the polynomials whose zeroes are:

   (i) $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$ (ii) $(2\alpha + 3\beta)$ *and* $(3\alpha + 2\beta)$

CASE STUDY QUESTIONS

CASE STUDY_1

ABC Construction Company got the contract of making speed humps on roads. Speed humps are parabolic in shape and prevents over speeding, minimize accidents and gives a chance of pedestrians to cross the road. The mathematical representation of a speed hump is shown in the given graph.



Based on the above information, answer the following questions.

(i) What is the type of polynomial represented by the graph? 1

(ii) What are the zeroes of the polynomial represented by the graph? 1

(iii) What shall be the sum of the zeroes and also the product of zeroes of the polynomial represented by the graph? 2

OR

If $\alpha \& \beta$ are the zeroes of the polynomial represented by the graph such that $\beta > \alpha$ then, find the value of $|8\alpha + \beta|$. 2

CASE STUDY_2

Refer to the above image. A teacher told 10 students to write a polynomial on black board. Students wrote as shown in the above image.

1. $x^2 + 2$	6. $x - 3$
2. $2x + 3$	7. $x^4 + x^2 + 1$
3. $x^3 + x^2 + 1$	8. $x^2 + 2x + 1$
4. $x^3 + 2x^2 + 1$	9. $2x^3 - x^2$
5. $x^2 - 2x + 1$	10. $x^4 - 1$

(i) How many students wrote cubic polynomial? 1

(ii) Find the zeroes of the polynomial $x^2 + 2x + 1$. 1

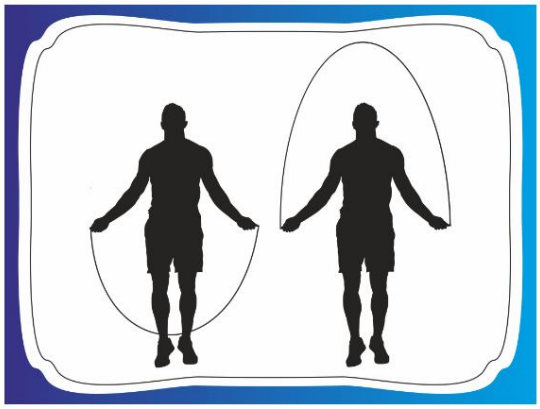
(iii) Find the sum of the zeroes and product of the zeroes of the polynomial $x^4 - 1$. 2

OR

Find the quadratic polynomial whose zeroes are $2 + \sqrt{3}$ *and* $2 - \sqrt{3}$. 2

CASE STUDY_3

During the skipping through skipping rope, its look like the in the form of parabola. It is a natural examples of parabolic shape which is represented by a quadratic polynomial. Similarly, we can observe in many other cases forming a in a variety of forms of different parabolas.



(i) Write the standard form of a polynomial of degree 2. 1

(ii) If $\alpha, -\alpha$ are the zeroes of the quadratic polynomial $2x^2-3(k-4)x-8$, then find the value of k . 2

OR

What are the zeroes of the polynomial $p(x)=x^2-1$? 2

(iii) If the sum of the zeroes is p and product of the zeroes is $-p$, then find the quadratic polynomial. 1

LEVEL_2

MCQ:

1. The quadratic polynomial, whose sum and the product of the zeroes are $\sqrt{2}$ & $\frac{1}{3}$ respectively is:

   a) $3x^2-3\sqrt{2}x+1$ b) $3x^2+3\sqrt{2}x+1$ c) $3x^2-3\sqrt{2}x-1$ d) $3x^2+3\sqrt{2}x-1$

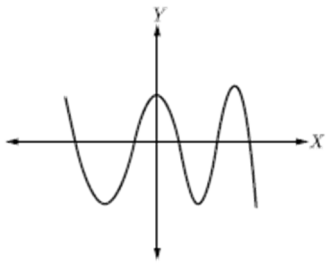
2. If one of the zero of the quadratic polynomial kx^2+3x+k is 3, then the value of k is:

   a) $\frac{9}{10}$ b) $\frac{-10}{9}$ c) $\frac{10}{9}$ d) $\frac{-9}{10}$

3. Sum of the zeroes of the polynomial $3x^2-10x+13$ is 5. Asha at once said, 'it is false'. Select the appropriate answer from the given below options.

   a) agree b) disagree c) both a) & b) d) can't say

4. The number of zeroes of the following graph $y=p(x)$ is.



   a) 1 b) 2 c) 5 d) 4

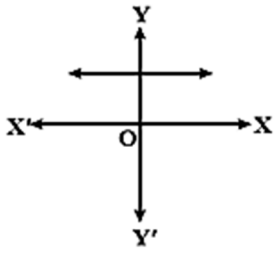
5. The lowest value of x^2+4x+2 is:

a) 2 b) -2 c) 3 d) -3

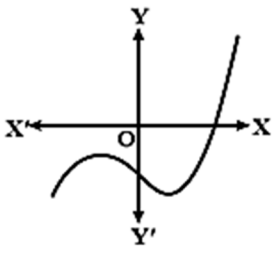
2-MARKS

1. If α,β are the zeroes of the polynomial x^2+px+q are double in value to the zeroes of $2x^2-5x-3$, then the values of p & q.
2. What are the zeroes of the polynomial $x^2-3x-m(m+3)$?
3. Find the value of k, such that the polynomial $x^2-(k+6)x+2(2k+1)$ has sum of its zeroes equal to half of their product.
4. The graph of $y=p(x)$ is given, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$ in each case.

(i)



(ii)



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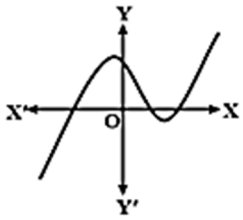

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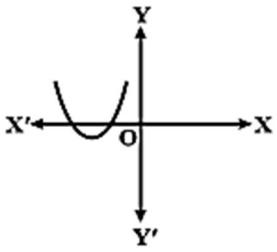

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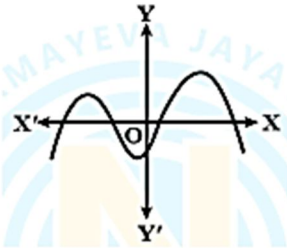
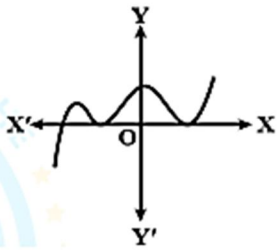


(vi)



5. If 2 and 3 are zeroes of polynomial $3x^2-2kx+2m$, then find the value of k & m.
6. Form a quadratic polynomial whose one of the zeroes is -15 and sum of the zeroes is 42.
7. Is the following statement true or false? Justify your answer.

"If the zeroes of a polynomial ax^2+bx+c are both negative then a, b and c will have the same sign".

3-MARKS

1. Find the zeroes of the polynomial $6x^2-3-7x$ and verify the relationship between the zeroes and the coefficients.

2. If $(x+a)$ is a factor of two polynomials x^2+px+q and x^2+mx+n, then prove that $a=\frac{n-q}{m-p}$.

3. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students:

$2x+3$, $3x^2+7x+2$, $4x^3+3x^2+2$, $x^3+\sqrt{3x}+7$, $7x+\sqrt{7}$, $5x^3-7x+2$, $2x^2+3-\frac{5}{x}$, $5x-\frac{1}{2}$, ax^3+bx^2+cx+d, $x+\frac{1}{x}$.

Answer the following question:

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

4. Find the zeroes of the quadratic polynomial $x^2-2\sqrt{2}x$ and verify the relation between the zeroes and the coefficients of the polynomial.

5. If α,β are the zeroes of quadratic polynomial such that $\alpha+\beta=24$ and $\alpha-\beta=8$. Find the quadratic polynomial having α *and* β as its zeroes.

6. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained.

5-MARKS

1. If the zeroes of the polynomial $p(x)=ax^2+bx+c$ are in the ratio of $m:n$, then find the value of $\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}$.

2. If α,β are the zeroes of the quadratic polynomial $f(x)=px^2+qx+r$, then find the value of $\frac{1}{p\alpha+q}+\frac{1}{p\beta+q}$.

3. If α,β are the zeroes of the quadratic polynomial $p(s)=3s^2-6s+4$, then find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$.

4. If α,β are the zeroes of the quadratic polynomial $f(x)=x^2-px+q$ then prove that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$.

5. If α,β are the zeroes of the polynomial $p(x)=2x^2+5x+k$ satisfying the relation $\alpha^2+\beta^2+\alpha\beta=\frac{21}{4}$, then find the value of k .

6. If α, β are the zeroes of the polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ *and* $\frac{1-\beta}{1+\beta}$.

CASE STUDY QUESTIONS

CASE STUDY_1:

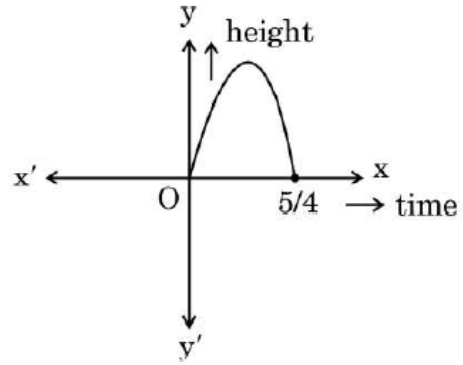
In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.

Based on the above, answer the following questions:

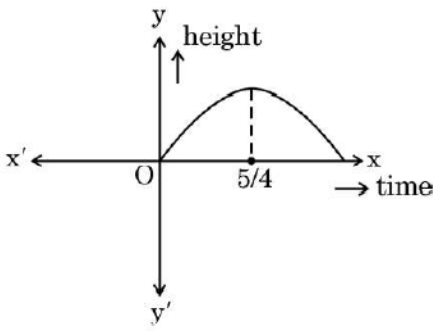
(i) Find zeroes of polynomial $p(t) = 20t - 16t^2$. 1

(ii) Which of the following types of graph represents p(t)? 1

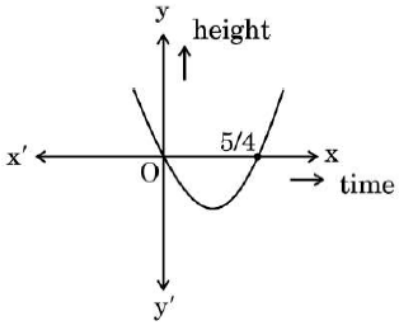
(a)



(b)



(c)



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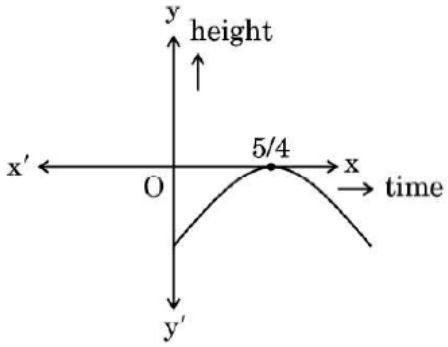


(iii) (a) What would be the value of h at $t = \frac{3}{2}$? Interpret the result. 2

OR

(b) How much distance has the dolphin covered before hitting the water level again? 2

LEVEL_3

MCQ:

1. If the product of the zeroes of the polynomial $(ax^2 - 6x - 6)$ is 4, then the value of 'a' is:

   a) $\frac{-3}{2}$ b) $\frac{3}{2}2$ c) $\frac{2}{3}$ d) $\frac{-2}{3}$

2. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, c ≠ 0 are equal, then

   a) c and a have opposite signs b) c and b have opposite signs

   c) c and a have the same sign d) c and b have the same sign

3. If the squared difference of the Zeros of Polynomial $x^2 + px + 45$ is 144, then the value of p is:

   a) 18 only b) -18 only c) ±18 d) 0

4. Ravi claims that the polynomial $p(x) = mx^a + x^{2b}$ has $4b$ zeroes. For Ravi's claim to be correct, which of these must be true?

   a) $a = 2b$ *or* $a = 4b$ b) $a = 2$ *or* $a = 4b$ c) $m = 2b$ d) $m = 4b$

5. The values of a and b, if they are the zeroes of the polynomial $x^2 + ax + b$.

   a) $a = 0, b = -2$ b) $a = -2, b = -2$ c) $a = -2, b = 0$ d) $a = 0, b = 2$

6. The quadratic polynomial whose sum and product of the zeroes are $\sqrt{2}+1, \frac{1}{\sqrt{2}+1}$ is:

   a) $x^2 - (2\sqrt{2})x + 1$ b) $x^2 - (2\sqrt{2})x - 1$ c) $x^2 - (3\sqrt{2})x + 1$ d) $x^2 - (2\sqrt{2})x + 4$

2-MARKS

1. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a.
2. A group of 18 average students and 7 best students of a class. Write a quadratic polynomials whose are equal to number of average students and number of students.
3. What is the value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously?
4. If α, β are the zeroes of polynomial $x^2 - p(x+1) + c$ such that $(\alpha + 1)(\beta + 1) = 0$, then find the value of c.
5. If α, β are the zeroes of the polynomial $2x^2 - 4x + 5$, find the value of :

   a) $\alpha^2 + \beta^2$ b) $(\alpha - \beta)^2$

3-MARKS

1. Obtain the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ and verify the relation between its zeroes and the coefficients.
2. If $\alpha \& \beta$ are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$ then find the value of $\alpha^4 + \beta^4$.
3. Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?
4. If α, β are the zeroes of the quadratic $f(x) = x^2 - x - 4$ polynomial find a polynomial whose zeroes are $2\alpha + 1$ and $2\beta + 1$.
5. If α, β are the zeroes of the quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$. Find a quadratic polynomial having $\alpha \& \beta$ as zeroes.
6. Find the zeroes of the quadratic polynomial $f(x) = abx^2 + (b^2 - ac)x - bc$, and verify the relationship between zeroes and the coefficients of the polynomial.

5-MARKS

1. If α, β are the zeroes of the polynomial $x^2 + 8x + 6$ then frame a quadratic polynomial whose zeroes are: a) $\frac{1}{\alpha} \& \frac{1}{\beta}$ b) $1 + \frac{\beta}{\alpha}, 1 + \frac{\alpha}{\beta}$
2. If α, β are the zeroes of the polynomial $2x^2 - 4x + 5$, find the values of

   (i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iii) $(\alpha - \beta)^2$ (iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$
3. If β and $\frac{1}{\beta}$ are the zeroes of the polynomial $(a^2 + a)x^2 + 61x + 6a$. Find the value of β and a.
4. Are the following statements 'True' or 'False'? Justify your answers.

(i) If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.

(ii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

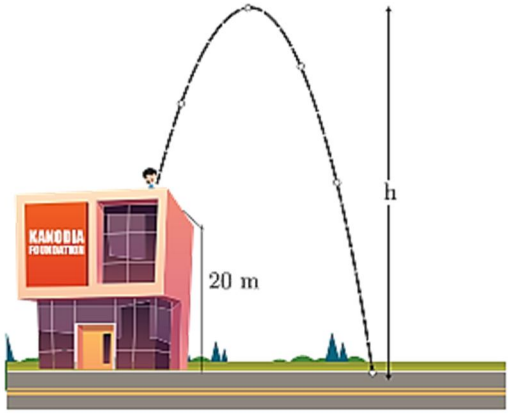
5. If $\alpha \; and \; \beta$ are the zeroes of the polynomial $f(x) = ax^2 + bx + c$, then evaluate:

   (i) $\alpha^4 + \beta^4$ (ii) $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$.

CASE STUDY QUESTIONS

CASE STUDY_1:

Lavanya throws a ball upwards, from a rooftop, which is 20 m above from ground. It will reach a maximum height and then fall back to the ground. The height of the ball from the ground at time t is h, which is given by $h = -4t^2 + 16t + 20$.



(i) What is the height reached by the ball after 1 second? 1

(ii) What is the maximum height reached by the ball? 1

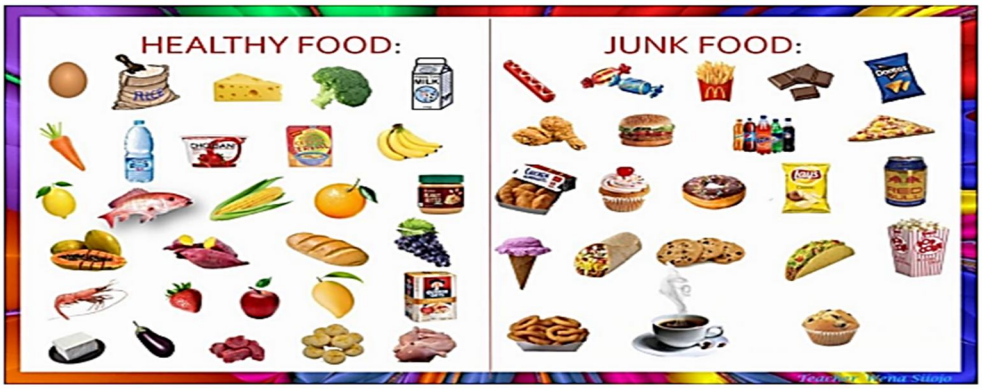
(iii) How long will the ball take to hit the ground? 2

OR

What are the two possible times to reach the ball at the same height of 32 m? 2

CASE STUDY_2:

Junk food is unhealthful food that is high in calories from sugar or fat, with little dietary fiber, protein, vitamins, minerals or other important forms of nutritional value. A sample of few students have taken. If α be the number of students who take junk, β be the number of students who take healthy food such that $\alpha > \beta$ and α, β are the zeroes of the quadratic polynomial $f(x) = x^2 - 7x + 10$, then answer the following questions:



(i) Find the number of students who take the junk food. 1

(ii) Find the number of students who take healthy food. 1

(iii) Find the quadratic polynomial whose zeroes are -3 and -4. 2

OR

If one of the zeroes of the polynomial $x^2 - 5x + 6$ is 2, find the other zero. 2

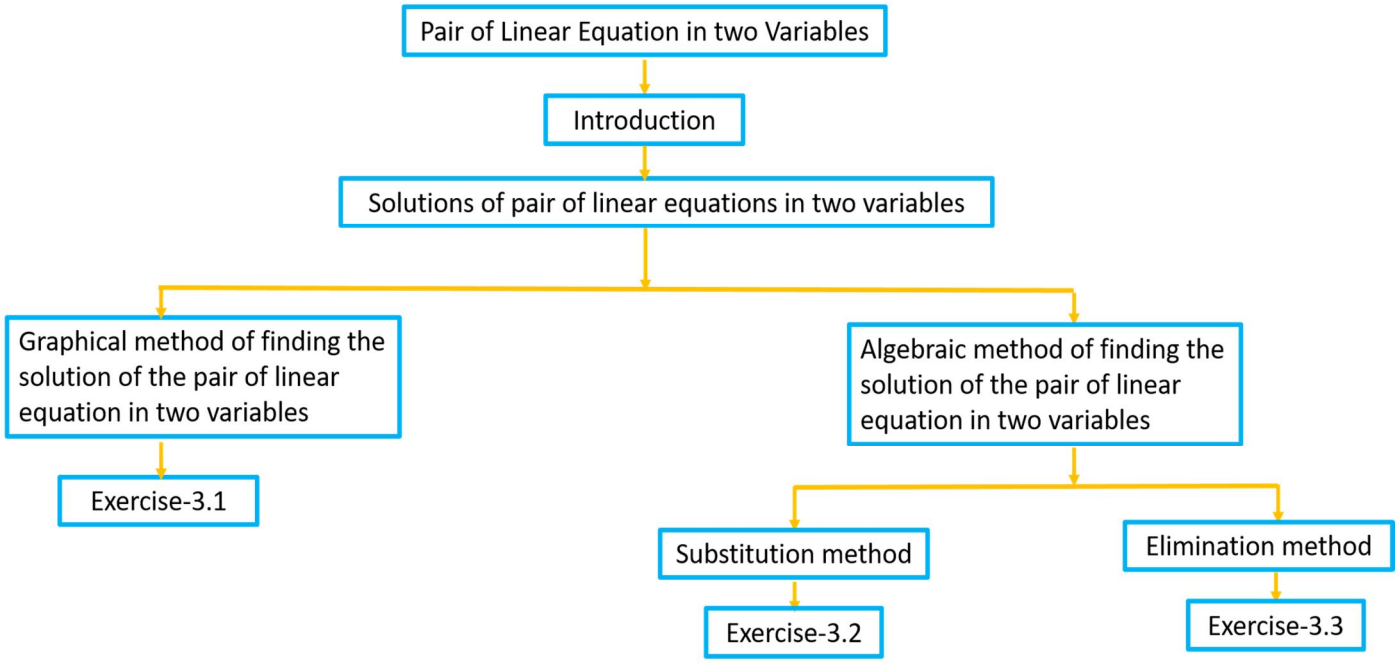
SKILL BASED QUESTIONS:

1. Sharmila went to a stationary shop and purchased 2pencils and 3 erasers for Rs.9. her friend Keerti saw the new variety of pencils and erasers with Sharmila, and also bought 4 pencils and 6 erasers of the same kind for Rs.18. Represent this situation algebraically and graphically.

2. Two rails are represented by the equation $x+2y-4=0$ and $2x+4y-12=0$. Represent this situation graphically.

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

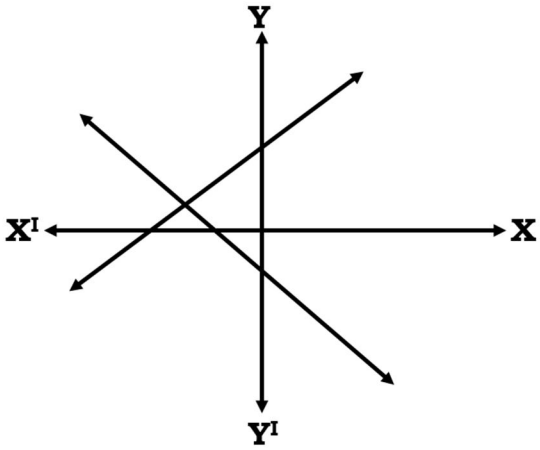
MIND MAPPING



Basic facts and formulae:

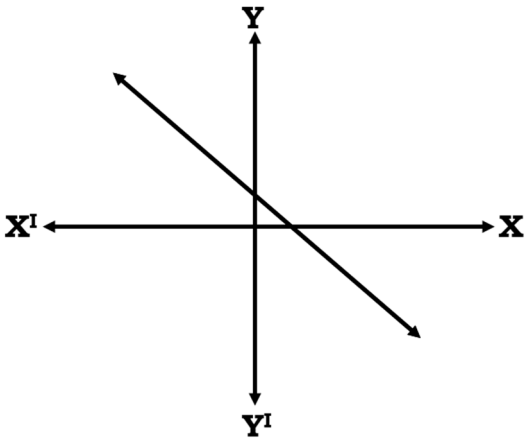
1. A linear equation is an equation of straight line in the form of ax + by + c = 0 where a ≠ 0, b ≠ 0 and a, b, c are real numbers. In this equation $a \& b$ are coefficients and c is a constant.
2. Two linear equations having two same variables are called pair of linear equation in two variables. The general form of pair of linear equations is $a_1x + b_1y + c_1 = 0 \&$ $a_2x + b_2y + c_2 = 0$. Where $a_1, a_2, b_1, b_2, c_1 \, and \, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0;\ a_2^2 + b_2^2 \neq 0$.
3. Every solution of the equation is a point on the line representing it.
4. Each solution (x, y) of linear equation in two variables, ax +by+ c=0 corresponds to a point on the line representing the equation, and vice versa.
5. A system of linear equations is said to be consistent if it has at least one solution.
6. A system of linear equations is said to be inconsistent if it has no solution.
7. Graphical method of pair of linear equation in two variables.

   a) If the two lines intersect each other at one particular point then that point will be the only solution of that pair of linear equations. It is said to be consistent pair of equations.



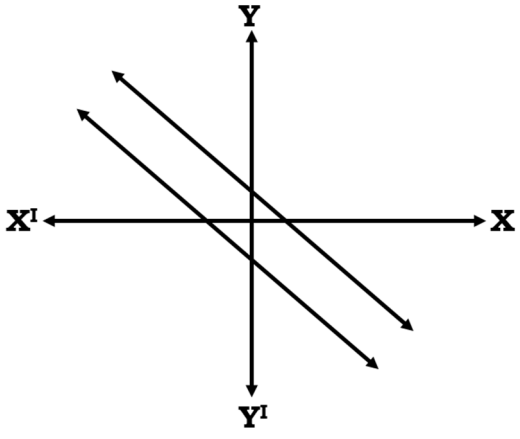
One Solution

b) If two lines coincide with each other, then there will be infinite solutions as all the points on the line will be the solution for the pair of linear equations. It is said to be dependent or consistent pair of equations.



Infinitely Many Solution

c) If the two lines are parallel then there will be no solution as the lines are not intersecting at any point. It is said to be inconsistent pair of equations.



No Solution

8. **Interpretation of the pair of equations:**

Pair of Equations	**Ratio Comparison**	**Graphical Representation**	**Algebraic Interpretation**
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$	Intersecting lines	Only one solution
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$	Coincident lines	Infinite solutions
$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$	Parallel lines	No solution

9. **Algebraic methods of solving a pair of linear equations:**

a) **Substitution method:**

If we have a pair of Linear Equations with two variables x and y, then we have to follow these steps to solve them with the substitution method-

Step 1: We have to choose any one equation and find the value of one variable in terms of other variable i.e. y in terms of x.

Step 2: Then substitute the calculated value of y in terms of x in the other equation.

Step 3: Now solve this Linear Equation in terms of x as it is in one variable only i.e. x.

Step 4: Substitute the calculate value of x in the given equations and find the value of y.

b) **Elimination method:**

In this method, we solve the equations by eliminating any one of the variables.

Step 1: Multiply both the equations by such a number so that the coefficient of any one variable becomes equal.

Step 2: Now add or subtract the equations so that the one variable will get eliminated as the coefficients of one variable are same.

Step 3: Solve the equation in that leftover variable to find its value.

Step 4: Substitute the calculated value of variable in the given equations to find the value of the other variable.

LEVEL_1

MCQ:

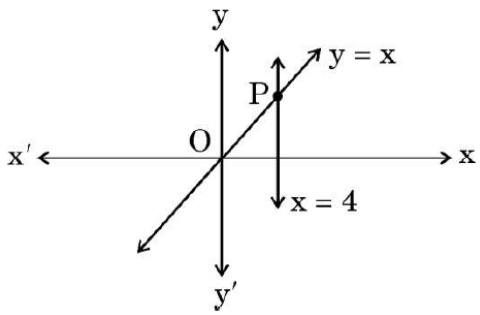
1. If the lines represented by equations $3x+2my=2$ and $2x+5y+1=0$ are parallel, then the value of m is:

   a) $\frac{2}{5}$ b) $-\frac{5}{4}$ c) $\frac{3}{2}$ d) $\frac{15}{4}$

2. The pair of linear equations $x+2y-5=0$ and $2x-4y+6=0$:

   a) is inconsistent
   b) is consistent with many solutions
   c) is consistent with a unique solution
   d) is consistent with two solutions

3. The lines represented by the linear equations $y=x$ and $x=4$ intersect at P. The coordinates of the point P are:



   a) $(4,0)$ b) $(4,4)$ c) $(0,4)$ d) $(-4,4)$

4. The value of 'k' for which the system of equations $kx+2y=5$ and $3x+4y=1$ have no solution, is:

   a) $k=\frac{3}{2}$ b) $k\neq\frac{3}{2}$ c) $k\neq\frac{2}{3}$ d) $k=5$

5. The pair of linear equations $2x=5y+6$ and $15y=6x-18$ represents two lines which are:

   a) intersecting
   b) parallel
   c) coincident
   d) either intersecting or parallel

6. Graphically pair of equations $6x-3y+10=0$ and $2x-y+9=0$ represents two lines which are:

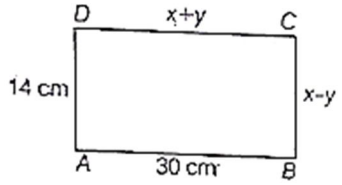
   a) intersecting at exactly one point
   b) intersecting exactly at two points
   c) coincident
   d) parallel

7. The point of intersection of the lines $y=3x$ and $x=3y$ is:

   a) $(0,0)$ b) $(0,3)$ c) $(3,0)$ d) $(3,3)$

2-Marks:

1. Solve the following pair of linear equations: $3x-5y=4$ and $2y+7=9x$.
2. In the fig. ABCD is a rectangle. Find the values of x and y.



3. Equation $2x=5y+4$ is given. Write another linear equation, so that the lines represented by the pair are: (i) intersecting (ii) coincident (iii) parallel
4. On comparing the ratios, $\frac{a_1}{a_2},\frac{b_1}{b_2}and\frac{c_1}{c_2}$ find out whether the following pair of equations are consistent or inconsistent:
   (i) $3x+2y=5;\ 2x-3y=7$ (ii) $5x-3y=11;-10x+6y=-22$
5. For which values of p does the pair of equations given below has unique solutions $2x+2y+2=0$ and $4x+py+8=0$.
6. Find a, if the line $3x+ay=8$ passes through the intersection of lines represented by the equations $3x-2y=10$ and $5x+y=8$.
7. Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.
8. For what values of k, will the following equations represent coincident lines? $2x+3y=4$ and $4x+(k+4)y=3k+2$.
9. Solve the system of equations by substitution method: $0.2x+0.3y=1.3$ and $0.4x+0.5y=2.3$.
10. Solve: $99x+101y=499;\ 101x+99y=501$.
11. Is the pair of equations $x+2y-3=0$ and $6y+3x-9=0$ consistent? Justify your answer.
12. The angles of a triangle are $x,y and 40^0$. The difference between two angles x and y is 30^0. Find x and y.

3-Marks:

1. Draw the graph of the equations $x-y+1=0$ and $3x+2y-12=0$. Using this graph find the values of x and y which satisfy both the equations.
2. Two numbers are in the ratio of 5 : 6. If 7 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

3. Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
   (i) intersecting lines (ii) parallel lines (iii) coincident lines.
4. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.
5. For what values of a and b does the following pair of linear equations have an infinite number of solutions? $2x+3y=7$ and $(a-b)x+(a+b)y=3a+b-2$.
6. The sum of the digits of a two digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the fraction.

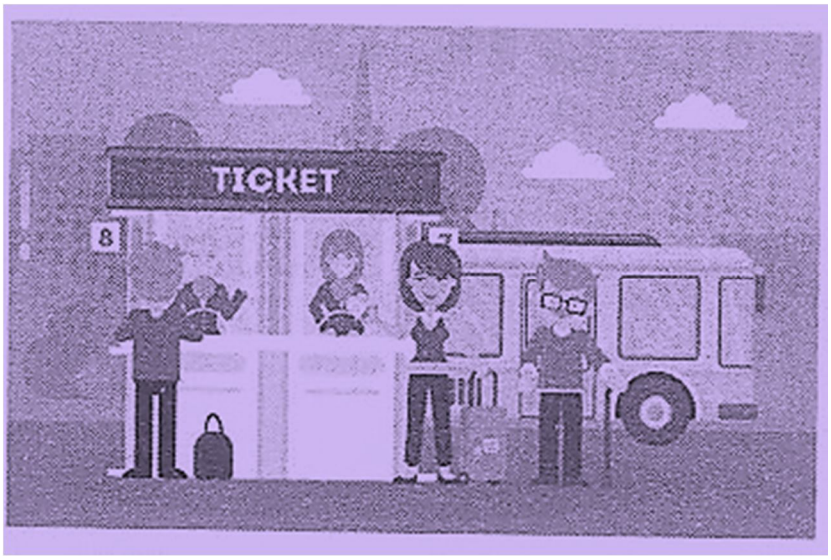
5-Marks:

1. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save Rs. 2000 per month, find their monthly incomes.
2. The area of the rectangle gets reduced by 9 square units, if its length reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 3 units, then the area increases by 67 sq.units. Find the dimensions of the rectangle.
3. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
4. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Shristi paid Rs. 27 for a book kept for seven days, while Rekha paid Rs. 21 for the book she kept for five days. Find the fixed charge and the additional charge paid by them?
5. Solve for $x \& y$: $mx-ny=m^2+n^2$, $x-y=2n$.
6. Angles of cyclic quadrilateral ABCD are $\angle A=(6x+10)^0$, $\angle B=(5x)^0$, $\angle C=(x+y)^0$, $\angle D=(3y-10)^0$. Find x and y, also the values of the four angles.

CASE STUDY QUESTIONS

CASE STUDY_1

From Bengaluru bus stand, if Riddhima buys 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, then total cost is Rs. 46, but if she buys 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur, then the total cost is Rs. 74.



Consider the fares from Bengaluru to Malleswaram and that to Yeshwanthpur as Rs x and Rs y respectively and answer the following questions.

(i) Represent 1st situation algebraically. 1

(ii) Represent 2nd situation algebraically. 1

(iii) What is the fare from Bengaluru to Malleswaram? 2

OR

What is the fare from Bengaluru to Yeshwanthpur?

CASE STUDY_2

Raman usually go to a dry fruit shop with his mother. He observes the following two situations.

On 1st day: The cost of 2 kg of almonds and 1 kg of cashew was Rs 1600.
On 2nd day: The cost of 4 kg of almonds and 2 kg of cashew was Rs 3000.
Denoting the cost of 1 kg almonds by Rs x and cost of 1 kg cashew by Rs y, answer the following questions.

(i) Represent algebraically the situation of day-I. 1

(ii) Represent algebraically the situation of day- II. 1

(iii) At what point the linear equation represented by day-I, intersect the x axis. 2

OR

At what point the linear equation represented by day-II, intersect the y-axis

CASE STUDY_3:

A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is Rs 9000 and from batch II is Rs 26,000. Assume that each poor child pays Rs. x per month and each rich child pays Rs. y per month.

Based on the above information, answer the following questions:

(i) Represent the information given above in terms of x and y. 1

(ii) Find the monthly fee paid by a poor child. 2

OR

Find the difference in the monthly fee paid by a poor child and a rich child. 2

(iii) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of the fees from batch II? 1

LEVEL_2

MCQ:

1. The pair of equations $y = 0$ and $y = -7$ has

   a) one solution
   b) two solutions
   c) infinitely many solutions
   d) no solution

2. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

   a) parallel
   b) intersecting at (b, a)
   c) coincident
   d) intersecting at (a, b)

3. For what values of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ has infinitely many solutions.

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

4. If the lines given by $3x + 2ky = 2$ and $2x + 5y = 1$ are parallel, then the value of k is:

   a) $-\frac{5}{4}$ b) $\frac{2}{5}$ c) $\frac{15}{4}$ d) $\frac{3}{2}$

5. One equation of a pair of dependent linear equation is $-5x + 7y - 2 = 0$. The second equation can be:

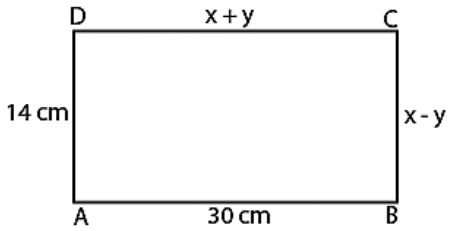
   a) $10x + 14y + 4 = 0$
   b) $-10x - 14y + 4 = 0$
   c) $-10x + 14y + 4 = 0$
   d) $10x - 14y + 4 = 0$

6. A pair of linear equation which has a unique solution $x = 2$ and $y = -3$ is:

   a) $x + y = 1$ & $2x - 3y = -5$
   b) $2x + 5y = -11$ & $4x + 10y = -22$
   c) $2x - y = 1$ & $3x + 2y = 0$
   d) $x - 4y - 14 = 0$ & $5x - y - 13 = 0$

2-MARKS

1. Determine the values of a and b for which the pair of linear equations has infinitely many solutions: $3x-(a+1)y=2b-1$ and $5x+(1-2a)y=3b$
2. Two straight paths are represented by the lines $7x-5y=3$ and $14x-10y=5$. Check whether the paths cross each other.
3. Solve for x and y: $\frac{x}{a}+\frac{y}{b}=2,\ ax-by=a^2-b^2$.
4. For what value of k will the following pair of linear equations have no solution? $3x+y=1;\ (2k-1)x+(k-1)y=2k+1.$
5. In the figure ABCD is a rectangle. Find the values of x and y.



