Circles

Previous Years' CBSE Board Questions

10.1 Introduction

MCQ

1. A chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is

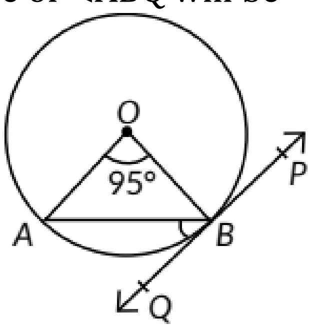
(a) $5\sqrt{2}$ (b) $10\sqrt{2}$

(c) $\frac{5}{\sqrt{2}}$ (d) $10\sqrt{3}$ *(AI 2014)*

10.2 Tangent to a Circle

MCQ

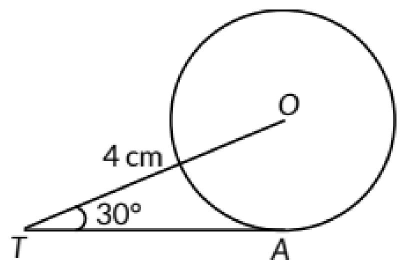
2. In the given figure, PQ is tangent to the circle centred at O. If <AOB = 95°, then the measure of <ABQ will be



(a) 47.5° (b) 42.5° (c) 85° (d) 95°

(2023)

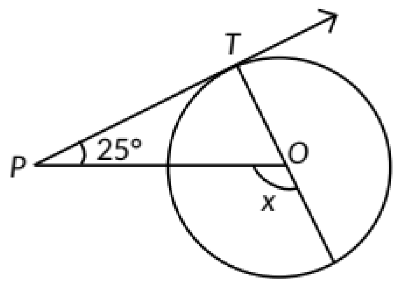
3. In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm, <OTA= 30°, then length of TA is



(a) $2\sqrt{3}$ cm (b) 2 cm

(c) $2\sqrt{2}$ cm (d) $\sqrt{3}$ cm *(2023)*

4. In the given figure, PT is a tangent at T to the circle with centre O. If <TPO = 25°, then x is equal to



(a) 25° (b) 65° (c) 90° (d) 115°

(2023)

5. The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is

(a) 40 cm

(b) 9 cm

(c) 41 cm

(d) 50 cm (2023)

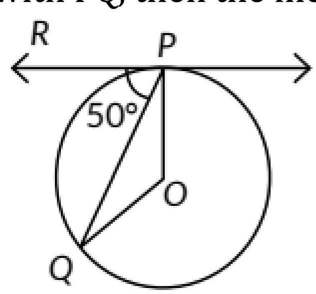
6. In the given figure, O is the centre of the circle and PQ is the chord. If the tangent PR at P makes an angle of 50° with PQ, then the measure of <POQ is

(a) 50°

(b) 40°

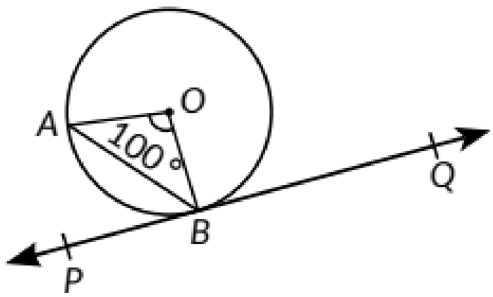
(c) 100°

(d) 130°



(2023)

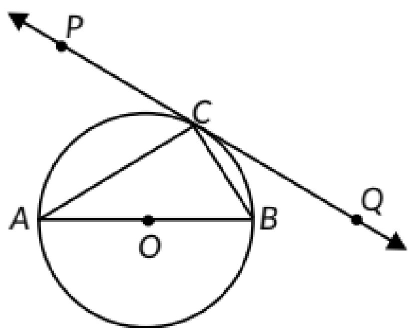
7. In figure, PQ is tangent to the circle with centre at O, at the point B. If ZAOB = 100°, then ZABP is equal to



(a) 50°
(b) 40°
(c) 60°
(d) 80° (2020)

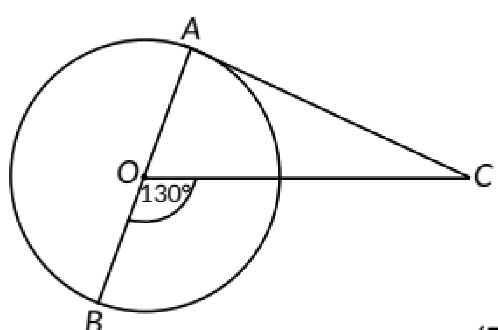
VSA (1 mark)

8. In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and <CAB = 30°, find <PCA.



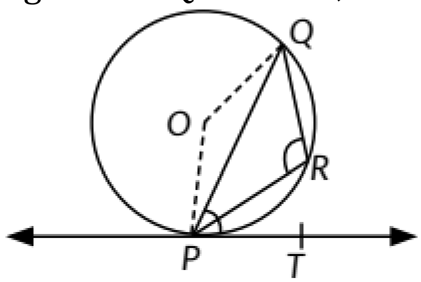
(AI 2016)

9. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. <BOC = 130°, then find <ACO.



(Foreign 2016)

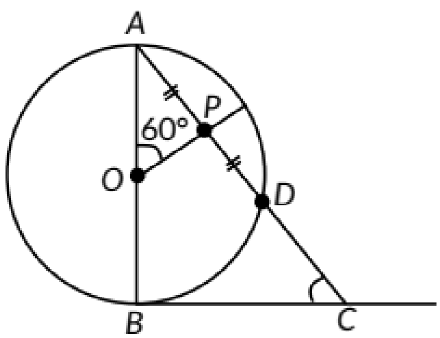
10. In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If <QPT = 60°, find <PRQ.



(AI 2015)

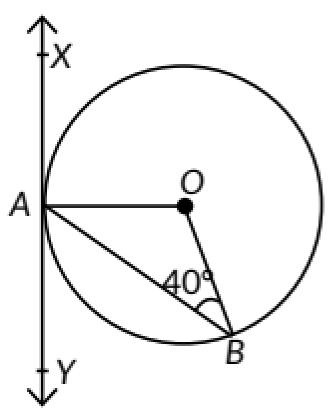
SAI (2 marks)

11. In Fig. AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and <AOP = 60°, then find m<C.



(Term II, 2021-22)

12. In Fig. XAY is a tangent to the circle centred at O. If <ABO = 40°, then find m<BAY and m<AOB.

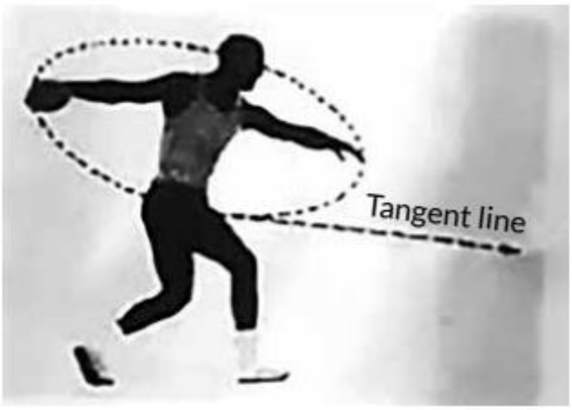


(Term II, 2021-22)

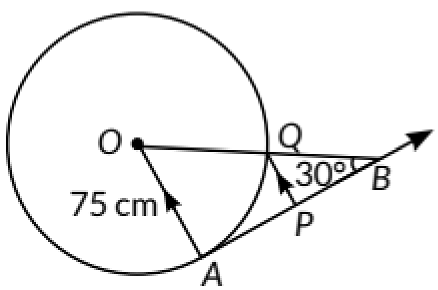
LA (4/5/6 marks)

(In this section, there are 3 case study/passage based questions. Each question is of 4 marks.)

14. Case Study: The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and $\angle ABO = 30°$. PQ is parallel to OA.

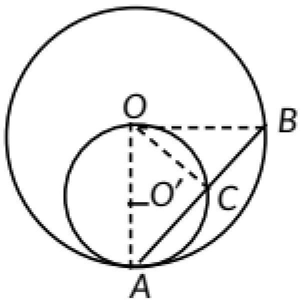


Based on above information:
(a) Find the length of AB.
(b) Find the length of OB.
(c) Find the length of AP.

OR

Find the length of PQ. (2023)

15. In Figure, two circles with centres at O and O' of radii 2r and r respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB.

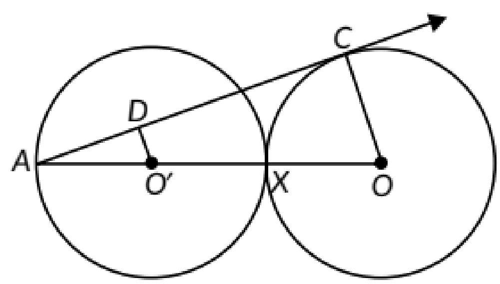


(Term II, 2021-22)

16. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. (Delhi 2016, 2015, 2014, AI 2016, 2015, 2014, Foreign 2016, 2015, 2014)

17. In the given figure, two equal circles, with centres O and O' touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to

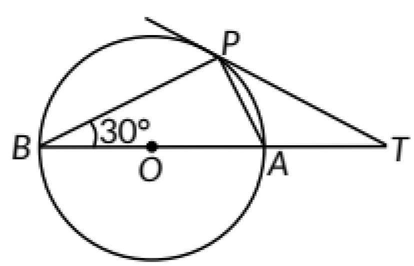
AC. Find the value of $\frac{DO'}{CO}$.



(AI 2016)

18. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. (AI 2015)

19. In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T.



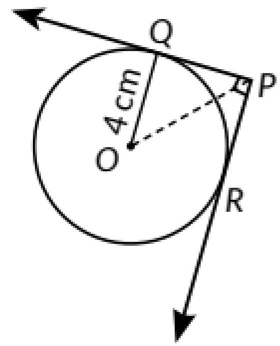
If $\angle PBT = 30°$, prove that $BA : AT = 2 : 1$.

(Foreign 2015)

10.3 Number of Tangents from a Point on a Circle

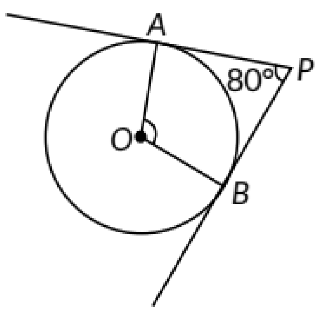
MCQ

20. In figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If <QPR 90°, then length of PQ is



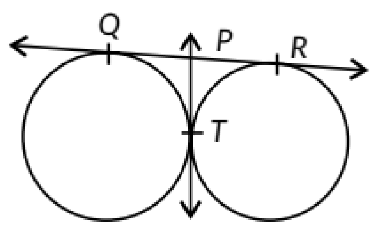
(a) 3 cm
(b) 4 cm
(c) 2 cm
(d) 2√2 cm (2020)

21. In Figure, if tangents PA and PB from an external point P to a circle with centre O, are inclined to each other at an angle of 80°, then ZAOB is equal to



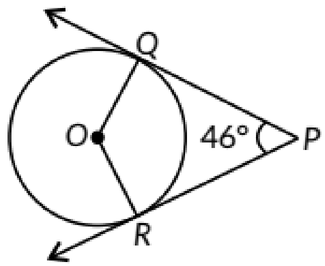
(a) 100° (b) 60° (c) 80° (d) 50°
(2020)

22. In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then the length of QR(in cm) is



(a) 3.8 (b) 7.6 (c) 5.7 (d) 1.9
(Delhi 2014)

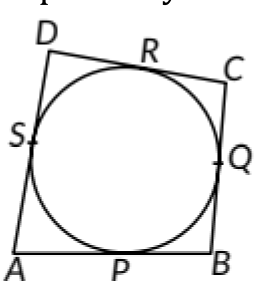
23. In the given figure, PQ and PR are two tangents to a circle with centre O. If <QPR = 46°, then <QOR equals



(a) 67°
(b) 134°
(c) 44°
(d) 46° (Delhi 2014)

24. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and
B. The value of <APB is
(a) 30°
(b) 45°
(c) 60°
(d) 90° (AI 2014)

25. In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm, find x.



(a) 10
(b) 9
(c) 8
(d) 7 (Foreign 2014)

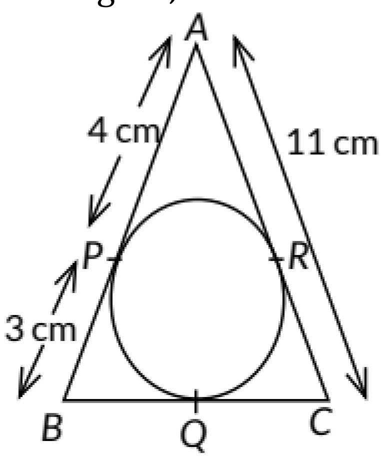
26. Two concentric circles are of radii 5 cm and 3 cm. Length of the chord of the larger circle, (in cm), which touches the smaller circle is
(a) 4

(b) 5
(c) 8
(d) 10 (Foreign 2014)

VSA (1 mark)

27. If tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of 70°, then find ZPOA. (2021)

28. In figure, ABC is circumscribing a circle, the length of BC is _________cm.

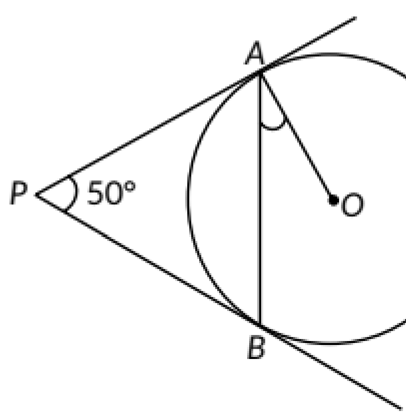


(2020)

29. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP. (AI 2017)

30. From an external point P, tangents PA and PB are drawn to a circle with centre O. If <PAB = 50°, then find <AOB. (Delhi 2016)

31. In the given figure, PA and PB are tangents to the circle with centre O such that <APB = 50°. Write the measure of <OAB.

