BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)
Grade X

DATE: 12.12.2024 **MAX MARKS: 80**
Subject: MATHEMATICS – STANDARD (041) **TIME : 3 hours**

SET - 1

MARKING SCHEME

Section A

1. (b) -6 only	**11. (b) 9**
2. (b) (-11/8, 7/4)	**12 (c) 18**
3. (d) 0, 8	**13** (b) $\sqrt{2}$
4. (a) 1	**14** (d) 4^{th} Quadrant
5. (b) 1	**15** (c) $\angle B = \angle D$
6. (c) 30 – 40	**16 (b) 70°**
7. (d) 18	**17 (c) 5/6**
8. (a) 8a **9. (a) $4\pi r^2$** **10. (c) 16 : 81**	**18 (a) 2.7 cm** **19. (c)** Assertion-TRUE, Reason: FALSE, **20 (d) Assertion is false but Reason is true**

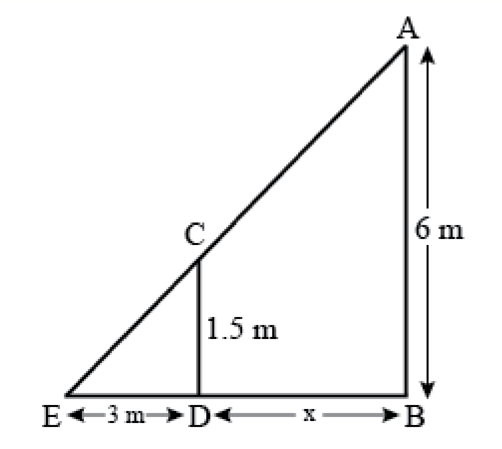
Section B

21. If α and β are the zeros of a quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, find a quadratic polynomial having α and βas its zeroes. Given $\alpha + \beta = 24$ ……(i) $\alpha - \beta = 8$ ……(ii) By subtracting equation (ii) from (i) we get $\alpha + \cancel{\beta} = 24$ $\alpha - \cancel{\beta} = 8$ $2\alpha = 32$ $\alpha = \frac{32}{2}$ $\alpha = 16$	½
Substituting $\alpha = 16$ in equation (i) we get, $\alpha + \beta = 24$ $16 + \beta = 24$ $\beta = 24 - 16$ $\beta = 8$	½
Let S and P denote respectively the sum and product of zeros of the required polynomial. then,	

$S = \alpha + \beta$ $= 16 + 8$ $= 24$ $P = \alpha\beta$ $= 16 \times 8$ $= 128$ Hence, the required polynomial if $f(x)$ is given by $f(x) = k(x^2 - Sx + P)$ $f(x) = k(x^2 - 24x + 128)$ Hence, required equation is $\boxed{f(x) = k(x^2 - 24x + 128)}$ where k is any non-zeros real number.	½ ½
22. If sec θ + tan θ = m and sec θ – tan θ = n, find the value of $\sqrt{mn}$. From the question it is given that, sec θ+tan θ=m sec θ−tan θ=n $\sqrt{mn}$ =(sec θ+tan θ)(sec θ–tan θ) $=\sec^2\theta - \tan^2\theta$ = 1	1 1
23. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have ? Given that 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If I each stack is of the same height and contains cartons of the same drink We need to find the greatest number of cartons, each stack would have Given that, Number of cartons of coke cans = 144 Number of cartons of Pepsi cans = 90. Therefore, the greatest number of cartons in one stack = H.C.F. of 144 and 90. HCF = **18** **(OR)**	1 1
(B) If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of "m" Number 144 = $2^4 \times 3^2$ Number 180= $2^2 \times 3^2 \times 5$ ∴H.C.F (180,144) = $2^2 \times 3^2$= 4 x 9 = 36 Given HCF = 13m - 3 13m - 3 = 36 ⇒13m = 39 ⇒m = 3 ∴ m = 3 Answer : m = 3	½ ½ ½ 1/2

24. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how far she is away from the base of the pole?

