

11(a) We want to solve

$$A \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 10 \end{bmatrix} \quad \text{where}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}$$

using least squares. This means we must solve

$$A^T A \begin{bmatrix} a \\ b \end{bmatrix} = A^T \begin{bmatrix} 5 \\ 9 \\ 10 \end{bmatrix}$$

$$\text{Now } A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 11 & 45 \end{bmatrix}$$

$$A^T \begin{bmatrix} 5 \\ 9 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 5 \\ 9 \\ 10 \end{bmatrix} = \begin{bmatrix} 24 \\ 96 \end{bmatrix}$$

Thus we have

$$\begin{bmatrix} 3 & 11 \\ 11 & 45 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 24 \\ 96 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 11 & 45 \end{bmatrix}^{-1} \begin{bmatrix} 24 \\ 96 \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 45 & -11 \\ -11 & 3 \end{bmatrix} \begin{bmatrix} 24 \\ 96 \end{bmatrix}$$
$$= \frac{1}{14} \begin{bmatrix} 24 \\ 24 \end{bmatrix} = \begin{bmatrix} 12/7 \\ 12/7 \end{bmatrix}$$

$$\text{Hence } R = \frac{12}{7} + \frac{12}{7} T$$

(b)

