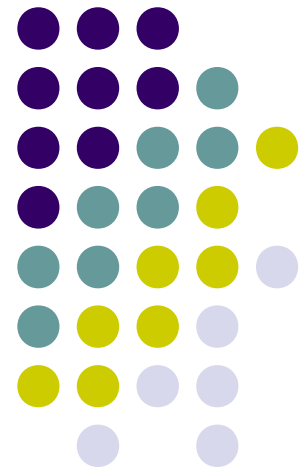


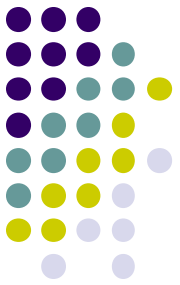
COMP20003

Algorithms and Data Structures

Introduction to Graphs

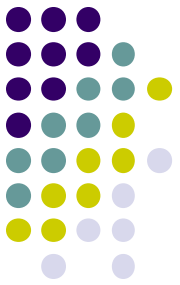
Nir Lipovetzky
Department of Computing and
Information Systems
University of Melbourne
Semester 2





Graph definition

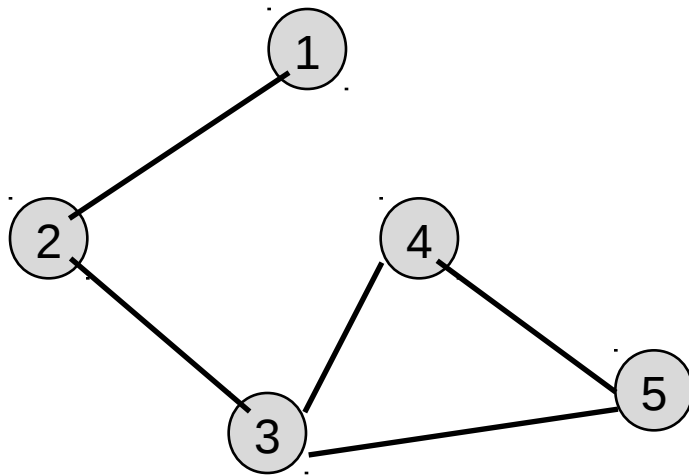
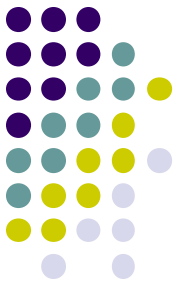
- Graph:
 - a representation of a set of objects
 - some pairs of objects are connected by links.



Graph definition

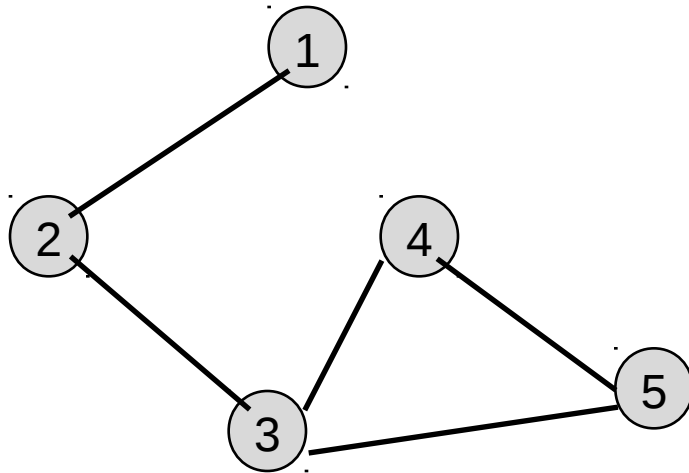
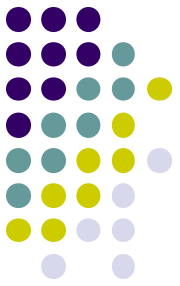
- Graph $G = \{V, E\}$
- Graph: a set of
 - Vertices V
 - Edges E (links between vertices)
- Comparing to trees, linked lists:
 - vertices = nodes
 - edges = links
- Vertices: can contain information
- Edges: can have direction and/or weight

