

# Coordination and Agreement

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# Aims

## **For a set of processes in a distributed system**

- Coordinate of actions of the independent processes to achieve common goals
- Agree on values
- Even if there are failures in the system reach these aims

## **Synchronous systems**

- Computation step of a process has known lower and upper bounds
- Message delivery times are bounded to a known value
- Each process has a clock whose drift rate from real time is bounded by a known value

## **Asynchronous systems**

- No bounds on process execution times, message delivery times clock drift rate

# Coordination Problems in DS

## **Asynchronous distributed systems**

- No single process has a view of the current global system state

## **Failure detection**

- How does a process determine whether a peer is dead or alive?

## **Mutual exclusion**

- No two process can access a shared resource in a critical section at the same time
- Simple solution: a single server and a locking mechanism to allow orderly access
- Sought solution: processes, among themselves, agree on how to access a resource or any other shared entity through passing messages

# Coordination Problems in DS

## **Election in master-slave systems**

- While booting up or when the master fails

## **Multicast**

- Reliability of multicast (correct delivery, only once, etc.)
- Order preservation

## **Consensus in the presence of faults (byzantine problems):**

- Determine whether acknowledgement was received over an unreliable communication medium
- Determine whether peer process knows about one's own intentions in the presence of a non-confidential communication channel

# Assumptions

**Processes are connected via reliable channels**

**Processes do not rely on others to communicate**

**Processes themselves can crash or act unexpectedly**

- A failure detector is a service that is used to realize that another process has failed
- Failure detectors may also be unreliable
  - We often use timeouts for detection, this may give false indications

# Failure Detection

## Failure Detector

- Service that can decide whether or not a particular process has crashed
- Can collaborate with other processes to detect failure

## Unreliable failure detector

- Unsuspected peer process
  - Failure is unlikely, e.g., failure detector has recently received communication from unsuspected peer
  - May be inaccurate
- Suspected peer process
  - Indication that peer process failed, e.g., no message received in quite some time
  - May be inaccurate, e.g., peer process has not failed, but the communication link is down or peer process is much slower than expected

## Reliable failure detector

- Unsuspected: potentially inaccurate as above
- Failed: accurate determination that peer process has failed

# Failure Detector

## Implementation of an unreliable failure detector

- Periodically, every  $T$  seconds each process  $p$  sends a “p is here” message to every other process
- If a local failure detector at  $q$  does not receive “p is here” from  $p$  within  $T+D$  ( $D$  = estimated maximum transmission delay), then  $p$  is suspected
- If message is subsequently received,  $p$  is declared OK

## Problem: calibration of transmission delay

- For small  $D$ , intermittent network performance downgrades will lead to suspected nodes
- For large  $D$  crashes will remain unobserved (crashed nodes will be fixed before timeout expires)

## Solution

- Variable  $D$  that reflect the observed network latencies

## Implementation of reliable failure detectors

- Only possible in a synchronous network

# Distributed Mutual Exclusion

## Task

- Distributed processes need to coordinate activities, e.g., for resource sharing
- Processes access shared resources, variables, etc. in a *critical section*
- Processes have to enter to this code part properly

## Assumptions

- There is only one critical section
- The system is asynchronous
- Message delivery is reliable and messages are delivered intact
- For this case: processes do not fail



# Mutual Exclusion Problems

## Prominent problem in multitasking operating systems

- Access to shared memory
- Access to shared resources
- Access to shared data
- Various algorithms to ensure mutual exclusion

## Mutual exclusion in distributed systems

- No shared memory
- Usually, no centralized instance like operating system kernel that would coordinate access
- Based on a synchronous or asynchronous, usually failure-prone network infrastructure

## Examples

- Consistent access to shared files (e.g., Network File Systems)
- Coordination of access to an access point in an IEEE 802.11 WaveLAN

# Application Level Protocol

## ***enter()***

- Enter critical section
- Block if necessary

## ***resourceAccess()***

- Access shared resources in critical section

## ***exit()***

- Leave critical section
- Other processes may now enter

# Conditions for Mutual Exclusion

## ME1: (Safety)

- At most one process may execute in the critical section at a time

## ME2: (Liveness)

- Requests to enter and exit the critical section eventually succeed (i.e., no deadlocks or starvation)
- Impossible for one process to enter critical section more than once while other processes are awaiting entry

## Use of happened-before relation

- It is not possible to order the access requests using time perfectly as there is no global clock

## ME3: ( $\rightarrow$ Ordering)

- If one request  $r_1$  to a resource happened before another request  $r_2$  then  $r_1$  should enter the critical section before  $r_2$

# Central Server Algorithm

## Central Server-based Algorithm

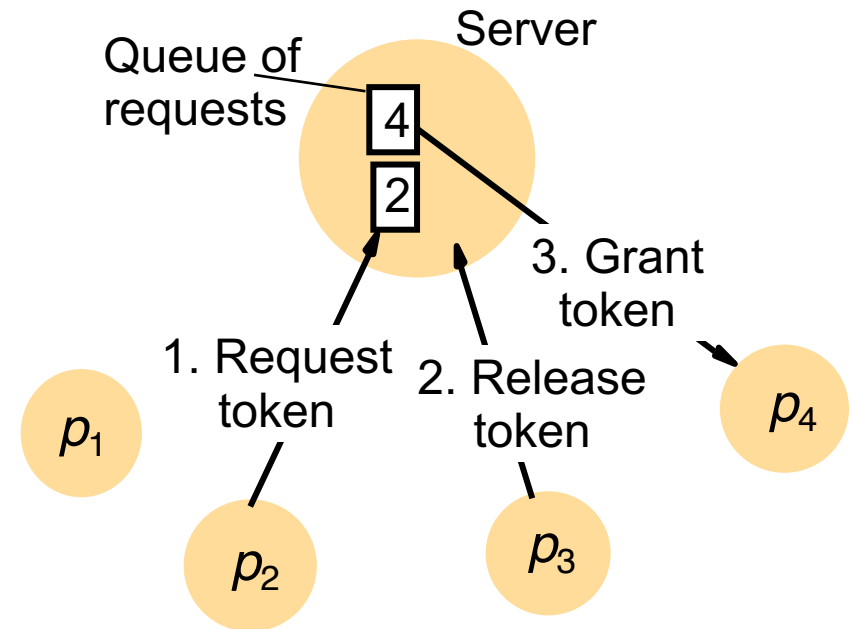
- Central server receives access requests
  - If no process in critical section, request will be granted
  - If process in critical section, request will be queued
- Process leaves critical section
- Grant access to next process in queue, or wait for new requests if queue is empty

**Safety and Liveness is guaranteed**

**Ordering (ME3) is not guaranteed**

- Network delays may reorder requests

**Performance and availability of server are the bottlenecks**



# Evaluation of Mutual Exclusion Algorithms

## Bandwidth consumption

- Number of messages sent in each entry and exit operation

## Client Delays

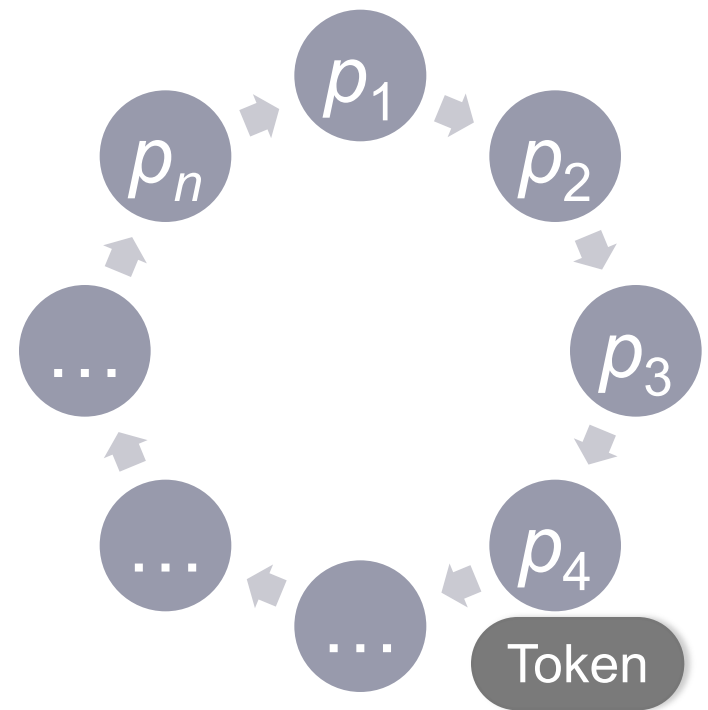
- At each entry and exit

## Throughput

- Number of critical region accesses that the system allows
- Measure effect using the *synchronization delay* of one process exiting the critical section and the next process entering it
- Throughput is greater when the synchronization delay is shorter

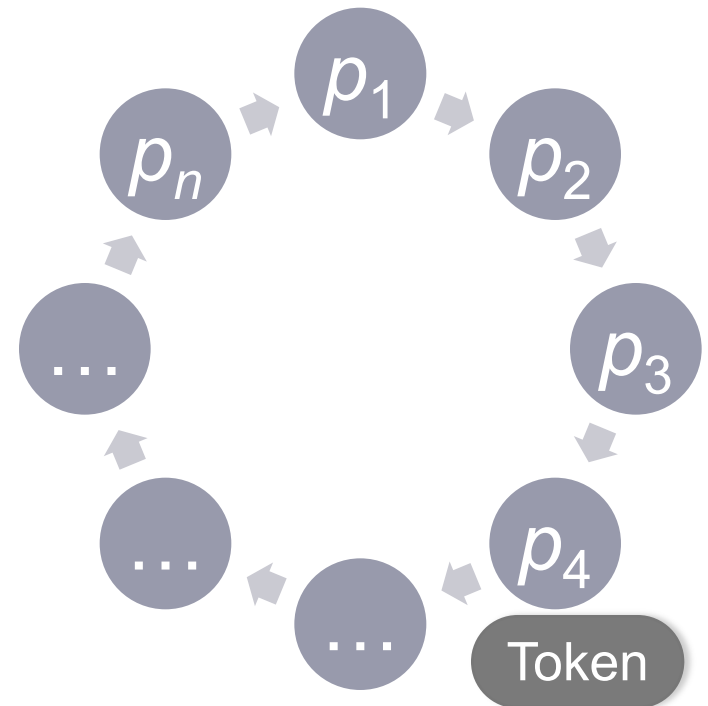
# Ring-based Algorithm

- Logical, not necessarily physical link: every process  $p_i$  has connection to process  $p_{(i+1) \bmod N}$
- Token passes in one direction through the ring
- Token arrival
  - Only process in possession of token may access critical region
  - If no request upon arrival of token, or when exiting critical region, pass token on to neighbor
- Algorithm satisfies ME1, ME2, but not ME3



# Ring-based Algorithm II

- Algorithm satisfies ME1, ME2, but not ME3 (happened before relation)
- Performance
  - Constant bandwidth consumption
  - Entry delay between 0 and  $N$  message transmission times
  - Synchronization delay between 1 and  $N$  message transmission times
  - The algorithm wastes bandwidth if no process requests the token



# Using Multicast and Logical Clocks

- Each process requests to enter a critical section via multicast
- When others reply to the multicast, then you can decide on the request
- All the conditions ME1 – ME3 are met
- Processes use Lamport clocks and their IDs
  - Timestamps are used to resolve simultaneous requests
  - If two timestamps are the same, then process IDs are used



