## COMP20003
## Algorithms and Data Structures
## Algorithms

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## Outline of the first few lectures

- Algorithms: general
- This subject: details
- Algorithm efficiency
- Computational complexity
- Data structures
  - Basic data structures
  - Algorithms on basic data structures
  - Complexity analysis of algorithms on basic ds's



## Revisit: What is an algorithm?

- A set of steps to accomplish a task.
- Computer algorithms must be:
  - Specific.
  - Correct.
  - Reasonably efficient.



## Algorithms and Efficiency

- An algorithm must:
  - be accurate (to within the required tolerance).
  - compute in a "reasonable" amount of time.
- The most accurate algorithm in the world is useless if it takes forever to compute.
  - We are particularly interested in efficiency as the size of the input grows.
- **Why?**

## Example algorithm: Compute Fibonacci numbers

- $F_n = F_{n-1} + F_{n-2}$
  - $F_0 = 0$
  - $F_1 = 1$

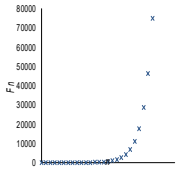
| n | F(n-1) | F(n-2) | F(n) |
|---|---|---|---|
| 0 | - | - | 0 |
| 1 | - | - | 1 |
| 2 | 1 | 0 | 1 |
| 3 | 1 | 1 | 2 |
| 4 | 2 | 1 | 3 |
| 5 | 3 | 2 | 5 |
| 6 | | | 8 |
| 7 | 8 | 5 | 13 |
| 8 | 13 | 8 | 21 |

- 0,1,1,2,3,5,8,13,21,34,55….



## Fibonacci numbers

- Fibonacci numbers grow very quickly:
  - $F_{10} = 55$
  - $F_{15} = 610$
  - $F_{20} = 6765$

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## Fibonacci numbers

- The original problem that Fibonacci was investigating (1202):
  - How fast can rabbits breed under ideal circumstances?
  - http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#Rabbits

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## How to compute Fibonacci numbers?

- Given the definition of Fibonacci numbers:
  - $F_n = F_{n-1} + F_{n-2}$
    - $F_0 = 0$
    - $F_1 = 1$
- Does this suggest an easy way to calculate $F_n$?

## Naïve Fibonacci algorithm

Definition:
$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

- Is the algorithm correct?
- How long does it take to compute?

## Fibonacci computation

- How to estimate computation effort:
  - Count operations.
  - Count operations as a function of input size.
  - Count operations as a proxy for time.
- *T(n)* = run time for input *n* ≈ *number of operations* for input *n*.
  - **Portable** between machines.
  - Can **compare** algorithms.

## Fibonacci computation

- Looking at *T(n)* as number of operations to calculate the nth Fibonacci number, then
  - *T(n) = T(n-1) + T(n-2) + 3 (operations)*
  - *T(1) = T(0) = 1*
- Example: unrolling the loop
  - T(4) = T(3) + T(2) + 3

## Fibonacci computation

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  - = T(2) + T(1) + 3 + T(2) + 3

## Fibonacci computation

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  - T(4) = T(3) + T(2) + 3
  - = T(2) + T(1) + 3 + T(2) + 3
  - = T(1) + T(0) + 3 + T(1) + 3 + T(1) + T(0) + 3 + 3
  - = 1+1+3+1+3+1+1+3+3 = 17 operations

## Fibonacci computation

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  - *T(n) = T(n-1) + T(n-2) + 3 (operations)*
  - *T(1) = T(0) = 1*
- Example:
  - T(5) = T(4) + T(3)
  - = 17 + T(2) + T(1) + 3
  - = 17 + 9
  - = 26

## Counting operations

- Count of operations as proxy for run time:
  - Advantages?
  - Caveats?
- How long does calculation of fib(2) take
  - Using the naïve algorithm?
- Do we care how long things take for small input *n*?

## Complexity analysis: general method

- Count operations for *T(n)* "time" (number of ops) taken for input *n*.
  - best to identify the most expensive operation and count that operation.
  - we can sometimes trade off space for time.



## Complexity analysis: a fine point for fib()

- Assumption:
  - addition of two numbers takes constant time.
  - True *if* both numbers can fit into one computer word: 32 bits, number < $2^{32}$.
- But Fibonacci numbers get very large:
  - $F_n$ takes approximately 0.694*n* bits, so
  - To fit in one word, n < 32/0.694 = 46
  - $F_{50}$ = 12,586,269,025 > 12*$10^9$ > $2^{33}$
  - Assumption not valid for large n.



## Closed form for Fibonacci numbers

- Binet's formula:

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

- For large *n*, $F_n \approx 2^{0.694n}$
- For more on Fibonacci numbers, *see :*
  - http://mathworld.wolfram.com/FibonacciNumber.html



## Complexity analysis: Intuition

- Fortunately, most algorithms do deal with smaller numbers.
  - Counting operations usually suffices.
- Always be aware of the assumptions:
  - What is the most expensive operation?
  - Are the operations really constant?
  - What are the inputs and outputs?

## Skiena: Algorithm Design Manual

- Chapter 1:
  - Algorithm correctness.
  - Example problems.
- Chapter 2:
  - Chapter 2.1: counting operations



## A small diversion: Which C standard?

- C standards:
  - ANSI C (C89)
  - C99 – "substantially" completely supported in gcc 4.5 (with `-std=C99` option on)
  - C11 (current C standard, from 2011) `gcc 4.8`
- On `nutmeg.eng.unimelb.edu.au:`
  - `gcc -v:`
  - `gcc version 4.4.7`



## C for gcc on nutmeg

- ANSI C with *some* of the features of C99, *e.g.*
  - Supported:
    - inline functions
    - long long int
  - Not supported:
    - Variable length arrays
    - Doesn't insist on explicit return type for function
- For all the new features in C99 see:
  http://www.open-std.org/jtc1/sc22/wg14/www/newinc9x.htm



## Next section

- Complexity analysis more formally.
- Big-O and related formalisms.