Undirected graph



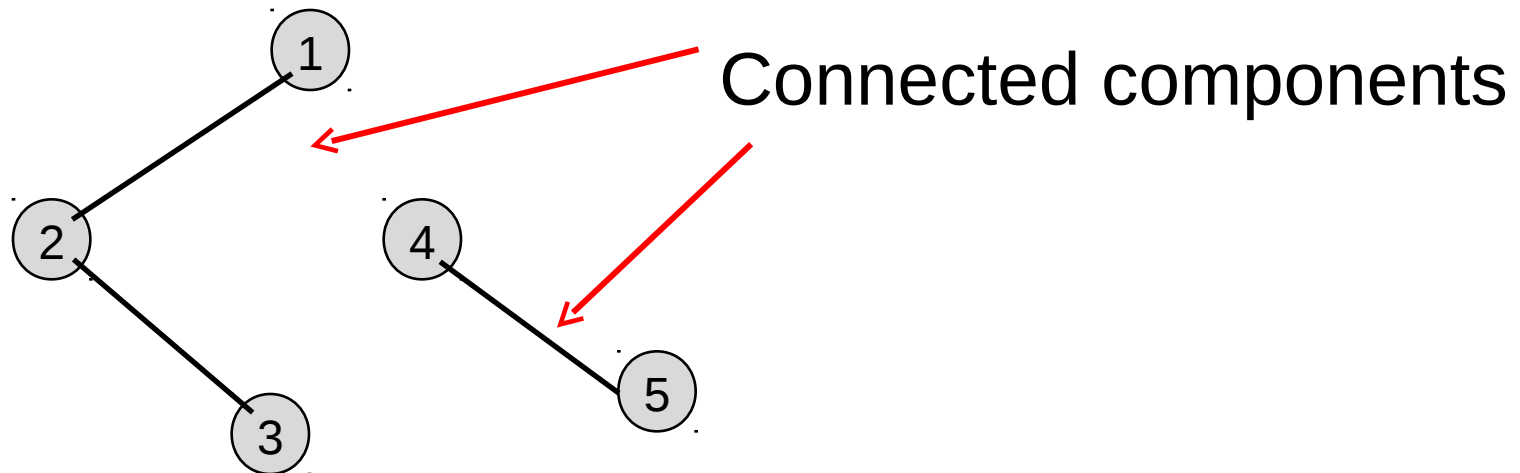
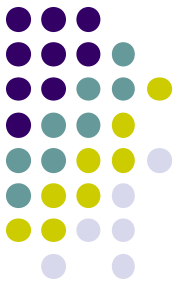
Edges have no direction specified.

