Matrices

Previous Years' CBSE Board Questions

3.2 Matrix

VSA (1 mark)

1. Construct a 2 × 2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = |(i)^2 - j|$. *(2020)* U

2. Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3. *(AI 2016)*

3. Write the element a_{23} of a 3 × 3 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$. *(Delhi 2015)* U

4. The elements a_{ij} of a 3 × 3 matrix are given by $a_{ij} = \frac{1}{2}|-3i+j|$. Write the value of element a_{32}. *(AI 2014C)*

3.3 Types of Matrices

VSA (1 mark)

5. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$. *(AI 2014)* Ap

6. If $\begin{pmatrix} a+4 & 3b \\ 8 & -6 \end{pmatrix} = \begin{pmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{pmatrix}$, write the value of $a - 2b$. *(Foreign 2014)* Ap

7. If $\begin{bmatrix} x \cdot y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, write the value of $(x + y + z)$. *(Delhi 2014C)* Ap

3.4 Operations on Matrices

MCQ

8. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals
(a) ±1 (b) –1 (c) 1 (d) 2 *(2023)*

9. If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then the value of $(2x + y - z)$ is
(a) 1 (b) 2 (c) 3 (d) 5 *(2023)*

10. If $x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$, then
(a) $x = 1, y = 2$ (b) $x = 2, y = 1$
(c) $x = 1, y = -1$ (d) $x = 3, y = 2$ *(2023)*

11. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to
(a) I (b) A (c) $2A$ (d) $3I$ *(2023)*

12. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then $(A - 2I)(A - 3I)$ is equal to
(a) A (b) I (c) $5I$ (d) O *(Term I, 2021-22)*

13. If order of matrix A is 2 × 3, of matrix B is 3 × 2, and of matrix C is 3 × 3, then which one of the following is not defined?
(a) $C(A + B')$ (b) $C(A + B')'$
(c) BAC (d) $CB + A'$ *(Term I, 2021-22)*

14. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then $A^5 - A^4 - A^3 + A^2$ is equal to
(a) $2A$ (b) $3A$ (c) $4A$ (d) O *(Term I, 2021-22)*

15. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to
(a) I (b) O
(c) $I - A$ (d) $I + A$ *(2020)*

16. If $A = [2 \ -3 \ 4]$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $X = [1 \ 2 \ 3]$ and $Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, then $AB + XY$ equals
(a) [28] (b) [24] (c) 28 (d) 24 *(2020)*

VSA (1 mark)

17. If $A = [1 \quad 0 \quad 4]$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, find AB. *(2021)* Ev

18. Find the order of the matrix A such that
$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$. *(2021)* R

19. If $B = \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find the matrix A. *(2021)*

20. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A = ________. *(2020)*

21. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$. *(NCERT Exemplar, Delhi 2016)* An

22. If $[2 \quad 1 \quad 3]\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then write the order of matrix A. *(Foreign 2016)* Ap

23. Solve the following matrix equation for x:

$$[x \ \ 1]\begin{bmatrix}1 & 0\\ -2 & 0\end{bmatrix}=O$$ *(Delhi 2014)*

24. If $2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$, find $(x-y)$. *(Delhi 2014)*

25. If A is a square matrix such that $A^2 = A$, then write the value of $7A-(I+A)^3$, where I is an identity matrix. *(AI 2014)* Ev

26. If $(2x \ \ 4)\begin{pmatrix}x\\ -8\end{pmatrix}=O$, find the positive value of x. *(AI 2014C)*

SA I (2 mark)

27. If $A=\begin{bmatrix}-3 & 2\\ 1 & -1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find scalar k so that $A^2+I=kA$. *(2020)*

28. For what value of x is $[1 \ \ 2 \ \ 1]\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=0$? *(2020)* Ev

29. Find a matrix A such that $2A-3B+5C=O$, where $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$. *(Delhi 2019)* Ev

SA II (3 mark)

30. If $A=\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}$, then show that $A^3-23A-40I=O$. *(2023)*

LA I (4 marks)

31. Let $A=\begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix}, B=\begin{pmatrix}5 & 2\\ 7 & 4\end{pmatrix}, C=\begin{pmatrix}2 & 5\\ 3 & 8\end{pmatrix}$, find a matrix D such that $CD-AB=O$. *(Delhi 2017)* An

32. Find matrix A such that

$$\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8\\ 1 & -2\\ 9 & 22\end{bmatrix}$$ *(AI 2017)*

33. If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$, find $A^2-5A+4I$ and hence find a matrix X such that $A^2-5A+4I+X=O$ *(Delhi 2015)* An

34. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given below.

Article/School	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also, find the total funds collected for the purpose.
Write one value generated by the above situation. *(Delhi 2015)*

35. To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls (ii) letters and (iii) announcements. The cost for each mode per attempt is given below:
(i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40
The number of attempts made in three villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organisation for the three villages separately, using matrices. Write one value generated by the organisation in the society. *(AI 2015)* Ev

36. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then find the values of a and b. *(Foreign 2015)*

37. In a parliament election, a political party hired a public relations firm to promote its candidates in three ways-telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$A=\begin{array}{c}\begin{bmatrix}140\\ 200\\ 150\end{bmatrix}\end{array}\begin{array}{l}\text{Telephone}\\ \text{House call}\\ \text{Letters}\end{array}$$

The number of contacts of each type made in two cities X and Y is given in matrix B as

$$B=\begin{array}{c}\begin{array}{ccc}\text{Telephone} & \text{House call} & \text{Letters}\end{array}\\ \begin{bmatrix}1000 & 500 & 5000\\ 3000 & 1000 & 10000\end{bmatrix}\end{array}\begin{array}{l}\\ \text{City } X\\ \text{City } Y\end{array}$$

