MAST10007 (Summer) Assignment 3 Due Mon. Jan. 25th 1.30pm

Please leave your assignment in your tutor's box located near the north entrance to the Richard Berry building. Make sure that you include your name, your student number and your tutor's name.

1. Let $\mathbf{x}_1 = (1, 2, -1, 1)$, $\mathbf{x}_2 = (-1, -1, -1, -1)$, $\mathbf{x}_3 = (1, 1, 1, 0)$, $\mathbf{x}_4 = (-2, -1, -4, -1)$.

   (a) Write the vector equation

   $$\alpha_1\mathbf{x}_1 + \alpha_2\mathbf{x}_2 + \alpha_3\mathbf{x}_3 + \alpha_4\mathbf{x}_4 = \mathbf{0}$$

   as an augmented matrix for a linear system.

   (b) Show that the matrix

   $$A = \begin{bmatrix} 1 & -1 & 1 & -2 \\ 2 & -1 & 1 & -1 \\ -1 & -1 & 1 & -4 \\ 1 & -1 & 0 & -1 \end{bmatrix} \quad \text{can be row reduced to} \quad A = \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   (c) Use (b) to find the general solution of the linear system in (a).

   (d) Further use (b) to show that $\mathbf{x}_1 = (1, 2, -1, 1)$, $\mathbf{x}_3 = (1, 1, 1, 0)$ and $\mathbf{x}_4 = (-2, -1, -4, -1)$ are linearly independent.

2. The following are non-empty sets of vectors in $\mathbb{R}^3$. Which are subspaces? For any which are subspaces use the definition of subspace to prove your answer. Otherwise show why the set is not a subspace of $\mathbb{R}^3$. Describe each of the sets geometrically.

   (a) $R = \{(x, y, z) : 2x + z = 0\}$

   (b) $S = \left\{(x, y, z) : \dfrac{x}{2} = y + 1 = 3z\right\}$

3. Write in vector, parametric and cartesian form the plane through the origin which contains the two vectors $(0, 1, 1)$ and $(1, 0, -2)$.