Connected Undirected graph



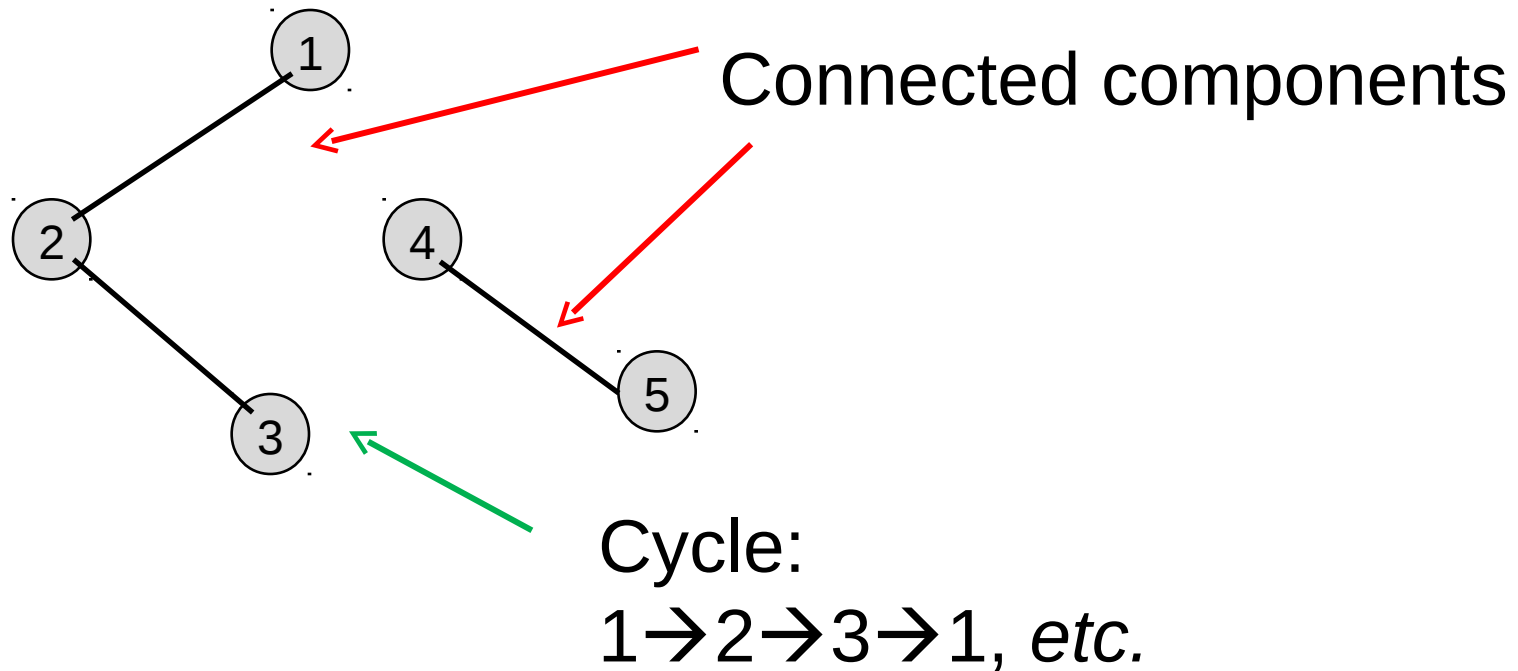
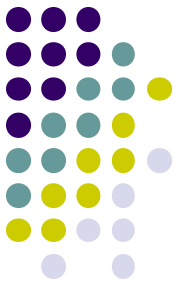
Every pair of vertices is connected (possibly indirectly).

Unconnected Undirected graph

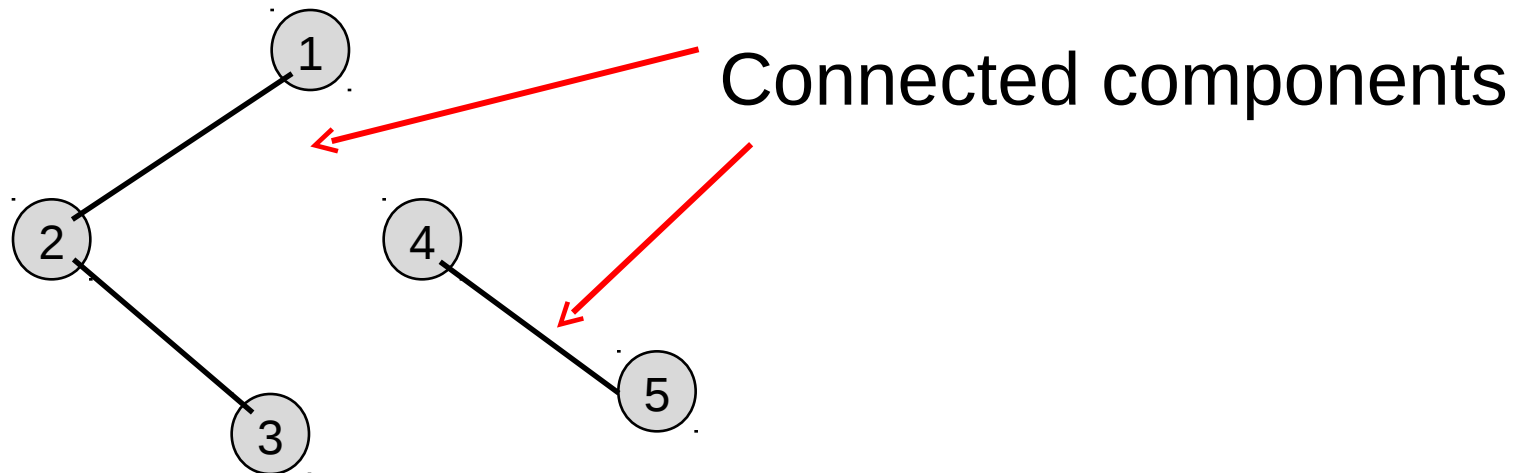
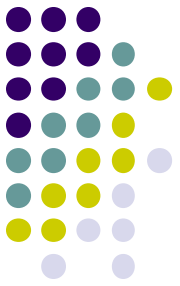