6. Find whether the lines representing the following pair of linear equations intersect at a point or parallel or coincident. $\frac{3}{2}x+\frac{5}{3}y=7$ and $\frac{3}{2}x+\frac{3}{2}y=6$
7. Solve the following equations by elimination method: $\frac{x}{7}+\frac{y}{3}=5;\ \frac{x}{7}-\frac{y}{9}=1$
8. Is it true to say that the pair of equations $-x+2y+2=0$ and $\frac{1}{2}x-\frac{1}{4}y-1=0$ has a unique solution? Justify your answer.
9. For the pair of equations $\lambda x+3y=-7$ & $2x+6y=14$ to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.
10. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
11. The sum of two numbers is 120 and one of numbers is 3 times the other. Find the numbers.

3-MARKS

1. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and denominator. If 3 is added to both the numerator and denominator, fraction becomes $\frac{3}{4}$. Find the fraction.
2. Seven times a two digit number is equal to four times the number obtained by reversing the order of the digits. If the sum of both the digits is 9, find the number.

3. The students of a class are made to stand in rows. If three students are extra in each row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

4. Solve: $\frac{4}{x}+3y=14;\ \frac{3}{x}-4y=23$

5. Write an equation of a line passing through the point representing solutions of the pair of linear equations $x+y=2$ and $2x-y=1$. How many such lines can be found?

6. The age of father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

5-MARKS

1. A two digit number is obtained by either multiplying the sum of digit by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the numbers.

2. Anuj decided to donate some money for books for the children living in an orphanage. If there are children less, everyone will get Rs. 20 more. If there are 7 children more everyone will get Rs.10 less. What is the number of children and how much does each get? What is the total amount distributed? Why Anuj decided to distribute money for books?

3. Raghav scored 70 marks in a test, getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been rewarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test?

4. Determine graphically, the vertices of the triangle formed by the lines $y=x$, $3y=x$ and $x+y=8$.

5. Solve graphically, the pair of equations $2x+y=6$ and $2x-y+2=0$. Find the ratio of the areas of the two triangle formed by the lines representing these equations with X-axis and the lines with Y-axis.

6. Draw the graph of the lines $x=-2$ and $y=3$. Write the vertices of the figure formed by these lines, the $x-axis$ and $y-axis$. Also find the area of the figure.

7. Determine, algebraically the vertices of the triangle formed by the lines. $5x-y=5$, $x+2y=1$ and $6x+y=17$.

CASE STUDY QUESTIONS

CASE STUDY_1:

A test consists of "True or False" questions. One mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

Type of Question	Marks given for correct answer	Marks deducted for wrong answer
True/False	1	0.25

(i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly? 1

(ii) How many questions did he guess? 1

(iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got? 2

OR

If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks? 2

LEVEL-3

MCQ:

1. Two numbers are in the ratio of 3:4. If 5 is subtracted from each other, then the ratio will be 2:3. What will be the smallest number?

   a) 15 b) 18 c) 20 d) 24

2. If $\frac{4x-3y}{7x-6y}=\frac{4}{13}$, then the value of $\frac{x}{y}$ is:

   a) $\frac{5}{8}$ b) $\frac{8}{5}$ c) $-\frac{5}{8}$ d) $-\frac{8}{5}$

3. In the equations shown below, a and b are unknown constants. $ax+2y=14$ & $2x+by=14$. If (-3, 4) is the solution of the given equations, what are the values of a and b?

   a) a = 2, b = 5 b) a = -2, b = 5 c) a = 5, b = 2 d) a = 5, b = -2

4. Consider the equations as shown: $(x-a)(y-b)=(x-2a)\left(y-\frac{b}{2}\right)$ and $x\left(x+\frac{1}{2b}\right)+y\left(y+\frac{a}{2}\right)-2xy=5+(x-y)^2$. On comparing coefficients, a student says these pair of equations is consistent. Is he/she correct? Which of these explains why?

a) Yes, because they are intersecting lines

b) Yes, because they are parallel lines

c) No, because they are parallel lines

d) No, because they are intersecting lines.

5. The solutions of the equation $\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$ is given by:

a) $x=2,\ y=1$ b) $x=1,\ y=1$ c) $x=-1,\ y=-1$ d) $x=1,\ y=2$

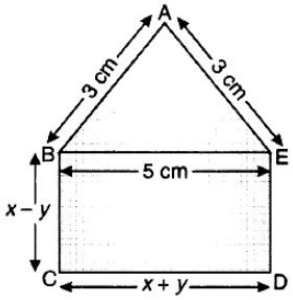
6. A farmer divides his herd of x cows among his 4 son's such that first son gets one-half of the herd, the second son gets one fourth, the third son gets one-fifth and the fourth son gets 7 cows, then the value of x is:

a) 100 b) 140 c) 160 d) 180

2-MARKS

1. Father's age is 3 times sum of the ages of his two children. After 5 years, his age will be twice the sum of the ages of two children. Find the age of the father.

2. In the adjacent figure ABCDE is a pentagon with $BE \parallel CD$ and $BC \parallel ED$. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm. Find the value of x & y.



3. Find the values of a and b for which the following system of linear equations has infinite solutions: $2x-(a-4)y=2b+1$ and $4x-(a-1)y=5b-1$.

4. Cars are parked in a parking place at a particular point of time in rows. If 3 cars are extra in a row, there would be one row less. If 3 cars are less in a row there would be 2 rows more. Find the number of cars in the parking place at that particular point of time.

5. At a certain time in a deer park, the number of heads and the number of legs of deer and human visitors were counted and it was found there were 39 heads and 132 legs. Find the number of deer and human visitors in the park.
6. Eight times a two digit number is equal to three times the number obtained by reversing the order if its digits. If the difference between the digit is 5, find the number.
7. The larger of two supplementary angles exceeds the smaller by 18^0, find them.
8. For what values of a and b will the following system of linear equations has infinitely many solutions? $2x-3y=7;\ (a+b)x-(9a+b-3)=4a+b$

3-MARKS

1. Find the value of p and q for which the system of equations represent coincident lines. $2x+3y=7$, and $(p+q+1)x+(p+2q+2)y=4(p+q)+1$
2. Solve the following system of linear equations by elimination method:
$2(ax-by)+(a+4b)=0$, $2(bc+ay)+(b-4a)=0$.
3. For what values of p and q will the following pair of linear equations has infinitely many solutions? $4x+5y=2;\ (2p+7q)x+(p+8q)y=2q-p+1$
4. Students are made to stand in rows. If one student is extra in a row there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.
5. For which value (s) of λ, do the pair of equations $\lambda x+y=\lambda^2$ and $x+\lambda y=1$ have (i) No solution? (ii) Infinitely many solutions? (iii) a unique solution?
6. Draw the graph of the pair of equations $2x+y=4$ and $2x-y=4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

5-MARKS

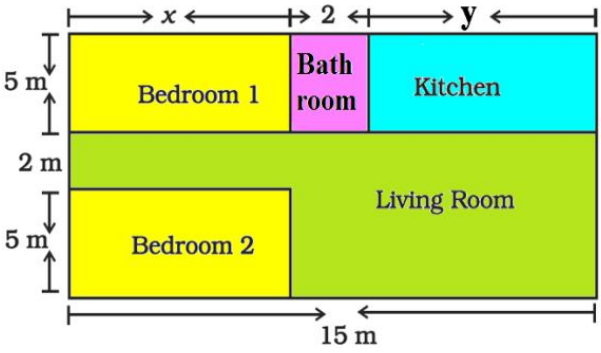
1. The length of the sides of a triangle are $2x+\frac{y}{2}, \frac{5x}{3}+y+\frac{1}{2}$ *and* $\frac{2}{3}x+2y+\frac{5}{2}$. If the triangle is equilateral. Find its perimeter.
2. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the coordinates of vertices of the triangle formed by these lines and x-axis.
3. Determine graphically, the coordinates of the vertices of a triangle whose equations are $2x+2y+6=0$, $2x+3y-18=0$, $y-2=0$. Also find the area of this triangle.

4. Check graphically, whether the pair of equations $x+3y=6$ and $2x-3y=12$ is consistent. If so, solve them graphically.
5. Ram decided to distribute some amount to poor students for their books. If there are 8 students less, everyone will get Rs. 10 more. If there are 16 students more everyone will get Rs. 10 less. What is the number of students and how much does each get? What is the total amount distributed? What is the reason that motivated Ram to distribute money for books?
6. The incomes of two persons A and B are in the ratio 8 : 7 and the ratio of their expenditures is 19 : 16. If their savings Rs. 2550 per month. Find their monthly income.
7. Jamila sold a table and a chair for Rs. 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit 25% on the table and 10% on the chair she would have got Rs. 1065. Find the cost price of each.
8. In an election contested between A and B, A obtained votes equal to twice the number of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 18000 persons on the electoral roll. How many voted for B.
9. When the son will be as old as the father today their ages will add upto 126 years. When the father was old as the son is today, their ages add upto 38 years. Find their present ages.
10. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of Rs.2 of 3 bananas and the second lot at the rate of Rs. 1 per banana and got a total of Rs. 400. If he had sold the first lot at the rate of Rs. 1 per banana, and the second lot at the rate of Rs. 4 for 5 bananas his total collection would have been Rs. 460. Find the total number of bananas he had.

CASE STUDY QUESTIONS

CASE STUDY_1:

In the below given layout, the design and measurements has been made such that area of two bedrooms and Kitchen together is 95 sq. m



(i) Find the area of two bedrooms and kitchen. 1

(ii) Find the length of the outer boundary of the layout. 1

(iii) Find the area of living room in the layout. 2

OR

Find the cost of laying tiles in Kitchen at the rate of Rs. 50 per sq.m 2

CASE STUDY_2:

Piyush sells a saree at 8% profit and a sweater at 10% discount, thereby getting a sum of Rs. 1008. If he had sold the saree at 10% profit and the sweater, he would have got Rs. 1028.

Denote the cost price of the saree and the list price (price before discount) of the sweater by Rs. x and Rs. y respectively and answer the following questions:

(i) Represent the 1st situation can be represented algebraically. 1

(ii) Represent the 2nd situation can be represented algebraically. 1

(iii) Name the point where the linear equation represented by 1st situation intersect the x-axis. 2

OR

Name the point where the linear equation represented by 2nd situation intersect the y-axis. 2

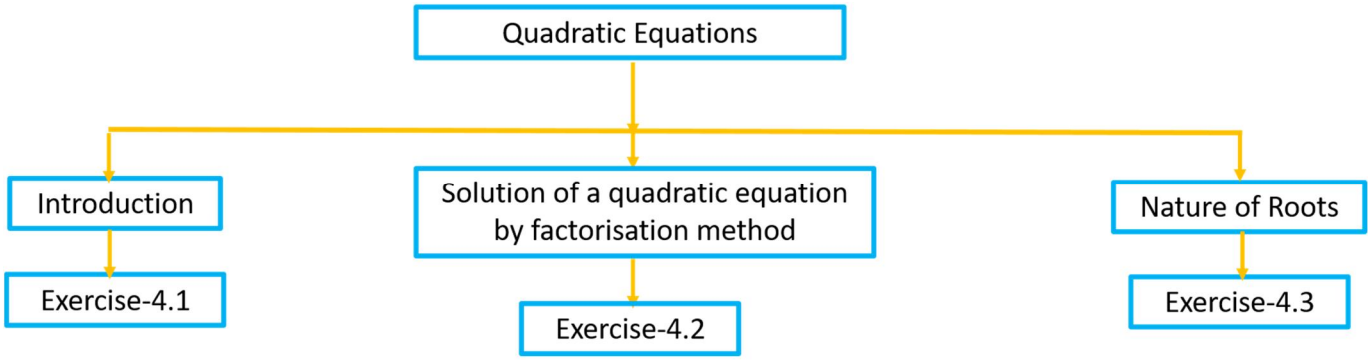
SKILL BASED QUESTIONS:

1. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

2. Draw the graphs of equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and Y-axis.

QUADRATIC EQUATIONS

MIND MAPPING:



Basic facts and formulae:

1. Standard form of a quadratic equation: The most general form of a quadratic equation, called the standard form is $ax^2 + bx + c = 0; a \neq 0$.

2. Roots (or solution) of a Quadratic equation: Those values of x, which satisfy a quadratic equation, are called roots (or solution) of the equation. Thus, a real number α is called a root of the quadratic equation if $a\alpha^2 + b\alpha + c = 0$.

3. **Discriminant of a quadratic equation:** It is a relationship between coefficients of a quadratic equation and is given by $D = b^2 - 4ac$.

4. Finding the roots of a quadratic equation: There are three methods to find the roots of a quadratic equation.

   (i) **By the factorisation method:** It is applied when the discriminant of a quadratic equation $D = b^2 - 4ac$ is a perfect square of a positive number.

   (ii) **By the quadratic formula:** It directly gives the two roots of a quadratic equation, provided discriminant $D \geq 0$ is given by the formula $x = \frac{-b \pm \sqrt{D}}{2a}$, where $D = b^2 - 4ac$

5. **Nature of Roots:** The nature of roots of a quadratic equation depends upon the nature of its discriminant

   (i) If D > 0, (i.e. positive), then the roots are real and unequal / distinct.

   (ii) If D = 0, then the roots are real and equal / coincident.

   (iii) If D < 0, then there are no real roots.

LEVEL_1

MCQ:

1. The value(s) of k for which the roots of quadratic equation $x^2+4x+k=0$ are real is:

   a) $k \geq 4$ b) $k \leq 4$ c) $k \geq -4$ d) $k \leq -4$

2. The discriminant of the quadratic equation $2x^2-5x-3=0$ is:

   a) 1 b) 49 c) 7 d) 19

3. If the quadratic equation $9x^2+bx+\frac{1}{4}=0$ has equal roots, then the value of b is:

   a) 0 b) -3 only c) 3 only d) ±3

4. A quadratic equation whose roots are $\left(2+\sqrt{3}\right)$ and $\left(2-\sqrt{3}\right)$ is:

   a) $x^2-4x+1=0$ b) $x^2+4x+1=0$ c) $4x^2-3=0$ d) $x^2-1=0$

5. Which of the following is a quadratic equation?

   a) $x^2+2x+1=(4-x)^2+3$ b) $-2x^2=(5-x)\left(2x-\frac{2}{5}\right)$

   c) $(k+1)x^2+\left(-\frac{3}{2}\right)x=7;\ where\ k=-1$ d) $x^3-x^2=(x-1)^3$

6. If $\frac{1}{2}$ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then the value of k is:

   a) 2 b) -2 c) $\frac{1}{4}$ d) $\frac{1}{2}$

7. Which of the following equations has sum of its roots as 3?

   a) $2x^2-3x+6=0$ b) $-x^2+3x-3=0$

   c) $\sqrt{2}x^2-\frac{3}{\sqrt{2}}x+1=0$ d) $3x^2-3x+3=0$

2-MARKS:

1. Find the values of p for which quadratic equation $x^2-2px+1=0$ has no real roots.
2. In the quadratic equation $kx^2-6x-1=0$, determine the values of k for which the equation does not have any real root.
3. Find the value of p, for which one root of the quadratic equation $px^2-14x+8=0$ is 6 times the other.
4. Solve for x: $\sqrt{6x+7}-(2x-7)=0$
5. Find two numbers whose sum is 27 and product is 182.

6. Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}\,x + 2 = 0$.

7. Find the roots of the quadratic equation $3x^2 - 5x + 2 = 0$ using the quadratic formula.

8. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$ and hence find the nature of its roots.

9. Find the values of k for the quadratic equation $2x^2 + kx + 3 = 0$ have two equal roots.

10. If one root of $2x^2 + kx - 6 = 0$ is 2, find the value of k and the other root.

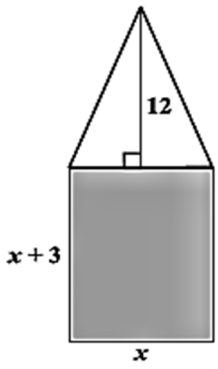
11. Check whether both $x = \frac{2}{3}$ and $x = -\frac{1}{3}$ are roots of $9x^2 - 3x - 2 = 0$ or not.

12. Does $(x-1)^2 + 2(x+1) = 0$ have a real root? Justify your answer.

3-MARKS:

1. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

2. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.



3. Solve for x: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

4. Solve for x: $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$.

5. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k.

6. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

5-MARKS:

1. Check whether the following are quadratic equations:

   (i) $(x-2)^2+1=2x-3$ (ii) $(x+2)^3=2x(x^2-1)$

   (iii) $x(x+1)+8=(x+2)(x-2)$ (iv) $x^2+3x+1=(x-2)^2$

2. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

3. Solve for $x: \frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3};\ x\neq 3,5$.

4. Using quadratic formula solve the quadratic equation: $p^2x^2+(p^2-q^2)x-q^2=0$

5. Find the roots of the quadratic equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30},\ x\neq -4,7$

6. Is the following situation possible? If so, determine their present ages.
   The sum of the ages of two friends is 20 years. Four years ago, the product of their ages is years was 48.

7. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

CASE STUDY QUESTIONS

CASE STUDY_1

Swati is a daughter of Varun. Seven years ago, Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two-fifth of Varun's age.

Based on the above, answer the following questions:

(i) If Swati's age seven years ago be x years, then what will be Varun's age? 1

(ii) What will be Swati's age after three years? 1

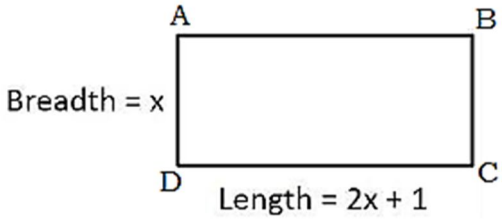
(iii) What is the quadratic equation related to the above given problem? 2

OR

What is the present age of Varun? 2

CASE STUDY_2

A prayer hall consists of a carpet area of a carpet area of 300 sq.m with its length $(2x+1)$ meters and breadth x meters.



(i) Find the length and breadth of the prayer hall? 2

OR

Find the value of BD in the prayer hall? 2

(ii) Find the perimeter of the hall. 1

(iv) From the above figure, compare the values of AB and AD. 1

CASE STUDY_3

John and Jivanti are playing with the marbles. They together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124.

Based on the above information, answer the following questions:

(i) How many marbles does Jivanti have, if John have x number of marbles ? 1

(ii) How many marbles are left with Jivanti, when she lost marbles? 1

(iii) What is the quadratic equation related to the given problem? 2

OR

How marbles do John have. 2

LEVEL_2

MCQ:

1. Which of the following equations has 2 as the root?

   a) $x^2-4x+5=0$ b) $x^2+3x-12=0$ c) $2x^2-7x+6=0$ d) $3x^2-6x-2=0$

2. Value(s) of k for which the quadratic equation $2x^2-kx+k=0$ has equal roots is/are:

   a) 0 b) 4 c) 8 d) 0, 8

3. The quadratic equation $2x^2-\sqrt{5}x+1=0$ has:

   a) two distinct real roots b) two equal real roots

   c) no real roots d) more than 2 real roots

4. Which of the following equations has two distinct real roots?

   a) $2x^2-3\sqrt{2}x+\frac{9}{4}=0$ b) $x^2+x-5=0$

   c) $x^2+3x+2\sqrt{2}=0$ d) $5x^2-3x+1=0$

5. Which of the following equations has no real roots?

   a) $x^2-4x+3\sqrt{2}=0$ b) $x^2+4x-3\sqrt{2}=0$

   c) $x^2-4x-3\sqrt{2}=0$ d) $3x^2+4\sqrt{3}x+4=0$

2-MARKS

1. Find the roots of the quadratic equation $3x^2-4\sqrt{3}x+4=0$.
2. Find the roots of the quadratic equation $2x^2-x+\frac{1}{8}=0$ by factorization.
3. Find the roots of the quadratic equation $2x^2-2\sqrt{2}x+1=0$ using the quadratic formula.
4. Find the value of k for the quadratic equation $kx(x-2)+6=0$ have two equal roots.
5. Find the value of m so that the equation $x^2+5mx+6=0$ has no real roots.
6. Find the roots of the quadratic equation $4x^2-4px+(p^2-q^2)=0$.
7. Find the values of p for which the equation $(p-12)x^2+2(p-12)x+2=0$ has two equal roots.
8. Solve the quadratic equation for x: $12x^2-6(a^2+b^2)x+3a^2b^2=0$
9. Is the following statement True or False? Justify your answer. “If in a quadratic equation the coefficient of x is zero, then the quadratic equation has no real roots.”

10. Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

11. Is 0.2 a root of the equation $x^2 + 0.4 = 0$? Justify.

3-MARKS

1. Find the values of k, for which the quadratic equation $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots.

2. The altitude of a right triangle is 7cm less than its base. If the hypotenuse is 13cm, find the other two sides.

3. Solve for x : $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$, where $x \neq 0, 1, 2$.

4. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

5. Solve for x : $\frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3}$; $x \neq 4, 6$

6. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

5-MARKS

1. Solve the equation for x : $\frac{4}{x} - 3 = \frac{5}{2x+3}$; $x \neq 0, \frac{-3}{2}$.

2. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

3. Solve for x : $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$, $x \neq 1, 2, 3$.

4. Using quadratic formula, solve the equation for x : $abx^2 + (b^2 - ac)x - bc = 0$.

5. Solve for x : $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$; where $x \neq \frac{-1}{2}, 1$.

6. Find two consecutive odd positive integers, sum of whose squares is 290.

7. Solve for x : $3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5$; $x \neq \frac{1}{5}, \frac{-3}{2}$.

CASE STUDY QUESTIONS

CASE STUDY_1

On a forest tour, camels and swans are spotted. One-fourth of a herd of camels were seen in the forest, twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. On the other hand, out of a group of swans, $\frac{7}{2}$ times the square root of the number were playing on the shore of a tank, while the two remaining ones were, playing, with amorous fight, in the water.

Based on the above information, answer the given below questions:

(i) What was the total number of camels? 2

OR

What was the total number of swans? 2

(ii) How many camels were at the mountains? 1

(iii) How many camels were at the forest? 1

LEVEL_3

MCQ:

1. $\left(x^2+1\right)^2 - x^2 = 0$ has:

a) four real roots b) two real roots c) no real roots d) one real root.

2. A teacher asks three students to complete the following statement about the nature of the roots of a quadratic equation. If $q^2 - 4pr > 0$, the roots of the quadratic equation $px^2 + qx + r = 0$ will be...

Zain answers, "always positive".

Vipul answers, "positive, if p, q, and r are positive".

Suman answers, "negative, if p, q, and r are positive".

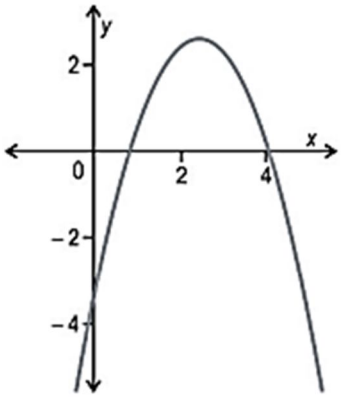
Who answered correctly?

a) Zain b) Vipul c) Suman d) none of them

3. Aman solved a quadratic equation and found its roots to be real.

Which of these could represent the graph of the equation Aman solved?

i



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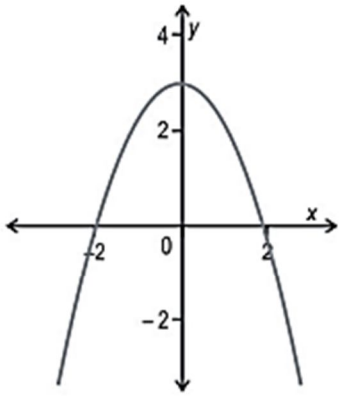

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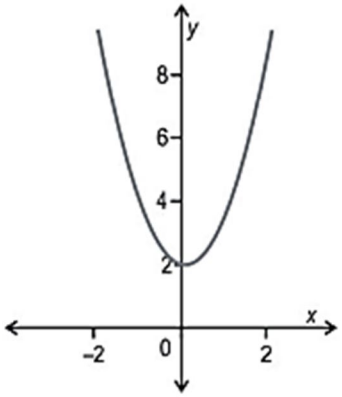

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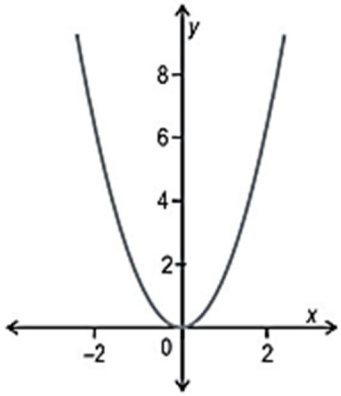


a) only iii) b) only i) and ii)

c) only iii) and iv) d) only i), ii) and iv)

4. A sum of ₹4000 was divided among x persons. Had there been 10 more persons, each would have got ₹80 less. Which of the following represents the above situation?

a) $x^2+10x-500=0$ b) $8x^2+10x-400=0$

c) $x^2+10x+500=0$ d) $8x^2+10x+400=0$

5. A teacher asked students to find the roots of the equation $\frac{x}{x+1}+\frac{x+1}{x}-\frac{34}{15}=0$.

Two students, Ravi and Ankit gave following answers. Ravi said one of the roots is $\frac{3}{2}$. Ankit said one of the roots is (-5)/2. Who is correct?

a) Ravi b) Ankit

c) Both Ravi and Ankit d) Neither Ravi nor Ankit

2-MARKS

1. Find the roots of the quadratic equation $\sqrt{2}x^2+7x+5\sqrt{2}=0$ by factorization.
2. Using quadratic formula, determine the roots of the equation $x-\frac{1}{x}=3$.
3. Solve the quadratic equation $36x^2-12ax+\left(a^2-b^2\right)=0$.
4. Solve the quadratic equation $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$.
5. Find the least positive value of k for which $x^2+kx+16=0$ has real roots.
6. Find the value of p for which the quadratic equation $2x^2+px+\frac{9}{2}=0$ has real and equal roots.
7. A quadratic equation with integral coefficient has integral roots. Justify your answer.
8. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?
9. If b = 0, c < 0, is it true that the roots of $x^2+bx+c=0$ are numerically equal and opposite in sign? Justify?
10. Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.
11. State whether the following quadratic equations have two distinct real roots. Justify your answer. $\sqrt{2}x^2-\frac{3}{\sqrt{2}}x+\frac{1}{\sqrt{2}}=0$

3-MARKS

1. Solve for x: $(a+b)^2x^2-4abx-(a-b)^2=0$.
2. For what values of k, does the $(4-k)x^2+(2k+4)x+8k+1=0$ have equal roots.
3. Find the roots of the quadratic equation $12abx^2-\left(9a^2-8b^2\right)x-6ab=0$ in the variable x.
4. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.
5. If the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are real and equal then prove that $2b=a+c$.
6. Solve for x : $9x^2-9(a+b)x+\left(2a^2+5ab+2b^2\right)=0$

5-MARKS

1. Solve for x : $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$.

2. Solve for x : $3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11;\ x\neq\frac{3}{5}.\frac{-1}{7}$.

3. A train takes 2 hours less for the journey of 3oo km, if its speed is increased by 5 km/h from its usual speed. Find the usual speed of the train.

4. Out of a number of Saras birds, one fourth of the number are moving about in lotus plants; $\frac{1}{9}^{th}$ coupled (along) with $\frac{1}{4}^{th}$ as well as 7 times the square root of the number move on a hill; 56 birds remain in vakula trees. What is the total number of birds?

5. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

CASE STUDY QUESTIONS

CASE STUDY_1:

Two water taps A and B together can fill a tank in 6 hours. The tap A of larger diameter takes 9 hours less than the smaller tap B to fill the tank separately. On the other hand two water taps P and Q together can fill a same tank $9\frac{3}{8}$ hours. The tap P of larger diameter takes 10 hours less than the smaller tap Q to fill the tank separately.

(i) Write the quadratic equation representing the above situation where taps A and B are considered. 1

(ii) Write the quadratic equation representing the above situation where taps P and Q are considered. 1

(iii) How much time will be taken by tap A and tap B individually to fill the tank? 2

OR

How much time will be taken by tap P and tap Q individually to fill the tank? 2

CASE STUDY_2:

Quadratic Equations started around 3000 B.C with the Babylonians. They were one of the world's first civilization, and came up with some great ideas like agriculture, irrigation and writing. There were many reasons why Babylonians needed to solve quadratic equations. For example to know what amount of crop you can grow on the square field.

Based on the above information, represent the following questions in the form of quadratic equations:

(i) Two numbers differ by 3 and their product is 504. 1

(ii) The sum of two numbers is 15 and sum of their reciprocals is $\frac{3}{10}$. 1

(iii) The sum of squares of two consecutive integers is 650. 2

OR

(iv) A natural number whose square diminished by 84 is thrice of 8 more of the given number. 2

SKILL BASED QUESTIONS

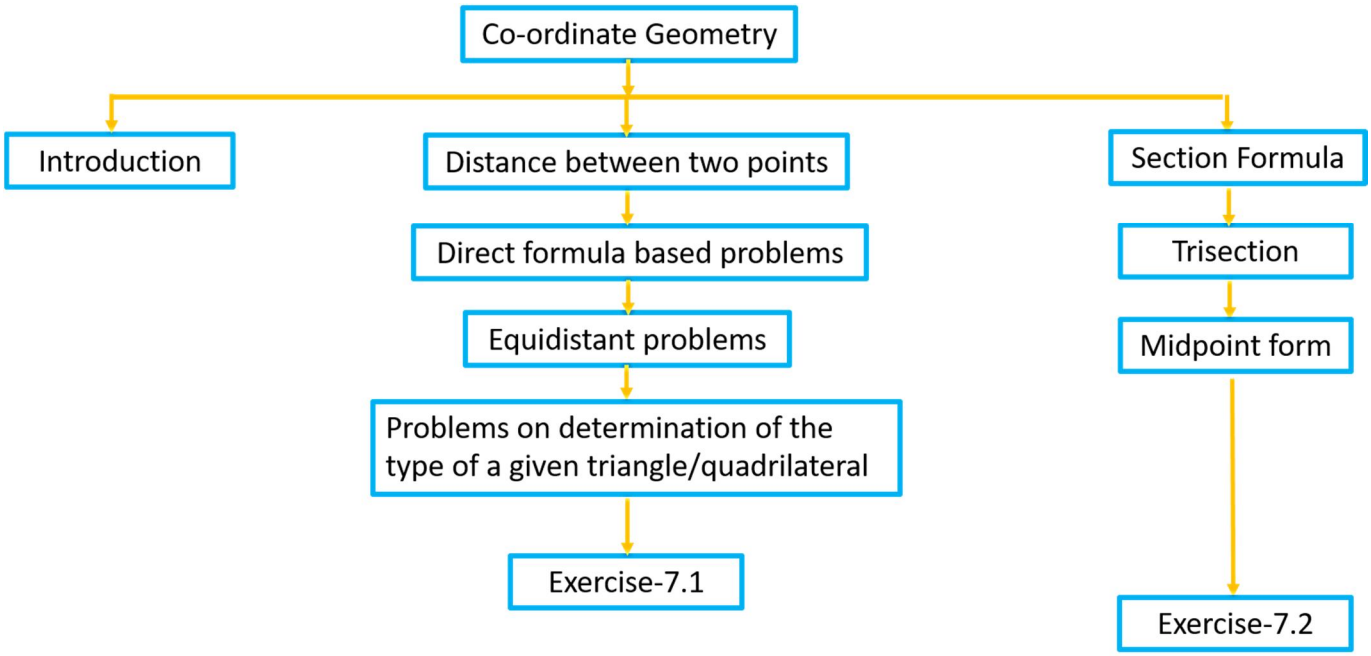
1. If the mean of the following distribution is 54, find the missing frequency x.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	16	14	24	26	x

2. Solve graphically the system of linear equations $4x-5y+16=0$ and $2x+y-6=0$. Determine the vertices of the triangle formed by these lines and the x-axis.

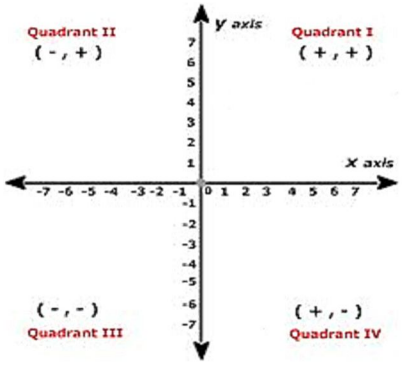
COORDINATE GEOMETRY

MIND MAPPING:



Basic facts and formulae:

1. In Cartesian coordinate system, there is a Cartesian plane which is made up of two number lines which are mutually perpendicular to each other. X-axis (horizontal) and Y-axis (vertical) which represents two variables. These two perpendicular lines are called coordinate axes.
2. The point of intersection of x-axis and y-axis i.e O is called the origin. The coordinates of Origin are (0, 0).
3. X – Coordinate of a point is called abscissa and y-coordinate of a point is called ordinate. A plane is divided by axes into four quadrants.



$Q_1:(+x,+y);\ Q_2:(-x,+y);\ Q_3:(-x,-y);\ Q_4:(+x,-y)$

4. The coordinates of the x- axis is $(x, 0)$.
5. The coordinates of the y-axis is $(0, y)$.

6. If $x \neq y$ then $(x, y) \neq (y, x)$ and if $x = y$ then $(x, y) = (y, x)$.

7. The distance between two points $P(x_1, y_1), Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

8. The distance of the point $P(x, y)$ from the Origin (0, 0) is $\sqrt{x^2 + y^2}$.

9. **Problems based on geometrical figure: To show that a given figure is a**

a) Parallelogram:– prove that the opposite sides are equal and diagonals bisect each other.

b) Rectangle:– prove that the opposite sides are equal and the diagonals are equal.

c) Parallelogram but not rectangle:– prove that the opposite sides are equal and the diagonals are not equal.

d) Rhombus:– prove that the four sides are equal

e) Square:– prove that the four sides are equal and the diagonals are equal.

f) Rhombus but not square:– prove that the four sides are equal and the diagonals are not equal.

g) Isosceles triangle:– prove any two sides are equal.

h) Equilateral triangle:– prove that all three sides are equal.

i) Right triangle:– prove that sides of triangle satisfies Pythagoras theorem

10. If $P(x, y)$ be any point on the line segment AB, which divides AB in the ratio of $m_1 : m_2$ internally then coordinates of $P(x, y)$ will be given by:

$$P(x, y) = \left[\frac{(m_1 x_2 + m_2 x_1)}{m_1 + m_2}, \frac{(m_1 y_2 + m_2 y_1)}{m_1 + m_2} \right].$$

11. The midpoint of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right]$

12. Let P and Q be the points of trisection (dividing into three equal parts) of the line segment AB, then for finding the coordinates of P the ratio is 1:2 and for finding the coordinates of Q the ratio is 2 : 1

13. The line drawn from the vertex of a triangle to the midpoint of the opposite side is called as median.

14. The point of concurrency of medians of a triangle is called as centroid.

15. The centroid divides the median in the ratio of 2:1

LEVEL_1

MCQ:

1. The distance of the point P(2, 3) from the X-axis is:

   (a) 2 (b) 3 (c) 1 (d) 5

2. The distance between the points A(0, 6) and B(0,- 2) is

   (a) 6 (b) 8 (c) 4 (d) 2

3. The distance between the points $(\cos\theta, \sin\theta)\ and (\sin\theta, -\cos\theta)$ is:

   (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1 (d) 0

4. The distance of the point P(- 6, 8) from the origin is

   (a) 8 (b) $2\sqrt{7}$ (c) 10 (d) 6

5. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

   (a) 5 (b) 12 (c)11 (d) $7+\sqrt{5}$

6. The point which divides the line segment joining the points (7, – 6) and (3, 4) in ratio 1: 2 internally lies in the

   (a) I quadrant (b) II quadrant

   (c) III quadrant (d) IV quadrant

7. If AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0), then the length of its diagonal is

   (a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

8. In what ratio does x-axis divide the line segment joining the points A(2, -3) and B(5, 6)?

   (a) 2 : 3 (b) 2 : 1 (c) 3 : 4 (d) 1 : 2

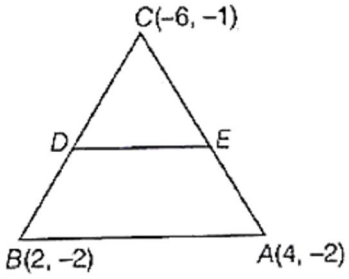
2-MARKS:

1. Points A(3, 1), B(5, 1), C(a, b) and D(4, 3) are the vertices of a parallelogram ABCD. Find the values of a and b.
2. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3). Hence find the value of m.
3. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the midpoint of PQ, then find the coordinates of P and Q.
4. Determine whether the given points (1, 5), (2, 3) and (-2, -11) are collinear.
5. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.
6. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

7. If the point P(2, 1) lies on the line segment joining the points A(4, 2) and B(8, 4), then find the relation between AP and AB.
8. ΔABC is a right angled triangle, in which A(0, 2) and B(2, 0) are given. Then, find the coordinates of C.
9. Prove that points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.
10. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3), then find the value of x^2+5.
11. If the point C(-1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio of 3 : 4. Find the coordinates of B.
12. If the point C(k, 4) divides the join of points A(2, 6) & B(5, 1) in the ratio of 2 : 3, find the value of k .