Find the total amount spent by the party in the two cities. What should one consider before casting his/her vote-party's promotional activity or their social activities? *(Foreign 2015)*

38. If $[2x \ \ 3]\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=O$, find x. *(Delhi 2015C)*

39. A trust fund, ₹ 35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to

orphanage and second bond pays 10% interest per annum which will be given to an N.G.O. (Cancer Aid Society). Using matrix multiplication, determine how to divide ₹ 35,000 among two types of bonds if the trust fund obtains an annual total interest of ₹ 3,200. What are the values reflected in this question? *(AI 2015C)* Ev

3.5 Transpose of a Matrix

MCQ

40. If $A=\begin{bmatrix}5 & x\\ y & 0\end{bmatrix}$ and $A = A^T$, where A^T is the transpose of the matrix A, then
(a) $x = 0, y = 5$ (b) $x = y$
(c) $x + y = 5$ (d) $x = 5, y = 0$ *(2023)*

41. If a matrix $A = [1\ 2\ 3]$, then the matrix AA' (where A' is the transpose of A) is
(a) 14 (b) $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$
(c) $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}$ (d) [14] *(2023)*

42. If P is a 3×3 matrix such that $P' = 2P + I$, where P' is the transpose of P, then
(a) $P = I$ (b) $P = -I$ (c) $P = 2I$ (d) $P = -2I$
(Term I, 2021-22)

43. If $A = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $A + A' = I$, then the value of α is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$
(Term I, 2021-22)

VSA (1 mark)

44. If A is a matrix of order 3×2, then the order of the matrix A' is ________. *(2020)* U

3.6 Symmetric and Skew Symmetric Matrices

VSA (1 mark)

45. A square matrix A is said to be symmetric, if ________.
(2020) U

46. Given a skew-symmetric matrix $A = \begin{bmatrix}0 & a & 1\\ -1 & b & 1\\ -1 & c & 0\end{bmatrix}$, the value of $(a + b + c)^2$ is ________. *(2020)*

47. If the matrix $A=\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'. *(2018)*

48. Matrix $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$ is given to be symmetric, find values of a and b. *(Delhi 2016)* Ap

49. If $A=\begin{bmatrix}3 & 5\\ 7 & 9\end{bmatrix}$ is written as $A = P + Q$, where P is a symmetric matrix and Q is a skew symmetric matrix, then write the matrix P. *(Foreign 2016)*

50. Express the matrix $A=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.
(AI 2015C) Ev

51. Write a 2×2 matrix which is both symmetric and skew symmetric. *(Delhi 2014C)*

SA I (2 marks)

52. If the matrix $\begin{bmatrix}0 & 6-5x\\ x^2 & x+3\end{bmatrix}$ is symmetric, find the value of x. *(2021)* Ev

53. For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, , verify that
(i) $(A + A')$ is a symmetric matrix.
(ii) $(A - A')$ is a skew-symmetric matrix. *(2020C)*

54. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that $AB - BA$ is a skew symmetric matrix. *(AI 2019)* An

55. Show that all the diagonal elements of a skew symmetric matrix are zero. *(Delhi 2017)* An

3.7 Invertible Matrices

MCQ

56. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals
(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$
(2023)

SA II (3 marks)

57. If $A = \begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}. *(2020)* U

CBSE Sample Questions

3.3 Types of Matrices

MCQ

1. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n, then
   (a) $a_{ij} = \frac{1}{a_{ji}} \forall i, j$ (b) $a_{ij} \neq 0 \ \forall i, j$
   (c) $a_{ij} = 0$, where $i = j$ (d) $a_{ij} \neq 0$ where $i = j$ R
   (2022-23)

2. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then value of $a + b - c + 2d$ is
   (a) 8 (b) 10 (c) 4 (d) –8
   (Term I, 2021-22) Ap

3. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is
   (a) 3×5 and $m = n$ (b) 3×5
   (c) 3×3 (d) 5×5 (Term I, 2021-22)

3.4 Operations on Matrices

MCQ

4. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{when } i \neq j \\ 0, \text{when } i = j \end{cases}$, then A^2 is
   (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   (Term I, 2021-22) U

5. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are
   (a) $-6, -12, -18$ (b) $-6, -4, -9$
   (c) $-6, 4, 9$ (d) $-6, 12, 18$
   (Term I, 2021-22) Ap

6. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to
   (a) A (b) $I + A$
   (c) $I - A$ (d) I (Term I, 2021-22)

7. Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$, then
   (a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$
   (c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$
   (Term I, 2021-22) Ap

VSA (1 mark)

8. If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix $5A - 3B$, given that it is defined. (2020-21) Ap

9. Given that A is a square matrix of order 3×3 and $|A| = -4$. Find $|\text{adj } A|$. (2020-21) An

3.7 Invertible Matrices

MCQ

10. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$
    (a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB
    (2022-23)

11. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then
    (a) $A^{-1} = B$ (b) $A^{-1} = 6B$
    (c) $B^{-1} = B$ (d) $B^{-1} = \frac{1}{6}A$
    (Term I, 2021-22) Ap

SA I (2 marks)

12. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}. (2020-21) Ap

Detailed SOLUTIONS

1. Here, $a_{11} = |(1)^2 - 1| = 0$, $a_{12} = |(1)^2 - 2| = 1$, $a_{21} = |(2)^2 - 1| = 3$ and $a_{22} = |(2)^2 - 2| = 2$

$\therefore$ Required matrix $= \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$.

Key Points

- A matrix is written as $A = [a_{ij}]_{m \times n}$, where a_{ij} is an element lying in the i^{th} row and j^{th} column.

2. As, matrix is of order 2×2, so there are 4 entries possible.
Each entry has 3 choices *i.e.*, 1, 2 or 3. So, the number of ways to make such matrices is $3 \times 3 \times 3 \times 3 = 81$.

