PART B - extended answer (60 marks)

Answer all Part B questions in the spaces provided on this paper.

Question 1 (20 marks)

1. [**3 marks**] Explain how analog signals are represented in digital circuits by binary numbers using the techniques of sampling and quantisation, and argue that digital circuits are more immune to noise than analog circuits.

2. **[2 marks]** Use the theorems of Boolean algebra to simplify the expression $F = XYZ + X\overline{Y}Z + \overline{X}$ for the Boolean variables X, Y and Z.

3. **[2 marks]** Convert the hexadecimal number $FACE_{16}$ to decimal.

4. **[3 marks]** The (7,4) Hamming code takes four information bits $(b_4\, b_3\, b_2\, b_1)$ and adds three parity check bits $(p_3\, p_2\, p_1)$ to give a codeword

$$(c_7\, c_6\, c_5\, c_4\, c_3\, c_2\, c_1) = (b_4\, b_3\, b_2\, p_3\, b_1\, p_2\, p_1).$$

The check bits $(p_3\, p_2\, p_1)$ are chosen as follows:

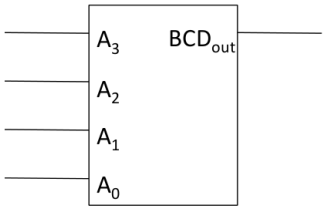
- p_3 is chosen so as to give an even number of 1s in the group $(c_7\, c_6\, c_5\, c_4) = (b_4\, b_3\, b_2\, p_3)$;
- p_2 is chosen so as to give an even number of 1s in the group $(c_7\, c_6\, c_3\, c_2) = (b_4\, b_3\, b_1\, p_2)$; and
- p_1 is chosen so as to give an even number of 1s in the group $(c_7\, c_5\, c_3\, c_1) = (b_4\, b_2\, b_1\, p_1)$.

After transmission across a communications channel, a single error occurs in a transmitted codeword leading to the 7-bit string 1100111 being received.

(a) **[2 marks]** Calculate the parity of each of the three code groups listed above for the received codeword 1100111 and use this to form the 3-bit syndrome $(s_3 s_2 s_1)$.

(b) **[1 mark]** What was the codeword that was transmitted?

5. **[10 marks]** You are to design a circuit, called a *BCD checker*, that takes a 4-bit number $A_3A_2A_1A_0$ as input and determines whether it is a valid Binary Coded Decimal (BCD) code. It can be represented by the logic block:



The output BCD_{out} is 1 if the 4-bit input number is a valid BCD code and 0 otherwise. For example:

- If the input number is 0111 then the output would be 1 as 0111 is a valid BCD code.
- If the input number is 1100 then the output would be 0 as 1100 is not a valid BCD code.

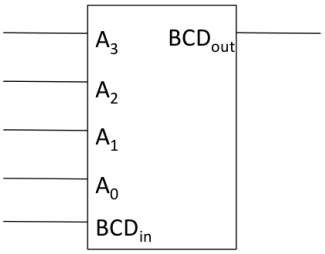
(a) **[3 marks]** Construct the truth table for the output BCD_{out} of the BCD checker.

(b) **[3 marks]** Use a Karnaugh map to obtain a minimal sum of products expression for the output BCD_{out}.

(c) **[1 marks]** Draw a circuit diagram for the BCD checker.

(d) **[3 marks]** Now suppose that the BCD checker circuit is to be modified to be used as a module in a larger n-bit BCD checker design capable of checking if the input of an n-bit number is a valid BCD code, where n is a multiple of 4.

Modify the circuit in part (c) using additional logic gates to yield a new module that has an additional input BCD_{in} as shown in the figure below



such that the output BCD_{out} is 1 if and only if the 4-bit input $A_3A_2A_1A_0$ is a valid BCD code AND the input BCD_{in} is 1. Show how several of these new modules could be used to check the validity of an 8-bit BCD code $X_7X_6X_5X_4X_3X_2X_1X_0$.

More space is provided for your answer on the following page.

Question 2 (20 marks)

1. **[10 marks]** The Euclidean distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The area of a triangle can be calculated based on the lengths of its sides as follows

$$area = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}$$

where a, b, and c are the lengths of the sides and s is equal to *half* the sum of the lengths of the three sides of the triangle, i.e. $s = (a + b + c)/2$.