Let A be the position of the street bulb fixed on a pole AB = 6m and CD = 1.5m be the height of a woman and the shadow be ED = 3m. Let the distance between pole and woman be x m.



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Here, woman and pole both are standing vertically,

So, CD II AB

In ΔCDE and ΔABE,

∠E=∠E [common angle]

∠ABE=∠CDE [each equal to 90∘]

∴ ΔCDE∼ΔABE [by AAA similarity criterion] ½

Then, EDEB=CDAB

⇒ 33+x=1.56=14

⇒ 3×4=3+x

⇒ 12=3+x

∴ x=9 m ½

Hence, she is at the distance of 9m from the base of the pole.

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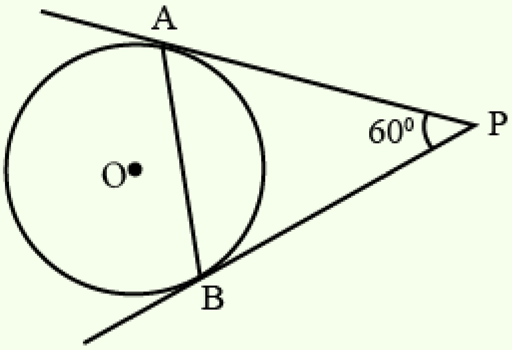
25. If PA and PB are tangents from an outside point P such that PA = 10 cm and ∠APB = 60˚. Find the length of the chord AB.

PA and PB are the tangents from a point P outside the circle with centre O.

PA = 10 cm and ∠APB = 60°



Tangents drawn from a point outside the circle are equal. 1/2

PA = PB = 10 cm

∠PAB = ∠PBA

(Angles opposite to equal sides)

But in ΔAPB,

∠APB + ∠PAB + ∠PBA = 180° (Sum of angles of a triangle) 1/2

=> 60° + ∠PAB + ∠PAB = 180°

=> 2 ∠PAB = 180° – 60° = 120°

∠PAB = 60°

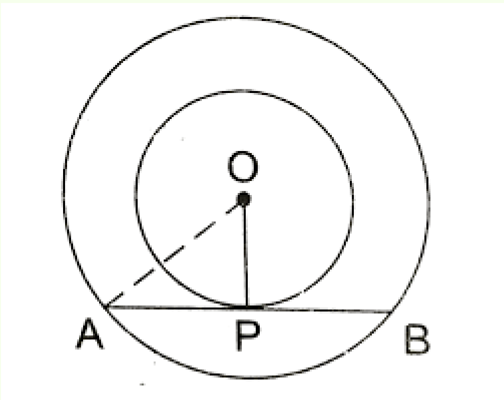
∠PBA = ∠PAB = 60°

PA = PB = AB = 10 cm 1/2

Hence, length of chord AB = 10 cm 1/2

(OR)

(B) Out of the two concentric circles with centre O, the radius of the outer circle is 5 cm and the chord AB of length 8 cm is tangent to the inner circle. Find the radius of the inner circle.



OP is perpendicular to AB and therefore bisects AB. Hence AP = ½ AB = 4cm 1

Consider Δ AOP

By Pythagoras Theorem, we find OP = 3cm

Ans : Radius of the inner circle = 3 cm 1

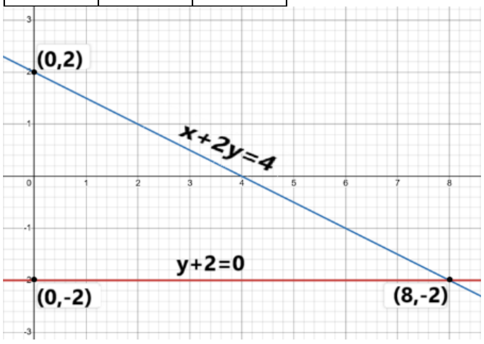
Section C

(Section C consists of 6 questions of 3 marks each)

26. (A) Find the solution $y+2=0$ and $x+2y=4$ graphically. Find the area the triangle formed by these lines and Y-axis.