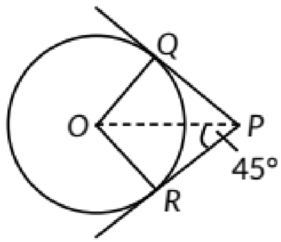


(Delhi 2015)

32. Two concentric circles of radii a and b (a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle. (Foreign 2015)

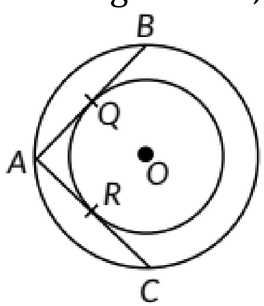
SAI (2 marks)

33. In Figure, PQ and PR are tangents to the circle centred at O. If ZOPR = 45°, then prove that ORPQ is a square.



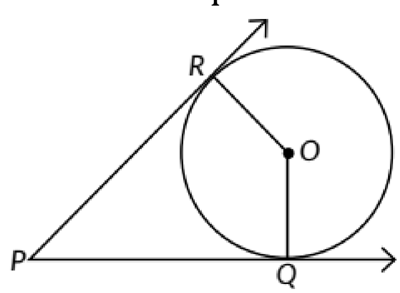
(Term II, 2021-22)

34. In Fig., there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if AQ = 5 cm.



(Term II, 2021-22,

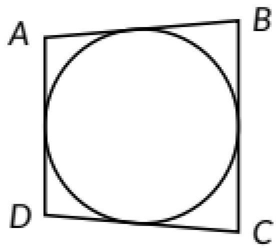
35. In Figure, O is the centre of the circle. PQ and PR are tangent segments. Show that the quadrilateral PQOR is cyclic.



(Term II, 2021-22)

36. In figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that

AB + CD = BC + AD



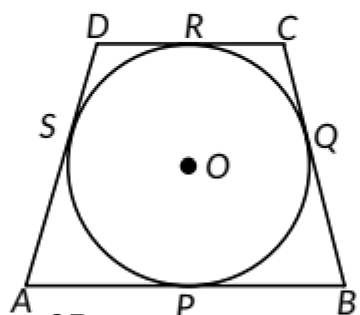
(2020)

OR

A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC+DA (AI 2017)

OR

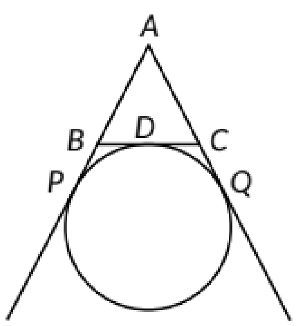
In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA. (AI 2016)



OR

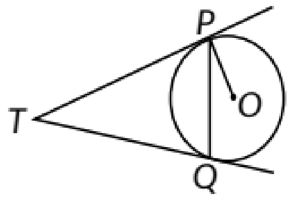
A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal. (Foreign 2014)

37. In figure, find the perimeter of AABC, if AP = 12 cm.



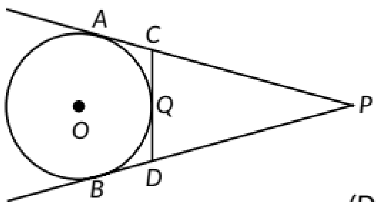
(2020)

38. In figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that <PTQ=2<OPQ.



(2020)

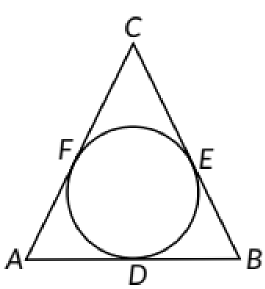
39. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm then find PC + PD.



(Delhi 2017)

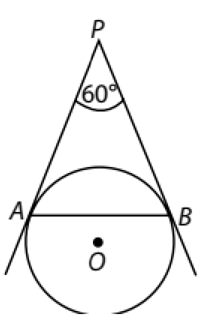
40. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. (AI 2017)

41. In the given figure, a circle is inscribed in a AABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



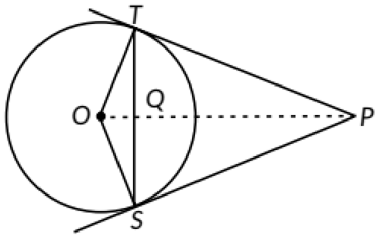
(Delhi 2016)

42. In the given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and <APB = 60°. Find the length of chord AB.



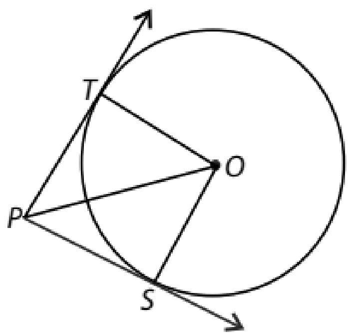
(Delhi 2016)

43. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP=2r, show that ZOTS = ZOST = 30°.



(AI 2016)

44. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that <SPT = 120°. Prove that OP = 2PS.

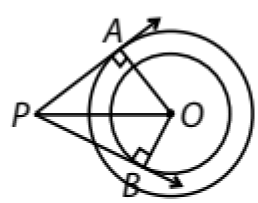


(Foreign 2016,

OR

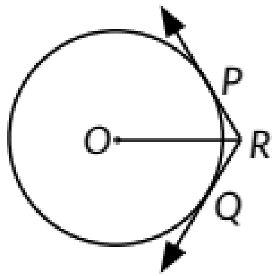
If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that ZQPR
= 120°, prove that 2PQ = PO. (Delhi 2014)

45. In the given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8 cm, find the length of BP. (Foreign 2016)

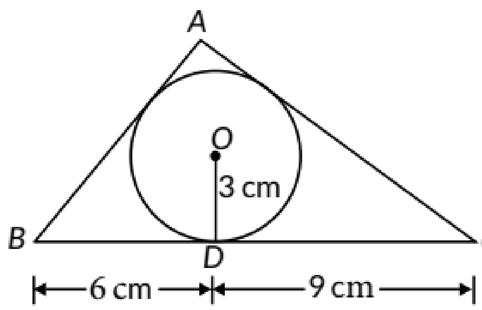


46. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ. (Delhi 2015)

47. In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If <PRQ 120°, then prove that OR = PR + RQ. (AI 2015)

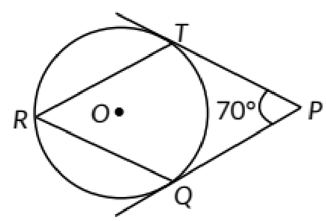


48. In the given figure, a .ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of .ΔABC is 54 cm^2, then find the lengths of sides AB and AC



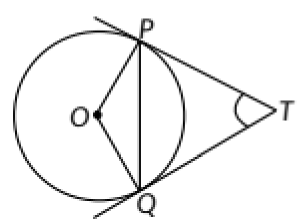
(AI 2015)

49. In the given figure O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ZTPQ = 70°, find /TRQ.



(Foreign 2015)

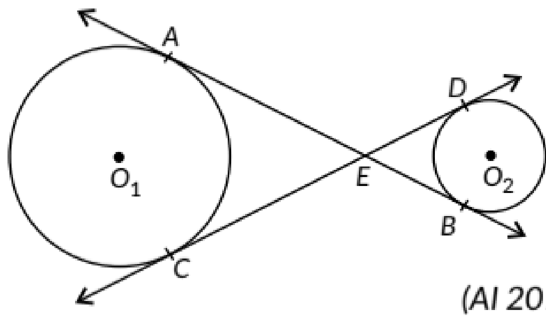
50. In the given figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ.



(Foreign 2015)

51. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre. (Delhi 2014)

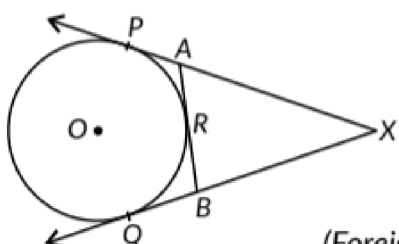
52. In the given figure, common tangents AB and CD to the two circles with centres O, and O_2 intersect at E. Prove that AB = CD.



(AI 2014)

53. The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC. (AI 2014)

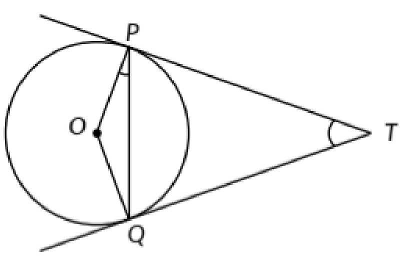
54. In the given figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA + AR = XB+ BR.



(Foreign 2014)

SA II (3 marks)

55. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that <PTQ =2<OPQ.

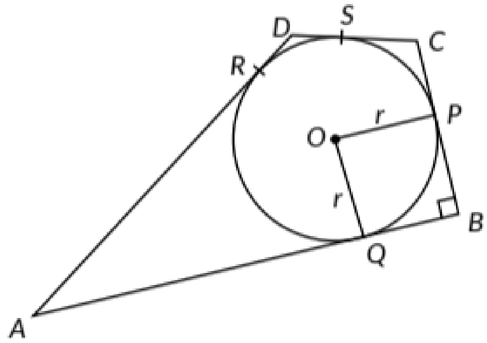


(2023)

OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that <PTQ = 2<OPQ (Delhi 2017)

56. In the given figure, a circle is inscribed in a quadrilateral ABCD in which ZB = 90°. If AD = 17 cm, AB = 20 cm and DS = 3 cm, then find the radius of the circle.



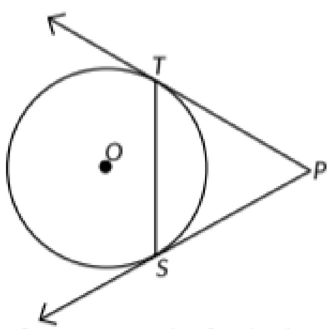
(2023)

57. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the centre of the circle bisects the angle between the two tangents. (2023)

58. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. (2023)

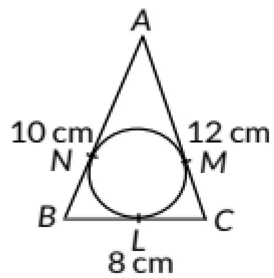
59. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line- segment joining the points of contact at the centre. (2023)

60. In the given figure, PT and PS are tangents to a circle with centre O, from a point P, such that PT = 4 cm and TPS = 60°. Find the length of the chord TS. Also, find the radius of the circle.



(2021)

61. In the given figure, a circle is inscribed in a AABC having sides BC = 8 cm, AB = 10 cm and AC = 12 cm. Find the lengths BL, CM and AN.



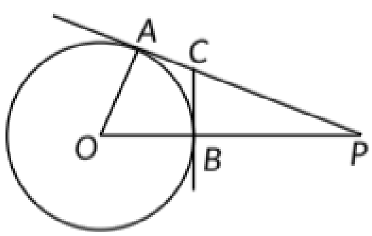
(Delhi 2019)

62. Prove that tangents drawn at the ends of a diameter of a circle are parallel.

63. (Al 2019, Delhi 2017, Foreign 2014)

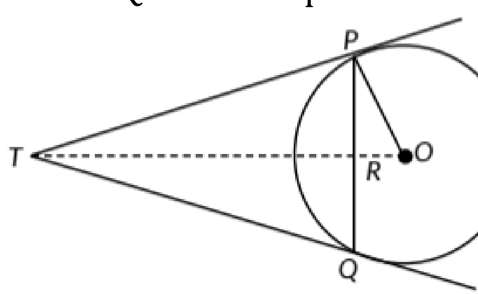
LA(4/5/6 marks)

63. In Figure O is centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP 13 cm, then find the length of tangents PA and BC.



(Term II, 2021-22)

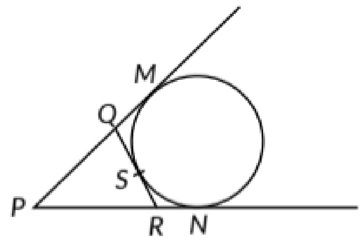
64. In fig. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length of TP.



(Term II, 2021-22)

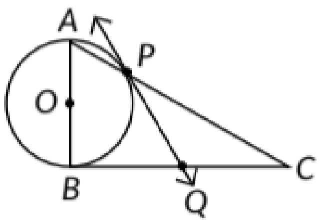
65. Prove that a parallelogram circumscribing a circle is a rhombus. (Term II, 2021-22, Delhi 2014)

66. In fig, if a circle touches the side QR of APQR at S and extended sides PQ and PR at M and N, respectively, then



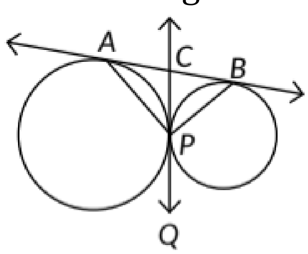
Prove that $PM = \frac{1}{2}(PQ+QR+PR)$ (Term II, 2021-22,

67. In figure, a triangle ABC with ZB 90° is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC.



(Term II, 2021-22)

68. In figure, two circles touch externally at P. A common tangent touches them at A and B and another common tangent is at P, which meets the common tangent AB at C. Prove that/APB = 90°.

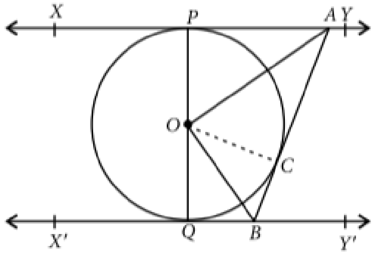


(Term II, 2021-22)

69. Prove that the length of tangents drawn from an external point to a circle are equal. (2018, Delhi 2017, 2016, 2015, 2014, AI 2017, 2016, 2015, Foreign 2016, 2015, 2014)

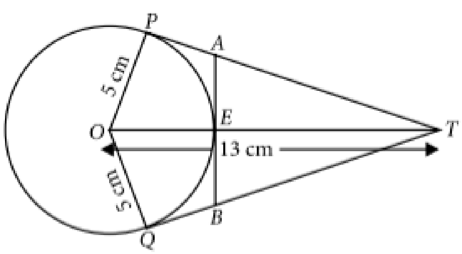
70. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at

A and X'Y' at B. Prove that ZAOB = 90°.