3-MARKS:

1. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).
2. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.
3. In the given figure of ΔABC, D and E are the midpoints of the sides BC and AC respectively.

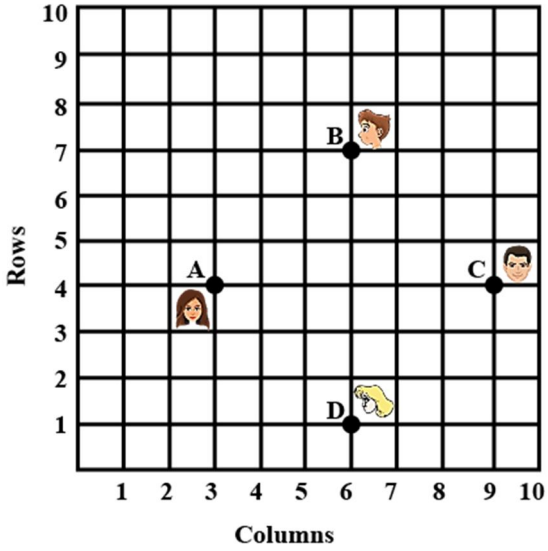


Find the length of DE and also prove that $DE=\frac{1}{2}AB$.

4. Show that the points A(5, -1), B(8, 3), C(4, 0) and D(1,-4) are the vertices of a rhombus.
5. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP=\frac{3}{7}AB$ and P lies on the line segment AB.
6. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

5-MARKS:

1. Find the coordinates of the points which divide the line segment joining the points A(-2, -2) and B(2, 8) into six equal parts.
2. In a class room, 4 friends are seated at the points A, B, C and D as shown in the figure. Champa and Chameli walk into the class and after observing for a few minutes, Champa asks Chameli, "Don't you think ABCD is a square"?



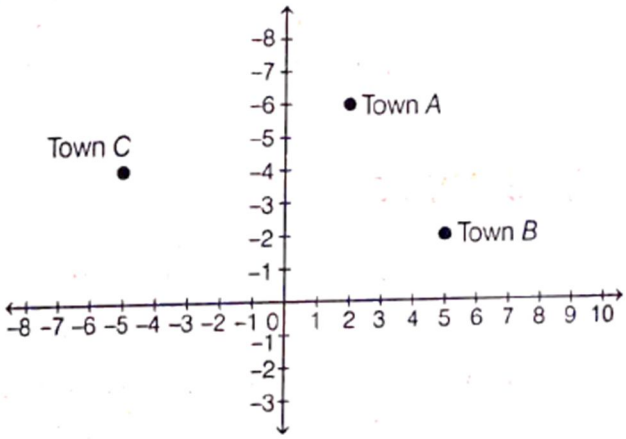
Chameli disagrees. Using distance formula, find which of them is correct.

3. Points P, Q, R, S and T divide the line segment joining the points A(1, 2) and B(6, 7) in 6 equal parts. Find the coordinates of the points P, Q, R, S and T.
4. The points (3, -4) and (-6, 5) are the end points of a diagonal of a parallelogram. If one end of the end points of the second diagonal is (-2, 1), then find its other end point.
5. Show that the quadrilateral PQRS formed by the vertices P(22, 5), Q(7, 10), R(12, 11) and S(3, 24) is not a parallelogram.
6. Show that the ΔABC, where $A(-2,0)$, $B(2,0)$, $C(0,2)$ and ΔPQR where P(-4,0), Q(4, 0) and R(0, 4) are similar triangles.

CASE STUDY QUESTIONS

CASE STUDY_1

Two friends Seema and Aditya work in the same office at Delhi. In the Vacation both decided to go to their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions.



(i) What are the locations of Town A (Seema) and Town B (Aditya)? 1

(ii) Locate the point K, which divides the line segment joining Town A and Town B in the ratio of 2 : 1. 1

(iii) Who will travel more distance, Seema or Aditya to reach their hometown? 2

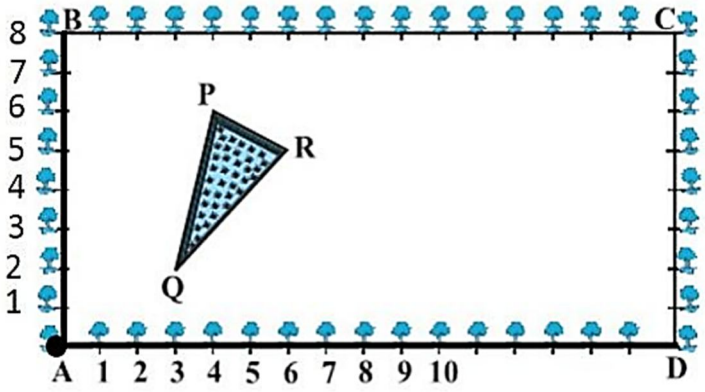
OR

Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B.

Find the coordinates of the point represented by the point D. 2

CASE STUDY_2

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the below figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of a triangle PQR. 1

(ii) What is the midpoint of the side PQ, when A is the origin? 1

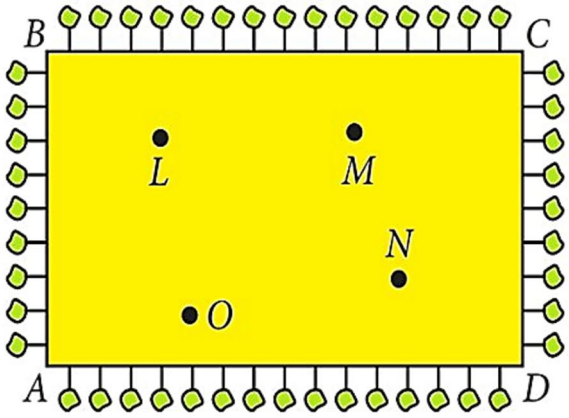
(iii) What will be the coordinates of the vertices of a triangle PQR if C is the origin? 2

OR

What is the midpoint of side QR, when C is the origin? 2

CASE STUDY_3

On the occasion of 'Diwali' a rectangular plot have been allotted for 'Diwali Mela' to students of secondary school in Hyderabad. In order to reduce smog and pollution they decided to keep little leaf linden plant on the boundary at a distance of 1 m from each other. Four air purifier machines have also been set up at points L, M, N, O. (Answer the following questions considering A as origin).



(i) What are the coordinates of L? 1

(ii) What are the coordinates of N? 1

(iii) What is the distance between L and O? 2

OR

Find the mid-point of the segment joining the points L and N. 2

LEVEL_2

MCQ:

1. The distance of the point p(x, y) from the origin is:

(a) $\sqrt{x^2+y^2}$ (b) $\sqrt{2x^2+2y^2}$ (c) $2\sqrt{x^2+y^2}$ (d) $\sqrt{x+y}$

2. The coordinates of the point, where the line $x-y=5$ cuts Y-axis is:

(a) $(0,5)$ (b) $(0,-5)$ (c) $(5,0)$ (d) $(-5,0)$

3. The distance between the points $(a\cos 30^0,\ 0)$ and $(0,\ a\cos 60^0)$ is:

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) a (d) 1

4. Sheena was asked to plot a point 10 unit on the left of the origin and other point 4 units directly above the origin. Which of the following are the two points?

(a) (10, 0) and (0,-4) (b) (-10, 0) and (4, 0)

(c) (10, 0) and (0, 4) (d) (-10, 0) and (0, 4)

5. If $\left(3, \frac{3}{4}\right)$ is the mid-point of the line segment joining the points $(k, 0)$ & $\left(7, \frac{3}{2}\right)$, then the value of k is:

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

6. On a graph, two-line segments, AB and CD of equal length are drawn. Which of these could be the coordinates of the points, A, B, C and D?

(a) A(-3,4) B(-1,2) and C(3,4) D(1,2) (b) A(-3,-4) B(-1,2) and C(3,4) D(1,2)

(c) A(-3,4) B(-1,-2) and C(3,4) D(1,2) (d) A(3,4) B(-1,2) and C(3,4) D(1,2)

2-MARKS

1. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).
2. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).
3. Find the distance between the points $P\left(\frac{\sin\theta}{2}, 0\right)$ and $Q\left(0, \frac{\cos\theta}{2}\right)$.
4. If the distance between the points (4, p) and (1, 0) is 5, then find the value of p.
5. Show that if a circle has its centre at the origin and a point P(5, 0) lies on it, then the point Q(6, 8) lies outside the circle.
6. P(-2, 5) and Q(3, 2) are two points. Find the coordinates of the point R on PQ such that PR = 2QR.
7. What will be the value of y, if the point $\left(\frac{23}{5}, y\right)$ divides the line segment joining the points (5, 7) and (4, 5) in the ratio 2 : 3 internally?
8. Point P divides the line segment joining the points A(2, 1) and B(5, -8) such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k .
9. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5). Find the coordinates of the other end of the diameter?
10. If the midpoint of the line joining (3, 4) and (k, 7) is (x, y) and lies on the line $2x + 2y + 1 = 0$, then find the value of k.
11. Point P divides the line segment joining the points A(-1, 3) and B(9, 8) such that $\frac{AP}{BP} = \frac{k}{1}$. If P lies on the line $x - y + 2 = 0$, then find the value of k.

3-MARKS

1. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).
2. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.
3. In what ratio does the Y-axis divide the line segment joining the points P(-4, 5) and Q(3, -7)? Also, find the coordinates of the point of intersection.
4. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{-1}{2}, \frac{3}{2}\right)$ and $B(2, -5)$.
5. If the centroid of the triangle formed by the points A(a, b), B(b, c) and C(c, a) is at the origin. What is the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$?
6. The coordinates of one end point of the diameter of a circle are (4, -1) and coordinates of the centre of the circle are (1, -3).
   (i) Find the coordinates of the other end of the diameter.
   (ii) Find the diameter of the circle.
   (iii) Calculate the area of the circle.
7. Prove that the diagonals of a rectangle ABCD, with vertices A(2, -1), B(5, -1), C(5, 6) and D(2, 6) are equal and bisect each other.
8. Determine the ratio in which the straight line $x-y-2=0$ divides the line segment joining (3, -1) and (8, 9).
9. Name the type of triangle formed by the points $P\left(\sqrt{2}, \sqrt{2}\right)$, $Q\left(-\sqrt{2}, -\sqrt{2}\right)$ and $R\left(-\sqrt{6}, \sqrt{6}\right)$.

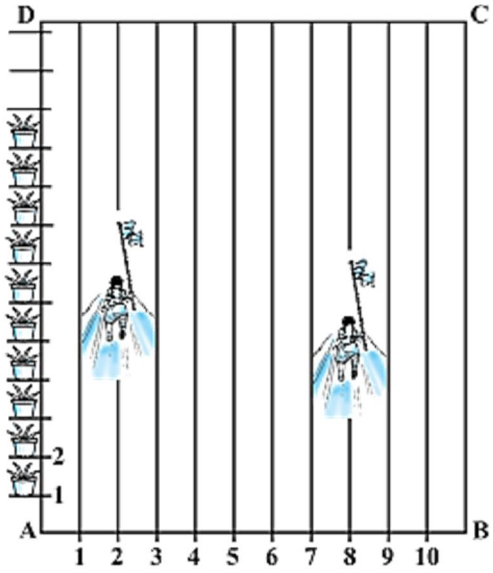
5-MARKS

1. Find the length of the medians RS and PT of a triangle PQR whose vertices are P(6, -2), Q(6, 3) and R(3, 1).
2. An equilateral triangle has two vertices at the points (3, 4) and (-2, 3). Find the coordinates of the third vertex.
3. Show that the points with coordinates (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square.
4. If R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a) then prove that $x+y=a+b$.

5. The coordinates of the vertices of ΔABC are A(1, -1), B(-4, 6) and C(-3, -5). Draw the figure and prove that ΔABC is a scalene triangle.

6. If (-4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

7. To conduct Sports Day activities in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD as shown in the figure. Niharika runs $\frac{1}{4}^{th}$ the distance AD in the 2nd line and posts a green flag. Preet runs $\frac{1}{5}^{th}$ the distance AD on the eighth line and posts a red flag.



(i) What is the distance between both the flags?

(ii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post the flag?

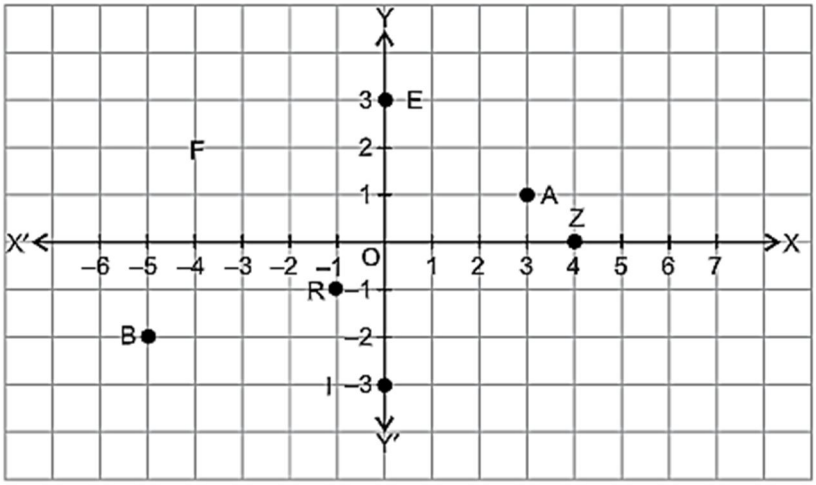
(iii) Which mathematical concept is used in the above problems?

(iv) Which value is depicted in the problem?

CASE STUDY QUESTIONS

CASE STUDY_1

For a sports event, certain points were marked on a rectangular ground denoting positions of different drills.



(i) Find the distance between the points A and E. 1

(ii) What type of triangle is formed by joining the points A, Z and R? 2

OR

What is the ratio in which X-axis divides the joing of A and R? 2

(iii) What is the distance of the point B from F? 1

LEVEL_3

MCQ:

1. The points (- 4, 0), (4, 0) and (0, 3) are the vertices of a:

   (a) right angled triangle (b) isosceles triangle

   (c) equilateral triangle (d) scalene triangle

2. If the point P(2, 1) lies on the line segment joining points A(4, 2) and 6(8, 4), then

   (a) $AP = \frac{1}{3}AB$ (b) AP = PB (c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

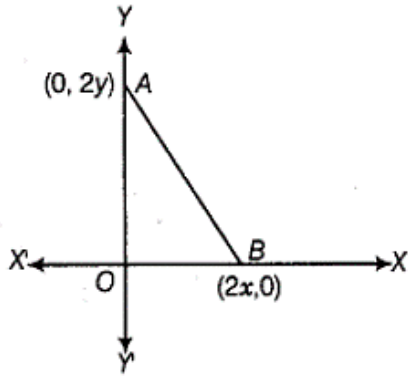
2. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$ is:

   (a) $\sqrt{a^2 + b^2}$ (b) $\sqrt{a^2 - b^2}$ (c) $2\sqrt{ab}$ (d) $3\sqrt{a^2 + b^2}$

3. The point which lies on the perpendicular bisector of the line segment joining the points A(-2, – 5) and B(2, 5) is:

   (a) (0,0) (b) (0, 2) (c) (2, 0) (d)(-2,0)

4. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is:



(a) (x,y) (b) (y,x) (c) $\left(\frac{x}{2},\frac{y}{2}\right)$ (d) $\left(\frac{y}{2},\frac{x}{2}\right)$

5. If a circle is drawn with origin as the centre passes through $\left(\frac{13}{2},0\right)$, then the point which does not lie in the interior of the circle is:

(a) $\left(-\frac{3}{4},1\right)$ (b) $\left(2,\frac{7}{3}\right)$ (c) $\left(5,-\frac{1}{2}\right)$ (d) $\left(-6,\frac{5}{2}\right)$

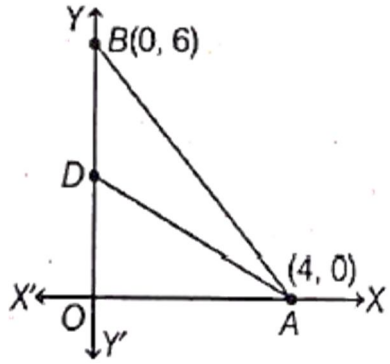
2-MARKS

1. Find the point which lies on the perpendicular bisector of the line segment joining the points A(-2, -5) and B(2, 5).
2. The vertices of a triangle are (a, b-c), (b, c-a) and (c, a-b). Prove that its centroid lies on X-axis.
3. Show that ΔABC with vertices A(-2, 0), B(0, 2) and C(2, 0) is similar to ΔDEF with vertices D(-4, 0), E(4, 0) and F(0, 4).
4. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). What is the length of its diagonals?
5. If P & Q are the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.
6. The line segment joining the points A(2, 1) and B(5, -8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x-y+k=0$. Find the value of k.
7. Find a relation between x and y such that the points (x, y) is equidistant from the point (3, 6) and (-3, 4).
8. If P(x, y) is any point on the line joining the points A(a, 0) and B(0, b) then show that $\frac{x}{a}+\frac{y}{b}=1$.

9. If $x-2y+k=0$ is a median of the triangle whose vertices are at points A(-1, 3), B(0, 4) and C(-5, 2) find the value of k .

10. Points A(-6, 10), B(-4, 6) and C(3, -8) are collinear such that $AB=\frac{2}{9}AC$. Justify your answer.

11. The vertices of a ΔOAB are O(0, 0), A(4, 0) and B(0, 6). The median AD is drawn on OB. Find the length of AD.



3-MARKS

1. Points P, Q, R and S divide the line segment joining the points A(1, 2) & B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.
2. If the points A(-2, 1), B(a, b) and C(4, -1) are collinear and $a-b=1$, find the values of a and b.
3. If the points A(1, 2), B(4, q), C(p, 6) & D(3, 5) are the vertices of a parallelogram ABCD, find the values of p and q.
4. If coordinates of midpoints of the sides of a triangle are (1, 2), (0, -1) and (2, -1). Find the coordinates of its vertices.
5. ABCD is a parallelogram with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$. Find the coordinates of the fourth vertex D in terms of x_1, x_2, x_3, y_1, y_2 and y_3.
6. The centre of the circle is (2a, a-7). Find the values of 'a' if the circle passes through (11, -9) and has the diameter $10\sqrt{2}$ units.
7. If $P(9a-2,-b)$ divides the line segment joining $A(3a+1,-3)$ and $B(8a,5)$ in the ratio of 3 : 1, find the values of a and b.
8. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points (8, -9) and (2, 1). Also find the coordinates of the point of division.

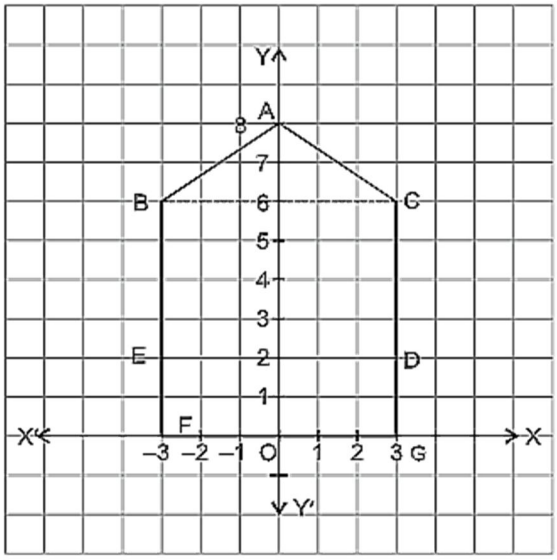
5-MARKS

1. The midpoint 'P' of the line segment joining the points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). Find the ratio in which P divides CD. Also find the values of y.
2. If two vertices of a parallelogram are (3, 2), (-1, 0) and the diagonals cut at (2, -5) then find the other vertices of the parallelogram.
3. If two opposite vertices of a square are (5, 4) and (1, -6) then find the coordinates of it's remaining the two vertices.
4. Using Analytical geometry, prove that diagonals of a rhombus bisect each other.
5. If the points A(1, -2), B(2, 3), C(a, 2) and D(-4, -3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.
6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from here to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching the office? (Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.
7. The mid-points D, E and F of the sides of a triangle ABC are (3, 4), (8, 9) & (6, 7). Find the coordinates of the vertices of the triangle.

CASE STUDY QUESTIONS

CASE STUDY_1

The front door of a guest house is drawn/shown on the coordinate plane as follows:



(i) What is the length of the line segment AB? 1

(ii) Is AB = AC? 1

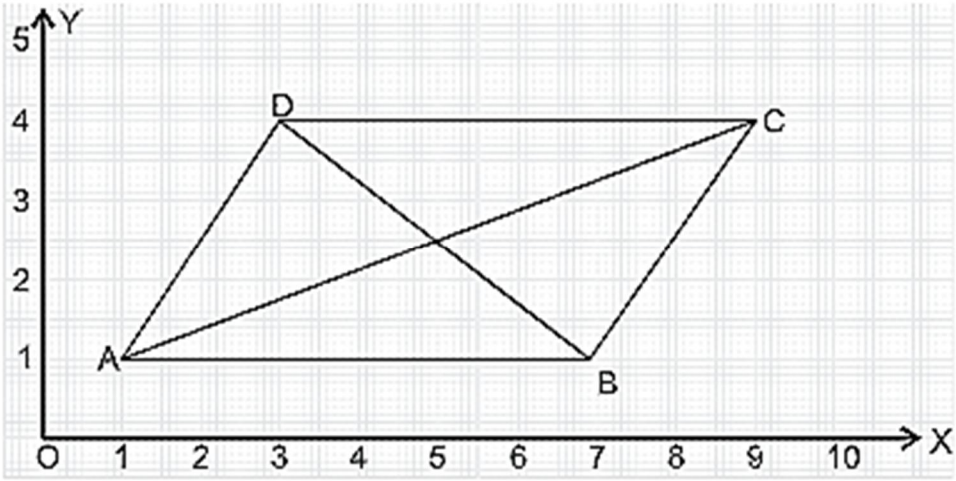
(iii) What are the coordinates of the midpoint of BE and midpoint of ED? 2

OR

What is the ratio in which Y-axis divides BD? (Join BD)

CASE STUDY_2

Ravi lives in a multistorey apartment. One day, from his balcony he observed that there is a flower bed on the ground in the shape of a parallelogram. He draws the sketch of the flower bed on a graph paper as shown:



(i) What are the coordinates of the vertex D? 1

(ii) What are the coordinates of the point of intersection of the diagonals? 1

(iii) What is the length of the sides AB and AD? 2

OR

If we take A as the origin and AB as the x-axis then what are the coordinates of M? 2

SKILL BASED QUESTIONS

1. In a health checkup, the number of heart beats of 40 women were recorded in the following table. Find the mean of the following by Assumed mean method.

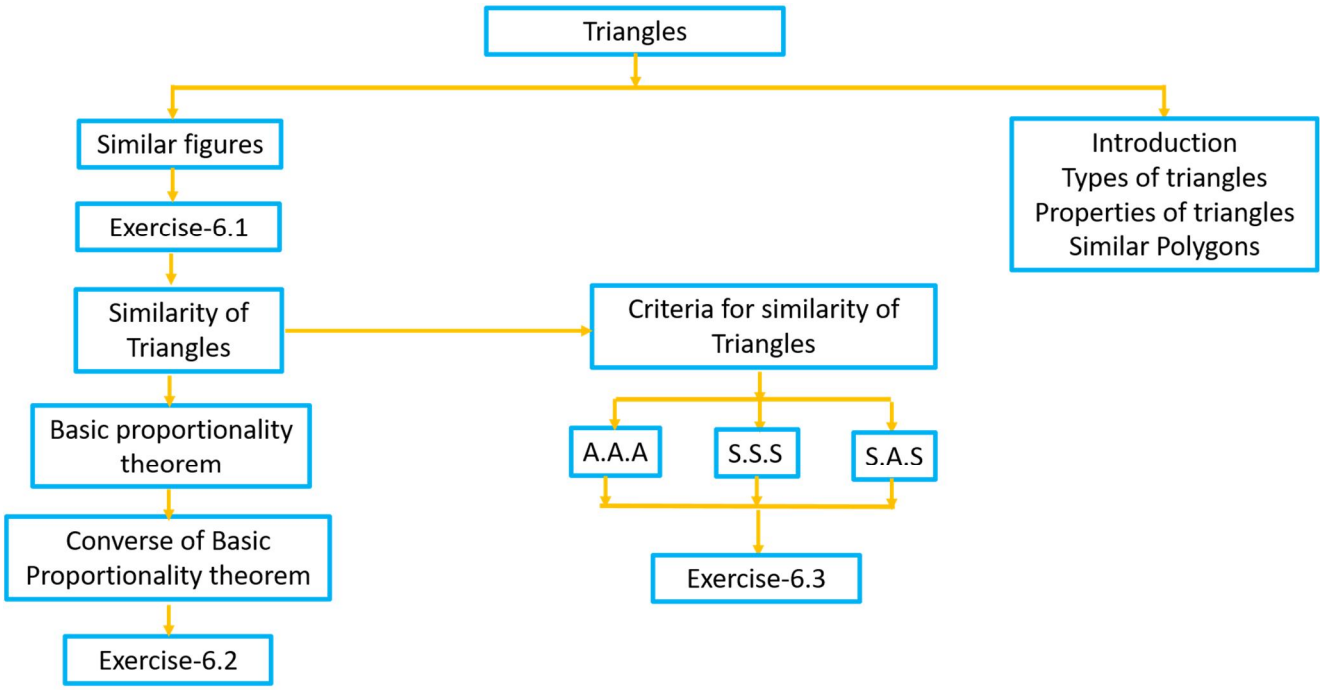
Number of heart beats/minute	65-69	70-74	75-79	80-84
Number of women	2	18	16	4

2. If the mode of the following series is 54, then find the value of f .

Class Interval	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	3	5	f	16	12	7

TRIANGLES

MIND MAPPING



Basic facts and formulae:

1. A polygon in which all sides and angles are equal is called a regular polygon. All regular polygons having the same number of sides are always similar.
2. Two polygons with the same number of sides are similar if:
   All the corresponding angles are equal and all the corresponding sides are in the same ratio (or in proportion).
3. All congruent triangles are-similar but converse need not be true.
4. All squares, circles, and equilateral triangles, etc., are examples of similar figures.
5. Basic proportionality Theorem (Thales Theorem): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
6. Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
7. **AAA criterion for similarity of triangles**: If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio, hence the two triangles are similar.
8. **SSS criterion for similarity of triangles:** If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal, hence the triangles are similar.

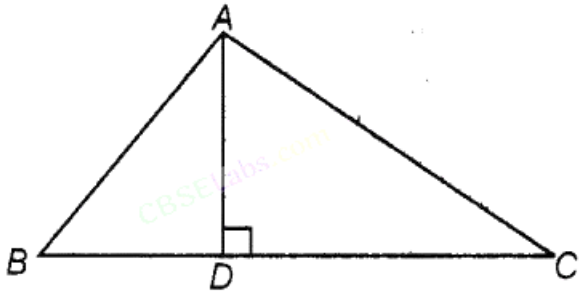
9. **SAS criterion for similarity of triangles:** If in two triangles one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

10. If two angles of a triangle are equal to the two angles of another triangle, then third angles of both triangles are equal by angle sum property of a triangle.

LEVEL_1

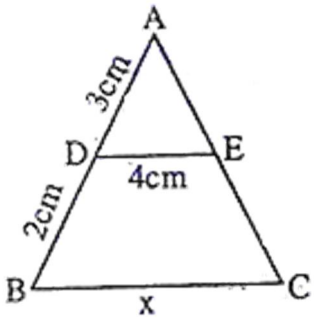
MCQ:

1. In the given below figure, if $\angle BAC = 90^0$ and $AD \perp BC$, then



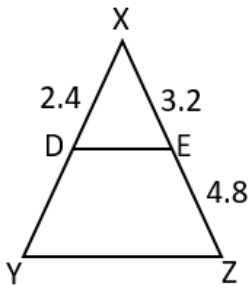
(a) $BD.CD = BC^2$ (b) $AB.AC = BC^2$ (c) $BD.CD = AD^2$ (d) $AB.AC = AD^2$

2. In the figure given below, if DE|| BC, then the value of x is:



(a) $\frac{3}{20}$ (b) $\frac{20}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{10}$

3. In the given figure, in △XYZ, DE||YZ, so that the lengths of sides XD, XE and EZ (in centimeters) are 2.4, 3.2 and 4.8 respectively. Then find the length of XY (in centimeters) is



(a) 5 (b) 6 (c) 7 (d) 8

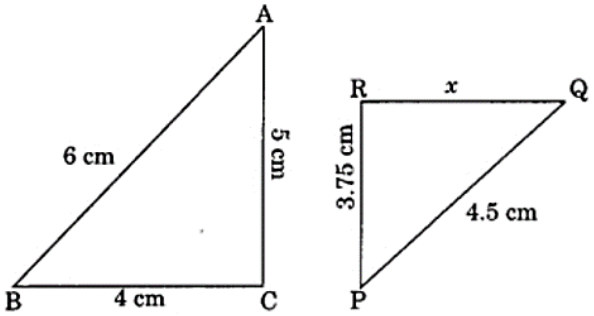
4. If $\Delta ABC \sim \Delta PQR$, perimeter of $\Delta ABC = 20cm$, perimeter of △PQR = 40cm & PR = 8 cm, then the length of AC is:

(a) 6 cm (b) 4 cm (c) 9 cm (d) 10 cm

5. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

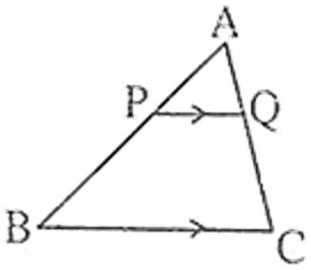
(a) $\Delta PQR \sim \Delta CAB$ (b) $\Delta PQR \sim \Delta ABC$ (c) $\Delta CBA \sim \Delta PQR$ (d) $\Delta BCA \sim \Delta PQR$

6. In the given figure if $\Delta ABC \sim \Delta PQR$, then the find the value of x is



(a) 2.5 cm (b) 3 cm (c) 3.5 cm (d) 4 cm

7. In the figure $PQ \| BC$, if $\frac{PQ}{BC} = \frac{2}{5}$, then the value of $\frac{AP}{PB}$ is:

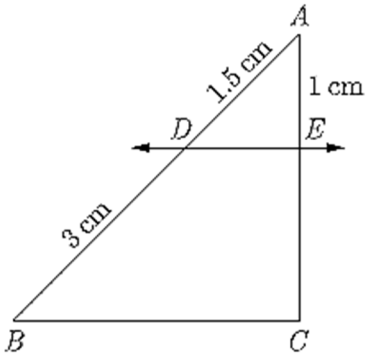


(a) $\frac{2}{5}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

8. In triangles ABC and DEF, $\angle A = \angle E = 40^0$, AB : ED = AC : EF and $\angle F = 65^0$, then find the value of $\angle B$.

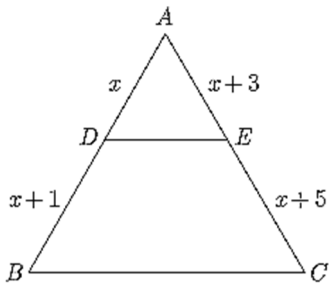
(a) 35^0 (b) 65^0 (c) 75^0 (d) 85^0

9. In the given figure, $DE \| BC$ then the value of EC is:



(a) 3 cm (b) 4 cm (c) 6 cm (d) 2 cm

10. In triangle ABC, $DE \| BC$ then the value of x is:

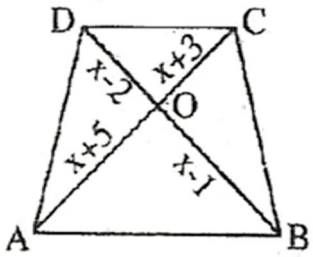


(a) 3 (b) 4 (c) 2 (d) 6

2-MARKS

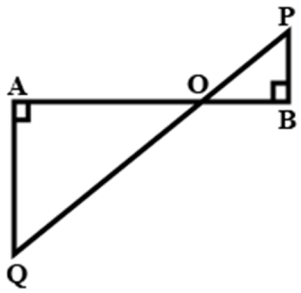
1. Prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

2. In the figure, if AB||DC, find the value of 'x'.



3. ABCD is a trapezium in which AB||DC and its diagonals intersect each other at O. Prove that $\frac{AO}{BO} = \frac{CO}{DO}$.

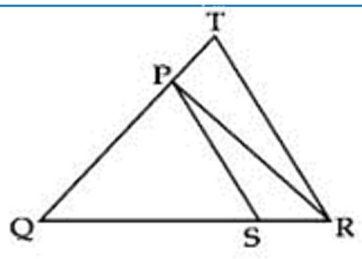
4. In figure QA and PB are perpendicular to AB. If AO = 10 cm, BO=6cm and PB=9 cm then find AQ.



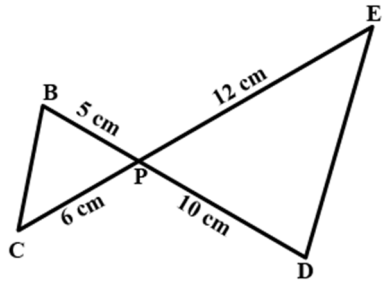
5. A vertical pole of length 6m casts a shadow 4m long on the ground and the same time a tower casts a shadow 28m long. Find the height of the tower.

6. If D, E are points on the sides AB and AC of ΔABC such that AD = 6cm, BD = 9cm, AE = 8cm, EC =12cm. Prove that DE||BC.

7. In the given figure, PQR is an isosceles triangle with PQ = PR. S is a point on QR and T is a point on QP produced such that $\frac{QT}{PR} = \frac{QR}{QS}$, prove that $\Delta PQS \sim \Delta TQR$.



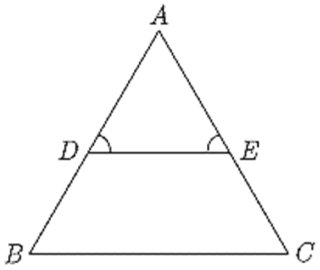
8. In figure, BD and CE intersect each other at the point P. Is $\Delta PBC \sim \Delta PDE$? Why?



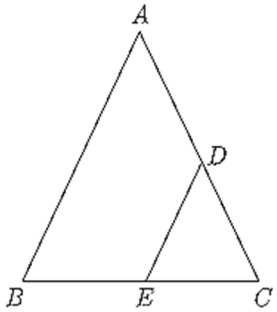
9. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles will be similar? Why?

10. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

11. In the figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that ΔBAC is an isosceles triangle.



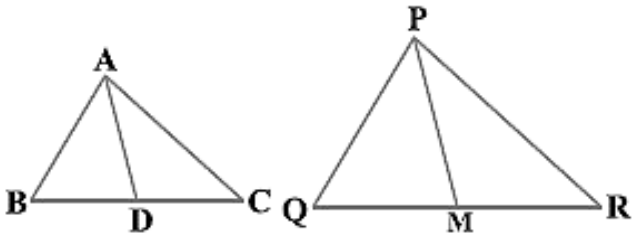
12. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that DE | | AB, AD = 2x, DC = x + 3, BE = 2x − 1 and CE = x. Then, find x.



