COMP20003
Algorithms and Data Structures
Dictionaries and Data Structures

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So far…

- We have:
  - Looked at algorithms, fast and slow.
  - Estimated computation time by counting operations.
  - Formalized a system for classifying algorithm efficiency.

Outline of the first few lectures

- Algorithms: general
- This subject: details
- Algorithm efficiency: intuitive
- Computational complexity
- Data structures
  - Basic data structures
  - Algorithms on basic data structures
  - Complexity analysis of algorithms on basic ds's

Textbook

- Skiena: Chapter 3, Data Structures

This section

- A lightning tour of fundamental data structures used for search:
  - Arrays
  - Linked Lists
  - Trees

Abstract Data Types *vs.* Data Structures

- *Abstract* data type: what it does
  - Stack, queue.
  - Dictionary: look up by key.
  - Does not specify an implementation
- *Concrete* data structure:
  - Array, linked list, tree
  - Implements an abstract data type.

Data structures

- Organizing data is important
- It is helpful to organize with the task in mind
- For searching, e.g.:
  - Some high-level languages have inbuilt “dictionaries”, or associative arrays (Python, awk).
  - In lower-level languages, dictionaries are implemented directly using a fundamental data structure.

Searching

- Search Question:
  - Given a search key,
  - Find the record(s) that correspond to this key.
  - Typically we describe record simplistically with fields key and info (or just key).
- Examples:
  - Students and seat numbers, Telephone books

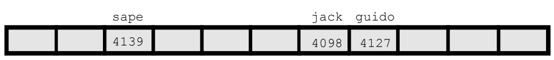
How you organize data is important for search!

Dictionaries

- Python built-in dictionary structure.

```
>>> tel = {'jack': 4098, 'sape': 4139}
>>> tel['guido'] = 4127
>>> tel
{'sape': 4139, 'guido': 4127, 'jack': 4098}
>>> tel['jack']
4098
```

- The dictionary is implemented using one of the underlying data structures.



Outline of the first few lectures

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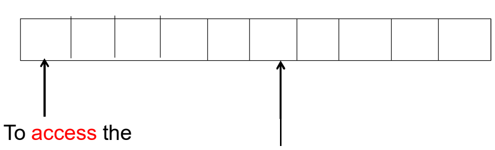
Array

- Def: given an index (location), we can retrieve any item in unit time.
- If items are in arbitrary order, finding a key in array of size n requires $O(n)$ time.

Mimics the structure of random access memory (RAM), where the index is the memory address

Arrays in C (lightening review)

```
int  A[10];
```



To access the value of the first element of the array:

```
A[0]
*A or *(A+0)
```

To access the value of the sixth element:

```
A[5]
*(A+5)
```

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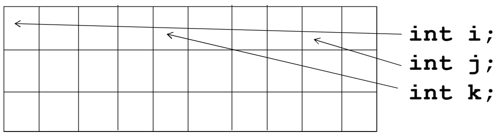
Random Access Memory



source: https://jdoodle.com/a/4eW

```
#include <stdio.h>
int main()
{
        int i,j,k;
        i = 5; j = 10; k = 15;
        printf("i: value = %d; address = %d\n", i, &i);
        printf("j: value = %d; address = %d\n", j, &j);
        printf("k: value = %d; address = %d\n", k, &k);
}
```

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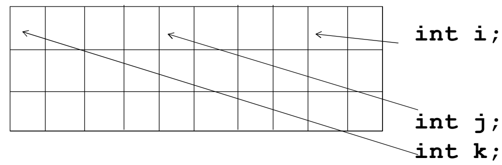
Random Access Memory




```
i: value =  5; address = 134509652
j: value = 10; address = 134509648
k: value = 15; address = 134509644
```

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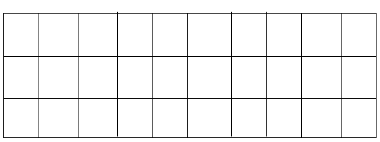
Random Access Memory