(a) **[2 marks]** Write a MATLAB function `side_length` that calculates and returns the length of the side formed by any two points x_1, y_1 and x_2, y_2 (i.e. the distance between them).

(b) **[5 marks]** Write a MATLAB function `triangle_area` that takes three coordinates of three points $x_1, y_1, x_2, y_2, x_3, y_3$ that determine a triangle (i.e. the x and y components of each of the three points in order) and returns the area of a triangle and is calculated by calling your `side_length` function from part (a).

(c) **[3 marks]** Write a MATLAB script that prompts a user to enter the coordinates of three points that determine a triangle (i.e. the x and y components of each point entered as $[x1, y1, x2, y2, x3, y3]$), calculates the area of the triangle and *prints it to the screen.* You may use your `triangle_area` function from part (b).

For example, running your script and providing it with some example input should yield:

```
Enter triangle coordinates [x1, y1, x2, y2, x3, y3] : [1, 2, 1, 6, 4, 2]
The area of the triangle is : 6
```

Hint : The MATLAB function to prompt for user input is `input`

2. [**10 marks**] In this question you must write a MATLAB function that finds duplicated elements in a vector. The output of the function must result in a new vector containing only unique instances of the duplicated values.

   For example, the duplicated element vector E of the vector `x=[2 3 2 4 5 6 5 2 1 1]` is

   ```
   E = [2 5 1]
   ```

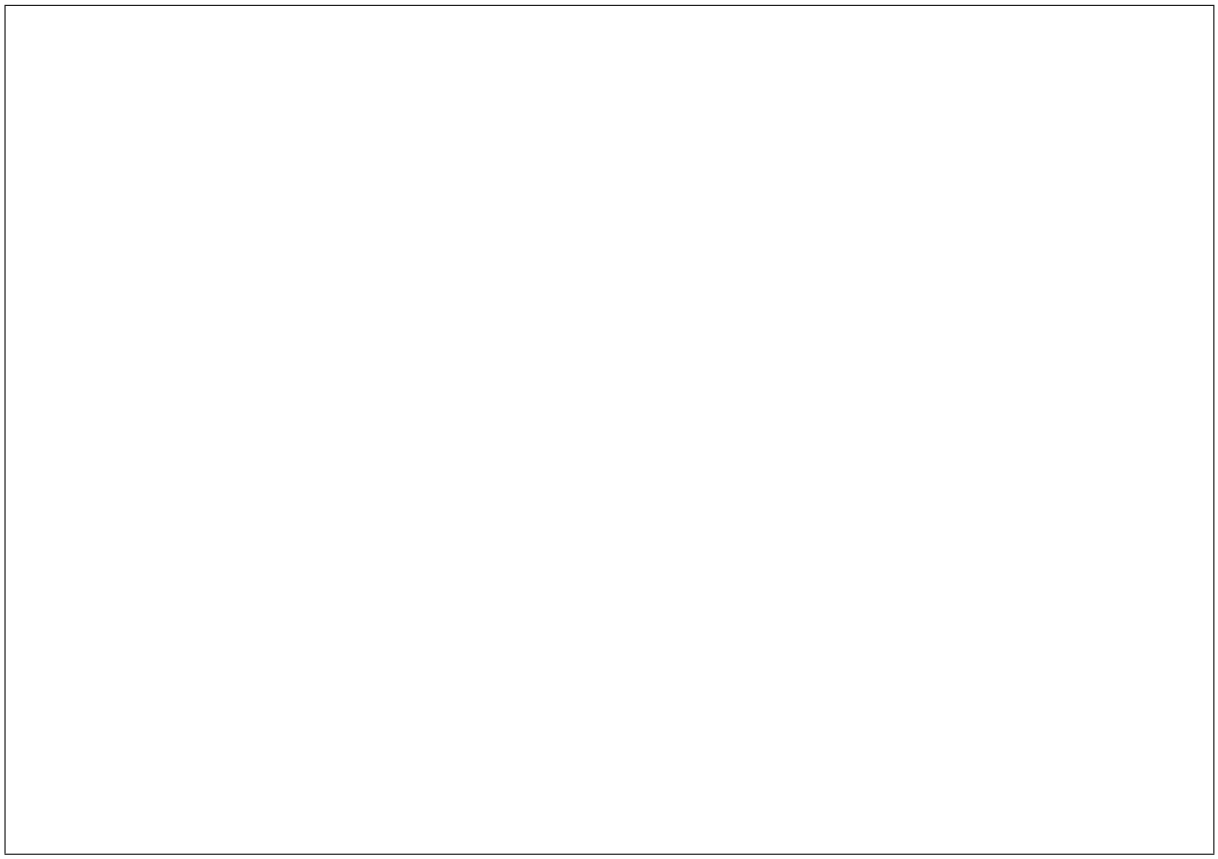
   Note that the duplicated element vector does not need to be sorted in any particular order, however it must contain only one of every duplicated value of the input vector.

