COMP20003
Algorithms and Data Structures
Introduction to Graphs

Nir Lipovetzky
Department of Computing and Information Systems
University of Melbourne
Semester 2

Graph definition

Graph:

- a representation of a set of objects
- some pairs of objects are connected by links

Graph definition

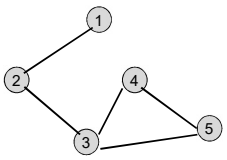
Graph G = {V,E} Set of:

- Vertices V: can contain information
- Edges E (links between vertices): can have direction and/or weight

Compared to trees and linked lists:

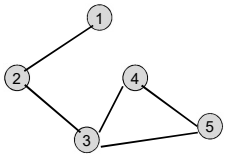
- vertices = nodes
- edges = links

Undirected graph



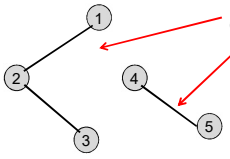
Edges have no direction specified

Connected Undirected graph



Every pair of vertices is connected (possibly indirectly)

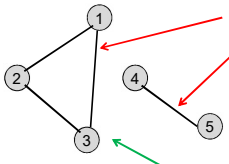
Unconnected Undirected graph



Connected components

.

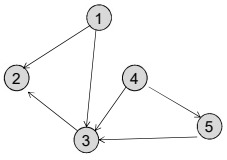
Unconnected Undirected graph with cycle



Connected components

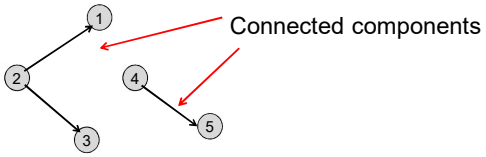
Cycle:
1→2→3 →1, *etc.*
.

Directed graph



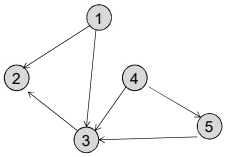
- Edge direction is specified
- Links are not symmetrical

Acyclic, unconnected directed graph



.

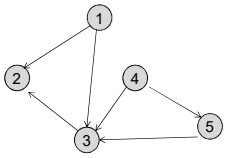
Directed graph



Reachability:

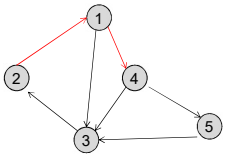
Can you get from Vertex 1 to Vertex 5?

Weakly connected Directed graph



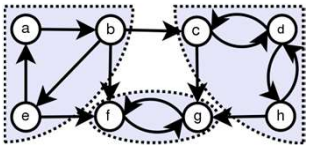
Replace all directed edges with undirected edges, to obtain a connected (undirected) graph

Strongly connected Directed graph



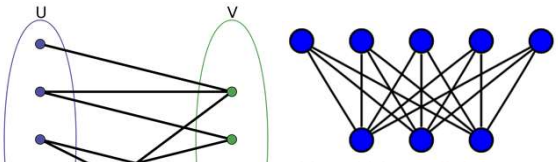
If every vertex is reachable from every other vertex

Strongly connected Components in a Digraph



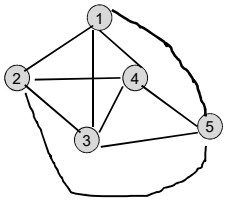
If every vertex is reachable from every other vertex within the same component

Bipartite Graph



- U and V are disjoint sets of vertexes
- Every vertex in U connects to a vertex in V, and viceversa

Complete graph



V(V-1)/2 edges

Trees and Graphs

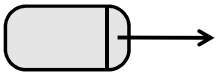
Tree, undirected graph that is:

- Connected
- Acyclic

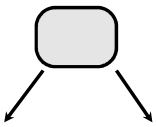
- *n.b.* Any two vertices are connected by exactly one simple path
- *n.b.* All vertices are connected

Representing graph vertices

- Linked list nodes:

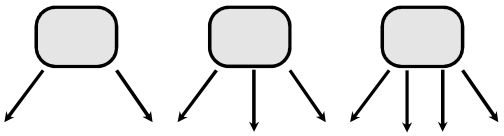


- Binary tree nodes:



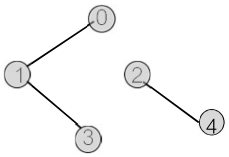
Vertices

- 2-3-4 tree nodes



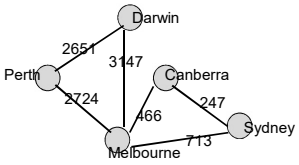
But what if the number of links is potentially large (up to V-1)?

Array representation



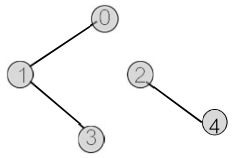
A	0	1	2	3	4
0	0	1	0	0	0
1	1	0	0	1	0
2	0	0	0	0	1
3	0	1	0	0	0
4	0	0	1	0	0

Weighted undirected graph



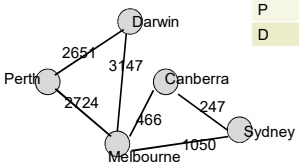
713

Array representation of weighted undirected graph



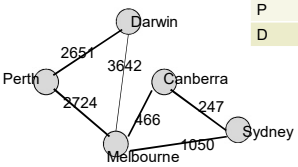
A	0	1	2	3	4
0		25			
1	25			40	
2					10
3		40			
4			10		

	M	C	S	P	D
M	0	466	1050	2724	3642
C	466	0	247		
S	1050	247	0		
P	2724			0	2651
D	3642			2651	0



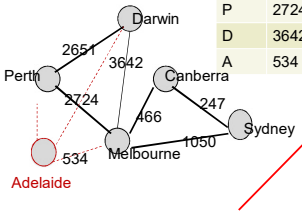
713

	M	C	S	P	D
M	0	466	1050	2724	3642
C	466	0	247		
S	1050	247	0		
P	2724			0	2651
D	3642			2651	0



Canberra to Darwin?
466+3642=4108

	M	C	S	P	D	A
M	0	466	1050	2724	3642	534
C	466	0	247			1160
S	1050	247	0			1163
P	2724			0	2651	2134
D	3642			2651	0	2620
A	534	1160	1163	2134	2620	0



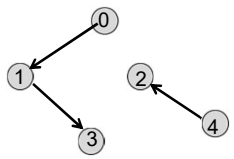
Canberra to Darwin?
Still 466+3642?
Or 466+534+2724?

Shortest route between Melbourne and Dubbo?

Considerations:
- Distance
- Kind of road
- Traffic points

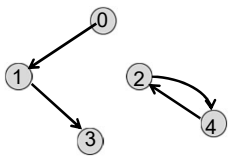
Shortest route between Melbourne and Dubbo?

Array representation: weighted directed graph



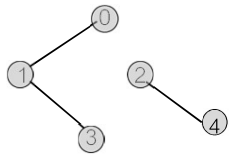
A	0	1	2	3	4
0		25			
1				40	
2					
3					
4			10		

Array representation: weighted directed graph



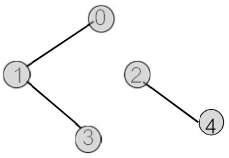
A	0	1	2	3	4
0		25			
1				40	
2					15
3					
4			10		

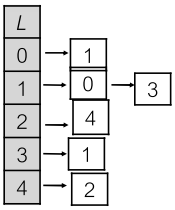
Array representation: initialization



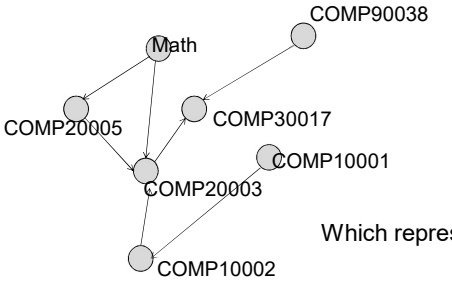
A	0	1	2	3	4
0	∞	25	∞	∞	∞
1	25	∞	∞	40	∞
2	∞	∞	∞	∞	10
3	∞	40	∞	∞	∞
4	∞	∞	10	∞	∞

Adjacency list representation





Directed graph: Subject prerequisites



Which representation?