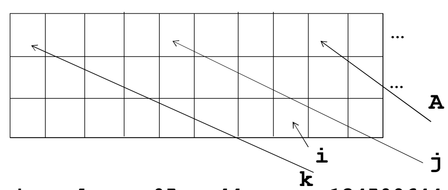


source: https://jdoodle.com/a/4eX

```
int i;
int  A[100];
int j,k;
i = 5; j = 10; k = 15;
printf("i: value = %.2d; address = %d\n", i, &i);
printf("A:           ; address = %d\n", A );
printf("j: value = %.2d; address = %d\n", j, &j);
printf("k: value = %.2d; address = %d\n", k, &k);
```

1-15

Random Access Memory

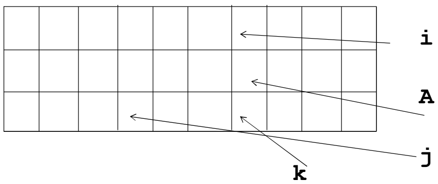



```
i: value = 05; address = 134509644
A:           ; address = 134509232
j: value = 10; address = 134509228
k: value = 15; address = 134509224

Element address =
  Addr Base + index * element_size
```

1-16

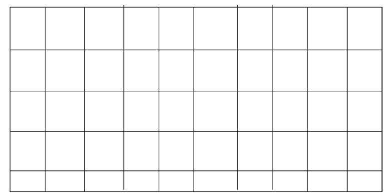
Random Access Memory



Although we often draw RAM to *look* like a 2-dimensional array,

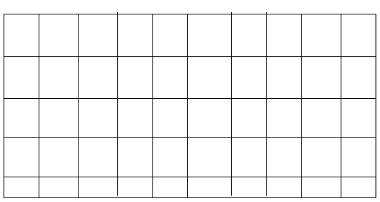
- *it is actually a linear (1-dimensional) array.*

A 2-Dimensional Array in C



- int A[rows][cols]
- int A[5][10]

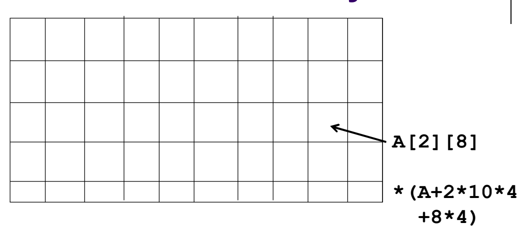
A 2-Dimensional Array in C



- int A[5][10]

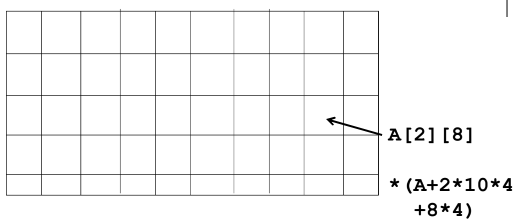
Actually, **A** is an array of size 5,
each element of **A** is an array of 10 ints.

A 2-Dimensional Array in C



- Address of element[thisrow][thiscolumn] =
  Address Base + thisrow * num_cols * sizeof(element)
  + thiscol * sizeof(element)

A 2-Dimensional Array in C



- Address of element[thisrow][thiscolumn] =

  Address Base + thisrow * num_cols * sizeof(element)
  + thiscol * sizeof(element)

- num_cols*sizeof(element) = size of one row

Homework

- What is the difference between:
  - `int a[10][20];`
  - `int *b[10];`
- ?

Back to arrays as dictionaries

- To sort or not to sort?
- How to determine which is best?

