## MATLAB Revision

The upcoming MATLAB test is designed both to evaluate your ability to calculate efficiently with the aid of MATLAB, and also to assess your general understanding of the concepts of the course to this stage.

In purpose of this MATLAB class is to provide revision questions that will help in your preparation for the test.

Some MATLAB commands:

- $A'$ is the transpose of the matrix $A$
- inv($A$) gives the inverse of $A$
- rref($A$) gives the fully reduced row echelon form of $A$
- eye($n$) gives the $n \times n$ identity matrix
- ones($p, q$) gives the $p \times q$ matrix of all 1's.
- zeros($p, q$) gives the $p \times q$ matrix of all 0's
- diag(v) gives the diagonal matrix with diagonal v.
- The command v = B(:,3) selects the third column of B, for example

**Q1**. Consider the matrix.

$$P = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 7 & 7 & 7 & 7 & 7 \\ 0 & 4 & 0 & 0 & 0 & 7 & 7 & 7 & 7 & 7 \\ 0 & 0 & 4 & 0 & 0 & 7 & 7 & 7 & 7 & 7 \\ 0 & 0 & 0 & 4 & 0 & 7 & 7 & 7 & 7 & 7 \\ 0 & 0 & 0 & 0 & 4 & 7 & 7 & 7 & 7 & 7 \\ 12 & 12 & 12 & 12 & 12 & 1 & 7 & 7 & 7 & 7 \\ 12 & 12 & 12 & 12 & 12 & 7 & 1 & 7 & 7 & 7 \\ 12 & 12 & 12 & 12 & 12 & 7 & 7 & 1 & 7 & 7 \\ 12 & 12 & 12 & 12 & 12 & 7 & 7 & 7 & 1 & 7 \\ 12 & 12 & 12 & 12 & 12 & 7 & 7 & 7 & 7 & 1 \end{bmatrix}$$

(i). Calculate the dot product of the 3rd row and 6th column of $P$.

(ii). Let $Q = 2P^{-1}$. Calculate $\det PQ$.

(iii). Calculate rank $(P + 6I)$.

(iv). Let $\mathbf{a}$ denote the vector corresponding to the first column of $P$, let $\mathbf{b}$ denote the vector corresponding to the sixth column, and let $\mathbf{c}$ denote the vector corresponding to the third row. Calculate the fourth entry of the linear combination $-\mathbf{a} + 2\mathbf{b} - 2\mathbf{c}$.

**Q2**. Let

$$A = \begin{bmatrix} -1 & 2 & 3 & -3 & 6 & 7 \\ 1 & -1 & -2 & 2 & -5 & -6 \\ -1 & 1 & 2 & -1 & 2 & 4 \\ -2 & 2 & 4 & -2 & 4 & 8 \end{bmatrix}$$

(i). Determine the vectors that are in the solution space of $A$

$$\mathbf{u}_1 = \begin{bmatrix} 4 \\ -3 \\ 2 \\ -1 \\ -1 \\ 1 \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ -3 \\ -2 \end{bmatrix} \quad \mathbf{u}_3 = \begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \\ -7 \\ 6 \end{bmatrix} \quad \mathbf{u}_4 = \begin{bmatrix} 8 \\ -2 \\ 1 \\ 0 \\ -2 \\ 3 \end{bmatrix}$$

(ii). What is the rank of $A$?

(iii). What is the dimension of the solution space of $A$?

(iv). Determine the vectors that are in the column space of A.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \\ -5 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 13 \\ -8 \\ 5 \\ 10 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -4 \\ 0 \\ 2 \\ 4 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} -3 \\ 2 \\ -1 \\ -2 \end{bmatrix}$$

(v). Let $\mathbf{u}_1, \dots, \mathbf{u}_4$ be as in (i). Suppose that $B$ is obtained from $A$ by deleting the final row. Determine which of the vectors $\mathbf{u}_1, \dots, \mathbf{u}_4$ are in the solution space of $B$.