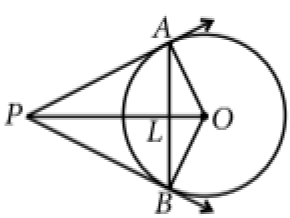
(AI 2017)

71. In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



(Delhi 2016)

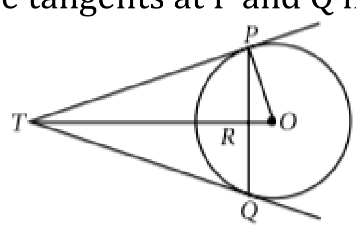
72. In the given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA.



(Foreign 2016)

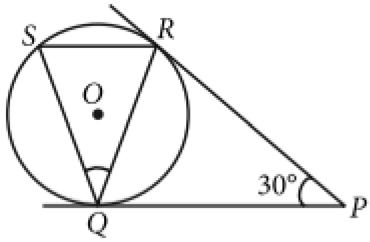
OR

In the given figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



(AI 2014

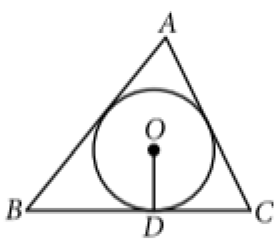
73. In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that <RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find <RQS.



(Delhi 2015)

74. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. (AI 2014)

75. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



(AI 2014) Ap

CBSE Sample Questions

10.2 Tangent to a Circle

VSA (1 mark)

1. PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then find <OQP. (2020-21)

10.3 Number of Tangents from a Point on a Circle

MCQ

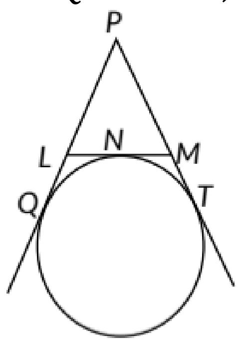
2. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(a) $\frac{3\sqrt{3}}{2}$ cm (b) 3 cm

(c) 6 cm (d) $3\sqrt{3}$ cm *(2022-23)*

VSA (1 mark)

3. If PQ = 28 cm, then find the perimeter of ΔPLM.



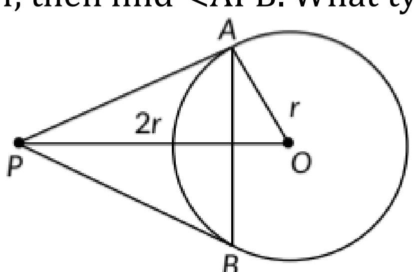
(2020-21)

4. If two tangents are inclined at 60° are drawn to a circle of radius 3 cm, then find length of each tangent. (2020-21)

SAI (2 marks)

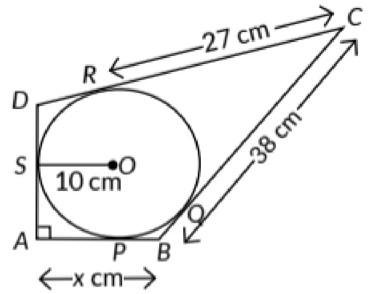
5. In the given figure, O is the centre of circle. Find <AQB, given that PA and PB are tangents to the circle and <APB = 75°.

6. From a point P, two tangents PA and PB are drawn to a circle C(0, r). If OP = 2r, then find <APB. What type of triangle is APB?



(Term II, 2021-22)

7. In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD AB. If radius of incircle is 10 cm, then the value of x is

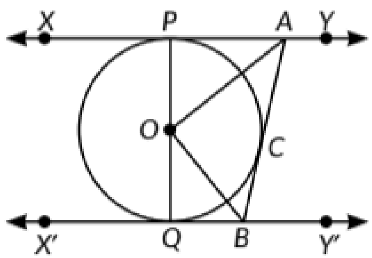


(2020-21)

SA II (3 marks)

8. Prove that a parallelogram circumscribing a circle is a rhombus. (2022-23)

9. In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, what is the measure of <AOB.

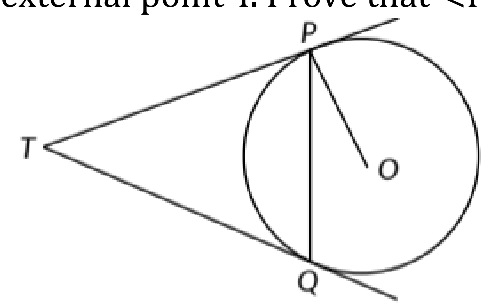


(2022-23)

LA (4/5/6 marks)

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact to the centre. (Term II, 2021-22)

11. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that <PTQ=2<OPQ.



(Term II, 2021-22)

SOLUTIONS

Previous Years' CBSE Board Questions

1. (b): Let AB is a chord of circle which subtends right angle at its centre.

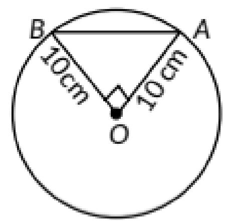
$\therefore$ In ΔOAB, by Pythagoras theorem, we have

$(AB)^2 = (OA)^2 + (OB)^2$

$\Rightarrow (AB)^2 = (10)^2 + (10)^2$

$\Rightarrow (AB)^2 = 200$

$\Rightarrow AB = 10\sqrt{2}$ cm



2.

(a): We have $\angle AOB = 95°$

In ΔAOB, $\angle OAB = \angle OBA$ (since $OA = OB$) ...(i)

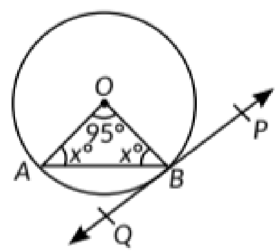
Now, $\angle OAB + 95° + \angle OBA = 180°$

(Angle sum property of a triangle)

$\Rightarrow \angle OAB = \frac{85°}{2} = 42.5°$

$\therefore \angle OAB = \angle OBA = 42.5°$ (From (i))

Now, OB is perpendicular to the tangent line PQ.



= <QBO = 90°

= <ABO+<ABQ = 90°

= 42.5° + <ABQ = 90°

= <ABQ = 47.5°

3. (a): Draw OAI TA.

In ΔOTA, <OAT = 90° (:- Tangent to a circle is perpendicular to the radius passing through the point of contact) and <OTA = 30°

$$\therefore \quad \frac{TA}{OT} = \cos 30° \Rightarrow TA = 4\cos 30° = 4 \times \frac{\sqrt{3}}{2}$$

$$TA = 2\sqrt{3} \text{ cm}$$

4. (d): Since, <TPO = 25° and <OTP = 90°
(:- Radius is perpendicular to the tangent T)
x = <OTP+<TPO
= 90° +25° = 115°

5. (a): OB AB [As tangent to a circle is perpendicular to the radius through the point of the contact] In ΔOAB,

$OA^2 = OB^2 + AB^2$ [By Pythagoras theorem]

$\Rightarrow \quad 41^2 = 9^2 + AB^2$

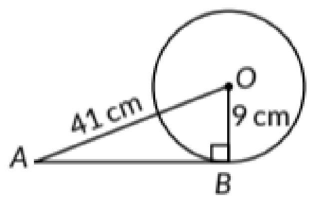
$\Rightarrow \quad AB^2 = 41^2 - 9^2$

$= (41 - 9)(41 + 9)$

$= (32)(50)$

$= 1600$

$\Rightarrow \quad AB = \sqrt{1600} = 40$ cm



6. (c): PR is tangent which touches circle at point P.
So, <OPR 90°
<OPQ=90° - <RPQ = 90° - 50° 40°
In, ΔPOQ,
OP=OQ (Radii of circle)
So, <OQP=<OPQ = 40°
:- <POQ=180°-40°-40° = 100°

7.

(a): In ΔOAB, $OA = OB$
($\because$ Radius of a circle are equal)

$\therefore \quad \angle OAB = \angle OBA$ ...(i)
($\because$ Angles opposite to equal sides are equal)

Now, by angle sum property of a triangle

$\angle OAB + \angle ABO + \angle AOB = 180°$

$\Rightarrow \quad \angle ABO + \angle ABO + 100° = 180°$ (Using (i))

$\Rightarrow \quad 2\angle ABO = 180° - 100° = 80°$

$\Rightarrow \quad \angle ABO = \frac{80°}{2} = 40°$

Here, $OBP = 90°$
(Radius is perpendicular to tangent at point of contact)

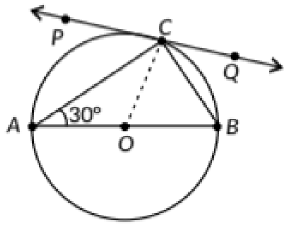
$\Rightarrow \quad \angle OBA + \angle ABP = 90°$

$\Rightarrow \quad 40° + \angle ABP = 90°$

$\Rightarrow \angle ABP = 90° - 40° = 50°$

8. Construction: Join OC

Now, in ΔAOC, AO = OC (radii of same circle)



$\Rightarrow$ $\angle OAC = \angle OCA = 30°$

[Angles opposite to equal sides are equal]

Also, $\angle OCP = 90°$ [Tangent to a circle is $\perp$ to radius at point of contact]

$\therefore$ $\angle PCA + \angle OCA = 90°$

$\Rightarrow$ $\angle PCA + 30° = 90°$ $\Rightarrow$ $\angle PCA = 60°$

9.

Given, $\angle BOC = 130°$

Since, AC is a tangent to the circle at A

$\therefore$ $\angle OAC = 90°$ [$\because$ Radius is perpendicular to the tangent at point of contact]

Now, $\angle AOC + \angle BOC = 180°$[Linear pair]

$\Rightarrow$ $\angle AOC = 180° - 130° = 50°$

In ΔAOC, $\angle AOC + \angle ACO + \angle OAC = 180°$

[Angle sum property]

$\Rightarrow$ $\angle ACO = 180° - 50° - 90° = 40°$

10.

Given, $\angle QPT = 60°$

$\because$ OP is the radius of the circle

$\therefore$ $\angle OPT = 90°$ ($\because$ Tangent is perpendicular to the radius through the point of contact.)

$\Rightarrow$ $\angle OPQ = \angle OPT - \angle QPT = 90° - 60° = 30°$

i.e., $\angle OPQ = \angle OQP = 30°$ ($\because$ $OP = OQ$)

In ΔOPQ, $\angle POQ = 180° - (30° + 30°) = 120°$

Now, reflex $\angle POQ = 360° - 120° = 240°$

As, we know that

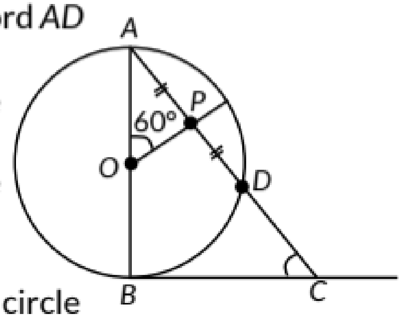
$\Rightarrow$ $\angle PRQ = \left(\frac{240}{2}\right)^{\circ} = 120°$

11.

Given, OP bisects the chord AD

$\therefore \quad OP \perp AD$

[$\because$ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]



$\Rightarrow \quad m\angle P = 90°$

Since, BC is the tangent to the circle at point B.

Therefore, OB is perpendicular to BC.

$\therefore \quad m\angle B = 90°$

Now, $m\angle BOP + m\angle AOP = 180°$ [Linear pair]

$\Rightarrow \quad m\angle BOP + 60° = 180°$

$\Rightarrow \quad m\angle BOP = 120°$ [$\because$ Given, $m\angle AOP = 60°$]

In a quadrilateral $BOPC$,

$m\angle B + m\,\angle BOP + m\angle P + m\angle C = 360°$

[$\because$ Sum of the angles of a quadrilateral is 360°]

$\Rightarrow \quad 90° + 120° + 90° + m\angle C = 360°$

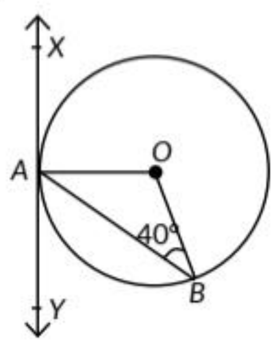
$\Rightarrow \quad m\angle C = 360° - 300° = 60°$

Hence, $m\angle C = 60°$

12. Since, XAY is a tangent to the circle centred at O. Therefore, OA is perpendicular to XAY

: m<XAO = 90°

$\therefore \quad m\angle XAO = 90°$



Now, $OA = OB$ [Radii of circle]

$\Rightarrow \quad m\angle OAB = m\angle OBA$ [Angles opposite to equal sides of a triangle are also equal]

Given, mZABO = 40°

Therefore, m/OAB = 40°

Now, m/BAY+mZOAB+mZXAO = 180° [ZXAY is a straight angle]

m<BAY + 40° + 90° = 180°
⇒ m<BAY = 180° - 130° = 50°
Now, in ΔAOB,
m<AOB+mZOAB+mZOBA = 180° [Angle sum property]
⇒ m<AOB+40° + 40° = 180°
⇒ m<AOB+80° = 180°
⇒ m<AOB = 180° - 80° = 100°

13. Since angle subtended by an arc at the centre is double the angle subtended by the same arc at the remaining part of the circle.
:- 2<ABQ = <AOQ

$$\Rightarrow \angle ABT = \frac{58^\circ}{2} \quad \Rightarrow \angle ABT = 29^\circ$$

Also, $\angle BAT = 90^\circ$ ($\because$ Tangent is perpendicular to the radius through the point of contact)

In ΔABT, $\angle ABT + \angle BAT + \angle ATB = 180^\circ$

$\Rightarrow \quad 29^\circ + 90^\circ + \angle ATQ = 180^\circ$

$\Rightarrow \quad \angle ATQ = 180^\circ - 119^\circ = 61^\circ$

14.

