Quiz 1

Solve the following systems of linear equations by Gaussian elimination method:

$$\begin{aligned} 2x - 2y + 3z &= 2 \\ x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \end{aligned}$$

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SOLUTION

(i) $2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$

The given system equations is

$$\begin{aligned} 2x - 2y + 3z &= 2 \quad \text{-----(1)} \\ x + 2y - z &= 3 \quad \text{-----(2)} \\ 3x - y + 2z &= 1 \quad \text{-----(3)} \end{aligned}$$

The augmented matrix of the above system is $[A \mid B] = \left[\begin{array}{ccc|c} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{array}\right]$

Transforming augmented matrix to echelon form we get

$$\left[\begin{array}{ccc|c} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -2 & 3 & 2 \\ 3 & -1 & 2 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -7 & 5 & -8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -1 & 0 & -4 \end{array}\right]$$

The given system of equations using this row-echelon form reduces to

$$\begin{aligned} x + 2y - z &= 3 \quad \text{-----(4)} \\ -6y + 5z &= -4 \quad \text{-----(5)} \\ -y &= -4 \quad \text{-----(6)} \\ \Rightarrow \quad y &= 4 \end{aligned}$$

Substituting $y = 4$ in equation (5) we get

$$-6 \times 4 + 5z = -4 \Rightarrow 5z = -4 + 24 = 20$$

$$z = \frac{20}{5} = 4$$

Substituting $y = 4$ and $z = 4$ in equation (1) we get

$$x + 2 \times 4 - 4 = 3 \Rightarrow x + 8 - 4 = 3 \Rightarrow x = 3 - 4 = -1$$

$\therefore$ The required solutions of the system is $x = -1$, $y = 4$, $z = 4$