COMP20003 Algorithms and Data Structures Algorithms

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Outline of the first few lectures

- Algorithms: general
- This subject: details
- Algorithm efficiency
- Computational complexity
- Data structures
  - Basic data structures
  - Algorithms on basic data structures
  - Complexity analysis of algorithms on basic ds's

Revisit: What is an algorithm?

- A set of steps to accomplish a task.
- Computer algorithms must be:
  - Specific.
  - Correct.
  - Reasonably efficient.

A small diversion: Which C standard?

- C standards:
  - ANSI C (C89)
  - C99 – “substantially” completely supported in gcc 4.5 (with `-std=C99` option on)
  - C11 (current C standard, from 2011) `gcc 4.8`
- On `nutmeg.eng.unimelb.edu.au:`
  - `gcc -v:`
  - `gcc version 4.4.7`

C for gcc on nutmeg

- ANSI C with *some* of the features of C99, *e.g.*
  - Supported:
    - inline functions
    - long long int
  - Not supported:
    - Variable length arrays
    - Doesn’t insist on explicit return type for function
- For all the new features in C99 see:

http://www.open-std.org/jtc1/sc22/wg14/www/newinc9x.htm

Algorithms and Efficiency

- An algorithm must:
  - be accurate (to within the required tolerance).
  - compute in a “reasonable” amount of time.
- The most accurate algorithm in the world is no use if it takes forever to compute.
- We are particularly interested in efficiency as the size of the input grows.
- Why?

Example algorithm: Compute Fibonacci numbers

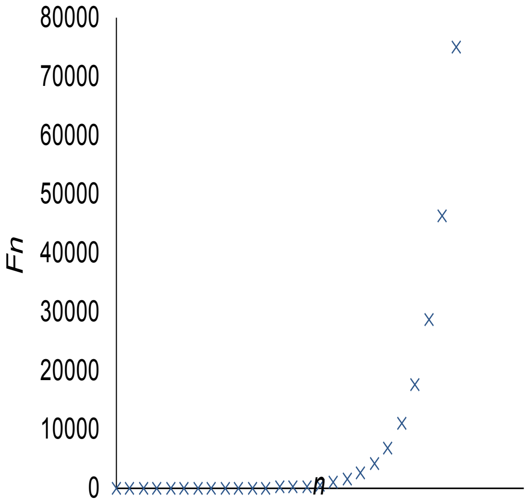
- $F_n = F_{n-1} + F_{n-2}$
  - $F_0 = 0$
  - $F_1 = 1$

n	F(n-1)	F(n-2)	F(n)
0	-	-	0
1	-	-	1
2	1	0	1
3	1	1	2
4	2	1	3
5	3	2	5
6			8
7	8	5	13
8	13	8	21

- 0,1,1,2,3,5,8,13,21,34,55….

Fibonacci numbers

- Fibonacci numbers grow very quickly:
  - $F_{10} = 55$
  - $F_{15} = 610$
  - $F_{20} = 6765$



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Fibonacci numbers

- The original problem that Fibonacci was investigating (1202):
  - How fast can rabbits breed under ideal circumstances?
  - http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#Rabbits

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How to compute Fibonacci numbers?

- Given the definition of Fibonacci numbers:
  - $F_n = F_{n-1} + F_{n-2}$
    - $F_0 = 0$
    - $F_1 = 1$
- Does this suggest an easy way to calculate F_n?

Computing Fibonacci Numbers: Scaffolding

```
main()
{
  int n, ans;

  printf("Enter a number:\n");
  scanf("%d", &n);

  if(DEBUG)
    printf("%d\n",n)
  ans = fib(n);
  printf("Fibonacci of %d is %d\n", n, ans);
}
```

Naïve Fibonacci algorithm

```
int
fib (int n)
{
    if(n==0) return 0;
    if(n==1) return 1;
    return fib(n-1) + fib(n-2);
}
```

Definition:

$F_0 = 0$

$F_1 = 1$

$F_n = F_{n-1} + F_{n-2}$

- Is the algorithm correct?

Naïve Fibonacci algorithm

```
int
fib (int n)
{
    if(n==0) return 0;
    if(n==1) return 1;
    return fib(n-1) + fib(n-2);
}
```

- How long does it take to compute?

Fibonacci computation

- Approach to estimating computation effort:
  - Count operations.
  - Count operations as a function of input size.
  - Count operations as a proxy for time.
- $T(n) =_{def}$ run time for input n.
- $T(n)$ = calculate as *number of operations* for input size n.
  - Portable between machines.
  - Can compare algorithms.

Fibonacci computation

- Looking at *T(n)* as number of operations to calculate the nth Fibonacci number, then
  - *T(n) = T(n-1) + T(n-2) + 3 (operations)*
  - *T(1) = T(0) = 1*
- Example: unrolling the loop
  - T(4) = T(3) + T(2) + 3

Fibonacci computation

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  - $T(4) = T(3) + T(2) + 3$
  - $\quad\quad = T(2) + T(1) + 3 + T(2) + 3$