(a): Given, $\angle ABO = 30^\circ$, $OA = 75$ cm

In ΔOAB, $\tan 30^\circ = \frac{OA}{AB}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{AB} \Rightarrow AB = 75\sqrt{3} \text{ cm}$$

(b) In ΔOAB, $\sin 30^\circ = \frac{OA}{OB}$

$$\Rightarrow \quad \frac{1}{2} = \frac{75}{OB} \Rightarrow OB = 150 \text{ cm}$$

(c) In ΔOAB, $PQ || OA$

$$\frac{QB}{QO} = \frac{BP}{AP}$$

$$\Rightarrow \quad \frac{150-75}{75} = \frac{AB}{AP} - 1 \Rightarrow 2 = \frac{AB}{AP} = \frac{75\sqrt{3}}{AP}$$

$$\Rightarrow \quad AP = 75 \times \frac{\sqrt{3}}{2} \Rightarrow AP = \frac{75\sqrt{3}}{2} \text{ cm}$$

OR

OA = OQ = 75 cm (∵ Radius)

In ΔOAB,

We have, $PQ || OA$

In ΔBQP and ΔBOA

$\angle BQP = \angle BOA$ (corresponding angles)

$\angle B = \angle B$ (common)

$\therefore \quad \Delta BQP \sim \Delta BOA$ (By AA similarity)

$$\therefore \quad \frac{BQ}{BO} = \frac{QP}{OA} = \frac{BP}{BA}$$

$$\Rightarrow \quad \frac{PQ}{75} = \frac{AB - AP}{AB}$$

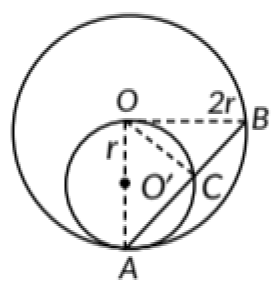
$$\Rightarrow \quad \frac{PQ}{75} = 1 - \frac{AP}{AB}$$

$$\Rightarrow \quad \frac{PQ}{75} = 1 - \frac{75\sqrt{3}}{2 \times 75\sqrt{3}}$$

$$\Rightarrow \quad \frac{PQ}{75} = \frac{1}{2} \quad \therefore \ PQ = \frac{75}{2} = 37.5$$

$\Rightarrow PQ = 37.5$ cm

15. Given: Two circles with centres O and O' of radii 2r and r respectively, touch each other internally at A. AB is the chord of bigger circle touches the smaller circle at C. To prove : C bisects AB i.e., AC = CB Here, for smaller circle (O', r)



$\angle ACO = 90°$

[∵ Angle in a semicircle is 90°]

$\therefore \quad OC \perp AC$

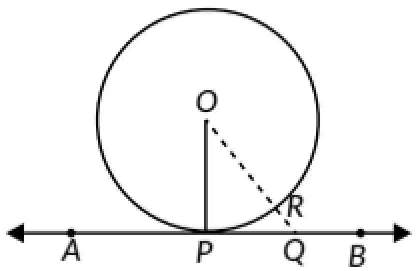
Now, in bigger circle $(O, 2r)$

Since, AB is a chord and $OC \perp AB$.

$\Rightarrow \quad AC = CB$ [∵ Perpendicular drawn from centre of the circle to a chord bisects the chord]

Hence, C bisects the chord AB.

16. Given: A circle C(O, r) and a tangent AB at a point P. To prove : OP AB. Construction:- Take any point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.



Proof: We know that among all line segments joining the point O to a point on AB, the shortest one is perpendicular to AB. So, to prove that OP AB, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB. Clearly, OP=OR [radii of the same circle]

Now, OQ = OR+RQ⇒ OQ > OR

⇒ OQ > OP [:- OP=OR]

⇒ OP < OQ

Thus, OP is shorter than any other segment joining O to any point of AB. Hence, OP AB.

17. Given Two equal circles O and O' touching each other at X. AC is tangent to the circle with centre O. <ADO' = 90°

To find: $\frac{DO'}{CO}$

Solution : Let $AO' = O'X = XO = r$

$\because$ Tangent to a circle is always perpendicular to its radius at the point of contact.

$\therefore$ $\angle ACO = 90°$

In $\Delta ADO'$ and ΔACO

$\angle DAO' = \angle CAO$ [Common]

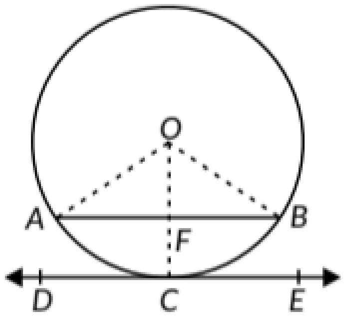
$\angle ADO' = \angle ACO$ [Each 90°]

$\therefore$ $\Delta ADO' \sim \Delta ACO$ (AA similarity criteria)

$$\Rightarrow \frac{DO'}{CO} = \frac{AO'}{AO} = \frac{r}{3r} = \frac{1}{3}$$

18. Given: A circle with centre O and C is the mid point of arc ACB and DE is a tangent to the circle. To prove : AB || DE Construction: Join OA, OB and OC.

Proof Since C is the midpoint of arc AB.



$\therefore \quad \angle AOF = \angle BOF$

($\because$ OA and OB are equally inclined with OC)

Now, in ΔOAF and ΔOBF,

$OA = OB$ (Radii of the circle)

$\angle AOF = \angle BOF$ (Proved above)

$OF = OF$ (Common)

$\therefore \quad \Delta OAF \cong \Delta OBF$ (By SAS criterion)

$\Rightarrow \quad \angle AFO = \angle BFO$ (By CPCT)

Now, $\angle AFO + \angle BFO = 180°$ (Linear pair)

$\Rightarrow \quad 2\angle AFO = 180° \Rightarrow \angle AFO = 90°$

Also, $\angle OCD = 90°$

Tangent is perpendicular to radius through the point of contact.

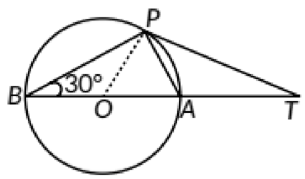
$\therefore \quad \angle AFO = \angle OCD$ (Each 90°)

But these are corresponding angles $\therefore$ $AB \parallel DE$

19.

Given : O is the centre of the circle and TP is the tangent to circle and $\angle PBT = 30°$

To prove : $\frac{BA}{AT} = \frac{2}{1}$



Construction : Join OP

Proof : $\angle BPA = 90°$

(Angle in a semi circle)

In ΔBPA, $\angle P + \angle PBA + \angle BAP = 180°$

[Angle sum property]

$\Rightarrow \quad 90° + 30° + \angle BAP = 180°$

$\Rightarrow \quad \angle BAP = 60°$

Also, $\angle OPT = 90°$

And $OP = OA$ (Radii of same circle) ...(i)

$\therefore \quad \angle OAP = \angle OPA = 60°$ ...(ii)

$\Rightarrow \quad \angle APT = 90° - 60° = 30°$

Now, $\angle OAP + \angle PAT = 180°$ (Linear pair)

$\Rightarrow \quad 60° + \angle PAT = 180°$ [Using (ii)]

$\Rightarrow \quad \angle PAT = 120°$

In ΔPAT, $\angle PAT + \angle APT + \angle PTA = 180°$

$\Rightarrow \quad 120° + 30° + \angle PTA = 180°$

$\Rightarrow \quad \angle PTA = 30°$

$\Rightarrow \quad PA = AT$ ...(iii)

Also in ΔOAP,

$\angle AOP = 180° - 60° - 60° = 60°$

$\therefore \quad \angle AOP = \angle OPA \quad \Rightarrow \quad PA = OA$ ...(iv)

Hence, $PA = AT = OA = OP$ [Using (i), (iii) and (iv)]

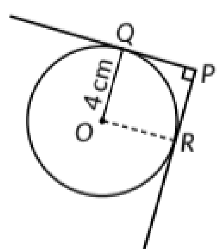
Now, $BA = BO + OA = 2OA$ ($\because OA = OB$)

$\Rightarrow \quad BA = 2AT$ ($\because OA = AT$)

$\Rightarrow \quad \frac{BA}{AT} = \frac{2}{1}$

20.

(b): Join OR.



We know that, tangent to a circle is $\perp$ to radius at the point of contact. So, $OQ \perp PQ$ and $OR \perp PR$.

Also, $\angle QPR = 90°$

Now, in quadrilateral $OQPR$,

$\angle QOR = 360° - (90° + 90° + 90°)$

$= 90°$ (By angle sum property of a quadrilateral)

Now, $\angle PQO = \angle QOR = \angle ORP = \angle RPQ = 90°$

Also, $PQ = PR$

($\because$ Tangents drawn from an external point are equal)

.. PQOR is a square.
Hence, PQ OQ = 4 cm

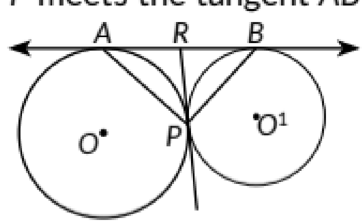
21. (a): In quadrilateral AOBP
<AOB+<OBP + <APB+<OAP = 360°
⇒ ZAOB+90° +90° +80° 360° [:- OAL PA and OBL PB)
⇒ AOB = 360°-260° = 100°

22. (b): It is known that the length of the tangents drawn from an external point to a circle are equal.
.. QP = PT = 3.8 cm and PR = PT = 3.8 cm
Now, QR = QP+PR = 3.8 cm +3.8 cm = 7.6 cm

23. (b): Given, <QPR = 46°
We have, OQ PQ and OR 1 RP [:: Radius isto the tangent through the point of contact]
= OQPZORP = 90°
In quadrilateral PQOR, we have
<OQP+<QPR + <PRO+ <ROQ = 360°
= 90° +46° +90° + = <ROQ = 360°
= ROQ = 360° - 226° = 134°

24.

(d) : Let common tangent at P meets the tangent AB at R.



Since, tangents drawn from an external point to a circle are equal.
$\therefore \quad AR = RP$
and $BR = RP$
$\Rightarrow \quad \angle RAP = \angle RPA = x$ (say) ...(i)
and $\angle RBP = \angle RPB = y$ (say) ...(ii)
Now, $\angle ARP + \angle BRP = 180°$ [Linear pair] ...(*)
In ΔARP, $\angle ARP + \angle RPA + \angle RAP = 180°$ ...(iii)
and in ΔBRP, $\angle BRP + \angle RPB + \angle RBP = 180°$ ...(iv)
Adding (iii) and (iv), we get
$\angle ARP + x + x + \angle BRP + y + y = 360°$ [Using (i) & (ii)]
$\Rightarrow \quad \angle ARP + \angle BRP + 2x + 2y = 360°$
$\Rightarrow \quad 2(x + y) = 360° - 180° = 180°$ [Using (*)]
$\Rightarrow \quad x + y = 90°$
i.e., $\angle RPA + \angle RPB = 90° \quad \Rightarrow \quad \angle APB = 90°$

25. (b):We know, tangents drawn from an external point to the circle aree qual in length.

i.e., AP = AS, BP = BQ, CQ = CR, DR = DS

So, CR=CQ CQ = 3 cm

Now, BC = 7 cm ⇒ CQ+BQ = 7 cm

⇒ BQ (7-3)cm = 4 cm

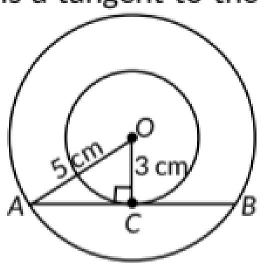
Also, BQ = BP⇒ BP = 4 cm

Also, ASAP and AS = 5 cm ⇒ AP = 5 cm

AB=AP+PB = (5+ 4) cm = 9 cm

So, x = 9

26. (c): Let chord *AB* of larger circle is a tangent to the smaller circle.



∴ $OC \perp AB$

In right ΔOAC,

$(OA)^2 = (OC)^2 + (AC)^2$

⇒ $(5)^2 = (3)^2 + (AC)^2$

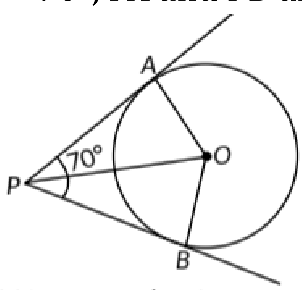
⇒ $AC^2 = 25 - 9 = 16$

⇒ $AC = 4$ cm

As, perpendicular drawn from the centre to the chord bisects the chord.

∴ Length of chord $AB = (2 \times 4)$ cm = 8 cm

27. Given APB = 70°, PA and PB are tangents to the circle.



$\angle PAO = \angle PBO = 90°$ $(\because OA \perp PA$ and $OB \perp PB)$

In quadrilateral *APBO*

$\angle AOB + \angle APB + \angle PBO + \angle PAO = 360°$

⇒ $\angle AOB + 90° + 90° + 70° = 360°$

⇒ $\angle AOB = 360° - 250° = 110°$

In ΔPAO and ΔPBO

PA =*PB* (Tangents drawn from an enternal point to the circle are equal)

$OA = OB$ (Radii of circle)

$OP = OP$ (common side)

∴ $\Delta PAO \cong \Delta PBO$ (*SSS*)

$\angle AOP = \angle BOP$ (CPCT)

Now, $\angle AOB = \angle POA + \angle POB$

⇒ $\angle AOB = \angle POA + \angle POA = 2\angle POA$

⇒ $\angle POA = \frac{1}{2} \times 110° = 55°$

28. Lengths of tangents drawn from an external point to the circle are equal.
:- AP = AR = 4 cm, BP = BQ = 3 cm and CQCRCA - AR-11-4=7cm
:- BC=BQ+CQ=3+7 = 10 cm

29. Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.