3-MARKS

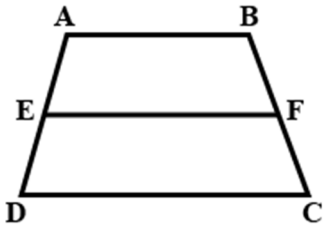
1. AD and PM are medians of triangles ABC & PQR respectively where $\Delta ABC \sim \Delta PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

2. In the figure, sides AB, BC and median AD of ΔABC are respectively proportional to sides PQ, QR and median PM of ΔPQR. Prove that $\Delta ABC \sim \Delta PQR$

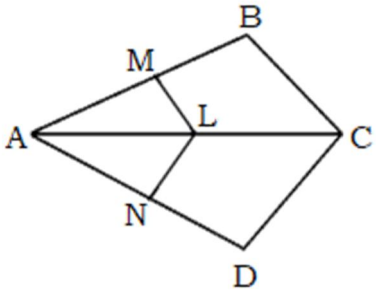


3. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. So that $CA^2 = CB.CD$.

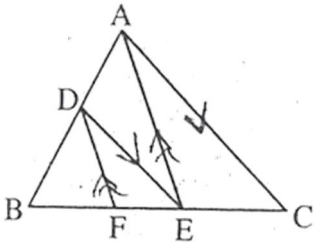
4. In figure, ABCD is a trapezium with AB||DC; E and F are the points on non-parallel sides AD and BC respectively such that EF||AB, prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



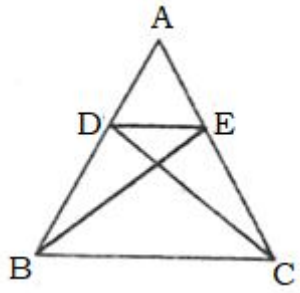
5. In the figure, if LM||CB and LN||CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



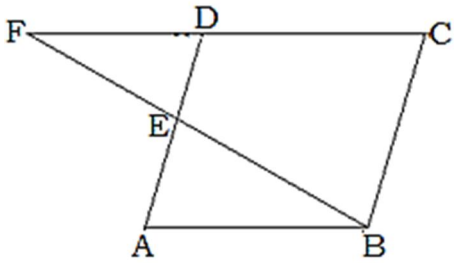
6. In the figure, if DE||AC and DF||AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



7. In figure $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



8. E is a point on side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

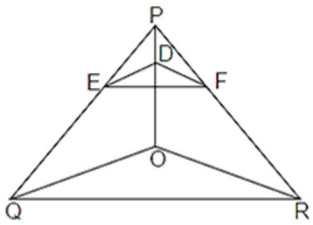


5-MARKS

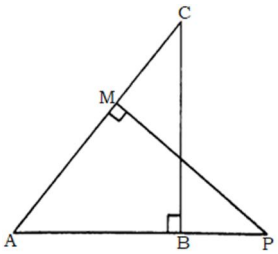
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

2. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

3. In figure DE||OQ and DF||OR. Show that EF||QR.

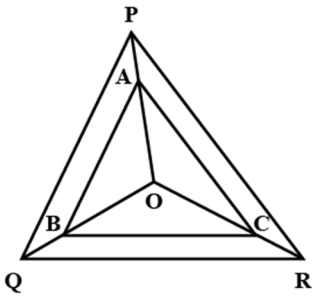


4. In figure, ABC and AMP are two right triangles, right angled at B & M respectively. Prove that

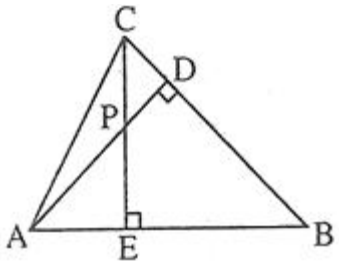


i) $\Delta ABC \sim \Delta AMP$ ii) $\frac{CA}{PA} = \frac{BC}{MP}$

5. In the figure, A, B and C are points on OP, OQ and OR respectively such that AB||PQ and AC||PR. Show that BC||QR.



6. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



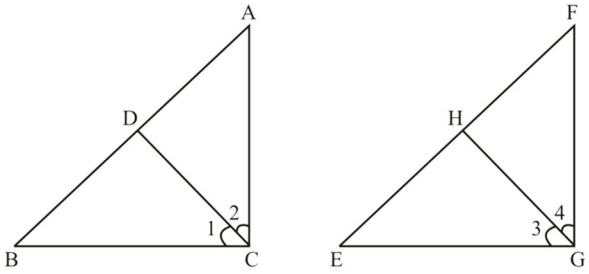
i) $\Delta AEP \sim \Delta CDP$

ii) $\Delta ABD \sim \Delta CBE$

iii) $\Delta AEP \sim \Delta ADB$

iv) $\Delta PDC \sim \Delta BEC$

7. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively.



If $\Delta ABC \sim \Delta FEG$, show that:

i) $\frac{CD}{GH} = \frac{AC}{FG}$

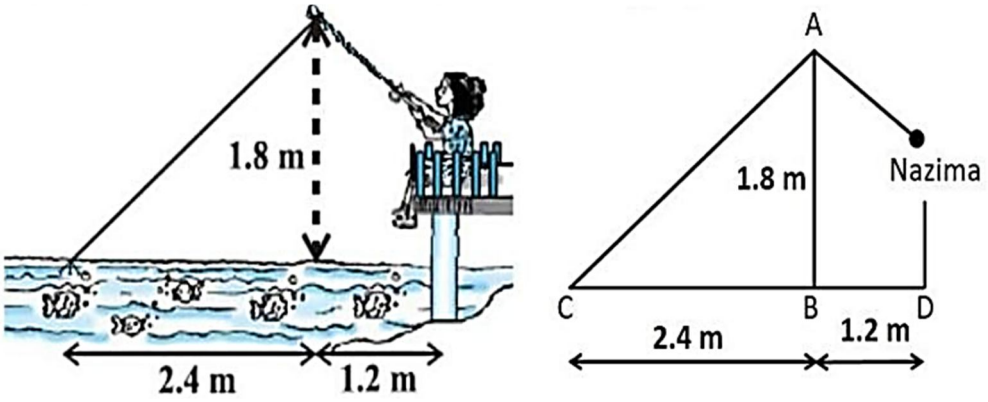
ii) $\Delta DCB \sim \Delta HGE$

iii) $\Delta DCA \sim \Delta HGF$

CASE STUDY QUESTIONS

CASE STUDY_1

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. She is pulling the string at the rate of 5 cm per second. Nazima's friend observe her position and draw a rough sketch by using A, B, C, and D positions of tip, point directly under the tip of the rod, fish and Nazima's position (see the below figure). Assuming that her string (from the tip of her rod to the fly) is taut, answer the following questions.



Based on the above information answer the following questions:

(i) What is the length of AC? 1

(ii) What is the length of the string pulled in 12 seconds? 1

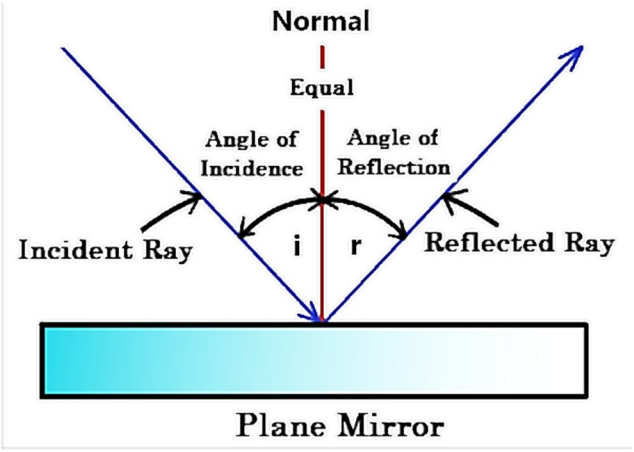
(iii) What is the length of the string after 12 seconds ? 2

OR

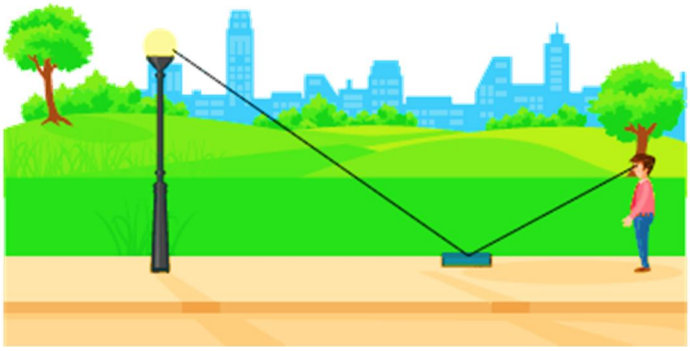
What will be the horizontal distance of the fly from her after 12 seconds? 2

CASE STUDY_2

The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.5 m above the ground. The distance of Ramesh and the pole from the mirror are 1.8 m and 6 m respectively.



Based on the above information answer the following questions:

(i) Which criterion of similarity is applicable to similar triangles? 1

(ii) What is the height of the pole? 1

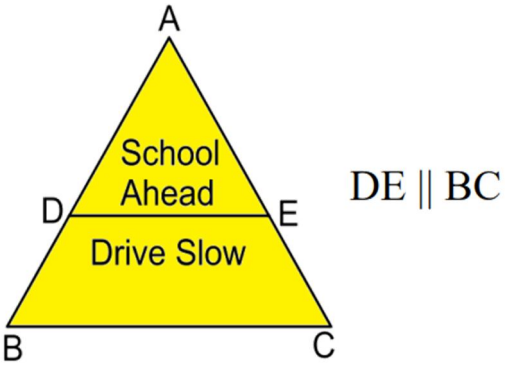
(iii) Now Ramesh move behind such that distance between pole and Ramesh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Ramesh? 2

OR

What is the distance between mirror and pole? 2

CASE STUDY_3

A group of students to volunteer are working in making a safety board for school. They prepared once triangular safety board for their school with title "School Ahead" and "Drive Slow" in two parts of the triangular board as shown in the figure.



DE || BC

Based on the above information answer the following questions:

(i) If AD = 2 cm, BD = 5 cm and AE = 3 cm then find the value of EC? 1

(ii) If AD = 3 cm, AB = 9 cm, BC = 6 cm then find the value of DE? 1

(iii) If $\angle A = 60^0$ and $\angle ADE = 50^0$ find the value of $\angle C$. 2

OR

D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 12.5 cm, AD = 5 cm, EC = 6 cm and AE = 4 cm. Is DE || EC? Give reason for your answer. 2

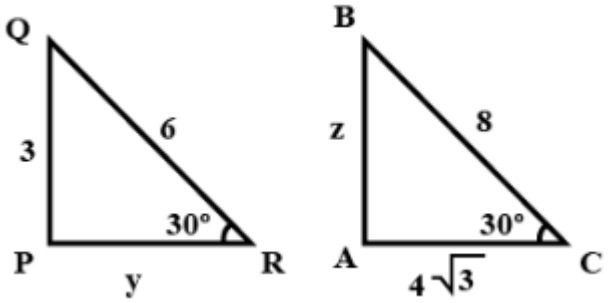
LEVEL_2

MCQ:

1. If in triangle ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar if:

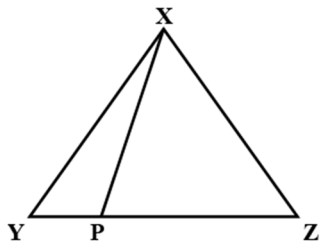
(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$

2. If $\Delta ABC \sim \Delta PQR$, then find the value of $y + z$.



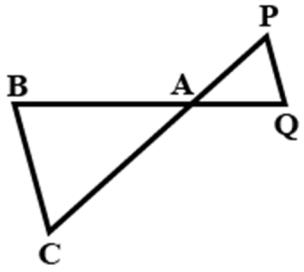
(a) $2 + \sqrt{3}$ (b) $4 + 3\sqrt{3}$ (c) $4 + \sqrt{3}$ (d) $3 + 4\sqrt{3}$

3. In the given figure, $\angle YXZ = \angle XPZ$ then the value of $\frac{ZX}{ZY}$ is.



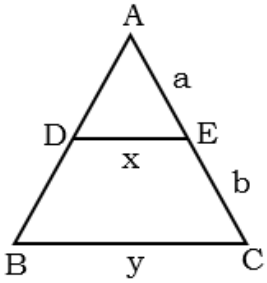
(a) PZ^2 (b) XZ^2 (c) $\frac{PZ}{XZ}$ (d) $ZY \times ZP$

4. In the given figure, $\Delta ACB \sim \Delta APQ$. If AB = 6cm, BC = 8cm and PQ = 4cm, then find the value of AQ.



(a) 2 cm (b) 2.5 cm (c) 3 cm (d) 3.5 cm

5. In the given figure, DE||AC. Which of the following is true?

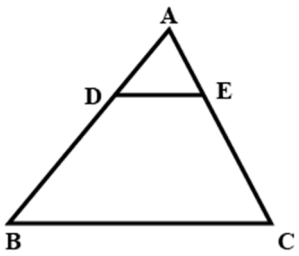


(a) $x = \frac{a+b}{ay}$ (b) $y = \frac{ax}{a+b}$ (c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$

6. If triangle ABC is similar to triangle DEF such that 2AB = DE and BC = 8cm, then find the value of EF.

(a) 16 cm (b) 12 cm (c) 8 cm (d) 4 cm

7. If in ΔABC, AB = 6cm, DE||BC such that $AE = \frac{1}{4}AC$, then the length of AD is:



(a) 3 cm (b) 1.5 cm (c) 6 cm (d) 8 cm

8. In a ΔABC, AD is the bisector of $\angle BAC$. If AB = 6cm, AC = 5cm and BD = 3cm, then the value of DC is:

(a) 11.3 cm (b) 2.5 cm (c) 3 : 5 (d) None of these

9. If triangle ABC and DEF are similar triangles such that $\angle A = 45^0$ and $\angle F = 56^0$ then $\angle C$ is equal to:

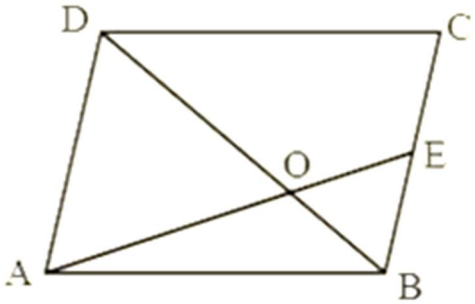
(a) 79^0 (b) 101^0 (c) 56^0 (d) 45^0

10. ABCD is a trapezium such that BC||AD and AD = 4 cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then the value of BC is:

(a) 6 cm (b) 7 cm (d) 8 cm (d) 9 cm

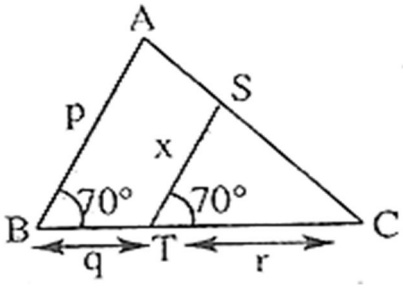
2-MARKS

1. In the figure ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm find BC.



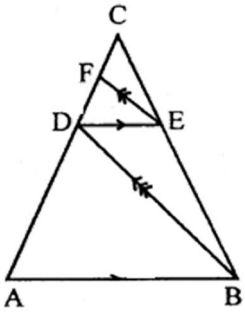
2. In ΔABC, D and E are the point on the side AB and AC respectively such that DE||BD. If AD = 6x – 7, DB = 4x – 3, AE = 3x – 3 and EC = 2x – 1, then find the value of x.

3. In the given figure, find 'x' in terms of p, q and r.



4. If one diagonal of a trapezium divides the other diagonal in the ratio 1:3. Prove that one of the parallel sides is three times the other.

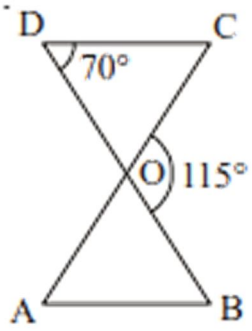
5. In the given figure ABC, AB||DE and BD||EF, prove that $DC^2 = CF \times AC$.



6. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

7. In ΔABC, AB = AC and D is a point on side AC such that $BC^2 = AC.CD$. Prove that BD = BC.

8. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115^0$ and $\angle CDO = 70^0$. Find



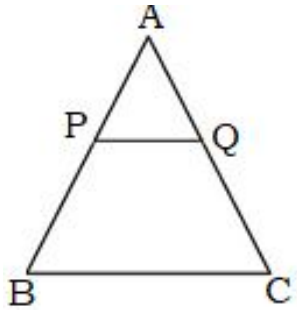
(i) $\angle DOC$ (ii) $\angle DCO$ (iii) $\angle OAB$ (iv) $\angle OBA$

9. It is given that $\Delta FED \sim \Delta STU$. Is it true to say that $\frac{DE}{ST} = \frac{EF}{TU}$.

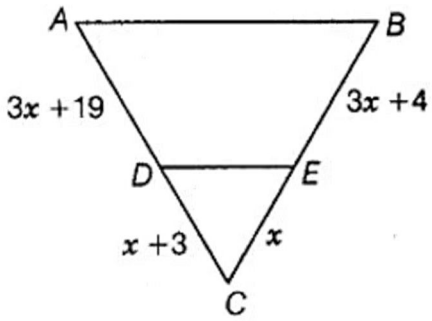
10. Is the following statement true? Why?
"Two quadrilaterals are similar, if their corresponding angles are equal"

3-MARKS

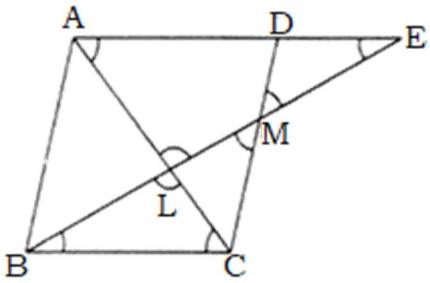
1. P & Q are points on sides AB and AC respectively of ΔABC. If AP = 3 cm, PB = 6cm, AQ = 5cm and QC = 10cm, show that BC = 3PQ.



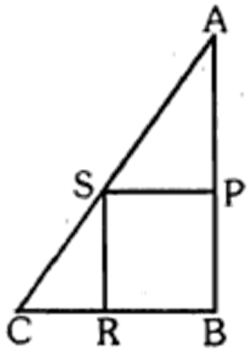
2. Find the value of x for which $DE \parallel AB$ in the given figure.



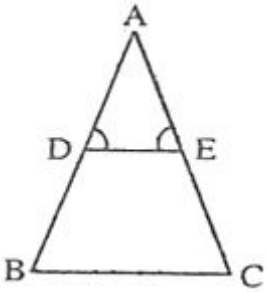
3. In the given figure, M is the mid-point of the side CD of parallelogram ABCD. The line segment BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL and BL = 2ML.



4. Legs (sides other than the hypotenuse) of a right triangle are of lengths 16cm and 8cm. Find the length of the side of the largest square that can be inscribed in the triangle.

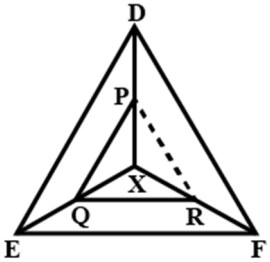


5. In figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that BAC is an isosceles triangle.

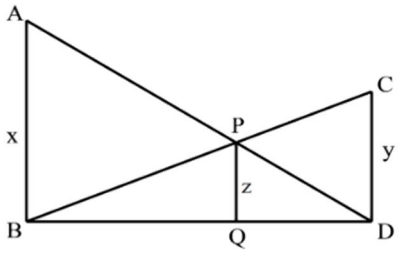


6. From airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h, then find the distance between two aeroplanes after 2 hours.

7. X is any point inside ΔDEF and is joined to all three vertices. From point P on DX, PQ||DE is drawn which meets EX at Q and from Q, QR||EF is drawn which meets XF at R. Prove that PR||DF.

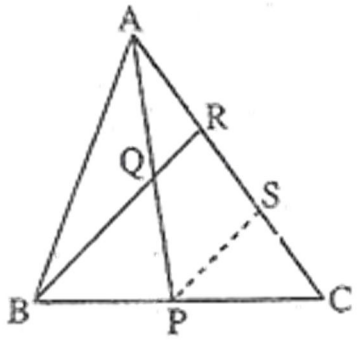


8. In the figure above, $AB \parallel PQ \parallel CD$, AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.

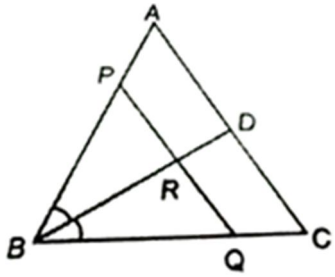


5-MARKS

1. In the figure, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that $RA=\frac{1}{3}CA$.

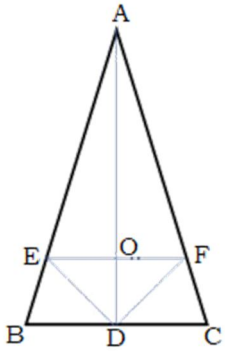


2. In ΔABC, the bisector of $\angle B$ meets AC at D. A line PQ||AC meets AB, BC and BD at P, Q and R respectively. Show that

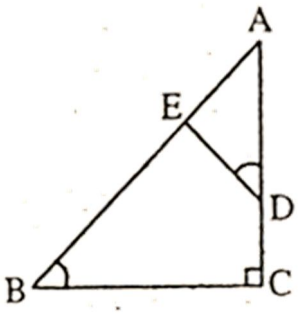


i) PR x BQ = QR x BP ii) AB × CQ = BC × AP

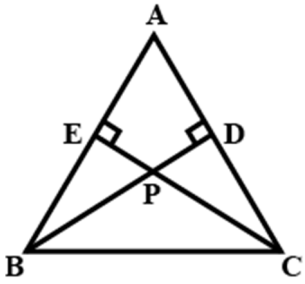
3. AD is a median of ΔABC. The bisector of $\angle ADB$ and $\angle ADC$ meet AB and AC in E and F respectively. Prove that EF||BC.



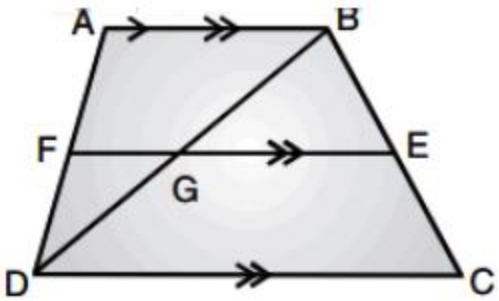
4. In the figure, if $\angle ADE = \angle B$ show that $\Delta ADE \sim \Delta ABC$. If AD = 3.8 cm, AE = 3.6 cm, BE = 2.1 cm and BC = 4.2 cm, find DE.



5. In the figure, considering triangles BEP and CPD, prove that BP × PD = EP × PC.



6. In trapezium ABCD, $AB \parallel DC$, DC=2AB. $EF \parallel AB$ where E & F lie on BC & AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Also diagonal DB intersects EF at G. Show that $7EF = 11AB$



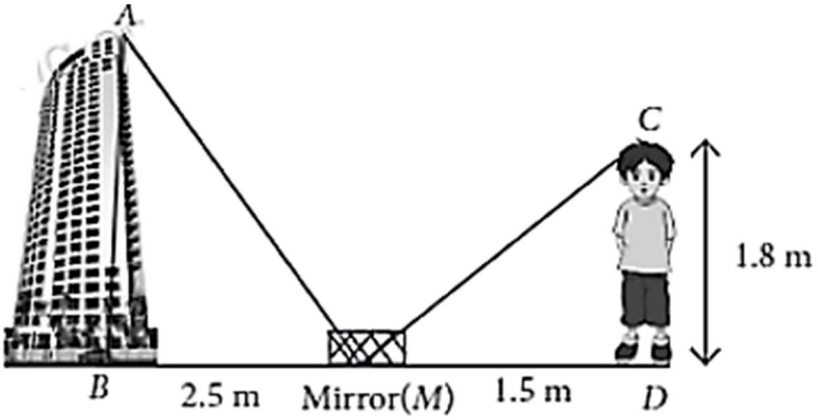
7. A street light bulb is fixed on a pole 6m above the level of the street. If a woman of height 1.5 cm casts shadow of 3 cm, find the base of the pole.

8. A vertical row of trees 12 m long casts a shadow 8 m long on the ground, at the same time a tower casts the shadow 40m long on the ground.
   i) Determine the height of the tower.
   ii) Which mathematical concept is used in this problem?

CASE STUDY QUESTIONS

CASE STUDY_1

Sujith's father is a mathematician. One day he gave Sujith an activity to measure the height of building. Rohit accepted the challenge and placed a mirror on ground level to determine the height of the building. He is standing at a certain distance so that he can see the top of the building reflected from mirror. Sujith eye level is at 1.8 m above the ground. The distance of Sujith from mirror and that of building from mirror are 1.5m and 2.5m respectively.



Based on the above information, answer the following questions:

(i) What are the two similar triangles formed in the given figure? 1

(ii) What is the height of the building? 1

(iii) In ΔABM, if $\angle BAM = 30^0$ then find the value of $\angle MCD$. 2

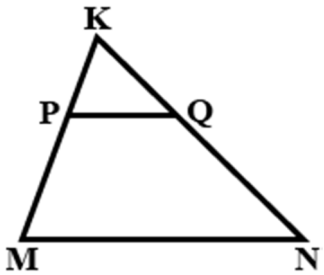
OR

If ΔABM *and* ΔCDM are similar where CD = 6cm, MD = 8cm, and BM = 24 cm then find the value of AB. 2

LEVEL_3

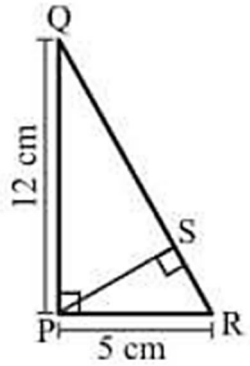
MCQ:

1. In the given below figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ then the value of KQ.



(a) 2.4 cm (b) 4.8 cm (c) 3.6 cm (d) 1.2 cm

2. In the figure below, PQR is a right-angled triangle, right angled at P. A perpendicular line PS is drawn from P to QR. PR = 5 cm and PQ = 12 cm.

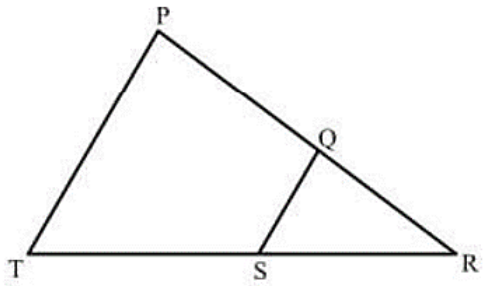


(Note: The figure is not to scale.)

What is RS : SQ?

(a) 5:12 (b) 13:17 (c) 13:60 (d) 25:144

3. In the following figure, Q is a point on PR and S is a point on TR. QS is drawn and ∠RPT = ∠RQS.

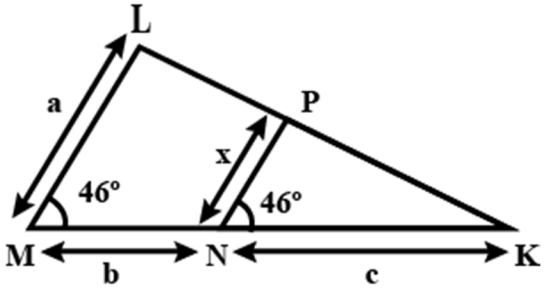


Which of these criteria can be used to prove that ΔRSQ is similar to ΔRTP?

(a) AAA similarity criterion (b) SAS similarity criterion

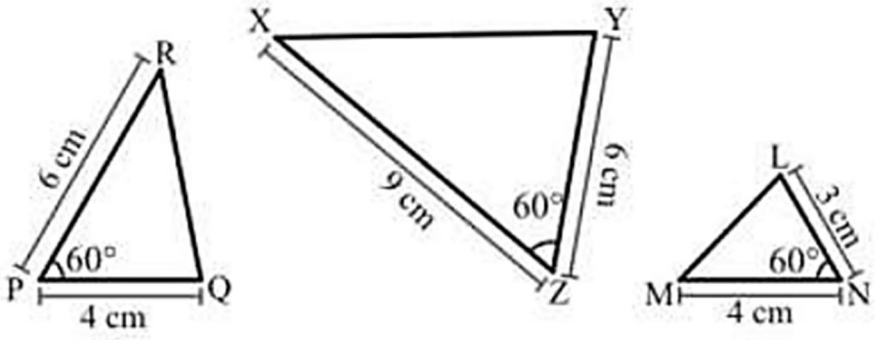
(c) SSS similarity criterion (d) RHS similarity criterion

4. What is the value of x?



(a) $\frac{ac}{b+c}$ (b) $\frac{b+c}{ac}$ (c) $\frac{ab}{a+c}$ (d) $\frac{bc}{a+c}$

5. Shown below are three triangles. The measures of two adjacent sides and included angle are given for each triangle.



(Note: The figure is not to scale.)

Which of these triangles are similar?

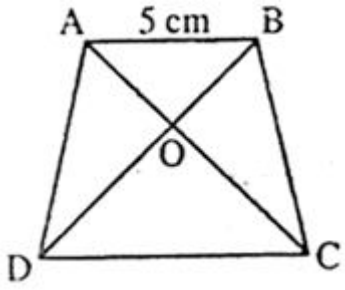
(a) ΔRPQ *and* ΔXZY (b) ΔRPQ *and* ΔMNL

(c) ΔXYZ *and* ΔMNL

(d) $\Delta RPQ, \Delta XYZ$ *and* ΔMNL are similar to one another.

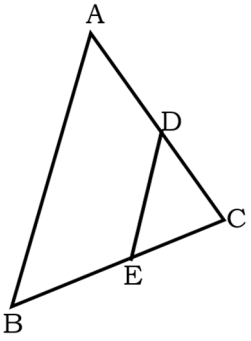
2-MARKS

1. In the figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5cm. Find the value of DC.

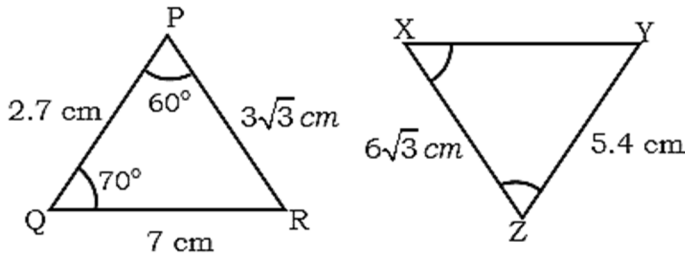


2. In ΔABC, $\angle B = 2\angle C$ and the bisector of $\angle B$ intersects AC of D. Prove that $\frac{BD}{DA} = \frac{BC}{BA}$.

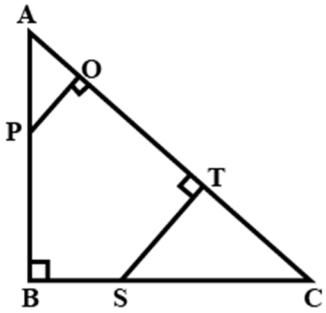
3. In the given figure, ABC is a triangle in which $\angle A = \angle B$ and AD=BE. Prove that DE||AB.



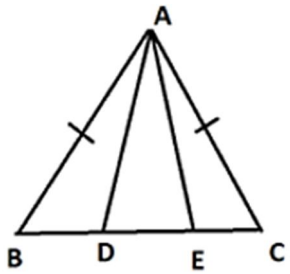
4. In the given figures, find the measure of $\angle X$, if $\Delta PQR \sim \Delta ZYX$.



5. In the given figure, ΔABC is isosceles right-angled triangle with such that $\angle B = 90^0$, $PQ \perp AC$ and $ST \perp AC$ where P lies on AB and S lies on BC. Prove that $\Delta AQP \sim \Delta CTS$.

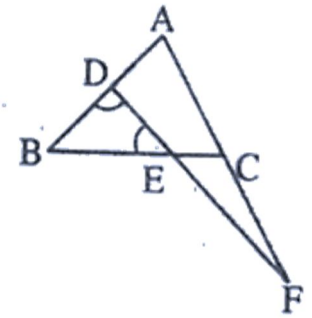


6. In an isosceles triangle ABC with AB = AC, D and E are the points on BC such that BE = CD. Find the value of $\frac{AD}{AE}$.



7. In triangle ABC, if AB = AC and D is a point on side AB such that $BC^2 = AC \times CD$ then, prove that BD = BC.

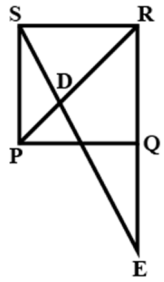
8. In the figure $\angle BED = \angle BDE$ and E is the middle point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



9. In ΔABC, if X and Y are the points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, AY = 5 and YC = 9, then state whether XY and BC are parallel or not.

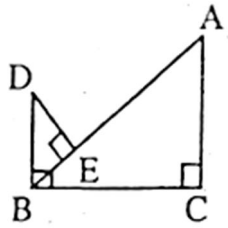
3-MARKS

1. In the given figure PQRS is a rhombus such that RQ is produced to E and SE is joined. SE and PR intersect at D. Prove that SD.RE = SR.DE

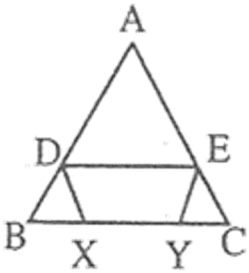


2. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that AB × BC = AE × CF.

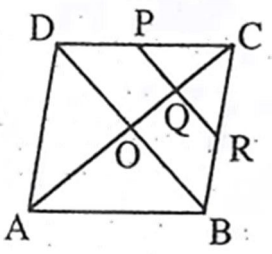
3. In the given figure, $DB \perp BC$; $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



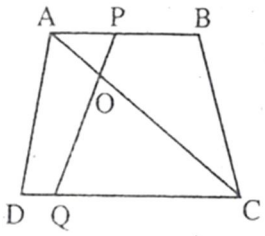
4. In given figure ΔABC, X and Y are two points lying on the side BC such that BX = CY. If DX||AC and YE||AB, then prove that DE||BC.



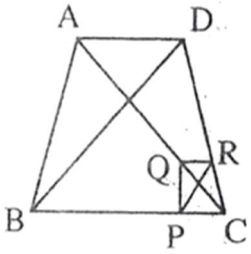
5. In given figure ABCD is a parallelogram such that diagonals AC, BD intersect at O. If P is mid-point of CD and $CQ = \frac{1}{4}AC$. Prove that R is mid-point of BC.



6. If base of a right-angled triangle is twice the base of another right-angled triangle and height of the first right angled triangle is one-fourth of the other's height, then find the ratio of areas of first triangle to second triangle.
7. In figure if AB||DC and AC and PQ intersect each other at the point O, prove that OA.CQ=OC.AP.

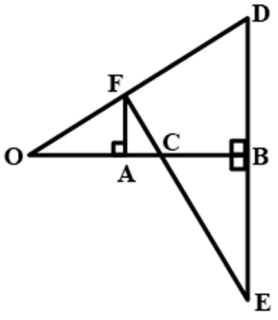


8. In the given figure, two triangles ABC and DBC lie on same side of BC. P is a point on BC such PQ || BA and PR || BD. Prove that QR || AD.

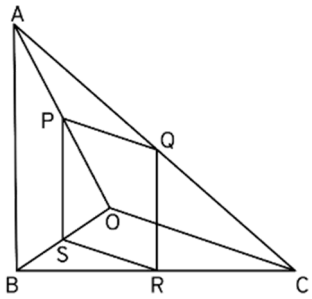


5-MARKS

1. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
2. Prove that the external bisector of an angle of a triangle divides the opposite side externally in the ratio of the side containing the angle.
3. If two scalene triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding angle bisector segments.
4. In figure, OB is the perpendicular bisector of the line segment DE, $FA \perp OB$ on FE intersects OB at the point C. Prove that $\frac{1}{OA}+\frac{1}{OB}=\frac{2}{OC}$.

