Q1 (a) (i)

$$3 = a - b + c$$
$$1 = a$$
$$4 = a + b + c$$

(ii) Introducing the augmented matrix form, and reducing to RE form, we have

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 4 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 + 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 2 & 5 \end{array}\right]$$

Since rank $= 3 =$ number of unknowns, there is a unique sol$^{\underline{n}}$. Back substitution gives

$$c = \frac{5}{2}$$
$$b = -2 + \frac{5}{2} = \frac{1}{2}$$
$$a = 3 - \frac{5}{2} + \frac{1}{2} = 1$$

(b) (i)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] R_2 \mapsto -\tfrac{1}{2}R_2 \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & -3/2 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 7/2 \\ 0 & 0 & 1 & -3/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(ii) No leading entry for β, so set $\beta = t$, $t \in \mathbb{R}$

Back substitution then gives

$\gamma = -3/2$, $\alpha = 7/2 - t$.

Q2 (a) To form ACB, A must have n columns and B must have n rows.

To form BCA, B must have n columns and A must have n rows.

Hence both A and B must be of size $n \times n$.

(b) (i) YZX is not possible as Y has one column while Z has two rows

(ii) $$ZZ^T = \begin{bmatrix} 1 & -3 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -4 \\ -4 & 2 \end{bmatrix}$$

(c) We have $\det AB = \det A \det B$

$= (\det A) \times 0$ since B is singular

$= 0$

Hence AB is singular.

Q3 (a) $$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_2 - R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_3 \leftrightarrow R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & -1 & -1 & 1 & 0 \end{array}\right] R_3 + 2R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 2 \end{array}\right] (-1)R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & -2 \end{array}\right] R_1 - R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & -2 \end{array}\right] R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & -2 \end{array}\right]$$

(b)

$$A^{-1}\begin{bmatrix} 15 & 1 & 19 \\ -4 & 0 & -3 \\ 8 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & -1 & -2 \end{bmatrix}\begin{bmatrix} 15 & 1 & 19 \\ -4 & 0 & -3 \\ 8 & 0 & 8 \end{bmatrix}$$

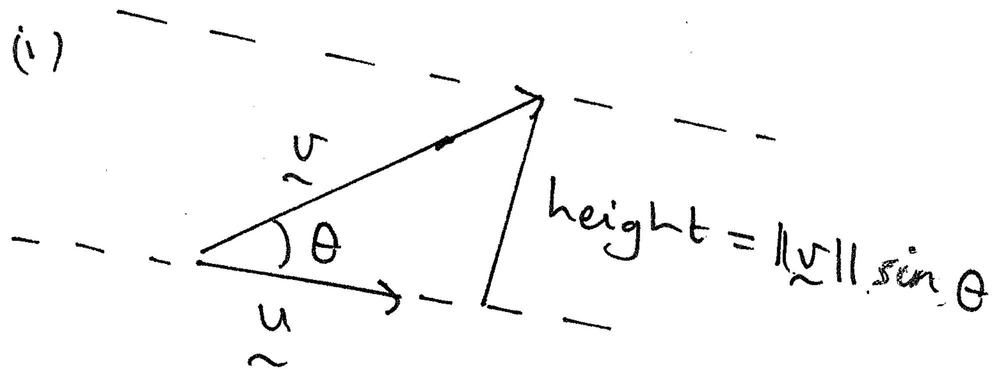
$$= \begin{bmatrix} 4 & 0 & 5 \\ 8 & 0 & 8 \\ 3 & 1 & 6 \end{bmatrix}$$

Hence the mobile number is 0483001586

(c) $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 1 & 6 \\ 0 & 7 & 7 \\ 2 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 14 & 22 \\ 4 & -6 & -1 \\ 0 & 7 & 7 \end{bmatrix}$$