3. Here, $a_{ij} = \frac{|i-j|}{2}$ $\therefore$ $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$ [For $i = 2, j = 3$]

4. Here, $a_{ij} = \frac{1}{2}|-3i + j|$

$\therefore \quad a_{32}=\frac{1}{2}|-3\cdot 3+2| \qquad$ [For $i=3, j=2$]

$=\frac{1}{2}|-9+2|=\frac{1}{2}|-7|=\frac{7}{2}$

5. Here, $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix}=\begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

By equality of two matrices, we get

$x-y=-1, z=4, 2x-y=0, w=5$

Solving these equations for x and y, we get

$x=1, y=2 \therefore x+y=1+2=3.$

Answer Tips

- Two matrices $A=[a_{ij}]$ and $B=[b_{ij}]$ are said to equal if they are of a same order and $a_{ij}=b_{ij} \ \forall i, j.$

6. Given, $\begin{pmatrix} a+4 & 3b \\ 8 & -6 \end{pmatrix}=\begin{pmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{pmatrix}$

By equality of two matrices, we get

$a+4=2a+2, 3b=b+2, -6=a-8b$

On solving these equations, we get $a=2, b=1$.

So, $a-2b=0$.

7. Here, $\begin{bmatrix} x\cdot y & 4 \\ z+6 & x+y \end{bmatrix}=\begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

By equality of two matrices, we get

$x\cdot y=8, w=4, z+6=0, x+y=6$

$\Rightarrow \quad z=-6, x+y=6 \Rightarrow x+y+z=6-6=0.$

8. (c): We have, $A=\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$

$\therefore \ B^2=\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$

Now, it is given that $A=B^2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$

On comparing, we get

$x^2=1$ and $x+1=2 \ \Rightarrow \ x=\pm 1$ and $x=1$

$\therefore \quad x=1$

9. (d) : $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

$\therefore \quad x+y+z=6 \qquad$ ...(i)

$y+z=3 \qquad$ ...(ii)

$z=2 \qquad$ ...(iii)

$\Rightarrow \quad y+2=3 \qquad$ [Using (ii) and (iii)]

$\Rightarrow \quad y=1 \qquad$ ...(iv)

$\Rightarrow \quad x+1+2=6 \qquad$ [Using (i), (iii) and (iv)]

$\Rightarrow \quad x=3$

So, $2x+y-z=(2\times 3)+1-2=6+1-2=5$

10. (b): We have, $x\begin{bmatrix} 1 \\ 2 \end{bmatrix}+y\begin{bmatrix} 2 \\ 5 \end{bmatrix}=\begin{bmatrix} 4 \\ 9 \end{bmatrix}$

$\begin{bmatrix} x+2y \\ 2x+5y \end{bmatrix}=\begin{bmatrix} 4 \\ 9 \end{bmatrix}$

$\Rightarrow \quad x+2y=4$ ...(i) and $2x+5y=9$ ...(ii)

Solving (i) and (ii), we get $x=2, y=1$

11. (a): Given that $A^2=A$

Consider $(I+A)^2-3A$

$=I^2+A^2+2AI-3A$

$=I+A+2A-3A \qquad [\because I^2=I, A^2=A \text{ (given)}]$

$=I$

12. (d)

13. (a) : Consider $C(A+B')$ i.e., $C_{3\times 3}(A_{2\times 3}+B'_{2\times 3})$

$=C_{3\times 3}(A+B')_{2\times 3}$

Here, number of columns in the matrix C is 3 and number of rows in the matrix $(A+B')$ is 2. So, it is not defined.

14. (d)

15. (a) : We have, $A^2=A$

Now, $(I-A)^3+A=(I-A)(I-A)(I-A)+A$

$=(I\cdot I-I\cdot A-A\cdot I+A\cdot A)(I-A)+A$

$=(I-A-A+A)(I-A)+A \qquad [\because I\cdot A=A\cdot I=A \text{ and } A^2=A]$

$=(I-A)(I-A)+A$

$=(I\cdot I-I\cdot A-A\cdot I+A\cdot A)+A$

$=(I-A-A+A)+A=(I-A)+A=I$

16. (a) : Consider, $AB=\begin{bmatrix} 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}=[6-6+8]=[8]$

and $XY=\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}=[2+6+12]=[20]$

$AB+XY=[8]+[20]=[28]$

17. Consider, $AB=\begin{bmatrix} 1 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}=[2+0+24]=[26]$

18. We have, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times 2} A=\begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}_{3\times 2}$

The order of matrix A should be 2×2.

19. Given, $B=\begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A+2B=\begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$

$\Rightarrow A+2\begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}=\begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$

$\Rightarrow A+\begin{bmatrix} 2 & -10 \\ 0 & -6 \end{bmatrix}=\begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix} \Rightarrow A=\begin{bmatrix} -2 & 14 \\ -7 & 11 \end{bmatrix}$

20. Given $A+B=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad$ ...(i)

and $A-2B=\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \qquad$ ...(ii)

(i) – (ii), we get

$3B=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}-\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow \quad 3B=\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow \quad B=\begin{bmatrix} 2/3 & -1/3 \\ 1/3 & 2/3 \end{bmatrix}$

From (i),

$$A=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}-B=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}-\begin{bmatrix}2/3 & -1/3\\1/3 & 2/3\end{bmatrix}=\begin{bmatrix}1/3 & 1/3\\2/3 & 1/3\end{bmatrix}$$

21. Given, $A^2 = I$

$\therefore$ The simplified value of $(A-I)^3+(A+I)^3-7A$

$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$

$= 2A^3 + 6AI^2 - 7A = 2AA^2 + 6AI - 7A$

$= 2AI + 6A - 7A = 2A - A = A$

Concept Applied

- $I\cdot A = A\cdot I = A$ and $A^2 = I$

22. Given, $A=\begin{bmatrix}2 & 1 & 3\end{bmatrix}\begin{bmatrix}-1 & 0 & -1\\-1 & 1 & 0\\0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$$=\begin{bmatrix}-2-1 & 1+3 & -2+3\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}=\begin{bmatrix}-3 & 4 & 1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}$$

$= [-3+0-1] = [-4]$

$\therefore$ The order of matrix $A = 1\times 1$

23. Given, $\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-2 & 0\end{bmatrix}=O \Rightarrow \begin{bmatrix}x-2 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\end{bmatrix}$

$\Rightarrow \quad x-2=0 \Rightarrow x=2$

Commonly Made Mistake

- Check the order of matrices before multiplying two matrices.

