PHYC90045 Introduction to Quantum Computing

Week 7

Lecture 13 – Introduction to IBM Quantum Experience
Introduction to IBM Quantum Experience: Guest Lecture

Lecture 14 – IBM and Optimizations
14.1 Rotation operators: QUI and IBM conversion
14.2 QASM and QISKit
14.3 Optimizing circuits

Lab 7
Using the IBM Q system

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IBM Q system and Optimization

Physics 90045
Lecture 14

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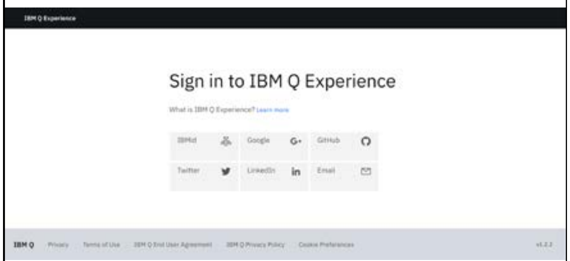
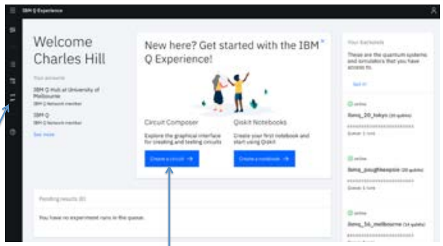
The IBM Q System

quantum-computing.ibm.com



Sign up using your university email before Thursday/Friday!

Starting a new circuit



Can also access circuit composer through menu here:

Click here to create a new circuit

IBM's Circuit Composer

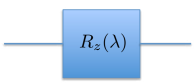
To add a Hadamard gate, drag and drop onto your circuit:

U1

U1 is a rotation around Z by angle lambda, which is equivalent to a rotation around the z-axis by an angle lambda

$$U_1 = \begin{bmatrix} 1 & 0 \\ 0 & \exp i\lambda \end{bmatrix}$$

Most easily understood as:



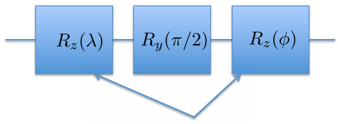
In the QUI, to emulate these z-rotations, use a global phase of lambda/2. No global phase for the y-rotation.

U2

The U2 operation is given by

$$U_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -\exp(i\lambda) \\ \exp(i\phi) & \exp(i\lambda + i\phi) \end{bmatrix}$$

Which can be represented as:



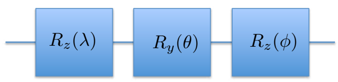
In the QUI, to emulate these z-rotations, use a global phase of theta/2.
No global phase for the y-rotation.

U3

The matrix of a U3 rotation is:

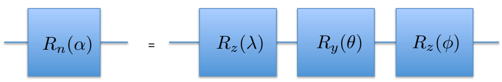
$$U_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos\theta/2 & -\exp(i\lambda)\sin(\theta/2) \\ \exp(i\phi)\sin(\theta/2) & \exp(i\lambda + i\phi)\cos(\theta/2) \end{bmatrix}$$

As a circuit:



Euler Angle Decomposition

Any rotation can be represented as a rotation around orthogonal axes:



QUI IBM Quantum Experience

Converting to and from Euler angles

General form of arbitrary rotation about an unit axis n=(n_x, n_y, n_z):

$$R_n(\alpha) = \cos\frac{\alpha}{2} I - i\sin\frac{\alpha}{2}\hat{n}\cdot\sigma$$
$$= \begin{bmatrix} \cos\frac{\alpha}{2} - in_z\sin\frac{\alpha}{2} & \sin\frac{\alpha}{2}(-in_x - n_y) \\ \sin\frac{\alpha}{2}(-in_x + n_y) & \cos\frac{\alpha}{2} + in_z\sin\frac{\alpha}{2} \end{bmatrix}$$

Euler angle rotations (with global phase = 0):

$$U_3 = \begin{bmatrix} e^{-i(\lambda+\phi)/2}\cos(\theta/2) & -e^{i(\lambda-\phi)/2}\sin(\theta/2) \\ e^{i(-\lambda+\phi)/2}\sin(\theta/2) & e^{i(\lambda+\phi)/2}\cos(\theta/2) \end{bmatrix}$$

Write out the matrix and equate elements.

