Linear Algebra

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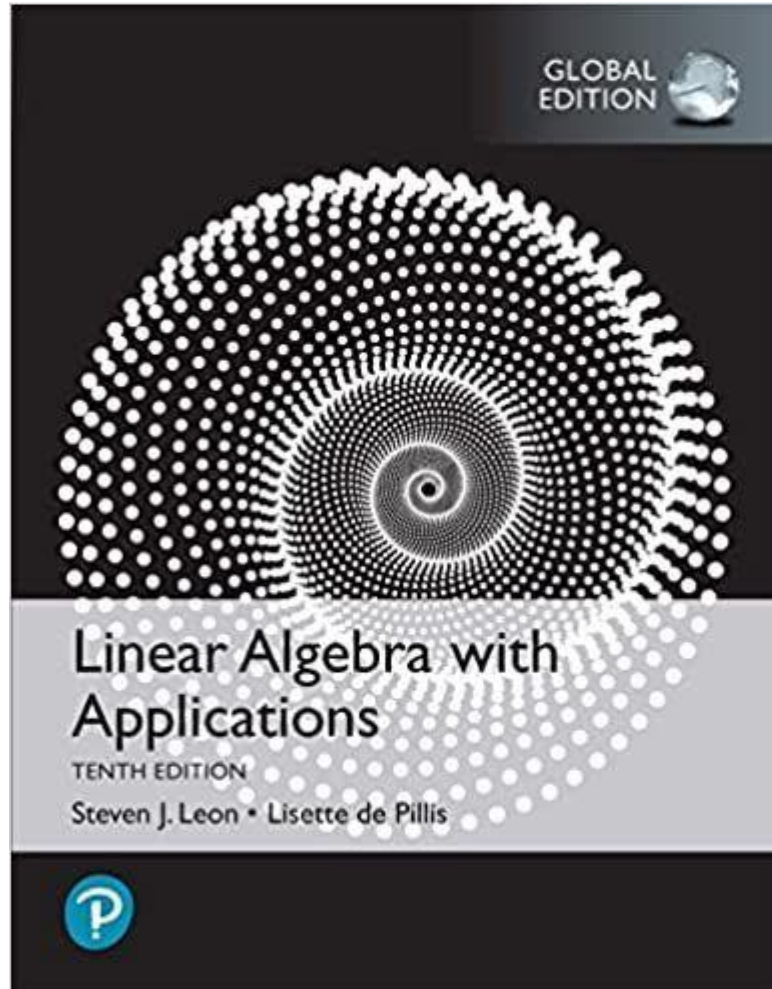
Linear Algebra