24. We have, $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}6 & 8\\10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}7 & 8+y\\10 & 2x+1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

On comparing the corresponding elements, we get

$8+y=0$ and $2x+1=5 \Rightarrow y=-8$ and $x=2$

$\therefore \quad x-y=2+8=10$

Key Points

- If $A = [a_{ij}]_{m\times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

25. Here, $A^2 = A$

Now, $7A-(I+A)^3 = 7A-(I+A)(I+A)(I+A)$

$= 7A-(I+A)(I\cdot I+I\cdot A+A\cdot I+A\cdot A)$

$= 7A-(I+A)(I+A+A+A) \qquad (\because I\cdot A=A\cdot I=A$ and $A^2=A)$

$= 7A-(I+A)(I+3A)$

$= 7A-(I\cdot I+I\cdot(3A)+A\cdot I+A\cdot(3A))$

$= 7A-(I+3A+A+3A) = 7A-I-7A=-I.$

26. Here, $\begin{pmatrix}2x & 4\end{pmatrix}\begin{pmatrix}x\\-8\end{pmatrix}=O$

$\Rightarrow \quad 2x\cdot x+4\cdot(-8)=0 \Rightarrow 2x^2-32=0$

$\Rightarrow \quad x^2=16=4^2 \Rightarrow x=4$

which is the required positive value of x.

27. We have, $A^2+I=kA$

$$\Rightarrow \begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=k\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}11 & -8\\-4 & 3\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=k\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}12 & -8\\-4 & 4\end{bmatrix}=k\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}$$

$$\Rightarrow -4\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}=k\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}$$

On comparing, we get $k=-4$

28. Given, $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0$

$$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=0 \Rightarrow 0+4+4x=0 \Rightarrow x=-1$$

29. Let $A=\begin{bmatrix}x & y & z\\p & q & r\end{bmatrix}$ [$\because$ B and C are matrices of order 2×3]

Given, $2A-3B+5C=O$

$\Rightarrow \quad 2A=3B-5C \Rightarrow A=\frac{1}{2}[3B-5C]$ ...(i)

Now, $3B-5C=3\begin{bmatrix}-2 & 2 & 0\\3 & 1 & 4\end{bmatrix}-5\begin{bmatrix}2 & 0 & -2\\7 & 1 & 6\end{bmatrix}$

$$=\begin{bmatrix}-6 & 6 & 0\\9 & 3 & 12\end{bmatrix}-\begin{bmatrix}10 & 0 & -10\\35 & 5 & 30\end{bmatrix}=\begin{bmatrix}-16 & 6 & 10\\-26 & -2 & -18\end{bmatrix}$$

From (i), we get $A=\begin{bmatrix}-8 & 3 & 5\\-13 & -1 & -9\end{bmatrix}$.

30. We have, $A=\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$

Now, $A^2=\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$

$$=\begin{bmatrix}1+6+12 & 2-4+6 & 3+2+3\\3-6+4 & 6+4+2 & 9-2+1\\4+6+4 & 8-4+2 & 12+2+1\end{bmatrix}=\begin{bmatrix}19 & 4 & 8\\1 & 12 & 8\\14 & 6 & 15\end{bmatrix}$$

Now, $A^3=\begin{bmatrix}19 & 4 & 8\\1 & 12 & 8\\14 & 6 & 15\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$

$$=\begin{bmatrix}19+12+32 & 38-8+16 & 57+4+8\\1+36+32 & 2-24+16 & 3+12+8\\14+18+60 & 28-12+30 & 42+6+15\end{bmatrix}=\begin{bmatrix}63 & 46 & 69\\69 & -6 & 23\\92 & 46 & 63\end{bmatrix}$$

Now, $A^3-23A-40I=\begin{bmatrix}63 & 46 & 69\\69 & -6 & 23\\92 & 46 & 63\end{bmatrix}-\begin{bmatrix}23 & 46 & 69\\69 & -46 & 23\\92 & 46 & 23\end{bmatrix}-\begin{bmatrix}40 & 0 & 0\\0 & 40 & 0\\0 & 0 & 40\end{bmatrix}$

$$=\begin{bmatrix}63-23-40 & 46-46-0 & 69-69-0\\69-69-0 & -6+46-40 & 23-23-0\\92-92-0 & 46-46-0 & 63-23-40\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=O$$

Hence proved.