Sorting: fine points

- Sorting assumes the keys are “sortable”, *i.e.* comparable. E.g.:
  - *Comparable:* categories,
  - Not *Comparable*: colours, unless you associate an identifier.
- The CS definition of sorting means to put things into a *well-defined* order.
  - Other ways of sorting without comparing keys: counting sort. (next lectures)

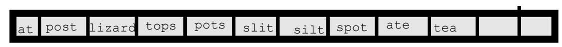
Sorting: fine points

- In CS applications of sorting, there is a *key*, and associated *information*.
- We sort by *key*, and the *information* comes along for the ride.
  - Student database, sorted on student ID.
  - Information: name, address, degree, *etc*.
- In our examples, we often do not show the *information* explicitly.

1-25

Unsorted arrays

- Just put the item at the end of the array:
  - Insertion is in $O(1)$



- How many comparisons you need to insert?

Unsorted array: search

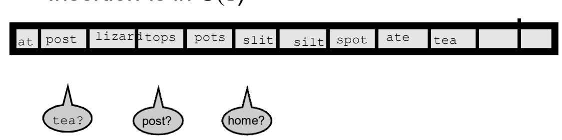
```
int i, cmps=0;   int EMPTY = 0;  int searchkey = 7;
int A[10] = {1,2,3,4,5,6,7,8,9,0};

for(i=0; A[i]!=EMPTY; i++)   /*A[] is an array of integers */
{
   cmps++;
   if (A[i] == searchkey)
   {
      printf("Found key %d at position %d\n", searchkey,i);
      printf("Key comparisons: %d\n", cmps);
      printf("FOUND");
      return 0;
   }
}
```

Source: https://jdoodle.com/a/4f1

Unsorted arrays

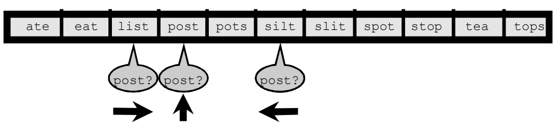
- Just put the item in at the end of the array:
  - Insertion is in $O(1)$



- What is the complexity of search?
  - O(?)

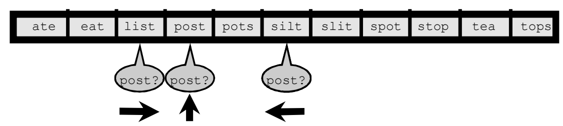
Sorted arrays

- How do you search for “post”?



Sorted arrays

- Binary search in sorted array:



Sorted array: binary search

Source: https://jdoodle.com/a/4f3

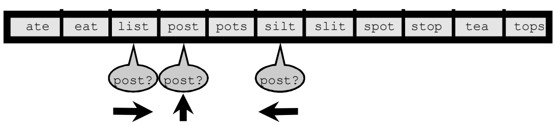
```
int start = 0;
int end = ARRAYSIZE-1;
while(start <= end)
{
   cmps++;
   mid = (start + end ) / 2; //DIVIDE: Get the middle position
   if(A[mid]==searchkey)
   {
      printf("Found key %d at position %d, comparisons: %d \n",searchkey,start,cmps);
      printf("FOUND");
      return 0;
   }
   //If middle postion smaller than key, then update the end, discard A[mid,..,end]
   if (searchkey < A[mid])
      end = mid-1;
   else // otherwise, updat the beggining and discard A[start,..,mid]
      start = mid+1;
}
```


Searching: analysis

- Unsorted array: O(?)
- Sorted array (binary search): O(?)

Sorted arrays

- Binary search in sorted array:
  - Search is in O(logn)



  - What about insertion?

Sorted arrays: insertion

```
int i = ArrayFill-1;
// Shift values to the right until finding the correct position,
while(A[i]>INSERTNUM && i>=0)
{
   A[i+1] = A[i];
   i--;
}
/** only get here when A[i] <= INSERTNUM, i.e. have already moved
* previous contents
**/
A[i+1] = INSERTNUM;

/* to accomodate the new item */
ArrayFill++;

/* Any Assumptions */
```

Source: https://jdoodle.com/a/4fj

Sorted and unsorted arrays

Key comparisons:

- usually the most expensive operation in searching.

