COMP20003
Algorithms and Data Structures
Topological Sort Algorithm

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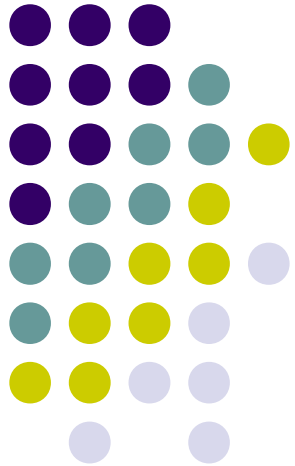
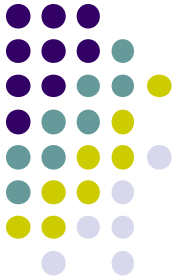


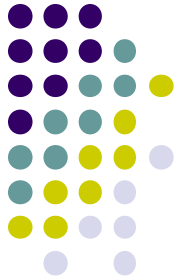
Directed Acyclic Graphs (DAGs)



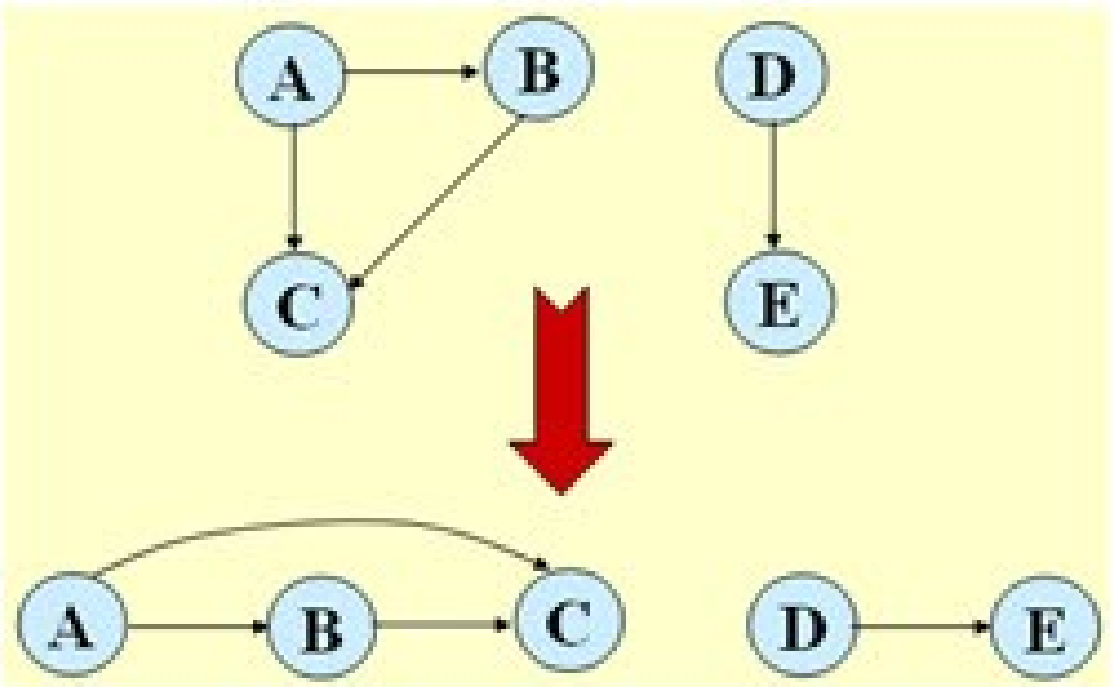
- Directed
- Acyclic

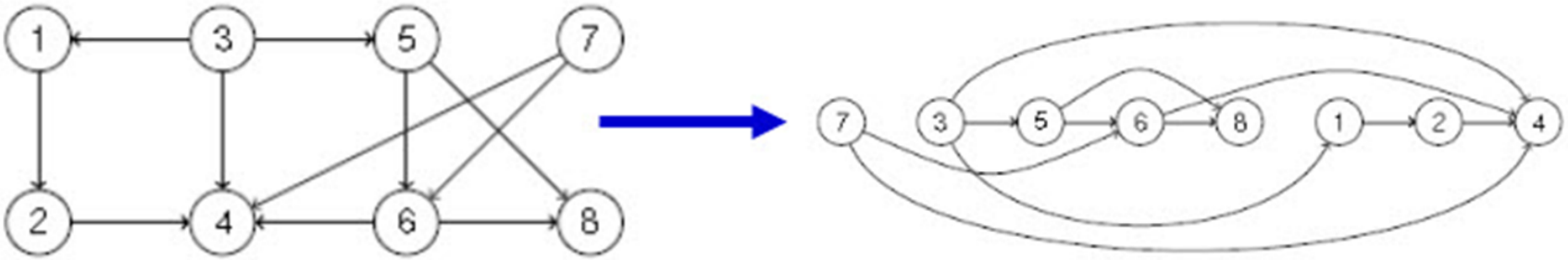
- Useful for modelling many problems:
  - Temporal dependencies
  - Causalities
  - Hierarchies
  - Compiling modularized programs

