COMP20003
Algorithms and Data Structures
Deletion from BST

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Binary search trees: Deletion

- Deletion?
- Deletion from a BST involves;
  - the in-order predecessor; or
  - the in-order successor
- In-order successor and in-order predecessor can be obtained from in-order traversal

Traverse

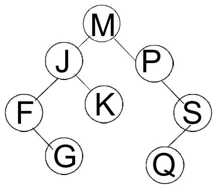
- Visit every node once
- Do something during the visit:
  - Print node value, or
  - Mark node as visited or
  - Check some property of node
- Use in any linked data structure
  - Tree
  - Graph
  - List

Traversal: recursive In-order traversal, tree

```
traverse(struct node *t)
{
        if(t!=NULL)
        {
            traverse(t->left);
            visit(t);
            traverse(t->right);
        }
}
```

Exercize

- Trace recursive in-order tree traversal on the following tree, with visit(t) as print.



In-order traversal, Application:

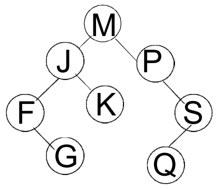
- For a binary search tree, an in-order traversal prints all nodes in:
  - key-order

Post-order Traversal

```
traverse(struct node *t)
{
       if(t!=NULL)
       {
           traverse(t->left);
           traverse(t->right);
           visit(t);
       }
}
```

Exercize

- Trace recursive post-order tree traversal on the following tree, with visit(t) as print.



Post-order traversal, Application:

Free all nodes in tree (free left and right nodes before freeing current node)

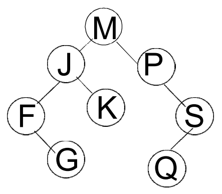
Can't free a tree by just freeing the root!

Pre-order Traversal

```
traverse(struct node *t)
{
       if(t!=NULL)
       {
           visit(t);
           traverse(t->left);
           traverse(t->right);
       }
}
```

Exercize

- Trace recursive pre-order tree traversal on the following tree, with visit(t) as print.

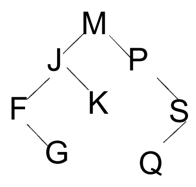


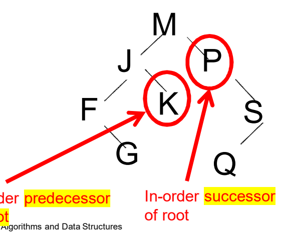
Pre-order traversal, Application:

Can be used to Copy a tree

In-order traversal, Application:

- For a binary search tree, an in-order traversal prints all nodes in key-order

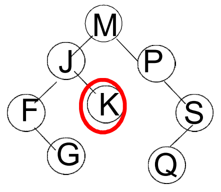




In-order successor and in-order predecessor

In-order **predecessor** of root M is **rightmost** node of **left** subtree.

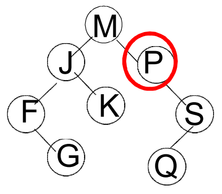
In-order traversal: FGJKMPQS



In-order successor and in-order predecessor

In-order **successor** of root M is **leftmost** node of **right** subtree.

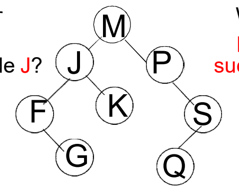
In-order traversal: FGJKMPQS



In-order successor and in-order predecessor

Every node has a predecessor (just before) and a successor (just after):

What are in-order predecessor and successor of node J?
G and K



What are in-order predecessor and successor of node P?
M and Q

In-order predecessor and in-order successor

- Just before (or after) in in-order traversal
  - Rightmost node in the left subtree; or
  - Leftmost node in the right subtree

Deletion from bst (finally)

Step 1: find the node to be deleted
Step 2: delete it!

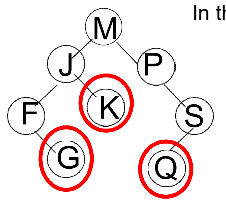
Three cases for deletion:

- Case 1: Node is a leaf
- Case 2: Node has *either* a left *or* right child, *not both*
- Case 3: Node has *both* a left child *and* a right child

Case 1: Node is a leaf

Just delete the node

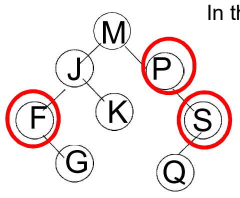
In this example: G, K, Q



Case 2: Node has *one* child

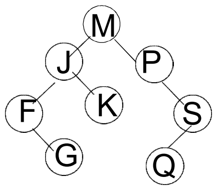
Replace node with the child

In this example: F, S, P



Case 3: Node has *two* children

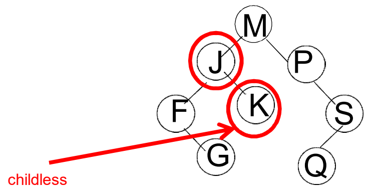
In this example: M, J



Case 3a: Node has *two* children, but…

… one of these children has no children

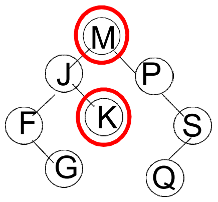
Replace node with the childless child



In this example: J, replaced by K

Case 3b: Node has *two* children, both have children

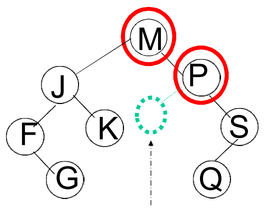
Replace node with *either* in-order successor or in-order predecessor.



In this example: M

Case 3b: Node has *two* children, both have children

Replace node with *either* in-order successor or in-order predecessor.



(if P had a left child, that node would be the In-order successor of M, so would replace M with that node)

Deletion from bst:

Step 1: find the node to be deleted.

Step 2: delete it!

- Replace the deleted node with:
  - Case 1: Node is a leaf: nothing
  - Case 2: Node has *either* a left *or* a right child, but *not both*: the single child
  - Case 3: Node has *both* a left child *and* a right child: in-order predecessor or successor.

Deletion from bst: Analysis

- Worst case:
  - Time to find the node: O(n)
  - Time to find the in-order predecessor or successor: O(n)
  - Total time: O(n)
- Average case:
  - Time to find the node: O(log n)
  - Time to find the in-order predecessor or successor:
    - O(n) if implemented using in-order traversal
    - O(log n) if implemented using specific call: FindMin in leftTree or FindMax in RightTree
  - Total time: O(n) or O(log n), depending on implementation