$y+2=0$			
x	0	8	½
y	-2	-2	

$x+2y=4$			
x	0	8	
y	2	-2	½



1 ½

Area of the triangle$=\frac{base X height}{2}=\frac{4X8}{2}=16\ sq\ units$ ½

(OR)

(B) Check if the pair of equations, $y-2x=1$ and $5y-x=14$, is consistent. If yes, find a solution of it graphically.

$$\frac{a_1}{a_2}=\frac{-2}{-1}=\frac{2}{1}$$

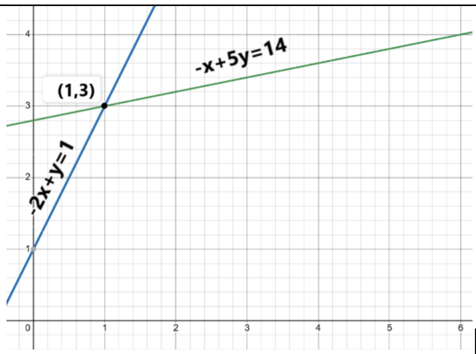
$$\frac{b_1}{b_2}=\frac{1}{5}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$ 1/2

So, the given is a pair of intersecting lines with a unique solution,

$y-2x=1$			
x	0	-1/2	
y	1	0	½

$5y-x=14$			
x	1	6	1/2
y	3	4	



[1M +1M] 1

The solution is (1,3) [1/2 M]

27. A juice seller serves his customers using a glass as shown in figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. [π = 3.14]

Solution : Since the inner diameter of the glass = 5 cm and height = 10 cm, 1

the apparent capacity of the glass = $\pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$ 1

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by $\frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3 = 32.71 \text{ cm}^3$

So, the actual capacity of the glass = apparent capacity of glass – volume of the hemisphere 1

$= (196.25 - 32.71) \text{ cm}^3$

$= 163.54 \text{ cm}^3$

28. Prove that $2 + \sqrt{5}$ is an irrational number

- Contradictory proof , assuming $\sqrt{5}$ is irrational
- Setting up the contradiction with all parameters of a rational number _ (1mark)
- Correct proof - (2 marks)

29. 17 cards numbered 1, 2, 3,……,16,17 are put in a box and mixed thoroughly. 1/2

One person draws a card from the box ½

The number of total possible outcomes = 17 ½

Number of favourable outcomes for a number to be odd = 9 [i.e., ½

1,3,5,7,9,11,13,15,17]

Therefore, p=9/ 17 ½

Similarly, for prime, p=7 / 17

For number divisible by 3, p=5 / 17 1

For number divisible by 3 and 2, p=2 / 17

30. (A)Prove that : $\sin\theta\,(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ 1

LHS = $\sin\theta\,(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta)$

= $\sin\theta + \sin\theta \,.\, \tan\theta + \cos\theta + \cos\theta \,.\, \cot\theta$

= $\sin\theta + \sin\theta \,.\, \dfrac{\sin\theta}{\cos\theta} + \cos\theta + \cos\theta \,.\, \dfrac{\cos\theta}{\sin\theta} = \sin\theta + \dfrac{\cos^2\theta}{\sin\theta} + \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$ 1

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta} + \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta}$

= $\operatorname{cosec}\theta + \sec\theta = \sec\theta + \operatorname{cosec}\theta$ = RHS

(OR)

1

(B) Prove that: $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

1

ANS: LHS = $(\sin^4\theta)^2 - (\cos^4\theta)^2$

= $(\sin^4\theta + \cos^4\theta)(\sin^4\theta - \cos^4\theta)$

= $(\sin^4\theta + \cos^4\theta)(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$ 1

= $(\sin^2\theta - \cos^2\theta)\,\{(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta\}$

= $(\sin^2\theta - \cos^2\theta)\,\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\}$

= $(\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$ = RHS

1/2

31. A ΔABC is drawn to circumscribe a circle of radius 4cm such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the lengths of sides AB and AC, when area of ΔABC is 84 cm^2. 1/2 1

