

ZONE 3 – NORTH CLUSTER (A unit of CSSC)

Revision Examination

Class : XII

Mathematics – 041 Set –A

Max.Marks :80

DATE: 28.12.2024

Time : 3 hours

General Instructions :

- i. Section A comprises of 20 questions of 1 mark each. Section –B comprises of 5 questions of 2 marks each. Section –C comprises of 6 questions of 3 marks each. Section – D comprises of 4 questions of 5 marks each. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
- ii. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 3 marks each and two questions of 5 marks each. You have to attempt only one of the alternatives in all such questions. **All questions are compulsory.**
- iii. This question paper contains 38 questions divided into 5 sections.
- iv. Use of calculators is not permitted.

SECTION –A

I. Choose the Correct Answer :

20 x 1 = 20

- ✓ Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and Symmetric but not transitive is

$$\{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1), (2, 3), (3, 2), (1, 2, 3), (2, 3, 1), (3, 1, 2)\}$$

- a. 1 b. 2 c. 3 d. 4

- ✓ The critical value of the function $f(x) = e^x \sin x$, $x \in [0, \pi]$ is

- a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$ d. $\frac{3\pi}{4}$

- ✓ If $\int_a^0 \frac{1}{1+4x^2} dx = \frac{\pi}{8}$ then find value of a.

- a. $-\frac{1}{2}$ b. $\frac{7}{2}$ c. $\frac{1}{2}$ d. 0

- ✓ What is the sum of order and degree of the following differential equation?

$$5x \left(\frac{dy}{dx} \right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$$

- (a) 1 (b) 2 (c) 3 (d) 4

OR

The number of arbitrary constants in a particular solution of a differential equation of third order is

- (a) 3 (b) 2 (c) 1 (d) 0

- ✓ The vector of the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

- a. $\hat{i} - 2\hat{j} + 2\hat{k}$ b. $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ c. $3(\hat{i} - 2\hat{j} + 2\hat{k})$ d. $9(\hat{i} - 2\hat{j} + 2\hat{k})$

- ✓ The solution set of the in equation $3x + 5y > 13$ is

- a) half plane that contains origin b) open half plane not containing origin
c) $xy =$ plane except the points lying on $3x + 5y = 13$ d) none of these

7. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is
- a. $\frac{33}{56}$ b. $\frac{9}{64}$ c. $\frac{1}{14}$ d. $\frac{3}{28}$

8. The function $f(x) = e^{|x|}$ is
- a) continuous everywhere but not differentiable at $x = 0$ b) continuous and differentiable everywhere.
c) not continuous at $x = 0$ d) none of these.

OR

The function $f(x) = \begin{cases} \frac{\sin 3x}{x} & x \neq 0 \\ \frac{k}{2} & x = 0 \end{cases}$ is continuous at $x = 0$. Then value of k is

- (a) 3 (b) 6 (c) 9 (d) 12
9. Set A has 3 elements and Set B has 4 elements. Then the number of injective mapping that can be defined from A to B

- a. 48 b. 24 c. 25 d. 28

10. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

- a. 9 b. 64 c. 512 d. 32

11. If f and g are continuous functions in $[0, 1]$ satisfying $f(x) = f(a-x)$ and $g(x) + f(a-x) = a$, then

$\int_0^a f(x) \cdot g(x) dx$ is equal to

- a. $\int_0^a f(x)$ b. $2 \int_0^a f(x)$ c. $\frac{a}{2} \int_0^a f(x)$ d. $\int_0^a f(x) \cdot g(x) dx$

12. The area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$ and $y = -1$ is

- a. 6 b. 5 c. 4 d. 12

13. The rate of change of volume of a sphere with respect to its surface area when the radius is 2cm is

- a. 2 b. 3 c. 5 d. 4

OR

If x is real, the minimum value of $f(x) = x^2 - 8x + 17$ is

- (a) -1 (b) 0 (c) 2 (d) 1

14. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

- a. 3 b. 2 c. 5 d. 2

15. The coordinates of the foot of the perpendicular drawn from the point $(2, 5, 7)$ on the x -axis are given by

- a. $(0, 0, 0)$ b. $(2, 0, 0)$ c. $(5, 0, 0)$ d. $(7, 0, 0)$

16. The value of λ for which the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is

- a. $-\frac{5}{2}$ b. 5 c. 2 d. $\frac{3}{2}$

The probability distribution of a discrete random variable X is given below:

