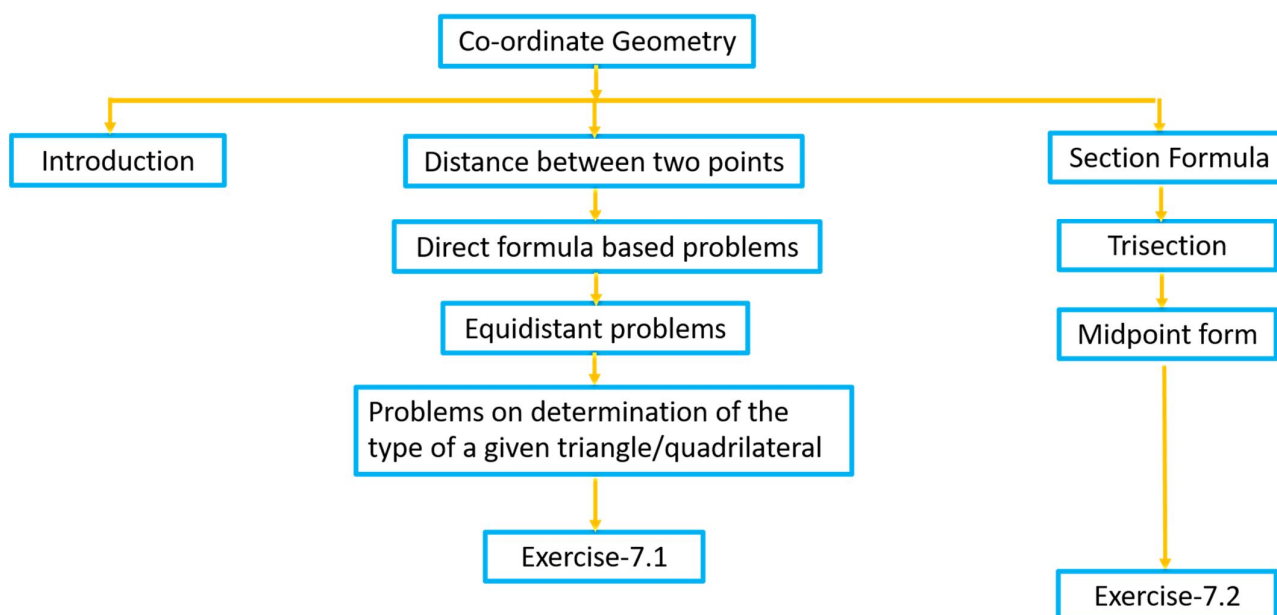# COORDINATE GEOMETRY

MIND MAPPING:

Basic facts and formulae:

1. In Cartesian coordinate system, there is a Cartesian plane which is made up of two number lines which are mutually perpendicular to each other. X-axis (horizontal) and Y-axis (vertical) which represents two variables. These two perpendicular lines are called coordinate axes.

2. The point of intersection of x-axis and y-axis i.e O is called the origin. The coordinates of Origin are (0, 0).

3. X – Coordinate of a point is called abscissa and y-coordinate of a point is called ordinate. A plane is divided by axes into four quadrants.
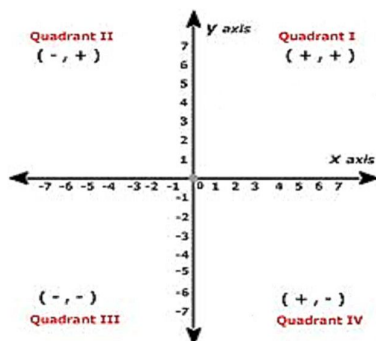

$$Q_1 : (+x, +y);\ Q_2 : (-x, +y);\ Q_3 : (-x, -y);\ Q_4 : (+x, -y)$$

4. The coordinates of the x- axis is $(x, 0)$.

5. The coordinates of the y-axis is $(0, y)$.

6.   If $x \neq y$ then $(x, y) \neq (y, x)$ and if $x = y$ then $(x, y) = (y, x)$.

7.   The distance between two points $P(x_1, y_1), Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

8.   The distance of the point $P(x, y)$ from the Origin (0, 0) is $\sqrt{x^2 + y^2}$.

9.   **Problems based on geometrical figure: To show that a given figure is a**

      a) Parallelogram:– prove that the opposite sides are equal and diagonals bisect each other.