31. We have, $A=\begin{bmatrix}2 & -1\\3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\7 & 4\end{bmatrix}, C=\begin{bmatrix}2 & 5\\3 & 8\end{bmatrix}$

Let $D=\begin{bmatrix}a & b\\c & d\end{bmatrix}$

Now, $CD-AB=O$

$\therefore \begin{bmatrix}2 & 5\\3 & 8\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}-\begin{bmatrix}2 & -1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\7 & 4\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2a+5c & 2b+5d\\3a+8c & 3b+8d\end{bmatrix}-\begin{bmatrix}10-7 & 4-4\\15+28 & 6+16\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2a+5c-3 & 2b+5d\\3a+8c-43 & 3b+8d-22\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

On comparing the corresponding elements, we get

$2a+5c-3=0$ ...(i)

and $3a+8c-43=0$ ...(ii)

Also, $2b+5d=0$ ...(iii)

and $3b+8d-22=0$ ...(iv)

Solving (i) and (ii), we get

$a=-191,\ c=77$

Solving (iii) and (iv), we get $b=-110,\ d=44$

$\therefore \quad D=\begin{bmatrix}-191 & -110\\77 & 44\end{bmatrix}$

32. Given that, $\begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8\\1 & -2\\9 & 22\end{bmatrix}$

Let $X=\begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}_{3\times 2}$ and $Y=\begin{bmatrix}-1 & -8\\1 & -2\\9 & 22\end{bmatrix}_{3\times 2}$

As order of X is 3×2, then A should be of order 2×2, so that we get Y matrix of order 3×2.

Let $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$

Now, $\begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}=\begin{bmatrix}-1 & -8\\1 & -2\\9 & 22\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2a-c & 2b-d\\a+0 & b+0\\-3a+4c & -3b+4d\end{bmatrix}=\begin{bmatrix}-1 & -8\\1 & -2\\9 & 22\end{bmatrix}$

On comparing the corresponding elements, we get

$2a-c=-1$ ...(i)

$2b-d=-8$ ...(ii)

$a=1$ ...(iii)

and $b=-2$ ...(iv)

Substituting $a=1$ in (i), we get $c=3$

and substituting $b=-2$ in (ii), we get $d=4$

So, $A=\begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$

33. Given, $A=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$

Now, $A^2-5A+4I$

$$=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}-5\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}+4\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}5 & -1 & 2\\9 & -2 & 5\\0 & -1 & -2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\10 & 5 & 15\\5 & -5 & 0\end{bmatrix}+\begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}9 & -1 & 2\\9 & 2 & 5\\0 & -1 & 2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\10 & 5 & 15\\5 & -5 & 0\end{bmatrix}=\begin{bmatrix}-1 & -1 & -3\\-1 & -3 & -10\\-5 & 4 & 2\end{bmatrix}$$

Since, $A^2-5A+4I+X=O \Rightarrow X=-(A^2-5A+4I)$

$\therefore \quad X=\begin{bmatrix}1 & 1 & 3\\1 & 3 & 10\\5 & -4 & -2\end{bmatrix}$

Answer Tips

- O is the additive identity.

34. The number of articles sold by each school can be written in the matrix form as

$X=\begin{bmatrix}40 & 25 & 35\\50 & 40 & 50\\20 & 30 & 40\end{bmatrix}$

The cost of each article can be written in the matrix form as $Y=[25\ \ 100\ \ 50]$

The fund collected by each school is given by

$YX=[25\ 100\ 50]\begin{bmatrix}40 & 25 & 35\\50 & 40 & 50\\20 & 30 & 40\end{bmatrix}=[7000\ \ 6125\ \ 7875]$

Therefore, the funds collected by schools A, B and C are ₹ 7000, ₹ 6125 and ₹ 7875 respectively.

Thus, the total funds collected

= ₹ (7000 + 6125 + 7875) = ₹ 21000

The situation highlights the helping nature of the students.

35. Let ₹A, ₹B and ₹C be the cost incurred by the organisation for villages X, Y and Z respectively. Then, we get the matrix equation as

$$\begin{bmatrix}400 & 300 & 100\\300 & 250 & 75\\500 & 400 & 150\end{bmatrix}\begin{bmatrix}50\\20\\40\end{bmatrix}=\begin{bmatrix}A\\B\\C\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}A\\B\\C\end{bmatrix}=\begin{bmatrix}400\times50+300\times20+100\times40\\300\times50+250\times20+75\times40\\500\times50+400\times20+150\times40\end{bmatrix}$$

$$=\begin{bmatrix}20{,}000+6{,}000+4{,}000\\15{,}000+5{,}000+3{,}000\\25{,}000+8{,}000+6{,}000\end{bmatrix}=\begin{bmatrix}30{,}000\\23{,}000\\39{,}000\end{bmatrix}$$

$\therefore$ A = ₹ 30,000, B = ₹ 23,000 and C = ₹ 39,000

These are the costs incurred by the organisation for villages X, Y and Z respectively.

The value generated by the organisation in the society is cleanliness.

Key Points

- The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.

36. We have, $A=\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}a & 1\\b & -1\end{bmatrix}$

Consider, $(A+B)=\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}+\begin{bmatrix}a & 1\\b & -1\end{bmatrix}=\begin{bmatrix}1+a & 0\\2+b & -2\end{bmatrix}$

Now, $(A+B)^2=\begin{bmatrix}1+a & 0\\2+b & -2\end{bmatrix}\begin{bmatrix}1+a & 0\\2+b & -2\end{bmatrix}$

$=\begin{bmatrix}(1+a)^2 & 0\\(2+b)(1+a-2) & 4\end{bmatrix}=\begin{bmatrix}(1+a)^2 & 0\\(2+b)(a-1) & 4\end{bmatrix}$

Now, consider $A^2=\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$

$=\begin{bmatrix}1-2 & -1+1\\2-2 & -2+1\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$

and $B^2=\begin{bmatrix}a & 1\\b & -1\end{bmatrix}\begin{bmatrix}a & 1\\b & -1\end{bmatrix}=\begin{bmatrix}a^2+b & a-1\\ab-b & b+1\end{bmatrix}$

$\therefore$ $A^2+B^2=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}+\begin{bmatrix}a^2+b & a-1\\ab-b & b+1\end{bmatrix}=\begin{bmatrix}a^2+b-1 & a-1\\ab-b & b\end{bmatrix}$

It is given that, $(A+B)^2=A^2+B^2$

$\therefore$ $\begin{bmatrix}(1+a)^2 & 0\\(2+b)(a-1) & 4\end{bmatrix}=\begin{bmatrix}a^2+b-1 & a-1\\ab-b & b\end{bmatrix}$

On comparing the corresponding elements, we get

$a-1=0 \Rightarrow a=1$ and $b=4$

And $(1+a)^2=a^2+b-1$ and $(2+b)(a-1)=ab-b$ are also satisfied by $a=1$ and $b=4$

Therefore, $a=1$ and $b=4$.