# Ricart and Agrawala's Algorithm

*On initialization*

*state := RELEASED;*

*To enter the section*

*state := WANTED;*

*Multicast request to all processes;*

*T := request's timestamp;*

*Wait until (number of replies received = (N - 1));*

*state := HELD;*

*On receipt of a request  $\langle T_i, p_i \rangle$  at  $p_j$  ( $i \neq j$ )*

*if (state = HELD or (state = WANTED and  $(T, p_j) < (T_i, p_i)$ ))*  
*then*

*queue request from  $p_i$  without replying;*

*else*

*reply immediately to  $p_i$ ;*

*end if*

*To exit the critical section*

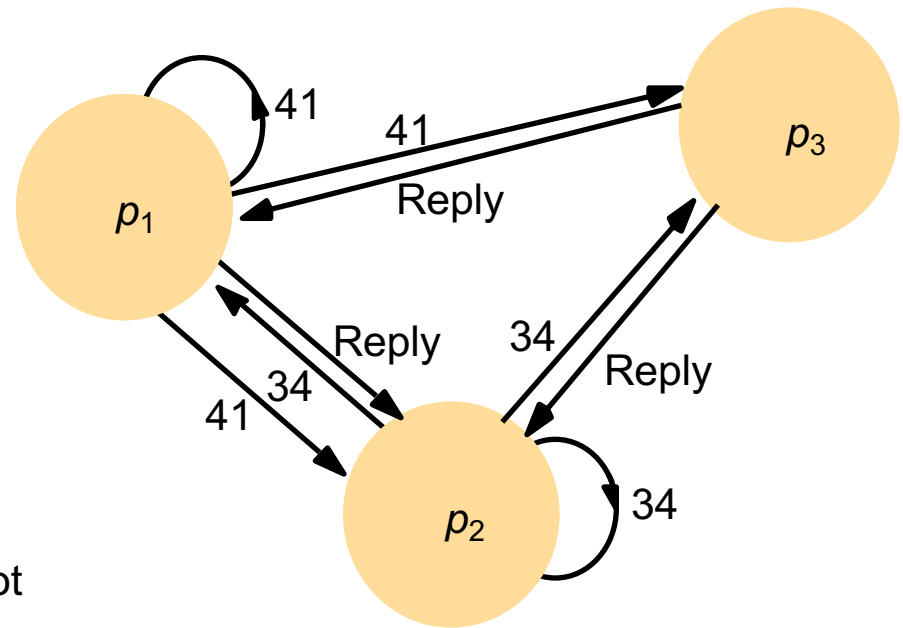
*state := RELEASED;*

*reply to any queued requests;*

# Multicast Synchronization

## Example

- $p_3$  does not want to enter the critical section
- $p_1$  and  $p_2$  request entry concurrently
- Timestamp of  $p_1$ 's request is 41, and that of  $p_2$  is 34
- $p_3$  replies immediately to  $p_1$ 's and  $p_2$ 's request
- $p_2$  receives  $p_1$ 's request: its own request has the lower timestamp and thus does not reply to  $p_1$
- $p_1$  finds that  $p_2$ 's request has a lower timestamp than that its own request and replies immediately
- Once  $p_2$  receives a reply from from  $p_1$ , receiving this second reply, it can enter the critical section
- When  $p_2$  exits the critical section, it will reply to  $p_1$ 's request



# Multicast-based Mutual Exclusion Contd

## Performance

- Expensive access requires  $2(N-1)$  messages per request
- Synchronization delay: just one message transmission time (previous algorithms: up to  $N$ )
- Improved versions exist for reducing bandwidth costs, e.g., repeated entry of same process without executing protocol

## ME1 – ME3:

- Algorithm satisfies ME1
  - Two processes  $p_i$  and  $p_j$  can only access critical section at the same time in case they would have replied to each other
  - BUT pairs  $\langle T_i, p_i \rangle$  are totally ordered (contradiction!)
- It can be shown that ME2 and ME3 are satisfied as well

# Maekawa's Voting Algorithm

## Idea

- Not all processes have to agree to decide on who enters to the critical section
- Subsets can be used: split set of processes up into overlapping subsets (“voting sets”)
- Processes vote for other processes to decide
- A candidate has to collect enough votes to enter to the critical section
- Sufficient condition: there is consensus within every subset

## Important observation

- Subsets used by any two process should overlap: the intersection processes in sets ensure the safety condition is met
- At most one process can enter the critical section, by casting their votes for only one candidate

# Maekawa's Voting Algorithm

## Model

- For  $N$  processes  $p_1, \dots, p_N$ , voting sets  $V_1, \dots, V_N$  are chosen such that:
  - $\forall i \forall j : i \neq j, 1 \leq i, j \leq N : V_i \cap V_j \neq \{\}$  (any two voting sets overlap)
  - $\forall i : 1 \leq i \leq N : p_i \in V_i$
  - $\forall i : 1 \leq i \leq N : |V_i| = K$  (fairness: all voting sets have equal size)
  - Each process  $p_j$  is contained in  $K$  of the voting sets  $V_i$

## Optimization goal

- Minimize  $K$  while achieving mutual exclusion
- Achieved when  $K \sim \sqrt{N}$

## Optimal voting sets

- Nontrivial to calculate, approximation: derive  $V_i$  so that  $|V_i| \sim 2\sqrt{N}$
- Place processes in a  $\sqrt{N}$  by  $\sqrt{N}$  matrix
- Let  $V_i$  the union of the row and column containing  $p$

# Maekawa's Algorithm

*On initialization*

*state := RELEASED;*

*voted := FALSE;*

*For  $p_i$  to enter the critical section*

*state := WANTED;*

*Multicast request to all processes in  $V_i$ ;*

*Wait until (number of replies received =  $K$ );*

*state := HELD;*

*On receipt of a request from  $p_i$  at  $p_j$*

*if (state = HELD or voted = TRUE)  
then*

*queue request from  $p_i$  without  
replying;*

*else*

*send reply to  $p_i$ ;*

*voted := TRUE;*

*end if*

*For  $p_i$  to exit the critical section*

*state := RELEASED;*

*Multicast release to all processes  
in  $V_i$ ;*

*On receipt of a release from  $p_i$  at  $p_j$   
if (queue of requests is non-empty)  
then*

*remove head of queue – say  $p_k$ ;*

*send reply to  $p_k$ ;*

*voted := TRUE;*

*else*

*voted := FALSE;*

*end if*

# Discussion of Maekawa's Voting Algorithm

## Maekawa's Voting Algorithm

### Satisfies ME1

- If two processes could enter a critical section, then the processes in the non-empty intersection of their voting sets would have both granted access
- Impossible, since all processes make at most one vote after receiving request

### Deadlocks are possible

- Given three processes with  $V_1 = \{p_1, p_2\}$ ,  $V_2 = \{p_2, p_3\}$ ,  $V_3 = \{p_3, p_1\}$
- Construct a cyclic wait graph
  - $p_1$  replies to  $p_2$ , but queues request from  $p_3$
  - $p_2$  replies to  $p_3$ , but queues request from  $p_1$
  - $p_3$  replies to  $p_1$ , but queues request from  $p_2$

# Voting Contd

## Algorithm is deadlock prone

- $p_1, p_2, p_3$  with sets  $\{p_1, p_2\}, \{p_2, p_3\}, \{p_3, p_1\}$  can deadlock if all three call for a critical section access and then they vote for themselves

## Extensions for satisfying ME2 and ME3

- Modification to ensure absence of deadlocks
- Use of logical clocks: processes queue requests in happened-before order

## Performance

- Bandwidth utilization:  $2\sqrt{N}$  per entry,  $\sqrt{N}$  per exit, total  $3\sqrt{N}$  is better than Ricart and Agrawala for  $N > 4$
- Client delay: same as for Ricart and Agrawala
- Synchronization delay: round-trip time instead of single-message transmission time in Ricart and Agrawala



# Failures

**All of the covered algorithms cannot tolerate a crash failure**

- Ring-algorithms cannot tolerate a single crash failure

## **Maekawa's algorithm**

- Tolerates crash failures if process is in a voting set not required, rest of the system not affected

## **Central-Server**

- Tolerates crash failures of a node that has neither requested access nor is currently in the critical section

## **Ricart and Agrawala algorithm**

- Tolerate crash failures if we assume that a failed process grants all requests immediately
- Requires reliable failure detector

# Elections

## Election algorithms

- Choosing a unique process to play a particular role
- For example: the server-based algorithm for mutual exclusion requires the election of a server process

## Fundamentals

- Each process can call an election (all processes can call simultaneously)
- Elected process should be unique
- The process with the largest value to an identifier needs to be found and elected
  - Example: elect process with lowest computational load, i.e., use  $\frac{1}{load}, i >$ , as its identifier, where  $load > 0$  and the process index  $i$  is used to order identifiers with the same load
- Use of special value such as “not yet defined” ( $\perp$ ) in cases where we do not hear from a process

# Elections

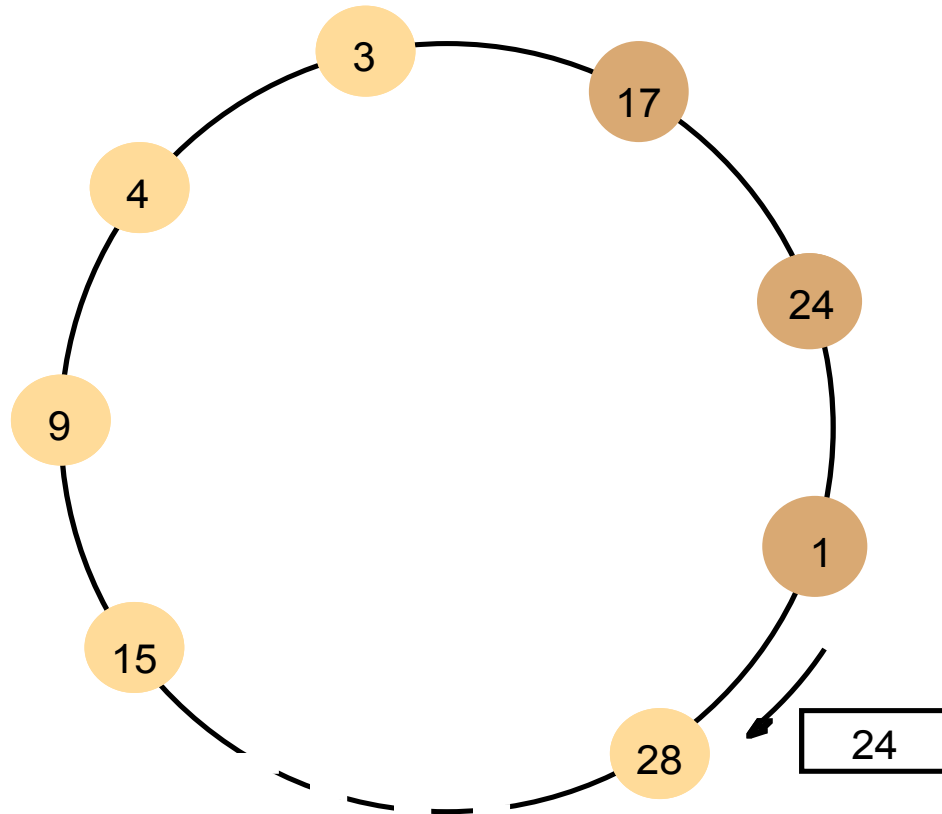
## Requirements for a run of an election algorithm

- E1: a process  $p_i$  has  $elected_i = \perp$  (undefined) or  $elected_i = P$  for some non-crashed process  $P$  that will be chosen at the end of the run with the largest identifier (*safety*)
- E2: all processes  $p_i$  will eventually set  $elected_i \neq \perp$  (*liveness*) or crash