      b) Rectangle:– prove that the opposite sides are equal and the diagonals are equal.

      c) Parallelogram but not rectangle:– prove that the opposite sides are equal and the diagonals are not equal.

      d) Rhombus:– prove that the four sides are equal

      e) Square:– prove that the four sides are equal and the diagonals are equal.

      f) Rhombus but not square:– prove that the four sides are equal and the diagonals   are not equal.

      g) Isosceles triangle:– prove any two sides are equal.

      h) Equilateral triangle:– prove that all three sides are equal.

      i) Right triangle:– prove that sides of triangle satisfies Pythagoras theorem

10.   If $P(x, y)$ be any point on the line segment AB, which divides AB in the ratio of

      $m_1 : m_2$ internally then coordinates of $P(x, y)$ will be given by:

      $$P(x, y) = \left[\frac{(m_1 x_2 + m_2 x_1)}{m_1 + m_2}, \frac{(m_1 y_2 + m_2 y_1)}{m_1 + m_2} \right].$$

11.   The midpoint of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

      given by $\left[\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right]$

12.   Let P and Q be the points of trisection (dividing into three equal parts) of the line segment AB, then for finding the coordinates of P the ratio is 1:2 and for finding the coordinates of Q the ratio is 2 : 1

              A   P   Q   B
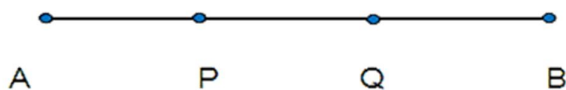
13.   The line drawn from the vertex of a triangle to the midpoint of the opposite side is called as median.

14.   The point of concurrency of medians of a triangle is called as centroid.

15.   The centroid divides the median in the ratio of 2:1

LEVEL_1

MCQ:

1. The distance of the point P(2, 3) from the X-axis is:

    (a) 2   (b) 3   (c) 1   (d) 5

2. The distance between the points A(0, 6) and B(0,- 2) is

    (a) 6   (b) 8   (c) 4   (d) 2

3. The distance between the points $(\cos\theta, \sin\theta)\ and\ (\sin\theta, -\cos\theta)$ is:

    (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 1   (d) 0

4. The distance of the point P(- 6, 8) from the origin is

    (a) 8   (b) $2\sqrt{7}$ (c) 10   (d) 6

5. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

    (a) 5   (b) 12   (c)11   (d) 7+√5

6. The point which divides the line segment joining the points (7, – 6) and (3, 4) in ratio 1: 2 internally lies in the

    (a) I quadrant   (b) II quadrant

    (c) III quadrant   (d) IV quadrant

7. If AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0), then the length of its diagonal is

    (a) 5   (b) 3   (c) √34   (d) 4

8. In what ratio does x-axis divide the line segment joining the points A(2, -3) and B(5, 6)?

   (a) 2 : 3   (b) 2 : 1   (c) 3 : 4   (d) 1 : 2

2-MARKS:

1. Points A(3, 1), B(5, 1), C(a, b) and D(4, 3) are the vertices of a parallelogram ABCD. Find the values of a and b.

2. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3). Hence find the value of m.

3. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the midpoint of PQ, then find the coordinates of P and Q.

4. Determine whether the given points (1, 5), (2, 3) and (-2, -11) are collinear.

5. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

6. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

7.   If the point P(2, 1) lies on the line segment joining the points A(4, 2) and B(8, 4), then find the relation between AP and AB.

8. ΔABC is a right angled triangle, in which A(0, 2) and B(2, 0) are given. Then, find the coordinates of C.