Unconnected

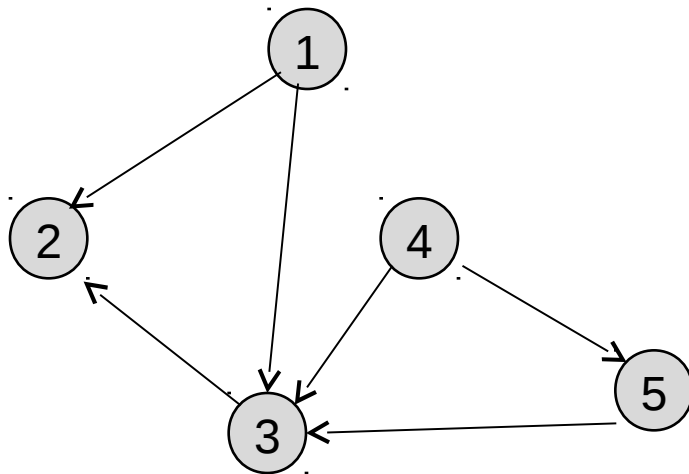
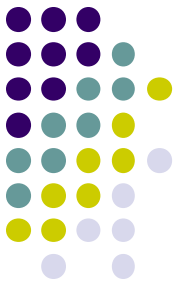
Undirected graph with cycle



Acyclic, unconnected Undirected graph

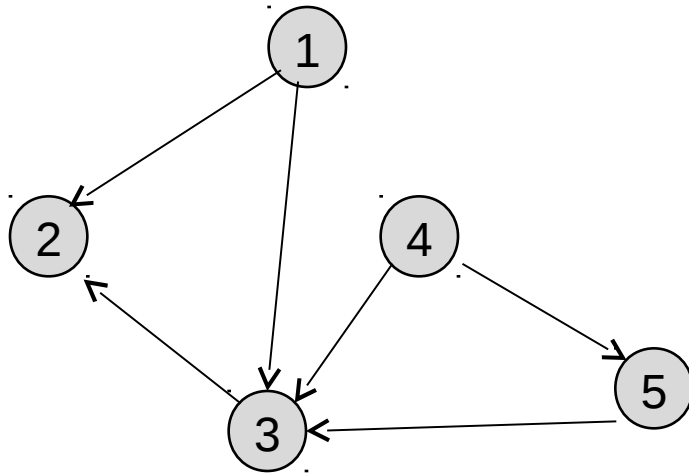
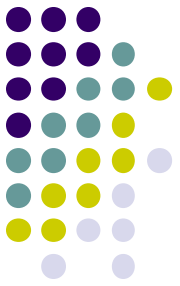


Directed graph



Edge direction is specified.
Links are not symmetrical.

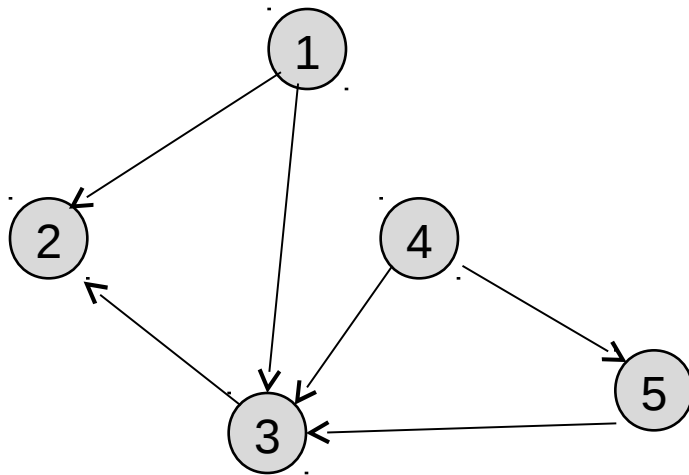
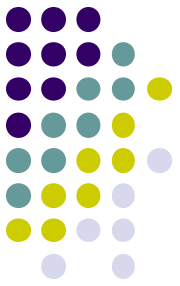
Directed graph