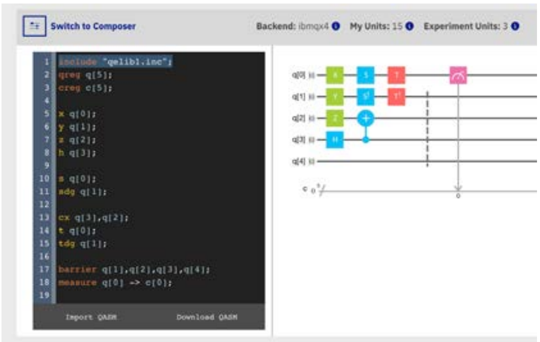
Product of single qubit unitaries

Euler angle rotations (with global phase = 0):

$$U_3 = \begin{bmatrix} e^{-i(\lambda+\phi)/2}\cos(\theta/2) & -e^{i(\lambda-\phi)/2}\sin(\theta/2) \\ e^{i(-\lambda+\phi)/2}\sin(\theta/2) & e^{i(\lambda+\phi)/2}\cos(\theta/2) \end{bmatrix}$$

Write out the matrix and equate elements.

QASM – Quantum Assembly language



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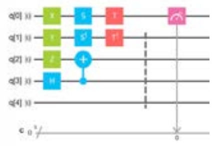
QASM Syntax

```
include "qelib1.inc";

// This is a comment
qreg q[5];
creg c[5];

x q[0];
y q[1];
z q[2];
h q[3];

s q[0];
sdg q[1];

cx q[3],q[2];
t q[0];
tdg q[1];

barrier q[1],q[2],q[3],q[4];
measure q[0] -> c[0];
```

Semi-Colons



Comment

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QASM

Hidden first line: **OPENQASM 2.0;**

```
include "qelib1.inc";
qreg q[5];
creg c[5];

x q[0];
y q[1];
z q[2];
h q[3];

s q[0];
sdg q[1];

cx q[3],q[2];
t q[0];
tdg q[1];

barrier q[1],q[2],q[3],q[4];
measure q[0] -> c[0];
```

Include standard definitions

Declare quantum register

Declare classical register

Single qubit gates

CNOT gate
(control first parameter, target second)

Dagger indicated by "dg"

Barrier (don't optimize across it)

Measure qubits to classical register

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Defining a new Function/Gate

Keyword: gate

```
// Controlled Phase gate
gate cz a,b
{
  h b;
  cx a,b;
  h b;
}
```

"cz" is name of gate

a and b are parameters

This gate can then be used like a native gate:

```
cz q[3],q[2];
```

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QASM Header File

```
// Quantum Experience (QE) Standard Header
// file: qelib1.inc

// --- QE Hardware primitives ---

// 3-parameter 2-pulse single qubit gate
gate u3(theta,phi,lambda) q { U(theta,phi,lambd
// 2-parameter 1-pulse single qubit gate
gate u2(phi,lambda) q { U(pi/2,phi,lambda) q; }
// 1-parameter 0-pulse single qubit gate
gate u1(lambda) q { U(0,0,lambda) q; }
// controlled-NOT
gate cx c,t { CX c,t; }
// idle gate (identity)
gate id a { U(0,0,0) a; }

// --- QE Standard Gates ---

// Pauli gate: bit-flip
gate x a { u3(pi,0,pi) a; }
// Pauli gate: bit and phase flip
gate y a { u3(pi,pi/2,pi/2) a; }
// Pauli gate: phase flip
gate z a { u1(pi) a; }
// Clifford gate: Hadamard
gate h a { u2(0,pi) a; }
// Clifford gate: sqrt(Z) phase gate
```

Also defines:

Rotations
rx, ry, rz

Toffoli
ccx

Controlled rotations
cu1, cu2, cu3, crz, ch

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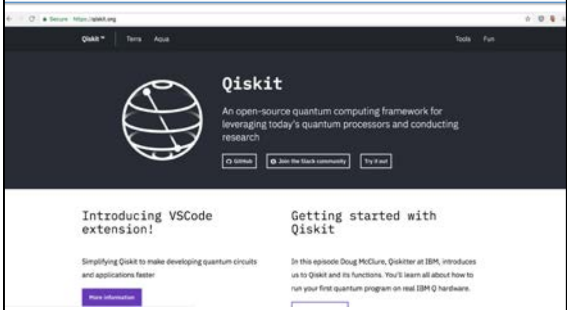
QISKit



Lots of examples in the github repository.

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QISKit

There is also a Python interface to IBM Quantum Experience.

It is required to make use of the larger machines.

You can:
- Authenticate with the system
- Construct circuits (ie. python which translates to QASM)
- Submit jobs, and check for results
- Receive the results of jobs

Python works well with Jupyter interface.

We will use this later when we use the 16 qubit quantum computer.

Python Primer (if required)

Some Python Basics

```
In [2]: a=6
        b=7
        life = a*b
        life
Out[2]: 42
```

Similar to many other imperative languages you may know for numerical work: (C/C++, MATLAB, R, FORTRAN, Julia) and often used for data processing.

