2015 exam (MAST20005), question 7

Suppose Y has the binomial distribution $\mathrm{Bi}(n, \theta)$. Show that a best region for testing the null hypothesis $H_0\colon \theta = \theta_0$ against $H_1\colon \theta > \theta_0$ is $\{y\colon y/n > c\}$. Then find c so that the test has significance $\alpha \approx 0.05$ when n is large.

$\boxed{\text{2015 exam (MAST20005), Q7}}$

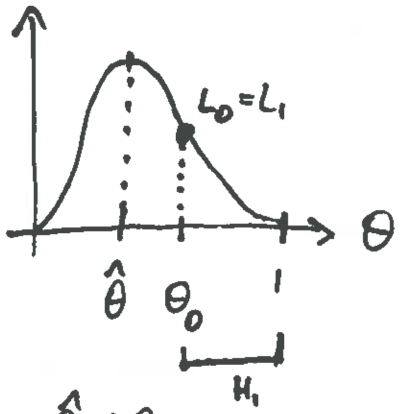
$Y \sim Bi(n, \theta)$

$$\begin{cases} H_0: \theta = \theta_0 \\ H_1: \theta > \theta_0 \end{cases}$$

$L(\theta) = \binom{n}{y} \theta^y (1-\theta)^{n-y} \quad \Rightarrow \quad \hat{\theta} = \frac{y}{n}$

$L_0 = L(\theta_0) = \binom{n}{y} \theta_0^y (1-\theta_0)^{n-y}$

L_1 ? 2 cases...



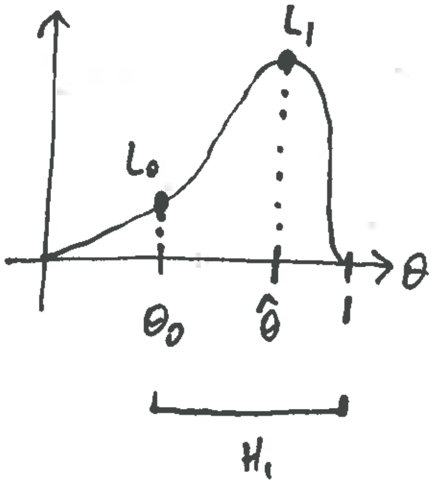
if $\hat{\theta} < \theta_0$

max. at $\theta = \theta_0$

$\Rightarrow L_1 = L(\theta_0) = L_0$

will never reject H_0, $(\lambda = 1)$

can ignore



if $\hat{\theta} > \theta_0$

max at $\theta = \hat{\theta}$

$\Rightarrow L_1 = L(\hat{\theta}) > L_0$

when $\hat{\theta} > \theta_0$,

$$\lambda = \frac{L_0}{L_1} = \frac{\binom{n}{y} \theta_0^y (1-\theta_0)^{n-y}}{\binom{n}{y} \hat{\theta}^y (1-\hat{\theta})^{n-y}}$$