COMP20003
Algorithms and Data Structures
Mergesort

Nir Lipovetzky
Department of Computing and Information Systems
University of Melbourne
Semester 2

Quicksort: Summary

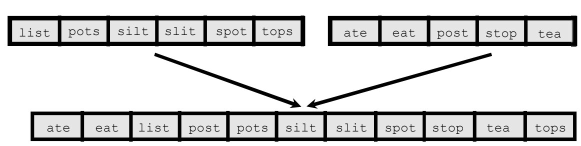
- http://www.youtube.com/watch?v=ywWBy6J5gz8

Mergesort

- Skiena Chapter 4.5

Merging

- We have two lists (stored as linked lists or arrays), each already in sorted order
- We would like to merge them into one sorted list that includes every element



How do you merge?

- 2 linked lists or arrays
  - Two pointers (or indices): to smallest element
  - Compare elements pointed to
  - Output smallest and move pointer
- How many comparisons?
- Code…

Merge Code

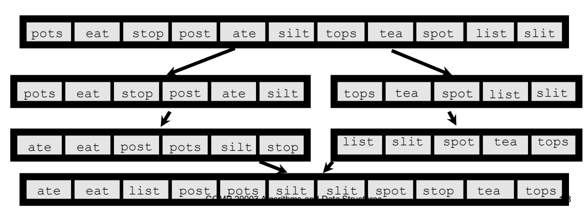
```
merge(item C[], item A[], item B[],
      int n, int m) /* n is size of A, m size of B */
{



}
```


Merge Code

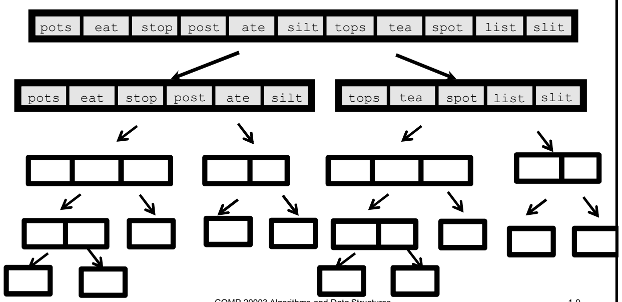
```
merge(item C[], item A[], item B[],
      int n, int m) /* n is size of A, m size of B */
{
   int i,j,k;
   for( i=0,j=0,k=0; k < n+m; k++ )
   {
     /* shortcut at the end of A or B*/
     if(i == n) { C[k] = B[ j++ ]; continue; }
     if(j == m) { C[k] = A[ i++ ]; continue; }
     if(A[i] <= B[j]) C[k] = A[ i++ ];
     else C[k] = B[ j++ ];
    }
}
```


Sorting using the merge operation

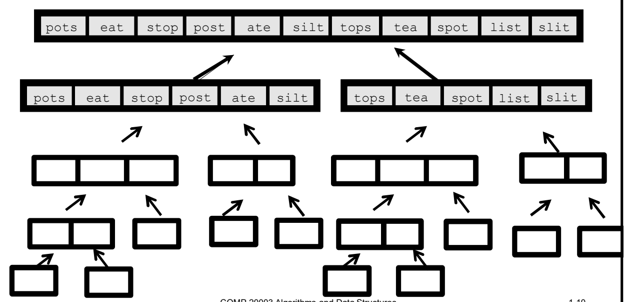
1. If list has one element, return
2. Split list into two equal-sized pieces (recursively, until singleton)
3. Sort each half
4. Merge two sorted halves



Mergesort: splitting



Mergesort: merging



Mergesort: topdown (recursive)

```
main() {/* code */ mergesort(A,0,n-1); /* more code */

mergesort(A,first,last)
{
        if( first < last){
                int i;
                item B[], item C[];
                mid = (int)(last-first+1)/2;
                for(i=0;i<mid;i++) B[i] = A[i];
                for(i=mid;i<=last;i++) C[i-mid] = A[i];
                B = mergesort(B,0,mid-1);
                C = mergesort(C,0,mid-1);
                A = merge(B,C);
        }
}
```


Analyzing mergesort

- We are concerned with:
  - Accuracy
    - Does mergesort work?
    - Is it stable?
  - Efficiency
    - Does it take extra space? How much?
    - Analyze time efficiency using recurrences

Recurrences

- Recurrence relation "mathematical defn":
  - an equation that recursively defines a sequence
  - each further term of the sequence is defined as a function of the preceding terms

Remember Fibonacci numbers (week 1)

Mergesort recurrences

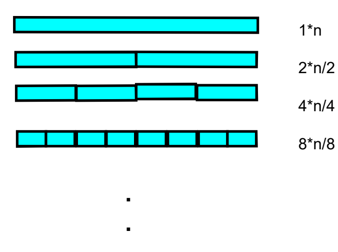
Recurrence for number of comparisons:

- Cost of sorting n items =
  - 2*Cost of sorting n/2 items + merge n items
  - $C(n) = 2C(n/2) + n-1$ (worst case)
  - $C(1) = 0$

Solving recurrence

- Approximate n as power of 2:
  - $C(n) = 2C(n/2) + n-1$
  - $= 2[2C(n/4) + (n/2-1)] + (n-1)$
  - $= 4C(n/4) + (n-2) + (n-1)$
  - $= 8C(n/8) + (n-4) + (n-2) + (n-1)$
  - … $\log_2 n$ splits
  - $2^{\log n}$ + n + n + …<log n times> n
  - ≈ ??

Intuition



Mergesort: Top-down (recursive)

- Top-down mergesort works well with arrays
  - Also with linked lists (pointer to find midpoint of list)
- Worst case O(   )
- Average case O(   )
- Stable?
- Requires O(   ) extra space

Bottom-up mergesort

- Break list into *n* singleton lists
- Insert single lists into a queue
- deQueue the first two items, merge them, and enQueue them

Merged Items go at the end

Mergesort: Implementation

- Top-down mergesort (recursive)
- Bottom-up mergesort (iterative)
- https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html

Mergesort: Analysis

- Analysis similar for recursive and non.
  - Θ( )
  - Stable ?
  - Reliable, and work with both arrays and lists
  - Can sort huge files on disk ☺
    - Use disk fetching just the portions of data you need
- Would be the perfect sort, except that:
  - Arrays require O(   ) extra space
  - Slower than quicksort ☹

Mergesort: Summary

- http://www.youtube.com/watch?v=XaqR3G_NVoo

Analyzing iterative mergesort

Running time is a bit tricky

- General principle:
  - there are $n/2^i$ lists of length 2^i
- Takes roughly n time to merge them all
- Since i runs from 0 to (log n) − 1

again we have *???* running time