Linear Algebra with applications

By

Steven J. Leon

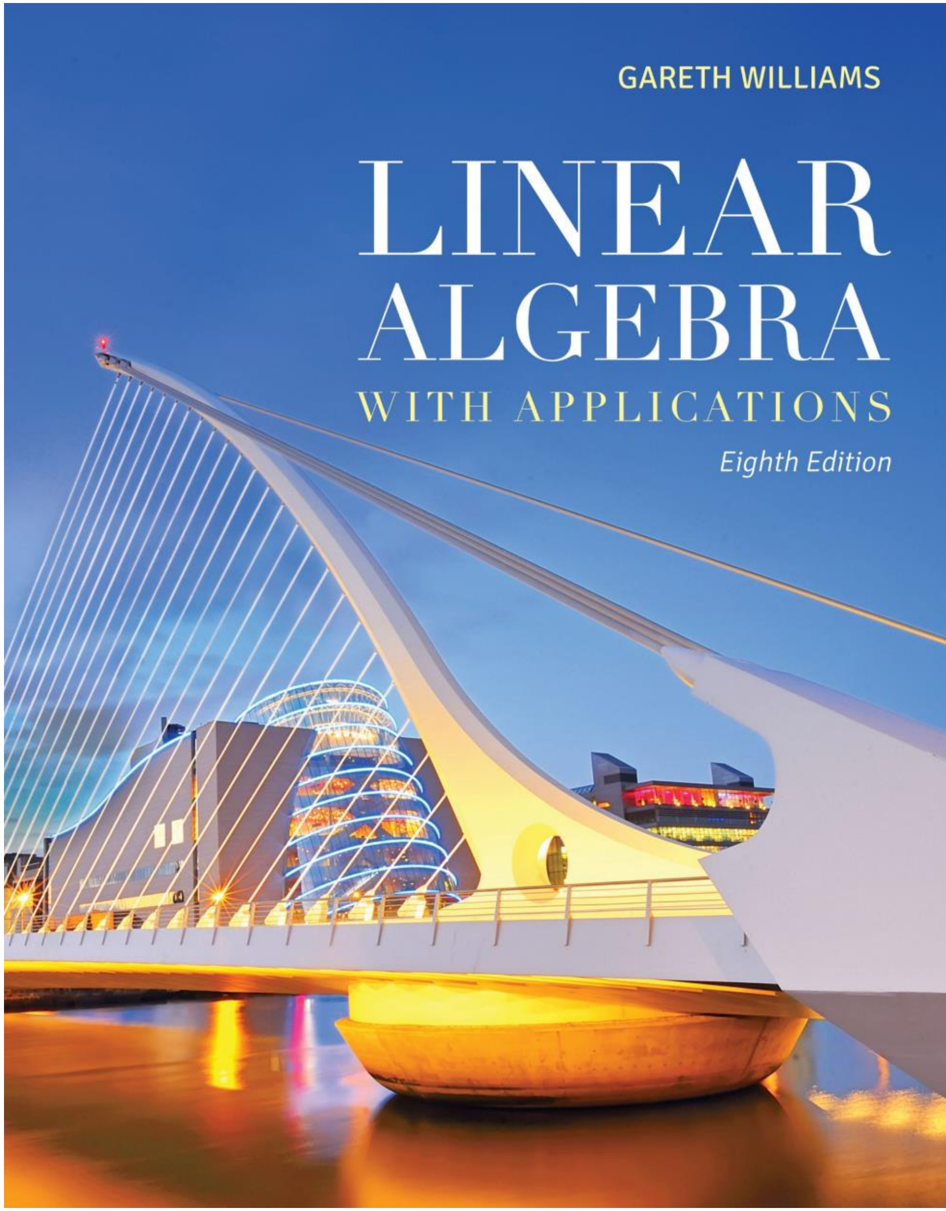


Linear Algebra

Linear Algebra with applications

By

Gareth Williams



Linear Algebra

Grading

- Quizzes
- Assignments
- Class activities
- Project
- Mid Term
- Final Term

Linear Algebra

- Chapter 1 Linear Equations and Vectors:
  - Solving systems of two linear equations to solving general systems.
  - The Gauss-Jordan method of forward elimination is used
  - Concepts of linear independence, basis, and dimension are discussed.
  - Applications:
    - Fitting a polynomial of degree n - 1 to n data points leads to a system of linear equations that has a unique solution.
    - The analyses of electrical networks and traffic flow give rise to systems that have unique solutions and many solutions.

- Chapter 2 Matrices and Linear Transformations:
  - matrix multiplication, transpose, and symmetric matrices
  - Solutions to a homogeneous system of linear equations forms a subspace
  - Applications

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- Chapter 3 Determinants and Eigenvectors:
  - Determinants and Eigenvectors
  - Applications weather prediction

- Chapter 4 General Vector Spaces:
  - Concepts of subspace, linear dependence, basis, and dimension are defined rigorously and are extended to spaces of matrices and functions
  - Linear transformations, kernel, and range are used to give the reader a geometrical picture of the sets **of solutions to systems of linear equations, both homogeneous and nonhomogeneous**

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- Chapter 5 Coordinate Representations:
  - Coordinate Representations of vectors and matrices

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- **Chapter 6** Inner Product Spaces：
  - The axioms of inner products are presented and inner products are to define norms of vectors, angles between vectors, and distances in general vector spaces.

- Chapter 7 Numerical Methods:
  - Solving linear systems of equations using Gaussian elimination, LU decomposition, and the Jacobi and Gauss-Seidel iterative methods.