37. The total amount spent by the party in two cities X and Y is represented in the matrix equation by matrix C as, $C=BA$

$$\Rightarrow \begin{bmatrix}X\\Y\end{bmatrix}=\begin{bmatrix}1000 & 500 & 5000\\3000 & 1000 & 10000\end{bmatrix}\begin{bmatrix}140\\200\\150\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}X\\Y\end{bmatrix}=\begin{bmatrix}1000\times140+500\times200+5000\times150\\3000\times140+1000\times200+10000\times150\end{bmatrix}$$

$$=\begin{bmatrix}990000\\2120000\end{bmatrix}$$

$\Rightarrow$ X = 990000 paise, Y = 2120000 paise

$\therefore$ X = ₹ 9900 and Y = ₹ 21200

i.e., Amount spent by the party in city X and Y are ₹ 9900 and ₹ 21200 respectively. One should consider about the social activities of a political party before casting his/her vote.

38. Here, $[2x\quad 3]\begin{bmatrix}1 & 2\\-3 & 0\end{bmatrix}\begin{bmatrix}x\\3\end{bmatrix}=O \Rightarrow [2x\quad 3]\begin{bmatrix}x+6\\-3x\end{bmatrix}=O$

$\Rightarrow$ $2x(x+6)+3(-3x)=0 \Rightarrow 2x^2+12x-9x=0$

$\Rightarrow$ $2x^2+3x=0 \Rightarrow x(2x+3)=0 \Rightarrow x=0,\dfrac{-3}{2}$.

39. Trust fund = ₹ 35,000

Let ₹ x be invested in the first bond and then ₹ $(35{,}000-x)$ will be invested in the second bond.

Interest paid on the first bond = 8% = 0.08

Interest paid on the second bond = 10% = 0.10

Total annual interest = ₹ 3,200

$\therefore$ In matrices, $[x\quad 35{,}000-x]\begin{bmatrix}0.08\\0.10\end{bmatrix}=[3{,}200]$

$\Rightarrow$ $x\times0.08+(35{,}000-x)\times0.10=3{,}200$

$\Rightarrow$ $x\times\dfrac{8}{100}+(35{,}000-x)\times\dfrac{10}{100}=3{,}200$

$\Rightarrow$ $8x+3{,}50{,}000-10x=3{,}20{,}000$

$\Rightarrow$ $2x=30{,}000 \Rightarrow x=15{,}000$

$\therefore$ ₹ 15,000 should be invested in the first bond and ₹ 35,000 – ₹ 15,000 = ₹ 20,000 should be invested in the second bond.

The values reflected in this question are :

(i) Spirit of investment.

(ii) Giving charity to cancer patients.

(iii) Helping the orphans living in the society.

40. (b): We have, $A=A^T$

$\Rightarrow \begin{bmatrix}5 & x\\y & 0\end{bmatrix}=\begin{bmatrix}5 & y\\x & 0\end{bmatrix}$

On comparing, we get $x=y$.

41. (d): $A=[1\ 2\ 3]$

$A'=\begin{bmatrix}1\\2\\3\end{bmatrix}$

So, $AA'=[1\ 2\ 3]\begin{bmatrix}1\\2\\3\end{bmatrix}=[1+4+9]=[14]$

42. (b): We have, $P'=2P+I$ ...(i)

Now, $(P')'=(2P+I)'=2P'+I$

$\Rightarrow$ $P=2(2P+I)+I$ [Using (i)]

$\Rightarrow$ $P=4P+3I \Rightarrow P=-I$

43. (b): We have, $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

and $A + A' = I$

$$\Rightarrow \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow 2\cos\alpha = 1 \Rightarrow \cos\alpha=\frac{1}{2} \Rightarrow \alpha=\frac{\pi}{3}$$

44. If A is a matrix of order 3×2, then the order of the matrix A' is 2×3.

45. A square matrix A is said to be symmetric, if $A' = A$.

46. Given, $A=\begin{bmatrix}0 & a & 1\\ -1 & b & 1\\ -1 & c & 0\end{bmatrix}$

A is a skew-symmetric matrix.

$\therefore$ $A' = -A$

$$\Rightarrow \begin{bmatrix}0 & -1 & -1\\ a & b & c\\ 1 & 1 & 0\end{bmatrix}=\begin{bmatrix}0 & -a & -1\\ 1 & -b & -1\\ 1 & -c & 0\end{bmatrix}$$

By comparing on both sides, we get $a = 1$,

$b = -b \Rightarrow 2b = 0 \Rightarrow b = 0; c = -1$

Now, $(a + b + c)^2 = (1 + 0 - 1)^2 = 0$

47. A square matrix A is said to be skew symmetric matrix if $A' = -A$ ...(i)

Now, $A=\begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}$ $\therefore$ $A'=\begin{bmatrix}0 & 2 & b\\ a & 0 & 1\\ -3 & -1 & 0\end{bmatrix}$

From (i), $A + A' = O$

$$\Rightarrow \begin{bmatrix}0 & a & -3\\ 2 & 0 & -1\\ b & 1 & 0\end{bmatrix}+\begin{bmatrix}0 & 2 & b\\ a & 0 & 1\\ -3 & -1 & 0\end{bmatrix}=O$$

$$\Rightarrow \begin{bmatrix}0 & 2+a & b-3\\ a+2 & 0 & 0\\ b-3 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\Rightarrow$ $a + 2 = 0$ & $b - 3 = 0$ $\therefore$ $a = -2$ & $b = 3$

Answer Tips

- If $A = [a_{ij}]$ be a m×n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A.