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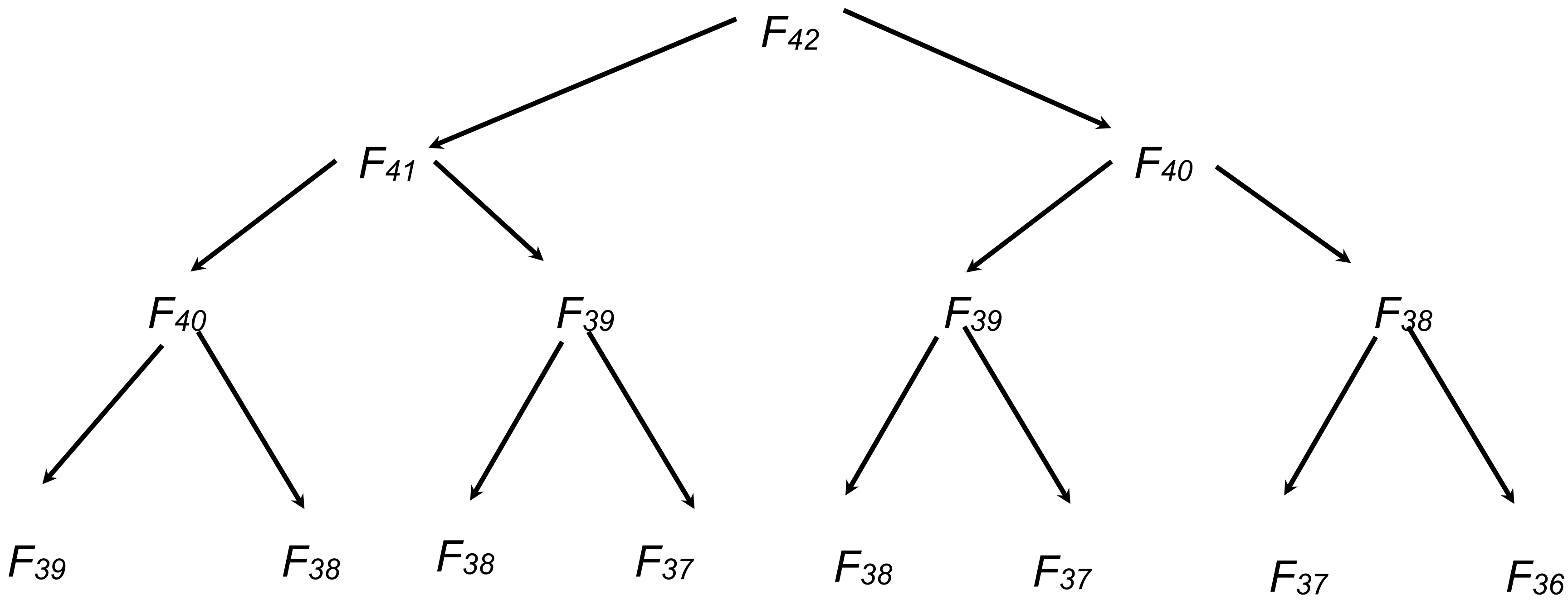
- Example: unrolling the loop
  - T(4) = T(3) + T(2) + 3
  - = T(2) + T(1) + 3 + T(2) + 3
  - = T(1) + T(0) + 3 + T(1) + 3 + T(1) + T(0) + 3 + 3
  - = 1+1+3+1+3+1+1+3+3 = 17 operations

Fibonacci computation

- Looking at *T(n)* as number of operations to calculate the nth Fibonacci number, then
  - *T(n) = T(n-1) + T(n-2) + 3 (operations)*
  - *T(1) = T(0) = 1*
- Example:
  - T(5) = T(4) + T(3)
  - = 17 + T(2) + T(1) + 3
  - = 17 + 9
  - = 26

How many operations to calculate F_{42}?



Any obvious inefficiencies?

Memoization

- Store previously computed values.

```
fib(n)
{
    int i;
    int fib[n+1];

    fib[0] = 0;
    fib[1] = 1;

    for(i = 2; i <= n; i++)
    {
          f[i] = f[i-1] + f[i-2];
    }
    return (fib[n]);
}/* how many operations to calculate fib(n)? */
```

…or without storing all the intermediate results

```
/* this one will work in C */
int fib(n)
{
        int result = 0;
        int preOldResult = 1; int oldResult = 1;

        if(n <= 0) return 0;
        if(n > 0 && n < 3) return 1;
        for (int i=3;i<n;i++)
        {
                result = preOldResult + oldResult;
                preOldResult = oldResult;
                oldResult = result;
        }
        return result;
}
```

Counting operations

- Count of operations used as proxy for run time:
  - Advantages?
  - Caveats?
- How long does calculation of fib(2) take
- Using the naïve algorithm?
- Using memoization?
- Do we care how long things take for small input n?

Complexity analysis: general method

- Count operations for $T(n)$ –
  "time" (number of ops) taken for input n.
- Note: best to identify the most expensive operation and count that operation.
  - *e.g.* count multiplications, ignore additions
- Note also that we can sometimes trade off space for time.

Complexity analysis: a fine point for fib()

- Assumption: addition of two numbers takes constant time.
- True *if* both numbers can fit into one computer word: 32 bits, number < 2^{32}.
- But Fibonacci numbers get very large:
  - F_n takes approximately $0.694n$ bits, so
  - To fit in one word, n < 32/0.694 = 46
  - $F_{50} = 12{,}586{,}269{,}025 > 12 * 10^9 > 2^{33}$
  - Last operations take longer. Assumption not valid for large n.

Closed form for Fibonacci numbers

- Binet's formula:

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

- For large *n*, $F_n \approx 2^{0.694n}$
- For more on Fibonacci numbers, *see :*

http://mathworld.wolfram.com/FibonacciNumber.html

Complexity analysis: Intuition

- Fortunately, most algorithms do not deal with such large numbers.
- Counting operations usually suffices.
- Always be aware of the assumptions:
  - What is the most expensive operation?
  - Are the operations really constant?
  - What are the inputs and outputs?

Skiena: Algorithm Design Manual

- Chapter 1:
  - Algorithm correctness.
  - Example problems.
- Chapter 2:
  - Chapter 2.1: counting operations

Next section

- Complexity analysis more formally.
- Big-O and related formalisms.