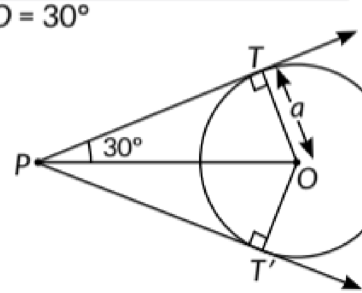
$\therefore \quad \angle TPT' = 60^\circ \Rightarrow \angle TPO = 30^\circ$

Also, $OT \perp TP$

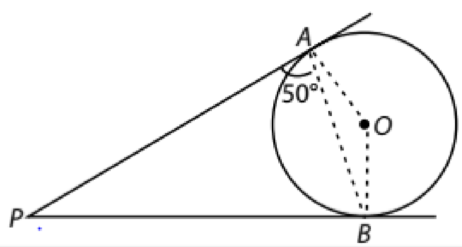
Now, in ΔTPO, $\sin 30^\circ = \dfrac{OT}{OP}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{a}{OP}$

$\Rightarrow \quad OP = 2a$



30. Given, PA and PB are tangents to a circle and ZPAB = 50°



Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

i.e. $\quad OA \perp PA$ and $OB \perp PB \Rightarrow \angle OAP = \angle OBP = 90^\circ$

$\therefore \quad \angle OAB = \angle OAP - \angle PAB$

$= 90^\circ - 50^\circ = 40^\circ = \angle OBA \qquad (\because OA = OB)$

In ΔAOB, $\angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$

31. PA = PB
(Tangents drawn from external point are equal)

$\Rightarrow \quad \angle ABP = \angle BAP = x$(say)

($\because$ Angles opposite to equal sides are equal)

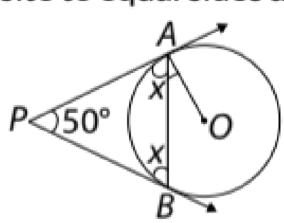
In ΔAPB,

$50^\circ + x + x = 180^\circ$

$\Rightarrow \quad 2x = 130^\circ$

$\Rightarrow \quad x = 65^\circ$

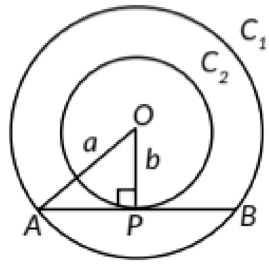
Now, $\angle OAP = 90^\circ$



($\because$ Tangent is perpendicular to the radius through the point of contact)

$\therefore \quad \angle OAB = 90^\circ - 65^\circ = 25^\circ$

32. Let O be the centre of concentric circles and AB be the chord for circle C, and tangent for circle C_2.
Let P be the point where AB meets C_2.
Join O to A and O to P.



:- Tangent at any point is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OPA = 90°$

Now, in ΔOPA, $OA^2 = OP^2 + AP^2$

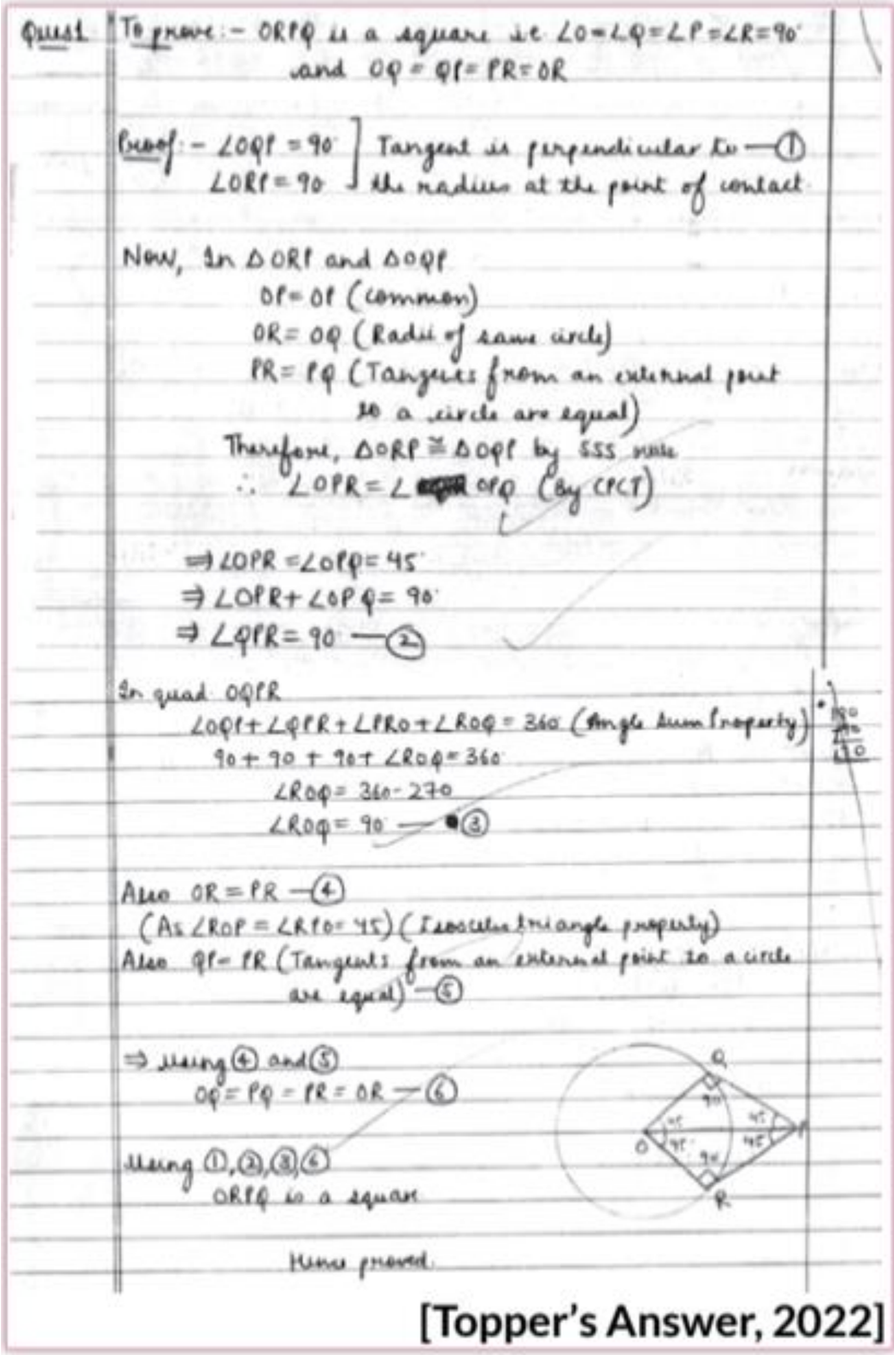
$\Rightarrow \quad a^2 = b^2 + AP^2$

$[\because OA = a,\ OP = b] \Rightarrow \ AP = \sqrt{a^2 - b^2}$

We know that, $AB = 2AP$

$\Rightarrow \ AB = 2\sqrt{a^2 - b^2}$.

33.



Ques1 To prove :- ORPQ is a square i.e $\angle O = \angle Q = \angle P = \angle R = 90^\circ$ and $OQ = QP = PR = OR$

Proof :- $\angle OQP = 90^\circ$, $\angle ORP = 90^\circ$] Tangent is perpendicular to the radius at the point of contact — (1)

Now, In ΔORP and ΔOQP

$OP = OP$ (common)

$OR = OQ$ (Radii of same circle)

$PR = PQ$ (Tangents from an external point to a circle are equal)

Therefore, $\Delta ORP \cong \Delta OQP$ by SSS rule

$\therefore \angle OPR = \angle OPQ$ (By CPCT)

$\Rightarrow \angle OPR = \angle OPQ = 45^\circ$

$\Rightarrow \angle OPR + \angle OPQ = 90^\circ$

$\Rightarrow \angle QPR = 90^\circ$ — (2)

In quad OQPR

$\angle OQP + \angle QPR + \angle PRO + \angle ROQ = 360^\circ$ (Angle sum Property)

$90 + 90 + 90 + \angle ROQ = 360^\circ$

$\angle ROQ = 360 - 270$

$\angle ROQ = 90^\circ$ — (3)

Also $OR = PR$ — (4)

(As $\angle ROP = \angle RPO = 45^\circ$) (Isosceles triangle property)

Also $QP = PR$ (Tangents from an external point to a circle are equal) — (5)

$\Rightarrow$ using (4) and (5)

$OQ = PQ = PR = OR$ — (6)

using (1), (2), (3), (6)

ORPQ is a square

Hence proved.

[Topper's Answer, 2022]

34. Given, AQ=5 cm

AQ=AR = 5 cm (Tangents drawn from an external point to the circle are equal)

Now, AC = AR + RC (OR is a perpendicular bisector of AC .. AR = RC)

$\Rightarrow$ AC = 10 cm

35. Given: PQ and PR are tangents from an external point P. To prove : PQOR is a cyclic quadrilateral. Proof OR and OQ are the radius of circle centred at O, and PR and PQ are tangents.

.. <ORP 90° and <OQP = 90°

In quadrilateral PQOR, we have

<P+<R+<O+<Q=360°

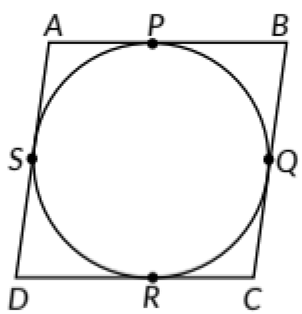
⇒ <P+90° +<O+90° = 360°

⇒ ZP+20=360°- 180° 180°

ZP and 20 are opposite angles of quadrilateral which are supplementary.

:- PQOR is a cyclic quadrilateral.

36. Let the circle touches the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively. Since, lengths of tangents drawn from an external point to the circle are equal.



∴ AP = AS ...(1) (Tangents drawn from A)

BP = BQ ...(2) (Tangents drawn from B)

CR = CQ ...(3) (Tangents drawn from C)

DR = DS ...(4) (Tangents drawn from D)

Adding (1), (2), (3) and (4), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ (AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC)

⇒ AB + CD = AD + BC

37. As we know that, tangents drawn from an external point are equal in length.

:- BP = BD and CD = CQ ...(i)

Also, AP = AQ= 12 cm

⇒ AB+BP = 12 cm and AC+ CQ = 12 cm

⇒ AB+ BD = 12 cm

AC+ CD = 12 cm ... (ii) [Using (i)]

Now, perimeter of AABC = AB + BC + CA

=AB+ BD + DC + AC
= 12+12 [Using (ii)]
= 24 cm

38. Since, tangents drawn from an external point are equal.
:- TP = TQ
= $<TPQ=ZTQP$ ...(i) (:- Angles opposite to equal sides are equal)
In ΔTPQ,
$<PTQ+<TQP+<TPQ = 180^\circ$
= $<PTQ+ <TPQ+<TPQ = 180^\circ$ [Using (i)]
= $<PTQ+2 <TPQ = 180^\circ$
= $<PTQ = 180^\circ - 2 <TPQ$ ...(ii)

Now, ZOPT 90° (:- Tangent is perpendicular to the radius through the point of contact)
:- $<TPQ=90^\circ - <OPQ$ ...(iii)
From (ii) and (iii), $<PTQ = 180^\circ -2(90^\circ - <OPQ)$
= $180^\circ - 180^\circ +2 <OPQ = 2 <OPQ$

39. As we know that, tangents drawn from an external
point are equal in length.
:- QC=CA; QD = BD and PA = PB
Since QC QD = 3 cm (given)
= CA = BD = 3 cm
Also, PC = PA - AC
= PC (12-3) cm = 9 cm
Similarly PD = 9 cm
:- PC + PD=9+9 = 18 cm

40. Given: Tangents PA and PB are drawn at end points of a chord AB of a circle C(O, r).
To prove: Tangents PA and PB make equal angles with the chord AB i.e.,
$<PAB = <PBA$
Proof: Since, lengths of tangents drawn from an external point to the circle are equal.
.. PA= PB
= $<PAB = ZPBA$ (:- Angles opposite to equal sides are equal)

41. Let AD = x => AF = x
[:- Tangents drawn from an external point to the circle are equal in lengths]
.. BD = BE = 12-x (... AB = 12 cm)
and FC = CE = 10-x (... AC = 10 cm)
But BC = 8 cm (Given)
:- 10-x+12-x=8
= -2x+22=8⇒ 2x=14⇒ x=7
Hence, length of AD = 7 cm
Length of BE = 12-7=5cm
Length of CF = 10-7=3 cm

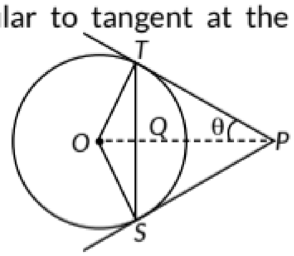
42. Given, PA and PB are tangents from an external point P.
PA = PB = 5 cm
= <PAB = ZABP
(:- Angles opposite to equal sides are equal)
In APB, by angle sum property
<APB = ZPAB = ZABP = 60°
= ΔPAB is an equilateral triangle.
:- AB = PB = PA = 5 cm

43.