Analysis

- Search:
  - Unsorted array:
  - Sorted array (binary search):
- Insert:
  - Unsorted array:
  - Sorted array:

Search: m lookups on n items

- Unsorted array, linear search:
  - n insertions @ ? operation → n operations, O(?)
  - m lookups @ ? operations → m*? (worst case)
  - O(n + m*?) = O(?)
- Sorted array, binary search:
  - n insertions @ ? comparisons and ? data movements each → $O(n^2)$
  - m searches @ ? comps each -> m*?
  - O(?)

Search: m lookups on n items

- Unsorted array, linear search:
  - ?
- Sorted array, binary search:
  - ?
- For m << n, unsorted arrays are better!
- But usually m > n, so use sorted array
  - Or even something better!?

Something better?

- What are the worst properties of a sorted array?

Limited size

- We can overcome the limited size problem using dynamic memory allocation
- C library functions:
  - void *calloc(size_t nobj, size_t size)
  - void *realloc(void *p, size_t size)
  - also, of course void *malloc(size_t size)
  - All defined in `stdlib.h`

malloc(): size_t

- malloc(size_t size)
- size_t is:
  - an unsigned integer type
  - the type returned by the sizeof operator
  - widely used in the standard library (**stdlib**) to represent sizes
- *e.g.* malloc(**sizeof**(int))

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malloc() example (part 1)

```
#define NUMBER 5
int main (argc, argv)
{
        int   var;
        var = NUMBER;
        printf("%d - %d\n", &var, var);
        return 0;
}
```

```
>a.out
134509940 - 5
```

1-42

malloc() example: (part 2)

```
#include<stdlib.h>
#include<stdio.h>
#define NUMBER 5
int main ()
{
   int* ptr;
   ptr =  (int *)malloc(sizeof(int));
   *ptr = NUMBER; /* note '*' */
   printf("%d - %d\n", ptr, *ptr);
   return 0;
}
>a.out
134613280 - 5
```

Source: https://jdoodle.com/a/4fjm

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malloc():check return value

- Be aware: malloc() can fail!
  - If malloc() fails, it returns NULL
  - Never use a pointer to something where the memory allocation has failed!

```
int* B;
B = (int*) malloc( NUMBER * sizeof(int));
/* always check return value of malloc()*/
If( B == NULL )
{
     printf("malloc() error\n");
     exit(1);
}
```

→ write a function safemalloc() that does this

Getting memory for an array using malloc()

```
int A[NUMBER];
/**
 * while insertions < NUMBER array is OK
 * BUT… has a limit
 */
int* B;
/* always check return value of malloc()*/
if( (B = (int *) malloc( NUMBER * sizeof(int) )) == NULL )
{
        printf("malloc() error\n");
        exit(1);
}
/**
 * B can now be used like A
 * better to use calloc(NUMBER,sizeof(int))
 */
```

1-45

Getting memory for an array using calloc()

```
int* B;
/* always check return value of malloc()*/
if( (B = (int*) calloc( NUMBER, sizeof(int) )) == NULL )
{
        printf("malloc() error\n");
        exit(1);
}

/* B now comes with each slot initialized to 0 */
```

1-46

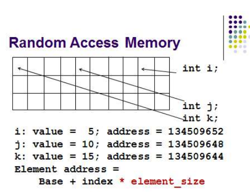
realloc()

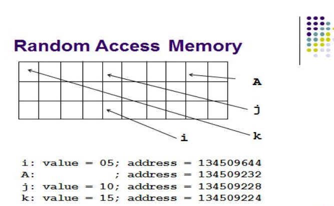
```
/**
 * as previously, used malloc(),calloc()
 * RESIZE when insertions == NUMBER
 */
B = realloc( B, ( NUMBER * 2 ) * sizeof(int) );
/* should also check realloc() for NULL*/

/* now initialize new part of array */
for( i = NUMBER; i < NUMBER * 2; i++ )
        B[ i ] = NULL;
/* now we have a bigger array, first half copied from the old B  */
```

1-47

Details about malloc() and friends

- malloc**()** returns a pointer to a place in memory.
- Argument to malloc**()** specifies how much space to reserve in memory





What's a pointer?