x	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is _____

a. 23

b. 32

c. 25

d. -32

18. If $f(x) = |\cos x - \sin x|$, then the value of $f'(\pi/6) =$ _____

a. $\frac{-1+\sqrt{3}}{2}$

b. $\frac{-1-\sqrt{3}}{2}$

c. $\frac{-1+\sqrt{3}}{-2}$

d. $\frac{\sqrt{3}}{2}$

ASSERTION AND REASON

- (a) Both A and R are individually true but R is the correct explanation of A.
 (b) Both A and R are individually true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

19. Assertion (A): For two sets $A = R - \{3\}$ and $B = R - \{1\}$ the function $f: A \rightarrow B$ defined as $f(x) = \frac{x-2}{x-3}$ is bijective. (b)

Reason (R): A function $f: A \rightarrow B$ is said to be surjective if $\forall y \in B, x \in A$ such that $f(x) = y$

20. Assertion (A): For a matrix A of order 3, if $\det(\text{adj } A) = 49$, then $\det(A) = \pm 7$

Reason (R): For a square matrix of order n , $|\text{adj } A| = |A|^{n-1}$ (a)

SECTION - B

II. Answer the following :

21. Show that the relation R on the set R of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive. $(\frac{1}{2}, \frac{1}{2})$ | $(2, 5)$ | $(1, -1)$
 Or $(5, 2)$ | $(-1, 0)$ | $(1, 0)$

Let $f: R \rightarrow R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}$, $\forall x \in R$. Then, find the range of f .

22. If $A = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$, show that $A - A^T$ is a skew symmetric matrix where A^T is the transpose of matrix A .

23. If A is a 3×3 invertible matrix, then show that for any scalar k (non zero), kA is invertible and $(kA)^{-1} = \frac{1}{k} A^{-1}$

24. Find the value of $\vec{a} \cdot \vec{b}$ if $|\vec{a}|=10$, $|\vec{b}|=2$ and $|\vec{a} \times \vec{b}|=16$. ± 12

25. If \vec{a}, \vec{b} and \vec{c} are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}|=2$, $|\vec{b}|=3$ and $|\vec{c}|=5$, then find the value

of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ -19

OR

Find the direction ratios and direction cosines of a line parallel to the equations $6x - 12 = 3y + 9 = 2z - 2$.

SECTION - C

III. Answer the following :

$$6x + 3 = 18$$

26. Solve the following linear programming problem graphically

$$\text{Maximize } Z = 22x + 18y \quad \text{subject to constraints } 3x + 2y \leq 48, x + y \leq 20, x, y \geq 0$$

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 $(3, 12)$

27. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of Rs. 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for Rs. 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs. 70. Using matrix method, find cost of each variety of pen.

$Rs. 28, 38$

28. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

29. Evaluate: $\int \sqrt{\tan x} + \sqrt{\cot x} dx = -\sqrt{2} \sin^{-1}(\cos x - \sin x) + C$
OR
 $20 - 24 = 16$

$$\text{Find: } \int \frac{1-x^2}{x(1-2x)} dx$$

30. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$, where A is a parameter.

Solve the differential equation : $x \frac{dy}{dx} + y - x + xy \cot x = 0$ and $x \neq 0$.
OR
 $xy \sin x = \frac{1}{-x \cos x} + C$
 $= \sin x - x \cos x + C$

31. Prove that $\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) = \frac{\pi}{4} - \frac{x}{2}$ $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$

SECTION - D

III. Answer the following :

32. A window is in the form of rectangle surmounted by a semi-circular opening. Total perimeter of the window is 10 m. What will be the dimensions of the whole opening to admit maximum through the whole opening?

$\frac{10}{4+\pi}, \frac{10}{4+\pi}$

33. Find the vector equation of the line that passes through point $(2, 3, 2)$ and parallel to the line $\vec{r} = 2\hat{i} + 3\hat{j} + \mu(\hat{i} - 3\hat{j} + 6\hat{k})$. Also find the distance between them.