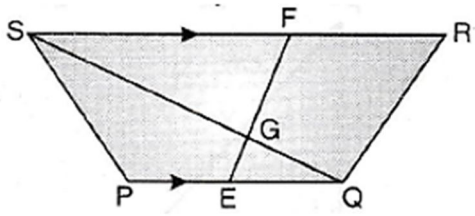


5. In figure, if PQRS is a parallelogram and AB||PS, then prove that OC||SR.

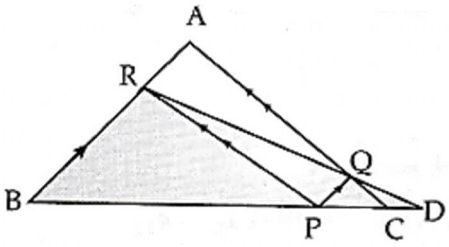


6. A 5m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4m high. If the foot of the ladder is moved 1.6m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

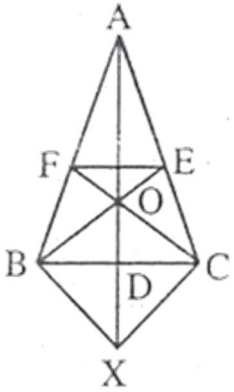
7. In the figure PQRS is a trapezium in which PQ||RS. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$



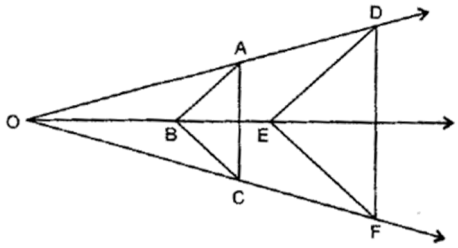
8. In the figure, P is any point on side BC of ΔABC, PQ||BA and PR||CA are drawn. RQ is extended to meet BC produced at D. Prove that DP^2 = DB.DC.



9. In ΔABC, AD is a median and O is any point on AD. OB and OC on producing meet AC and AB at E and F respectively. Now, AD is produced to X such that OD = DX as shown in figure. Prove that: i) EF||BC ii) AO : AX = AF : AB



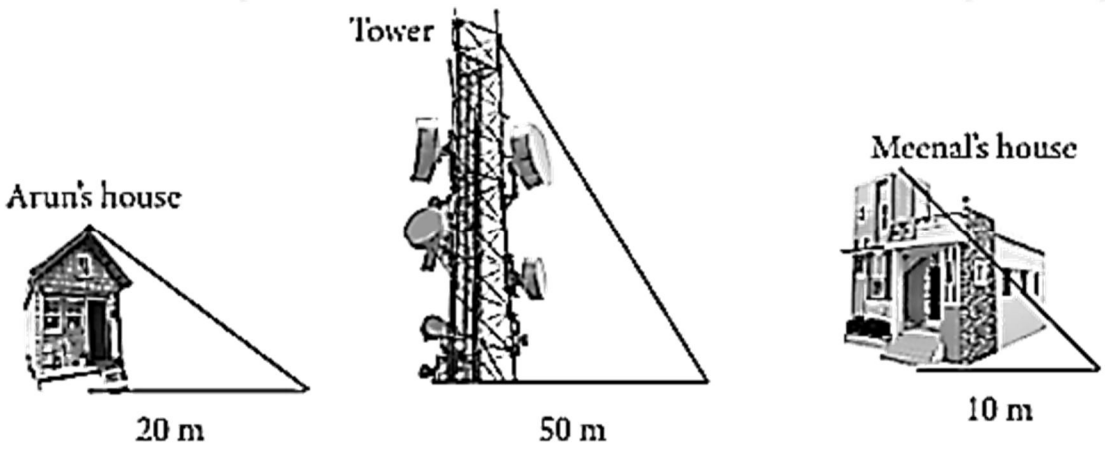
10. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. Using the above, prove the following. In the figure, AB||DE and BC||EF. Prove that AC||DF.



CASE STUDY QUESTIONS

CASE STUDY_1

Meenal was trying to find the height of the tower near his house. She is using the properties of similar triangles. The height of Meenal's house is 20m. When Meenal's house casts a shadow is 10m long the ground, at the same time, tower casts a shadow of 50m long and Arun's house casts a shadow of 20m long on the ground as shown below.



Based on the above information, answer the following questions:

(i) What is the height of the tower? 1

(ii) What will be the length of the shadow of tower when Meenal's house caste a shadow 15m? 1

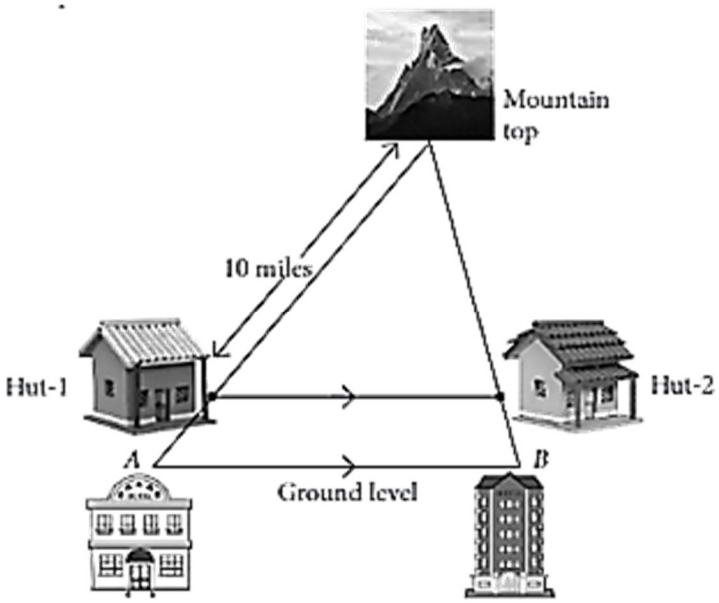
(iii) If the tower casts a shadow of 40m, then find the length of the shadow of Arun's house? 2

OR

If the tower casts a shadow of 40m, then what will be the length of shadow of Meenal's house? 2

CASE STUDY_2

Two hotels are at the ground level on the either sides of a mountain. On moving a certain distance towards the top of the mountain two huts are situated as shown in the figure. The ration between the distance from hotel B to hut-2 and that of hut-2 to mountain top is 3 : 7.



Based on the above information, answer the following questions:

(i) What is the ratio of perimeters of the triangle formed by both hotels and mountain top to the triangle formed by both huts and mountain top? 1

(ii) What is the distance between hotel A and hut-1? 1

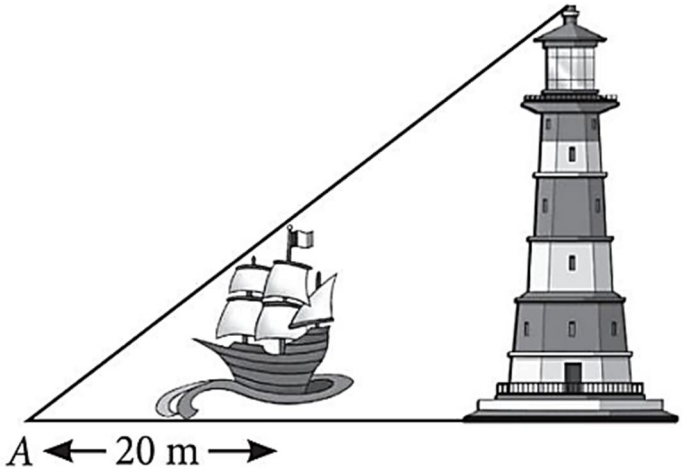
(iii) If the horizontal distance between the hut-1 and hut-2 is 8 miles, then what is the distance between the two hotels? 2

OR

If the distance from mountain top to hut-1 is 5 miles more than that of distance from hotel B to mountain top, then what is the distance between hut-2 and mountain top?

CASE STUDY_3

Shweta went to a beach with her uncle. From a point A where Shweta was standing, a ship and light house come in a straight line as shown in the figure.

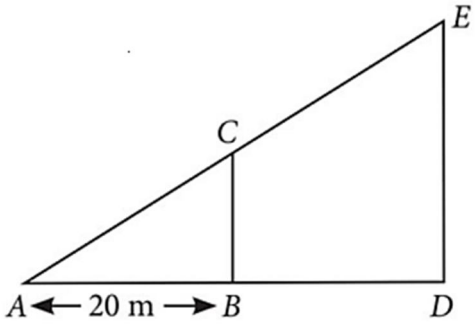


Based on the above information, answer the following questions:

(i) Which similarity criteria can be seen in this case, if ship and lighthouse are considered as straight lines? 1

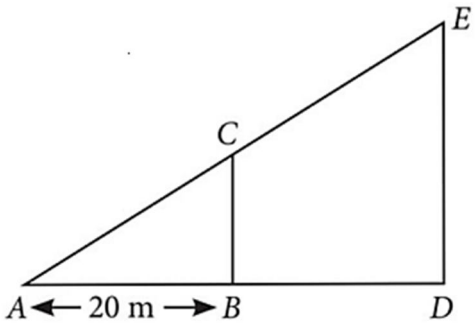
(ii) The distance between Shweta and Ship is twice as much as the height of the ship. What is the height of the ship? 1

(iii) If the distance of Shweta from the lighthouse is twelve times the height of the ship, then what is the ratio of heights of the ship and lighthouse? 2



OR

What is the ratio of the distance between Shweta and top of ship to the distance between the tops of the ship and lighthouse? 2



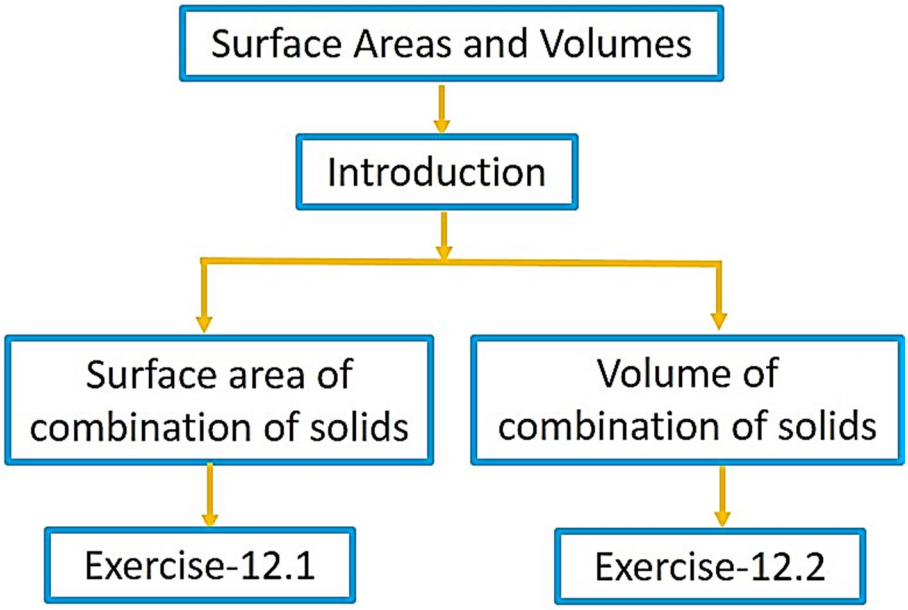
SKILL BASED QUESTIONS

1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

2. If a line divides any two sides of a triangle in the same ratio, then prove that the line is parallel to the third side.

SURFACE AREAS AND VOLUMES

MIND MAPPING

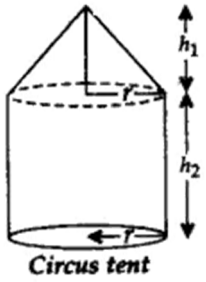


Basic Facts and Formulae:

Shape	Parameters	Surface Area (Sq.units)	Volume (Cu.units)
Cuboid	Length = l, Breadth = b Height = h	$TSA = 2(lb + bh + lh)$ $LSA = 2h(l + b)$	$V = l \times b \times h$
Cube	Length = Breadth = Height = l	$TSA = 6l^2$ $LSA = 4l^2$	$V = l^3$
Cylinder	Radius = r Height = h	$CSA = 2\pi rh$ $TSA = 2\pi r(h + r)$	$V = \pi r^2 h$
Cone	Radius = r height = h Slant Height = l	$CSA = \pi rl$ $TSA = \pi r(l + r)$	$V = \frac{1}{3}\pi r^2 h$
Sphere	Radius = r	$CSA = TSA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Hemisphere	Radius = r	$CSA = 2\pi r^2$, $TSA = 3\pi r^2$	$V = \frac{2}{3}\pi r^3$

Surface Area and Volume of Combination of solids:

(i) **Cone on a Cylinder:**



r = radius of cone & cylinder;

h_1 = height of cone

h_2 = height of cylinder

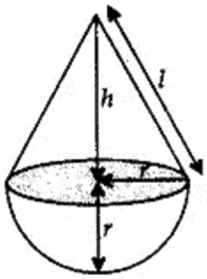
(a) Total Surface area = CSA of cone + CSA of cylinder + area of circular base

$$= \pi rl + 2\pi rh_2 + \pi r^2$$

(b) Slant height, $l = \sqrt{r^2 + h^2}$

Total Volume = Volume of cone + Volume of cylinder = $\frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$.

(ii) **Cone on a Hemisphere:**

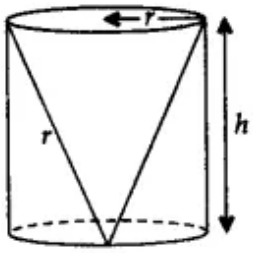


h = height of cone, l = slant height $= \sqrt{r^2 + h^2}$, r = radius of cone and hemisphere

(a) TSA = CSA of cone + CSA of hemisphere = $\pi rl + 2\pi r^2$

(b) Volume = Volume of cone + Volume of hemisphere = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

(iii) **Conical Cavity in a Cylinder:**



r = radius of cone and cylinder, h = height of cylinder and conical cavity

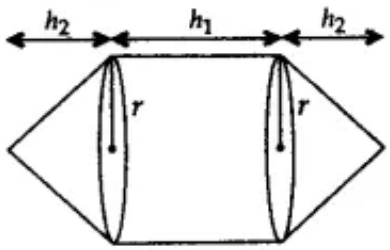
l = Slant height

TSA = CSA of cylinder + Area of bottom face of cylinder + CSA of cone

$= 2\pi rh + \pi r^2 + \pi rl$

Volume = Volume of cylinder – Volume of cone = $\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

(iv) **Cones on Either Side of Cylinder:**



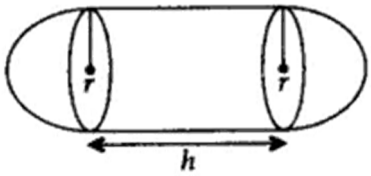
r = radius of cylinder and cone, h_1 = height of cylinder, h_2 = height of cones

Slant height of cone, $l = \sqrt{r^2 + h^2}$

(a) Surface area = CSA of 2 cones + CSA of cylinder = $2\pi rl + 2\pi rh_1$

(b) Volume = 2(Volume of cone) + Volume of cylinder = $\frac{2}{3}\pi r^2 h_2 + \pi r^2 h_1$

(v) **Cylinder with Hemispherical Ends:**

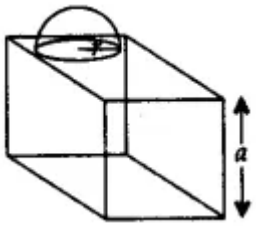


r = radius of cylinder and hemispherical ends;
h = height of cylinder

(a) TSA = CSA of cylinder + CSA of 2 hemispheres = $2\pi rh + 4\pi r^2$

(b) Volume = Volume of cylinder + Volume of 2 hemispheres = $\pi r^2 h + \frac{4}{3}\pi r^3$

(vi) **Hemisphere on Cube:**



a = side of cube

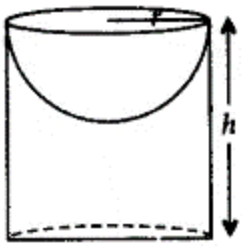
r = radius of hemisphere.

(a) Surface area = SA of cube – Area of hemisphere face + CSA of hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

(b) Volume = Volume of cube + Volume of hemisphere = $a^3 + \frac{2}{3}\pi r^3$

(vii) **Hemispherical Cavity in a Cylinder:**



r = radius of hemisphere, h = height of cylinder

(a) TSA = CSA of cylinder + SA of base + CSA of hemisphere

$= = 2\pi rh + \pi r^2 + 2\pi r^2 = 2\pi rh + 3\pi r^2$

(b) Volume = Volume of cylinder – Volume of hemisphere = $\pi r^2 h - \frac{2}{3}\pi r^3$

LEVEL_1

MCQ:

1. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

   (a) $47\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

2. A right circular cylinder of radius r cm and height h cm (where, h>2r) just encloses a sphere of diameter is:

   (a) r cm (b) 2r cm (c) h cm (d) 2h cm

3. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3 then, the ratio of their volumes is:

   (a) 22 : 23 (b) 16 : 19 (c) 25 : 29 (d) 20 : 27

4. If volumes of two spheres are in the ratio of 64 : 27, then the ratio of their surface areas is:

   (a) 4 : 3 (b) 16 : 9 (c) 64 : 27 (d) 3 : 4

5. If the surface areas of two spheres are in the ratio of 16 : 9, then the ratio of their volumes.

   (a) 4 : 3 (b) 16 : 9 (c) 64 : 27 (d) 3 : 4

6. A cylindrical pencil sharpened at one edge is the combination of:

   (a) a cone and a cylinder (b) frustum of a cone and a cylinder

   (c) a hemisphere and a cylinder (d) two cylinders

7. The radius of the hemispherical bowl, given that the circumference of its edge is 132 cm (Use $\pi = \frac{22}{7}$) is:

   (a) 14 cm (b) 21 cm (c) 7 cm (d) 28 cm

8. Ratio of volumes of two cylinders with equal heights is:

   (a) H : h (b) R : r (c) R^2: r^2 (d) None of these

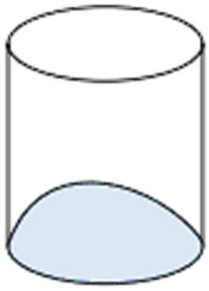
9. Ratio of lateral surface areas of two cylinders with equal heights is:

   (a) 1 : 2 (b) H : h (c) R : r (d) None of these

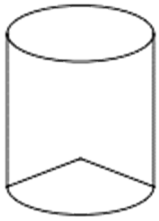
2-MARKS

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
2. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes is 1 : 2 : 3.
3. Find the volume of a right circular cylinder of height 21 cm and base radius 5 cm.
4. What will be the approximate volume of the largest circular cone that can be cut out from a cube of edge 4.2 cm?
5. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.
6. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?
7. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice. Aisha, a juice seller, was serving juice to her customers in two types of glasses. Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.

   First Type: A glass with hemispherical raised bottom.



Second Type: A glass with conical raised bottom of height 1.5 cm.



Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much?

8. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

9. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

10. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm3. Find the height of the cylinder. (Use $\pi = \frac{22}{7}$).

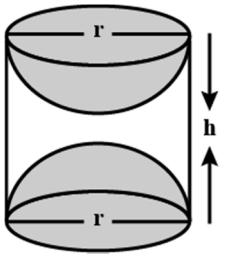
11. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of spheres respectively, then find the value of $(R-r):r$.

12. If a rectangular sheet of paper 40 cm x 22 cm is rolled to form a hollow cylinder of height 40 cm, then find the radius of the cylinder (in cm).

3-MARKS

1. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

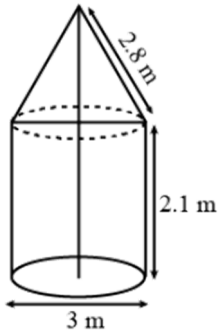
2. A wooden article is made by scooping out a hemisphere from each end of a solid cylinder as shown in the fig. If the height of the cylinder is 10cm and its base radius is 3.5cm. Find the total surface area of the article.



3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

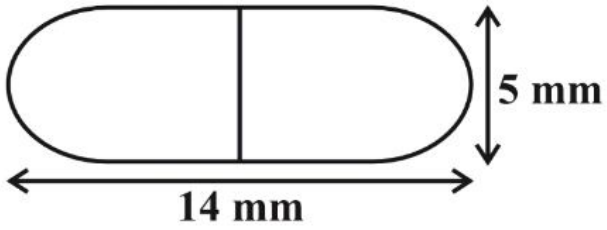
4. From a solid right circular cylinder of height 2.4cm and radius 0.7cm, a right circular cone of same height and same radius is cut out. Find the total surface area of the remaining solid.

5. In the figure, a tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1m and 3m respectively and slant height of the conical part is 2.8m, then find the cost of canvas needed to make the tent, if the canvas is available at the rate of 500 m^2. (Use $\pi = \frac{22}{7}$).



6. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

7. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see below fig.). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



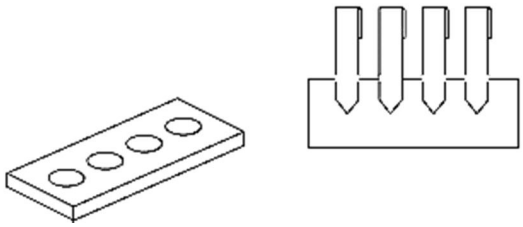
8. A sphere of diameter 12cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}cm$. Find the diameter of the cylindrical vessel.

9. The radii of two right circular cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 4. Calculate the ratio of their curved surface area and ratio of their volumes.

10. A rectangular sheet of paper of dimensions 44cm x 18cm is rolled along its length and a cylinder is formed. Find the volume of the cylinder so formed (Use $\pi = \frac{22}{7}$).

5-MARKS

1. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The diameter of each of the depressions is 1 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

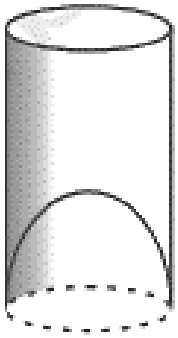


2. A gulab jamun, contains sugar syrup up to about 30% of its volume.

(i) Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

(ii) Which mathematical concept is used in the above problem?

3. A juice seller was serving his customers using glasses as shown in Fig. given below. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use $\pi = 3.14$).



4. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm, then

(i) If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

(ii) Which mathematical concept is used in the above problem?

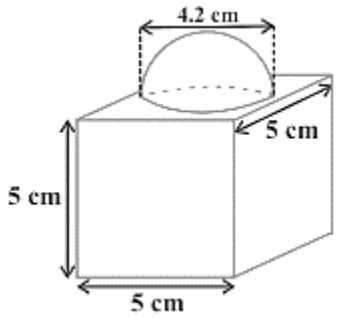
5. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

6. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem? (Use $\pi = \frac{22}{7}$).

7. The decorative block shown in the given below fig. is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find

(i) The total surface area of the block.

(ii) The volume of the block formed. (Take $\pi = \frac{22}{7}$).



CASE STUDY QUESTIONS

CASE STUDY_1

Mathematics teacher of a school took her 10th standard students to show Red fort. It was a part of their Educational trip. The teacher had interest in history as well. She narrated the facts of Red fort to students. Then the teacher said in this monument one can find combination of solid figures. There are 2 pillars which are cylindrical in shape. Also 2 domes at the corners which are hemispherical.7 smaller domes at the centre. Flag hoisting ceremony on Independence Day takes place near these domes.

Based on the above information answer the following questions:

(i) Write the formula to find the volume of a cylindrical pillar. 1

(ii) How much cloth material will be required to cover 2 big domes each of radius 2.5 metres (take $\pi = \frac{22}{7}$)? 2

OR

What is the ratio of sum of volumes of two hemispheres of radius 2 cm each to the volume of a sphere of radius 4 cm? 2

(iii) Find the lateral surface area of two pillars if height of the pillar is 7m and radius of the base is 1.4m. 1

CASE STUDY_2

Advertising columns are cylindrical outdoor sidewalk structures with a characteristic style that are used for advertising and other purposes. They are common throughout Germany including its capital Berlin, where the first 100 columns were installed in 1855. Advertising columns are typically used to display advertisements in the form of posters, traditionally chiefly theatre, and cinema, nightclub, and concert announcements. Some are motorized and rotate very slowly.

Rajesh has been given the task of designing a advertising column for a client. It consist of a cylindrical part surmounted by hemisphere part on top. The base diameter of column is 7 feet and height of cylindrical part is 11 feet.

Based on the above information answer the following questions:

(i) Find the surface area of cylindrical part of column? 1

(ii) Find the total surface area of advertising column? 2

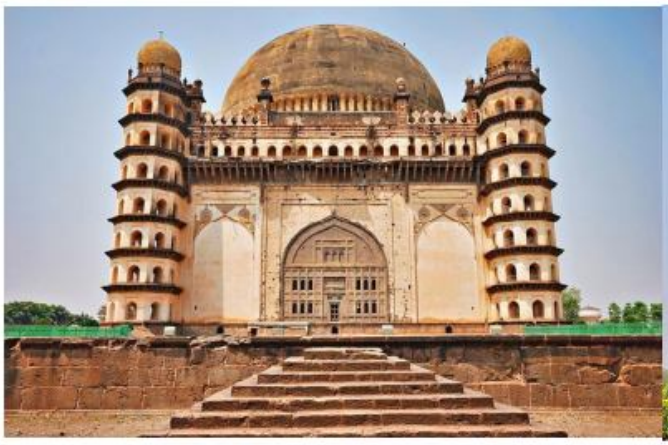
OR

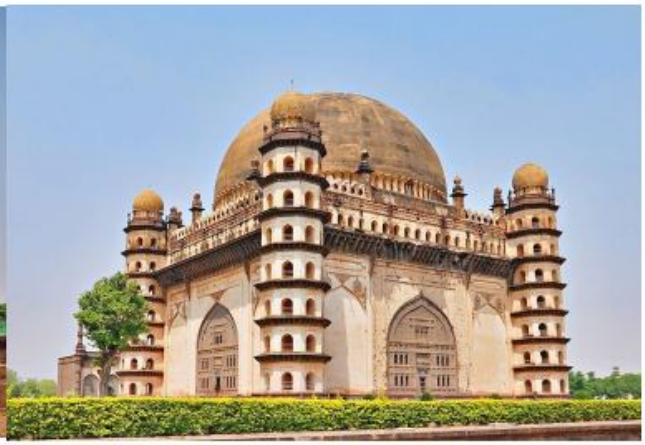
Find the volume of advertising column? 2

(iii) If cost of construction of column is Rs 75 per sq feet, find total cost of column. 1

CASE STUDY_3

Mathematics teacher of a school took her 10th standard students to show Gol Gumbaz. It was a part of their Educational trip. The teacher had interest in history as well. She narrated the facts of Gul Gumbaz to students. Gol Gumbaz is the tomb of king Muhammad Adil Shah, Adil Shah Dynasty. Construction of the tomb, located in Vijayapura Karnataka, India, was started in 1626 and completed in 1656. Then the teacher said in this monument one can find combination of solid figures. She pointed that there are cubical bases and hemispherical dome is at the top.





Based on the above information answer the following questions:

(i) What is the diagonal of the cubic portion of the Gol Gumbaz, if one side of the cubical portion is 23m? 1

(ii) A block of Gol Gumbaz is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the shape is 2 cm. Find the volume of the block. (Use $\pi = 3.14$) 2

OR

A block of Gol Gumbaz is in the shape of a cylinder of diameter 1 cm with two hemispheres stuck to each of its ends. The length of the shape is 4 cm. Find the Total surface area of the block. (Use $\pi = 3.14$) 2

(iii) If two solid hemispheres of same base radii r, are joined together along their bases, then find the curved surface area of this new solid. 1

LEVEL_2

MCQ:

1. If the radius of the sphere is increased by 100%, the volume of corresponding sphere is increased by:

   (a) 200% (b) 500% (c) 700% (d) 800%

2. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then the volume of remaining solid is

   (a) 280 π cm^3 (b) 330 π cm^3 (c) 240 π cm^3 (d) 440 π cm^3

3. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes (taken in same order as given in the question).

   (a) 3 : 1 : 2 (b) 2 : 1 : 3 (c) 1 : 1 : 1 (d) 1 : 2 : 3

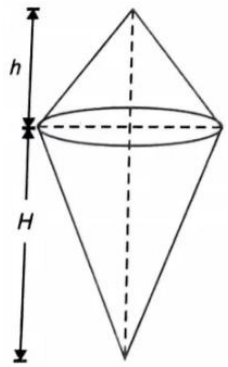
4. The volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

   (a) 1.9404 cm^3 (b) 19.404 cm^3 (c) 194.04 cm^3 (d) 1940.4 cm^3

5. If the radius of the base of a right circular cylinder is halved, keeping the height same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder.

   (a) 1:2 (b) 2:1 (c) 1:4 (d) 4:1

6. A solid metallic object is shaped like a double cone as shown in the figure. Radius of the base of both cones is same but their heights are different. If this cone is immersed in water, then the quantity of water it will be displaced is:



(a) $\frac{\pi}{3}r^2(h_1+h_2)$ (b) $2\pi r(h_1+h_2)$ (c) $\pi r^2(h_1+h_2)$ (d) All of these

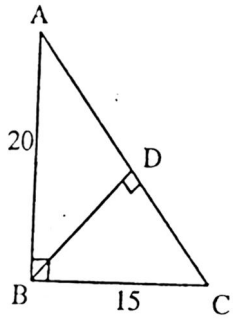
2-MARKS

1. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid
2. Two cubes of 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.
3. If total surface area of the solid hemisphere is 462 cm^2, then find its volume (Use $\pi = \frac{22}{7}$).
4. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.
5. A right triangle with sides 3cm, 4cm and 5cm is rotated about the side of 3cm to form a cone. Find the volume of the cube so formed.
6. Find the volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm.
7. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r.
8. If four times the sum of areas of two faces of cylinder of height 8cm is equal to twice the curved surface area, then find the diameter of the cylinder.
9. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r\left[\sqrt{r^2+h^2}+r+2h\right]$. Justify your answer.

10. Two identical solid cubes of side 'a' are joined end to end. Then the total surface area of the resulting cuboid is $12a^2$. Justify your answer.

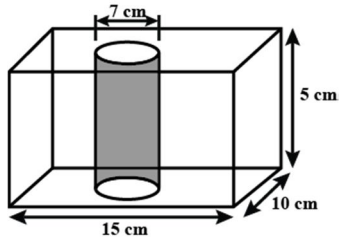
3-MARKS

1. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm^3. (Use $\pi = \frac{22}{7}$).
2. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then find the volume of remaining solid?
3. Find the volume of the largest right circular cone that can be cut from a cube whose edge is 14cm. (Use $\pi = \frac{22}{7}$).
4. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder. The diameter of the hemisphere is 14cm and the total height of the vessel is 13cm. Find the capacity of the vessel.
5. A toy is in the form of a cone mounted on a hemisphere of common base radius 7cm. The total height of toy is 30cm. Find the total volume of toy (Use $\pi = \frac{22}{7}$).
6. The cost of painting the total outer surface area of a closed cylindrical oil tank at 60 paise per cm^2 is Rs. 237.60. The height of the tank is 6 times the radius of the base of the tank. Find the height and the radius of the tank. (Use $\pi = \frac{22}{7}$).
7. The circumference of the base of the conical tent is 44m. If the height of the tent is 24m, then find the length of the canvas used in making the tent of the width of the canvas is 2m.
8. A right triangle whose sides are 15cm and 20 cm is made to revolve about its hypotenuse. Find volume and surface area of double cone so formed.(Use $\pi = 3.14$).



5-MARKS

1. From the cuboidal solid metallic block of dimensions 15cm x 10cm x 5cm, a cylindrical hole of diameter 7cm is drilled out. Find the surface area of the remaining block. (Use $\pi = \frac{22}{7}$)



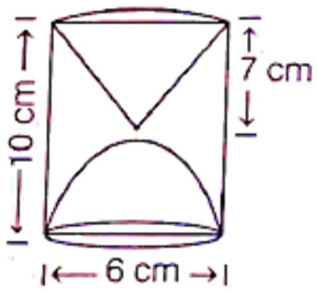
2. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy if a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Take $\pi = 3.14$).

3. A solid right circular cylinder has a total surface area of 462 cm^2. Its curved surface area is one third of its total surface area. Find the volume of cylinder. (Use $\pi = \frac{22}{7}$)

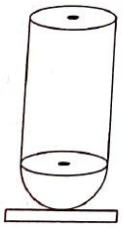
4. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 100cm and the diameter of the hemispherical ends is 42cm, then find the cost of polishing the surface of the solid at the rate of 5 paise per cm^2.

5. Due to sudden floods, some welfare associations jointly requested the govt. to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2m and height 4 m with the conical upper part of same diameter but of height 2.8 m and the canvas to be used cost Rs.100 per m2. Find the amount, the associations will have to pay. What values are shown by these associations? (Use $\pi = \frac{22}{7}$)

6. A wooden article as shown in the figure was made from a cylinder by scooping out a hemisphere from one end and a cone from other end. Find the total surface area of the article.



7. A trophy awarded to the best student in the class is in the form of a solid cylinder mounted on a solid hemisphere with the same radius and is made from some metal. The trophy is mounted on a wooden cuboid as shown in the figure. The diameter of the hemisphere is 21cm and the total height of the trophy is 24.5 cm. Find the weight of the metal used in making the trophy, if the weight of 1 cm3 of metal is 1.2g. (Take $\pi = \frac{22}{7}$).

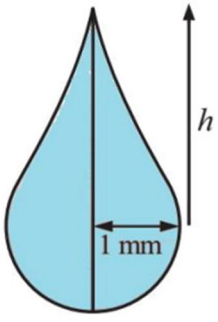


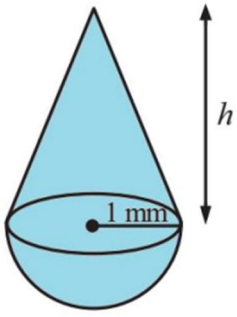
8. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm is surmounted by another cylinder of height 60cm and radius 8cm. Find the mass of the pole, given that $1cm^3$ of iron has approximately 8 g mass (Use $\pi = 3.14$).

CASE STUDY QUESTIONS

CASE STUDY_1

In the month of December 2020, it rained heavily throughout the day over the city of Hyderabad. Anil observed the raindrops as they reached him. Each raindrop was in the shape of a hemisphere surmounted by a cone of the same radius of 1 mm. Volume of one of such drops is 3.14 mm^3. Anil collected the rain water in a pot having a capacity of 1099 cm^3. [Use $\sqrt{2} = 1.4$]





Based on the above information, answer the following questions:

(i) Find the total height of the drop. 1

(ii) What is the total surface area of a hemisphere of radius r? 1

(iii) How many drops will be collected in the pot? 2

OR

Find the C.S.A of the drop. 2

LEVEL_3

MCQ:

1. A vessel having 30 m^3 of water is emptied through two openings, one small and the other large. Water flows out through the smaller opening at rate of U m^3/h and through the larger one at the rate of V m^3/h. Given that 3U + 2V = 70 and that the vessel gets fully emptied in 1 hour, what is V?

   (a) 10 m^3/h (b) 20 m^3/h (c) 30 m^3/h (d) 50 m^3/h

2. Figure-1 below is a solid cuboid made of unit cubes. Figure-2 is obtained after removing some unit cubes from figure-1.

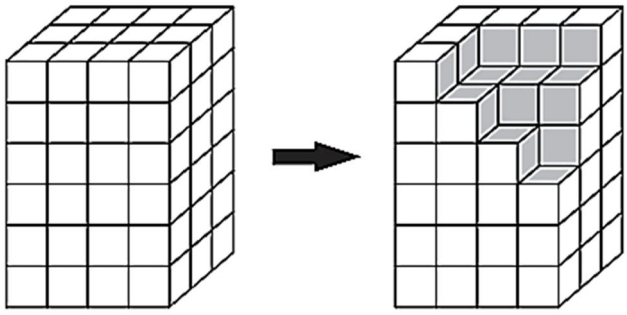


Figure 1 Figure 2

(Note: The figures are not to scale.)

