MAST10007 (Summer) **Assignment 5** **Due Mon. Feb. 8th 1.30p.m.**

Please leave your assignment in your tutor's box located near the north entrance to the Richard Berry building. Make sure that you include your name, your student number and your tutor's name.

1. You are given that the transition matrix $P_{\mathcal{C},\mathcal{B}}$ from a basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ to a basis $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ is

$$\frac{1}{2}\begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

   (a) Compute the vector $\mathbf{u} = \mathbf{b}_1 + \mathbf{b}_2 + 2\mathbf{b}_3$ as a linear combination of the vectors in $\mathcal{C}$, and from this write down $[\mathbf{u}]_{\mathcal{C}}$.

   (b) Calculate $P_{\mathcal{B},\mathcal{C}}$.

   (c) Suppose

$$\mathbf{c}_1 = (1, 2, 3), \quad \mathbf{c}_2 = (1, 2, 0), \quad \mathbf{c}_3 = (1, 0, 0).$$

   Compute $P_{\mathcal{S},\mathcal{B}}$ where $\mathcal{S}$ is the standard basis, and from this read off the explicit form of the vectors $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$.

2. Define $T : \mathbb{R}^3 \to \mathbf{R}^3$ by

$$T\mathbf{x} = (\mathbf{x} \cdot (1, 0, -1))(1, 0, -1) + (\mathbf{x} \cdot (1, 1, 1))(1, 1, 1)$$

   (a) Compute the action of T on the unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

   (b) Write down the standard matrix representation A_T of T.

   (c) Calculate ImT and dim(ImT).

   (d) Calculate Ker$T$ and dim(Ker$T$).

   (e) Calculate the volume of the image of the unit cube under the action of T.