Defining and calling Functions

def keyword indicates a new function

No types on parameters

Colon

Comment

```
def square(x):
    # This is a comment
    return x*x
```

Whitespace is significant in python. Indentation indicates a new block.

No semicolons. Newline is the end of a statement

Calling a function:

```
square(4)
square(x=4)
```

Named parameters

Lists and for loops

Lists store a sequence of values. Square brackets indicate a list:

```
["This", "is", "a", "list"]

primes = [2, 3, 5, 7, 11]
```

Eg. For loops often use lists:

```
for p in primes:
    print(p)
```

```
2
3
5
7
11
```

Accessing an individual element. 0-based!

```
primes[2]
```

```
5
```


Dictionaries

Dictionaries store key-value pairs.

Curly braces indicate a dictionary

key

value

```
me = {"name": "Charles", "height":1.79, "favourite_food": "pizza"}
me["favourite_food"]
```

```
'pizza'
```

```
me["favourite_food"] = "sweet and sour pork"
me["favourite_food"]
```

```
'sweet and sour pork'
```


Importing other libraries

```
import numpy as np

X = np.matrix([[0,1],
               [1,0]])
```

Importing a module ("as np" is optional). numpy gives similar functionality to MATLAB

Calling functions from that module. Here creating an X matrix.

Or import individual functions and classes:

```
from qiskit import QuantumProgram
from qiskit import available_backends, execute, get_backend, compile
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister, QISKitError
```

qiskit is an Python library/API for interacting with IBM's quantum computers remotely.

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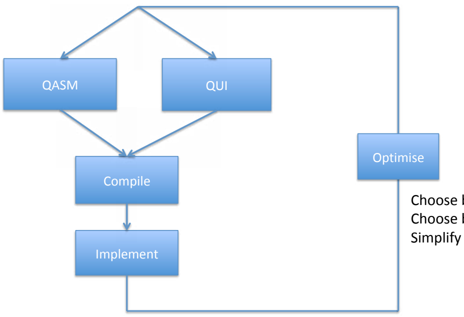
Optimisations and Gate Construction



Choose better qubits
Choose better gates
Simplify circuit

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Optimizing Circuits

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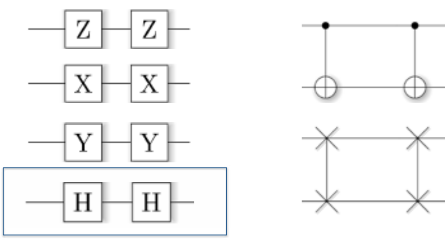
Many gates square to identity

Pro Tip: Most physicists looking at quantum circuit diagrams aren't multiplying matrices in their head. They're identifying common patterns.



All of these combinations square to the identity (do nothing)

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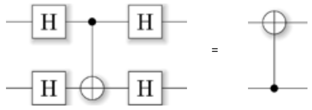
Circuit identity: Inverted CNOT



Exercise: You can verify this by writing out the matrices and multiplying!

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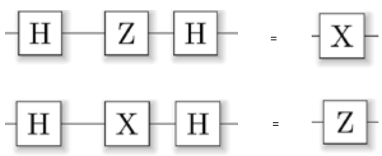
Conjugating with Hadamard



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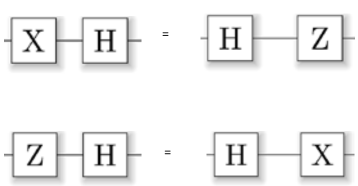
Commuting through Hadamard



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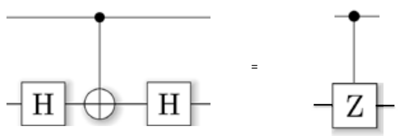
Control-Z from CNOT



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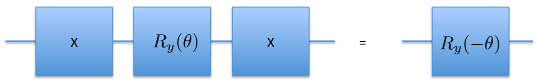
Conjugation with Pauli



$$
\begin{aligned}
XR_y(\theta)X &= X\left(\cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y\right)X \\
&= \cos\frac{\theta}{2}XX - i\sin\frac{\theta}{2}XYX \\
&= \cos\frac{\theta}{2}I + i\sin\frac{\theta}{2}Y \\
&= R_y(-\theta)
\end{aligned}
$$

Paulis **anticommute**
XY = -YX

Works with any two orthogonal axes.

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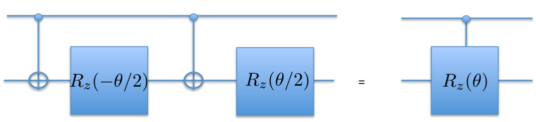
Controlled Angle Rotation



If the control is a zero, the rotations cancel.
If the control is one, the rotations add.