9.   Prove that points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

10.   If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3), then find the value of $x^2 + 5$.

11.   If the point C(-1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio of 3 : 4. Find the coordinates of B.

12.   If the point C(k, 4) divides the join of points A(2, 6) & B(5, 1) in the ratio of 2 : 3, find the value of k.

3-MARKS:

1.   Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

2.   Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

3.   In the given figure of ΔABC, D and E are the midpoints of the sides BC and AC respectively.

C(-6, –1)

D   E

B(2, –2)   A(4, –2)
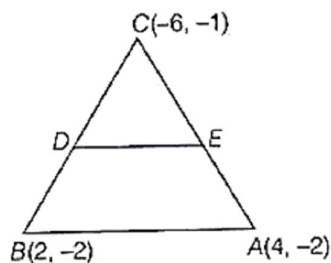
Find the length of DE and also prove that $DE = \dfrac{1}{2} AB$.

4.   Show that the points A(5, -1), B(8, 3), C(4, 0) and D(1,-4) are the vertices of a rhombus.

5.   If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \dfrac{3}{7} AB$ and P lies on the line segment AB.

6.   Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

5-MARKS:

1. Find the coordinates of the points which divide the line segment joining the points A(-2, -2) and B(2, 8) into six equal parts.

2. In a class room, 4 friends are seated at the points A, B, C and D as shown in the figure. Champa and Chameli walk into the class and after observing for a few minutes, Champa asks Chameli, "Don't you think ABCD is a square"?
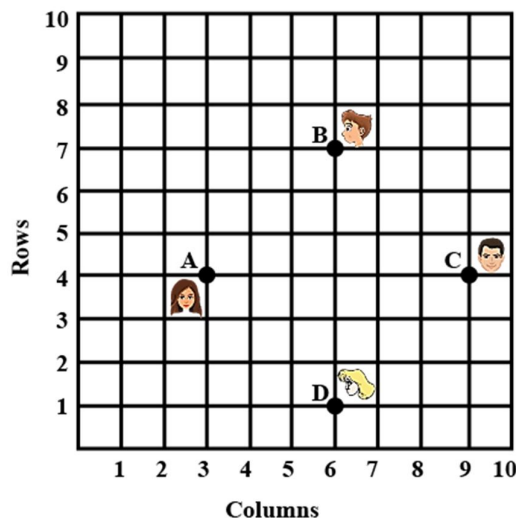

Chameli disagrees. Using distance formula, find which of them is correct.

3. Points P, Q, R, S and T divide the line segment joining the points A(1, 2) and B(6, 7) in 6 equal parts. Find the coordinates of the points P, Q, R, S and T.

4. The points (3, -4) and (-6, 5) are the end points of a diagonal of a parallelogram. If one end of the end points of the second diagonal is (-2, 1), then find its other end point.

5. Show that the quadrilateral PQRS formed by the vertices P(22, 5), Q(7, 10), R(12, 11) and S(3, 24) is not a parallelogram.

6. Show that the ΔABC, where $A(-2,0)$, $B(2,0)$, $C(0,2)$ and ΔPQR where P(-4,0), Q(4, 0) and R(0, 4) are similar triangles.

CASE STUDY QUESTIONS

CASE STUDY_1

Two friends Seema and Aditya work in the same office at Delhi. In the Vacation both decided to go to their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions.
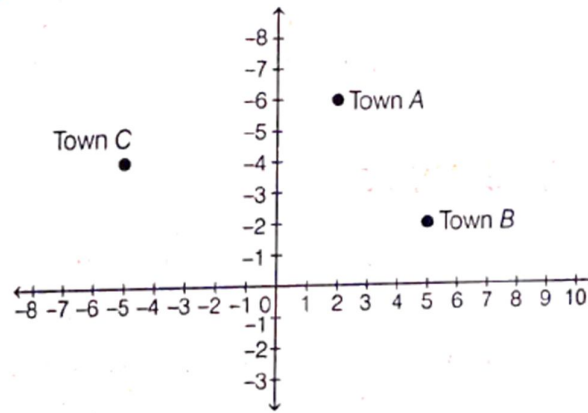
(i)   What are the locations of Town A (Seema) and Town B (Aditya)?   1

(ii)   Locate the point K, which divides the line segment joining Town A and Town B in the ratio of 2 : 1.   1

(iii) Who will travel more distance, Seema or Aditya to reach their hometown?   2

OR

Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B.