$\Rightarrow$ coded mobile number 6, 4, 0, 14, −6, 7, 22, −1, 7

Q4 (a) (i)



$$\begin{aligned}
\text{area} &= \text{base times height} \\
&= \|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\| \sin\theta \\
&= \|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\| \sqrt{1-\cos^2\theta} \\
&= \|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\| \sqrt{1-\left(\frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\|}\right)^2} \\
&= \sqrt{\|\underset{\sim}{u}\|^2\|\underset{\sim}{v}\|^2-(\underset{\sim}{u}\cdot\underset{\sim}{v})^2}
\end{aligned}$$

Hence, with $\underset{\sim}{u}=(3,-1,4)$, $\underset{\sim}{v}=(2,1,2)$ so that

$$\|\underset{\sim}{u}\|^2 = 9+1+16 = 26 \qquad \Rightarrow \|\underset{\sim}{u}\|^2\|\underset{\sim}{v}\|^2 = 234$$

$$\|\underset{\sim}{v}\|^2 = 4+1+4 = 9$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = 3\times 2 - 1\times 1 + 4\times 2 = 13 \quad \Rightarrow (\underset{\sim}{u}\cdot\underset{\sim}{v})^2 = 169$$

we have

$$\text{area} = \sqrt{234-169} = \sqrt{65}$$

(b) The volume is the absolute value of the determinant

$$\begin{vmatrix} k & k+1 & k+2 \\ k+3 & k+4 & k+5 \\ k+6 & k+7 & k+8 \end{vmatrix}_{\substack{R_2-R_1 \\ R_3-R_1}} = \begin{vmatrix} k & k+1 & k+2 \\ 3 & 3 & 3 \\ 6 & 6 & 6 \end{vmatrix}_{R_3-2R_2}$$

$$= \begin{vmatrix} k & k+1 & k+2 \\ 3 & 3 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0.$$

5(a)(i) Since, with the given vectors along the rows of a matrix:

$$\begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

the rank is 2, we have that the vectors are linearly independent. The dimension of the span is therefore equal to 2.

(ii) The vectors are in $\mathbb{R}^4$, so the span is a subspace of $\mathbb{R}^4$.

(b) With $p(x) = a + bx + cx^2$

we have $p'(x) = b + 2cx$

Hence $p'(1) = 0 \Rightarrow b + 2c = 0$. Since we have the correspondence

$$a + bx + cx^2 \leftrightarrow (a, b, c)$$

we see that in $\mathbb{R}^3$, S is equivalent to the set

$$S = \{ (a, b, c) \in \mathbb{R}^3 : b + 2c = 0 \}$$

To write this as a span, we note

$$S = \{ (a, b, -b/2), \ a, b \in \mathbb{R} \}$$

$$= \{ a(1, 0, 0) + b(0, 1, -1/2), \ a, b \in \mathbb{R} \}$$

$$= \text{Span}\{ (1, 0, 0), (0, 1, -1/2) \}$$

Since all spans are subspaces, it follows that this form of S is a subspace of $\mathbb{R}^3$.

(c) Let $(x_1, y_1, -2x_1)$ where $x_1, y_1 \in \mathbb{R}$

and $(x_2, y_2, -2x_2)$ " $x_2, y_2 \in \mathbb{R}$

be arbitrary vectors in the set R.

Then we have

$$(x_1, y_1, -2x_1) + (x_2, y_2, -2x_2)$$

$$= (x_1+x_2, y_1+y_2, -2(x_1+x_2))$$

$$= (x_3, y_3, -2x_3) \quad \text{where } x_3 = x_1+x_2 \in \mathbb{R}$$

$$\in S \qquad\qquad y_3 = y_1+y_2 \in \mathbb{R}$$

Hence S is closed under vector addition.

6. (a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -3 \\ 12 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \\ -14 \end{bmatrix} \right\}$

(b) No, since there are only 3 vectors in the column spaces whereas 4 are required to span $\mathbb{R}^4$.