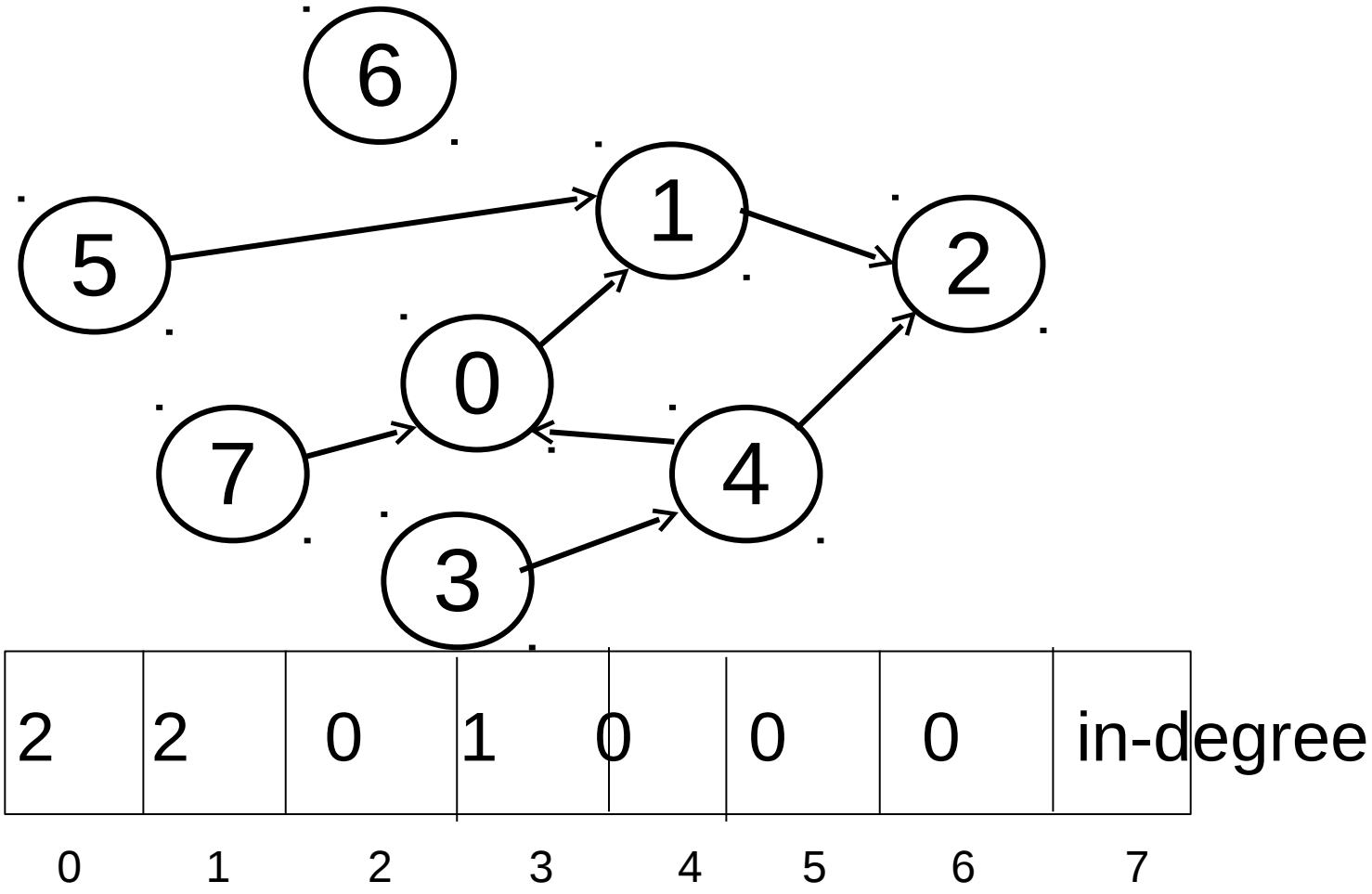
Topological sort



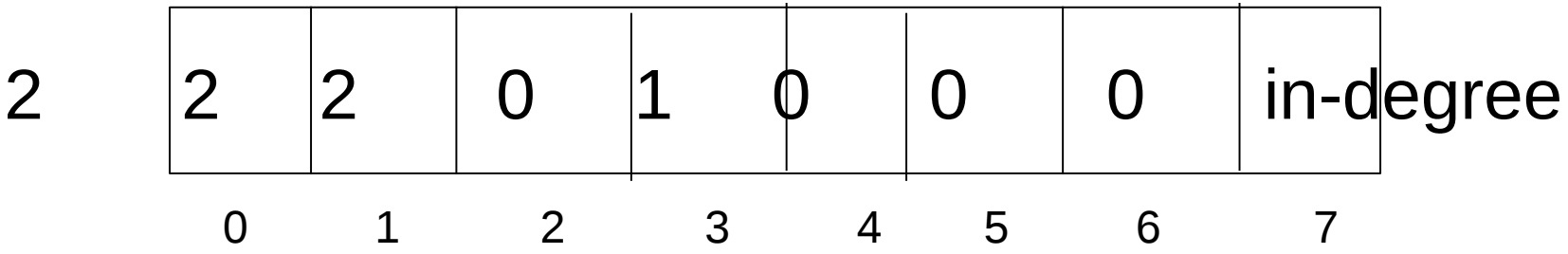
- Topological sort: a partial ordering that fulfils certain constraints.
- A topological ordering of a DAG is an ordering of the nodes such that for every edge (i,j) the direction is i→j.
- The output can be the schedule for:
  - A builder
  - A course plan
  - *etc*.