   Write a MATLAB function `find_duplicates(x)` that computes and returns the duplicated elements vector of the vector `x`. You must make sure that the vector `x` contains only numbers and if not return an appropriate error message.

   NOTE : You may NOT use the MATLAB functions `unique` or `find` in your answer.

   Hints :

   - The MATLAB function `isnumeric(A)` returns 1 if `A` is a numeric array and 0 otherwise.
   - The MATLAB function `ismember(A,B)` where `A` is a number and `B` is a vector, returns 1 if `A` is contained in `B` and 0 otherwise.



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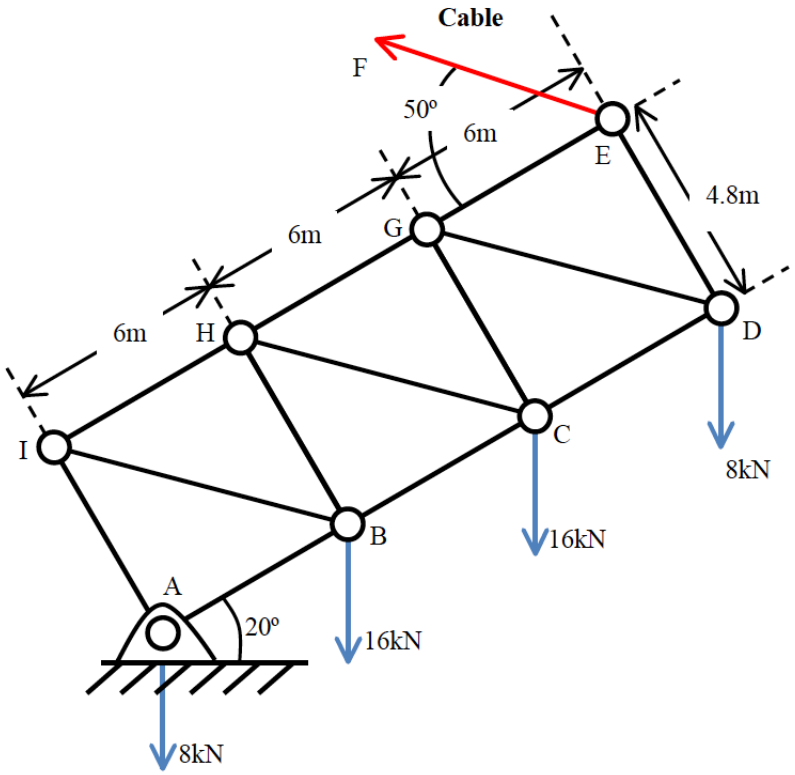
Question 3 (20 marks)