(c) No. We have

$$-\underset{\sim}{v}_2 + 2\underset{\sim}{v}_1 = \underset{\sim}{v}_3$$

(d) We have that

dim (column space) + dim (sol$^{\underline{n}}$ space) = # columns
= 5

From (a),

dim (column space) = 3

and so dim (sol$^{\underline{n}}$ space) = 2.

(e) Let the unknowns be denoted x_1, x_2, x_3, x_4, x_5.

We have no leading entry for x_5, so we set
$$x_5 = t, \quad t \in \mathbb{R}$$
We have no leading entry for x_3, so we set
$$x_3 = s, \quad s \in \mathbb{R}$$
Now using back substitution shows that
$$x_4 = -t$$
$$x_2 = s - t$$
$$x_1 = -2s - t$$
Hence the sol^{n} space is equal to
$$\{(-2s-t, s-t, s, -t, t) : s, t \in \mathbb{R}\}$$
$$= \{s(-2, 1, 1, 0, 0) + t(-1, -1, 0, -1, 1) : s, t \in \mathbb{R}\}$$
$$= \text{Span}\{(-2, 1, 1, 0, 0), (-1, -1, 0, -1, 1)\}$$
Hence a basis is
$$\{(-2, 1, 1, 0, 0), (-1, -1, 0, -1, 1)\}$$

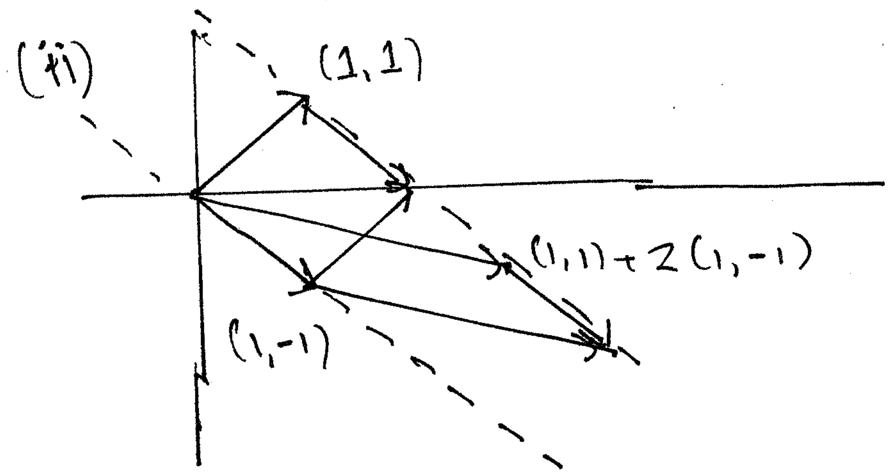
(f) We observe that
$$\begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -5 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix} \begin{array}{l} \\ R_2 \times (-1) \\ \\ R_4 \times 2 \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ 2 & 5 & -1 & 1 & 5 \\ 0 & -3 & 3 & 4 & 1 \\ 6 & 12 & 0 & -14 & 4 \end{bmatrix} = A$$
Thus the fully reduced row echelon form is equal to B.

Q7 (a) (i)

$$[T]_{B,B} = \left[[T(1,1)]_B \quad [T(1,-1)]_B \right]$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

(ii)



(iii) We know that $\det[T]_{B,B}$ is the signed factor by which the area of a parallelogram changes under the transformation T. According to the formula base×height, we see that the area doesn't change, and so $\det[T]_{B,B} = 1$.

(b) (i) We have

$$A_S = [S\underset{\sim}{i} \quad S\underset{\sim}{j}] = [\underset{\sim}{j} \quad \underset{\sim}{i}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(ii) $\{t(1,1) : t \in \mathbb{R}\}$ i.e. the line $y = x$

$\{t(1,-1) : t \in \mathbb{R}\}$ i.e. " " $y = -x$

(iii) We have

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \text{eigenvector with eigenvalue } 1$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \text{eigenvector with eigenvalue } -1.$$

Q8 (a) (i) $A_T = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(ii) $\operatorname{Im} T =$ column space of A_T.