Let AE = AF = x cm ; BE = BD = 8 cm ; CD = CF = 6 cm 1

(length of tangents from an external point are equal

Area of Δ **ABC = ½ r (Perimeter of Δ ABC)**

84 cm^2 = ½ x 4 x (8+6 + 6 + x + 8 + x)

Solving the above equation , we get x = 7

Ans : Hence AB = 15 cm and AC = 13 cm

32. In a stream running at 2 km per hour, a motor boat goes 10 km upstream and back again to the starting point in 55 minutes. Find the speed of the motor boat in still water.

speed of the stream= 2 km/h
Let the speed of the boat in still water = x km/h [1/2 M]
Speed upstream = x-2 km/h
Speed downstream = x+2 km/h [1/2 M]
ATQ: $\frac{10}{x+2}+\frac{10}{x-2}=\frac{55}{60}$ [1/2 M]

$\frac{10(x-2+x+2)}{(x+2)(x-2)}=\frac{11}{12}$

$\frac{20x}{x^2-4}=\frac{11}{12}$

$240x = 11x^2 - 44$

$11x^2 - 240x - 44 = 0$ [1M]

$11x^2 - 242x + 2x - 44 = 0$ [1/2 M]

$11x(x-22)+2(x-22)=0$

$(x-22)(11x+2)$

$x = 22$ or $x = -\frac{2}{11}$ [1 M]

Speed cannot be negative. So $-\frac{2}{11}$ is not admissible.[1/2 M]

$\therefore x = 22$

Speed of the boat in still water is 22 km/h [1/2 M]

(OR)

At present, Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

ANS: Let mom's present age be x years mom's age, $x = 2+y^2$ [1/2M]
And Daughter's present age be y years [1/2 M]
Daughter becomes mom's age in x-y years
After x-y years, Mom's age= $x+x-y \ = \ 10y-1$

$2x—y-10y+1=0$

$2x-11y=-1$ [1/2 M]

Substituting x in the above

$2(2+y^2)-11y=-1$ [1/2 M]

$4+2y^2-11y+1=0$

$2y^2-11y+5=0$ [1 M]

$2y^2-10y-y+5=0$

$2y(y-5)-1(y-5)=0$

$(y-5)(2y-1)=0$

$y=5$ or $y=\frac{1}{2}$ [1 M]

When y=5.

$x = 2+5^2$

$x = 27$

When $y=\frac{1}{2}$

$x = 2+\frac{1}{4} \ = \frac{13}{4}$ years, this age is not suitable for mother's age.

So $y=\frac{1}{2}$ is not admissible. [1/2 M]

$\therefore$ Asha's age = 27 years.

Nisha's age = 5 years. [1/2 M]

33. The median of the distribution given below is 15.75.

Class interval	0-6	6-12	12-18	18-24	24-30	30-36	36-42
frequency	4	x	8	y	4	2	1

(i) Find the values of x and y, if the total frequency is 30.
(ii) Then, find the mode of the data.

Class interval	frequency	Cumulative frequency
0-6	*4*	*4*
6-12	*x*	*4+x*
12-18	*8*	*12+x*
18-24	*y*	*12+x+y*
24-30	*4*	*16+x+y*
30-36	*2*	*18+x+y*
36-42	*1*	*19+x+y*
$N = 30$		

[1 M]
Median = 15.75
Median class = 12-18, [1/2 M]
h=6, $\frac{n}{2} = 15$
$f = 8, cf = 4 + x$

median= $l + \frac{\frac{n}{2} - cf}{f} Xh$

15.5= 12+ $\frac{15-4-x}{8} X6$ [1/2 M]

$3.5 = \frac{11 - x}{4} X3$

$\frac{3.5X4}{3} = 11 - x$

$5 = 11 - x$

$x = 6$ [1/2 M

$x + y + 19 = 30$

$6 + y + 19 = 30$

$y = 30 - 25$

$y = 5$ [1/2 M]