In ΔOTP, $OT = r$, $OP = 2r$ [Given]
$\angle OTP = 90°$[Radius is perpendicular to tangent at the point of contact]
Let $\angle TPO = \theta$

$$\therefore \sin\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$\Rightarrow \theta = 30°$
$\therefore$ In ΔTOP, $\angle TOP = 60°$ [By angle sum property]
$\angle TOP = \angle SOP$ [As Δ's are congruent]
$\Rightarrow \angle SOP = 60°$ $\therefore$ $\angle TOS = 120°$
In ΔOTS, as $OT = OS$ (Radii of same circle)
$\therefore$ $\angle OST = \angle OTS$



Now, ZOTS + ZOST+ZSOT = 180°
⇒ 2/OST + 120° = 180° :- <OTS = <OST = 30°

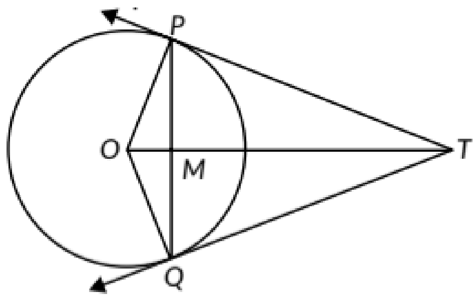
44. Given, <SPT = 120°
Since, radius is perpendicular to the tangent at the point of contact.
.. <OSP = 90° Also, ZSPO= 60° [:- Tangents drawn to a circle from an external

point are equally inclined to the segment, joining the centre to that point]

In ΔSPO, $\cos 60° = \frac{PS}{OP} \Rightarrow \frac{1}{2} = \frac{PS}{OP} \Rightarrow OP = 2PS$

45. In right AAOP, OP2 = $AP^2 + OA^2$ = 82 +6^2 = 100
In right ABOP, $OP^2 = BP^2 + OB^2$
⇒ 100 $BP^2 + 4^2$
⇒ BP2 = 100-16=84 ⇒ BP = $2\sqrt{21}$cm

46. Given: A circle with centre O, two tangents TP and TQ drawn from external point T to the circle.



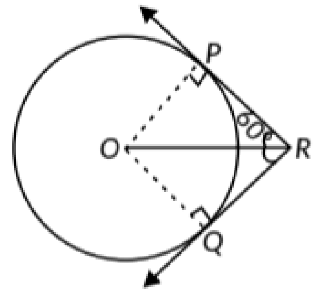
To prove : OT is right bisector of PQ *i.e*,
$\angle TMP = 90°$, $PM = MQ$
Proof : In ΔTPM and ΔTQM, we have
$TP = TQ$ (∵ Tangents drawn from an external point are equal)
$TM = TM$ (Common)
$\angle MTP = \angle MTQ$ [∵ TP and TQ are equally inclined to OT]
∴ $\Delta TPM \cong \Delta TQM$ (By SAS congruency)
⇒ $PM = MQ$ (By CPCT) ...(i)
Also, $\angle TMP = \angle TMQ$ (By CPCT) ...(ii)
But $\angle TMP + \angle TMQ = 180°$ [Linear Pair]
⇒ $2\angle TMP = 180°$ ⇒ $\angle TMP = 90°$
Hence, OT is the right bisector of line segment PQ.

47. Given: Two tangents PR and QR are drawn from an
external point R and ZPRQ = 120°
To prove : OR = PR + RQ.
Construction: Join OP and OQ.
Proof: In ΔOPR and ΔOQR,
OP = OQ (Radii of the circle)
<OPR = <OQR = 90° (∵ Radius is perpendicular to the tangent at the point of contact)
RP=RQ (Tangents from an external point are equal)

$\therefore \quad \Delta OPR \cong \Delta OQR$ (By SAS)

So, $\angle PRO = \angle QRO$ (By CPCT)

$\therefore \quad \angle PRO = \frac{1}{2}\angle PRQ = \frac{1}{2}\times 120° = 60°$



In ΔOPR, we have

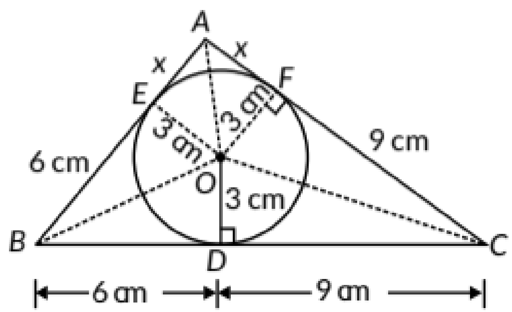
$\cos 60° = \frac{PR}{OR}$

$\Rightarrow \frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow \quad OR = 2PR \Rightarrow OR = PR + PR$

$\Rightarrow \quad OR = PR + RQ$ ($\because$ $PR = RQ$)

48. Let E and F be the points where the tangents AB and AC touches the circle respectively. Join OE and OF. Now, radius is perpendicular to tangent at the point of contact.



So, $OD \perp BC$, $OE \perp AB$ and $OF \perp AC$.

Join OA, OB and OC.

Since, tangents drawn from an external point to a circle are equal.

$\therefore \quad BD = BE = 6$ cm, $CD = CF = 9$ cm and $AE = AF = x$ cm (say)

Now, area of ΔABC = area of ΔAOB + area of ΔBOC + area of ΔAOC

$\Rightarrow 54 = \frac{1}{2}\times AB\times OE + \frac{1}{2}\times BC\times OD + \frac{1}{2}\times AC\times OF$

$\Rightarrow 54 = \frac{1}{2}\times(x+6)\times 3 + \frac{1}{2}\times 15\times 3 + \frac{1}{2}\times(x+9)\times 3$

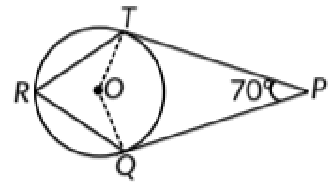
$\Rightarrow 54 = \frac{1}{2}\times(3x+18+45+3x+27)$

$\Rightarrow \quad 6x + 90 = 108 \Rightarrow 6x = 108 - 90 = 18 \Rightarrow x = \frac{18}{6} = 3$

$\therefore \quad AB = 3 + 6 = 9$ cm and $AC = 3 + 9 = 12$ cm

49.

Given : O is the centre of a circle. PT and PQ are tangents to the circle from an external point P.



$\angle TPQ = 70°$

Construction : Join T to O and Q to O

$\because$ $OT \perp PT$ and $OQ \perp QP$ $\therefore$ $\angle OTP = \angle OQP = 90°$

In quadrilateral $PTOQ$,

$\angle OTP + \angle OQP + \angle TOQ + \angle TPQ = 360°$

$\Rightarrow$ $90° + 90° + \angle TOQ + 70° = 360°$

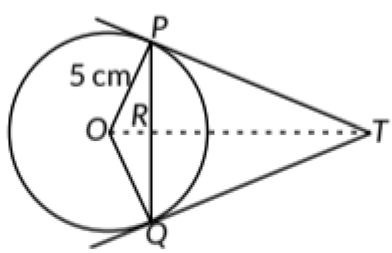
$\Rightarrow$ $\angle TOQ + 70° = 180° \Rightarrow \angle TOQ = 110°$

Also, $\angle TOQ = 2\angle TRQ$

[$\because$ Angle subtended by an arc at centre of the circle is double the angle subtended by the same arc at the remaining part of the circle]

$\therefore$ $110° = 2\angle TRQ \Rightarrow \angle TRQ = \frac{110°}{2} = 55°$

50. Join OT and let PQ intersect OT at R. We know that, tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point.



So, $\angle OTP = \angle OTQ$

In ΔTPR and ΔTQR,

$TP = TQ$ (Tangents drawn from an external point are equal in length)

$\angle RTP = \angle RTQ$ ($\because$ $\angle OTP = \angle OTQ$)

$RT = RT$ (common)

$\therefore$ $\Delta TPR \cong \Delta TQR$ (SAS congruence)

Now, $PR = QR$ (By CPCT)

Also, $OR \perp PQ$ or $\angle ORP = \angle ORQ = 90°$

Now, $PQ = PR + QR$

$\Rightarrow$ $PQ = PR + PR$ [$\because$ $PR = QR$]

$\Rightarrow$ $PR = 4$ cm

Now, in ΔOPR, $OR^2 + PR^2 = OP^2$

$\Rightarrow \quad OR^2 = OP^2 - PR^2$

$\Rightarrow OR^2 = 5^2 - 4^2 \Rightarrow OR = 3$ cm

Let $RT = x$

In ΔPRT, $PT^2 = PR^2 + RT^2 = 4^2 + x^2 = 16 + x^2$

In ΔOPT, $PT^2 = OT^2 - OP^2$

$\Rightarrow \quad 16 + x^2 = (3 + x)^2 - 5^2 \Rightarrow 16 + x^2 = 9 + 6x + x^2 - 25$

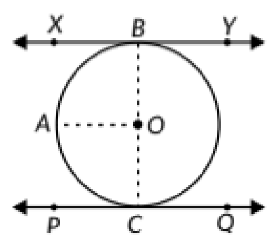
$\Rightarrow \quad 6x = 32 \Rightarrow x = \frac{32}{6} = \frac{16}{3}$

$\therefore \quad PT^2 = 16 + \left(\frac{16}{3}\right)^2 \Rightarrow PT^2 = 16 + \frac{256}{9}$

$\Rightarrow PT^2 = \frac{144+256}{9} = \frac{400}{9} \Rightarrow PT = \sqrt{\frac{400}{9}} = \frac{20}{3}$ cm

So, $TP = TQ = \frac{20}{3}$ cm

51. Let XBY and PCQ be two parallel tangents to a circle with centre O.



Construction : Join OB and OC.

Draw $OA \parallel XY \parallel PQ$.

Now, $XB \parallel AO$

$\Rightarrow \quad \angle XBO + \angle AOB = 180°$

Now, $\angle XBO = 90°$ ($\because$ Tangent to a circle is perpendicular to the radius through the point of contact)

$\therefore \quad 90° + \angle AOB = 180° \Rightarrow \quad \angle AOB = 180° - 90° = 90°$

Similarly, $\angle AOC = 90°$

$\therefore \quad \angle AOB + \angle AOC = 90° + 90° = 180°$

Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

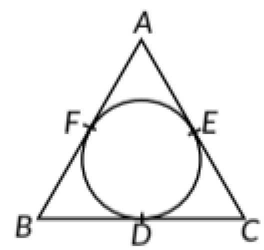
52. Since, we know that tangents drawn from an external point are equal in length.

.. AE = EC and ED = EB ...(i)

Now, AB = AE+EB ...(ii)

Using (i) in (ii), we get AB = CE + ED = CD
⇒ AB = CD

53. Since, tangents drawn from an external point are equal in length
:- AF = AE, BF = BD, CD = CE
and AB = AC (given) ...(i)
Now, subtracting AF on both the sides of (i)
AB - AF = AC-AF
= AB- AF-AC-AE [:: AF = AE]
= BF = EC
= BD = CD [:: BF = BD, EC = CD]



54. Since lengths of tangents drawn from an exterior point to a circle are equal.
:- XP = XQ ...(i), APAR ...(ii), BQ = BR ...(iii)
From (i), XP = XQ
= XA+AP = XB+BQ
= XA + AR = XB+ BR [Using (ii) and (iii)]

55. Since, tangents drawn from an external point are equal.
:- TP = TQ
+ <TPQ = <TQP ...(i)
(. Angles opposite to equal sides are equal)
In TPQ,
<PTQ+<TQP+<TPQ = 180°
⇒ <PTQ+ <TPQ+ <TPQ = 180° [Using (i)]
⇒ <PTQ+2 <TPQ = 180°
⇒ <PTQ = 180° - 2 <TPQ ...(ii)
Now, <OPT = 90° (:- Tangent is perpendicular to the radius through the point of contact)
:- <TPQ=90° - <OPQ ...(iii)
From (ii) and (iii), <PTQ = 180° - 2(90° - <OPQ)
= 180° 180° +2 <OPQ = 2 <OPQ

56. Given, B = 90°, AD = 17 cm, AB = 20 cm, DS = 3 cm Now, DS = DR and AR = AQ
[.. Tangents drawn from an external point to the circle are equal]
:- DR = 3 cm

AR = AD-DR = 17 - 3 = 14 cm
:- AQ = 14 cm
Now, BQ=AB - AQ=20-14= 6 cm
OQ BQ, OP BP (. Tangent at any point of a circle is perpendicular to the radius through the point of contact)
:- Quadrilateral BQOP is a square
:- BQ=OQ=r=6cm
Hence, the radius of the circle = 6 cm.
So, ΔOAP = OBP [By SSS congruency criterion]
So, <APO = <BPO
Hence, OP bisects <APB

58. Let the centre of the two concentric circles is O and AB be the chord of the larger circle which touches the smaller circle at point P as shown in figure.
:- AB is a tangent to the smaller circle at point P

$\Rightarrow \quad OP \perp AB$

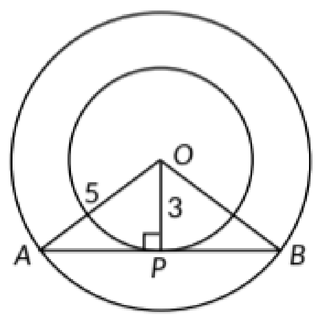
By Pythagoras theorem, in ΔOPA

$OA^2 = AP^2 + OP^2$

$\Rightarrow \quad 5^2 = AP^2 + 3^2$

$\Rightarrow \quad AP^2 = 5^2 - 3^2 = 25 - 9$

$\Rightarrow \quad AP^2 = 16 \Rightarrow AP = 4$ cm



In ΔOPB

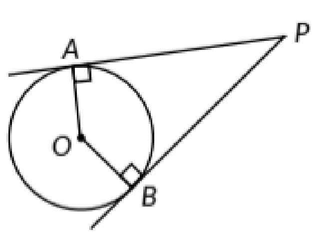
Since, $OP \perp AB$

$AP = PB$ ($\because$ Perpendicular drawn from the centre of the circle bisects the chord)

$\therefore \quad AB = 2AP = 2 \times 4 = 8$ cm

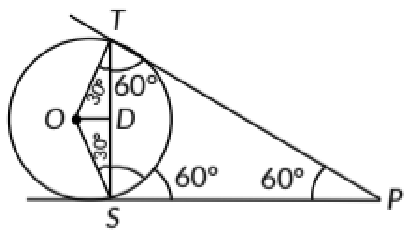
$\therefore$ The length of the chord of the larger circle is 8 cm.

59. Let PA and PB are two tangents on a circle from point P as shown in the figure. It is known that tangent to a circle is perpendicular to the radius through the point of contact.



:- ZOAP ZOBP = 90° ...(i)
In quadrilateral AOBP,
<OAP + <APB+<PBO+<BOA = 360°
= 90° + <APB +90° + <BOA = 360° [Using (i)]
= <APB+<BOA = 180° Hence proved

60. Given TP and SP are tangents from an external point P.
:- PT = PS = 4 cm (:- Tangents drawn from an external point to the circle are equal)
= <PTS = ZPST
(:- Angles opposite to equal sides are equal)
In ATPS, by angle sum property
<TPS = <PTS = <PST = 60°
⇒ ΔTPS is an equilateral triangle.



