MAST10007 (Summer) Assignment 4 Due Monday February 1st 1.30pm

Please leave your assignment in your tutor's box located near the north entrance to the Richard Berry building.

1. It is known that

$$\begin{bmatrix} -10 & 3 & 8 & 13 & -104 \\ -2 & 1 & 3 & 3 & 65 \\ -1 & 1 & 0 & 2 & -78 \\ 2 & 2 & 1 & 0 & 143 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 & 58 \\ 0 & 1 & 0 & 1 & -20 \\ 0 & 0 & 1 & 0 & 67 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   Let the matrix on the left be denoted A.

   (a) What is the dimension of the row space of A?

   (b) Write down a basis for the row space of A, and state the piece of theory being used.

   (c) Write down a basis for the column space of A.

   (d) Do the vectors $(-10,-2,-1,2),(3,1,1,2),(8,3,0,1),(13,3,2,0)$ span $\mathbb{R}^4$? Give a reason.

   (e) What is the dimension of
   $\text{Span}\{(-10,-2,-1,2),(3,1,1,2),(13,3,2,0)\}$?

   (f) Write $(13,3,2,0)$ as a linear combination of $(-10,-2,-1,2)$ and $(3,1,1,2)$.

   (g) Find a basis for the solution space of A.

2. (a) A matrix is said to be antisymmetric if $A^T = -A$. Show that a general 2×2 antisymmetric matrix is of the form

$$\begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}$$

   (b) Let the set of 2×2 antisymmetric matrices be denoted S. By making a correspondence between elements of $M_{2,2}$ and four-tuples (a,b,c,d), write the subset of $\mathbb{R}^4$ corresponding to S as a span. What is the dimension of this subspace?

   (c) Show from first principles that the set S is closed under scalar multiplication.

3. Let the standard matrix representation of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$A_T = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

   (a) Compute the image of the unit vectors $\mathbf{e}_1 = (1,0)$, $\mathbf{e}_2 = (0,1)$.

   (b) Compute the image of the unit square $\{(x,y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

   (c) Describe T geometrically.