48. Given, $A=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$

$\because$ A is symmetric. $\therefore$ $A' = A$

$$\Rightarrow \begin{bmatrix}0 & 3 & 3a\\ 2b & 1 & 3\\ -2 & 3 & -1\end{bmatrix}=\begin{bmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{bmatrix}$$

On comparing the corresponding elements, we get

$$a=\frac{-2}{3} \text{ and } b=\frac{3}{2}.$$

49. Given, $A=\begin{bmatrix}3 & 5\\ 7 & 9\end{bmatrix} \Rightarrow A'=\begin{bmatrix}3 & 7\\ 5 & 9\end{bmatrix}$

$\because$ P is symmetric matrix. So, $P=\frac{1}{2}(A+A')$

$$\therefore \quad P=\frac{1}{2}\left(\begin{bmatrix}3 & 5\\ 7 & 9\end{bmatrix}+\begin{bmatrix}3 & 7\\ 5 & 9\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}3+3 & 5+7\\ 7+5 & 9+9\end{bmatrix}=\frac{1}{2}\begin{bmatrix}6 & 12\\ 12 & 18\end{bmatrix}=\begin{bmatrix}3 & 6\\ 6 & 9\end{bmatrix}$$

Hence, the matrix $P=\begin{bmatrix}3 & 6\\ 6 & 9\end{bmatrix}$

Concept Applied

- A square matrix $A = [a_{ij}]$ is said to be symmetric if $A' = A$, that is $[a_{ij}] = [a_{ji}]$ for all possible values of i and j.

50. We know that a square matrix A can be written as

$$A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$$

Out of which $\frac{1}{2}(A+A^T)$ is symmetric and $\frac{1}{2}(A-A^T)$ is skew symmetric matrix,

$\therefore$ For the given matrix

$$A=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix} \text{and } A^T=\begin{bmatrix}2 & 7 & 1\\ 4 & 3 & -2\\ -6 & 5 & 4\end{bmatrix}$$

$$\therefore \quad A+A^T=\begin{bmatrix}4 & 11 & -5\\ 11 & 6 & 3\\ -5 & 3 & 8\end{bmatrix} \text{and } A-A^T=\begin{bmatrix}0 & -3 & -7\\ 3 & 0 & 7\\ 7 & -7 & 0\end{bmatrix}$$

Hence, $A=\frac{1}{2}(A+A^T)+\frac{1}{2}(A-A^T)$

$$=\begin{bmatrix}2 & 11/2 & -5/2\\ 11/2 & 3 & 3/2\\ -5/2 & 3/2 & 4\end{bmatrix}+\begin{bmatrix}0 & -3/2 & -7/2\\ 3/2 & 0 & 7/2\\ 7/2 & -7/2 & 0\end{bmatrix}$$

In above case, first is symmetric and the second is skew symmetric matrix.

51. $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ is a 2×2 symmetric as well as skew symmetric matrix.

52. Let, $A=\begin{bmatrix}0 & 6-5x\\ x^2 & x+3\end{bmatrix}$

A is symmetric, then $A' = A$

$$\therefore \quad \begin{bmatrix}0 & x^2\\ 6-5x & x+3\end{bmatrix}=\begin{bmatrix}0 & 6-5x\\ x^2 & x+3\end{bmatrix}$$

On comparing both sides, we get

$\Rightarrow$ $x^2 = 6 - 5x \Rightarrow x^2 + 5x - 6 = 0$

$\Rightarrow$ $(x + 6)(x - 1) = 0 \Rightarrow x = -6, 1$

53. Given, $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$

(i) $A'=\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$

$A+A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}+\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$

$\therefore \quad (A+A')'=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$

$\Rightarrow \quad (A+A')'=A+A'$

$\therefore \quad (A+A')$ is a symmetric matrix.

(ii) $A-A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}-\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$(A-A')'=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}=-\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}=-(A-A')$

$\Rightarrow \quad (A-A')$ is a skew symmetric matrix.

Concept Applied

- A square matrix $A=[a_{ij}]$ is said to be skew symmetric if $A'=-A$ i.e. $[a_{ji}]=-[a_{ij}]$ for all possible values of i and j

54. Given, A and B are symmetric matrices.

$\therefore \quad A'=A$ and $B'=B$

Now, $(AB-BA)'=(AB)'-(BA)' \;=(B'A')-(A'B')$

$=(BA-AB) \qquad [\because A'=A \text{ and } B'=B]$

$=-(AB-BA)$

Thus, $(AB-BA)'=-(AB-BA)$

Hence, $(AB-BA)$ is a skew symmetric matrix.