## Performance

- Network bandwidth utilization (proportional to total number of messages sent)
- Turnaround: number of serialized message transmission times between initiation and termination of a single run

# Ring-based Election Algorithm



Note: The election was started by process 17.  
The highest process identifier encountered so far is 24.  
Participant processes are shown darkened

# Ring-based Election

## Assumptions

- All processes have a unique integer ID
- All  $N$  nodes communicate on a uni-directional ring structure
- Each process  $p_i$  has a communication channel to the next process in the ring,  $p_{(i+1) \bmod N}$

## Algorithm

- Initially all processes marked as non-participants
- Any process can start an election
- Election start: a process marks itself as a participant and places its ID in a message and sends it clockwise to the next

# Ring-based Election II

- If a message is received, a process compares the received ID with its own
  - If the ID is greater then forward it to the next process
  - If the ID is smaller then
    - If own status is “non-participant”, then substitute own ID in election message and forward on ring
    - otherwise, do not forward message (as “participant”)
  - If received ID is identical to own ID
    - This process ID must be greatest and it becomes elected
    - Marks own status as “non-participant”
    - Sends out “elected” message
- If a message is forwarded, upon any forwarding, mark own state as “participant”
- When receiving “elected” message
  - Mark own status as “non-participant”
  - Sets its variable *elected<sub>i</sub>* to the ID in the message (unless it is the new coordinator)
  - Forward *elected* message

# Ring-based Election Contd

## E1: Safety

- Met as all identifiers are considered
- Even if elections start simultaneously the smaller identifier cannot pass the larger identifier

## E2: Liveness

- Met due to reliable communication

## Worst-case behavior:

- $3N - 1$  messages in total and sequentially
  - At most  $2N - 1$  messages for electing the left-hand neighbor
  - Another  $N$  *elected* messages

## Failures

- Not tolerated

# The Bully-Algorithm

## Synchronous networks

- Nodes can crash
- Crashes will be detected reliably

## Assumptions

- Each node knows identifiers of *all* other nodes
- Every node can communicate with every other node

## Message types

- Election: announce an election
- Answer: reply to an election message
- Coordinator: announce identity of elected process



# The Bully-Algorithm II

## Initiation of algorithm

- Reliable failure detection
- Failure: no answer to request within  $T = 2T_{\text{trans}} + T_{\text{process}}$

## Coordinator

- Process decides whether to become coordinator by comparing own ID with all other IDs (highest wins)
- Announce its role by sending coordinator message to all other nodes with lower ID
- Process with lower ID can bid to become coordinator by sending election message to all processes with higher ID
  - If no response within  $T$ , considers itself elected coordinator, sends coordinator message to all processes with lower ID
  - Otherwise: wait for another  $T'$  time units for a coordinator message to arrive from new coordinator
    - If no response, then begin another election process

# The Bully-Algorithm III

## Receipt of an election message

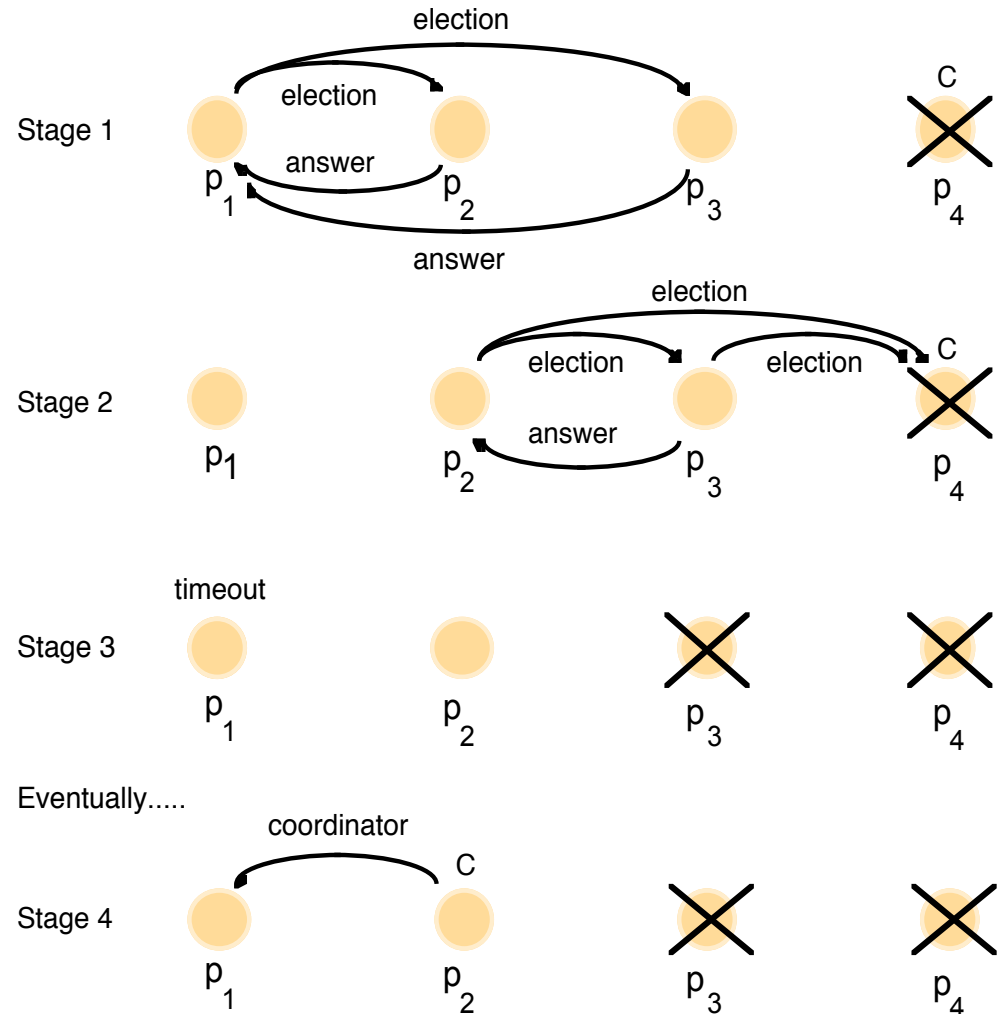
- Process sets variable  $election_i$  to the ID of the coordinator received in the election message
- Process sends back an answer message and begins another election – unless an election was already initiated

## New process replacing crashed process

- If it has the highest ID, it will immediately send coordinator message and “bully” current coordinator to resign

# The Bully Algorithm

The election of coordinator  $p_2$ , after the failure of first  $p_4$  and then  $p_3$



# The Bully Algorithm: Evaluation

## Properties

- E1 satisfied if no process is replaced and the timeout  $T$  estimate is accurate
- E2 satisfied for a synchronous network and reliable transmission
- E1 not satisfied if crashed process replaced at the same time while another process has announced that it is the new coordinator

## Performance

- Best case: process with the second highest identifier detects coordinators failure and elects itself coordinator and sends  $N-2$  coordinator messages
- requires  $O(N^2)$  messages in worst case when lowest ID detects failure  
⇒  $N-1$  processes with higher IDs start election

# Multicast Communication

## Group communication

- Sending and delivery of messages to a subset of (broadcast is to all) processes
- Receiving of message: queuing of arriving message in network interface buffer
- Delivery of message: passing message from network interface buffer to target application
- Membership in recipient group is transparent to sender: a single send operation without having to send individual messages to all group members
  - Example: IP address for which first 4 bits are XXXX in IPv4

## Issues

- Addressing
- Delivery guarantees that messages are received by a group
- Coordination: delivery ordering amongst group members

## Applications of multicast

- Computer Supported Collaborative Work (CSCW)
- Communication with replicated servers (fault-tolerance)
- Event notification in networks

# Multicast Communication: System Model

## Message $m$ :

- Contains ID of sender ( $\text{sender}(m)$ ) and of destination group ( $\text{group}(m)$ )

## **`multicast(g, m)`**

- Multicast message  $m$  to group  $g$

## **`delivery(m)`**

- Delivery of a message at recipient

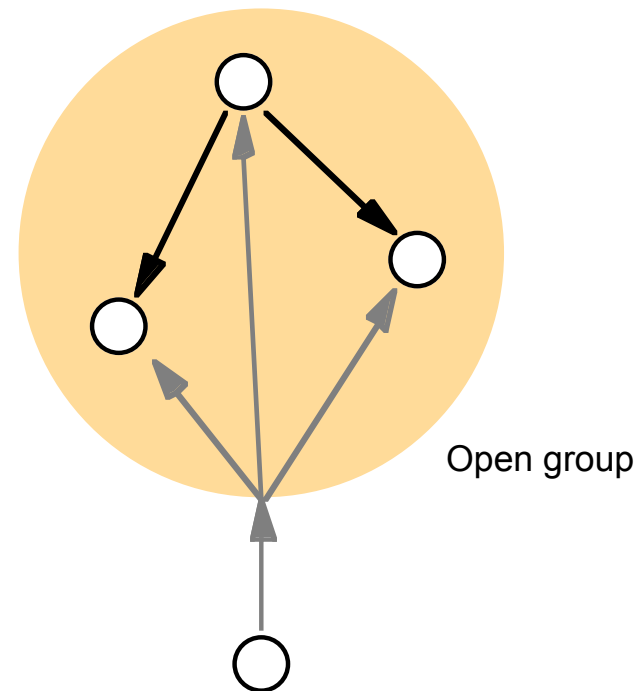
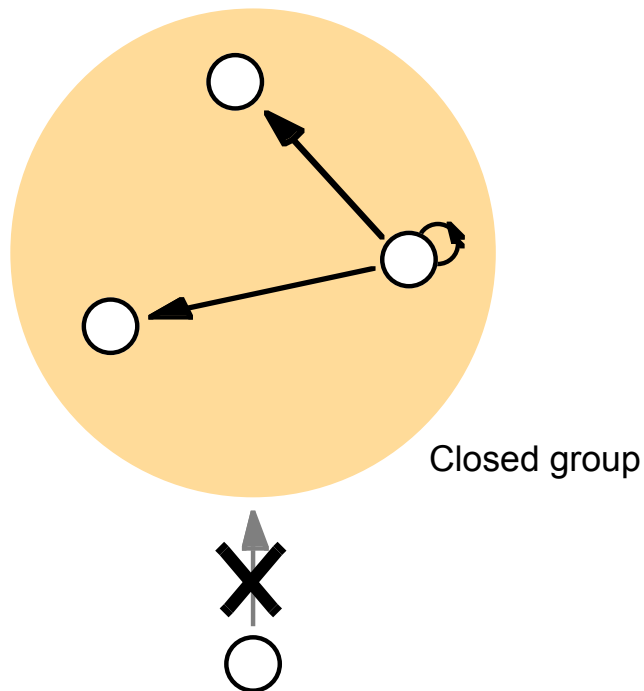
# Open and Closed Multicast Groups

## Closed

- Multicast only within group possible

## Open

- Processes outside the group may send to it



# Basic Multicast

## Property

- Guaranteed delivery, unless multicasting process crashes (different to IP multicast)
- Primitive *B-multicast* and its basic delivery primitive *B-deliver*