Mode:
Modal class=12-18 [1/2 M]

$l = 12, h = 6, f_0 = 6, f_1 = 8, f_3 = 5$

$mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} Xh$

$= 12 + \frac{8-6}{2(8)-6-5} X6$ [1/2 M]

$= 12 + \frac{2}{5} X6$

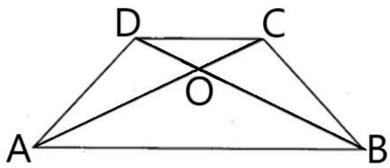
$= 12 + 2.4$

$= 14.4$ [1 M]

34. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then other two sides are divided in the same ratio.

- Given, To prove, Construction, Diagram – 2 marks
- Correct Proof - 3 marks

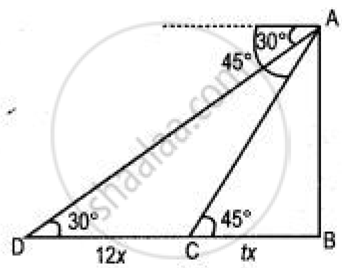
Prove that the diagonals of a trapezium divide each other proportionally. Using this result, find the value of x, if in the given figure, it is given that $AB \,||\, DC$, $OA = 3x - 19$, $OC = x - 5$; OD = 3 and $OB = x - 3$.



Solution : for correct proof for the first part (3 marks)

Since the diagonals of a trapezium divide each other proportionally.
∴ AO / OC = BO/ OD
⇒ 3x−19 / x−5 = x−3 / 3 (1/2 mark)
⇒ 3(3x - 19) = (x - 5) (x - 3)
⇒ 9x - 57 = x^2 - 8x + 15
⇒ x^2 - 17x + 72 = 0 (1 mark)
⇒ (x - 8) (x - 9) = 0
⇒ x- 8 = 0 or x - 9 = 0
⇒ x = 8 or x = 9 (1/2 mark)

35. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. It it takes 12 minutes for the angle of depression to change from 30˚ to 45˚ , how soon after this, will the car reach the tower ? Give your answer to the nearest second.



Let AB be the tower of height H metres. Let C be the initial position of the car and let after 12 minutes the car be at D. It is given that the angles of depression at C and D are 30˚ and 45˚ respectively. Let the speed of the car be v metres per minute.
Then , CD = Distance travelled by the car in 12 minutes = 12 v metres
Suppose the car takes t minutes to reach the tower AB from D. Then, DA = vt metres.
In Δ DAB, we obtain
tan 45˚ = AB/AD ⇒ 1 = h/vt ⇒ h = vt …….(i)
In ΔCAB, we obtain
Tan 30˚ = AB/AC ⇒ $1/\sqrt{3} = \frac{h}{vt+12v}$ ⇒ $\sqrt{3}$ h = vt + 12v ……..(ii)
Substituting the value of h from (i) and (ii), we get
$\sqrt{3}$ vt = vt + 12 v
Solving for t we get t = 6($\sqrt{3}$ + 1) = 16.39 minutes = 16 minutes 23 seconds

Section E

(Section E consists of 3 case study based questions of 4 marks each)

36. Subba Rao started to work in a company in 1995 and his annual salary in the n^{th} year is calculated by the expression $200(29 + n)$. He worked in that company for 35 years with an increment of Rs. 200 each year. Answer the following questions based on the above information.

(i) Write the first four terms of the AP representing his annual salaries in first four years. Write the common difference.

ANS: AP: 6000, 6200, 6400, 6600 [1/2 M]

d=200 [1/2 M]

(ii) In which year, his annual salary becomes 9000 rupees?

ANS:

$a_n = a + (n-1)d = 9000$ [1/2 M]

$6000 + (n-1)200 = 9000$

$$n = \frac{9000 - 6000}{200} + 1$$

$n = 16$ [1 M]

in 16 years, in the year 2010. [1/2 M]

(iii) Find the total amount Subba Rao received in the form of salary in these 35 years.

