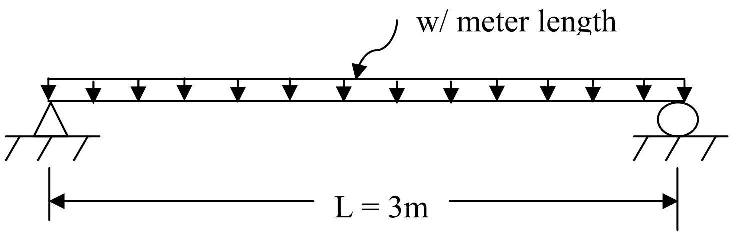
The University of Melbourne
CVEN30008 Engineering Risk Analysis

Tutorial 11

Engineering Reliability

1. A simply supported timber beam of length 3 m is loaded with a uniformly distributed load w with μ = 5 kN/m and σ = 1 kN/m. The bending strength of similar beams has been found to have a mean strength μ_R = 10 kNm with a coefficient of variation (COV) of 0.2. Assuming that the beam self-weight and any variation in the length of beam can be ignored, evaluate the probability of failure.

   Hint: The applied moment is: $S = \frac{wL^2}{8}$



Solution:

The applied moment, $S = \frac{wL^2}{8}$

$\mu_S = \frac{3^2}{8}\ \mu_w = \frac{9}{8}\ \mu_w = \frac{9}{8} \times 5 = 5.625\ kNm$

$$\sigma_S^2 = \left\{\frac{9}{8}\right\}^2 \sigma_w^2 = \frac{81}{64} \times\ 1^2 = 1.27\ (kNm)^2$$

$\mu_R = 10\ kNm$

$$COV = \frac{\sigma}{\mu}$$

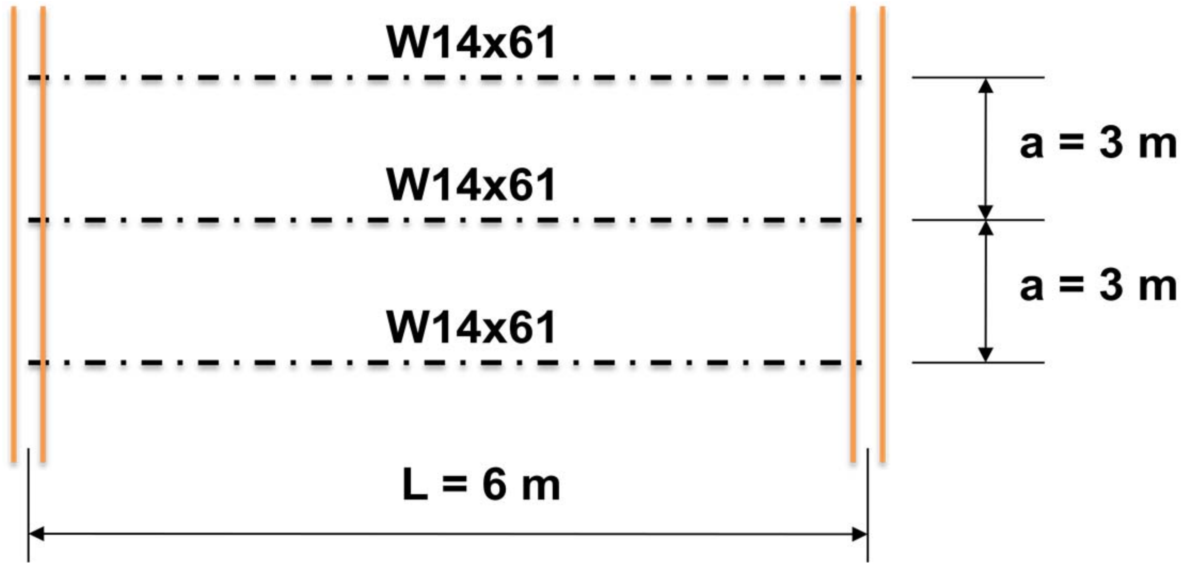
$\sigma_R^2 = [COV\ \mu_R]^2$ = (0.2 x 10)^2 = 4 (kNm)^2

Safety index (Reliability index), $\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{10-5.625}{\sqrt{4+1.27}} = 1.91$

Probability of failure, $p_f = 1 - \Phi(\beta) = 1 - 0.9719 = 0.0281$

2. A simply supported steel beam W14x61 (capacity μ_R = 360.7 kNm, σ_R = 72.9 kNm) with a 6 m span has been designed to carry a dead load (μ_D = 2.6 kN/m^2, σ_D = 0.35 kN/m^2) and a live load (μ_L = 2.75 kN/m^2, σ_L = 1 kN/m^2). Assuming dead load (D), live load (L) and beam capacity (R) are statistically independent normal variables, evaluate the probability of failure.

Hint: The applied moment is: $S = \frac{T \times a \times L^2}{8}$; where T is the total load (T=D+L).



Solution:

Total load, T = D+L

$$\mu_T = \mu_D + \mu_L = 2.6 + 2.75 = 5.35\ kN/m^2$$

$$\sigma_T = \sqrt{(\sigma_D)^2 + (\sigma_L)^2} = \sqrt{0.35^2 + 1^2} = 1.06\ kN/m^2$$

$$The\ applied\ moment, S = \frac{T \times a \times L^2}{8}$$

$$\mu_S = \frac{\mu_T \times a \times L^2}{8} = \frac{5.35 \times 3 \times 6^2}{8} = 72.225\ kNm$$

$$\sigma_S = \sqrt{\left[\frac{a \times L^2}{8}\right]^2 \sigma_T^2} = \sqrt{\left[\frac{3 \times 6^2}{8}\right]^2 \times 1.06^2} = 14.31\ kNm$$

Safety index (Reliability index), $\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{360.7 - 72.225}{\sqrt{72.9^2 + 14.31^2}} = 3.88$

$$p_f = 1 - \Phi(\beta) = 1 - 0.9999478 = 5.2 \times 10^{-5}$$

3. Consider a case of a steel bridge that deteriorates continuously with time as a result of corrosion. The initial structure performance is 100% with a threshold limit of 25%. Estimate the probability of failure of the bridge after 35 years if the progressive deterioration of the bridge can be modelled as:

(a) Graceful (linear) deterioration with a rate K = 0.70% per year.

(b) Exponential deterioration with a rate $\alpha = 0.08$/year.

Assume the remaining structural capacity is governed by an exponential distribution with an average rate of = 0.05.

Solution:

The initial structure performance is $u_0 = 100\%$

Threshold limit $a^* = 25\%$

t=35 years

(a) Graceful (linear) deterioration with a rate $K = 0.7$/year

$V(t=35) = u_0 - a^* - K \times t = 100-25-0.7\times35 = 50.5$

Cumulative distribution function (CDF): $\mathrm{CDF}(V) = 1 - e^{-\theta V}$

Probability of failure (PoF):

$$\mathrm{PoF(V)} = 1 - \mathrm{CDF}(V) = 1 - (1 - e^{-\theta V}) = 1 - (1 - e^{-0.05\times 50.5}) = 8\%$$

(b) Exponential deterioration with a rate $\alpha = 0.08$/year.

$V(t=35) = u_0 - a^* - (e^{\alpha \times t} - 1) = 100 - 25 - (e^{0.08 \times 35} - 1) = 59.5$

$\mathrm{PoF(V)} = 1 - \mathrm{CDF}(V) = 1 - (1 - e^{-\theta\, V}) = 1 - (1 - e^{-0.05\times 59.5}) = 5.1\%$