## Primitives and implementation

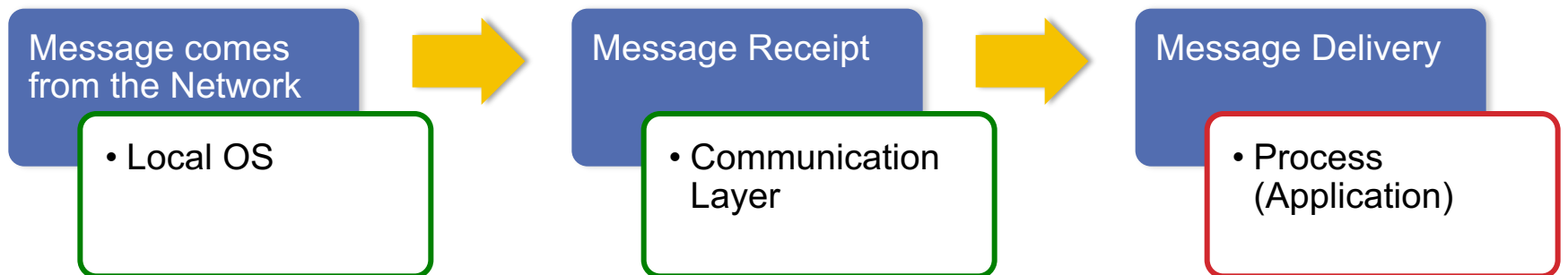
- Use a reliable one-to-one `send( )` operation
  - `B-multicast(g, m)`: for each process  $p \in g$ , `send(p, m)`
  - `B-deliver(m)` at  $p$ : when `receive(m)` at  $p$ , for all  $p$

## ack-implosion:

- Problem in using concurrent `send(p, m)` operations
  - All recipients acknowledge receipt at about same time
  - Buffer overflow leads to dropping of ack messages
  - Retransmits, even more ack messages



# Multicast: Layered Architecture



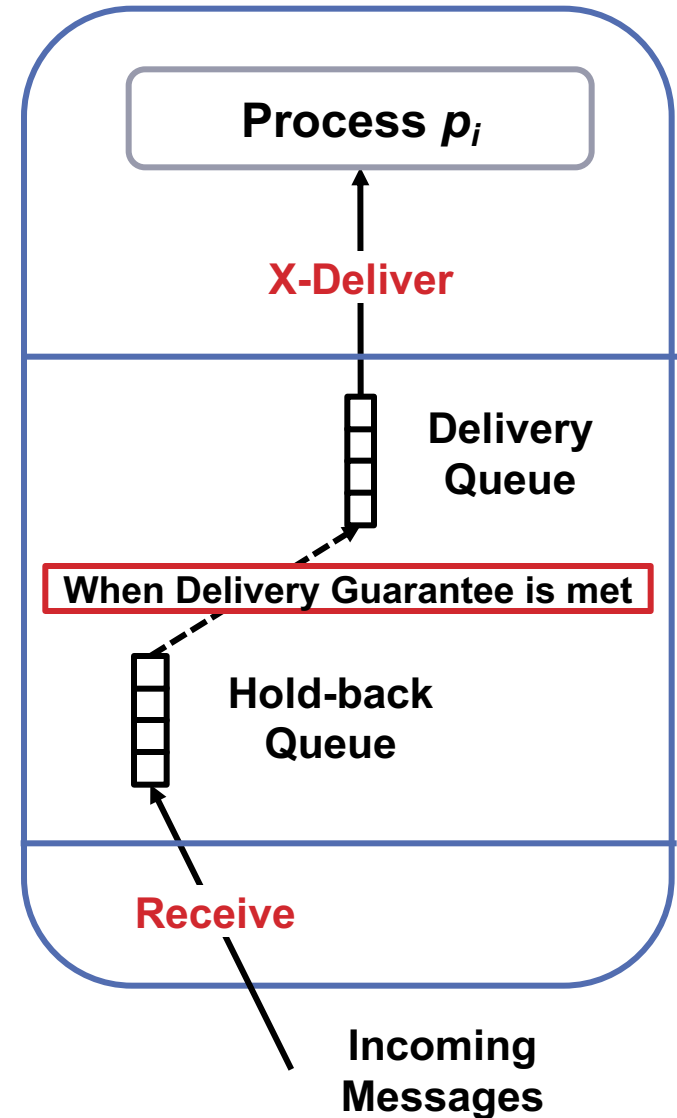
# X-Multicast

**X-multicast(g, m)**

**X-deliver(m)**

**X is**

- B: basic
- R: reliable
- FO: FIFO order
- CO: causal order
- TO: total order



# Reliable Multicast

## Primitives

- `R-multicast(m, g)`
- `R-deliver(m)`

## Desired properties

- Integrity
  - A correct process  $p$  delivers a message at most once
  - Safety: a delivered message is identical to the message sent in the multicast send operation
- Validity
  - If a correct process multicasts message  $m$ , then it will eventually deliver  $m$  (liveness)
- Agreement
  - If a correct process delivers a message  $m$ , then all other correct processes in the target group of message  $m$  will also deliver message  $m$

## Note

- Validity and agreement ensure overall liveness: if one process delivers a message  $m$ , then  $m$  will eventually be delivered to all group members

# Reliable Multicast

## Properties

- Validity: a correct process will eventually B-deliver to itself
- Integrity: based on underlying communication medium
- Agreement: B-multicast to all other processes after B-deliver
- Inefficient: each message is sent  $|g|$  times to each process

*On initialization*

`Received := {};`

*For process  $p$  to R-multicast message  $m$  to group  $g$*

`B-multicast( $g, m$ ); ( $p \in g$  is included as destination)`

*On B-deliver( $m$ ) at process  $q$  with  $g = \text{group}(m)$*

`if ( $m \notin \text{Received}$ ): Integrity`

`Received := Received  $\cup$  { $m$ };`

`if ( $q \neq p$ ):`

`B-multicast( $g, m$ ); Agreement`

`R-deliver( $m$ ) Integrity, Validity`

***Idea:  $q$  sends  $m$  to other working nodes before it delivers  $m$  to process***

# Reliable Multicast over IP Multicast

## R-IP-multicast

- Observation: multicast successful in most cases
- No separate acknowledgement messages
  - Piggyback acknowledgements on sent messages
  - Use negative acknowledgement (absence of a message) to indicate non-delivery

## Basic idea

- Assumption: closed multicast groups
- $S_g^p$ : sequence number for group  $g$  that process  $p$  belongs to (number for the next message to be sent)
- $R_g^q$ : sequence number of latest message that a process has delivered from process  $q$  and that was sent to group  $g$  (last message delivered from  $q$ )

# Hold-back Queue for Arriving Multicast Messages

## Basic algorithm

- p R-multicasts message to group g and piggybacks onto message
  - $S_g^p$
  - Acknowledgements  $\langle q, R_g^q \rangle$  for all q
- IP-multicast message with its piggybacked values to g
- Increment  $S_g^p$  by one
- R-deliver message from p:
  - Only if received sequence number  $S = R_g^p + 1$
  - Then increment  $R_g^p$  by 1
  - Retain any message that cannot yet be delivered in *hold-back-queue*

# Reliable Multicast over IP Multicast

## Basic idea

- R-deliver message from p
  - If  $S \leq R_g^p$ , then message is already delivered, discard
  - If  $S > R_g^p + 1$  or  $R > R_g^p$  for any enclosed acknowledgement  $\langle q, R \rangle$ 
    - Then receiver has missed one or more messages
    - Request retransmit through negative acknowledgement

## Integrity

- Follows from detection of duplicates and properties of IP multicast (e.g., checksum to detect message corruption)

## Validity

- Message loss can only be detected when a successor message is eventually transmitted
- Requires processes to multicast messages indefinitely

## Agreement

- Requires unbounded history for multicast messages so that retransmit is always possible

## Note

- There exist practical variants that ensure validity and agreement

# Ordered Multicast

## Example

- Order ignored in multicasting until now
- Bulletin-boards, chat rooms, instant messaging

## Assumption

- Every process belongs to at most one group

## FIFO ordering

- If a correct process issues a  $\text{multicast}(g, m)$  and then  $\text{multicast}(g, m')$ , then every correct process that delivers  $m'$  will deliver  $m$  before  $m'$

## Causal ordering

- If  $\text{multicast}(g, m) \rightarrow \text{multicast}(g, m')$ , where  $\rightarrow$  is induced by message passing only, then every correct process that delivers  $m'$  will deliver  $m$  before  $m'$



# Ordered Multicast II

## Total ordering

- If a correct process delivers  $m$  before it delivers  $m'$ , then any other correct process that delivers  $m'$  will deliver  $m$  before  $m'$

## Notes

- Causal ordering implies FIFO ordering
- FIFO ordering and causal ordering are partial (pre-)orders
- Total order allows arbitrary ordering of deliver events relative to multicast events, as long as this order is identical in all correct processes
- Atomic multicast: reliable, totally ordered multicast

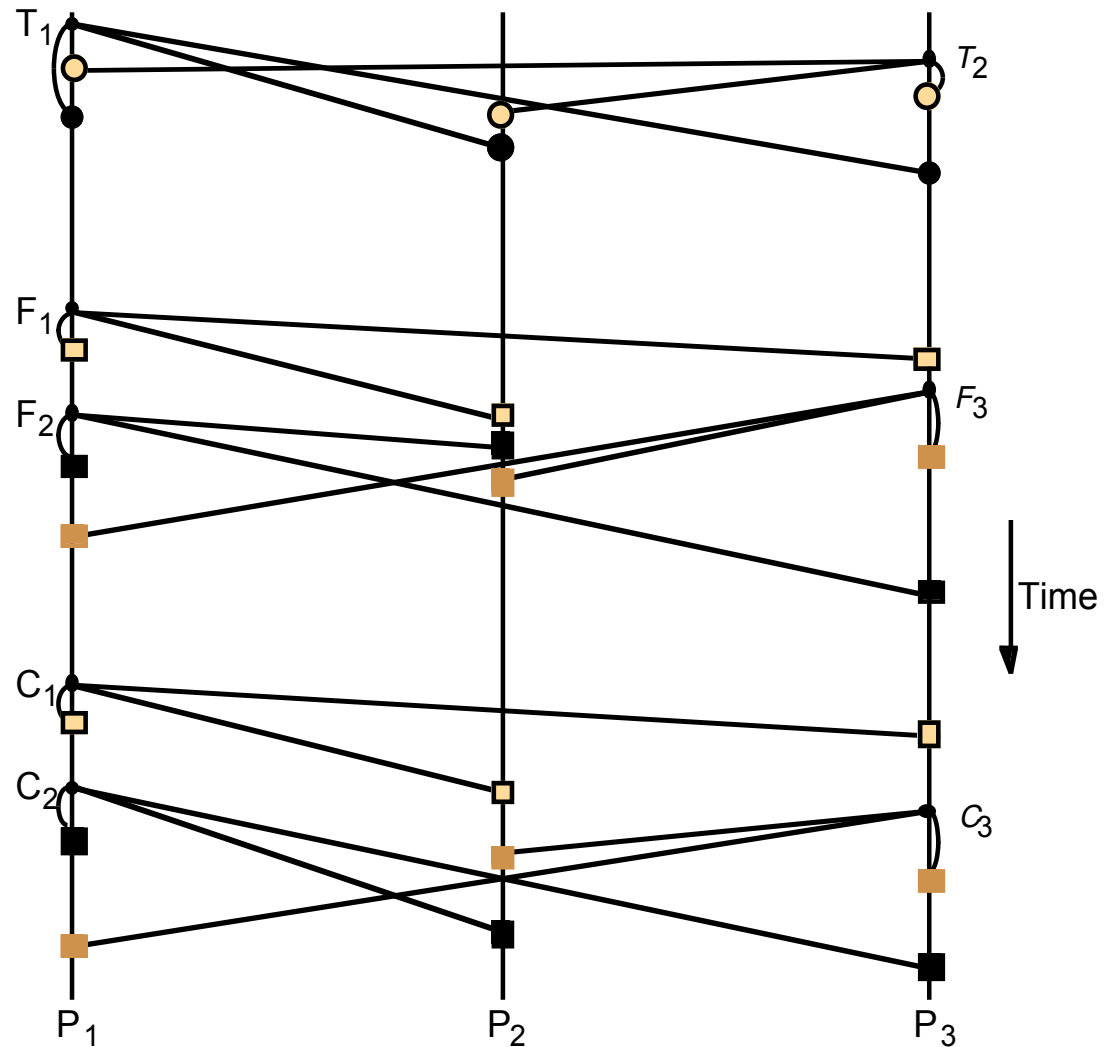
# Total, FIFO and Causal Ordering

The consistent ordering of totally ordered messages  $T_1$  and  $T_2$

The FIFO-related messages  $F_1$  and  $F_2$

The causally related messages  $C_1$  and  $C_3$

The otherwise arbitrary delivery ordering of messages



# Display from a Bulletin Board Program

- FIFO: Every message from same sender should be ordered
- Causal: Responses should appear after the posting
- Total: All the left-hand numbers should be the same on all users

