COMP20003
Algorithms and Data Structures
Hash Tables

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So far…

- Dictionary search has been based on key comparisons.
  - Linked list (sorted and unsorted)
  - Array (sorted and unsorted)
  - Binary Search Tree
  - Balanced Trees

This section

- A way of going directly to the desired item
- Hash tables:
  - Search usually takes only 1 (or few) operations.
    - (on average)
    - (if managed well)
    - (but very bad worst case)
- Probabilistic data structure

Textbook

- Skiena: Chapter 3, Section 3.7

Direct storage and search (a hypothetical fast method)

Task: Store in dictionary items with keys within the range 0 to **RANGE-1**.

- Data structure: Array

```
itemtype*    A[RANGE];
```

- Operations:

```
void initialize( itemtype* A) {
      for(i=0;i<RANGE;i++)
         A[i] = NULL;
   }
void insert( itemtype* item ) { A[item->key] = item; }
itemtype* search( int key )   { return A[key]; }
```


Direct storage and search (a hypothetical fast method)

Create table and initialize:

```
#define RANGE 10000
#define EXAMPLEKEY 8179
struct item{
  int key;
  char *info;
} item;
item  *A[RANGE];
item  *newitem;

for( i=0;i < RANGE; i++ ) A[i] = NULL;
```


Direct storage and search (a hypothetical fast method)

- Insert item with key=**EXAMPLEKEY**:

```
newitem = (item *)malloc……
newitem->key = EXAMPLEKEY;
newitem->info = …malloc….strcpy…
A[EXAMPLEKEY] = newitem;
```

- Search for item with key=**EXAMPLEKEY**:

```
return A[EXAMPLEKEY];
```


Limitations?

Use the idea: make it practical

- Solution: circular array
  - Squash the keys to fit into an array:
    - A[100]
    - Store key in A[key%100]
- Issue: Collisions
    - If key1 = 200 and key2 = 400, both map to A[0]
    - Collisions are *always* possible, so *must* have a plan
    - Solution: Patterns
      - Use complicated mapping of keys to disrupt patterns

Patterns Excercise

Key = Input % modulo

Fill the table below with key values

Input	Modulo 8	Modulo 7
0		
4		
8		
12		
16		
20		
24		
28		

Hash Functions

Hash function:

```
int hash(keytype key);
```

maps item's key to an array slot

```
A[ hash(item->key) ] = item;
```

Desirable features and requirements of a hash function:

- Output value within bounds of the array
- Should minimize collisions, as far as possible
- Should spread items throughout the table

Hash Functions

Some bad hash functions

- `A[100]; hash(key) = key%10`
- `A[100]; hash(key) = key%100`

Better:

- `A[97]; hash(key) = (key * BIGPRIME)%97`

Prime numbers:

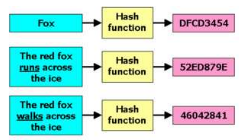
- disrupt patterns in data
- spread it throughout the table.

Hash Functions

Student numbers example:

- 3 first numbers
- 3 last numbers
- 0-9 buckets

Hash functions for strings



Record with string key *s*, array dictionary size **SIZE** might be stored in location:

Example:

```
A[ (17*s[0] + 37*s[2] +101*length(s))%SIZE ]
```


Hash functions for strings

- Skiena (p.89) shows mapping strings to number, base alphabet size α:

$$H(S) = \sum_{i=0}^{|S|-1} \alpha^{|S|-(i+1)} \times char(s_i)$$

- `H("cat") = 26`^{2} `* 3 + 26 * 1 + 1 * 20`
- Does this work for longer strings?
- Is this efficient?

Hash functions for strings

More efficient:

- Use a power of 2 instead of alphabet size:

Implementation:

```
H("cat") = 32^2 * 3 + 32 * 1 + 1 * 20;
    hashcat = (('c')*(1<<10)) +
              (('a')* (1<<5)) +
               ('t'))%TABLESIZE;
      OR
    hashcat = (('c' << 10) +
               ('a' <<5)  +
               ('t'))%TABLESIZE;
```

Hash functions for strings

More efficient *and* prevent overflow:

- Use a power of 2 instead of alphabet size:

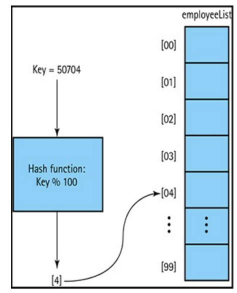
```
H("cat") = 32^2 * 3 + 32 * 1 + 1 * 20;
        hashcat =((((‘c’* 1<<10) %TABLESIZE)
                +('a' * 1 <<5)%TABLESIZE)
                +'c')%TABLESIZE;
```

Principle:

$$(a+b) \bmod n = ((a \bmod n) + (b \bmod n)) \bmod n$$

Hash tables: key idea

- Huge range of possible keys
  - e.g. space of possible surnames: 26^n
    - 26^{10} = 141,167,095,653,376
- Map to a smaller set of array indexes, 0..m-1
  - hash function: *h*
  - easily computed
  - even distribution



Collisions

Collision: Two keys map to the same array index

$$h(k_1) == h(k_2)$$

When array `SIZE` < number of records

- *definitely* have collisions

Whern array `SIZE` > number of records

- often have collisions – and we *must* handle them

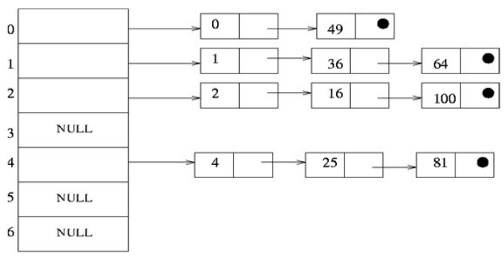
Good hash functions have fewer collisions, but *we can never assume there will be none*