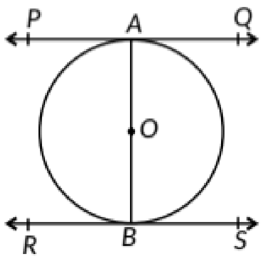
$\therefore$ $TP = PS = TS = 4$ cm
$\angle OSP = 90°$ and $\angle TSP = 60°$
$\therefore$ $\angle OSD = 30°$

Now, $\frac{DS}{OS} = \cos 30° \Rightarrow \frac{2}{OS} = \frac{\sqrt{3}}{2} \Rightarrow OS = \frac{4\sqrt{3}}{3}$cm

61. Let BL = x⇒ BN = x
[:: Tangents drawn from an external point to the circle are equal in length]
:- CL=CM=8-x [:- BC= 8 cm]
and AN = AM = 10-x [:: AB = 10 cm]
But AC 12 cm [Given]
:- AM+MC = 12
= 10-x+8-x=12
= 18-2x=126=2x⇒x=3
Length of BL = 3 cm

Length of CM = 8-3=5cm
Length of AN = 10-3=7cm

62. Given: A circle C(O, r)with diameter AB and let PQ and RS be the tangents drawn to the circle at point A and B. To
prove : PQ || RS
Proof : Since tangent at a point to a circle is perpendicular to the radius through the point of contact.



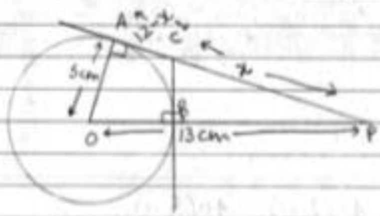
$\therefore \quad AB \perp PQ$ and $AB \perp RS$

$\Rightarrow \quad \angle PAB = 90°$ and $\angle ABS = 90° \quad \Rightarrow \quad \angle PAB = \angle ABS$

$\Rightarrow \quad PQ \parallel RS$ [$\because$ $\angle PAB$ and $\angle ABS$ are alternate interior angles]

63.

Ques12.



$\angle OAP = 90^\circ$ (Tangent is perpendicular to radius at the point of contact)

Using pythagoras theorem in ΔOAP

$H^2 = B^2 + P^2$

$OP^2 = AP^2 + OA^2$

$13^2 = AP^2 + 5^2$

$169 = AP^2 + 25$

$169 - 25 = AP^2$

$\Rightarrow AP^2 = 144$

$\boxed{AP = \sqrt{144} = 12\text{ cm}}$

Let PC be x and AC be 12-x

$AC = BC = 12 - x$ (Tangents from point C to the circle are equal)

Also $OP = 13$ cm

$OB = 5$ cm

$BP = OP - OB$

$BP = 8$ cm

$\angle OBC = 90^\circ$ (Tangent is perpendicular to the radius at the POC)

$\Rightarrow \angle CBP = 90^\circ$ (Linear Pair)

Using pythagoras theorem in ΔCBP

$H^2 = B^2 + P^2$

$CP^2 = BP^2 + BC^2$

$x^2 = 8^2 + (12 - x)^2$

$x^2 = 64 + 144 + x^2 - 24x$

$24x = 64 + 144$

$24x = 208$

$x = \cancel{\frac{208}{24}} = \cancel{\frac{104}{12}} = \frac{52}{6} = \frac{26}{3}$

$BC = 12 - x$

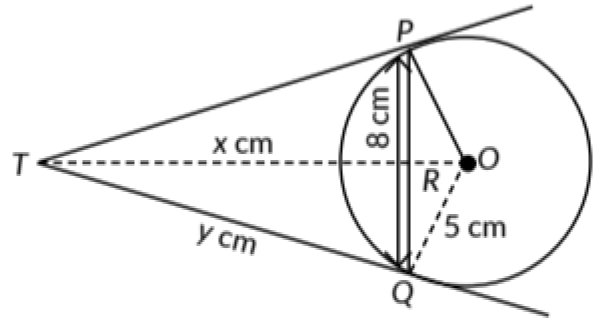
$BC = 12 - \frac{26}{3}$

$BC = \frac{36 - 26}{3} = \frac{10}{3} = \boxed{3.34\text{ m}}$

[Topper's Answer, 2022]

64. We have, PQ = 8 cm, OQ = 5 cm

:- PR=RQ=4 cm



In ΔORQ, by Pythagoras Theorem, we have

$$OQ^2 = OR^2 + RQ^2$$

$\Rightarrow \quad 5^2 = OR^2 + 4^2 \Rightarrow 25 = OR^2 + 16$

$\Rightarrow \quad OR^2 = 25 - 16 = 9 = 3^2$

$\Rightarrow \quad OR = 3$ cm

Let $TR = x$ cm and $TQ = y$ cm

Then, $OT = (x + 3)$ cm

In ΔTRQ, $TQ^2 = TR^2 + RQ^2$ [By Pythagoras Theorem]

$\Rightarrow \quad y^2 = x^2 + 4^2$

$\Rightarrow \quad y^2 = x^2 + 16$ ...(i)

Now, $OQ \perp TQ$

In ΔOQT, $OT^2 = OQ^2 + TQ^2$ [By Pythagoras Theorem]

$\Rightarrow \quad (x + 3)^2 = 5^2 + y^2$

$\Rightarrow \quad x^2 + 9 + 6x = 25 + y^2$

$\Rightarrow \quad x^2 + 9 + 6x = 25 + x^2 + 16$ [By (i)]

$\Rightarrow \ 6x = 32 \ \Rightarrow \ x = \frac{32}{6} = \frac{16}{3}$

$\therefore \ y^2 = \left(\frac{16}{3}\right)^2 + 16 = \frac{256 + 144}{9} = \frac{400}{9}$

$\Rightarrow y = \frac{20}{3}$

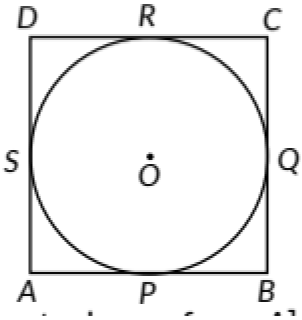
Hence, $TP = TQ = \frac{20}{3}$ cm

[$\because$ Tangents drawn from an external point to a circle are equal in length]

65. Given: A parallelogram ABCD circumscribing a circle with centre O.

To prove : ABCD is a rhombus.

Proof: We know that the tangents drawn to a circle from
an external point are equal in length.



⇒ AP = AS [Tangents drawn from A] ...(i)
⇒ BP = BQ [Tangents drawn from B] ...(ii)
⇒ CR = CQ [Tangents drawn from C] ...(iii)
⇒ DR=DS [Tangents drawn from D] ...(iv)
Adding (i), (ii), (iii) and (iv) we get
AP+BP + CR+DR = AS+BQ + CQ+ DS
⇒ (AP+BP) + (CR+DR) = (AS + DS) + (BQ + CQ)
⇒ AB+CD=AD+BC
⇒ 2AB = 2BC[Opposite sides of the given parallelogram are equal. ABDC and AD = BC]
:- AB = BC=DC=AD
Hence, ABCD is a rhombus.

66. PM = PN
(:- Tangents drawn from an external point are equal)
PQ+QM = PR+RN
PQ+QS = PR+RS ...(i)
Now, Perimeter of ΔPQR = PQ+ QR + PR
⇒ PQ+QR+PR=PQ+QS+SR + PR=PQ+QS+ PQ+QS
=2(PQ+QS)=2(PQ+QM) = 2 PM

$$\Rightarrow PM = \frac{1}{2}(PQ + QR + PR)$$

Hence proved.

67. Construction: Join B and P
PQ and BQ are tangents to the circle centred at O from
external point Q.
⇒ PQ = BQ ....(i) [Length of tangents drawn from
external point to the circle are equal]

In ΔPBQ, <PBQ = <BPQ .....(ii)

AB is the diameter of the circle

$\therefore \quad \angle APB = 90°$ [Angle in a semi circle is 90°]

Now, $\angle APB + \angle BPC = 180°$ [Linear pair]

$\Rightarrow \angle BPC = 180° - 90° = 90°$ ...(iii)

Now In ΔBPC

$\angle BPC + \angle PBC + \angle PCB = 180°$

$\Rightarrow \quad \angle PBC + \angle PCB = 180° - 90° = 90°$ ...(iv)

Now, $\angle BPQ + \angle CPQ = 90°$ [Using (iii) ...(v)

$\angle PBC + \angle PCB = \angle BPQ + \angle CPQ$

$\Rightarrow \angle PCQ = \angle CPQ$

[$\because \angle BPQ = \angle PBQ, \angle PCB = \angle PCQ, \angle PBQ = \angle PBC$

In ΔPQC

$\angle PCQ = \angle CPQ$

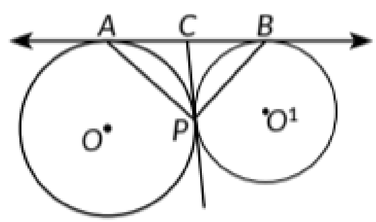
$PQ = QC$

From (i) and (iv), we get $BQ = QC$

Hence, PQ bisects BC.

68. Let common tangent at P meets the tangent AB at C.

Since, tangents drawn from an external point to a circle are equal.



$\therefore \quad AC = CP$

and $BC = CP$

$\Rightarrow \quad \angle CAP = \angle CPA = x$ (say) ...(i)

and $\angle CBP = \angle CPB = y$ (say) ...(ii)

Now, $\angle ACP + \angle BCP = 180°$ [Linear pair] ...(*)

In ΔACP, $\angle ACP + \angle CPA + \angle CAP = 180°$ ...(iii)

and in ΔBCP, $\angle BCP + \angle CPB + \angle CBP = 180°$ ...(iv)

Adding (iii) and (iv), we get

$\angle ACP + x + x + \angle BCP + y + y = 360°$

[Using (i) & (ii)]

$\Rightarrow \quad \angle ACP + \angle BCP + 2x + 2y = 360°$

$\Rightarrow \quad 2(x + y) = 360° - 180° = 180°$ [Using (*)]

$\Rightarrow \quad x + y = 90°$

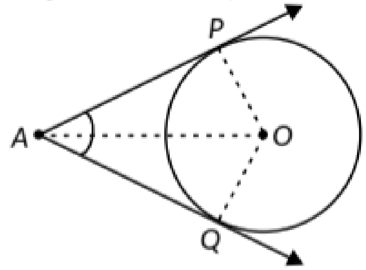
i.e., $\angle CPA + \angle CPB = 90° \quad \Rightarrow \quad \angle APB = 90°$

69.

Given : *AP* and *AQ* are two tangents from a point *A* to a circle *C*(*O*, *r*)

To Prove : *AP* = *AQ*

Construction : Join *OP*, *OQ* and *OA*



Proof : In order to prove that *AP* = *AQ*, we shall first prove that $\Delta OPA \cong \Delta OQA$

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

:- OP LAP and OQLAQ

= <OPA = ZOQA = 90° ...(i)

Now, in right triangle OPA and OQA, we have

OP=OQ [Radii of circle]

<OPA = <OQA [Each 90°]

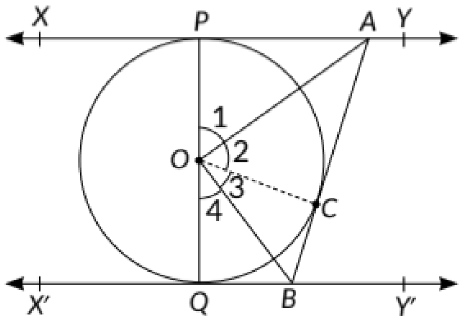
and OA = OA [Common]

So, by RHS-criterion of congruence, we get

ΔOPA=ΔOQA⇒ AP = AQ [By CPCT]

Hence, lengths of two tangents drawn from an external point are equal.

70. Given: Two parallel tangents XY and X'Y' to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.



To prove: $\angle AOB = 90°$

Proof: In ΔAPO and ΔACO,

$AP = AC$ (Tangents drawn from an external point are equal in length)

$OP = OC$ (Radii of same circle)

$OA = OA$ (Common)

$\therefore$ $\Delta APO \cong \Delta ACO$ (SSS congruence criterion)

$\therefore$ $\angle 1 = \angle 2$ (By CPCT) ...(i)

Similarly, $\Delta OCB \cong \Delta OQB$

$\therefore$ $\angle 3 = \angle 4$ (By CPCT) ...(ii)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

(Angles on a straight line)

$\Rightarrow$ $2\angle 2 + 2\angle 3 = 180°$ (Using (i) and (ii))

$\Rightarrow$ $\angle 2 + \angle 3 = \dfrac{180°}{2} = 90°$ $\Rightarrow$ $\angle AOB = 90°$

71. We know, tangent to a circle is perpendicular to its radius at the point of contact.

So, $OP \perp PT$ and $OQ \perp QT$

In ΔOPT, $(OP)^2 + (PT)^2 = OT^2$ $\Rightarrow$ $PT^2 = (OT)^2 - (OP)^2$

$\Rightarrow$ $(PT)^2 = 169 - 25 = 144$ $\Rightarrow$ $PT = 12$ cm

$\Rightarrow$ $PT = QT = 12$ cm

($\because$ Tangents drawn from an external point are equal)

Let $PA = x$ cm $\Rightarrow$ $AT = (12 - x)$cm

Hence, in right angled ΔAET

$(AE)^2 + (ET)^2 = (AT)^2$ ($\because$ $OE \perp AB$)

$\Rightarrow$ $(x)^2 + (8)^2 = (12 - x)^2$ ($\because$ $PA = AE$ and $ET = OT - OE$)

$\Rightarrow$ $x^2 + 64 = 144 + x^2 - 24x$

$\Rightarrow$ $x = \dfrac{80}{24} = 3.33$

In ΔAET and ΔBET

$\angle ETA = \angle ETB$

$ET = ET$ (Common)

$\angle AET = \angle BET$ (90° each, as $OE \perp AB$)

$\Rightarrow$ $\Delta AET \cong \Delta BET$ (By ASA congruence)

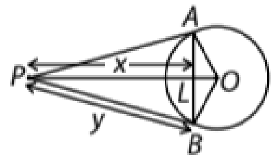
$\Rightarrow$ $AE = EB$ (By CPCT)

Now, AB = AE + EB
= AB = AE + AE
= AB = 2AE= 2 × 3.33 = 6.66 cm

72.