$$\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - R_1 \end{matrix} \sim \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Leading entry is in the first column, so

$\operatorname{Im} T = \operatorname{span}\{(1,1,1)\}$ $\quad \dim \operatorname{Im} T = 1$

(iii) $\operatorname{Ker} T =$ sol^n space of A_T. Let the unknowns be x, y, z. No leading entry for y or z so we set $y = s$, $z = t$ where $s, t \in \mathbb{R}$. Back substitution gives $x = -s-t$

Hence

$$\operatorname{Ker} T = \{(-s-t, s, t) : s, t \in \mathbb{R}\}$$
$$= \operatorname{Span}\{(-1,1,0), (-1,0,1)\}$$

(b)
(i) $P_{S,B}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Hence $\underset{\sim}{x} = \underset{\sim}{b}_1 + \underset{\sim}{b}_2 + \underset{\sim}{b}_3$

(ii) To compute $P_{S,B}$,

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] R_2 + R_1 \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] R_3 + R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

Hence $\underset{\sim}{b}_1 = (1,1,1)$ $\quad \underset{\sim}{b}_2 = (0,1,1)$ $\quad \underset{\sim}{b}_3 = (0,0,1)$

(iii) We observe

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} R_2 + R_1 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

There are only two leading entries, so the columns are not linearly independent, and thus cannot be a basis.

Q 9 (a) (i) The vector $\frac{1}{\sqrt{2}}(1,1,0)$ is normalized. According to the Gram-Schmidt algorithm, with $\underset{\sim}{y} = \frac{1}{\sqrt{2}}(1,0,1)$ we construct the vector

$$\underset{\sim}{v} = \underset{\sim}{y} - (\underset{\sim}{y} \cdot \underset{\sim}{a})\underset{\sim}{a}$$

$$= \frac{1}{\sqrt{2}}(1,0,1) - \frac{1}{2}\frac{1}{\sqrt{2}}(1,1,0)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}, -\frac{1}{2}, 1\right) = \frac{1}{2\sqrt{2}}(1,-1,2)$$

$$\Rightarrow \hat{\underset{\sim}{v}} = \frac{1}{\sqrt{6}}(1,-1,2) = \underset{\sim}{b}$$

(ii) We know that the closest point $\underset{\sim}{p}$ say is the orthogonal projection. Hence

$$\underset{\sim}{p} = (\underset{\sim}{x} \cdot \underset{\sim}{a})\underset{\sim}{a} + (\underset{\sim}{x} \cdot \underset{\sim}{b})\underset{\sim}{b}$$

$$= \frac{2}{2}(1,1,0) + \frac{2}{6}(1,-1,2)$$

$$= \frac{1}{3}(4,2,2)$$

(b) The axiom requires

(i) that $\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle \geq 0$

(ii) that the only vector for which $\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle = 0$ is $\underset{\sim}{x} = \underset{\sim}{0}$.

To check (i) : with $\underset{\sim}{x} = (x_1, x_2)$
we have

$$\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle = x_1^2 + x_1 x_2 + \frac{1}{3} x_2^2$$
$$= \left(x_1 + \frac{x_2}{2}\right)^2 - \left(\frac{x_2}{2}\right)^2 + \frac{1}{3} x_2^2$$
$$= \left(x_1 + \frac{x_2}{2}\right)^2 + \frac{1}{12} x_2^2$$

Thus $\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle \geq 0$ since both terms on the RHS are non-negative

(ii) For $\langle \underset{\sim}{x}, \underset{\sim}{x} \rangle = 0$, the above gives
$x_1 + \frac{x_2}{2} = 0$ and $x_2 = 0$
Thus both $x_1 = 0$ and $x_2 = 0$
which implies $\underset{\sim}{x} = \underline{0}$ as required.

