Tutorial 1: Solutions

Q1. (a) Linear: All the variables appear to the first power and multiplied by constants.

(b) Non Linear: In the second and third equations the variables are multiplied together.

(c) Linear: The data gives equations such as

$$3 = \alpha + 10\beta + 100\gamma + 1000\delta.$$

All the variables appear to the first power and multiplied by constants.

Q2. (a) $\left[\begin{array}{cc|c} -2 & -1 & 44 \\ 5 & 8 & -22 \end{array}\right]$

(b) $\left[\begin{array}{cc|c} -2 & -1 & 44 \\ 5 & 8 & -22 \end{array}\right] \begin{array}{l} \\ R_2 + \frac{5}{2}R_1 \end{array} \sim \left[\begin{array}{cc|c} -2 & -1 & 44 \\ 0 & \frac{11}{2} & 88 \end{array}\right]$

(Other row echelon forms are also possible.)

(c) From the second line of the row echelon form we read off that

$$\frac{11}{2}y = 88 \quad \Rightarrow \quad y = 16.$$

Using this in the first row gives

$$-2x - y = 44 \quad \Rightarrow \quad -2x = 44 + y = 44 + 16 = 60 \quad \Rightarrow \quad x = -30.$$

Q3. Let

$$\begin{aligned} A &= \text{number of truffles} \\ B &= \text{number of torrone} \\ C &= \text{number of fudge.} \end{aligned}$$

From the given information

$$\begin{array}{lrcl} \textit{dark} & 75A + 50B + 150C & = & 650 \\ \textit{plain} & 75A + 170B + 80C & = & 560 \\ \textit{white} & 75A + 240B + 10C & = & 420 \end{array}$$

To solve these equations write as an augmented matrix and reduce to row-echelon form.

$$\left[\begin{array}{ccc|c} 75 & 50 & 150 & 650 \\ 75 & 170 & 80 & 560 \\ 75 & 240 & 10 & 420 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \end{array} \sim \left[\begin{array}{ccc|c} 75 & 50 & 150 & 650 \\ 0 & 120 & -70 & -90 \\ 0 & 190 & -140 & -230 \end{array}\right] \begin{array}{l} \\ \\ R_3 - \frac{19}{12}R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 75 & 50 & 150 & 650 \\ 0 & 120 & -70 & -90 \\ 0 & 0 & -\frac{175}{6} & -\frac{175}{2} \end{array}\right]$$

Using back substitution:

$$-\frac{175}{6}C = -\frac{175}{2} \quad \Rightarrow \quad C = 3$$

$$120B - 70C = -90 \quad \Rightarrow \quad 120B = -90 + 70C = 120 \quad \Rightarrow \quad B = 1$$

$$75A + 50B + 150C = 650 \quad \Rightarrow \quad 75A = 650 - 50B - 150C = 150 \quad \Rightarrow \quad A = 2$$

Hence there were 2 Truffles, 1 Torrone and 3 Fudge made.

Q4. (i). This system is consistent. The number of non-zero rows equals the number of variables so there is a unique solution. Using back substitution, with the variables called (x, y, z).

$$z = 2$$

$$y - z = -1 \quad \Rightarrow \quad y = 1$$

$$x + 2y + 3z = 0 \quad \Rightarrow \quad x = -2 \times 1 - 3 \times 2 = -8$$

(ii). This system is inconsistent, due to the final row. There is no solution.

(iii). This system is consistent. The number of non-zero rows is less than the number of variables so there are infinitely many solutions. Let the variables be called w, x, y, z. The leading entries correspond to w and y and so x and z are not determined. Set

$$\begin{aligned} z &= t \\ x &= s \end{aligned}$$

The non-zero lines give

$$\begin{aligned} y + 2z = -1 &\quad\Rightarrow\quad y = -1 - 2t \\ w + 2x + 2y - z = 7 &\quad\Rightarrow\quad w = -2s + 5t + 9 \end{aligned}$$

Solution set $\{(w, x, y, z) = (-2s + 5t + 9, s, -1 - 2t, t),\ s, t \in \mathbb{R}\}$

(iv). This system is consistent. The number of non-zero rows is less than the number of variables so there are infinitely many solutions. Let the variables be called x, y, z. The leading entries correspond to x and y and so z is not determined. Set $z = t$. The non-zero lines give

$$\begin{aligned} y - 3z = 1 &\quad\Rightarrow\quad y = 1 + 3t \\ x + 2y + 2z = 0 &\quad\Rightarrow\quad x = -2 - 6t - 2t = -2 - 8t \end{aligned}$$

Solution set $\{(x, y, z) = (-2 - 8t, 1 + 3t, t),\ t \in \mathbb{R}\}$

Q5. (i).

$$\left[\begin{array}{ccc|c} 2 & -4 & 4 & 12 \\ 3 & 1 & -8 & 4 \\ -5 & 11 & k & -32 \end{array}\right] \begin{array}{l} \\ R_2 - \frac{3}{2}R_1 \\ R_3 + \frac{5}{2}R_1 \end{array} \sim \left[\begin{array}{ccc|c} 2 & -4 & 4 & 12 \\ 0 & 7 & -14 & -14 \\ 0 & 1 & k+10 & -2 \end{array}\right] \begin{array}{l} \\ \\ R_3 - \frac{1}{7}R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 2 & -4 & 4 & 12 \\ 0 & 7 & -14 & -14 \\ 0 & 0 & k+12 & 0 \end{array}\right]$$

(ii). For $k \neq -12$ the number of non-zero rows equals the number of variables, so there is a unique solution. For $k = -12$ the number of non-zero rows is less than the number of variables, so there are an infinite number of solutions.

(iii). For the case $k = -12$, the leading entries correspond to x and y so set $z = t$ and then

$$\begin{aligned} 7y - 14z = -14 &\quad\Rightarrow\quad y = 2t - 2 \\ 2x - 4y + 4z = 12 &\quad\Rightarrow\quad 2x = 12 + 8t - 8 - 4t = -4 + 2t \\ &\quad\Rightarrow\quad x = 2 + 2t \end{aligned}$$

Solution set $\{(x, y, z) = (2 + 2t, -2 + 2t, t),\ t \in \mathbb{R}\}$