Find the coordinates of the point represented by the point D.   2

CASE STUDY_2

The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the below figure. The students are to sow seeds of flowering plants on the remaining area of the plot.
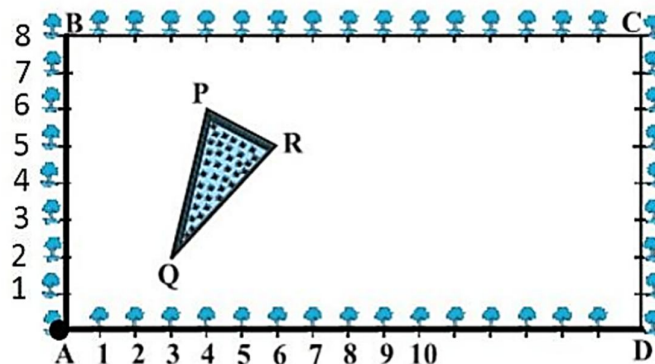

(i)   Taking A as origin, find the coordinates of the vertices of a triangle PQR.   1

(ii)   What is the midpoint of the side PQ, when A is the origin?   1

(iii) What will be the coordinates of the vertices of a triangle PQR if C is the origin? 2

OR

What is the midpoint of side QR, when C is the origin?   2

CASE STUDY_3

On the occasion of 'Diwali' a rectangular plot have been allotted for 'Diwali Mela' to students of secondary school in Hyderabad. In order to reduce smog and pollution they decided to keep little leaf linden plant on the boundary at a distance of 1 m from each other. Four air purifier machines have also been set up at points L, M, N, O. (Answer the following questions considering A as origin).
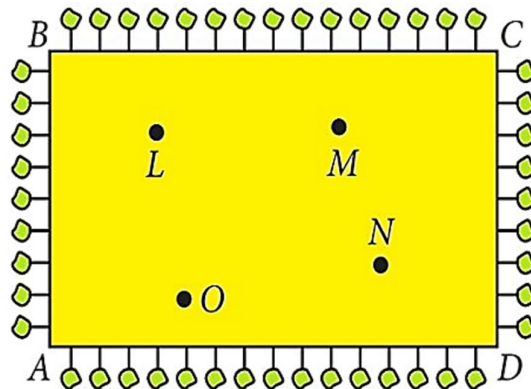
(i)   What are the coordinates of L?   1

(ii)   What are the coordinates of N?   1

(iii) What is the distance between L and O?   2

OR

Find the mid-point of the segment joining the points L and N.   2

<u>LEVEL_2</u>

MCQ:

1.   The distance of the point p(x, y) from the origin is:

(a) $\sqrt{x^2 + y^2}$ (b) $\sqrt{2x^2 + 2y^2}$ (c) $2\sqrt{x^2 + y^2}$ (d) $\sqrt{x + y}$

2.   The coordinates of the point, where the line $x - y = 5$ cuts Y-axis is:

(a) $(0, 5)$ (b) $(0, -5)$ (c) $(5, 0)$ (d) $(-5, 0)$

3.   The distance between the points $(a \cos 30^0, 0)$ and $(0, a \cos 60^0)$ is:

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{1}{2}$ (c) a   (d) 1

4.   Sheena was asked to plot a point 10 unit on the left of the origin and other point 4 units directly above the origin. Which of the following are the two points?

(a) (10, 0) and (0,-4)   (b) (-10, 0) and (4, 0)

(c) (10, 0) and (0, 4)   (d) (-10, 0) and (0, 4)

5. If $\left(3, \dfrac{3}{4}\right)$ is the mid-point of the line segment joining the points $(k, 0)$ & $\left(7, \dfrac{3}{2}\right)$,

then the value of k is:

(a) 1   (b) -1   (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{2}$

6. On a graph, two-line segments, AB and CD of equal length are drawn. Which of these could be the coordinates of the points, A, B, C and D?