- A pointer is an address in memory
- What is the output of this code?

```
int* ptr;
ptr = (int *)malloc(sizeof(int));
*ptr = 5;
printf("%d, %d", ptr, *ptr);
```

- Now what is the output of *this* code? Try it:https://jdoodle.com/a/4fP

```
int* ptr;
ptr = (int*) malloc(sizeof(int));
ptr = 5;
printf("%d, %d", ptr, *ptr);
```

- Int* p; or int *p; ? What the inventor of C++ thinks:
  - http://www.stroustrup.com/bs_faq2.html#whitespace

1-49

Details about malloc() and friends

- #include<stdlib.h>
- Read the documentation for fine points:
  - `malloc()` returns uninitialized space
  - `calloc()` returns space initialized to 0
  - `realloc(void *p, size_t size)` returns space where the start is copied from p and the rest is unintialized.
- Check return value of all memory alloc functions!

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malloc() and free()

- `malloc()` allocates memory.
- `free()` use to deallocate memory

```
void* ptr;
 ptr = malloc(NUMBER_OF_BYTES);
 /* do things until finished with the
    contents pointed to by ptr */
 free(ptr);
```

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Back to sorted arrays…

- Space limitations:
  - Can use `realloc()`.
  - Or can use linked list (sorted linked list).

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More about Pointers

- For an excellent exposition of pointers in C, see the excellent tutorial by Ted Jensen:
  - LMS Resources → Pointers and Arrays in C

101 on Pointers

- Declaration: pointer * in C is an address
- Operators:

*	Value at Operator (dereferencing)	Gives Value stored at Particular address
&	Address of Operator	Gives Address of Variable

```
int k;
int* ptr;

k=5;
ptr = &k;
printf("%d", *ptr);
```

<5>

101 on Pointers

- A pointer in C is an address.
- When A is the name of an array, A is a pointer to the array, and
  - A[0] is equivalent to *(A+0),
  - A[5] is equivalent to *(A+5)…

Memory Allocation: Summary

- `malloc(),calloc(),and realloc()` return:
  - The (untyped) address of allocated memory;
  - *i.e.* a pointer to allocated memory.

```
/*allocates just enough room for an address */
struct node* ptr;
/* allocates enough room for the node */
ptr = (struct node*) malloc(sizeof(struct node));
```

We have discussed the strong and weak points of sorted arrays as data strucutres for search

Linked lists: flexibility, but overheads

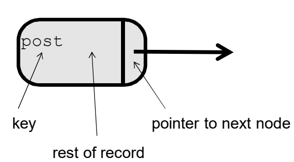
- In a linked list:
  - each item (or key) is located in an arbitrary place in memory,
  - with a link (pointer) to the next item
- Search Operations:
  - If unsorted, finding item is still $\Theta(n)$-time.
  - Once insertion point has been determined, easy to insert (or delete) a new item, by rearranging links.

Linked lists: flexibility, but overheads

- Takes extra space for each item in the list.
- Takes extra time to allocate the memory for the node for each item.

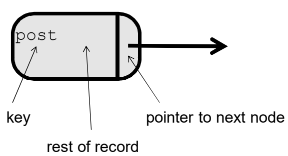
The node

```
struct node{
    record r;
    struct node *next;
    };
```



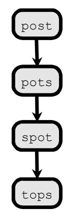
The node

```
typedef struct node{
        record r;
        struct node *next;
        } node_t;
```



1-61

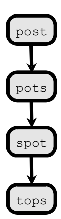
A linked list of nodes




```
struct node
{
   char* key;
   char* info;
   struct node* next;
};

struct node* newnode;

newnode = /*malloc space and put in the key and info */

/* suggested declaration style for beginner and intermediate */
```