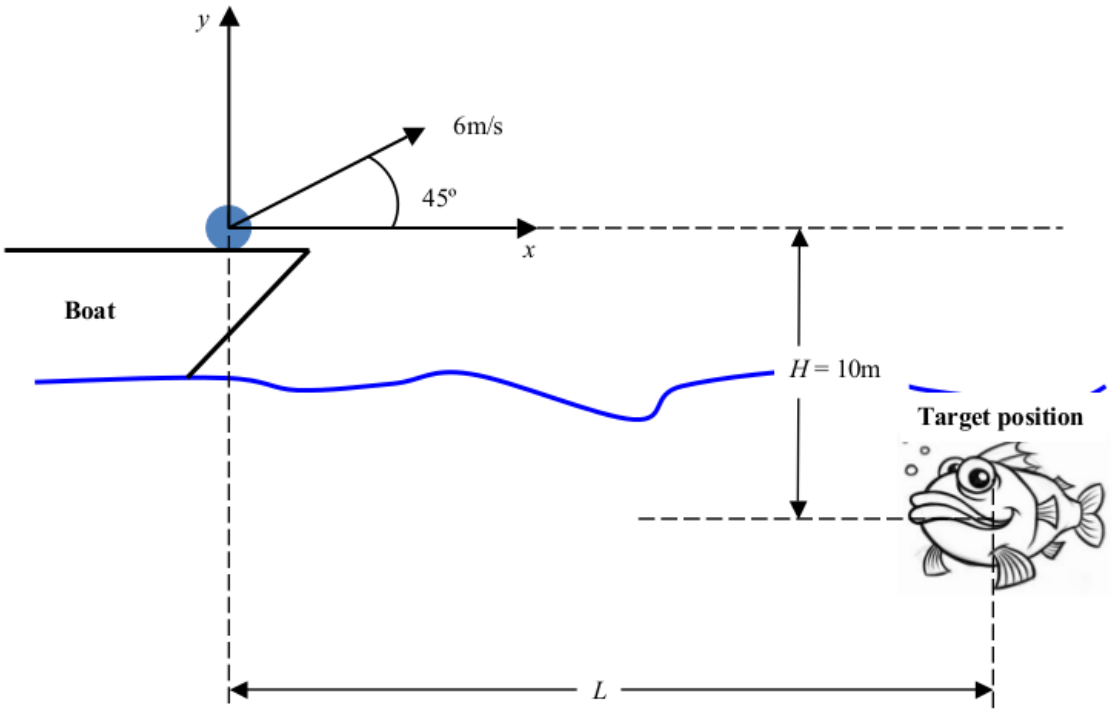
1. **[10 marks]** A cable between points E and F is raising a drawbridge as shown in the diagram below. The weight of the drawbridge is assumed to be a series of point loads acting at pin joints A, B, C and D with the forces as indicated.



(a) **[2 marks]** Sketch the free body diagram of the system described above.

(b) **[8 marks]** Using the 'Method of Joints', evaluate the force in each of the truss members EG, DE, DG, CD and GH. In order to get full marks, you need to specify whether each member is in 'Compression' or in 'Tension'.

2. **[10 marks]** The School of Engineering is organising an end-of-year fishing trip. Prof. Andrew Ooi and Dr. Gavin Buskes have decided to compete with each other and see who can catch the most fish for the day. In order to ensure victory, Andrew needs to calculate the distance that the fishing spear travels given an initial velocity and angle so that he can achieve a 100% chance of hitting the fish. To simplify his calculation, the fishing spear is modelled as a spherical particle as shown in the figure below.



The aerodynamic drag of the spherical particle is

$$F = \frac{C_D}{2}\rho V^2 A \tag{1}$$

where V is the velocity of the spherical particle, C_D is the coefficient of fluid/air drag, ρ is the density of the fluid/air and A is the spherical particle's frontal surface area. The mass of the object is m.

(a) [**4 marks**] Derive the governing equations that define the 2-dimensional motion of the spherical particle under the effect of fluid/air dynamic drag and gravity. Assume that the drag coefficients in air and sea water are the same and there is no effect as the spherical particle enters the sea water from air.

(b) **[4 marks]** Assume ZERO fluid/air dynamic drag in the set of ordinary differential equations that you derived in part (a). Further assume that the fish is modelled as a single point located $H = 10m$ below the lauch location, and let the spear be fired at an initial velocity of $6m/s$ at an angle of $45°$ to the horizontal.

Determine the flight time t_{flight} (i.e. how long it would take for the spear to hit the fish), the distance L the fish must be located at in order to hit it and the maximum height, y_{max}, that the particle reaches after being launched in the air.

More space is provided for your answer on the following page.

(c) **[2 marks]** Is it possible to point the spear gun downward and still able to catch the fish? Explain your answer.

END OF EXAMINATION