ANS: The amount he received in 35 years= s_{35} [1/2 M]

$= \frac{35}{2}(2X6000 + 34X200)$ [1/2 M]

$= \frac{35}{2}(12000 + 6800)$

$= \frac{35}{2}(18800)$

$= 35X9400$

$=$ 3,29,000 rupees. [1M]

(OR)

How many two-digit numbers are divisible by 7? Find their sum.

AP: 7, 14, 21, 28, … …98 [1/2 M]

a=7, d=7

$a_n = a + (n-1)d$

$7 + (n-1)7 = 98$

$7n = 98$

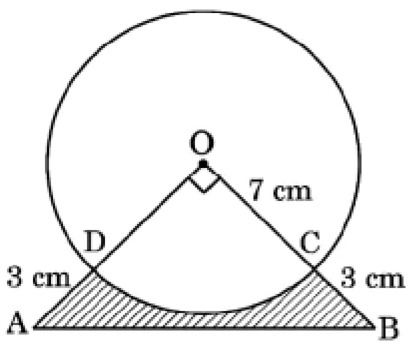
$n = 14$ [1 M]

There are 14 two-digit numbers divisible by 7.

Their sum = $s_{14} = \frac{14}{2}(7 + 98)$ $\quad \because s_n = \frac{n}{2}(a + a_n)$

$= 7X105 = 735$ [1/2 M]

37. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is 20 per cm^2. Based on the above, answer the following questions:



(i) What is the area of the quadrant ODCO? (1)

(ii) Find the area of ΔAOB. (1)

(iii) What is the total cost of silver plating the shaded part ABCD? (2)

(OR)

(iii) What is the length of arc CD? (2)

Ans:

(i) Area of sector ODCO $= \frac{90^0}{360^0} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} = 38.5cm^2$

(ii) ar $(\Delta AOB) = \frac{1}{2} \times 10 \times 10 = 50cm^2$

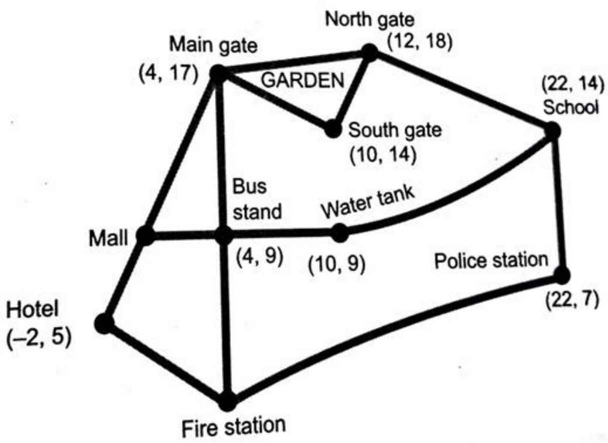
(iii) Required cost $= (50 - 38{\cdot}5) \times 20 =$ Rs. 230

OR

(iii) Length of arc CD $= \frac{90^0}{360^0} \times 2 \times \frac{22}{7} \times 7 = 11cm^2$

38. Answer the questions based on the information given. Shown below is a map of Giri's neighborhood. Giri did a survey of his neighborhood and collected the following information:

* The hotel, mall and the main gate of the garden lie in a straight line.
* The distance between the hotel and the mall is half the distance between the mall and the main gate of the garden
* The bus stand is exactly midway between the main gate of the garden and the fire station
* The mall, bus stand and the water tank lies in a straight line



(i) What is the x-coordinate of the mall's location? **Ans : Using Midpoint formula, Mall = (1,6)** (1)

(ii) What are the coordinates of the fire station? Ans : Using Midpoint formular , we get Fire Station = (4,1) (1)

(iii) What is the shortest distance between the water tank and the school?
Ans : Using distance formula we get the distance is 13 units (2)

(OR)

(iii) How much more is the shortest distance of the school from the water tank than the distance of the school from the police station? Ans : compare the two distances, calculate using distance formula and then subtract = 6 units (2)