Reachability:

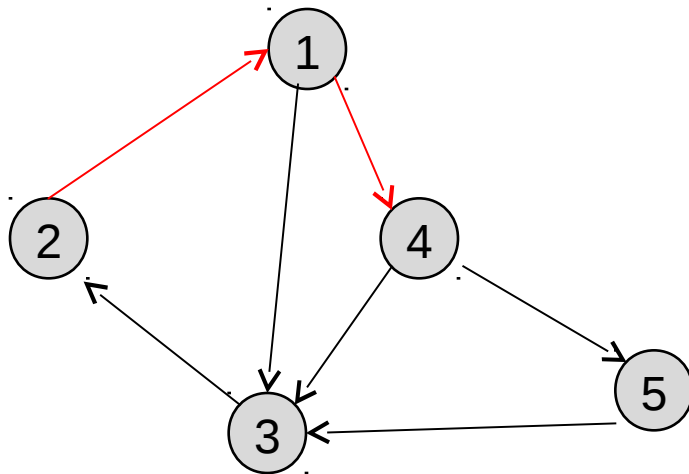
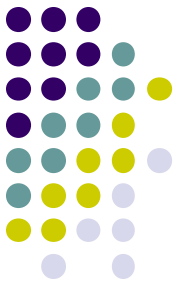
Can you get from Vertex 1 to Vertex 5?

Weakly connected Directed graph



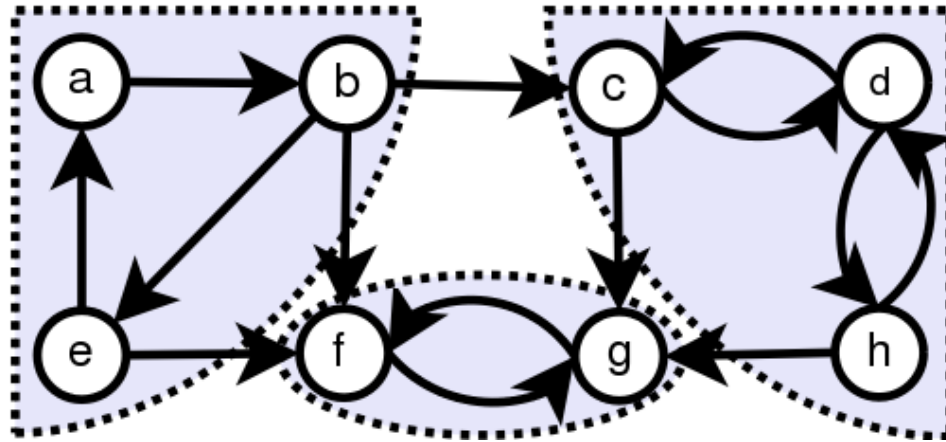
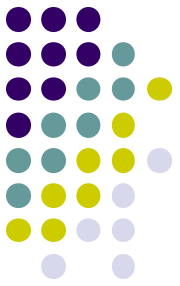
Replace all directed edges with undirected edges, to obtain a connected (undirected) graph