- Chapter 8 Linear Programming:

What is a “Flipped Classroom”?

Flipping Learning is Upside Down

- The basic idea is to reverse the structure of traditional teaching.
- Traditional teaching usually is based on:
  - lectures that are delivered in a classroom by a lecturer
  - homework carried out by students by themselves, not in the classroom
- With the flipped approach, we will do the opposite:
  - you will listen to the online presentations at home
  - you will be in the classroom to do your homework (that we will call lab sessions)

The Flipped Classroom Model

- Students watch lectures at home at their own pace, communicating with peers and teachers via email or via the platform.

Learning Process

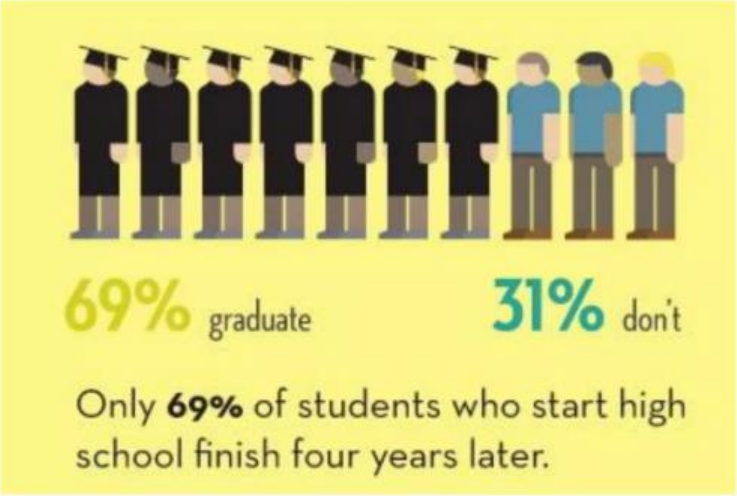
- Passive phase: that we can call the *receptive* phase, where the student/learner opens the mind by listening, reading and receiving new information. In this phase the student lets new knowledge come in.

- Active phase: that we can call the *production* phase, where the student/learner processes the new knowledge, constructs a personal concept map, creates cross-references with previous knowledge. In this phase, the student will become able to apply the new knowledge and to solve practical tasks.

Learning Process

Research says that …

… often with traditional teaching, where the passive phase is carried out in the classroom, learning outcomes are poor. For ex:



Learning Process

Thanks to Technology and eLearning…

eLearning: thanks to the availability and success of online videos used for pedagogical purposes, and the increased access to technology, it is now possible to stop this negative trend.

Learning Process

The benefits

- It allows students to personalize the learning at their own pace.
- You can replay the videos as many time as you like, you stop them and resume them if you need to look up a word in a dictionary, or if you need to brush up a concept, or if you are tired or hungry, etc.
- Therefore there is both a cognitive and physical advantage in doing the passive phase at home.

Learning Outcomes

What is a learning outcome?

- Learning outcomes describe what students are able to demonstrate in terms of knowledge, skills, and values upon completion of a course.

Learning Outcomes

Upon successful completion, students will have the knowledge and skills to:

- Basic understanding of linear equation and matrices. Solution to these equations
- Finding determinants of matrices
- Understanding of vector spaces in terms of linear independence, dimensions etc
- Understanding of linear transformation
- Understanding of Orthogonality
- Finding Eigen values and eigen vectors and their applications
- Numerical linear algebra
- Canonical forms

Linear Algebra

Chapter 1

Systems of Linear Equations

1.1 Matrices and Systems of Linear Equations

Definition

- An equation in the variables x and y that can be written in the form **ax + by = c**, where **a, b,** and **c** are real constants (a and b not both zero), is called a linear equation.

1.1 Matrices and Systems of Linear Equations

Definition

- An equation such as $x+3y=9$ is called a ***linear equation*** (in *two* variables or unknowns).
- The graph of this equation is a straight line in the xy-plane.
- A pair of values of x and y that satisfy the equation is called a ***solution***.