Collision Resolution Methods

1. Chaining
2. Open addressing methods
   - Linear probing
   - Double hashing

Chaining

Average length of the lists?




```
void insert( HT, item )
{
   new newnode = /* ... make a list node */
   /* put --item-- in the list node */

   index = hash(item->key);

   if( HT[ index ] == NULL)
      HT[ index ] = newnode;
   else
   {
      newnode->next = HT[ index ]->node;
      HT[ index ] = newnode;
   }
}
```


Linear chaining

What happens if:

- you forget to null the table initially?
- all the items hash to the same location?
- number of items is much bigger than the table?

Chaining: analysis

- Insertion:
  - Best case
  - Worst case
  - Average Case
- Search
  - Best case
  - Worst case
  - Average Case

Chaining: analysis

Average case:

- fast lookup when table is not heavily loaded

Performance degrades when table gets crowded

- Eventually degenerates to a linked lists

Extra time and space for pointers

Open addressing: Linear probing

If there is a collision, put the item in the next available slot

```
while( HT[ index ] != NULL )
  index = (index + 1)%TABLESIZE
  /* only get out of this loop when
  get to a vacant spot */
```


Open addressing: linear probing

m = 20, f(k) = k % m

Initial Situation

–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

After inserting 34, 55, 12, 8, 45, 37, 32, 88, 98, 54

–1	–1	–1	–1	–1	45	–1	–1	8	88	–1	–1	12	32	34	55	54	37	98	–1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
					0			0	1			0	1	0	0	2	0	0	

After inserting 21, 42, 56, 74, 52, 33, 16

74	21	42	52	33	45	16	–1	8	88	–1	–1	12	32	34	55	54	37	98	56
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
6	0	0	11	11	0	10		0	1			0	1	0	0	2	0	0	3

Linear probing

- What happens when:
  - HT lightly loaded?
  - HT heavily loaded?
  - HT full?

Linear probing: Biggest problems

Catastrophic failure when table full

Clustering:

- Once things start to go bad in part of the table…

Double hashing

Choose a second hash function

- Reduces clustering

```
jumpnum = hash2(key);
while (HT[index] != NULL)
  index=(index+jumpnum)%TABLESIZE
```

Example hash2 function:

```
hash2(key) = key%SMALLNUMBER + 1;
```


Open addressing: Analysis

- Consider load factor α
  - for n keys
  - in m cells
  - $\alpha = n/m$

Open Addressing: Analysis

Average case, under some simplifying assumptions, expected time for insertion is:

- Double hashing: $1/(1-\alpha)$
- Linear probing: $1/(1-\alpha)^2$
- Example: $\alpha = 0.75$
  - Double hash insertion: 4 probes
  - Linear probing insertion: 16 probes

A nice explanation of the assumptions, by Tim Roughgarden:

- https://www.youtube.com/watch?v=nWQv4BCEhjM&list=PLXFMmlk03Dt7Q0xr1PIAriY5623cKiH7V&index=73

Open Addressing: Analysis

- Average case lookup:
  - Double hash ~ ½(1 + 1/(1-α))
  - Linear probing ~ ½(1 + 1/(1-α)^2)

	Double hash	Linear probe
α	$\frac{1}{2}(1+\frac{1}{1-\alpha})$	$\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$
50%	1.5	2.5
75%	2.5	8.5
90%	5.5	50.5

Open Addressing: Analysis

Degraded performance as table nears full.

α	$\frac{1}{2}(1+\frac{1}{1-\alpha})$	$\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$
50%	1.5	2.5
75%	2.5	8.5
90%	5.5	50.5

Catastrophic failure when table full.

- Performance depends on α (*n/m*), so choice of table size must be appropriate

Open Addressing: Analysis

Degraded performance as table nears full.

α	$\frac{1}{2}(1+\frac{1}{1-\alpha})$	$\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$
50%	1.5	2.5
75%	2.5	8.5
90%	5.5	50.5

Catastrophic failure when table full.

- How and why do people use open addressing?

Open Addressing: Analysis

Degraded performance as table nears full.

α	$\frac{1}{2}(1+\frac{1}{1-\alpha})$	$\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$
50%	1.5	2.5
75%	2.5	8.5
90%	5.5	50.5

Catastrophic failure when table full.

- How might you prevent degraded performance?

Hash tables: Summary

O(1) lookup!!

- But only on average
- And only for small α

Some bad worst cases:

- Table full (open addressing)
- Table near full (open addressing)
- Everything hashes to same/similar slot (all)

Hash tables: Summary

Performance degrades:

- For linear chaining, degrades gracefully
- For open address chaining, degrades, then can fail catastrophically.

Cannot retrieve items in sorted order

A nice review of hashing, including some advanced topics:

- http://courses.csail.mit.edu/6.006/fall10/lectures/lecture5.pdf
- http://courses.csail.mit.edu/6.006/fall10/lectures/lecture6.pdf
- http://courses.csail.mit.edu/6.006/fall10/lectures/lecture7.pdf

Some notes about hash tables

Hash tables show fast lookup

- O(1) lookup
- Better than log n

Used in non-time critical applications

Often used in CS applications

Other uses of hashing

Duplicate detection, *e.g.* for documents:

- If hash signatures are different, documents can't be duplicates
- Only have to thoroughly check a few documents

Plagiarism detection