A
B
C
D
E
A
B
C
D
E


1
3
5
7
2
4
6
8
7
3
5
6
8
1
2
4

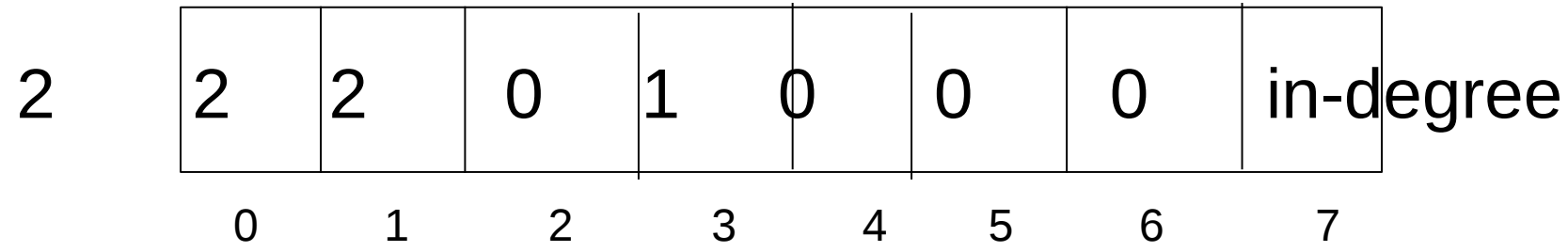

6
5
1
2
0
7
4
3
2 2 0 1 0 0 0
in-degree
0 1 2 3 4 5 6 7

Topological sort construction: source removal algorithm

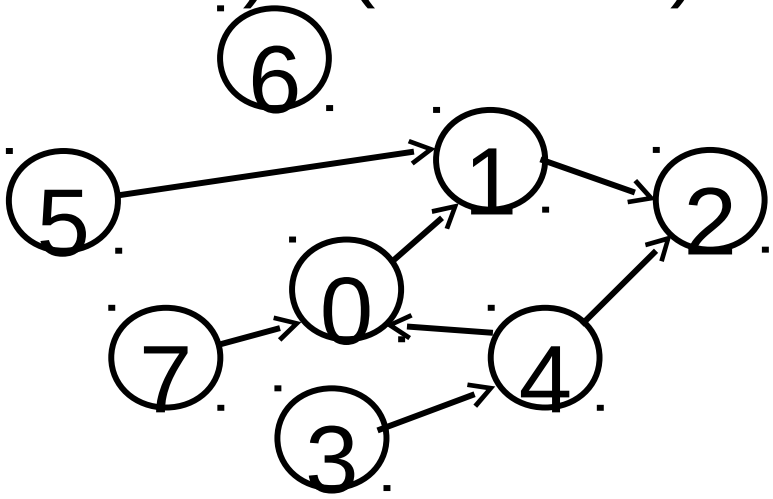


- Identify a source (in-degree = 0)
  - put that node in the topsort output,
  - remove that node from DAG,
  - update in-degree matrix.
- Indentify another source…

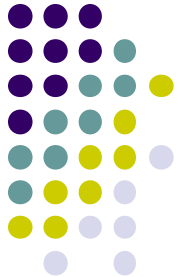
Topological sort: sink removal



- 7-6-5-3-4(new src)-0(new src)-1(new src) -2(new src)
- 6-3-4-7-0-5-1-2
- 5-7-3-4-0-1-2-6
- *etc.*



Topological sort: Assumptions

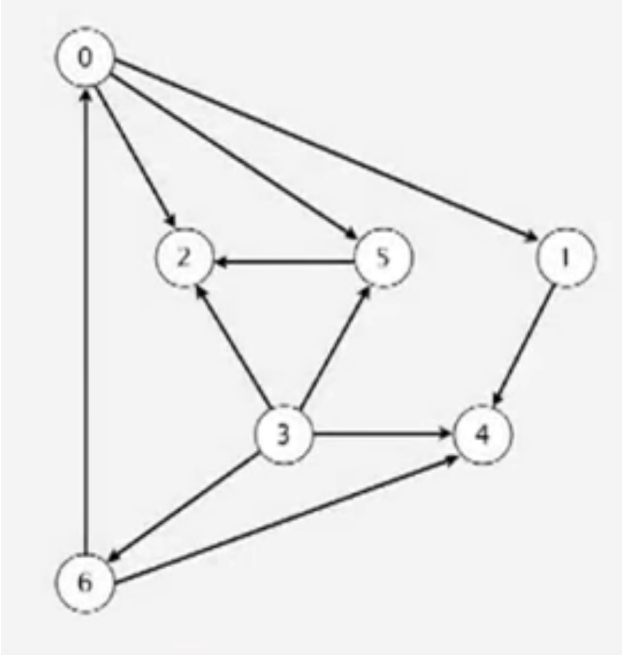


- There must be at least one source and one sink for this topological sorting algorithm to work.
  - Is this a valid assumption?

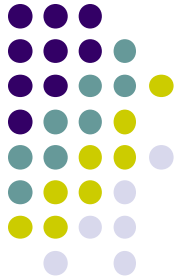
- Directed acyclic graph: must have at least one source and one sink.

Topological sort: Assumptions

- Any Alternative algorithm?
  - We can use a DFS search and read the reverse post-order
- Complexity O()?
  - Exercise->



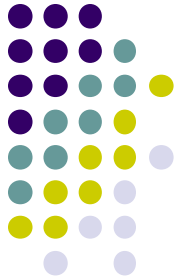
Topological sort: Uniqueness



- If a Hamiltonian path exists in the DAG, then the topological sort is unique.
- Finding a Hamiltonian Path is NP-Hard

- How hard is to prove Uniqueness?
  - Have to solve the decision problem: Given a DAG, Does a Hamiltonian Path exists?

Topological sort: Uniqueness



- The Hamiltonian Path problem has a property called polynomial verifiability.
  - Verifying the existence of a Hamiltonian path can be easier than determining its existence.

  Given a topological sort, if two consecutive vertices are not connected, then you can swap them. Implies: Non Unique and No Ham. Path. Can be done in linear time.