Definition

A ***linear equation*** in n variables $x_1, x_2, x_3, \ldots, x_n$ has the form $a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots + a_n x_n = b$

where the coefficients $a_1, a_2, a_3, \ldots, a_n$ and b are real numbers.

Solutions for system of linear equations

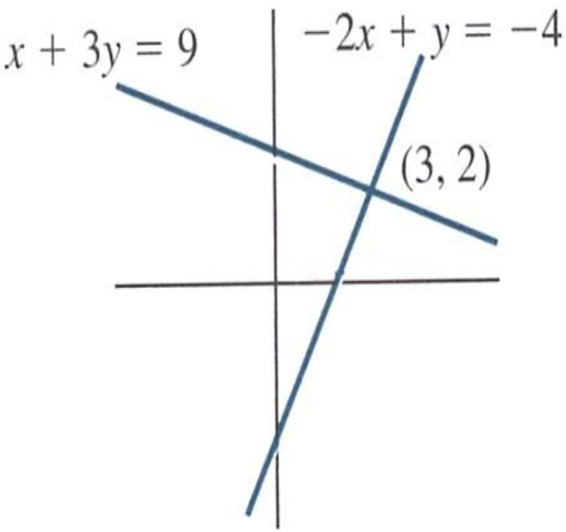


Figure 1.1

Unique solution

$x + 3y = 9$

$-2x + y = -4$

Lines intersect at (3, 2)

Unique solution:

$x = 3, y = 2.$

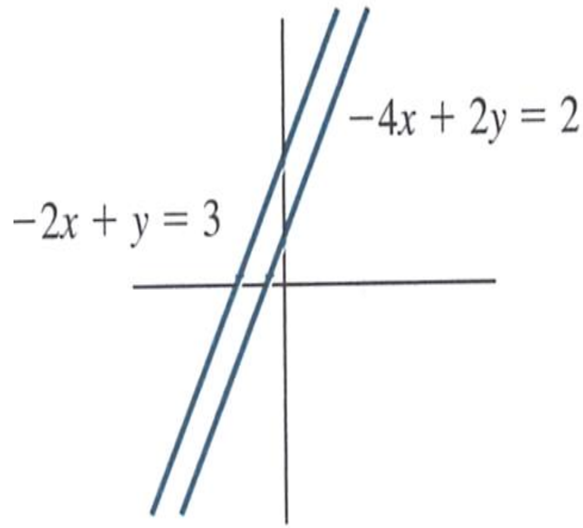


Figure 1.2

No solution

$-2x + y = 3$

$-4x + 2y = 2$

Lines are parallel.

No point of intersection.

No solutions.