Based on the figures shown above, the surface area of the cuboid in figure-1 is _____ the surface area of the solid in the figure-2.

(a) Less than (b) More than (c) Equal to

(d) Cannot be concluded with the given information.

3. A cylinder whose height is two third of its diameter has the same volume of sphere of radius 4cm. Calculate the radius of base of cylinder.

   (a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

4. A container with a grey hemispherical lid has radius R cm. In figure-1, it contains water upto a height of R cm. It is then inverted as shown in the figure-2.

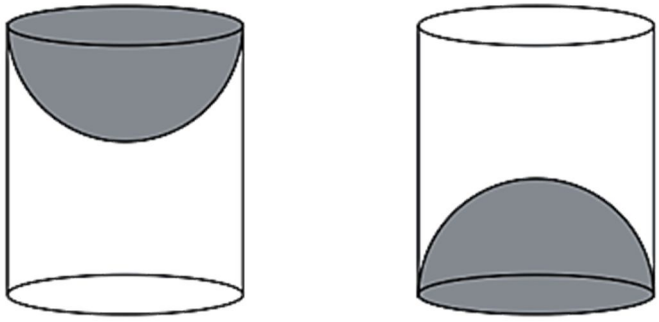


Figure 1 Figure 2

What is the height of the water in figure-2?

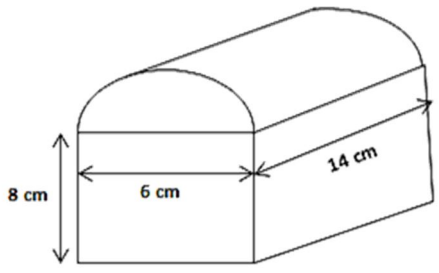
(a) R cm (b) $\frac{5R}{3}cm$ (c) 2R cm (d) $\frac{7R}{3}cm$

5. If the volume of a hemisphere is $2425\frac{1}{2}cm^3$, then its Curved Surface Area is:

(a) 639 cm^2 (b) 693 cm^2 (c) 369 cm^2 (d) 963 cm^2

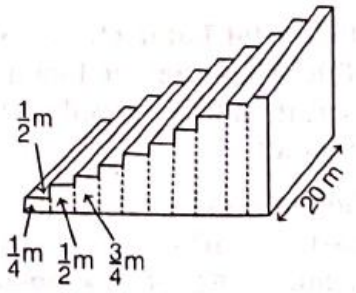
2-MARKS

1. Find the ratio of internal volume of a cubical box to that of the largest sphere which will fit inside it?
2. A cubical block of side 7cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the surface area of the solid so formed. (Use $\pi = \frac{22}{7}$).
3. A cylinder whose height is two third of its diameter has the same volume of sphere of radius 4cm. Calculate the radius of base of cylinder.
4. A solid ball is exactly fitted inside the cubical box of side 'a'. The volume of the ball is $\frac{4}{3}\pi a^3$. Justify your answer.
5. A spherical glass vessel has a cylindrical neck 7cm long, 4 cm in diameter, the diameter of spherical part is 21cm. Find the quantity of water it can hold. (Use $\pi = \frac{22}{7}$).
6. A warehouse is used as a granary. It is in the shape of a cuboid surmounted by a half-cylinder. The base of the warehouse is 6 m x 14 m and its height is 8m. Find the surface area of the non-cuboidal part of the warehouse.



7. 50 circular plates, each of radius 7 cm and thickness 0.5 cm, are placed one above another to form a solid right circular cylinder. Find the total surface area and the volume of the cylinder so formed.

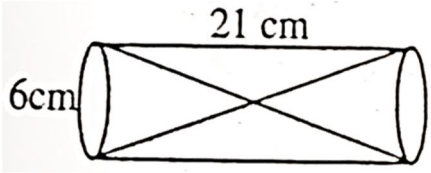
8. A small terrace at a hockey ground comprises of 10 steps each of which 20 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}m$ and a tread of $\frac{1}{2}m$. Calculate the total volume of concrete required to build the terrace.



9. A cubical block of side 7cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the surface area of the solid so formed. (Use $\pi = \frac{22}{7}$).

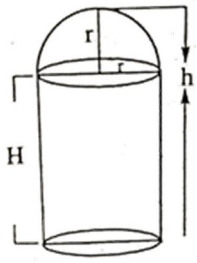
3-MARKS

1. Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is Rs. 40 per meter, find the amount by which Rampal helped the money.
2. A toy is in the form of a cylinder of diameter $2\sqrt{2}m$ and height 3.5m surmounted by a cone whose vertical angle is 600. Find the total surface area of the top.
3. A circus tent is cylindrical upto a height of 3m and conical above it. The diameter of the base of cone and cylinder is 105m and the slant height of the conical part is 53m. Find the total canvas used in making the tent.
4. A cone of maximum size is carved out from a cube of edge 14cm. Find the surface area of the cone and of the remaining solid left out after the cone carved out.
5. Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of the capacities are 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



5-MARKS

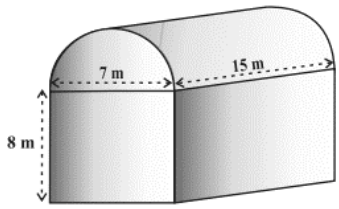
1. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2cm and the total height of the toy is 10.2cm, then find the volume of the wooden toy.
2. The cost of painting the total outside surface area of a closed cylindrical oil tank at the rate of 50 paise per square metre is 198. The height of the tank is 6 times the radius of the base of the tank. Find the volume of the tank.
3. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of the hemisphere is 3.5cm and the total wood in the making of toy is $166\frac{5}{6}cm^3$. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of Rs. 10 per cm^2. (Use $\pi = \frac{22}{7}$).
4. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of the cube and the toy. Also, find the total surface area of the toy.
5. A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building if it contains $67\frac{1}{21}m^3$ of the air.



6. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surface area of the pencils manufactured in one day at Rs.0.05 per dm^2.
7. A solid is in the form of cone mounted on a hemisphere in such a way that the centre of the base of the cone just coincide with the centre of the base of the hemisphere. Slant height of the cone is l and the radius of the base of the cone is $\frac{1}{2}r$, where r is the radius of the hemisphere.(See the below figure). Prove that

the total surface area of the solid is $\frac{\pi}{4}(11r+2l)r \ sq.units$.

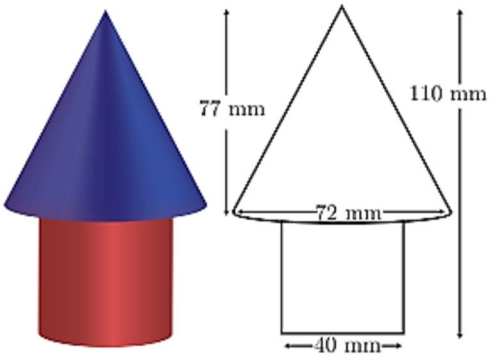
8. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder (see below figure). If the base of shed is of dimension 7 m × 15 m, and the height of the cuboidal portion is 8 m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300 m^3, and there are 20 workers, each of whom occupy about 0.08 m3 space on an average. Then, how much air is in the shed? (Use $\pi = \frac{22}{7}$).



CASE STUDY QUESTIONS

CASE STUDY_1

In a toys manufacturing company, wooden parts are assembled and painted to prepare a toy. For the wood processing activity center, the wood is taken out of storage to be sawed, after which it undergoes rough polishing, then is cut, drilled and has holes punched in it. It is then fine polished using sandpaper. For the retail packaging and delivery activity center, the polished wood sub-parts are assembled together, then decorated using paint.



One specific toy is in the shape of a cone mounted on a cylinder. The total height of the toy is 110 mm and the height of its conical part is 77 mm. The diameters of the base of the conical part is 72 mm and that of the cylindrical part is 40 mm.

(i) If its conical part is to be painted blue, what is the surface area need to be painted? 1

(ii) How much of the wood have been used in making the toy ? 1

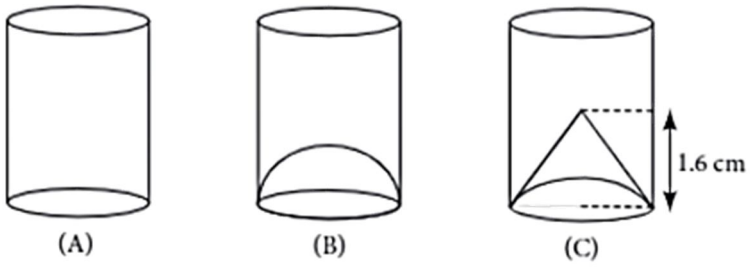
(iii) If the cost of painting the toy is 2 paise for 8π mm^2, then what is the cost of painting of a box of 100 toys? 2

OR

If the toy manufacturer company charge 3 paise for 32π mm^3 of wood, what is the price of a box of 100 toys? 2

CASE STUDY_2

Pinki's class teacher explained the students about the benefits of drinking fruit juice in the morning. So, Pinki went to a juice stall with her friend Bipin. On the stall, they observed that shopkeeper has three types of glasses of inner diameter 4.6 cm to serve customers. The height of each glass is 11 cm. Seeing this, certain questions came into their mind. Help Pinki and Bipin to solve these questions.



Based on the above information answer the following questions:

(i) Which mathematical concept has been used in the above problem? 1

(ii) What is the volume of type (A) glass and type (B) glass? 2

OR

How much more juice can be filled in type (A) glass than glass of type (C)? 2

(iii) Which glass has the maximum capacity? 1

SKILL BASED QUESTIONS

1. In a class test, marks scored by students are given in the following distribution.

Marks	0-6	6-12	12-18	18-24	24-30
No.of students	1	4	9	3	3

Find the mean and median of the data.

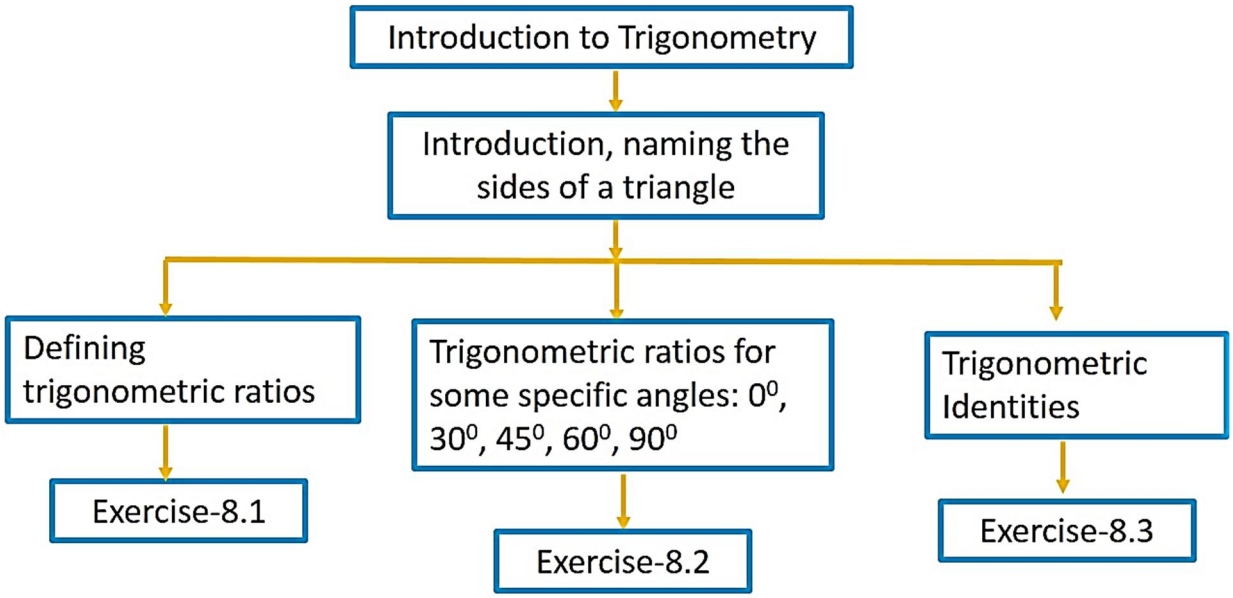
2. Following distribution gives the marks obtained, out of 150, by the students of class IX in their class test.

Marks	0-25	25-50	50-75	75-100	100-125	125-150	150-175	175-200
No.of students	10	15	22	30	28	27	12	6

Find the mean and mode of the data.

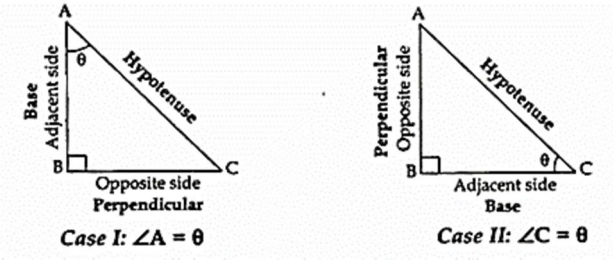
INTRODUCTION TO TRIGONOMETRY

MIND MAPPING



Basic facts and formulae:

1. Let us look at both cases:



In a right triangle ABC, right-angled at B. Once we have identified the sides, we can define six t-Ratios with respect to the sides.

CASE-I	CASE-II
$\text{sine } A = \frac{perpendicular}{Hypotenuse} = \frac{BC}{AC}$	$\text{sine } C = \frac{perpendicular}{Hypotenuse} = \frac{AB}{AC}$
$\text{Cosine } A = \frac{base}{Hypotenuse} = \frac{AB}{AC}$	$\text{Cosine } C = \frac{base}{Hypotenuse} = \frac{BC}{AC}$
$\text{tangent } A = \frac{Perpendicular}{base} = \frac{BC}{AB}$	$\text{tangent } C = \frac{Perpendicular}{base} = \frac{AB}{BC}$
$\text{cosecant } A = \frac{Hypotenuse}{Perpendicular} = \frac{AC}{BC}$	$\text{cosecant } C = \frac{Hypotenuse}{Perpendicular} = \frac{AC}{AB}$
$\text{secant } A = \frac{Hypotenuse}{Base} = \frac{AC}{AB}$	$\text{secant } C = \frac{Hypotenuse}{Base} = \frac{AC}{BC}$
$\text{cotangent } A = \frac{Base}{Perpendicular} = \frac{AB}{BC}$	$\text{cotangent } C = \frac{Base}{Perpendicular} = \frac{BC}{AB}$

2. **Reciprocal Relation between Trigonometric Ratios:**

Cosec A, sec A, and cot A are the reciprocals of sin A, cos A, and tan A respectively.

$\sin\theta = \frac{1}{\text{cosec}\,\theta}$	$\text{cosec}\,\theta = \frac{1}{\sin\theta}$	$\sin\theta\,\text{cosec}\,\theta = 1$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cos\theta\sec\theta = 1$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\tan\theta\cot\theta = 1$

3. **Trigonometric ratios for some specific angles:**

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\text{cosec}\,\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. **Trigonometric Identities:**

(i) $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

(ii) $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$

(iii) $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec^2\theta = 1 + \tan^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$

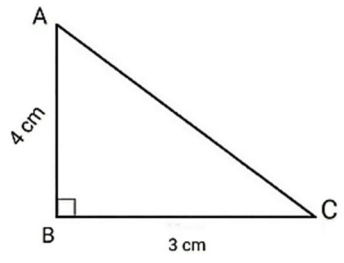
LEVEL_1

MCQ:

1. The value of $5\tan^2\theta - 5\sec^2\theta$ is:

(a) 5 (b) -5 (c) 1 (d) -1

2. In figure, AB = 4cm, BC = 3cm, then the value of $\cot A$ is:



(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

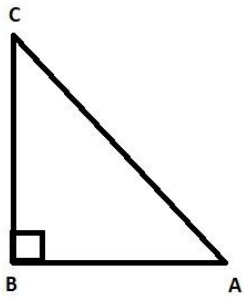
3. The simplified value of [sec A+ tan A) (1-sinA)] is:

(a) $\cos A$ (b) $\cos^2 A$ (c) $\sin A$ (d) $\sin^2 A$

4. If $\sin\theta = \cos\theta$, then the value of $\cos ec\theta$ is:

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\frac{\sqrt{3}}{2}$

5. In figure below ΔABC is right angled at B and $\tan A = \frac{4}{3}$. If AC = 15cm then the length of AB is:



(a) 3 cm (b) 6 cm (c) 9 cm (d) 12 cm

6. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^2\theta + \cot^2\theta$ is:

(a) 4 (b) 2 (c) 3 (d) 4

7. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is:

(a) 60^0 (b) 30^0 (c) 90^0 (d) 120^0

8. If $\tan A = \frac{3}{4}$, then the value of $\frac{1}{\sin A} + \frac{1}{\cos A}$ is:

(a) $\frac{35}{12}$ (b) $\frac{12}{35}$ (c) $\frac{5}{3}$ (d) $\frac{5}{4}$

9. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is:

(a) 60^0 (b) 30^0 (c) 90^0 (d) 45^0

10. If $4x = \cos ec\theta$ and $\frac{4}{x} = \cot\theta$, then the value of $4\left[x^2 - \frac{1}{x^2}\right]$ is:

(a) $\frac{1}{4}$ (b) 4 (c) 16 (d) $\frac{1}{16}$

2-MARKS

1. If $\cos A + \cos^2 A = 1$, find the value of $\sin^2 A + \sin^4 A$.

2. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then find $x + y$.

3. If $\sqrt{3}\tan\theta = 3\sin\theta$ then find the value of $\sin^2\theta - \cos^2\theta$.

4. If A = 15^0, verify that $4\sin 2A\cos 4A\sin 6A = 1$

5. Find the value of $4\cos ec^2 60^0 - 16\tan^2 30^0$.

6. If $\sin(A+B) = \frac{\sqrt{3}}{2}$ & $\cos(A-B) = \frac{\sqrt{3}}{2}$,where $0^0 < A+B \le 90^0$; A > B find A & B.

7. If $k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$, find the value of k.

8. The value of $\sin\theta + \cos\theta$ is always greater than 1. Justify your answer.

9. The value of $\tan\theta$ $(\theta < 90^0)$ increases as θ increases. Justify your answer.

10. If $\sin A = \frac{3}{4}$, calculate cosA and tanA.

3-MARKS

1. In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, then find the value of

   (i) $\sin A\cos C + \cos A\sin C$ (ii) $\cos A\cos C - \sin A\sin C$

2. In a triangle PQR, right angled at Q, PR + QR = 25 cm and PQ = 5cm. Determine the values of sinP, cosP and tanP.

3. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0^0 < A+B \le 90^0, A > B$, find the values of A and B.

4. State whether the following are true or false. Justify your answer.

   (i) $\sin(A+B) = \sin A + \sin B$

   (ii) The value of $\sin\theta$ increases as θ increases

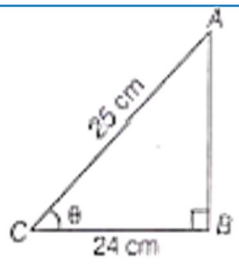
   (iii) The value of $\cos\theta$ increases as θ increases

5. If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$.

6. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$.

7. With the help of the following figure, find the value of

   (i) $\sec^2\theta + \tan^2\theta$ (ii) $\cos ec^2\theta - \cot^2\theta$



8. Evaluate $\frac{3\tan^3 30^0 + \tan^2 60^0 + \cos ec30^0 - \tan 45^0}{\cot^2 45^0}$.

9. Given in a right angled triangle ABC, right angled at C in which $\tan A = \sqrt{3}$ and $\tan B = \frac{1}{\sqrt{3}}$. Show that $\sin A \cos B - \cos A \sin B = \frac{1}{2}$.

5-MARKS

1. Prove the following identity: $(\cos ec\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$, where the angles involved are acute angles for which the expressions are defined.

2. Prove the following identity: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta.\cos ec\theta$, where the angles involved are acute angles for which the expressions are defined.

3. Prove the following identity: $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$, where the angles involved are acute angles for which the expressions are defined.

4. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

5. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$.

6. Prove the following: $\frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\sec^2\theta} + \frac{1}{1+\cos ec^2\theta} = 2$.

7. Prove that: $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\cos ec\theta - 2\sin\theta\cos\theta$.

8. If $\theta = 30^0$, verify the following:

   (i) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (ii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

9. Prove that $\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$.

10. Prove that $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ecA + \cot A$.

11. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \dfrac{p^2 - 1}{p^2 + 1}$.

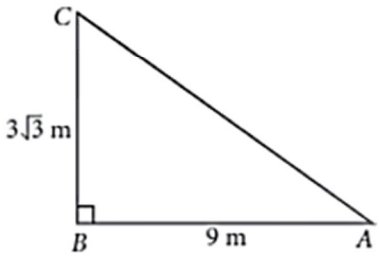
12. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, then prove that $x^2 - y^2 = a^2 - b^2$.

13. Prove that: $(\sin A + \cos ecA)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

CASE STUDY QUESTIONS

CASE STUDY_1

Three friends - Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that $AB = 9$ m, $BC = 3\sqrt{3}$ m and $\angle B = 90°$, then answer the following questions.



Based on the above information, answer the following questions:

(i) What is the measure of $\angle A$? 1

(ii) What is the measure of $\angle C$? 1

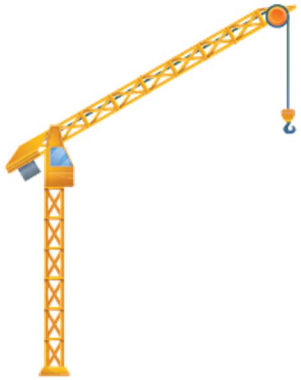
(iii) What is the length of AC? 2

OR

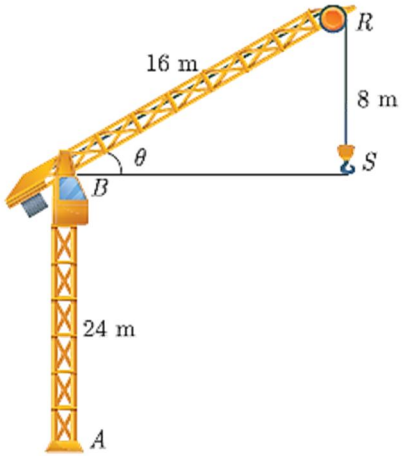
From the above figure, what are the values of $\cos 2A$ and $\sin\left(\dfrac{C}{2}\right)$ 2

CASE STUDY_2

Tower cranes are a common fixture at any major construction site. They're pretty hard to miss, they often rise hundreds of feet into the air, and can reach out just as far. The construction crew uses the tower crane to lift steel, concrete, large tools like acetylene torches and generators, and a wide variety of other building materials.



A crane stands on a level ground. It is represented by a tower AB, of height 24 m and a jib BR. The jib is of length 16 m and can rotate in a vertical plane about B. A vertical cable, RS, carries a load S. The diagram shows current position of the jib, cable and load.



(i) What is the distance BS? 1

(ii) What is the measure of the angle BRS? 1

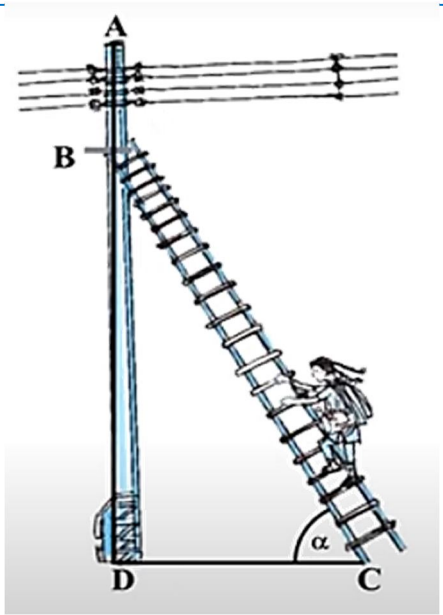
(iii) Now the jib BR, has been rotated and the length RS is increased. The load is now on the ground at a point 8m from A. What is the angle through which the jib has been rotated? 2

OR

What is the length by which RS has been increased? 2

CASE STUDY_3

Raj is an electrician in a village. One day power was not there in entire village and villagers called Raj to repair the fault. After thorough inspection he found an electric fault in one of the electric pole of height 5 m and he has to repair it. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work.



Based on the above information, answer the following questions:

(i) When the ladder is inclined at an angle of α such that $\sqrt{3}\tan\alpha + 2 = 5$ to the horizontal, find the angle α. 1

(ii) How far from the foot of the pole should he place the foot of the ladder? (Use $\sqrt{3} = 1.73$). 2

OR

From the above situation, find the value of $\sin\alpha.\cos\frac{\alpha}{2} - \cos\alpha.\sin\frac{\alpha}{2}$ 2

(iii) In the above situation, if BD = 3 cm and BC = 6 cm. Find α. 1

LEVEL_2

MCQ:

1. Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to:

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

2. In triangle OPQ, right angled at P, OP = 7cm and OQ-PQ = 1 cm then the value of sinQ is:

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d) None of these

3. If $x = 3\sec^2\theta - 1$, $y = \tan^2\theta - 2$ then the value of $x - 3y$ is:

(a) 8 (b) 4 (c) 5 (d) 3

4. The value of $\frac{\sin 90^0}{\cos 45^0}+\frac{1}{\cos ec 30^0}$ is:

(a) $\frac{2\sqrt{3}}{\sqrt{2}+1}$ (b) $\frac{2\sqrt{2}+1}{2}$ (c) $\frac{2\sqrt{2}-1}{2}$ (d) $\frac{1}{\sqrt{2}}$

5. The value of $\frac{\cos 45^0}{\sec 30^0}+\frac{1}{\sec 60^0}$ is:

(a) $\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{2}}$ (c) $\frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (d) $\frac{2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

6. The value of $\left(\sin 30^0+\cos 30^0\right)-\left(\sin 60^0+\cos 60^0\right)$ is equal to:

(a) $-\sin 90^0$ (b) $\sin 0^0$ (c) $\sin 90^0$ (d) $\cos 60^0$

7. The value of $\cos 2\theta$, if $2\sin 2\theta=\sqrt{3}$ is:

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

8. If $\sin A+\sin^2 A=1$ then the value of $\left(\cos^2 A+\cos^4 A\right)$ is:

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

2-MARKS

1. Simplify: $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$

2. If $\tan\theta=1$ and $\sin\phi=\frac{1}{\sqrt{2}}$, find the value of $\cos\left(\theta+\phi\right)$, where θ *and* ϕ are both acute angles.

3. Prove that $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$.

4. Two famous bowlers of Indian Cricket team Navdeep Saini and Jaspreet Bumrah throw a ball at an angle of A and B respectively in such a way that

$\sin\left(A-B\right)=\frac{1}{2}$ and $\cos\left(A+B\right)=0$, $0^0<A+B\le 90^0$, $A>B$.

(i) Find the angles of both bowlers at which they throw a ball.

(ii) Find the values of the following trigonometric ratios $\tan A$, $\cos ec\left(A-B\right)$ and $\sec B$

5. Prove that $\sqrt{\sec^2\theta+\cos ec^2\theta}=\tan\theta+\cot\theta$.

6. Prove that $\frac{\sin\theta}{\cot\theta+\cos ec\theta}=2+\frac{\sin\theta}{\cot\theta-\cos ec\theta}$.

7. If $7\sin^2 A + 3\cos^2 A = 4$, prove that $\tan A = \frac{1}{\sqrt{3}}$.

8. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

9. Ram was trying hard to prove $a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$, when $a\sin\theta + b\cos\theta = c$. His classmate Swati gave her a hint of squaring both sides of $a\sin\theta + b\cos\theta = c$ and proceed further. With her hint, Ram was able to solve the problem and he thanks Swati for this hint. So, write the solution of the above problem.

10. If $\tan\theta = \frac{20}{21}$, then prove that $\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{3}{7}$.

3-MARKS

1. Show that $2\left(\cos^4 60^0 + \sin^4 30^0\right) - \left(\tan^2 60^0 + \cot^2 45^0\right) + 3\sec^2 30^0 = \frac{1}{4}$.

2. Prove that $\frac{\sin^4\theta + \cos^4\theta}{1 - 2\sin^2\theta\cos^2\theta} = 1$.

3. If $m\sin\theta + n\cos\theta = p$ and $m\cos\theta - n\sin\theta = q$, prove that $m^2 + n^2 = p^2 + q^2$.

4. Prove that $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$.

5. If $\cos\theta - \sin\theta = x$ and $\sin\theta + \cos\theta = y$, then $x^2 + y^2 = 2$.

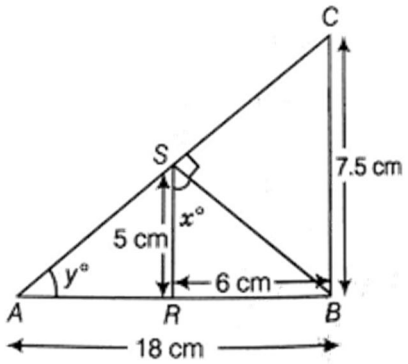
6. Calculate A + B when $\tan A = \frac{2}{3}$ & $\tan B = \frac{3}{2}$, given $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ where A and B are acute angles.

7. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $\left(m^2 + n^2\right)\cos^2\beta = n^2$.

8. Prove that $\sqrt{\frac{\sec A + \tan A}{\sec A - \tan A}}.\sqrt{\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} = 1$.

9. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$ find the value of $\tan^2\theta + \cot^2\theta$.

10. In the given figure, ΔABC is right angled at B. ΔBSC is right angled at S and ΔBRS is right angled at R, AB = 18cm, BC = 7.5cm, RS = 5cm, RB = 6cm, $\angle BSR = x^0$ and $\angle SAB = y^0$. Find the following: (i) $\tan x^0$ (ii) $\sin y^0$



11. If $\sin\theta + 2\cos\theta = 1$, prove that $2\sin\theta - \cos\theta = 2$.

12. If $\sin A = \frac{1}{\sqrt{5}}$ and $\sin B = \frac{1}{\sqrt{10}}$, find the values of cosA and cosB. Hence, using the formula $\cos(A+B) = \cos A\cos B - \sin A\sin B$. Show that $A + B = 45^0$.

13. Prove that $1 + \frac{\cot^2\alpha}{1+\cos ec\,\alpha} = \cos ec\alpha$.

5-MARKS

1. Prove that $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

2. Prove that $\cot^2 A\left[\frac{\sec A - 1}{1+\sin A}\right] + \sec^2 A\left[\frac{\sin A - 1}{1+\sec A}\right] = 0$.

3. Prove that $\left[\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$.

4. Prove that $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta - \cos ec^3\theta} = \sin^2\theta\cos^2\theta$.

5. If $\sec A = \frac{17}{8}$, show that $\frac{3-4\sin^2 A}{4\cos^2 A - 3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$.

6. Show that $(\sin^8 A - \cos^8 A) = (2\sin^2 A - 1)(1 - 2\sin^2 A\cos^2 A)$.

7. If $\cos ecA - \cot A = q$, show that $\frac{q^2-1}{q^2+1} + \cos A = 0$.

8. Prove that $2\sec^2\theta - \sec^4\theta - 2\cos ec^2\theta + \cos ec^4\theta = \cot^4\theta - \tan^4\theta$.

9. If $x = r\sin A\cos B$, $y = r\sin A\sin B$ and $z = r\cos A$, show that $x^2 + y^2 + z^2 = r^2$.

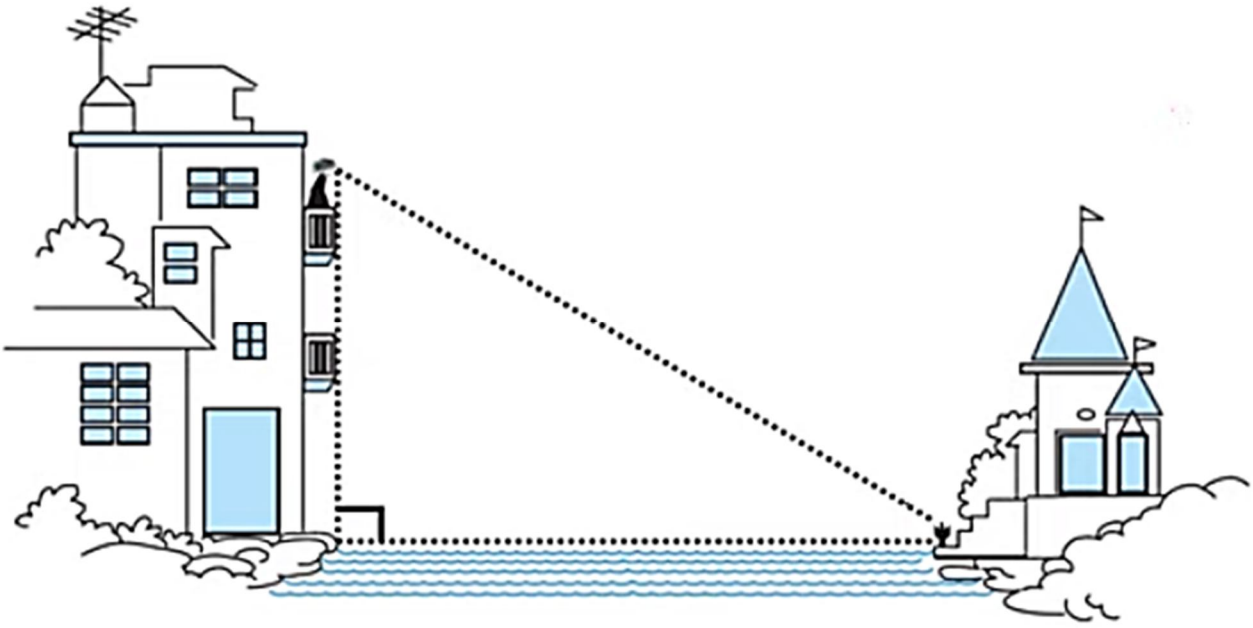
10. Prove that $\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$.

11. Prove that $l^2m^2\left(l^2+m^2+3\right)=1$, if $\operatorname{cosec}\theta-\sin\theta=l$ and $\sec\theta-\cos\theta=m$.

CASE STUDY QUESTIONS

CASE STUDY_1

Suppose a girl is sitting on the balcony of her house located on the bank of a river. She is looking down at a flower pot placed on a stair of a temple situated nearby on the other bank of the river. A right triangle is imagined to be made in this situation as shown in the figure. If the height at which the person is sitting is known, the width of the river can be calculated.



Based on the above situation, answer the following questions:

(i) If the height of her house is 12m, and the distance between her house and the river is 5m, then what will be the value of $\cos\theta$? 1

(ii) If the width of the river is 15m, and $\angle C=60^0$ then what will be the height of the building? 1

(iii) What is the value of $\left(\sin 45^0-\tan 30^0\right)\left(\cos 45^0+\frac{1}{\tan 60^0}\right)$ 2

OR

Find the value of x: $\tan x=\cos 30^0.\sin 90^0+\sin 60^0.\cos 0^0$ 2

LEVEL_3

MCQ:

1. If $\tan(3x+30^0)=1$, then the value of x is:

   (a) 3 (b) 4 (c) 5 (d) 7

2. If $x=3\sin\theta+4\cos\theta$ and $y=3\cos\theta-4\sin\theta$ then the value of x^2+y^2 = ___.

   (a) 12 (b) 23 (c) 25 (d) 24

3. If $\cos A=\frac{2}{5}$, then the value of $4+4\tan^2 A$ is:

   (a) 25 (b) 23 (c) 21 (d) 24

4. The value of $\sin^2 60^0-2\tan^2 45^0-\cos^2 30^0$ is _____.

   (a) 2 (b) -2 (c) 1 (d) -1

5. If $3\tan^2\theta-1=0$, then the value of $\frac{2\tan\theta}{1-\tan^2\theta}=$_____.

   (a) $\tan 2\theta$ (b) $\tan\theta$ (c) $\sin 2\theta$ (d) $\cos 2\theta$

6. If $\cos\theta=\frac{3}{5}$, then the value of $\frac{\sin\theta-\cot\theta}{2\tan\theta}$ is:

   (a) $\frac{3}{20}$ (b) $\frac{3}{60}$ (c) $\frac{3}{160}$ (d) none of these

7. If $0<\theta<\frac{\pi}{4}$, then the simplest form of $\sqrt{1-2\sin\theta\cos\theta}$ is:

   (a) $\pm(\sin\theta-\cos\theta)$ (b) $\pm(\sin\theta+\cos\theta)$ (c) $\pm\sin\theta.\cos\theta$ (d) 1

8. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$ then the value of x^2+y^2 is:

   (a) 0 (b) 1 (c) -1 (d) 2

9. If $\sin\theta+\cos\theta=\sqrt{2}\cos\theta,\ (\theta\neq 90^0)$, then the value of $\tan\theta$ is:

   (a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

10. If $\sin\theta=\frac{3}{4}$, then the value of $\cot\theta$ is:

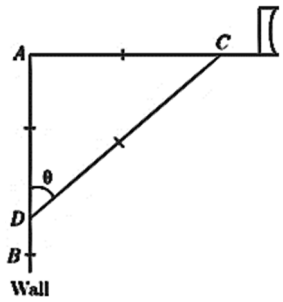
    (a) $\frac{\sqrt{7}}{3}$ (b) $\frac{7}{\sqrt{3}}$ (c) $3\sqrt{3}$ (d) $7\sqrt{3}$

2-MARKS

1. If $\cos(A-B)=\frac{\sqrt{3}}{2}$ and $\sin(A+B)=\frac{\sqrt{3}}{2}$, find A and B, where $(A+B)$ & $(A-B)$ are acute angles.

2. Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=4$; $\theta \le 90^0$.

3. The rod of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Figure. If AC = 1.5 m long and CD = 3 m, find



(i) $\tan\theta$ (ii) $\sec\theta+\cos ec\theta$

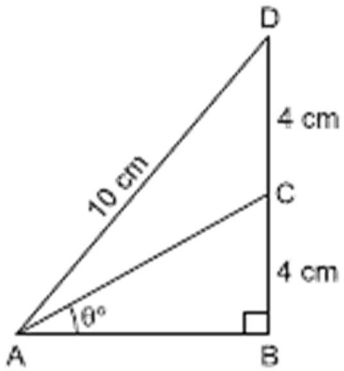
4. If $b\cos\theta=a$, then show that $\cos ec\theta+\cot\theta=\sqrt{\frac{b+a}{b-a}}$.

5. Evaluate: $\sin^2 30^0\cos^2 45^0+4\tan^2 30^0+\frac{1}{2}\sin 90^0-2\cos^2 90^0+\frac{1}{24}$

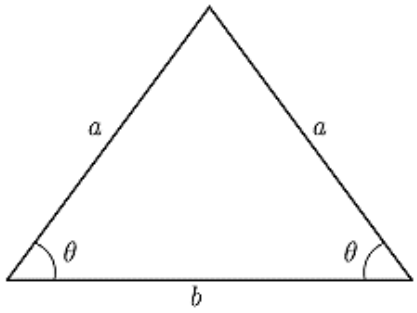
6. If $15\tan^2\theta+4\sec^2\theta=23$ then find the value of $(\sec\theta+\cos ec\theta)^2-\sin^2\theta$

7. In an acute angled triangle ABC, if $\sin(A+B-C)=\frac{1}{2}$ and $\cos(B+C-A)=\frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ *and* $\angle C$.

8. In the adjoining figure, $\angle B=90^0$, $\angle BAC=\theta^0$, BC = CD = 4cm and AD = 10cm. Find (i) $\sin\theta$ and (ii) $\cos\theta$.



9. Area of an Isosceles Triangle: Consider the following isosceles triangle. The length of each of the two equal sides of the triangle is a, and each of the base angles has a measure of θ. Verify that the area of the triangle is $A = a^2 \sin\theta \cos\theta$



10. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.

3-MARKS

1. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if:

   (i) $x = a\sec\theta,\ y = b\tan\theta$ (ii) $x = a\,\text{cosec}\,\theta,\ y = b\cot\theta$

2. Prove that: $\dfrac{\cot^3\theta \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \dfrac{\tan^3\theta \cos^3\theta}{(\cos\theta + \sin\theta)^2} = \dfrac{\sec\theta \cos ec\theta - 1}{\cos ec\theta + \sec\theta}$

3. Prove that $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = 2\cos ec\theta$

4. If $\tan\theta = \dfrac{1}{\sqrt{5}}$ (i) Evaluate: $\dfrac{\cos ec^2\theta - \sec^2\theta}{\cos ec^2\theta + \sec^2\theta}$ (ii) Verify: $\sin^2\theta + \cos^2\theta = 1$

5. Evaluate of the following: $\dfrac{2\cos^2 60^0 + 3\sec^2 30^0 - 2\tan^2 45^0}{\sin^2 30^0 + \cos^2 45^0}$

6. If $\cos ec\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$, prove that $\left(m^2n\right)^{2/3} + \left(mn^2\right)^{2/3} = 1$

7. If $\tan\theta + \sec\theta = l$, then prove that $\sec\theta = \dfrac{l^2 + 1}{2l}$.

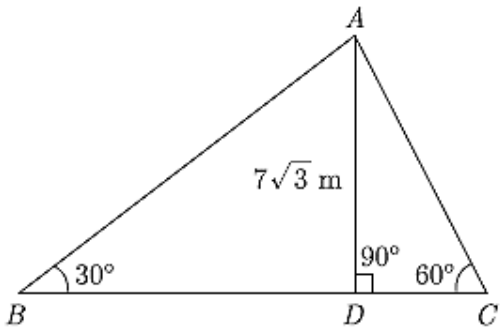
8. Evaluate: $\tan^2 30^0 \sin 30^0 + \cos 60^0 \sin^2 90^0 \tan^2 60^0 - 2\tan 45^0 \cos^2 0^0 \sin 90^0$

9. Using the formula, $\sin A = \sqrt{\dfrac{1 - \cos 2A}{2}}$ find the value of $\sin 30^0$, it is being given that $\cos 60^0 = \dfrac{1}{2}$.

10. If $x = \cot A + \cos A$ and $y = \cot A - \cos A$, prove that $\left(\dfrac{x - y}{x + y}\right)^2 + \left(\dfrac{x - y}{2}\right)^2 = 1$

5-MARKS

1. In the given figure, if $AD = 7\sqrt{3}\,m$, find the value of BC.



2. If

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ represents each side of an equilateral triangle, then prove that each side is equal to ± 1

3. Taking $\theta = 30^0$, verify each of the following:

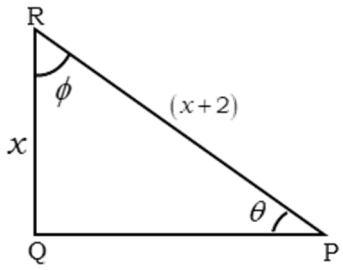
(i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

(iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

4. In the figure of triangle PQR, $\angle P = \theta$ and $\angle R = \phi$, find:

(i) $\left(\sqrt{x+1}\right)\cot\phi$ (ii) $\left(\sqrt{x^3+x^2}\right)\tan\theta$ (iii) $\cos\theta$



5. Show that:

(i) $\dfrac{1-\sin 60^0}{\cos 60^0} = \dfrac{\tan 60^0 - 1}{\tan 60^0 + 1}$ (ii) $\dfrac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \cos 30^0$

6. Prove that $(\cos ecA - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$.

7. Prove that $\left(\tan^2 A - \tan^2 B\right) = \dfrac{\left(\sin^2 A - \sin^2 B\right)}{\cos^2 A\cos^2 B} = \dfrac{\left(\cos^2 B - \cos^2 A\right)}{\cos^2 B\cos^2 A}$

8. Prove that $\frac{\sin\theta}{(\sec\theta+\tan\theta-1)}+\frac{\cos\theta}{(\cos ec\theta+\cot\theta-1)}=1$.

9. Prove that $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2}{(\sin^2\theta-\cos^2\theta)}=\frac{2}{(2\sin^2\theta-1)}$.

10. Prove that:

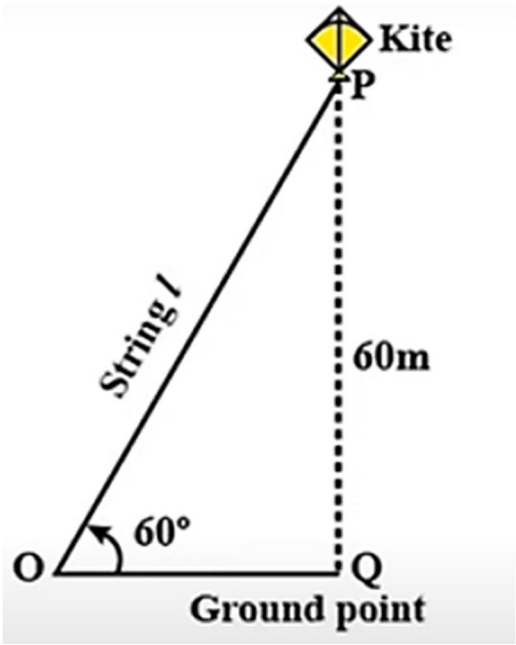
(i) $\frac{\cos ec\theta+\cot\theta}{\cos ec\theta-\cot\theta}=(\cos ec\theta+\cot\theta)^2=1+2\cot^2\theta+2\cos ec\theta\cot\theta$

(ii) $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}=(\sec\theta+\tan\theta)^2=1+2\tan^2\theta+2\sec\theta\tan\theta$

CASE STUDY QUESTIONS

CASE STUDY_1

Kite flying is also a major part of Makar Sankranti, although the states of Gujarat and Rajasthan indulge in this with a lot more enthusiasm. Makar Sankranti is a major harvest festival celebrated in India and is dedicated to the Sun god, Surya. It is the first major festival to be celebrated in India and usually takes place in January, this year the festival will be celebrated on January 14. Aditya flying a kite at a height of 60m above the ground. He attached a string to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^0.



Based on the above information, answer the following questions:

(i) In the above situation, find the length of the string, assuming that there is no slack in the string. 1

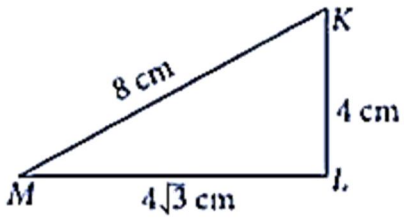
(ii) The string of a kite is 100m long and it makes an angle of 60^0 with the horizontal. If there is no slack in the string, then find the height of the kite from the ground. 1

(iii) A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60m. Assuming that there is no slack in the string, find angle POQ. 2

OR

CASE STUDY_2

Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle, with sides $4cm$, $4\sqrt{3}cm$ and $8cm$.



On the basis of the above information, answer the following questions:

(i) Find the value of $\angle M$. 1

(ii) Find the value of $\angle K$. 1

(iii) Find the value of $\tan M$ and $\sec^2 M - 1$ 2

OR

Find the value of $\dfrac{\tan^2 45^0 - 1}{\tan^2 45^0 + 1}$

SKILL BASED WORKSHEET

1. Find the median and mode of the following data and then find the mean from the empirical relationship between them.

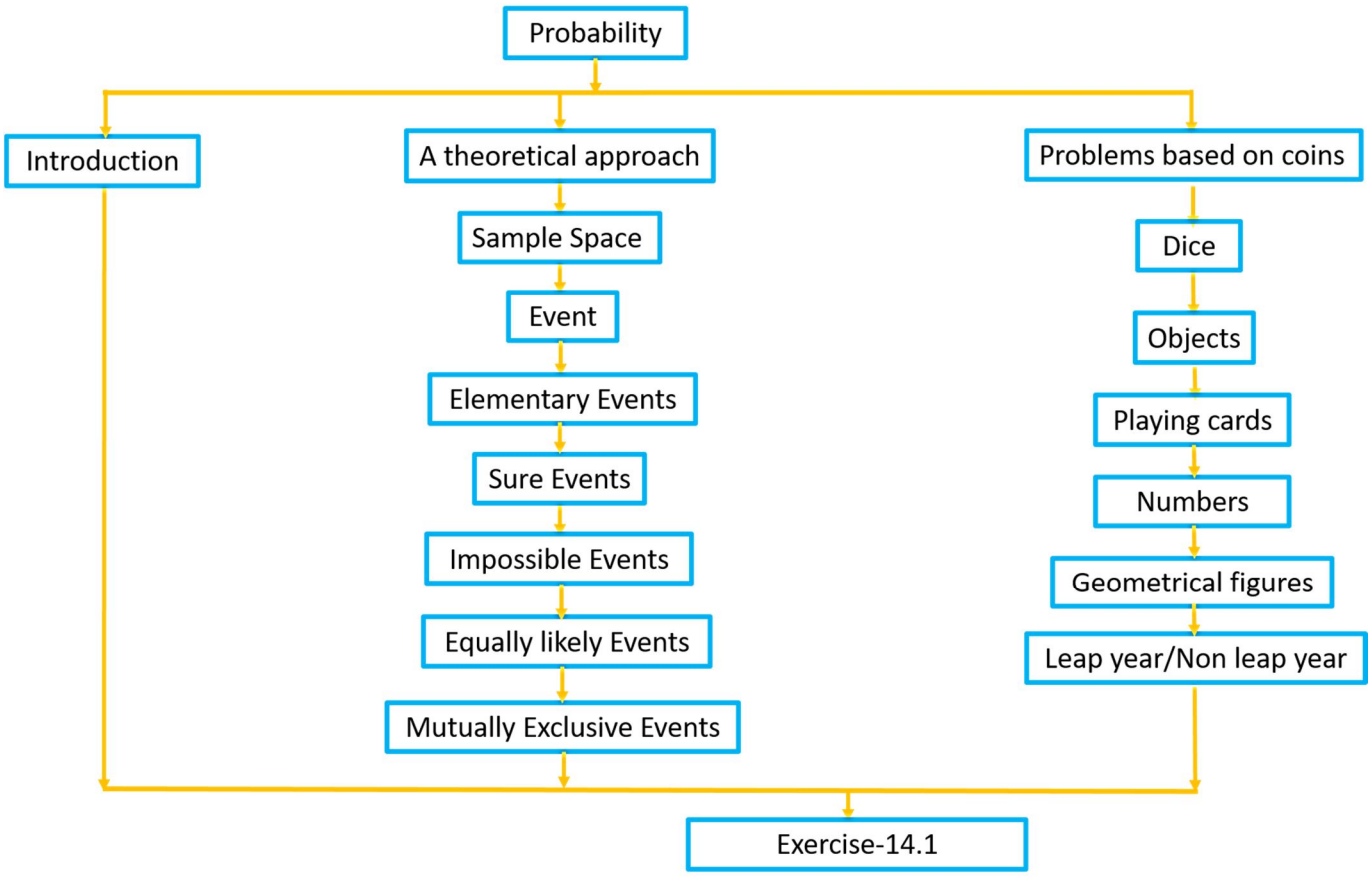
Class Interval	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

2. If the median of the distribution given below is 28.5 Find the value of x and y.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	y	5	60

PROBABILITY

MIND MAPPING



Basic facts and formulae:

1. Experimental or Empirical Probability
2. The result of probability based on the actual experiment is called experimental probability. In this case, the results could be different if we do the same experiment again.

$$P(E) = \frac{\text{Number of the trials in which the event happened}}{\text{Total number of trials}}$$

3. **Probability — A Theoretical Approach:** In the theoretical approach, we predict the results without performing the experiment actually. The other name of theoretical probability is classical probability.

$$P(E) = \frac{\text{Number of outcomes favourable to F}}{\text{Number of all possible outcomes}}$$, Where outcomes are equally likely.

4. **Equally Likely Outcomes:** If we have the same possibility of getting each outcome then it is called equally likely outcomes.
5. If P(E) = 1, then it is called a 'Certain Event'.
6. If P(E) = 0, then it is called an 'Impossible Event'.

7. The probability of an event E is a number P(E) such that: $0 \le P(E) \le 1$

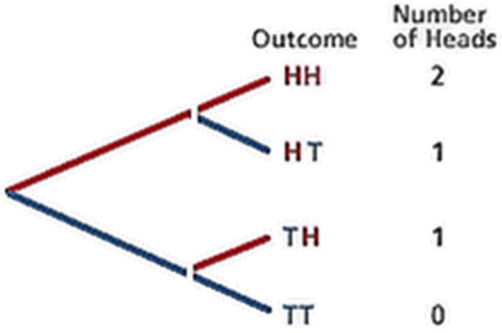
8. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

9. For any event E, $P(E)+P(\overline{E})=1$, where $\overline{E}$ stands for 'not E'. E and $\overline{E}$ are called complementary events.

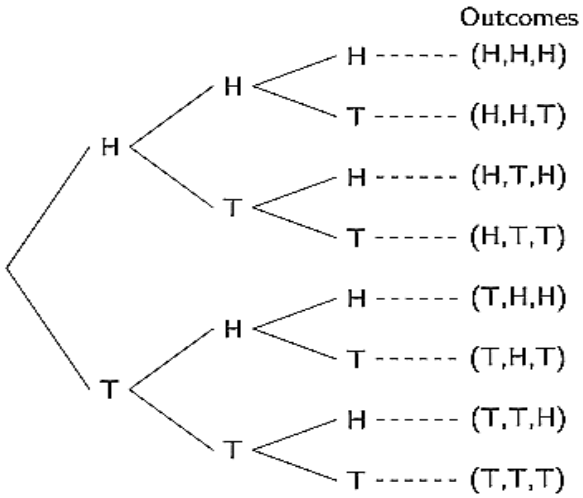
10. A collection of all possible outcomes of an experiment is known as sample space. It is denoted by 'S' and represented in curly brackets.

11. If a coin is tossed once total possible outcomes are 2. {H, T}

12. If a two coins are tossed at once or one coin tossed twice, then the number of possible outcomes are 4.



13. If a three coins are tossed at once or one coin tossed thrice, then the number of possible outcomes are 8.

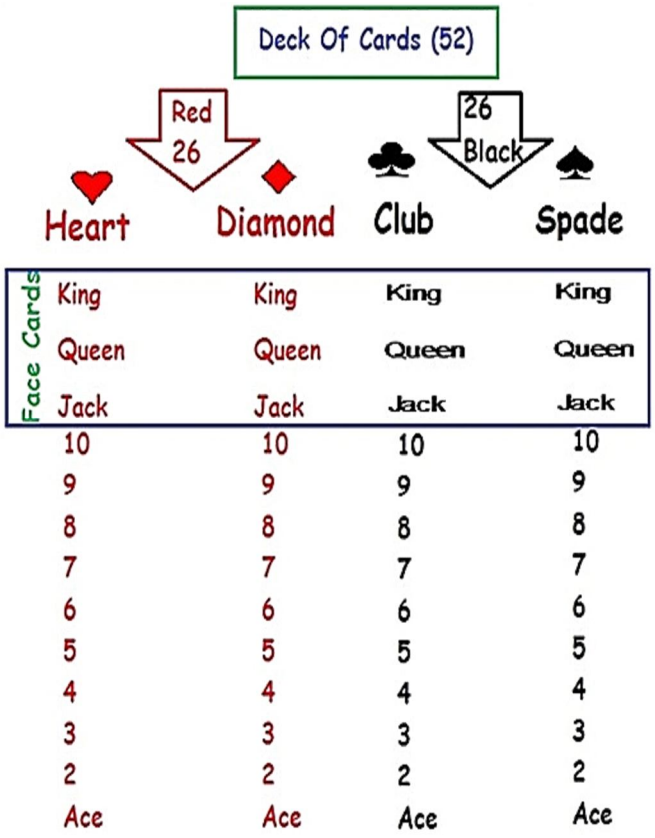


14. If a die is thrown once, then the number of possible outcomes are 6. They are 1, 2, 3, 4, 5, 6.

15. If two dice are thrown or a die is thrown twice, then the number of possible outcomes are 36.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

16. Playing cards Chart:



LEVEL_1

MCQ:

1. If a die is rolled once, then the probability of obtaining a number greater than 3 is.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{5}{6}$

2. In a single throw of a pair of dice, the probability of getting the sum 9 is.

(a) $\frac{1}{9}$ (b) $\frac{1}{26}$ (c) $\frac{2}{9}$ (d) $\frac{5}{36}$

3. If probability of happening of an event is $\frac{5}{9}$ then the probability of non-happening of this event is:

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

4. If an event cannot occur then its probability is:

(a) 1 (b) 3/4 (c) 1/2 (d) 0

5. If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; then, the probability that the digit is even, is.

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

6. A bag contains three green marbles, four blue marbles, and two orange marbles. If a marble is picked at random, then the probability that it is an Orange marble is.

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

7. A card is accidentally dropped from a pack of 52 playing cards. The probability that it is an ace is:

(a) 1/4 (b) 1/13 (c) 1/52 (d) 12/13

8. If $P(E) = 0.05$ then the value of $\mathrm{P}(\mathrm{not\,E})$ is:

(a) 0.05 (b) 0.095 (c) 0.95 (d) 1

9. The probability of an impossible event is:

(a) 1 (b) 0 (c) -1 (D) ½

10. One card is drawn from a well shuffled deck of 52 playing cards. What is the probability of getting '4 of hearts'?

(a) 1/52 (b) 1/13 (c) 1/26 (d) 1/6

2-MARKS:

1. A die is thrown once. Find the probability of getting:

i) a composite number ii) a prime number

2. Cards numbered 7 to 40 were put in a box. Poonam selects a card at random. What is the probability that Poonam selects a card which is a multiple of 7?

3. An integer is chosen at random 1 and 100. Find the probability that it is

i) divisible by 8 ii) not divisible by 8

4. Two different dice are tossed together. Find the Probability of:

   i) getting a doublet ii) getting a sum 10, of the numbers on the two dice

5. A card is drawn from a well shuffled pack of 52 playing cards. What is the probability that the card drawn is:

   i) Either a red or a king ii) a black card.

6. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting

   i) a non-face card ii) a black king

7. A pack of 52 playing cards pack is shuffled well. A card is drawn at random from the pack of cards. Find the probability of getting:

   i) a black face card ii) a queen

8. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a queen.

9. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting

   a) a face card of red colour b) a jack of spades

10. A card is drawn at random from well shuffled deck of playing cards. Find the probability that the card drawn is:

    i) a spade ii) a red king

11. A card is drawn from a well shuffled deck of 52 cards. Find the probability that it is not an ace.

12. Two dice are thrown at the same time. Find the probability of getting same numbers on both the dice.

13. Two dice are thrown simultaneously at the same time. Find the probability of getting different numbers on both the dice.

14. In a single throw of two dice, find the probability of getting a total of 8.

15. Two dice are together thrown once. Find the probability of getting such numbers on tops so that their sum is 3.

16. If there are 2 children in a family, find the probability that there is at least one boy in the family.

17. A bag contains 5 red, 8 green & 7 white balls. One ball is drawn at random from the bag. Find the probability of getting:

    i) Not a white ball

    ii) neither a green nor a red ball.

18. Two unbiased coins are tossed simultaneously. Find the probability of getting.
    a) At least one head b) at most one head
19. 17 cards numbered 1, 2, 3,.....16, 17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is: i) A prime number ii) divisible by 3
20. A letter of English alphabet is choosen at random, find the probability that the letter so choosen is: i) a vowel ii) a consonant

3-MARKS:

1. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, then find the number of black balls in the bag.
2. There are 100 cards in a bag on which numbers from I to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card (i) is divisible by 9 and is a perfect square (ii) is a prime number greater than 80.
3. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange ball, then find the total number of balls in the jar.
4. One card is drawn at random from a well - shuffled deck of 52 cards. Find the probability of getting:
    i) a king of red colour ii) a face card iii) a red face card
    iv) The jack of hearts v) a spade vi) a queen of diamonds
5. Cards marked with numbers 13, 14, 15.....60 are placed in a box and mixed thoroughly, one card is drawn at random from the box. Find the probability that the sum of digits on the card drawn is 5.
6. Cards marked with numbers 5 to 50 are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the card taken out is:
    i) A prime number less than 20
    ii) a perfect square number
    iii) A multiple of 5 or 6

7. Cards numbered 2 to 101 are placed in a box. A card is selected at random from the box, find the probability that the card selected.
   i) Has a number which is a perfect square
   ii) Has an odd number which is not less than 70
8. From 30 tickets marked with numbers 2 to 31, one ticket is drawn at random. Find the probability that it is
   i) A multiple of 7 ii) an even number iii) a prime number
9. The king, queen and jack of diamonds are removed from a pack of 52 cards and then the pack is well shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of:
   i) diamonds ii) a jack iii) a heart
10. 15 cards numbered 1, 2, 3.....15, are put in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the card drawn bears
    i) An even number ii) a number divisible by 2 or 3
11. Cards with the numbers 2 to 103 are placed in a box. A card is selected at random from the box. Find the probability that the card selected is a number which is a multiple of 4.

5-MARKS:

1. Two different dice are thrown together. Find the probability that the numbers Obtained have
   i) Even sum ii) Even product
2. A number x is selected at random from the numbers 1, 4, 9, 16 and another number y is selected at random from the numbers 1, 2, 3, 4. Find the probability that the value of xy is more than 16.
3. A card is drawn at random from a well- shuffled deck of playing cards. Find the probability that the card drawn is
   i) A card of spade or an ace ii) a black king
   iii) Neither a jack nor a king iv) either a king or a queen.
4. Two dice one blue and one grey, are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is 8?
5. A die is thrown twice. Find the probability of getting the sum of the numbers on the top both the times is a prime number.

6. Find the probability that a non-leap year chosen at random has

(i) 52 Sundays (ii) 53 Sundays.

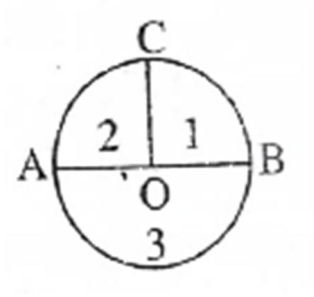
7. A pair of dice is thrown once. Find the probability of getting

i) The same number on each die

ii) Sum of the numbers on the two dice is greater than 12.

8. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the card drawn is neither a black card nor a king.

9. A game of chance consists of an arrow which comes to rest, pointing at one of the regions I or 2 or 3. O is the center of the circle, $OC \perp AB$. Find the probability that



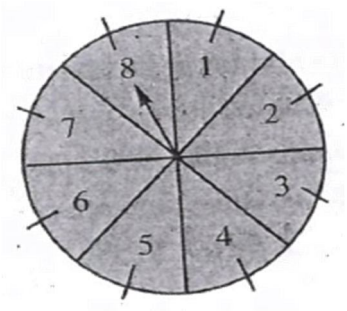
a) Arrow is resting on 3 b) arrow is resting on 1 c) arrow is not resting on 2

10. Two dice are thrown together and the score is recorded.

i) What is the probability of getting a total of 7?

ii) What is probability of getting product as 12?

11. A game of chance consists of spinning an arrow which comes of rest pointing at one of the number 1, 2, 3, 4, 5, 6, 7, 8 (see figure) and there are equally likely outcomes. What is the probability that it will point at:



i) 8? ii) An odd number?

iii) A number greater than 2? iv) A number less than 9?

v) Which mathematical concept is used in the above problem?

vi) What is its value?

12. A piggy bank contains hundred 50 P coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. It is equally likely that one of the coins will fall out when the bank is tuned upside down, what is the probability that the coin:

i) Will be a 50 p coin?

ii) Will not be a ₹5 coin?

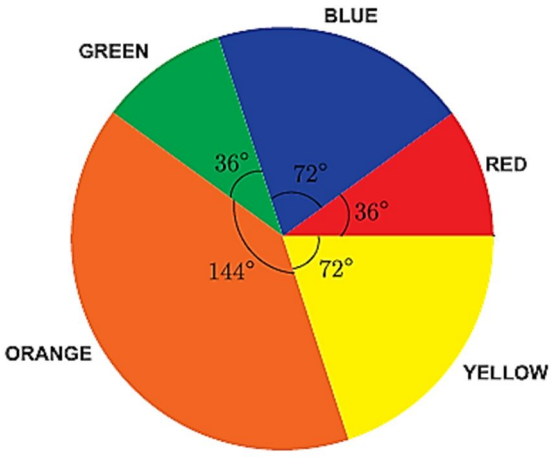
iii) Which mathematical concept is used in the above problem?

iv) Which value is discussed above?

CASE STUDY QUESTIONS

CASE STUDY_1

A survey was taken at a high school, and the results were put in a circle graph. The students were asked to list their favourite colours. The measurement of each central angle is shown. **If a person is chosen at random from the school, find the probability of each response.**



(i) What is the probability of favourite colour being red? 1

(ii) What is the probability of favourite colour being blue or green? 1

(iii) What is the probability of favourite colour not being red or blue? 2

OR

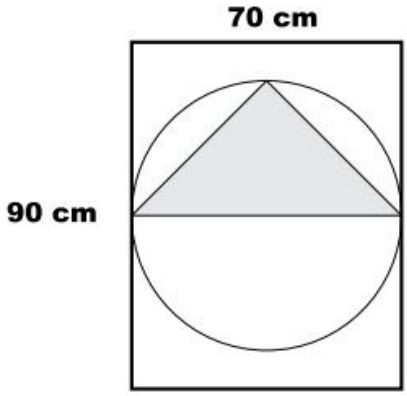
What is the probability of favourite colour not being orange or green? 2

CASE STUDY_2

Deepika Kumari is an Indian athlete who competes in the event of archery, is currently ranked World No. 9, and is a former world number one. She won a gold medal in the 2010. Commonwealth games in the women's individual recurve event. She also won a gold medal in the same competition in the women's team recurve event along with Dola Banerjee and Bombayala Devi. Kumari qualified for the 2012 Summer Olympics in London, where she competed in the Women's Individual and Women's team events, finishing in eighth place in the latter.

Deepika can hit the rectangular target shown 100% of the time at a range of 80 m.

Assuming the probability the target is hit is related to its area, answer the following.



(i) What is the probability the Deepika hits within the triangle? 1

(ii) What is the probability the Deepika hits within the circle? 1

(iii) What is the probability the Deepika hits within the circle but outside the triangle? 2

OR

What is the probability the Deepika hits within the rectangle but outside the circle? 2

CASE STUDY_3

Insurance: An insurance company selected 1000 drivers at random in a particular city to determine a relationship between age and accidents. The data obtained are listed in following table.

Age	0	1	2	3	Over 3
Under 20	50	62	53	35	20
20–29	64	93	67	40	36
30–39	82	68	32	14	4
40–49	38	32	20	7	3
Over 49	43	50	35	28	24

Compute the probabilities of the following events for a driver chosen at random in the city:

(i) E_1 : being under 20 years old and having exactly three accidents in 1 year 1

(ii) E_2 : being 30–39 years old and having one or more accidents in 1 year 1

(iii) E_3 : having no accidents in 1 year 2

OR

E_4 : being under 20 years old or having exactly three accidents in 1 year 2

LEVEL_2

MCQ:

1. The probability of guessing the correct answer to a certain test questions is $\frac{x}{12}$ If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then the value of x is:

(a) 4 (b) 5 (c) 6 (d) 7

2. Which of the following cannot be the probability of an event?

(a) $\frac{1}{2}$ (b) 0.1 (c) 3 (d) $\frac{17}{16}$

3. Aarushi sold 100 lottery tickets in which 5 tickets carry prizes. If Priya purchased a ticket, what is the probability of Priya winning a prize?

(a) 19/20 (b) 1/25 (c) 1/20 (d) 17/20

4. A number is selected from first 50 natural numbers. What is the probability that it' is a multiple of 3 or 5?

(a) 13/25 (b) 21/50 (c) 12/25 (d) 23/50

5. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime number less than 23, is:

(a) 7/20 (b) 10/90 (c) 4/45 (d) 9/89

6. A number x is chosen at random from the numbers -3, -2, -l, 0, l, 2, 3, find the probability that $|x| < 2$.

(a) 1/7 (b) 2/7 (c) 3/7 (d) 5/7

7. The probability expressed as a percentage of a particular occurrence can never be

(a) less than 100 (b) less than 0

(c) greater than 1 (d) anything but a whole number

8. Two numbers 'a' and 'b' are selected successively without replacement in that order from the integers 1 to 10. Find the probability that $\frac{a}{b}$ is an integer.

(a) 17/45 (b) 1/5 (c) 17/90 (d) 8/45

9. What is the probability that a leap year has 52 Mondays?

(a) 2/7 (b) 4/7 (c) 5/7 (d) 6/7

10. In a family of 3 children, the probability of having at least one boy is:

(a) 7/8 (b) 8/7 (c) 5/7 (d) 4/7

2-MARKS

1. A card is drawn from a well shuffled pack of 52 cards. Find the probability of getting:

i) a red card ii) a king or queen.

2. Two dice are thrown at the same time. Find the probability of getting an even number on the first die.

3. A pair of dice is thrown once. What is the probability that 5 will come at least on one die?

4. There are three children in a family. Find the probability of that there is at most one girl in the family
5. What is the probability that there are 53 Wednesdays in a leap year?
6. Two coins are tossed simultaneously. Find the probability of getting at least one head.
7. Two coins are tossed together. Find the probability of getting both Heads and both Tails.
8. A girl calculated that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?
9. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Rama wins if he gets either all heads or all tails, and loses otherwise. Find the probability that he will lose the game.
10. A bag contains 20 balls out Of which x balls are red.

    i) If one ball is drawn at random from the bag, then find the probability that it is not a red.

    ii) If 4 more red balls are out into the bag, then the probability of drawing a red ball will be $\frac{5}{4}$ times the probability of drawing a red ball in the first case. Find the value of x.
11. A number x is chosen from 25, 24, 23, -2, -l, 0, l, 2, 3. Find the probability that $|x| < 3$.
12. In any situation that has only two possible outcomes, each outcome will have probability $\frac{1}{2}$. True or false? Why?
13. A student says that if you throw a die, it will show up 1 or not l. Therefore, the probability of getting 1 and the probability of getting 'not 1 ' each is equal to $\frac{1}{2}$. Is this correct? Give reasons.
14. If you toss a coin 6 times and it comes down heads on each occasion. Can you Say that the probability of getting a head is 1? Give reasons.
15. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since this situation has only two possible outcomes, so the probability of each is $\frac{1}{2}$. Justify.

16. If three coins are tossed simultaneously, find the probability of getting at least two heads.

17. If a number x is chosen from the numbers 1, 2, 3, and a number y is selected from the numbers 1, 4, 9. Find the value of $P(xy < 9)$.

3-MARKS

1. Cards marked with numbers 5, 6, 7...., 30 are placed in a box and mixed thoroughly and one card is drawn at random from the box. What is the probability that the number on the card is:

   i) A prime number? ii) A multiple of 3 or 5?

   iii) Neither divisible by 5 nor by 10?

2. A coin is tossed 3 times. Find the probability of getting:

   i) At least 2 heads

   ii) Not getting the same result in all the tosses

   iii) Exactly 1 tail

3. Two dice (one blue and one grey) are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of dice is:

4. A bag contains 5 red balls and some blue balls. The probability of drawing a blue ball from the bag is thrice that of a red ball. Find the number of blue balls in the bag.

5. There coins are tossed simultaneously once. Find the probability of getting

   i) At least one tail ii) no tail

6. A jar contains 24 marbles. Some of them are green and other are blue. If a marble is drawn at random from the jar, then the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

7. Two dice are thrown together. Find the probability that a multiple of 2 occurs on one dice and a multiple of 3 occurs on the other.

8. The king, queen and jack of clubs are removed from a pack of 52 playing cards and then the remaining pack is well shuffled. One card is selected from the remaining cards. Find the probability of getting:

   i) A heart ii) a king iii) a club iv) a diamond

   v) a jack

5-MARKS

1. Two dice are numbered l, 2, 3, 4, 5, 6 and l, l, 2, 2, 3, 3 respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.
2. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.
3. An integer is chosen between 0 and 100. What is the probability that it is
   i) Divisible by 7?
   ii) Not divisible by 7?
4. Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has
   i) an even number ii) a square number
5. There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of ₹100 each, 100 of them contain a cash prize of ₹50 each and 200 of then contain a cash prize of ₹10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize?
6. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Han if wins if all the tosses give the same result (three heads or three tails), and loses otherwise. Calculate the probability that Hanif will lose the game.
7. Cards marked with number 3, 4, 5,……,50 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that the number on the drawn card is:
   i) Divisible by 5 ii) a number which is a perfect square.
8. A box contains 27 marbles, some are red and others are yellow. If a marble is drawn at random from the box, then the probability of that it is a red is $\frac{1}{3}$. Find the number of yellow marbles in the box.