Strongly connected Directed graph



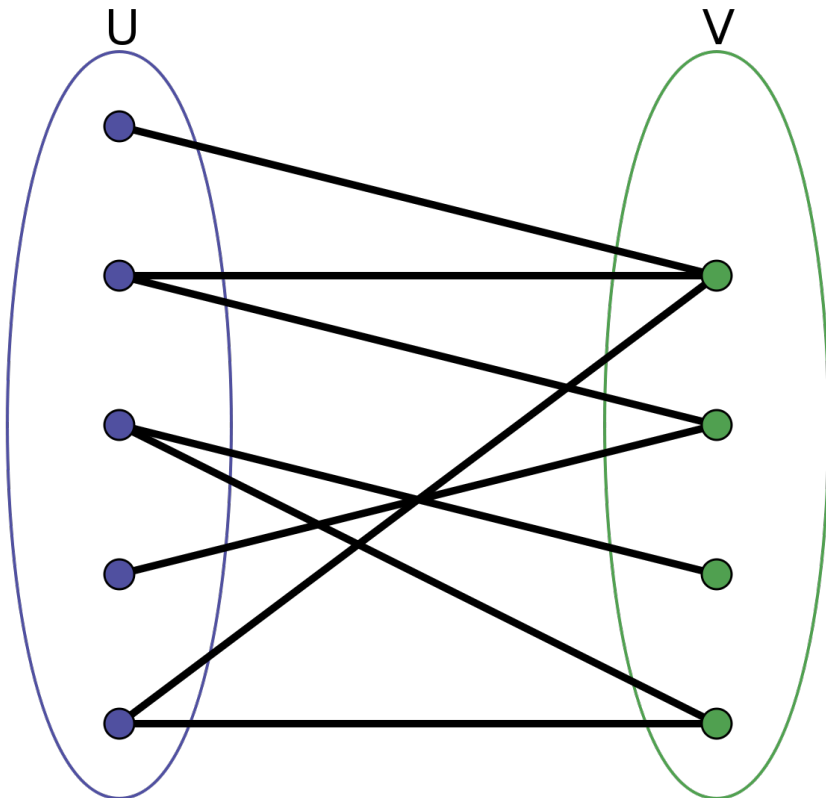
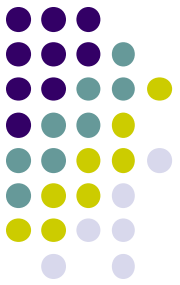
If every vertex is reachable from every other vertex

Strongly connected Components in a Digraph

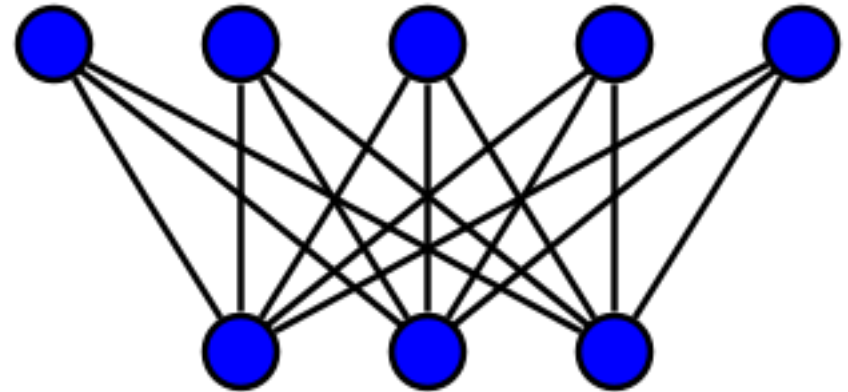


If every vertex is reachable from every other vertex within the same component

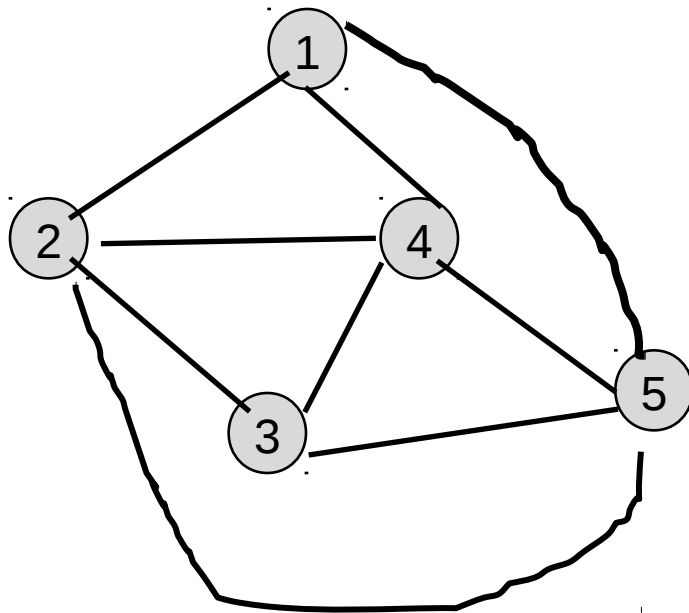
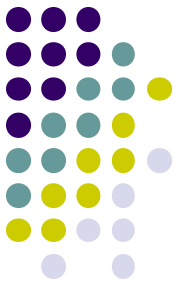
Bipartite Graph