(vi). Let $\mathbf{a}_j$ denote the vector corresponding to the $j$th column of $A$. Express $\mathbf{a}_3$ as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$. Also express $\mathbf{a}_5$ as a linear combination of $\mathbf{a}_1$, $\mathbf{a}_3$ and $\mathbf{a}_4$.

**Q3**. In addition to the matrix $A$ of the previous question, for this question you require the matrix

$$B = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & -9 \\ -3 & 3 & 16 \\ -2 & 0 & 3 \end{bmatrix}$$

(i). Let $\mathbf{b}_j$ denote the vector corresponding to the $j$th column of $B$. Find a basis for

$$\text{Span}\{\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\}$$

Choose the vectors with the smallest subscripts possible.

(ii). Is $(-3457109, -444555778, 17171510986, -3470990091)$ a member of the above span?

**Q4**. Indicate if the following statements are true or false.

(i). For $A + A^T$ to be well defined it must be that $A$ is square.

(ii). For $AA^T$ to be well defined it must be that $A$ is square.

(iii). Let the square matrix $B$ be related to the matrix $A$ by multiplication of the first row by $-2$, and interchange of the first and second column. We have that $\det B = -2 \det A$.

(iv). It is always true that rank $A$ = rank $A^T$.

(v). The linear system specified by the augmented matrix $[A|\mathbf{0}]$ can never be inconsistent.

(vi). The matrix product

$$\begin{bmatrix} a \\ c \end{bmatrix} \begin{bmatrix} 3 & 2 \end{bmatrix}$$

is not defined.

(vii). Let $A$ be a $p \times q$ matrix and suppose $\mathbf{0}$ is in the solution set of $A$. Then the solution set of $A$ is a subspace of $\mathbb{R}^q$.

(viii). If $A$ and $B$ are square matrices of the same size, and $A$ is invertible, but $B$ is singular, it may be that $AB$ is invertible.

(ix). For square matrices $A$ and $B$ it is always true that $(A+B)^2 = A^2 + 2AB + B^2$.

(x). Let $A$ be a $p \times q$ matrix and suppose $A\mathbf{v} = \mathbf{0}$ for all vectors $\mathbf{v} \in R^q$. It must be that all entries of $A$ are zero.

(xi). The line with cartesian equation $(x-3)/2 = (y+7)/5 = z-1$ is parallel to the line with vector equation $(x,y,z) = (0,0,1) + t(2,5,1)$.

(xii). The rank of a $4 \times 3$ matrix cannot equal 4.

(xiii). The line with cartesian equation $(x-3)/2 = (y+7)/5 = z-1$ is perpendicular to the line with vector equation $(x,y,z) = t(3,-7,1)$.

(xiv). The plane with vector equation $(x,y,z) = t(2,1,0) + s(-1,1,2) + (1,2,2)$ does not pass through the origin.

(xv). The linear system corresponding to the augmented matrix $[A|\mathbf{b}]$ is inconsistent if rank $(A)$ $>$ rank $[A|\mathbf{b}]$.

(xvi). Let $A$ be an $N \times N$ matrix, and suppose the dimension of the solution space of $A$ is zero. Then $A$ is singular.

(xvii). The cross product of vectors in $\mathbb{R}^4$ is undefined.

(xviii). For vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ in $\mathbf{R}^3$, the volume of the corresponding parallelpiped is given by $||(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}||$.

(xix). For vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ in $\mathbf{R}^3$, the volume of the corresponding parallelpiped is 1/6 of that for $\mathbf{a}$, $\mathbf{b}+\mathbf{a}$, $\mathbf{c}+\mathbf{b}+\mathbf{a}$.

(xx). For a matrix $P$ with the property that $P$ is different from the identity and $P^2 = P$, it is true that $\det P = 1$.

(xxi). If $A$ is a $7 \times 5$ matrix, then the solution space of $A$ is a subspace of $\mathbb{R}^7$.

(xxii). The plane $x + y - z = 3$ is perpendicular to the plane $3x - y + 2z = 4$.

(xxiii). Let $A$ be a $p \times q$ matrix. The dimensions of the row and column spaces adds up to $q$.

(xxiv). The row space of $A$ is equal to the column space of $A^T$.