School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 9

18–22 September 2017

Plan

Time to start working with DFAs and NFAs! Most of the exercises on automata are taken from Sipser, *Introduction to the Theory of Computation*, Thomson, 2006. (There is a more recent edition, but I don't have that.) Chapter 1, on regular languages, is available as Readings Online. The book has many examples and it contains many more exercises, plus answers to selected exercises.

There are lots of questions below, to make sure you have enough to practice with, especially during that long boring teaching-free week.

The exercises

62. Each of the following languages is the intersection of two simpler languages. First construct the DFAs for the simpler languages, then combine them using the following idea: If the set of states for DFA D_1 is Q_1 and the set of states for D_2 is Q_2, we let the set of states for the combined DFA D be $Q_1 \times Q_2$. We construct D so that, having consumed a string s, D will be in state (q_1, q_2) iff D_1 is in state q_1, and D_2 is in state q_2 when they have consumed s. Throughout this question, assume that the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

    (a) $\{w \mid w$ has at least three as and at least two bs$\}$

    (b) $\{w \mid w$ has an even number of as and one or two bs$\}$

    (c) $\{w \mid w$ has an odd number of as and ends with b$\}$

    (d) $\{w \mid w$ has an odd number of as and has even length$\}$

63. Each of the following languages is the complement of a simpler language. Again, the best way to proceed is to first construct a DFA for the simpler language, then find a DFA for the complement by transforming that DFA appropriately. Throughout this question, assume that the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

    (a) $\{w \mid w$ does not contain the substring bb$\}$

    (b) $\{w \mid w$ contains neither the substring ab nor ba$\}$

    (c) $\{w \mid w$ is any string not in $\mathtt{a}^*\mathtt{b}^*\}$

    (d) $\{w \mid w$ is any string not in $\mathtt{a}^* \cup \mathtt{b}^*\}$

    (e) $\{w \mid w$ is any string that doesn't contain exactly two as$\}$

    (f) $\{w \mid w$ is any string except a and b$\}$

64. Draw DFAs recognising the following languages. Assume that the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

    (a) $\{w \mid w$ begins with a 1 and ends with a 0$\}$

    (b) $\{w \mid w$ contains the substring 0101$\}$ (so $w = x\mathtt{0101}y$ for some strings x and y)

    (c) $\{w \mid w$ has length at least 3 and its third symbol is 0$\}$

    (d) $\{w \mid$ the length of w is at most 5$\}$

    (e) $\{w \mid w$ is any string except 11 and 111$\}$

(f) $\{w \mid$ every odd position of w is a $\texttt{1}\}$

(g) $\{w \mid w$ contains at least two $\texttt{0}$s and at most one $\texttt{1}\}$

(h) $\{\epsilon, \texttt{0}\}$

(i) The empty set

(j) All strings except the empty string

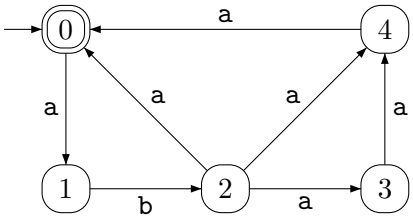
65. Give transition diagrams for NFAs for the following languages, with the specified number of states. Throughout this question, assume that the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$.

(a) $\{w \mid w$ ends with $\texttt{00}\}$ using three states

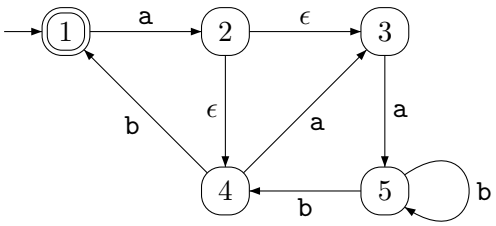
(b) $\{w \mid w$ contains the substring $\texttt{0101}\}$ using five states

(c) The language $\{\epsilon\}$ with one state

66. Use the subset construction method to turn this NFA into an equivalent DFA:



67. Use the subset construction method to turn this NFA into an equivalent DFA:



68. Give regular expressions for the following languages.

(a) $\{w \mid w$ begins with a $\texttt{1}$ and ends with a $\texttt{0}\}$

(b) $\{w \mid w$ contains the substring $\texttt{0101}\}$ (so $w = x\texttt{0101}y$ for some strings x and y)

(c) $\{w \mid w$ has length at least 3 and its third symbol is $\texttt{0}\}$

(d) $\{w \mid$ the length of w is at most 5$\}$

(e) $\{w \mid w$ is any string except $\texttt{11}$ and $\texttt{111}\}$

(f) $\{w \mid$ every odd position of w is a $\texttt{1}\}$

(g) $\{w \mid w$ contains at least two $\texttt{0}$s and at most one $\texttt{1}\}$

(h) $\{\epsilon, \texttt{0}\}$

(i) The empty set

(j) All strings except the empty string