(a) A(-3,4) B(-1,2) and C(3,4) D(1,2)   (b) A(-3,-4) B(-1,2) and C(3,4) D(1,2)

(c) A(-3,4) B(-1,-2) and C(3,4) D(1,2)   (d) A(3,4) B(-1,2) and C(3,4) D(1,2)

2-MARKS

1. Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

2. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

3. Find the distance between the points $P\left(\dfrac{\sin\theta}{2}, 0\right)$ and $Q\left(0, \dfrac{\cos\theta}{2}\right)$.

4. If the distance between the points (4, p) and (1, 0) is 5, then find the value of p.

5. Show that if a circle has its centre at the origin and a point P(5, 0) lies on it, then the point Q(6, 8) lies outside the circle.

6. P(-2, 5) and Q(3, 2) are two points. Find the coordinates of the point R on PQ such that PR = 2QR.

7. What will be the value of y, if the point $\left(\dfrac{23}{5}, y\right)$ divides the line segment joining the points (5, 7) and (4, 5) in the ratio 2 : 3 internally?

8. Point P divides the line segment joining the points A(2, 1) and B(5, -8) such that $\dfrac{AP}{AB} = \dfrac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k .

9. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5). Find the coordinates of the other end of the diameter?

10. If the midpoint of the line joining (3, 4) and (k, 7) is (x, y) and lies on the line $2x + 2y + 1 = 0$, then find the value of k.

11. Point P divides the line segment joining the points A(-1, 3) and B(9, 8) such that $\dfrac{AP}{BP} = \dfrac{k}{1}$. If P lies on the line $x - y + 2 = 0$, then find the value of k.

3-MARKS

1. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

2. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

3. In what ratio does the Y-axis divide the line segment joining the points P(-4, 5) and Q(3, -7)? Also, find the coordinates of the point of intersection.

4. Find the ratio in which the point $P\left(\dfrac{3}{4}, \dfrac{5}{12}\right)$ divides the line segment joining the points $A\left(\dfrac{-1}{2}, \dfrac{3}{2}\right)$ and $B(2, -5)$.

5. If the centroid of the triangle formed by the points A(a, b), B(b, c) and C(c, a) is at the origin. What is the value of $\dfrac{a^2}{bc} + \dfrac{b^2}{ca} + \dfrac{c^2}{ab}$?

6. The coordinates of one end point of the diameter of a circle are (4, -1) and coordinates of the centre of the circle are (1, -3).
    (i) Find the coordinates of the other end of the diameter.
    (ii) Find the diameter of the circle.
    (iii) Calculate the area of the circle.

7. Prove that the diagonals of a rectangle ABCD, with vertices A(2, -1), B(5, -1), C(5, 6) and D(2, 6) are equal and bisect each other.

8. Determine the ratio in which the straight line $x - y - 2 = 0$ divides the line segment joining (3, -1) and (8, 9).

9. Name the type of triangle formed by the points $P\left(\sqrt{2}, \sqrt{2}\right)$, $Q\left(-\sqrt{2}, -\sqrt{2}\right)$ and $R\left(-\sqrt{6}, \sqrt{6}\right)$.

5-MARKS

1. Find the length of the medians RS and PT of a triangle PQR whose vertices are P(6, -2), Q(6, 3) and R(3, 1).

2. An equilateral triangle has two vertices at the points (3, 4) and (-2, 3). Find the coordinates of the third vertex.

3. Show that the points with coordinates (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square.

4. If R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a) then prove that $x + y = a + b$.

5. The coordinates of the vertices of $\triangle ABC$ are A(1, -1), B(-4, 6) and C(-3, -5). Draw the figure and prove that $\triangle ABC$ is a scalene triangle.