Bulletin board: <i>os.interesting</i>		
Item	From	Subject
23	A.Hanlon	Mach
24	G.Joseph	Microkernels
25	A.Hanlon	Re: Microkernels
26	T.L' Heureux	RPC performance
27	M.Walker	Re: Mach
end		

# How Do We Achieve Ordering?

## **FIFO**

- Use sequence numbers and delay delivery until this number is reached

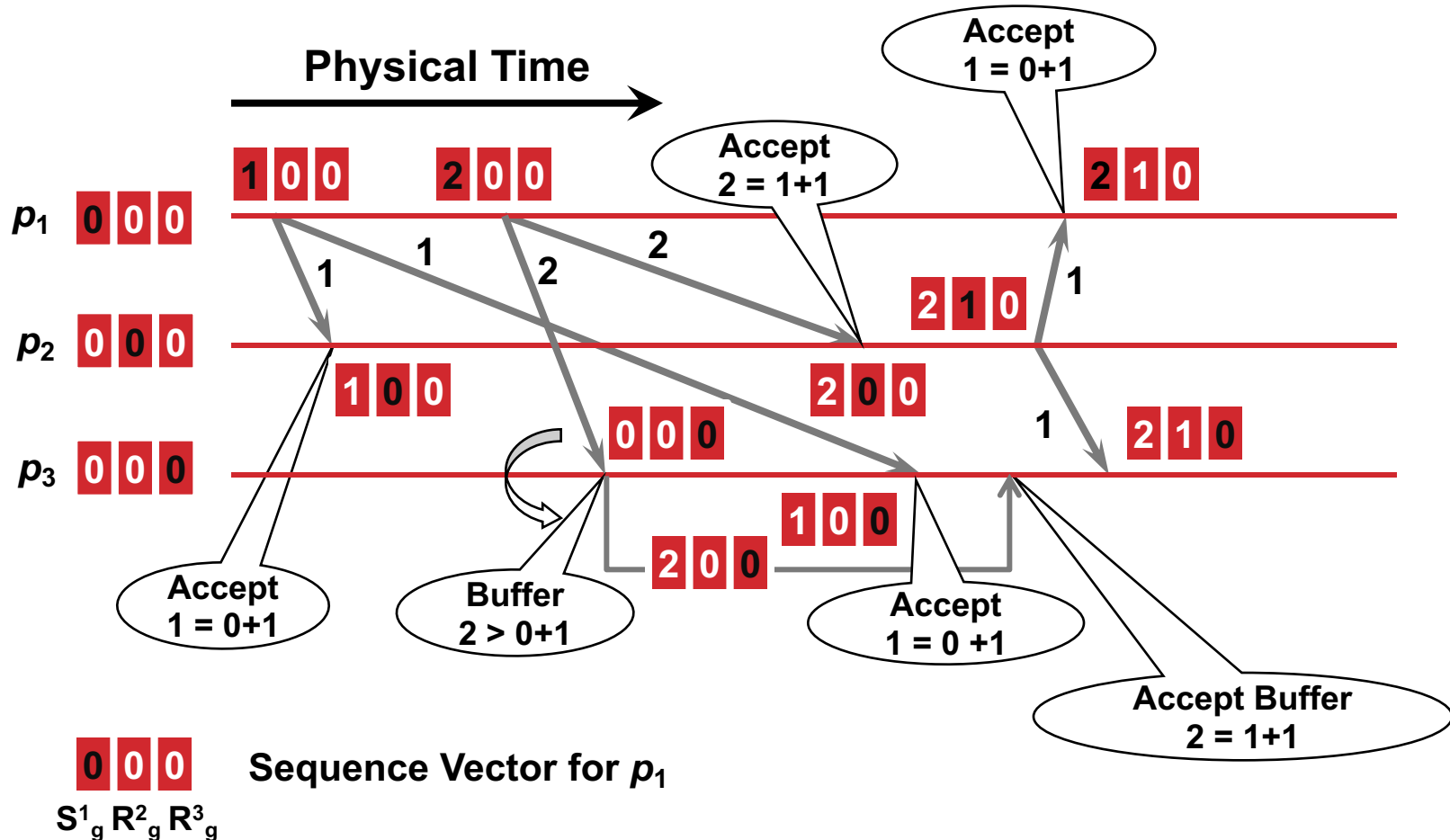
**For total ordering we need non-process specific sequence numbers**

# Ordered Multicast

## Implementing FIFO ordering

- $S_g^p$ : sequence number for group  $g$  that process  $p$  belongs to  $g$
- $R_g^p$ : sequence number of latest message that a process has delivered  $g$  from process  $p$  and that was sent to group  $g$
- *Assumption: non-overlapping groups*
- FO-multicast( $m, g$ )
  - Increment  $S_g^p$  by 1
  - B-multicast( $m, g, \langle S_g^p \rangle$ )
- Receipt of a message from  $q$  with sequence number  $S$ 
  - If  $S = R_g^p + 1$ , then this is the next message,
    - Therefore FO-deliver( $m$ )
    - $R_g^p := S$
  - If  $S > R_g^p + 1$ , then
    - Place message on hold-back queue until intervening messages have been delivered and  $S = R_g^p + 1$

# Example: FIFO Multicast



# Ordered Multicast

## Implementing total ordering

- Idea: assign totally ordered identifiers to multicast messages so that every process makes the same delivery decision based on these identifiers
- Delivery similar to FIFO delivery, only that group-specific sequence numbers rather than process-specific sequence numbers are used
- Assumption: non-overlapping groups
- Two main methods for the assignment of identifiers
  - Sequencer
  - Collective agreement on the assignment of message identifiers

# Ordered Multicast

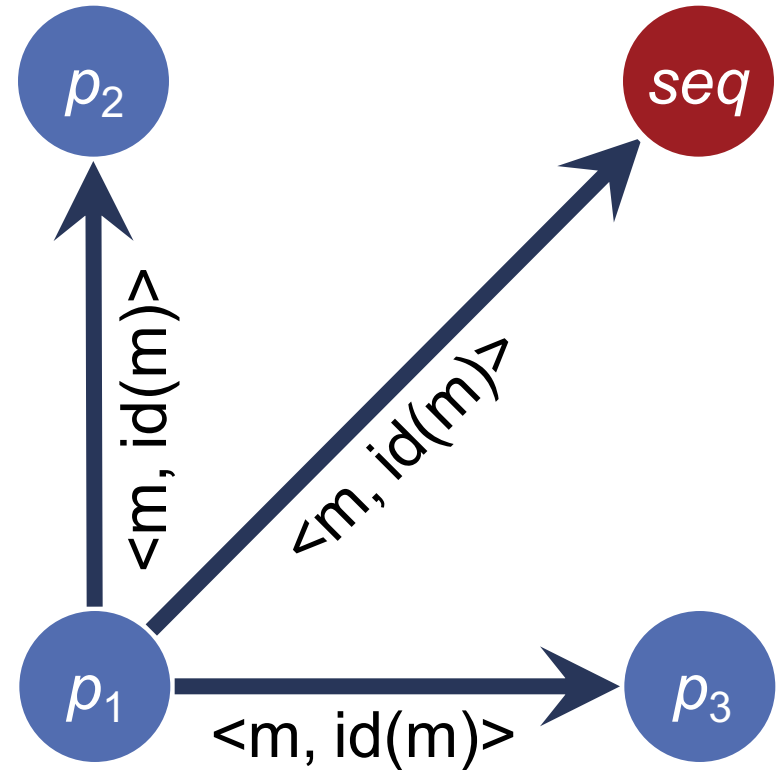
## Implementing total ordering

- Sequencer
- Process wishing to TO-multicast attaches a unique identifier  $ID(m)$  to the message
- Message is sent to sequencer as well as all members of  $g$
- Sequencer maintains group-specific sequence number  $s_g$  which it uses to assign increasing and consecutive sequence numbers to the messages it B-delivers
- Announces the order in which members of  $g$  have to deliver these messages using a B-multicast order message



# Total Ordering Using a Sequencer

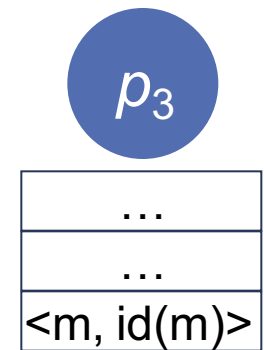
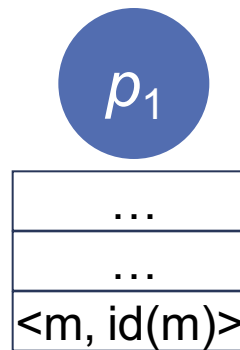
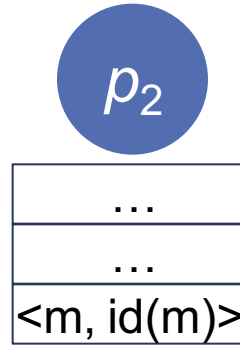
- To TO-multicast a message  $m$  to a group  $g$ ,  $p_1$  attaches a unique identifier  $\text{id}(m)$  to it
- The messages for  $g$  are sent to the sequencer for  $g$  and to the members of  $g$
- The sequencer may be a member of  $g$



# Total Ordering Using a Sequencer

## On B-deliver( $\langle m, \text{id}(m) \rangle$ )

- A process (but NOT the sequencer) places the message  $\langle m, \text{id}(m) \rangle$  in its hold-back queue



# Total Ordering Using a Sequencer

## Sequencer

- Maintains a group-specific sequence number  $s_g$
- Assigns increasing and consecutive sequence numbers to the messages that it B-delivers