10. (a) In the usual notation for least squares we have

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad \underset{\sim}{y} = \begin{bmatrix} 20 \\ 30 \\ 30 \end{bmatrix}$$

and

$$A^T A \begin{bmatrix} a \\ b \end{bmatrix} = A^T \underset{\sim}{y}$$

Now $A^T A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

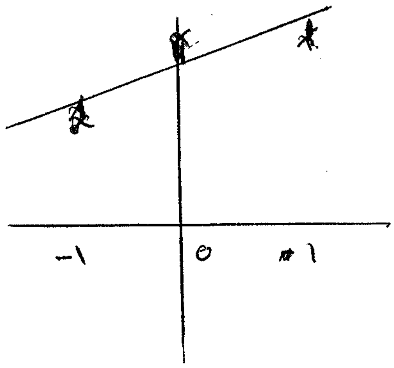
$$A^T \underset{\sim}{y} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 30 \end{bmatrix} = \begin{bmatrix} 80 \\ 10 \end{bmatrix}$$

Thus $3a = 80 \Rightarrow a = \frac{80}{3}$
$2b = 10 \Rightarrow b = 5$

Hence the line of best fit is

$$y = \frac{80}{3} + 5x$$

(b)



The sum squared of the distance in the y-direction from the data to the line is being minimized.

11. (a) $\begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (2-\lambda)(4-\lambda) - 3 = 0$

$\Rightarrow \quad \lambda^2 - 6\lambda + 5 = 0$

$\Rightarrow \quad \lambda = 5 \text{ or } \lambda = 1$

(b) When $\lambda = +5$

$$\begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}_{R_2 + \frac{1}{3}R_1} \sim \begin{bmatrix} -3 & 3 \\ 0 & 0 \end{bmatrix}$$

Let the unknowns be x and y. No leading entry for y, so put $y = t$.

Back substitution gives $x = t$. Hence

sol^{n} space $= \{ t(1,1) : t \in \mathbb{R} \}$

$\Rightarrow$ eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

When $\lambda = 1$

$$\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \underset{R_2 - R_1}{\sim} \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

Proceeding as above, we put $y = t$, and back substitute to get $x = -3t$. Hence

sol^{n} space $= \{ t(-3,1) : t \in \mathbb{R} \}$

$\Rightarrow$ eigenvector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

12 (a) sum of the diagonal entries of A = sum of the eigenvalues of A

(b) Let λ_3 denote the 3^{rd} eigenvalue. From the above formula and the given information

$$7 = 2 \times 3 + \lambda_3 \Rightarrow \lambda_3 = 1$$

(c) $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}_{R_3+R_1} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let the unknowns be denoted x, y, z. No leading entry for z, so set $z = t$, $t \in \mathbb{R}$.
Back substitution gives

$$y = 0 \quad x = t$$

$\Rightarrow$ sol^n space $= \{ t(1,0,1) : t \in \mathbb{R} \}$

$\Rightarrow$ Normalized eigenvector $\frac{1}{\sqrt{2}}(1,0,1)$

(d) $\lambda_1 = 3 \quad \underset{\sim}{v}_1 = \frac{1}{\sqrt{2}}(-1,0,1)$

$\lambda_2 = 3 \quad \underset{\sim}{v}_2 = (0,1,0)$

$\lambda_3 = 1 \quad \underset{\sim}{v}_3 = \frac{1}{\sqrt{2}}(1,0,1)$

we have $\underset{\sim}{v}_1 \underset{\sim}{v}_1^T = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$\underset{\sim}{v}_2 \underset{\sim}{v}_2^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \underset{\sim}{v}_3 \underset{\sim}{v}_3^T = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Thus

$$\text{RHS} = \frac{3}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= A$$

(e) $$A\underset{\sim}{x} = 2A\underset{\sim}{v}_1 - A\underset{\sim}{v}_2 + A\underset{\sim}{v}_3$$

$$= 6\underset{\sim}{v}_1 - 3\underset{\sim}{v}_2 + \underset{\sim}{v}_3$$

$$\Rightarrow [A\underset{\sim}{x}]_V = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$$