6. If (-4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

7. To conduct Sports Day activities in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD as shown in the figure. Niharika runs $\frac{1}{4}^{th}$ the distance AD in the 2nd line and posts a green flag. Preet runs $\frac{1}{5}^{th}$ the distance AD on the eighth line and posts a red flag.
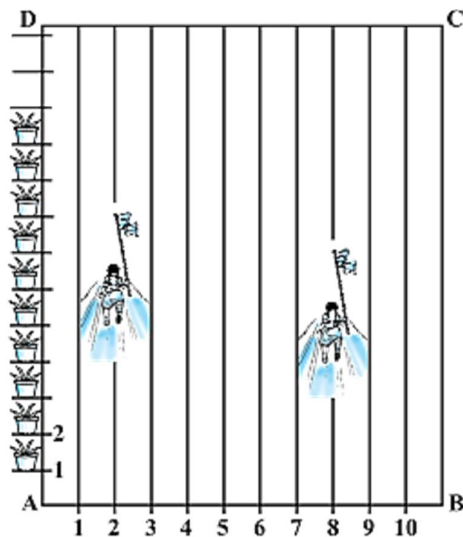
(i) What is the distance between both the flags?

(ii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post the flag?

(iii) Which mathematical concept is used in the above problems?

(iv) Which value is depicted in the problem?

CASE STUDY QUESTIONS

CASE STUDY_1

For a sports event, certain points were marked on a rectangular ground denoting positions of different drills.
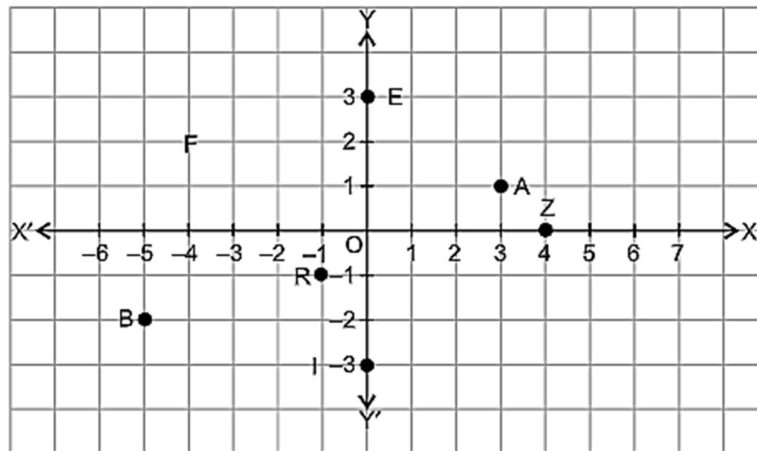
(i)   Find the distance between the points A and E.   1

(ii) What type of triangle is formed by joining the points A, Z and R?   2

OR

What is the ratio in which X-axis divides the joing of A and R?   2

(iii) What is the distance of the point B from F?   1

LEVEL_3

MCQ:

1.   The points (- 4, 0), (4, 0) and (0, 3) are the vertices of a:

(a) right angled triangle   (b) isosceles triangle

(c) equilateral triangle   (d) scalene triangle

2.   If the point P(2, 1) lies on the line segment joining points A(4, 2) and 6(8, 4), then

(a) $AP = \dfrac{1}{3}AB$ (b) AP = PB   (c) $PB = \dfrac{1}{3}AB$ (d) $AP = \dfrac{1}{2}AB$

2.   The distance between the points $(a\cos\theta + b\sin\theta,\ 0)$ and $(0,\ a\sin\theta - b\cos\theta)$ is:

(a) $\sqrt{a^2 + b^2}$ (b) $\sqrt{a^2 - b^2}$ (c) $2\sqrt{ab}$ (d) $3\sqrt{a^2 + b^2}$

3.   The point which lies on the perpendicular bisector of the line segment joining the points A(-2, − 5) and B(2, 5) is:

(a) (0,0)   (b) (0, 2)   (c) (2, 0)   (d)(-2,0)