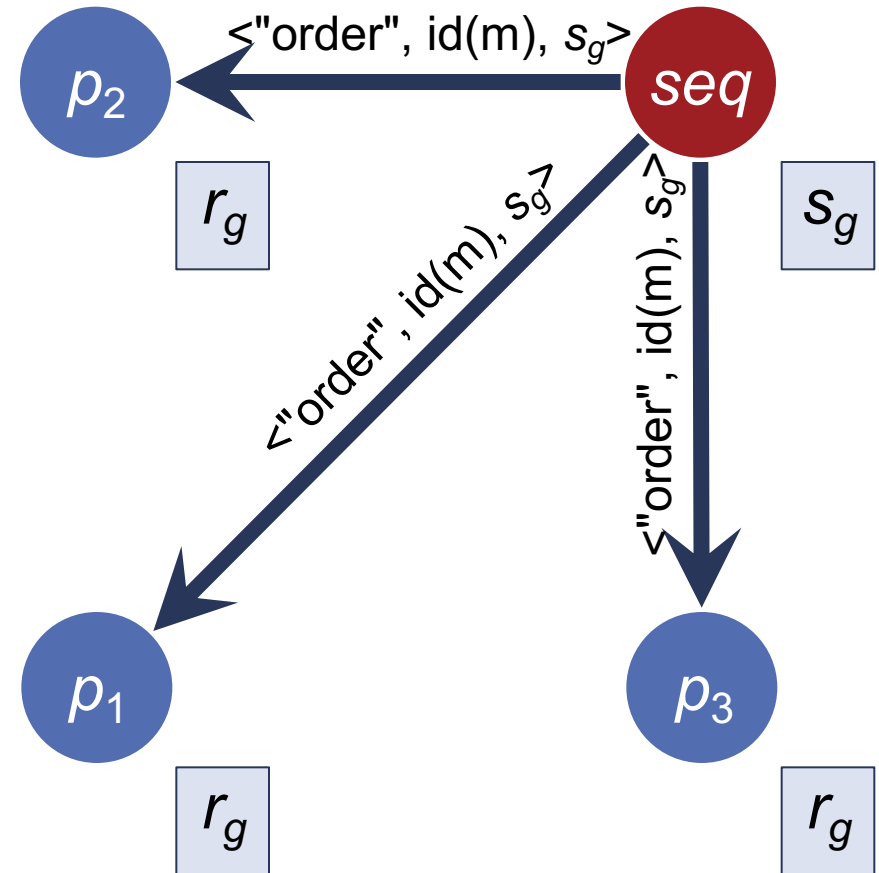
## Processes

- Have their local group-specific sequence number  $r_g$



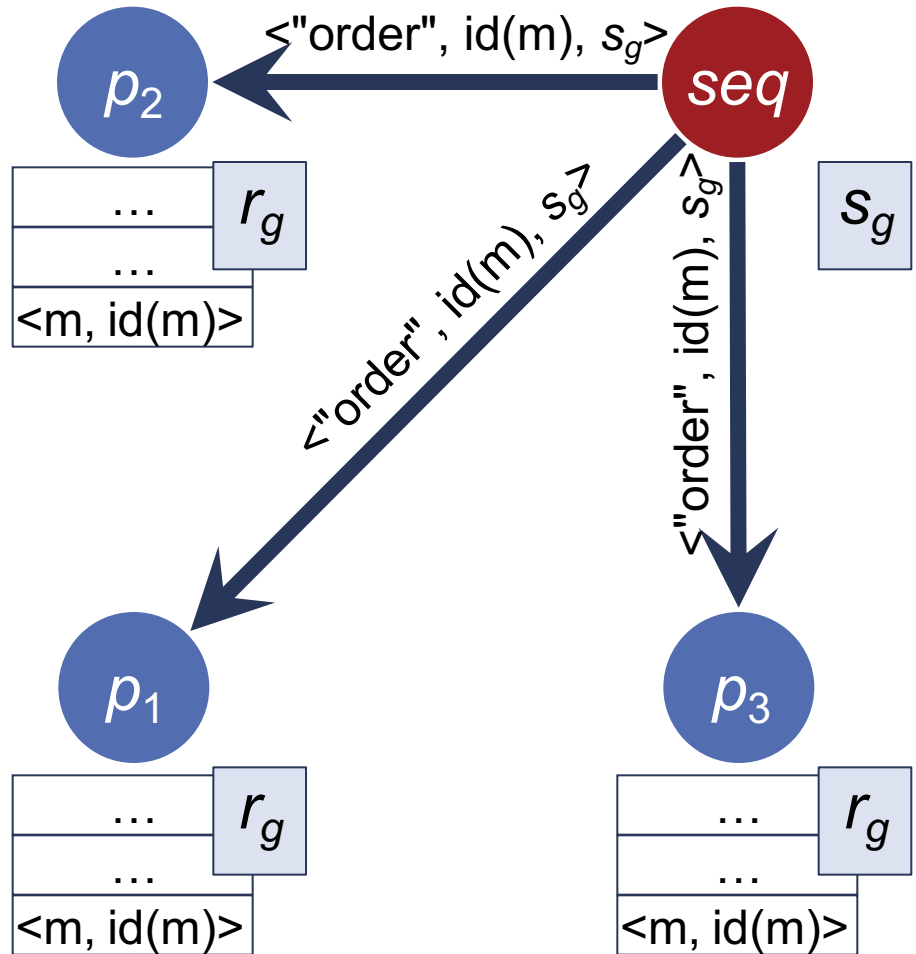
# Total Ordering Using a Sequencer

- On  $B\text{-deliver}(\langle m, \text{id}(m) \rangle)$  the sequencer announces the sequence numbers
- B-multicasting “order” messages to  $g$ .



# Total Ordering Using a Sequencer

- A message will remain in a hold-back queue until it can be TO-delivered
- To-delivery: if the corresponding sequence number  $s_g = r_g$



# Total Ordering Using a Sequencer

## Algorithm for group member p

*On initialization:*  $r_g := 0$ ;

*To TO-multicast message m to group g*

$B\text{-multicast}(g \cup \{\text{sequencer}(g)\}, \langle m, i \rangle)$ ;

*On B-deliver( $\langle m, i \rangle$ ) with  $g = \text{group}(m)$*

Place  $\langle m, i \rangle$  in hold-back queue;

*On B-deliver( $m_{\text{order}} = \langle \text{"order"}, i, S \rangle$ ) with  $g = \text{group}(m_{\text{order}})$*

wait until  $\langle m, i \rangle$  in hold-back queue and  $S = r_g$ ; **Ensures total ordering**

$TO\text{-deliver } m$ ; // (after deleting it from the hold-back queue)

$r_g = S + 1$ ;

## Algorithm for sequencer of g

*On initialization:*  $s_g := 0$ ;

*On B-deliver( $\langle m, i \rangle$ ) with  $g = \text{group}(m)$*

$B\text{-multicast}(g, \langle \text{"order"}, i, s_g \rangle)$ ;

$s_g := s_g + 1$ ;

# Total Ordering Using Distributed Agreement

## Sequencer-based approach

- Sequencer may become a bottleneck
- Sequencer is a critical point of failure
- There are algorithms to address this problem

## Approach without a sequencer

- Processes collectively agree on the assignment of sequence numbers to messages in a distributed fashion
- Receiving processes bounce proposed sequence numbers to sender
- Sender returns agreed sequence numbers
- Each process  $q$  in group  $g$  maintains
  - $A_g^q$ : the largest agreed sequence number it has observed so far for group  $g$
  - $P_g^q$ : own largest proposed sequence number

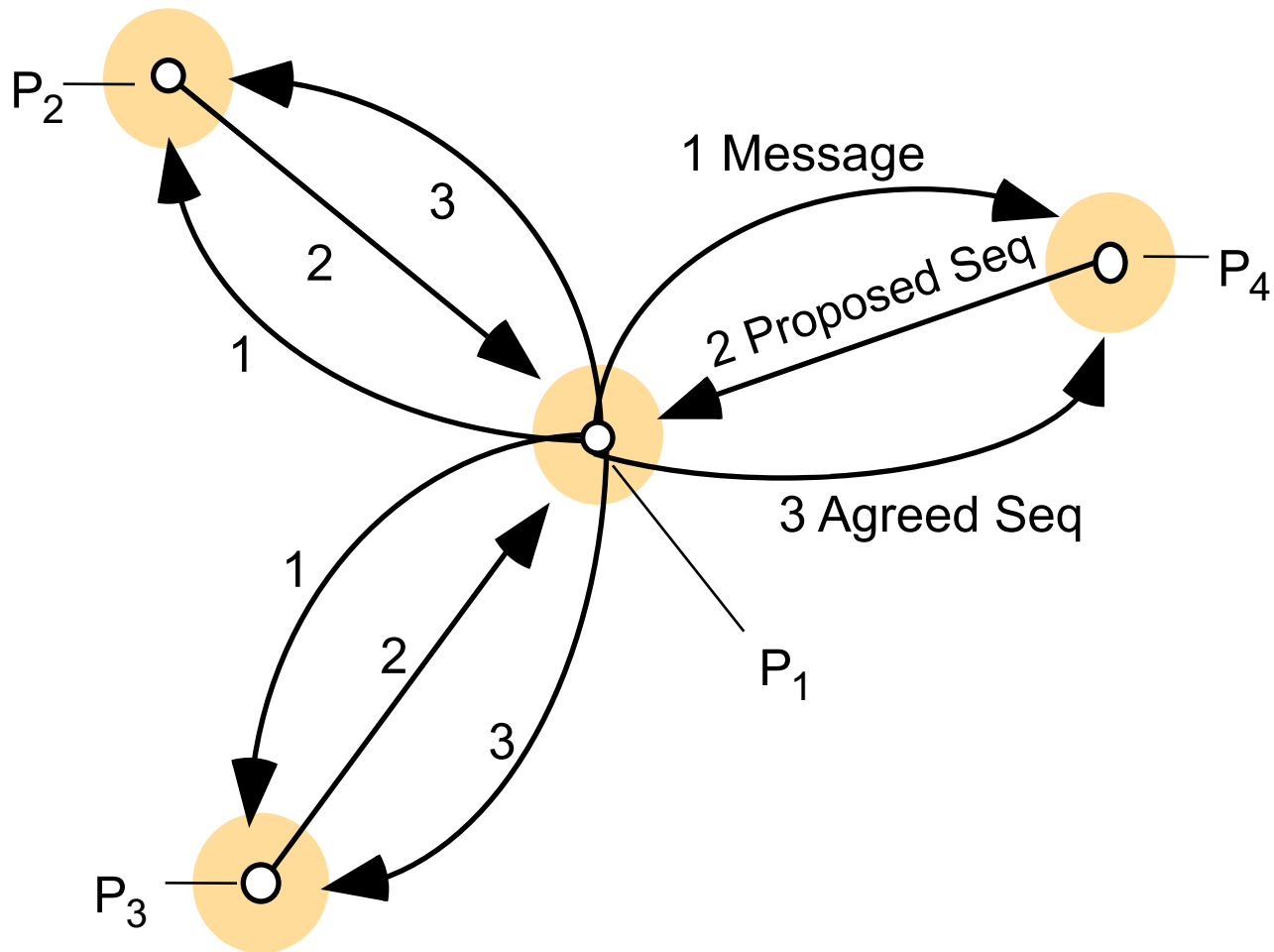
# Total Ordering Using Distributed Agreement

## Collective agreement on message identifiers

- Each process  $q$  in group  $g$  keeps  $A_g^q$ , its largest observed agreed sequence number, and  $P_g^q$ , its own largest proposed number
- $p$  B-multicasts  $\langle m, i \rangle$  to  $g$ , where  $i$  is unique identifier for  $m$
- Each recipient  $q$  replies to  $g$  with proposed agreed sequence number
  - $P_g^q := \max(A_g^q, P_g^q) + 1$
  - Each process  $q$  provisionally assigns own proposed sequence number to message and queues message in hold back queue, ordered according to proposed sequence number
- $p$  chooses largest proposed number as sequence number  $a$
- $p$  B-multicasts  $\langle i, a \rangle$  to  $g$
- Each process  $q$  in group
  - Sets  $A_g^q := \max(A_g^q, a)$
  - Reorders received message in hold-back queue if received sequence number differs from proposed number
  - Only when message at head of hold-back queue is assigned an agreed sequence number, it will be queued in delivery queue