Size of matrix and list

- Size of representation in terms of
  - |V| number of vertices
  - |E| number of edges
- Matrix
  - O(??)
- Adjacency list
  - O(??)

Some interesting graph path problems

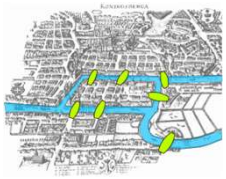
- Reachability
- Single shortest path
- Single source shortest path
- All pairs shortest paths
- Travelling Salesman Problem

Other interesting graph problems

- Minimum Spanning Tree
- Topological sort
- Map coloring
- Matching

History

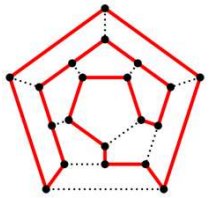
- Graph theory started with Euler (1736) who was asked to find a nice path across the seven Königsberg bridges



- The (Eulerian) path should cross over each of the seven bridges (edge) exactly once

History

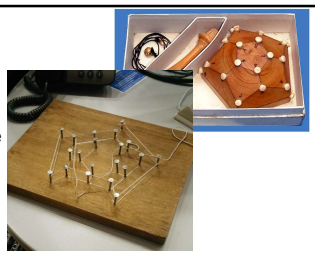
- Another early precursor was Sir William Rowan Hamilton (1805-1865)



In 1859 he developed a toy based on finding a path visiting all cities (vertexes) in a graph exactly once and sold it to a toy maker in Dublin. It never was a big success

History

Build your own game

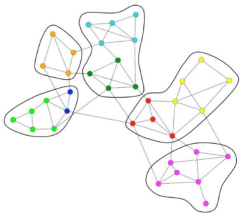


Or play it here: http://neamar.fr/Res/Icosien/ (Flash)

- Euler Game:
  - Traverse an Edge once: https://youtu.be/ztGEIutUIJQ
- Hamiltonian Cycle:
  - Traverse an Vertex once: https://youtu.be/PU7ZIA0_Z5w
- Analysis: https://www.naturelovesmath.com/en/games/icosian-a-graph-theory-game/

Applications

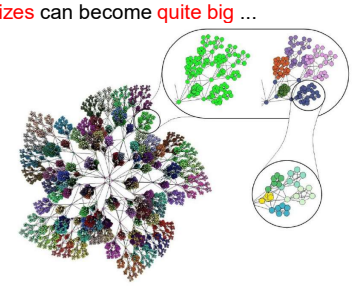
But now graph theory is used for finding communities in networks



where we want to detect hierarchies of substructures

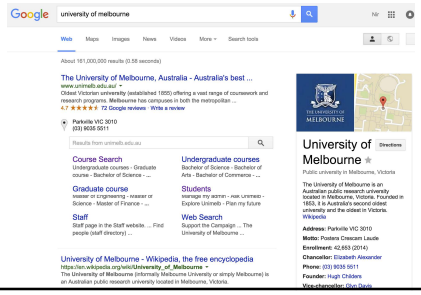
Applications

and their sizes can become quite big ...



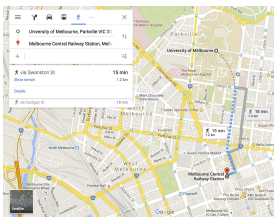
Applications

It is also used for ranking (ordering) hyperlinks



Applications

or by your GPS to find the shortest path home ...



… and the list should go on for hours

COMP 20003 Algorithms and Data Structures 1-41