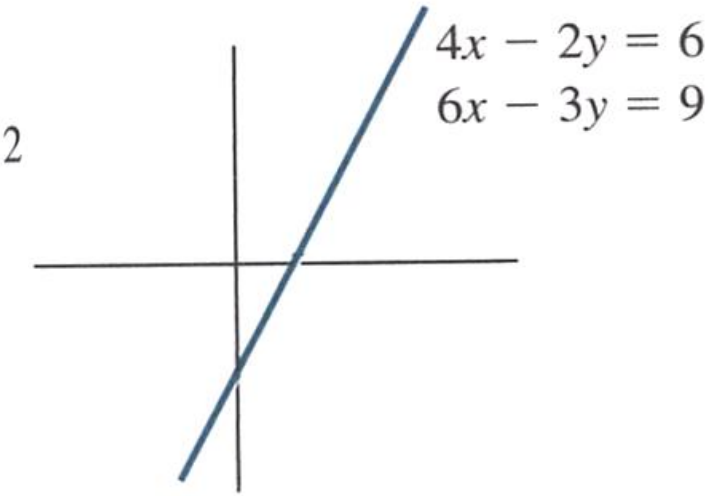


Figure 1.3

Many solution

$4x - 2y = 6$

$6x - 3y = 9$

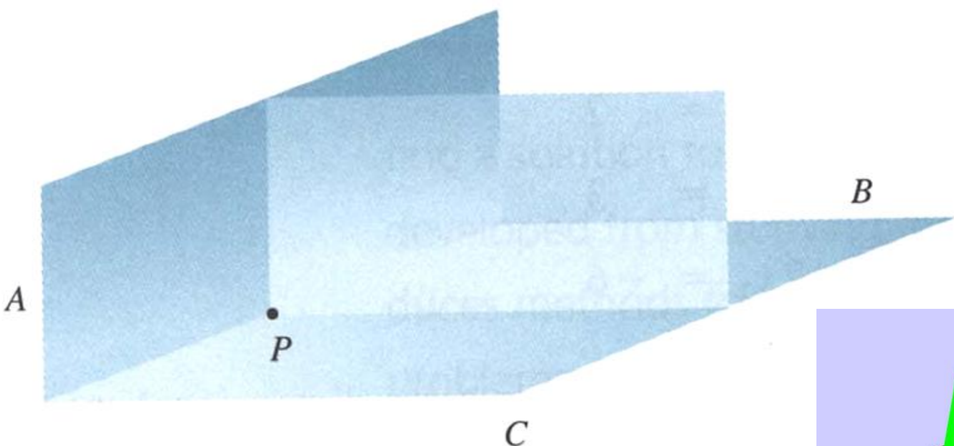
Both equations have the same graph. Any point on the graph is a solution.

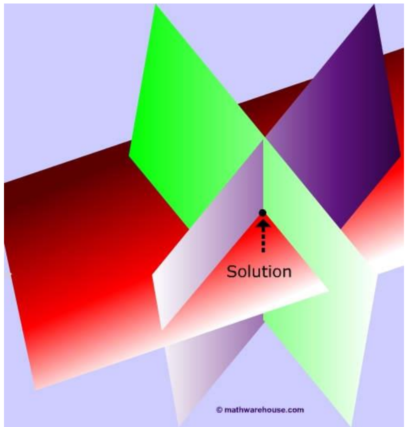
Many solutions.

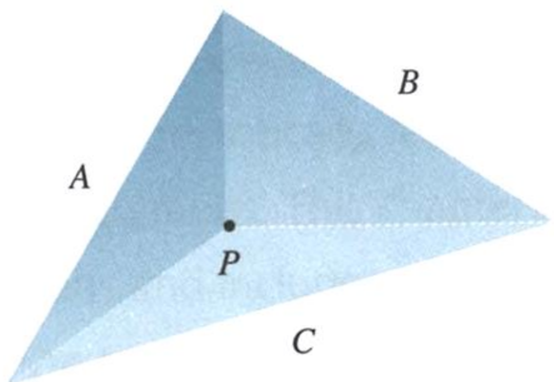
A linear equation in three variables corresponds to a plane in three-dimensional space.

※ Systems of three linear equations in three variables:

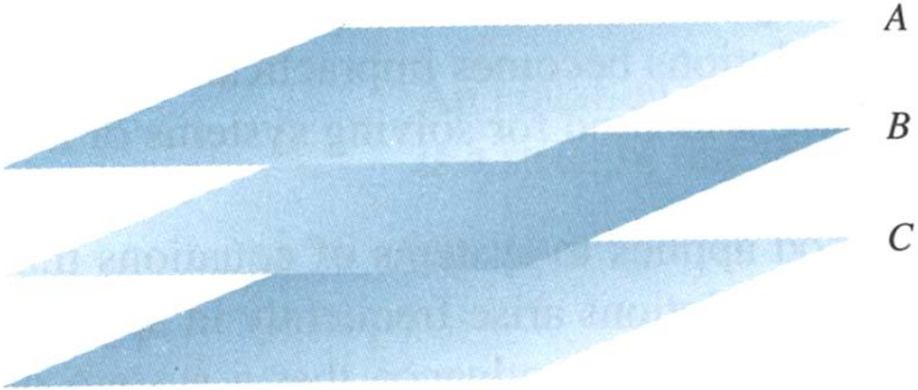
- *Unique solution*

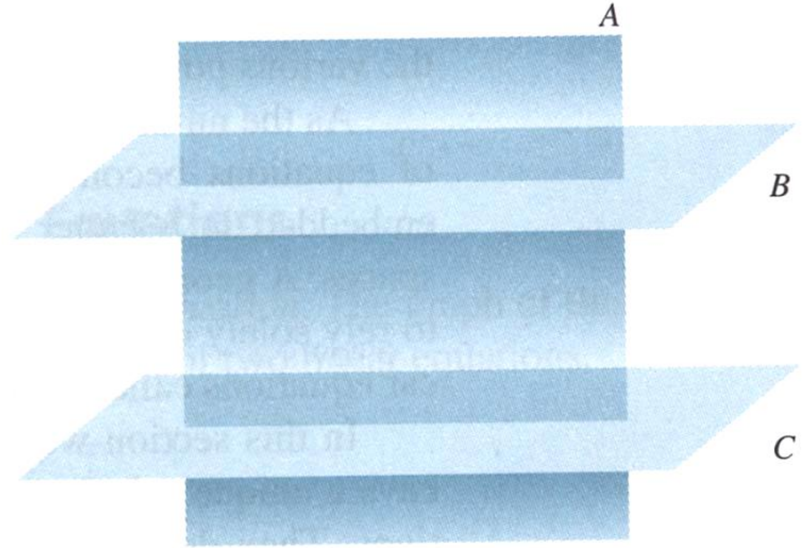




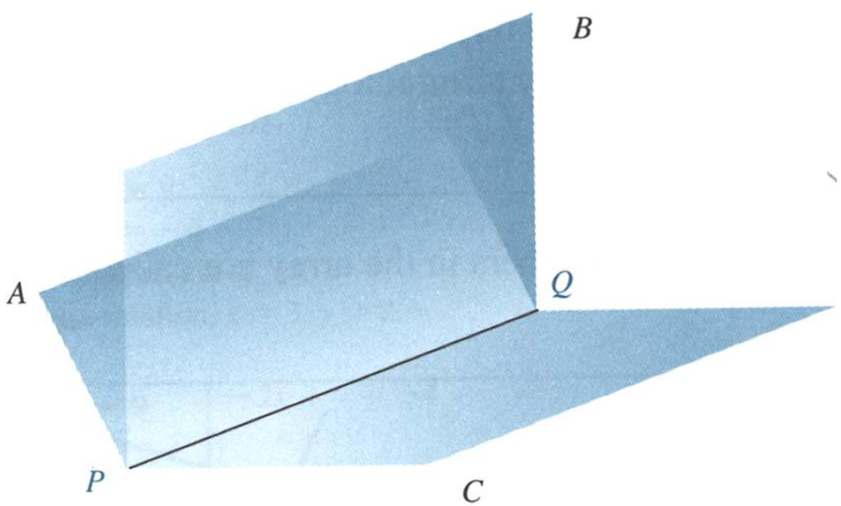


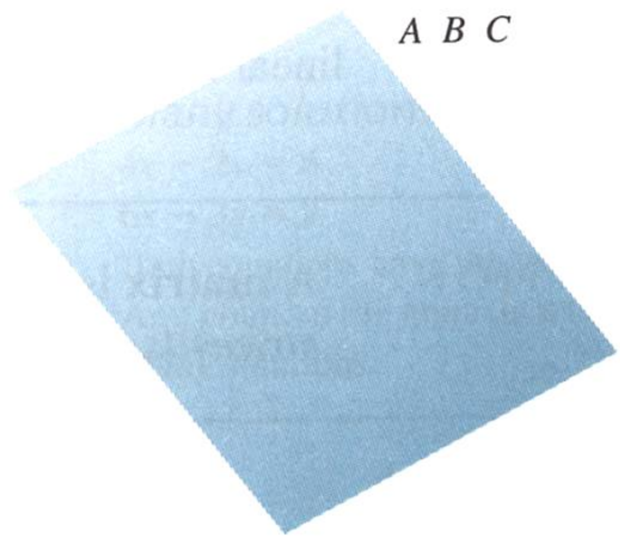
- *No solutions*





- *Many solutions*





A solution to a system of a three linear equations will be points that lie on all three planes.