55. Let $A=[a_{ij}]$ be a skew symmetric matrix.

Then, $a_{ji}=-a_{ij}\ \forall\ i,j$

$\Rightarrow \quad a_{ii}=-a_{ii}\ \forall\ i \Rightarrow 2a_{ii}=0 \Rightarrow a_{ii}=0\ \forall\ i$

$\Rightarrow \quad a_{11}=a_{22}=a_{33}=\ldots\ldots=a_{nn}=0$

56. (c): We have, $A^2-A+I=O$

Pre-multiplying with A^{-1} on both sides, we get

$(A^{-1}A)\cdot A-A^{-1}\cdot A+A^{-1}\cdot I=A^{-1}\cdot O$

$\Rightarrow \quad I\cdot A-I+A^{-1}=O$

$\Rightarrow \quad A^{-1}=-(A-I)=I-A$

57. $A^2=\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}9 & 7 & 5\\ 1 & 4 & 1\\ 8 & 9 & 9\end{bmatrix}$

$A^3=\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}\begin{bmatrix}9 & 7 & 5\\ 1 & 4 & 1\\ 8 & 9 & 9\end{bmatrix}=\begin{bmatrix}28 & 37 & 26\\ 10 & 5 & 1\\ 35 & 42 & 34\end{bmatrix}$

Now, $A^3-4A^2-3A+11I=\begin{bmatrix}28 & 37 & 26\\ 10 & 5 & 1\\ 35 & 42 & 34\end{bmatrix}-4\begin{bmatrix}9 & 7 & 5\\ 1 & 4 & 1\\ 8 & 9 & 9\end{bmatrix}$

$-3\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}+11\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}28 & 37 & 26\\ 10 & 5 & 1\\ 35 & 42 & 34\end{bmatrix}-\begin{bmatrix}36 & 28 & 20\\ 4 & 16 & 4\\ 32 & 36 & 36\end{bmatrix}-\begin{bmatrix}3 & 9 & 6\\ 6 & 0 & -3\\ 3 & 6 & 9\end{bmatrix}$

$+\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$

$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

Hence, $A^3-4A^2-3A+11I=O$

Now, $A^{-1}[A^3-4A^2-3A+11I]=A^{-1}O$

$\Rightarrow \quad A^2-4A-3A^{-1}A+11A^{-1}I=O \Rightarrow A^2-4A-3I+11A^{-1}=O$

$\Rightarrow \quad A^{-1}=\dfrac{-A^2+4A+3I}{11}$

$\Rightarrow A^{-1}=\dfrac{1}{11}\left(\begin{bmatrix}-9 & -7 & -5\\ -1 & -4 & -1\\ -8 & -9 & -9\end{bmatrix}+\begin{bmatrix}4 & 12 & 8\\ 8 & 0 & -4\\ 4 & 8 & 12\end{bmatrix}+\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}\right)$

$\Rightarrow A^{-1}=\dfrac{1}{11}\left(\begin{bmatrix}-2 & 5 & 3\\ 7 & -1 & -5\\ -4 & -1 & 6\end{bmatrix}\right)=\begin{bmatrix}-2/11 & 5/11 & 3/11\\ 7/11 & -1/11 & -5/11\\ -4/11 & -1/11 & 6/11\end{bmatrix}$

CBSE Sample Questions

1. (c): In a skew-symmetric matrix, the $(i,j)^{th}$ element is negative of the $(j,i)^{th}$ element. Hence, the $(i,i)^{th}$ element $=0$. **(1)**

2. (a): From the definition of equality of two matrices, we have

$2a+b=4 \quad \ldots(i) \qquad a-2b=-3 \quad \ldots(ii)$

$5c-d=11 \quad \ldots(iii) \qquad 4c+3d=24 \quad \ldots(iv)$

Solving (i) and (ii), we get

$5a=5 \Rightarrow a=1, b=2$

Solving (iii) and (iv), we get

$19c=57 \Rightarrow c=3, d=4$

$\therefore \quad a+b-c+2d=1+2-3+8=8$ **(1)**

3. (b): We know that the sum of two matrices is defined only if both matrices have same order.

Here $5A+3B$ is defined if A and B have same order.

$\Rightarrow \quad 3\times n=m\times 5 \Rightarrow n=5, m=3$

So, order of matrix C is 3×5 and $m\neq n$. **(1)**

4. (d): We have, $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

$\therefore \quad A^2=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}0+1 & 0+0\\ 0+0 & 1+0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ **(1)**

5. (b): We have, $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix} \Rightarrow kA=\begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}$

$\Rightarrow \begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}=\begin{bmatrix}0 & 2k\\ 3k & -4k\end{bmatrix}$ (Given)

$\Rightarrow \quad -4k=24,\ 3a=2k,\ 2b=3k$

$\Rightarrow \quad k=-6,\ a=-4,\ b=-9$ **(1)**

6. (d) : We have, $(I+A)^3 - 7A$

$= I^3 + A^3 + 3I^2A + 3IA^2 - 7A = I + A \cdot A + 3A + 3A - 7A$

$(\because A^2 = A)$

$= I + A + 3A + 3A - 7A = I$ (1)

7. (c) : We have, $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \gamma\beta+\alpha^2 \end{bmatrix}$

But $A^2 = 3I$

$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \alpha^2+\beta\gamma \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow \alpha^2 + \beta\gamma = 3$

$\Rightarrow 3 - \alpha^2 - \beta\gamma = 0$ (1)

8. For addition or subtraction of two matrices to be defined, the two matrices should be of same order.

$\therefore 3 \times n = m \times 5 \Rightarrow m = 3$ and $n = 5$

So, order of matrix $(5A - 3B)$ is 3×5 and $m \neq n$. (1)

9. We know, $|\text{adj}A| = |A|^{n-1}$, where $n \times n$ is the order of non-singular matrix A.

$\therefore |\text{adj } A| = (-4)^{3-1} = 16$ (1)

10. (c) : We know that if A and B are non-singular matries of same order, then

$(AB)^{-1} = B^{-1}A^{-1}$; $(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$ (1)

11. (d) : We have,

$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$

$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I \Rightarrow B^{-1} = \frac{1}{6}A$ (1)

12. We have, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Also, $-5A = \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix}$ and $7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

Now, $A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ (1)

Now, $A^{-1}(A^2 - 5A + 7I) = A^{-1}O$

$\Rightarrow A - 5I + 7A^{-1} = O$

$\Rightarrow 7A^{-1} = 5I - A$

$\Rightarrow A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right) \Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (1)