School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 7

4–8 September 2017

Plan

Next week is the week of the mid-semester test, so make sure you are well prepared. If you skipped important questions in previous weeks, or if you have doubts about how to approach particular types of questions, now is the time to catch up, and to ask those questions in your tute, or on the LMS.

The exercises

46. Start with the Week 7 worksheet on Grok. It contains Haskell exercises related to primes and to sets.

47. The Grok worksheet asked you to use Haskell to find counter-examples to certain conjectures, or, equivalently, to prove "existential" claims. Can we program our way out of proving "universal" claims with the same ease?

    Recall that a prime triple is a triple $(p, p+2, p+4)$ with p, $p+2$, and $p+4$ all prime. The Grok worksheet's Haskell script `Numeric.hs` includes an expression `primeTriples` denoting the list of all prime triples. From your work there, you will know the first prime triple. (Okay, what is it?) Now show that this triple is in fact the only prime triple.

48. (If you were comfortable with exercise 39, skip this one, otherwise spend a bit more time thinking about politicians.) Last week we looked at the statement "some politicians are not honest" and we translated it as $\exists x\ (P(x) \land \neg H(x))$. However, the formula $\exists x\ (P(x) \Rightarrow \neg H(x))$ also looks like a plausible translation. But the two formulas are not logically equivalent. Explain why the latter does not capture what we mean when we say "some politicians are not honest".

49. Let A, B, and C be sets. Show:

    (a) $A \not\subseteq B \Leftrightarrow A \setminus B \neq \emptyset$.
    (b) $A \cap B = A \setminus (A \setminus B)$.

    Hint: Use the formal (logical) definitions of the concepts involved.

50. Recall that the *symmetric difference* of sets A and B is the set $A \oplus B = (A \setminus B) \cup (B \setminus A)$. For each of the following set equations, give an equivalent equation that does not use $\oplus$. However, do not simply replace $\oplus$ by its definition; instead try to find the simplest equivalent equation.

    (a) $A \oplus B = A$
    (b) $A \oplus B = A \setminus B$
    (c) $A \oplus B = A \cup B$
    (d) $A \oplus B = A \cap B$
    (e) $A \oplus B = A^c$

51. The *Cartesian product* of two sets A and B is defined $A \times B = \{(a, b) \mid a \in A \land b \in B\}$. That is, a pair whose first component comes from A and whose second component comes from B is an element of $A \times B$ (and no other pairs are). Recall that $\cap$ and $\cup$ are absorptive, commutative and associative. Does $\times$ have any of those properties?

52. Consider this statement: For all sets S and T, $S \times T = T \times S$ iff $S = T$.

    If the statement is true, prove it. Otherwise provide a counter-example.