1-62

A (sorted) linked list of nodes




```
/* record struct contains key and other info */

typedef struct node{
        record r;
        struct node* next;
        } node_t;

typedef node_t* node_ptr;

node_ptr   newnode;

/* more advanced style */
```

1-63

Traverse the list

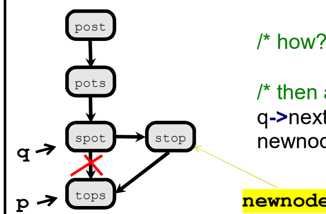
```
p = listhead;
if(p!=NULL){ /* empty list */
   while( p->next !=NULL)
   {
      printf("%d\n", p->key);
      p = p->next;
   }
   printf("%d\n",p->key);
}
```

1-64

Traverse the list

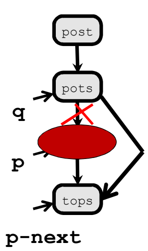
```
p = listhead;
if(p==NULL)
{
   printf("List empty\n");
   return;
}

/* traverse and print key */
while( p->next !=NULL)
{
   printf("%d\n", p->key);
   p = p->next;
}
printf("%d\n",p->key);
```


Inserting a new node into a sorted linked list




```
/* find correct place in list to insert */

/* how? */

/* then adjust pointers*/
q->next = newnode;
newnode->next=p;
```


Deleting a node




```
/* p points to the node we want to delete */
q->next = p->next;
free(p);
```

Fill the missing code: https://jdoodle.com/a/4fT

Linked lists: sorted *vs.* unsorted

- What are the advantages of keeping a linked list in sorted order?
- What are the disadvantages?

Search: Arrays vs. Linked Lists

- Sorted arrays:
  - Fast search (binary search), but
  - Slow insertion (keeping sorted order)
- Sorted array:
  - Fixed size, but
  - Can grow with realloc()
- Array needs (in general) only 1 memory allocation
  - Linked list needs many

Table of "running times"

	One Search	One Insert
Unsorted array		
Sorted array		
Unsorted linked list		
Sorted linked list		

Exercize

How many operations are needed for *m* searches in a dictionary of *n* items?

	Each Insertion	Each Search	Build + Search
Unsorted array			
Sorted array			
Unsorted linked list			
Sorted linked list			

Practical complexity and algorithms

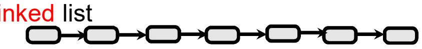
- O(1): Execute instructions once (or a few times), independent of input
  - Example: pick a lottery winner
- O(log n): keep splitting the input, and only operate on *one* section of the input.
  - Example:
- O(n): Execute instruction(s) once for each item:
  - Example:

Practical complexity and algorithms

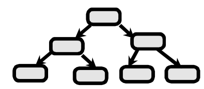
- O(n log n): split the input repeatedly, and do something to *all* the segments
  - Example: Many sorting algorithms
- $O(n^2)$: For each item, do something to all the others. (Nested loops.)
  - Example:
    - Note: getting slow for large data…
- $O(n^3)$:
- $O(2^n)$:

Breaking out of linearity

- Compare:
  - Linked list



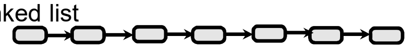
  - Binary tree



- If we reliably know whether the desired item is in the left subtree or the right subtree, we could find it more quickly!

Breaking out of linearity

- Compare:
  - Linked list



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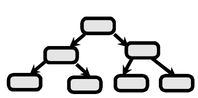
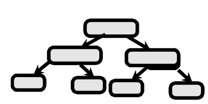


- Note: for a complete binary tree, half the nodes are at the bottom level…

What is a complete binary tree?

- A binary tree is **complete** if **every level, except** possibly the **last**, is:
  - completely filled, and
  - **all nodes are as far left** as possible.