4. The coordinates of the point which is equidistant from the three vertices of the $\triangle AOB$ as shown in the figure is:
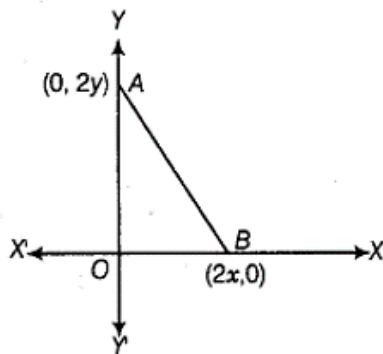

(a) (x, y) (b) (y, x) (c) $\left(\dfrac{x}{2}, \dfrac{y}{2}\right)$ (d) $\left(\dfrac{y}{2}, \dfrac{x}{2}\right)$

5. If a circle is drawn with origin as the centre passes through $\left(\dfrac{13}{2}, 0\right)$, then the point which does not lie in the interior of the circle is:

(a) $\left(-\dfrac{3}{4}, 1\right)$ (b) $\left(2, \dfrac{7}{3}\right)$ (c) $\left(5, -\dfrac{1}{2}\right)$ (d) $\left(-6, \dfrac{5}{2}\right)$

2-MARKS

1. Find the point which lies on the perpendicular bisector of the line segment joining the points A(-2, -5) and B(2, 5).

2. The vertices of a triangle are (a, b-c), (b, c-a) and (c, a-b). Prove that its centroid lies on X-axis.

3. Show that $\triangle ABC$ with vertices A(-2, 0), B(0, 2) and C(2, 0) is similar to $\triangle DEF$ with vertices D(-4, 0), E(4, 0) and F(0, 4).

4. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). What is the length of its diagonals?

5. If P & Q are the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.

6. The line segment joining the points A(2, 1) and B(5, -8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$. Find the value of k.

7. Find a relation between x and y such that the points (x, y) is equidistant from the point (3, 6) and (-3, 4).

8. If P(x, y) is any point on the line joining the points A(a, 0) and B(0, b) then show that $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

9. If $x - 2y + k = 0$ is a median of the triangle whose vertices are at points A(-1, 3), B(0, 4) and C(-5, 2) find the value of k .

10. Points A(-6, 10), B(-4, 6) and C(3, -8) are collinear such that $AB = \dfrac{2}{9}AC$. Justify your answer.

11. The vertices of a ΔOAB are O(0, 0), A(4, 0) and B(0, 6). The median AD is drawn on OB. Find the length of AD.
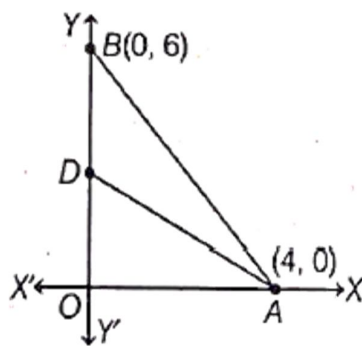

3-MARKS

1. Points P, Q, R and S divide the line segment joining the points A(1, 2) & B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

2. If the points A(-2, 1), B(a, b) and C(4, -1) are collinear and $a - b = 1$, find the values of a and b.

3. If the points A(1, 2), B(4, q), C(p, 6) & D(3, 5) are the vertices of a parallelogram ABCD, find the values of p and q.

4. If coordinates of midpoints of the sides of a triangle are (1, 2), (0, -1) and (2, -1). Find the coordinates of its vertices.

5. ABCD is a parallelogram with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Find the coordinates of the fourth vertex D in terms of x_1, x_2, x_3, y_1, y_2 and y_3.

6. The centre of the circle is (2a, a-7). Find the values of 'a' if the circle passes through (11, -9) and has the diameter $10\sqrt{2}$ units.

7. If $P(9a - 2, -b)$ divides the line segment joining $A(3a + 1, -3)$ and $B(8a, 5)$ in the ratio of 3 : 1, find the values of a and b.

8. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points (8, -9) and (2, 1). Also find the coordinates of the point of division.

5-MARKS

1. The midpoint 'P' of the line segment joining the points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). Find the ratio in which P divides CD. Also find the values of y.

2. If two vertices of a parallelogram are (3, 2), (-1, 0) and the diagonals cut at (2, -5) then find the other vertices of the parallelogram.