$9\sqrt{23}$ units

34. A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from the bag I to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red, then find the probability that one red and one white are transferred from the bag I to the bag II.

OR

Find the probability distribution of number of doublets in three throws of a pair of coins.

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$

35. Using integration find the area of the triangle ABC, whose vertices are $A(2, 5)$, $B(4, 7)$ and $C(6, 2)$.

OR

Using integration, find the area bounded by $|x - 1|$ and $y = 1$.

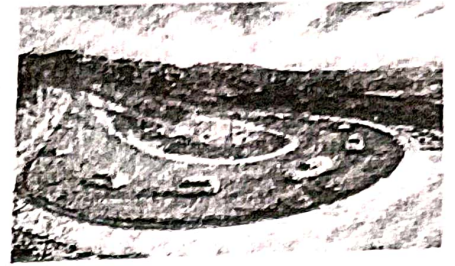
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SECTION - E

Case study:

36. A car is moving on the curvy roads of a beautiful mountain. The slope of the tangent to a curve at any point is reciprocal of the twice the ordinate of that point. The curve passes through $(4, 3)$.

Based on the above information, answer the following questions :



1. The differential equation related to the problem is

- (a) $\frac{dy}{dx} = 2x$ (b) $\frac{dy}{dx} = 2y$
~~(c) $\frac{dy}{dx} = \frac{1}{2x}$~~ (d) $\frac{dy}{dx} = \frac{1}{2y}$

2. By which method can this differential equation be solved

- (a) Variable Separable method
 (b) Homogeneous Differential equation method
 (c) Linear differential equation method
 (d) Any of above three method

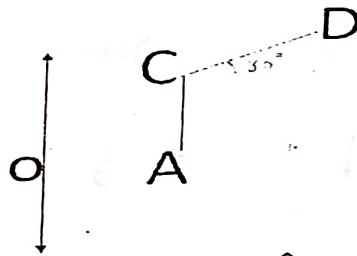
3. The solution of the differential equation is

- (a) $y^2 = x^2 + C$ (b) $y^2 = x + C$ (c) $y = x^2 + C$ (d) $y = x^3 + C$
 (Or)

The particular solution of differential equation is

- (a) $y^2 = x^2 + 5$ (b) $y^2 = x + 2$ (c) $y = x^2 + 3$ (d) $y = x^3 + 4$

37. Ramesh's house is situated at point O, He first travel 8 km in the east direction and reaches at point A, from A he takes an auto and goes 6 km in north direction to reach at his office at point C. Mahesh's house is situated at point D which is 30 degree north of east and 6 km from the point C.



1. Write the vector OC using standard notation ($\hat{i}, \hat{j}, \hat{k}$)

- a. $8\hat{i} + 6\hat{j}$ b. $8\hat{i} - 6\hat{j}$ c. $-8\hat{i} + 6\hat{j}$ d. $-8\hat{i} - 6\hat{j}$

2. Write the magnitude of vector OC

- a. 10 units b. 100 units c. ± 10 units d. None of these

3. Write the vector CD using standard notation ($\hat{i}, \hat{j}, \hat{k}$).

- a. $\sqrt{3}\hat{i} + \hat{j}$ b. $3\sqrt{3}\hat{i} + 3\hat{j}$ c. $6\hat{i}$ d. $6\hat{j}$

OR

Write the total distance between Ramesh's house and Mahesh's house.

- a. 11 km b. 13 km c. 20 km d. 25 km

38. A pharmaceutical company wants to advertise a new product on T.V., where the product is specially designed for women. For that an advertising executive is hired to study television-viewing habits of married couples during prime-time hours. Based on past viewing records he has determined that during prime-time husbands are watching television 70% of the time. It has also been determined that when the husband is watching television, 30% of the time the wife is also watching. When the husband is not watching television, 40% of the time the wife is watching television.

Based on the above information, answer the following questions.

(i) The probability that the husband is not watching television during prime time, is

- (a) 0.6 (b) 0.3 (c) 0.4 (d) 0.5

(ii) If the wife is watching television, the probability that husband is also watching television, is

- (a) 2/11 (b) 7/11 (c) 5/11 (d) 8/11