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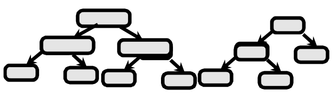

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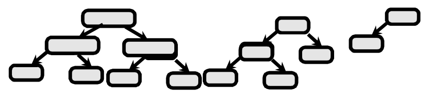

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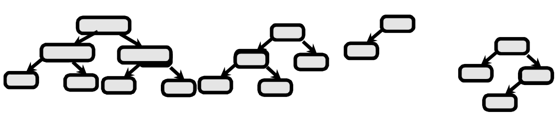
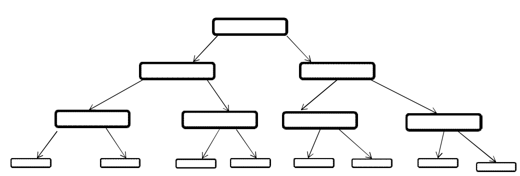


complete binary tree

Nodes

2^0



2^1

2^2

2^3

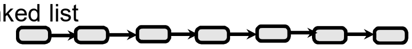
A complete binary tree of n nodes has depth approximately $\log_2 n$

….

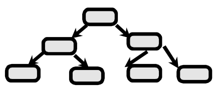
$\sim 2^{\log n}$

Breaking out of linearity

- Compare:
  - Linked list

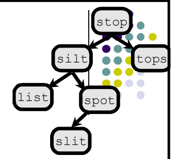


  - Binary tree



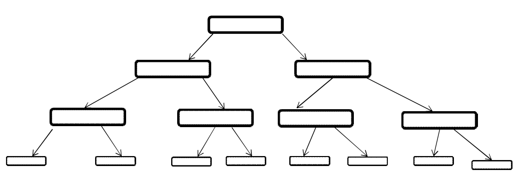
- As we will see, both insertion and search are O(log *n*) operations.

How a binary search tree work?



- In a sorted linked list:
  - `next` links to a record with a key ≥ this one.
- In a BST
  - `left` links to items with key < current key
  - `right` links to items with key ≥ current key.
- Insert node with key `slit` in this tree.

Looking at a complete binary tree



Nodes

2^0

2^1

2^2

2^3

….

$2^{\log n-1}$

For a complete binary tree of n nodes, to get to the bottom takes not more than $\log_2 n$ key comparisons

linked list, to get to the end, we need how many comparisons?

Binary tree exercizes

- Put the following numeric keys into a bst:
  - 45, 37, 86, 90, 50, 16, 37
  - How long (how many key comparisons) does it take to search for key=5?
- Put the following numeric keys into a bst:
  - 90, 86, 50, 45, 37, 32, 16
  - How long does it take to search for key=5?

Best case run time in bst: Perfectly balanced tree

- Best case for BST: perfectly balanced
- Height of tree with n items: $\log_2 n$
- Path from root to any node:
  - Maximum length: $\log_2 n$
  - Average length: $\log_2 n$
- Insertion/search/deletion are all O(logn) for a well-balanced tree

Worst case run time in bst: Stick

- Worst case for BST: a stick.
  - e.g. when items are inserted in sorted order.
  - The BST degenerates to a linked list!
- Height of tree with n items: n
- Path from root to any node:
  - maximum length: n
  - average length: n/2
- Insertion/search/deletion are O(n)!

Binary search trees

- Deletion?

Something to think about

- Why don't we just randomize the order of the items we insert into the binary search tree, to prevent worst case behavior?

Binary search trees

- Good average case behavior – logn
- Bad worst case behavior – n
- So overall BST O(n).
  - Actual behavior usually not linear
  - But potentional linearity
- Balanced trees: AVL, red-black; 2,3,4; B+tree.

Dictionaries: Summary

- We have looked at various underlying data structures for implementing dictionaries:

Dictionaries: Summary

- We have analyzed the computational complexity for these data structures:

Dictionaries: Summary

- So far the best we have done is log n search, where either:
  - Insertion is O(n); or
  - O(log n) average case but O(n) worst case.
- We can do better…

Next section

- Balanced trees.