We have, $AB = 16$ cm

$\therefore \quad AL = BL = 8$ cm



In ΔOLB, we have

$OB^2 = OL^2 + LB^2$

$\Rightarrow \quad 10^2 = OL^2 + 8^2 \quad \Rightarrow \quad OL^2 = 100 - 64 = 36$

$\Rightarrow \quad OL = 6$ cm

Let $PL = x$ cm and $PB = y$ cm

Then, $OP = (x + 6)$ cm

In ΔPLB, $PB^2 = PL^2 + BL^2 \Rightarrow y^2 = x^2 + 64$

Now, $OB \perp PB$.

In ΔOBP, $OP^2 = OB^2 + PB^2$

$\Rightarrow \quad (x + 6)^2 = 100 + y^2$

$\Rightarrow \quad x^2 + 36 + 12x = 100 + x^2 + 64 \qquad [\because y^2 = x^2 + 64]$

$$\Rightarrow 12x = 128 \Rightarrow x = \frac{32}{3}$$

$$\therefore \; y^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1600}{9} \Rightarrow y = \frac{40}{3}$$

Hence, $PA = PB = \dfrac{40}{3}$ cm

73. We know that, tangents drawn from an external point to a circle are equal in length.

So, $PQ = PR$

$\therefore \quad \angle PRQ = \angle PQR$

In ΔPQR,

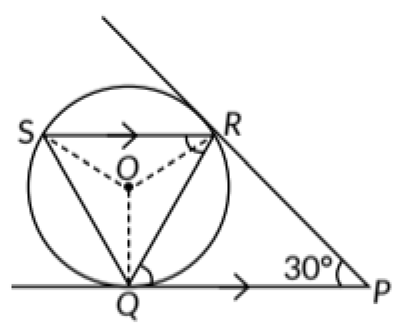
$\angle PQR + \angle PRQ + \angle QPR = 180°$

$\Rightarrow \quad \angle PQR + \angle PQR + 30° = 180°$

$\Rightarrow \quad 2\angle PQR = 150° \quad \Rightarrow \quad \angle PQR = 75°$

Since $SR \parallel QP$ and QR is a transversal,

$\therefore \quad \angle SRQ = \angle PQR = 75°$

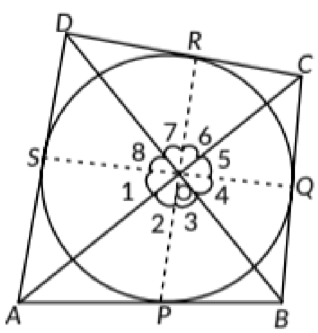


Now, join OR, OS and OQ

We know that, angle subtended by an arc at the centre is double the angle

subtended by same arc at any point on the remaining part of the circle.
So, $<SOQ=2<SRQ \Rightarrow <SOQ = 2 \times 75° = 150°$
Now, in ΔOSQ, OS = OQ
$\Rightarrow <OQS = <OSQ$
Now, by angle sum property,
$2/OQS = 180° - 150° \Rightarrow ZOQS = 15°$
We know that, tangent is perpendicular to radius at the point of contact.
So, $<OQP = 90°$
$\Rightarrow <OQR + <PQR = 90°$
$\Rightarrow <OQR = 90° - 75° = 15°$
Now, $<RQS = <OQR + <OQS$
$\Rightarrow <RQS = 15° +15° = 30°$

74. Given: ABCD is a quadrilateral circumscribing a circle whose sides AB, BC, CD and DA touches the circle at P, Q, R and S respectively.
To prove : $ZAOB + COD = 180°$
and $<BOC+<AOD = 180°$
Construction Join OP, OQ, OR and OS.
Proof : Since we know that tangents drawn from an external point to a circle subtend equal angles at the centre.

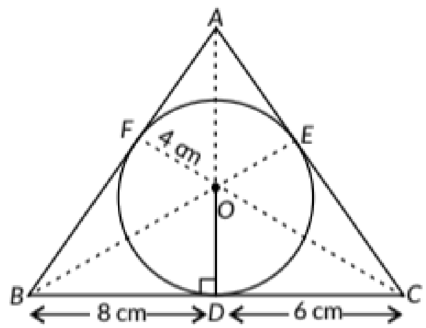


$\therefore \quad \angle 1 = \angle 2,$
$\angle 3 = \angle 4,$
$\angle 5 = \angle 6$ and
$\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
[Sum of all angles around a point is 360°]

$\Rightarrow \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$ and
$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$
$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$ and
$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$
$\Rightarrow \quad \angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

75. Given, AABC circumscribing a circle of radius 4 cm with centre O.
Join OA. Draw BE and CF such that these passes through the centre O.
Now, radius OD = OE = OF = 4 cm



Since, tangents drawn from an external point are equal
Also $BD = BF = 8$ cm, $DC = EC = 6$ cm
$AF = AE = x$ cm (say)
Semi-perimeter of ΔABC,

$$s = \frac{(x+8)+(14)+(x+6)}{2} = (x+14)\text{cm}$$

Area of ΔABC

$$= \sqrt{(x+14)(x+14-x-8)(x+14-14)(x+14-x-6)}$$

$$= \sqrt{(x+14)(6)(x)\times 8} = \sqrt{48x(x+14)}\ \text{cm}^2 \qquad ...(i)$$

Also,
Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOAC

$$= \frac{1}{2}\times(x+8)\times(4) + \frac{1}{2}\times(14)\times(4) + \frac{1}{2}\times(x+6)\times(4)$$

$= (4x + 56)$ cm^2 ...(ii)

From (i) and (ii), we have

$$\sqrt{48x(x+14)} = 4x+56$$

$\Rightarrow\ 48x(x + 14) = (4x + 56)^2$ (Squaring on both sides)

$\Rightarrow\ 48x(x + 14) = 16(x + 14)^2$

$\Rightarrow\ 3x(x + 14) = (x + 14)^2 \Rightarrow 3x = x + 14$

$\Rightarrow\ 2x = 14 \Rightarrow x = 7 \quad \therefore \quad AB = x + 8 = 7 + 8 = 15$ cm

and $AC = x + 6 = 7 + 6 = 13$ cm

CBSE Sample Questions

1.

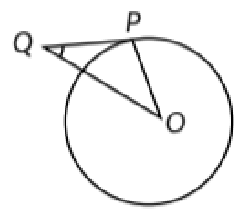
In ΔOPQ

$\angle P + \angle Q + \angle O = 180°$

$\Rightarrow \quad 2\angle Q + 90° = 180°$ **(1/2)**

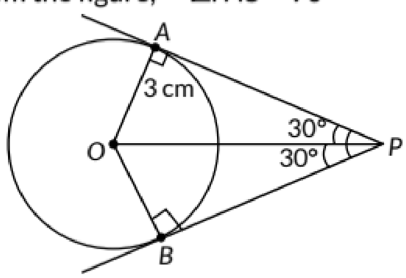
$[\because \Delta OPQ$ is an isosceles triangle$]$

$\Rightarrow \quad 2\angle Q = 90° \Rightarrow \angle Q = 45°$ **(1/2)**



2.

(d): From the figure, $\angle PAO = 90°$



In ΔPAO, $\tan 30° = \dfrac{AO}{AP}$ **(1/2)**

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{3}{AP} \Rightarrow AP = 3\sqrt{3}$ cm **(1/2)**

3. Here PQ = PT

[:- Length of tangents drawn from an external point to the circle are equal]

=PL+LQ=PM+MT

= PL+LN=PM + MN ...(i)

Now, perimeter (ΔPLM) PL+LM+PM (1/2)

=PL+LN+MN + PM [:- LM = LN + NM]

= 2(PL+LN) [Using (i)]

=2(PL+LQ) = 2 × PQ

=2×28 = 56 cm [:- PQ = 28 cm (Given)] (1/2)

4.

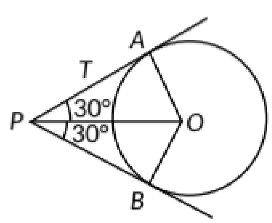
In ΔPAO

$\tan 30° = AO/PA$ **(1/2)**

$\Rightarrow \quad 1/\sqrt{3} = 3/PA$

$[\because$ Radius = 3 cm (Given)$]$

$\Rightarrow \quad PA = 3\sqrt{3}$ cm $= PB$



(1/2)

5. We have, ZPAO = /PBO = 90° (1)
(angle between radius and tangent)
Since, APBO is quadrilateral and ZA+ P+ZB+20=360°
= 90° +75° +90° +20=360° = 20=105° (1/2)
= <AQB = 1/2 × 105° = 52.5° (Angle at the remaining part of the circle is half the angle subtended by the arc at the centre) (1/2)

6.

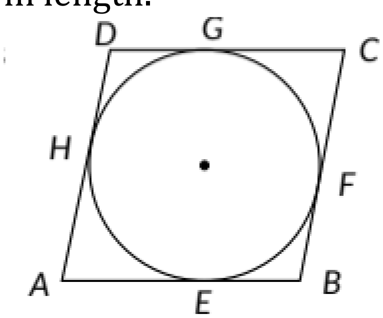
Let $\angle APO = \theta$

$$\Rightarrow \quad \sin\theta = \frac{OA}{OP} = \frac{1}{2} = \sin 30° \Rightarrow \theta = 30° \qquad \textbf{(1/2)}$$

= <ΔPB20 60° (1/2)
Also, ZPAB = <PBA = 60° (:: PA = PB) (1/2)
⇒ AAPB is an equilateral triangle. (1/2)

7. Clearly, ZAZOPA = ZOSA = 90° (1/2)
<SOP = 90°
Also, AP = AS
[Length of tangents from the external point A]
So, OSAP is a square
=> AP = AS = 10 cm [OS 10 cm (Given)] (1/2)
Again, CRCQ= 27 cm
[Length of tangents from the external point C]
=> BQ=BC-CQ=38-27=11cm
=> BP = BQ = 11 cm (1/2)
:- x = AB = AP + BP = 10+11=21 cm (1/2)

8. Let ABCD be the rhombus circumscribing the circle with centre O, such that AB, BC, CD and DA touch the circle at points E, F, G and H respectively.
We know that the tangents drawn to a circle from an exterior point are equal in length.



$\therefore \quad AE = AH$ ...(i)

$BE = BF$ ...(ii)

$CG = CF$ ...(iii)

$DG = DH$ ...(iv)

Adding (i), (ii), (iii) and (iv) we get

$AE + BE + CG + DG = AH + BF + CF + DH$

$\therefore \quad AB + CD = AD + BC$ ...(v)

Since $AB = DC$ and $AD = BC$

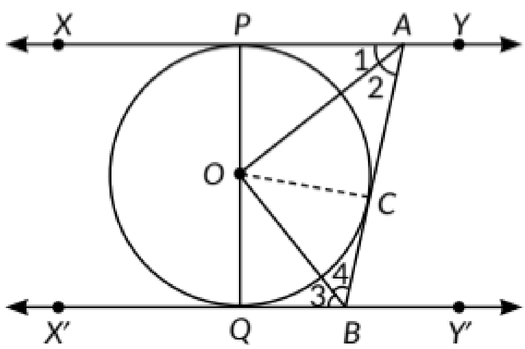
Putting in (v) we get, $2AB = 2AD$ **(1)**

or $\quad AB = AD$

$\therefore \quad AB = BC = DC = AD$

Since a parallelogram with equal adjacent sides is a rhombus, so $ABCD$ is a rhombus. **(1)**

9.



Join OC

In ΔOPA and ΔOCA

$OP = OC$ (radii of same circle)

$PA = CA$ (length of two tangents from an external point)

$AO = AO$ (Common)

Therefore, $\Delta OPA \cong \Delta OCA$ (By SSS congruency criterion)

Hence, $\angle 1 = \angle 2$ (CPCT) **(1)**

Similarly $\angle 3 = \angle 4$

$\angle PAB + \angle QBA = 180°$(co-interior angles are supplementary as $XY \parallel X'Y'$)

$\Rightarrow \quad 2\angle 2 + 2\angle 4 = 180°$

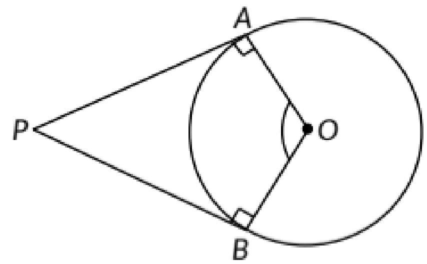
$\Rightarrow \quad \angle 2 + \angle 4 = 90°$ ...(i) **(1)**

In ΔAOB,

$\angle 2 + \angle 4 + \angle AOB = 180°$ (Angle sum property) **(1)**

Using (i), we get $\angle AOB = 90°$

10. Let PA and PB be the two tangents drawn to a circle with centre O.



(1)

$\Rightarrow \quad \angle OAP = \angle OBP = 90^\circ \quad$ (Tangents to a circle)

To prove : $\angle APB + \angle AOB = 180^\circ$

Now, $\angle OAP + \angle OBP + \angle APB + \angle AOB = 360^\circ$ **$(1\frac{1}{2})$**

$\Rightarrow \quad 90^\circ + 90^\circ + \angle APB + \angle AOB = 360^\circ$

$\Rightarrow \quad \angle APB + \angle AOB = 180^\circ$ **$(1\frac{1}{2})$**

Hence proved.

11.

Let $\angle PTQ = \theta$

Here, ΔTPQ is an isosceles triangle. $\quad (\because TP = TQ)$

$\Rightarrow \quad \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{\theta}{2}$ **$(1\frac{1}{2})$**

Also, $\angle OPT = 90^\circ$ $\quad$ ($\because$ Tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow \quad \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left(90^\circ - \frac{\theta}{2}\right) = \frac{\theta}{2}$ **$(1\frac{1}{2})$**

$\Rightarrow \quad \angle OPQ = \frac{1}{2}\angle PTQ$

$\Rightarrow \quad \angle PTQ = 2\angle OPQ$ **(1)**

Hence proved.