# The ISIS algorithm for total ordering



# Implementing Causal Ordering

## Causal ordering

- Takes into account of the happened-before relationship only established by multicast messages

## Idea

- Each process  $p$  maintain its own vector timestamp for the group  $g$
- $|g|$  entries count the number of multicast messages from each process that happened-before the next message to be multicast
- $V_i^g[j]$  counts the number of group  $g$  messages from  $p_j$  to  $p_i$

## Review Vector Timestamps

- If process  $p_i$  receives a message  $\langle m, V_j^g \rangle$  from process  $p_j$ , then
  - $V_i^g[k] = \max(V_i^g[k], V_j^g[k])$  if  $k \neq i$
  - $V_i^g[k] = V_i^g[k] + 1$  if  $k = i$
- Remember  $V(a) < V(b)$  iff event  $a$  happens before event  $b$

# Causal Ordering Using Vector Timestamps

Algorithm for group member  $p_i$  ( $i = 1, 2, \dots, N$ )

*On initialization*

→  $V_i^g[j] := 0$  ( $j = 1, 2, \dots, N$ );

The number of messages in group  $g$  from process  $p_j$  that have been seen at process  $p_i$  so far

*To CO-multicast message  $m$  to group  $g$*

$V_i^g[i] := V_i^g[i] + 1$ ;

$B\text{-multicast}(g, \langle V_i^g, m \rangle)$ ;

$p_i$  has delivered any message that  $p_j$  had delivered

*On  $B\text{-deliver}(\langle V_j^g, m \rangle)$  from  $p_j$ , with  $g = \text{group}(m)$*

place  $\langle V_j^g, m \rangle$  in hold-back queue;

wait until  $V_j^g[j] = V_i^g[j] + 1$  and  $V_j^g[k] \leq V_i^g[k]$  ( $k \neq j$ );

$CO\text{-deliver } m$ ; // after removing it from the hold-back queue

$V_i^g[j] := V_i^g[j] + 1$ ;

$p_i$  has delivered any earlier message sent by  $p_j$

*Example:*  $V^{g_2} = [3, 6, 2]$  from  $p_2$  is received by  $p_3$  with  $V^{g_3} = [2, 5, 2]$ , i.e.,  $p_3$  needs to deliver a message from  $p_1$  first

# Overlapping Groups

**We assumed that the multicast groups do not overlap**

- Obvious solution: multicast to all but this is not practical
- More details in the textbook ...

# Consensus

## Agreement on a value or action to be taken

- Mutual exclusion: processes agree on the process that enters critical section
- Election: processes agree on elected process
- Totally ordered multicast: processes agree on the order of message delivery
- Banking: processes agree on whether or not to perform a transaction such as debit or credit

## Byzantine process failures

- Processes fail, but may still respond with arbitrary, erratic behavior
- Examples: software bugs or malicious attacks

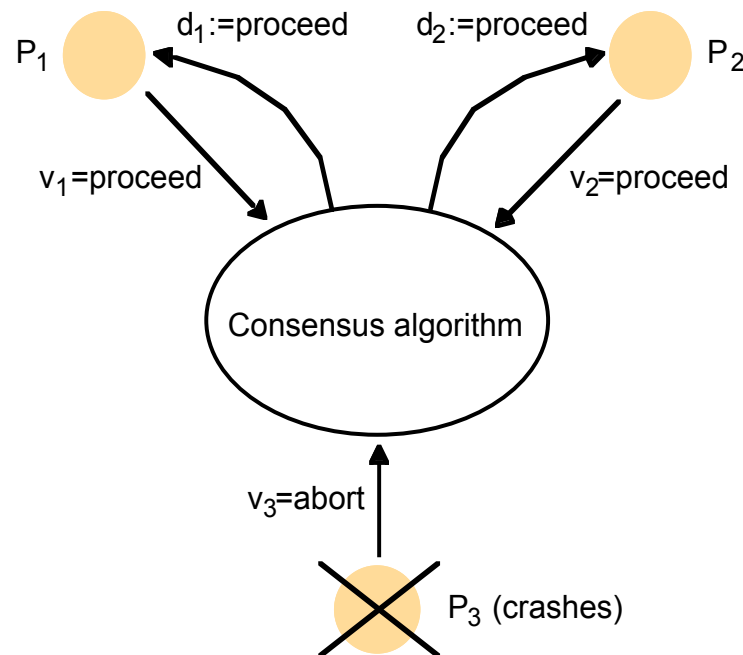
## Digital signatures

- During an agreement algorithm a faulty process cannot make a false claim about the values that a correct process has sent to it

# Consensus for three processes

**Agreement in the value of a decision variable amongst all correct processes**

- $p_i$  is in state undecided and proposes a single value  $v_i$  drawn from a set  $D$
- Processes communicate with each other to exchange values
- $p_i$  sets decision variable  $d_i$  and enters the decided state after which the value of  $d_i$  remains unchanged



# Consensus Problem: Properties

## Termination

- Eventually each process sets its decision variable

## Agreement

- The decision value is the same on all processes
- For all correct  $p_i$  and  $p_j$  holds  $d_i = d_j$

## Integrity

- If the correct processes all proposed the same value, then any correct process in the decided state has chosen that value

# Consensus: Majority

## Majority algorithm

- Solve consensus in a *failure-free* environment
- Each process *reliably multicasts* proposed values
- After receiving response, solves consensus function
  - `majority( $v_1, \dots, v_N$ )`
  - Returns most often proposed value or undefined ( $\perp$ ) if there is no majority

## Properties

- Termination guaranteed by reliability of multicast
- Agreement, integrity: definition of majority, and integrity of reliable multicast (all processes apply the same function to the same data)

## Other functions

- Minimum or maximum instead of majority



# Byzantine Generals

## Problem

- Three or more generals are to agree on an attack or retreat
- The commander issues order and the lieutenants have to decide to attack or retreat
- One of the generals may be treacherous

## Treacherous commander

- Proposes attack to one general and retreat to the other

## Treacherous lieutenants

- Tell one of their peers that commander ordered to attack, and others that commander ordered to retreat

## Difference to consensus problem

- One process supplies a value that others have to agree on (other processes cannot propose a value)

## Integrity

- *If* the commander is correct, then all correct processes decide on the value proposed by the commander (but the commander may not be correct)

# Interactive Consistency

## Goal

- All correct processes agree on a *vector of values*, each component corresponding to one process' agreed value
- Example: agreement about each process' local state

## Requirements

- Termination: eventually each correct process sets its decision variable
- Agreement: the decision vector of all correct processes is the same
- Integrity: if  $p_i$  is correct, then all correct processes decide on  $v_i$  as the  $i$ -th component of their vector

# Relationship of Consensus to Other Problems

## Decision variables

- $C_i(v_1, \dots, v_N)$  returns the decision value of  $p_i$  as a solution to the consensus problem, where  $v_1, \dots, v_N$  are the proposed values of the processes
- $BG_i(j, v)$  returns the decision value of  $p_i$  where  $p_j$  is the commander proposing value  $v$
- $IC_i(v_1, \dots, v_N)[j]$  returns the  $j$ -th value in the decision vector of  $p_i$  where  $v_1, \dots, v_N$  are the proposed values of the processes

# Relationship of Consensus to Other Problems

## BG $\rightarrow$ IC

- Run BG  $N$  times, once with each  $p_i$  acting as commander  
 $IC_i(v_1, \dots, v_N)[j] = BG_i(j, v_j)$

## IC $\rightarrow$ C

- Run IC to produce a vector of values at each process
- Apply an appropriate function on the vector's values to derive a single value  $C_i(v_1, \dots, v_N) = \text{majority}(IC_i(v_1, \dots, v_N)[1], \dots, IC_i(v_1, \dots, v_N)[N])$

## C $\rightarrow$ BG

- Commander  $p_j$  sends its proposed value  $v$  to itself and of the remaining processes
- All processes run C with the values  $v_1, \dots, v_N$  that they receive
- $BG_i(j, v) = C_i(v_1, \dots, v_N)$

# Consensus from RTO Multicast

## Idea

- Implementing consensus through a reliable and totally ordered multicast operation RTO-multicast

## Algorithm

- Assumption: all processes form a group
- $p_i$  performs RTO-multicast  $(v_i, g)$
- $p_i$  sets  $d_i = m_i$ , where  $m_i$  is the first value that  $p_i$  RTO-delivers

## Properties

- Termination guaranteed by reliable multicast
- Agreement and validity because reliability and total ordering of multicast delivery

## RTO multicast from consensus

- Chandra and Toueg [1996]

# Consensus in Synchronous system: Multicast

## Assumption

- No more than  $f$  of the  $N$  processes crash

## Algorithm proceeds in $f+1$ rounds

- Processes B-multicast values between them
- At the end of  $f+1$  rounds, all surviving processes agree

Algorithm for process  $p_i \in g$ ; algorithm proceeds in  $f + 1$  rounds

*On initialization*

$Values_i^1 := \{v_i\}; Values_i^0 = \{\};$

*In round  $r$  ( $1 \leq r \leq f + 1$ )*

$B\text{-multicast}(g, Values_i^r - Values_i^{r-1});$  // Send only values that have not been sent

$Values_i^{r+1} := Values_i^r;$

*while (in round  $r$ )*

{

*On B-deliver( $V_j$ ) from some  $p_j$*   
 $Values_i^{r+1} := Values_i^{r+1} \cup V_j;$

}

*After  $(f + 1)$  rounds*

Assign  $d_i = \text{minimum}(Values_i^{f+1});$

# Consensus in Synchronous system: Multicast

## Dolev-Strong algorithm

- $Values_i^r$ : set of proposed values known to process  $p_i$  before round  $r$
- Every process multicasts the set of values it has not sent in previous rounds
- Every process takes delivery of values from other processes
- At the end of  $f+1$  rounds: each process chooses minimum value

Algorithm for process  $p_i \in g$ ; algorithm proceeds in  $f + 1$  rounds

*On initialization*

$Values_i^1 := \{v_i\}; Values_i^0 = \{\};$

*In round  $r$  ( $1 \leq r \leq f + 1$ )*

$B\text{-multicast}(g, Values_i^r - Values_i^{r-1});$  // Send only values that have not been sent

$Values_i^{r+1} := Values_i^r;$

*while (in round  $r$ )*

{

*On B-deliver( $V_j$ ) from some  $p_j$*   
 $Values_i^{r+1} := Values_i^{r+1} \cup V_j;$

}

*After  $(f + 1)$  rounds*

Assign  $d_i = \text{minimum}(Values_i^{f+1});$

# Dolev-Strong Algorithm

## Correctness

- Is every process going to arrive at the same set of values?
- If yes: integrity and agreement will follow, since processes consistently apply the minimum function to this set

## Proof sketch

- Assume two processes differ in their final set of values (with  $f$  crashes)
- Thus: some correct process  $p_i$  has a value  $v$  that another correct process  $p_j$  ( $i \neq j$ ) does not have
- This means that some other process  $p_m$ , which sent  $v$  to  $p_i$  crashed before  $v$  could be delivered to  $p_j$
- In turn: any process sending  $v$  in the previous round must have crashed, i.e., at least one crash per round
- We have  $f+1$  rounds, at most  $f$  crashes ... hence contradiction!