3. If two opposite vertices of a square are (5, 4) and (1, -6) then find the coordinates of it's remaining the two vertices.

4. Using Analytical geometry, prove that diagonals of a rhombus bisect each other.

5. If the points A(1, -2), B(2, 3), C(a, 2) and D(-4, -3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from here to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching the office? (Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.

7. The mid-points D, E and F of the sides of a triangle ABC are (3, 4), (8, 9) & (6, 7). Find the coordinates of the vertices of the triangle.

CASE STUDY QUESTIONS

CASE STUDY_1

The front door of a guest house is drawn/shown on the coordinate plane as follows:
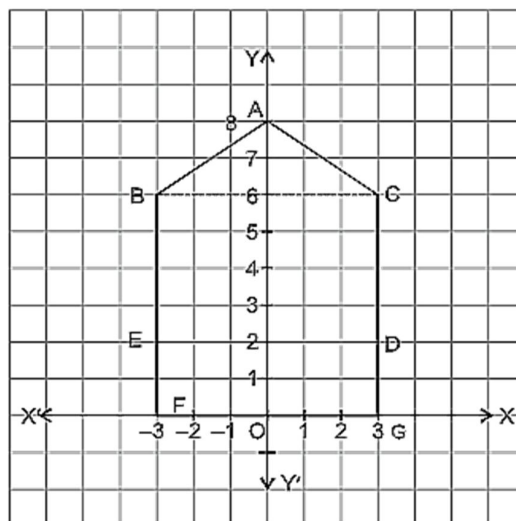
(i)   What is the length of the line segment AB?   1

(ii)   Is AB = AC?   1

(iii)   What are the coordinates of the midpoint of BE and midpoint of ED?   2

OR

What is the ratio in which Y-axis divides BD? (Join BD)

CASE STUDY_2

Ravi lives in a multistorey apartment. One day, from his balcony he observed that there is a flower bed on the ground in the shape of a parallelogram. He draws the sketch of the flower bed on a graph paper as shown:
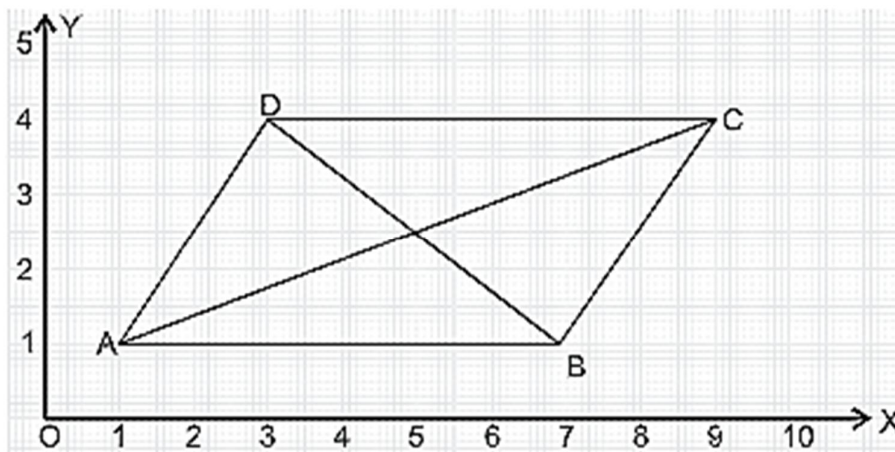
(i)   What are the coordinates of the vertex D?   1

(ii)   What are the coordinates of the point of intersection of the diagonals?   1

(iii)   What is the length of the sides AB and AD?   2

OR

If we take A as the origin and AB as the x-axis then what are the coordinates of M?   2

SKILL BASED QUESTIONS

1. In a health checkup, the number of heart beats of 40 women were recorded in the following table. Find the mean of the following by Assumed mean method.

Number of heart beats/minute	65-69	70-74	75-79	80-84
Number of women	2	18	16	4

2. If the mode of the following series is 54, then find the value of f.

Class Interval	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	3	5	f	16	12	7