The following is an example of a system of three linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ x_1 - x_2 - 2x_3 &= -6 \end{aligned}$$

How to solve a system of linear equations? For this we introduce a method called Gauss-Jordan elimination. (Section1.2)

Definition

- A ***matrix*** is a rectangular array of numbers.
- The numbers in the array are called the ***elements*** of the matrix.

Matrices

$$A=\begin{bmatrix} 2 & 3 & -4 \\ 7 & 5 & -1 \end{bmatrix} \qquad \mathrm{B}=\begin{bmatrix} 7 & 1 \\ 0 & 5 \\ -8 & 3 \end{bmatrix} \qquad \mathrm{C}=\begin{bmatrix} 3 & 5 & 6 \\ 0 & -2 & 5 \\ 8 & 9 & 12 \end{bmatrix}$$

Row and Column

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 7 & 5 & -1 \end{bmatrix}$$

$$\underset{\text{row 1}}{\begin{bmatrix} 2 & 3 & -4 \end{bmatrix}} \quad \underset{\text{row 2}}{\begin{bmatrix} 7 & 5 & -1 \end{bmatrix}} \quad \underset{\text{column 1}}{\begin{bmatrix} 2 \\ 7 \end{bmatrix}} \quad \underset{\text{column 2}}{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} \quad \underset{\text{column 3}}{\begin{bmatrix} -4 \\ -1 \end{bmatrix}}$$

Submatrix

$$A = \begin{bmatrix} 1 & 7 & 4 \\ 2 & 3 & 0 \\ 5 & 1 & -2 \end{bmatrix}$$

matrix A

$$P = \begin{bmatrix} 1 & 7 \\ 2 & 3 \\ 5 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 4 \\ 5 & -2 \end{bmatrix}$$

submatrices of A

Size and Type

$$\begin{bmatrix} 1 & 0 & 3 \\ -2 & 4 & 5 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & 7 \\ -9 & 0 & 1 \\ -3 & 5 & 8 \end{bmatrix} \qquad \begin{bmatrix} 4 & -3 & 8 & 5 \end{bmatrix} \qquad \begin{bmatrix} 8 \\ 3 \\ 2 \end{bmatrix}$$

$\text{Size}: 2\times 3$ $\qquad 3\times 3\ \text{matrix}$ $\qquad 1\times 4\ \text{matrix}$ $\qquad 3\times 1\ \text{matrix}$

a square matrix $\qquad \text{a row matrix}$ $\qquad \text{a column matrix}$

Location

$$A=\begin{bmatrix} 2 & 3 & -4 \\ 7 & 5 & -1 \end{bmatrix} \qquad a_{13}=-4,\ a_{21}=7$$

The element a_{ij} *is in* row i, column j

The element in location (1,3) is –4

Identity Matrices

diagonal size

$$I_2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Relations between system of linear equations and matrices

- **matrix of coefficients and augmented matrix**

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ x_1 - x_2 - 2x_3 &= -6 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ 1 & -1 & -2 & -6 \end{bmatrix}$$

matrix of coefficients augmented matrix

Observe that the matrix of coefficients is a submatrix of the augmented matrix. The augmented matrix completely describes the system.

Relations between system of linear equations and matrices

- Transformations called elementary transformations can be used to change a system of linear equations into another system of linear equations that has the same solution.
- These transformations are used to solve systems of linear equations by eliminating variables.
- In practice it is simpler to work in terms of matrices using analogous transformations called elementary row operations.

Elementary Row Operations of Matrices

- These transformations are as follows:

- Elementary Transformation

1. Interchange two equations.
2. Multiply both sides of an equation by a nonzero constant.
3. Add a multiple of one equation to another equation.

- Elementary Row Operation

1. Interchange two rows of a matrix.
2. Multiply the elements of a row by a nonzero constant.
3. Add a multiple of the elements of one row to the corresponding elements of another row.

Example 1

Solving the following system of linear equation.

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ x_1 - x_2 - 2x_3 &= -6 \end{aligned}$$