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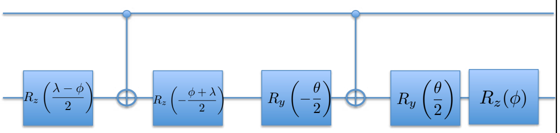
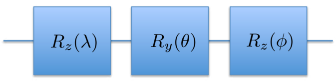
Any Controlled U

For U_3 Euler angle rotation (on IBM's system):



Controlled version of a U3 gate:



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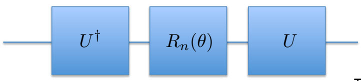
Conjugation with Rotation

Conjugation with a rotation:



Changes the axis of rotation, but not the rotation angle.

This rotates the axis itself

$$\begin{aligned} SR_x(\theta)S^\dagger &= \cos\left(\frac{\theta}{2}\right) I - i\sin\left(\frac{\theta}{2}\right) SXS^\dagger \\ &= \cos\left(\frac{\theta}{2}\right) I - i\sin\left(\frac{\theta}{2}\right) Y \\ &= R_y(\theta) \end{aligned}$$

Conjugation with Hadamard is a special case of this.

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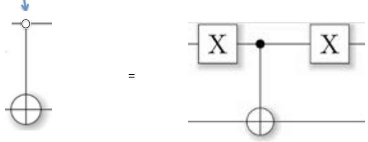
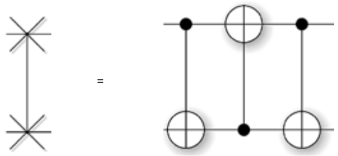
Control from "0" state

Open circle =
Only apply when
the control is "0"



We've seen this trick in labs: for example in the oracle for Grover's algorithm.

Swap gate from three CNOTs



Let's check:

00 -> 00
01 -> 10
10 -> 01
11 -> 11

Square root of SWAP

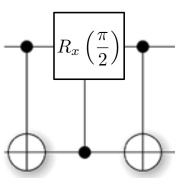
SWAP

$$U_{Swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Square root of SWAP

$$U_{SS} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Square Root Swap Construction



Similar to SWAP

More general version of Gray Code construction.

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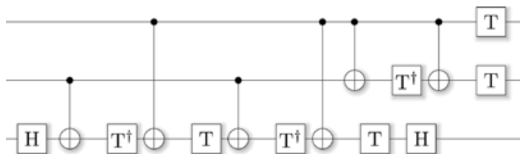
Toffoli from CNOTS



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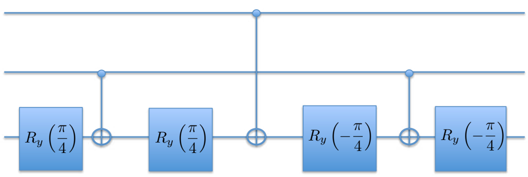
Toffoli with Incorrect Phase



If neither control is 1, then no net rotation.
If only first control is a 1, then becomes Z rotation (Ry by pi).
If only second control qubit is a 1, then both halves cancel.
If both controls are 1, then bottom qubit flips because of middle CNOT.

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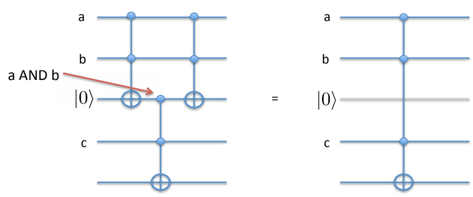
Multiply Controlled Gates



We can use Toffoli and an ancilla qubit to define multiply controlled gates.

Mocking up gates

If you use controlled operations on all qubits except the target, then you isolate a single 2x2 subspace, with the rest of the matrix untouched.

$$CU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}$$

Or controlled off the zero state:

$$C_0U = \begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

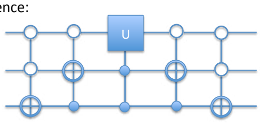
Using Gray codes

Imagine we wanted a 2x2 matrix between the 000 and 111 states:

A	B	C
0	0	0
0	0	1
0	1	1
1	1	1

Gray code

Corresponding sequence:



In this way, complicated multi-qubit gates can be built, piece by piece.

Week 7

Lecture 13 – Introduction to IBM Quantum Experience

Introduction to IBM Quantum Experience: Guest Lecture

Lecture 14 – IBM and Optimizations

14.1 Rotation operators: QUI and IBM conversion

14.2 QASM

14.3 Optimizing circuits

Lab 7

Using the IBM Q system