CASE STUDY QUESTIONS

CASE STUDY_1

In two dice game, the player take turns to roll both dice, they can roll as many times as they want in one turn. A player scores the sum of the two dice thrown and gradually reaches a higher score as they continue to roll. If a single number 1 is thrown on either die, the score for that whole turn is lost. Two dice are thrown simultaneously.

(i) What is the probability of getting the sum as an even number? 1

(ii) What is the probability of getting the sum as a prime number? 1

(iii) What is the probability of getting a doublet of even number? 2

OR

What is the probability of getting a product of numbers greater than 16? 2

LEVEL_3

MCQ:

1. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn is a number which is multiple of 3 or 5?

   (a) 1/2 (b) 2/5 (c) 8/15 (d) 9/20

2. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn are blue?

   (a) 10/21 (b) 11/21 (c) 2/7 (d) 5/7

3. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

   (a) 1/3 (b) 3/4 (c) 7/19 (d) 8/21

4. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is:

   (a) 1/13 (b) 2/13 (c) 1/26 (d) 1/52

5. There are 30 cards of the same size in a bag on which natural numbers 1 to 30 are written. One card is taken out of the bag at random. Then the probability that the number on the selected card is not divisible by 3 is:

   (a) 1/3 (b) 3/4 (c) 2/3 (d) 1/4

2-MARKS

1. A number x is selected from the numbers 1, 2, 3 and then a second number y is selected from the numbers 1, 4, 9. What is the probability that the product xy of the two numbers will be less than 9?
2. A coin is tossed three times. Find the probability of getting exactly two tails.
3. 1000 tickets of a lottery were sold and there are 5 prizes on these tickets. If saket has purchased one lottery ticket, then what is the probability of winning a

prize?

4. What is the probability that a number selected from the numbers 1, 2, 3,..... 25 is a prime number, when each of the given numbers is equally likely to be selected?

5. There are total of 45 beads of blue, green and white colour in a jar. The probability of selecting a blue bead is $\frac{1}{3}$ and probability of selecting a green bead is $\frac{4}{9}$. How many white beads are there in the jar?

6. A child has a die whose 6 faces show the letters given below:

A	B	C	A	D	A

The die is thrown once. What is the probability of getting (i) A, (ii) D?

7. A card is drawn from a well-shuffled pack of 52 cards. Find the probability of getting (i) a red face card (ii) a black king.

8. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25?

9. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the drawn card is neither a king nor a queen.

3-MARKS

1. A bag contains 12 balls out of which x are white.

   i) If one ball is drawn at random, what is the probability that it will be a white ball?

   ii) If 6 more white bails are put in the bag, then the probability of drawing a white ball will be double than that in (i). Find the value of x.

2. Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?

3. A card is drawn at random from a well - shuffled deck of playing cards. Find the probability that the card drawn is

   i) A card of spades or an ace. ii) A red king

   iii) Either a king or a queen iv) neither a king nor a queen

4. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.
5. Two dice are thrown at the same time, determine the probability that the difference of the numbers on the two dice is 2.
6. A number is selected from the numbers 1, 2, 3, 4 and a second number is selected from the numbers 1, 5, 6, 12. Find the probability that the product of two numbers selected is less than 12.

5-MARKS

1. A jar contains blue and green marbles. The number of green marbles is 5 more than twice the number of blue. If probability of drawing a blue one at random is $\frac{2}{7}$,then how many blue and green marbles are there in the jar?
2. A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that the drawn card bears:

   a) A two digit number

   b) A number which is a perfect square.

3. A bundle of cards numbered from 5 to 20 are put in a box. One card is drawn at random from the box. What is the probability that the number on the card is?

   i) A single digit number? ii) A multiple of both 2 and 5?

4. A child's game has 8 triangles of which 3 are blue and rest are red and 10 squares of which 6 are blue and' rest are red. One piece is lost at random. Find the probability that it is a

   i) Triangle ii) square

   iii) Square of blue colour iv) triangle of red colour.

5. In a game, the entry fee is ₹ 5. The game consists of a tossing a coin 3 times. If one or two heads shown, Sweat gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she

   i) loses the entry fee

   ii) gets double entry fee

   iii) Just gets her entry fee

6. A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Veronika will buy a phone if it is good but the trader will only buy a mobile if it has no major defect. One phone is selected at random from the lot. What is the probability that it is?

   i) Acceptable to Veronika? ii) Acceptable to the trader?

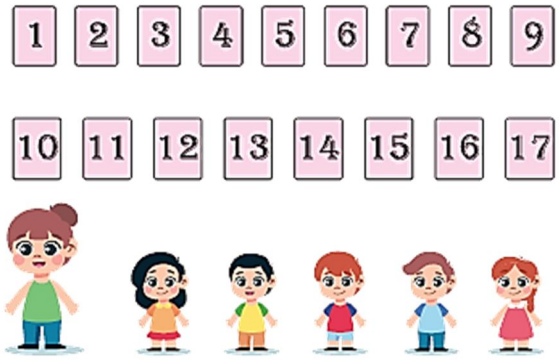
7. A bag contains 24 balls of which x are red, 2x are white and 3x are blue. A ball is selected at random. What is the probability that it is:

   i) Not red? ii) White?

CASE STUDY QUESTIONS

CASE STUDY_1

Five friends and one of their mother are having a picnic. The mother deicide to play card game. 17 cards numbered 1, 2, 3 ... 17 are put in a box and mixed thoroughly. The mother asks each boy to draw a card and after each draw she shows some magic tricks based on card number.



(i) What is the probability of drawing an odd number card in the first draw by the first boy? 1

(ii) Now in second draw, card drawn in first draw is replaced. What is the probability of drawing a prime number card by the second boy? 1

(iii) If in second draw, boy got number 2 and the card is not replaced, what is the probability of drawing a card bearing a multiple of 3 greater than 5 by the third boy? 2

OR

(iv) If the card is replaced after the third draw, what is the probability of drawing a card bearing a number greater than 17 by the fourth boy? 2

CASE STUDY_2

A game at a stall in New Year carnival involves spinning a wheel first as a first step to complete the game with certain rules. If the wheel stops at a particular number, then the player is allowed to roll a 6 faced unbiased dice.

Rules of Game:

(1) If the wheel stops at a particular number, then the player is allowed to roll a unbiased dice.

(2) If the wheel stops at any other number, player get to try again and only one extra try allowed.

(3) If player reach the next stage and roll a dice, he may get a prize depending on the number on dice.

(i) What is the probability of getting an even number on the wheel? 1

(ii) If getting an odd number on the wheel allows a player to roll the die, then what is the probability of his rolling the die? 1

(iii) If the player is allowed to roll the dice and getting a number greater than 4 entitles him to get prize, what is the probability of his winning the prize? 2

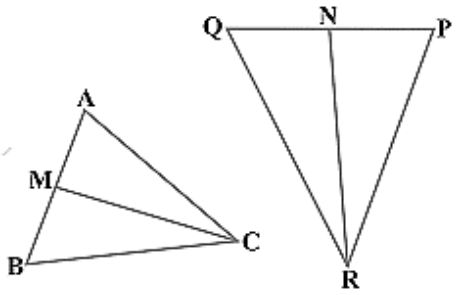
OR

If getting a square number on the wheel allows a player to roll the dice, then what is the probability of his rolling the dice?

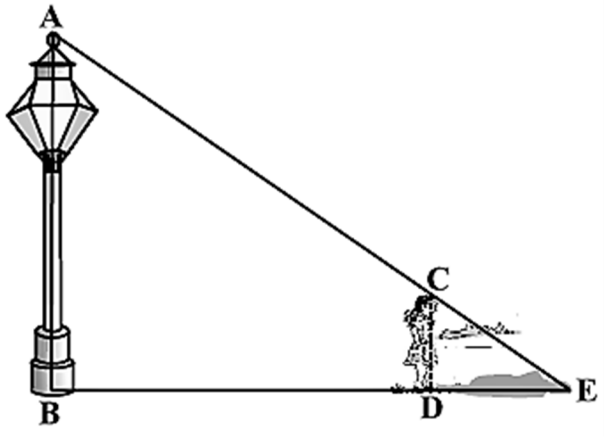
SKILL BASED QUESTIONS

1. In the below figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$, Prove that:

(i) $\Delta AMC \sim \Delta PNR$ (ii) $\frac{CM}{RN} = \frac{AB}{PQ}$ (iii) $\Delta CMB \sim \Delta RNQ$

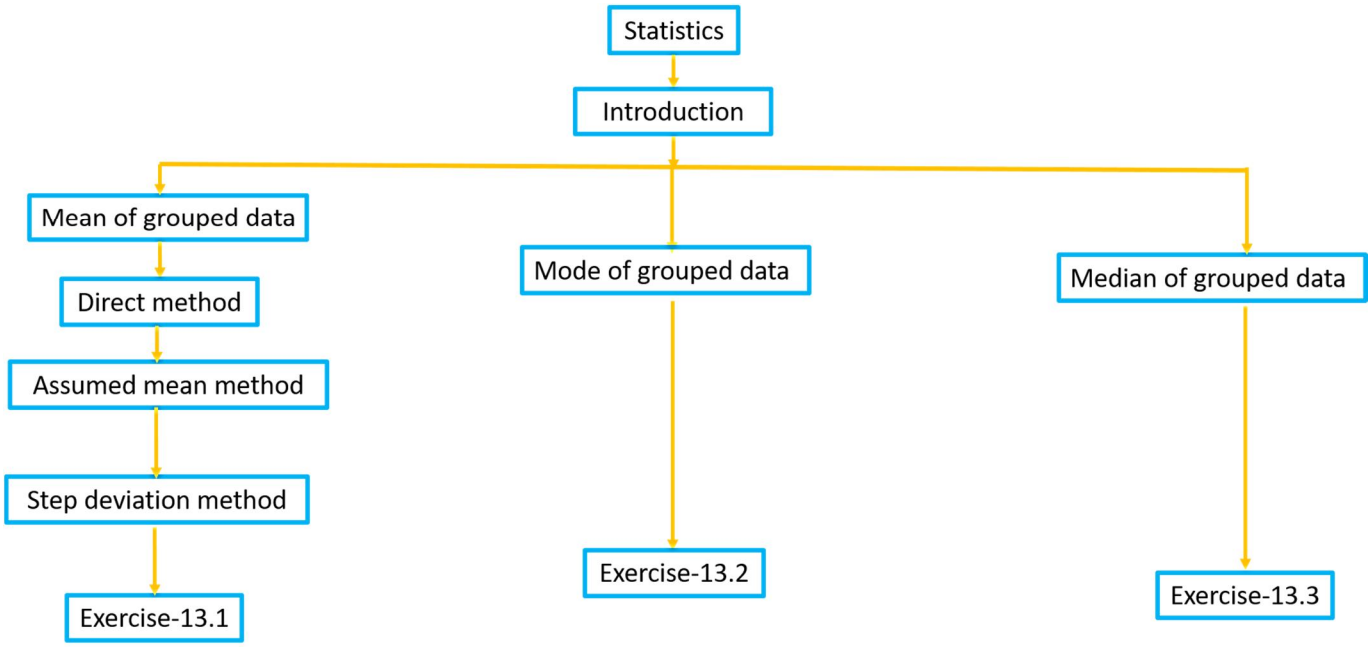


2. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



STATISTICS

MIND MAPPING



Basic facts and formulae:

1. Class mark = (Upper Class Limit + Lower Class Limit)/2
2. **Mean [Ungrouped Data]:** Mean of n observations, x1, x2, x3 … xn, is

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{1}{n}\sum x \qquad \therefore \qquad \overline{X} = \frac{\sum x}{n}$$

3. **MEAN [Grouped Data]:** The mean for grouped data can be found by the following three methods:

(i) **Direct method:** $\overline{X} = \dfrac{\sum f_i x_i}{\sum f_i}$

(ii) **Assumed Mean Method:** In this, an arbitrary mean 'a' is chosen which is called, 'assumed mean', somewhere in the middle of all the values of *x*.

$\overline{X} = a + \dfrac{\sum f_i d_i}{\sum f_i}$ … [where $d_i = (x_i - a)$]

(iii) **Step Deviation Method:** $\overline{x} = a + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) \times h$ where $u_i = \dfrac{x_i - a}{h}$; 'a' is the assumed mean and h is the class size.

4. Median is a measure of central tendency which gives the value of the middle-most observation in the data.

(i) **Ungrouped data:** If n is odd; Median = $\left(\frac{n+1}{2}\right)^{th}$ *observation*

If n is even; Median = $\frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$

(ii) **Grouped data:** Median = $l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$

Where [l = Lower limit of median class; n = Number of observations; f = Frequency of median class; c.f. = Cumulative frequency of preceding class; h = Class size].

5. **Mode of Ungrouped Data:** The value of the observation having maximum frequency is the mode.

6. **Mode of Grouped Data:** $Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

Where [l = Lower limit of modal class; f_1 = Frequency of modal class; f_0 = Frequency of the class preceding the modal class; f_2 = Frequency of the class succeeding the modal class; h = Size of class interval. c.f. = Cumulative frequency of preceding class]

7. Mode = 3 Median – 2 Mean

LEVEL_1

MCQ:

1. If the mode of a data is 45, mean is 27, then the value of median is:

(a) 30 (b) 27 (c) 33 (d) All of these

2. The mean $\left(\bar{x}\right)$ of a frequency distribution is 45. If the value of $\sum f_i = 20$ then the value of $\sum f_i x_i$ is:

(a) 800 (b) 900 (c) 2.25 (d) 4/9

3. The mean of first five prime numbers is:

(a) 5 (b) 5.6 (c) 6.5 (d) 7.8

4. What is the class mark of the class 10-25?

(a) 17.5 (b) 17 (c) 19.5 (d) 18.5

5. If the mean of five observations $x, x+2, x+4, x+6, x+8$ is 11, then find the value of x.

(a) 6 (b) 5 (c) 8 (d) 7

6. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of grouped data d_i's are deviation from 'a' is:

(a) lower limits of the classes (b) upper limits of the classes

(c) mid-points of the classes (d) frequencies of the class marks

7. If the mean of the following distribution is 6, then find the value of a.

x_i	2	4	6	10	a+5
f_i	3	2	3	1	2

(a) 4 (b) 3 (c) 5 (d) 9

8. If the mean of the following distribution is 2.6, then find the value of y.

Variable	1	2	3	4	5
Frequency	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

9. In the following distribution, find the number of families having income range 16000-19000 (in Rs).

Monthly income range in (Rs.)	**Number of families**
Income more than Rs. 10000	100
Income more than Rs. 13000	85
Income more than Rs. 16000	69
Income more than Rs. 19000	50
Income more than Rs. 22000	33
Income more than Rs. 25000	15

(a) 31 (b) 26 (c) 13 (d) 19

10. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of lower limits of the median class and modal class is:

(a) 15 (b) 25 (c) 30 (d) 35

2-MARKS:

1. Find the mean of the following data:

x	10	20	30	40	50
f	2	3	2	3	1

2. Mean of the following data is 20 then, find p.

x	15	17	19	20 + p	23
f	2	3	4	5p	6

3. The mean of the following data is 50. Find the missing frequencies.

x	10	30	50	70	90	Total
f	2	f_1	32	f_2	19	120

4. The median of the following distribution is 24, then find the missing frequency.

x	0-10	10-20	20-30	30-40	40-50
f	5	25	p	18	7

5. A survey was conducted by a group of students as a part of their environment awareness programme in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	7	2	3

6. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	1	2	1	5	7	2

Form the "Less than type" cumulative frequency distribution table.

7. Convert the following distribution to more than type cumulative frequency distribution.

Class	50-60	60-70	70-80	80-90	90-100
Frequency	1	2	1	5	5

8. Write the frequency distribution table for the data given below.

Marks	0 or more	100 or more	200 or more	300 or more	400 or more	500 or more	600 or more	700 or more	800 or more	900 or more
No.of students	100	98	93	84	72	55	35	20	11	4

9. In the following data, find the values of p and q. Also find the median class and modal class.

Class	**Frequency (f)**	**Number of families**
100-200	11	11
200-300	12	p
300-400	10	33
400-500	q	46
500-600	20	66
600-700	14	80

10. Find the mean of the following frequency distribution by direct method.

Marks	2-4	4-6	6-8	8-10
No.of students	2	5	5	3

11. The mean of 10 observations is 15.3. If two observations 6 and 9 are replaced by 8 and 14 respectively. Find the new mean.

12. The median of the following observations given in order 16, 18, 20, 24-x, 22 + 2x, 28, 30, 32 is 24. Find the value of x.

3-MARKS:

1. The daily income of a sample of 50 employees are tabulated as follows:

Marks	1-200	201-400	401-600	601-800
No.of students	14	15	14	7

Find the mean daily income of employees.

2. Find the median for the given frequency distribution.

Class	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Frequency	2	3	8	6	6	3	2

3. A class teacher has the following absentee record of 40 students of a class for the whole team. Find the mean number of days a student was absent.

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	10	11	7	4	4	3	1

4. The table below shows the salaries of 280 persons.

Salary (in thousands)	**No.of persons**
5-10	49
10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
45-50	1

Calculate the median salary of the data.

5. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table. Find the median length of leaves.

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No.of leaves	3	5	9	12	5	4	2

6. The table shows the daily expenditure on food of 25 household in the locality.

Daily expenditure	100-150	150-200	200-250	250-300	300-350
No.of house holds	10	11	7	4	4

Find the mean daily expenditure on food by a suitable method.

7. Find the mode of the following distribution of student marks.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No.of Students	8	20	45	58	70

8. Calculate the median for the following distribution:

Marks obtained	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No.of students	6	15	29	41	60	70

9. Find the mean of the following data:

Class	Less than 20	Less than 40	Less than60	Less than80	Less than 100
Frequency	15	37	74	99	120

10. The mean of the following table is 50. Find the missing frequencies f_1 and f_2.

Class	10-30	30-50	50-70	70-90	90-110	Total
Frequency	90	f_1	30	f_2	40	200

5-MARKS:

1. The mean of the following given distribution is 18. Find the frequency f of the class 19 – 21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

2. The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class-interval	**Frequency**
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

3. The following table shows the ages of the patient admitted in a hospital during a year. Find the mean by step deviation method.

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No.of patients	6	11	21	23	14	5

4. Find the mean of the following frequency distribution using assumed mean method.

Classes	2-8	8-14	14-20	20-26	26-32
Frequency	6	3	12	11	8

5. Find the mean of children per family from the data given below.

No.of children	0	1	2	3	4	5
No.of families	5	11	25	12	5	2

(i) Which mathematical concept is used in this problem?

(ii) What is its value?

6. Find the median for the following data.

Profit (in lakh of rupee)	**No.of shops**
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

7. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

8. Calculate the mode from the following data.

Monthly salary (in Rs)	**No.of employees**
Less than 5000	90
Less than 10000	240
Less than 15000	340
Less than 20000	420
Less than 25000	490
Less than 30000	500

9. Calculate the mode of the following frequency distribution table.

Marks	No.of Students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

CASE STUDY QUESTIONS

CASE STUDY_1

Transport department of a Jaipur wants to buy some Electric buses for the city. For which they wants to analyse the distance travelled by existing public transport buses in a day.

Daily distance travelled (in km)	200-209	210-219	220-229	230-239	240-249
No.of buses	4	14	26	10	6

The following data shows the distance travelled by 60 existing public transport buses in a day.

Based on the above information, answer the following questions.

(i) Find the median class of daily distance travelled? 1

(ii) What is the cumulative frequency of the class preceding the median class? 1

(iii) Find the median of the distance travelled. 2

OR

If the mode of the distance travelled is 223.78 km, find the mean of the distance travelled by the bus. 2

CASE STUDY_2

The Kendriya Vidyalaya Sangathan is a system of premier central government schools in India that are instituted under the aegis of the Ministry of Education (MHRD), Government of India. As of October 2020, it has a total of 1239 schools. It is one of the world's largest chains of schools. The system came into being in 1963 under the name 'Central Schools'. Later, the name was changed to Kendriya Vidyalaya. Its schools are all affiliated to the Central Board of Secondary Education (CBSE). The objective of KVS is to cater to the educational needs of the children of transferable Central Government employees including Defence and Para-Military personnel by providing a common programme of education.

Commissioner of Regional office Jaipur prepare a table of the marks obtained of 100 students which is given below:

Marks obtained	0-20	20-40	40-60	60-80	80-100
No.of students	15	18	21	29	p

He was told that mean marks of a student is 53.

Based on the above information, answer the following questions:

(i) What is the value of p? 1

(ii) What is the lower limit of modal class? 1

(iii) What is the value of modal marks? 2

OR

What is the value of median marks? 2

CASE STUDY_3

Amul, is an Indian dairy cooperative society, based at Anand in the Gujarat. Formed in 1946, it is a cooperative brand managed by a cooperative body, the Gujarat Co-operative Milk Marketing Federation Ltd. (GCMMF), which today is jointly owned by 36 lakh (3.6 million) milk producers in Gujarat. Amul spurred India's White Revolution, which made the country the world's largest producer of milk and milk products.

Survey manager of Amul dairy has recorded monthly expenditures on milk in 100 families of a housing society. This is given in the following frequency distribution:

Monthly expenditure (in Rs)	Number of families
0-175	10
175-350	14
350-525	15
525-700	x
700-875	28
875-1050	7
1050-1225	5

Based on the above information, answer the following questions:

(i) How many families spend between Rs 350- 700 on milk? 1

(ii) What is the upper limit of median class? 1

(iii) What is the median expenditure on milk? 2

OR

What is the modal expenditure on milk? 2

LEVEL_2

MCQ:

1. The median of the following data:

x:	10	20	30	40	50
y:	2	3	2	3	1

(a) 20 (b) 30 (c) 40 (d) 50

2. Consider the following distribution:

Marks obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
No.of students	63	58	55	51	48	42

Find the frequency of the class 30-40.

(a) 3 (b) 4 (c) 5 (d) 6

3. What is the median in the arranged series of an even number of 2n terms?

(a) $\left(\frac{n^{th}term+(n+1)^{th}\ term}{2}\right)$ (b) $\left(\frac{n^{th}term-(n+1)^{th}\ term}{2}\right)$

(c) $\left(\frac{2n^{th}term+(2n+1)^{th}\ term}{2}\right)$ (d) $\left(\frac{2n^{th}term+(2n+1)^{th}\ term}{3}\right)$

4. Consider the following frequency distribution:

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is:

(a) 7 (b) 17.5 (c) 18 (d) 18.5

5. Consider the following frequency distribution of the height of 60 students of a class.

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
No.of students	15	13	10	8	9	5

Find the sum of the lower limit of the modal class and the upper limit of the median class.

(a) 165 (b) 160 (c) 325 (d) 235

6. For the following distribution,

Marks	No.of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

The modal class is:

(a) 10-20 (b) 20-30 (c) 30-40 (d) 40-50

7. If the median of the data 6, 7, x-2, x, 17, 20 written in ascending order is 16, then find the value of x.

(a) 15 (b) 16 (c) 17 (d) 18

8. If the mean of the frequency distribution is 8.1 and $\sum f_i x_i = 132 + 5k$, $\sum f_i = 20$ then find the value of k.

(a) 3 (b) 4 (c) 5 (d) 6

9. If the mean of n observations is $\bar{x}$. If the first observation is increased by 1, the second by 2, the third by 3, and so on, then find the new mean.

(a) $x + \left(\frac{n-1}{2}\right)$ (b) $x + \left(\frac{n+1}{2}\right)$ (c) $x + \frac{n(n-1)}{2}$ (d) $x + n$

10. If the difference of mode and median of a data is 24, then find the difference of median and mean.

(a) 12 (b) 24 (c) 8 (d) 36

2-MARKS

1. Sum of 12 observations is 256, if one of the observation 14 is deleted, find the mean of the remaining observations.
2. Find the modal class and the median class for the following distribution:

C.I	0-10	10-20	20-30	30-40	40-50
Frequency	6	10	12	8	7

3. Write the frequency distribution table for the following data:

Marks	Above 0	Above 10	Above 20	Above 30	Above 40	Above 50
No.of students	40	38	31	25	20	0

4. Find the sum of lower limit of median class and upper limit of the modal class.

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3

5. Find the mean of the following data and hence find the mode, given that the median of the data is 42.5.

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	4	8	10	12	10	4	2

6. The monthly income of 100 families are given below:

Income (in Rs)	0 -5000	5000 -10000	10000 -15000	15000 -20000	20000 -25000
No.of families	4	8	10	12	10

Calculate the modal income.

7. The mean and median of 100 observations are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110 and 100. Find the true mean the median.

8. Find the unknown entries a, b, c, d, e, f in the following distribution of heights the students in a class.

Height (in cm)	**Frequency**	**Cumulative Frequency**
150–155	12	a
155–160	b	25
160–165	10	c
165–170	d	43
170–175	e	48
175–180	2	f
Total	50	

3-MARKS

1. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y:

Class	0-30	30-60	60-90	90-120	120-150	150-180
Frequency	12	21	x	52	y	11

2. Calculate the average daily income (in Rs.) of the following data about men working in a company:

Daily income (Rs)	< 100	< 200	< 300	< 400	< 500
Number of men	12	28	34	41	50

3. Find the mean of the following distribution:

Height (in cm)	Less than 75	Less than 100	Less than 125	Less than 150	Less than 175	Less than 200	Less than 225	Less than 250	Less than 275	Less than 300
No.of students	5	11	14	18	21	28	33	37	45	50

4. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

No.of letters	1-4	4-7	7-10	10-13	13-16	16-19
No.of surnames	6	30	40	16	4	4

5. Calculate the median for the following distribution:

Marks obtained	**Number of students**
Less than 10	14
Less than 20	22
Less than 30	37
Less than 40	58
Less than 50	67
Less than 60	75

6. The median of the distribution given below is 35. Find the value of x and y, if the sum of all the frequencies is 170.

Variable	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	10	20	x	40	y	25	15

7. The annual profits earned by 60 shops of a shopping complex in a given locality is described in the following distribution: Calculate median profit.

Profit (in thousands)	**Class Frequency**
More than 10	60
More than 20	56
More than 30	32
More than 40	28
More than 50	20
More than 60	14
More than 70	6

8. The median class of a frequency distribution is 125-145. The frequency and cumulative frequency of the class preceding to the median class are 20 and 22, respectively. Find the sum of the frequencies, if its median is 137.

5-MARKS

1. Find the mean, median and mode of the following data:

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	3	2	4	5	14

2. A health officer took an initiative organizing a medical camp in a remote village. The medical checkup of 35 students of the age group of 10 year and their weights were recorded as follows:

Weight (in Kg)	38-40	40-42	42-44	44-46	46-48	48-50	50-52
No.of Students	3	2	4	5	14	4	3

(i) Find the mean weight of the students using assumed mean method.

(ii) Calculate the median of the given data.

3. The median of the distribution given below is 14.4. Find the values of x and y, if the sum of the frequency is 20.

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	5	y	4	x	1

4. The distribution of heights (in cm) of 96 children is given below: Calculate the median.

Height (in cm)	**No.of children**
124-128	5
128-132	8
132-136	17
136-140	24
140-144	16
144-148	12
148-152	6
152-156	4
156-160	3
160-164	1

5. The following is the cumulative frequency distribution (of less than type) of 1000 persons each of age 20 years and above. Determine the mean age.

Age below(in years)	30	40	50	60	70	80
No.of persons	100	220	350	750	950	1000

CASE STUDY QUESTIONS

CASE STUDY_1

Formula one Portuguese Grand Prix technical team at the Algarve International Circuit are analysing last year data of drivers' performance to provide valuable inferences to commentators on how the drivers can improve this year.

The length of time taken by 80 drivers to complete a journey is given in the table below:

Time (in mins)	70-80	80-90	90-100	100-110	110-120	120-130
No.of drivers	4	10	14	20	24	8

(i) What is the estimate of the mean time taken to complete the journey? 2

OR

What is the median time taken to complete journey? 2

(ii) In which interval does the median of the distribution lie? 1

(iii) In which interval does the mode of the distribution lie? 1

LEVEL_3

MCQ:

1. Observations of some data are $\frac{x}{5}, x, \frac{x}{3}, \frac{2x}{3}, \frac{x}{4}, \frac{2x}{5} and \frac{3x}{4}$ where $x>0$. If the median of the data is 4, then what is the value of x?

   (a) 4 (b) 6 (c) 8 (d) 10

2. If the mean of the squares of first n natural numbers is 105, then find the first n natural numbers.

   (a) 8 (b) 9 (c) 10 (d) 11

3. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. What is the mode of this set of numbers?

   (a) 4 (b) 5 (c) 8 (d) 10

4. The mean weight of 9 students is 25 kg. If one more student is joined in the group the mean is unaltered, then find the weight of the 10th student (in kg).

   (a) 25 (b) 24 (c) 26 (d) 23

5. The mean and median of the data a, b and c are 50 and 35 respectively, where $a<b<c$. If $c-a=55$, then find the value of $(b-a)$.

   (a) 8 (b) 7 (c) 3 (d) 5

6. If median is 137 and mean is 137.05, then what is the value of the mode?

   (a) 136.90 (b) 135.75 (c) 134.70 (d) 135.79

7. If x_i's are the midpoints of the class intervals of grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then find $\sum\left(f_i x_i - \bar{x}\right)$.

   (a) 0 (b) -1 (c) 1 (d) 2

8. What is the modal class for the following distribution?

Marks	Number of Students	Marks	Number of students
Below 10	3	Below 40	57
Below 20	12	Below 50	75
Below 30	28	Below 60	80

(a) 40-50 (b) 30-40 (c) 50-60 (d) 20-30

2-MARKS

1. Convert the following data into more than type distribution:

Class interval	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	2	8	12	24	38	16

2. Construct the cumulative frequency distribution of the following distribution and find the median class.

Class interval	12.5-17.5	17.5-22.5	22.5-27.5	27.5-32.5	32.5-27.5
Frequency	2	22	19	14	13

3. The marks obtained by 60 students, out of 50 in a Mathematics examination, are given below.

Marks	0-10	10-20	20-30	30-40	40-50
No.of Students	2	22	19	14	13

Write the above distribution as "Less than type cumulative frequency distribution".

4. The mean of ungrouped data and the mean calculated when the same data is grouped are always the same. Do you agree with this statement? Give reason for your answer.

5. The mean of the following data is 7.5. Find the value of p.

x_i	3	5	7	9	11	13
f_i	6	8	15	p	8	4

6. The median of ungrouped data and the median calculated when the same data is grouped are always the same. Do you agree with this statement? Give reason for your answer.

7. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

8. Will the median class and modal class of grouped data always be different? Justify your answer.

3-MARKS:

1. 100 surnames were randomly picked up from a telephone directory, distribution of the number of letters of the English alphabet in the surnames are obtained as follows:

No.of letters	1-4	4-7	7-10	10-13	13-16	16-19
No.of surnames	6	30	40	16	4	4

Determine the median and mean number of letters in the surnames. Also, find the modal size of surnames.

2. The mean of the following distribution is 314. Determine the missing frequency x.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	10	12	7	8

3. Prove that $\sum\left(x_i-\overline{x}\right)=0$.

4. Find the mean of the following data:

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	15	37	74	99	120

5. Compute the median from the following data:

Mid-values	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

6. The mode of a distribution is 55 and the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

7. The sum of deviations of a set of values $x_1,x_2,x_3,....x_n$ measured from 50 is -10 and the sum of deviations of the values from 46 is 70. Find the value of n and the mean.

8. The arithmetic mean of the following frequency distribution is 53. Find the value of k .

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	k	13

5-MARKS

1. A class teacher has the following absents record of 40 students of a class for the whole term. Find the mean number of days a student was absent (Solve it by Step deviation method).

No.of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40
No.of students	11	10	7	4	4	3	1

2. The mean of the following frequency distribution is 62.8. Find the missing frequency f.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	f	12	7	8

3. The following distribution given the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

No.of students per teacher	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No.of states/U.T	3	8	9	10	3	0	0	2

4. A survey was conducted to give the percentage distribution of doctors in hospitals of rural areas of various states and various Union Territories (UT) of India are given in the following table.

Percentage of doctors	**Number of states/UT**
15-25	6
25-35	11
35-45	7
45-55	4
55-65	4
65-75	2
75-85	1

(i) Find the mean percentage of doctors of rural areas of various states and union territories.

(ii) Suppose there are two persons Ram and Shyam. If Ram find out the mean by direct method and Shyam find out the mean by Assumed mean method, then whether both of them get the same value. Explain the reason.

5. Find the mean and median marks of the students for following distribution:

Marks	Number of Students
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

6. Following is the cumulative frequency distribution of less than type of 1000 persons each of age 20 years and above. Determine the mean age.

Age (in years)	Number of persons
Below 30	100
Below 40	220
Below 50	350
Below 60	750
Below 70	950
Below 80	1000

CASE STUDY QUESTIONS

CASE STUDY_1

Apples are most widely planted and are commercially the most important fruit crop in Jammu and Kashmir. The cultivation of apple crop in Jammu and Kashmir shows particular interest for a number of reasons. In terms of both area and production, apple is very beneficial fruit crop. This provides a major source of income and employment in Jammu and Kashmir.

Horticultural department has tasked their statistical officer to create a model for farmers to be able to predict their produce output based on various factors.

A box containing 250 apples was opened and each apple was weighed. The distribution of the weight of the apples is given in the following table:

Weight (in grams)	No. of f_i Apples
80-100	20
100-120	60
120-140	70
140-160	40
160-180	60

Based on the above information, answer the following questions:

(i) How many apples are in the range 140-160 grams? 1

(ii) What is the upper limit of the median class? 1

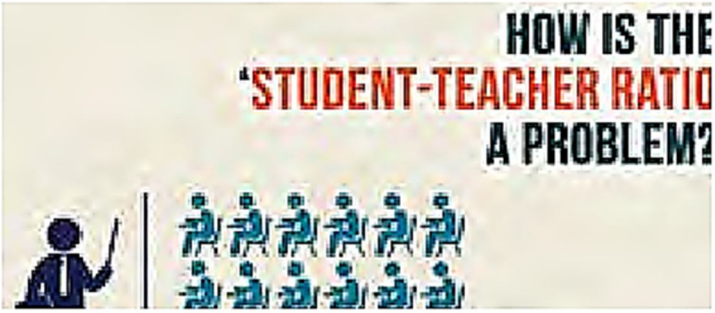
(iii) What is the modal mass of the apples? 2

OR

What is the median mass of the apples? 2

CASE STUDY_2

Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number teachers employed by the school. Student-teacher ratio is important for a number of reasons. It can be used as a tool to measure teacher workload as well as the allocation of resources. A low student-teacher ratio indicates the burden on a single teacher of teaching multiple students as well as the lack of time that each student gets.



A survey was conducted in the 100 secondary school of Rajasthan and following frequency distribution table was prepared:

Students per teacher	Number of School
20-25	5
25-30	15
30-35	25
35-40	30
40-45	15
45-50	10

Based on the above information, answer the following questions:

(i) What is the upper limit of median class? 1

(ii) What is the median value of students per teacher? 2

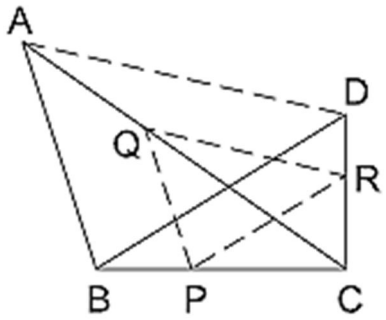
OR

What is the mean value of students per teacher? 2

(iii) What is the lower limit of modal class?

SKILL BASED QUESTIONS

1. ΔABC and ΔDBC lie on the same side of BC, as shown in the figure. From a point P on BC, $PQ \parallel AB$ and $PR \parallel BD$ are drawn, meeting AC at Q and CD at R respectively. Prove that $QR \parallel AD$.



2. Find the area of the segment AYB as shown in the below figure. If the radius of the circle is 21 cm and $\angle AOB = 120^0$ (Use $\pi = \frac{22}{7}$).