## General property in synchronous systems

- Any algorithm to reach consensus, tolerating up to  $f$  crashes or byzantine failures, requires at least  $f+1$  rounds

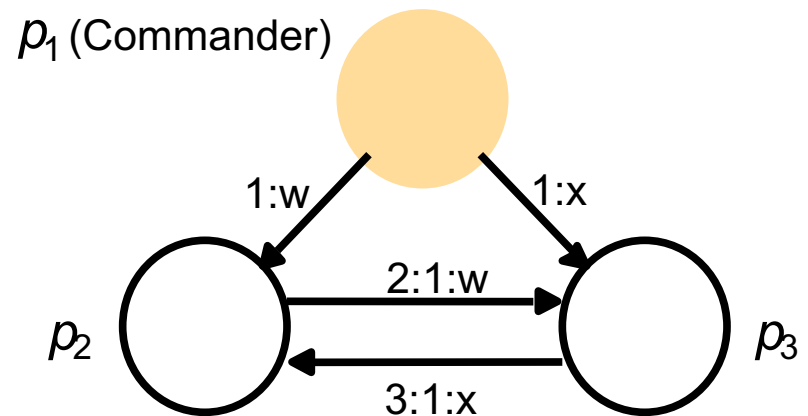
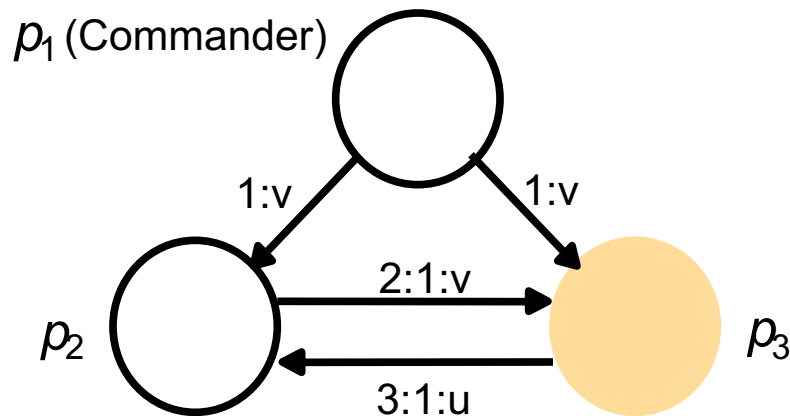


# Impossibility of three Byzantine Generals

## Impossibility for $N = 3$ and $f = 1$

- '3:1:u' is the message '3 says (1 says u)'
- Both scenarios show two rounds of messages
- Left:  $p_2$  knows is that it has received two different values ( $p_3$  is faulty)
- Right: same situation, but commander is faulty
- Assume a solutions exists:  $p_2$  decides on w,  $p_3$  on x. Contradiction!

## Impossibility for $N \leq 3f$



Faulty processes are shown coloured

# Solution For $N \geq 4$ Byzantine Generals

## Correct generals reach agreement in two rounds

- First, commander sends value to each lieutenant
- Second, each lieutenant sends value it received to all peers
- Lieutenant receives
  - Value from commander
  - $N-2$  values from peers

## Commander is faulty

- All lieutenants correct: each will gather exactly the set of values the commander sent out

## 1 lieutenant is faulty

- each of its peers receives  $N-2$  copies of the value the commander sent out, plus the faulty lieutenant value

# Solution For 4 Byzantine Generals

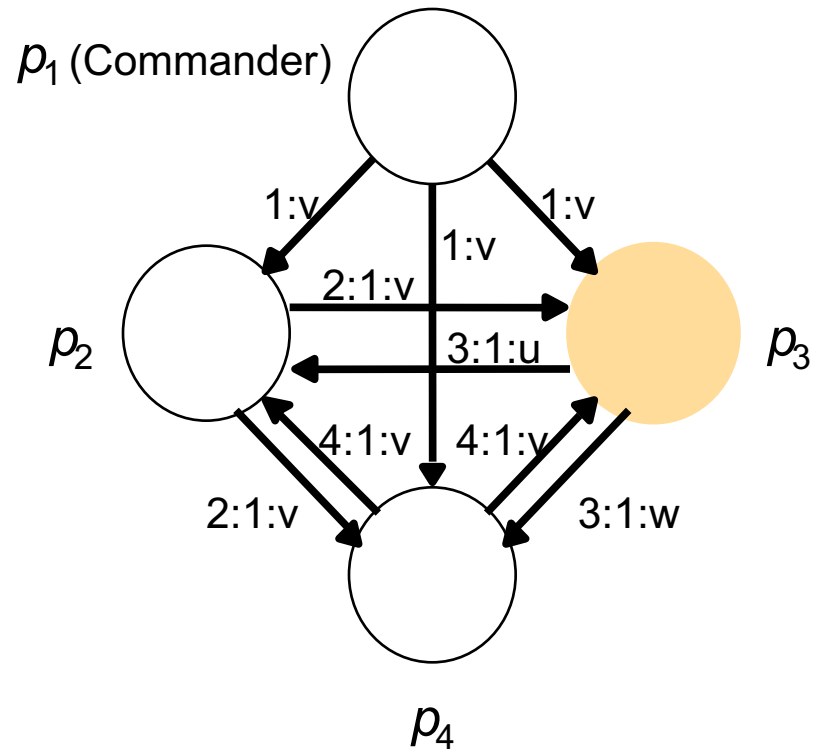
## Reaching agreement

- Simple majority function
- Commander is correct
  - Since  $N \geq 4$ ,  $N-2 \geq 2$ , majority function will ignore value of faulty lieutenant
  - Majority function produces value of commander
- Commander is incorrect
  - There is no majority and the majority function will produce  $\perp$
  - Note: BG requires agreement only if commander correct

# Solution For 4 Byzantine Generals

$p_2$ : majority( $\{v, u, v\}$ ) =  $v$

$p_4$ : majority( $\{v, v, w\}$ ) =  $v$



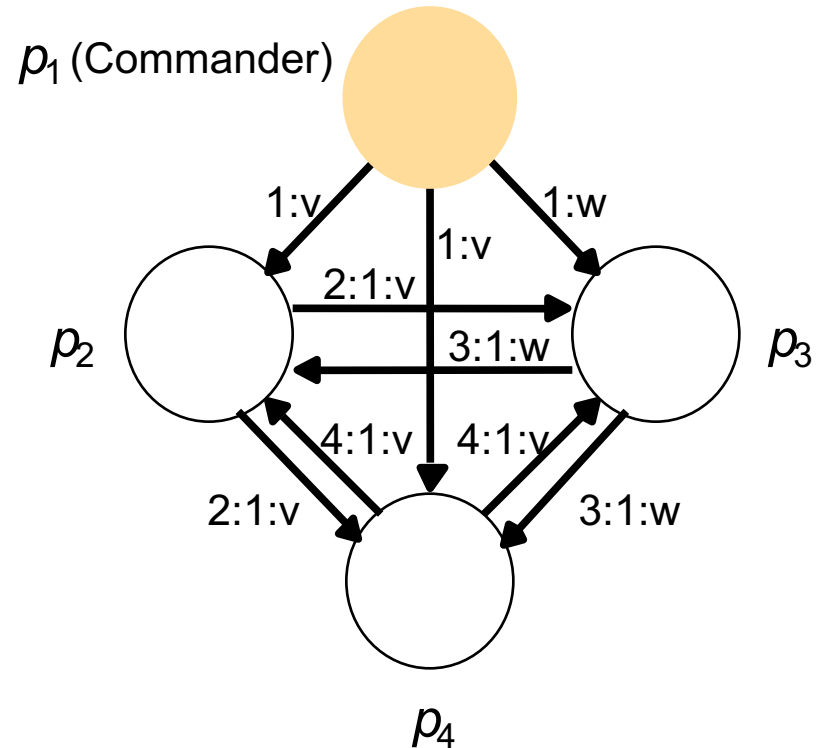
Faulty processes are shown coloured

# Solution For 4 Byzantine Generals

$p_2$ : majority( $\{v, w, v\}$ ) =  $v$

$p_3$ : majority( $\{v, v, w\}$ ) =  $v$

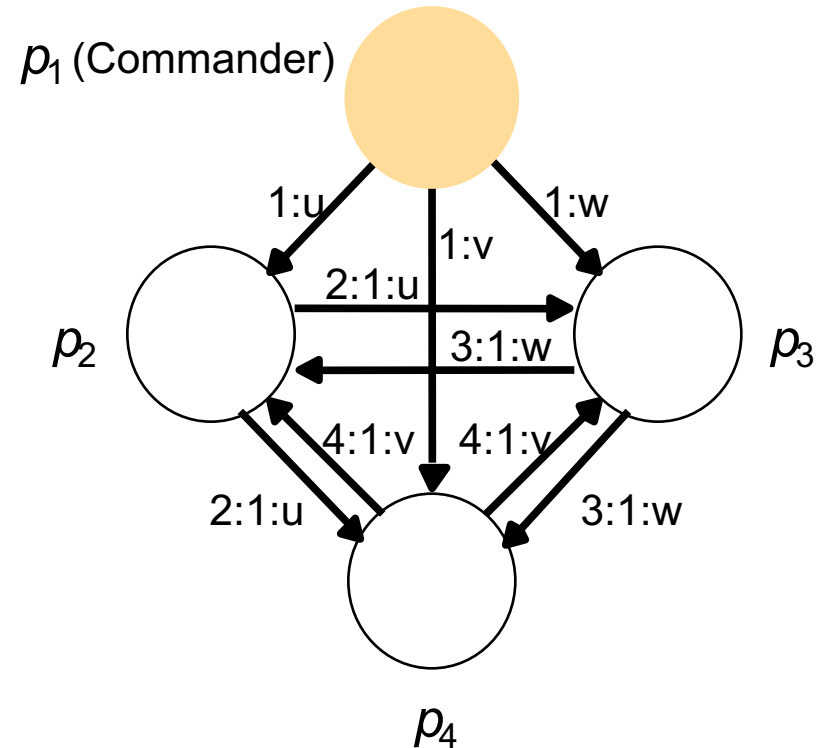
$p_4$ : majority( $\{w, v, v\}$ ) =  $v$



Faulty processes are shown coloured

# Solution For 4 Byzantine Generals

$p_2, p_3, p_4$ :  
 $\text{majority}(\{u, v, w\}) = \perp$



# Impossibility in Asynchronous Systems

## Previous algorithms: synchronous systems

- Message exchanges in rounds
- Timeouts

## No algorithm can guarantee consensus

- Even with a single process crash failure (Fischer, Lynch and Paterson, 1985)
- Proof idea: show that there is always some continuation of the process's execution that avoids consensus being reached
- But: in practice consensus can often be reached, but a small probability remains that consensus cannot be reached

# Asynchronous Systems ...

## Reaching consensus by weakening system assumptions

- *Masking faults*
  - Design system so that failures appear like intermittent slowdown in processing of messages
    - Store system state on persistent storage before crash
    - Restart system in that state after recovery
- *Modified failure detectors*
  - Deem process that has not responded as failed
  - Treat this process as fail-safe, i.e., discard any subsequent messages from this process
- Problems
  - Long timeouts necessary
  - False negatives possible that reduce effectiveness of system
- Turn an asynchronous system into a synchronous system



# Asynchronous Systems ...

## Consensus using randomization

- Adversary: interferes with the processes' attempts to reach consensus
  - Manipulates the network to delay messages so that they arrive at just the wrong time,
  - Slows down or speeds up the processes just so that they are in the 'wrong' state when they receive a message
- Use element of chance
  - Adversary cannot exercise its counter-strategy
  - Consensus might still not be reached in some cases
  - But: processes can reach consensus in a finite expected time