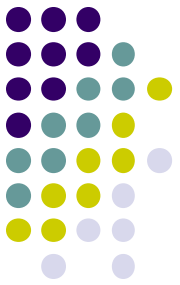
- U and V are disjoint sets of vertexes
- Every vertex in U connects to a vertex in V, and viceversa.



Complete graph

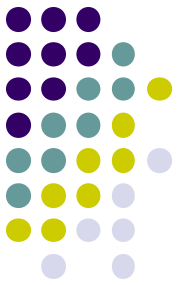


$V(V-1)/2$ edges



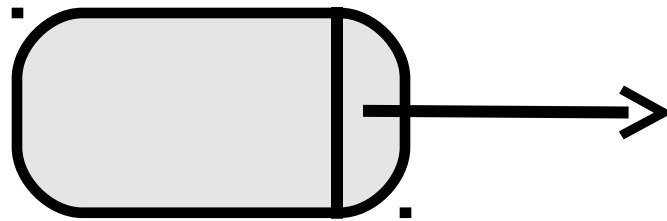
Trees and Graphs

- Tree: a undirected graph that is:
 - Connected
 - Acyclic
- *n.b.* Any two vertices are connected by exactly one simple path.
- *n.b.* All vertices are connected.

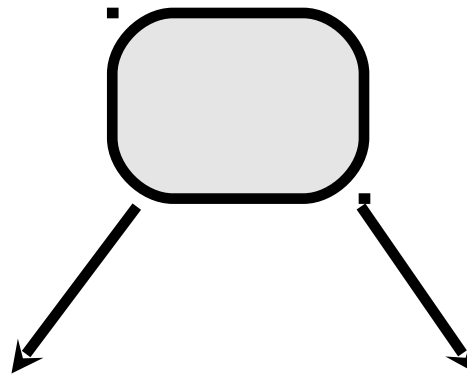


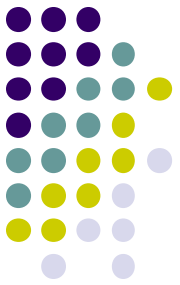
Representing graph vertices

- Linked list nodes:



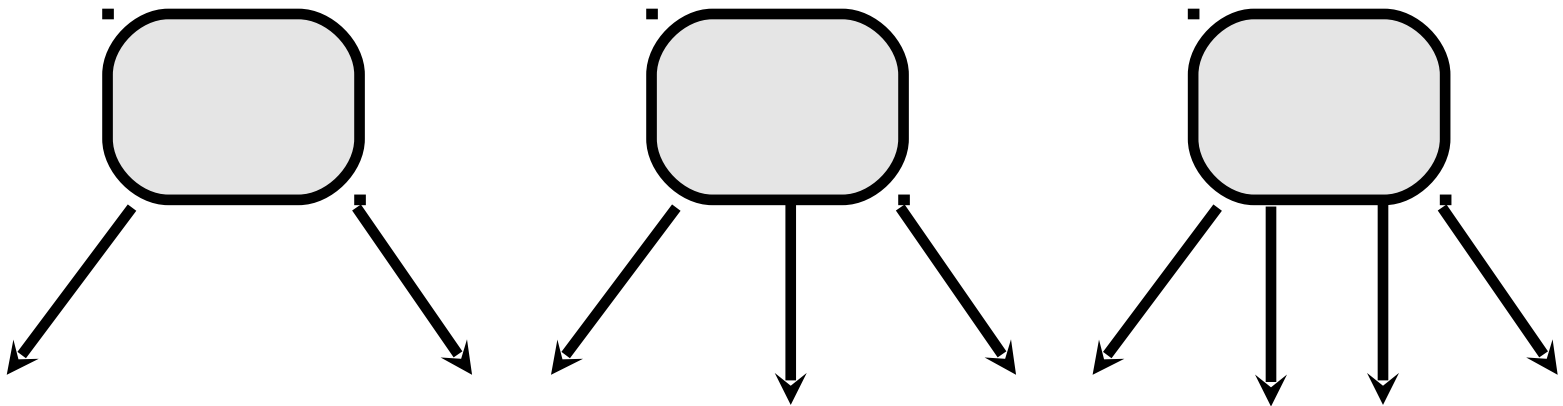
- Binary tree nodes:



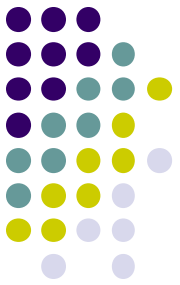


Vertices

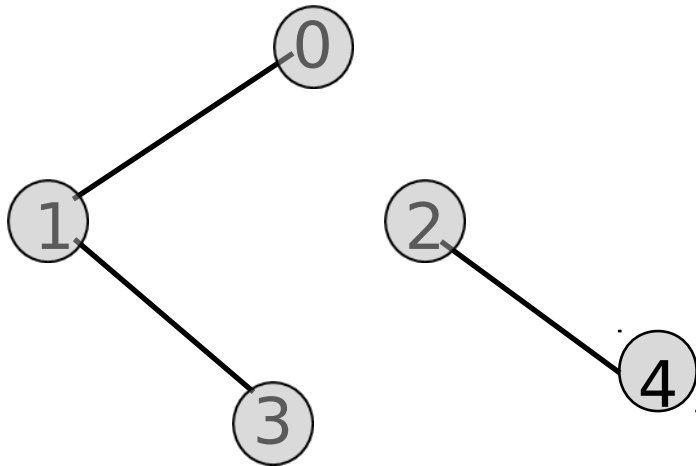
- 2-3-4 tree nodes



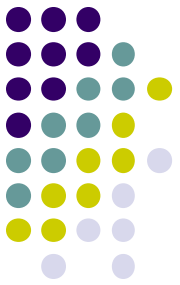
- But what if the number of links is potentially large (up to $V-1$)?



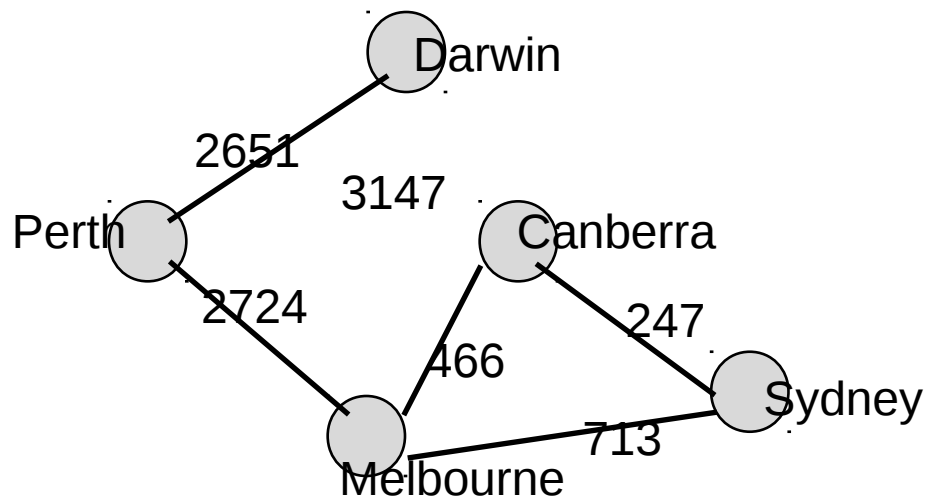
Array representation



A	0	1	2	3	4
0	0	1	0	0	0
1	1	0	0	1	0
2	0	0	0	0	1
3	0	1	0	0	0
4	0	0	1	0	0

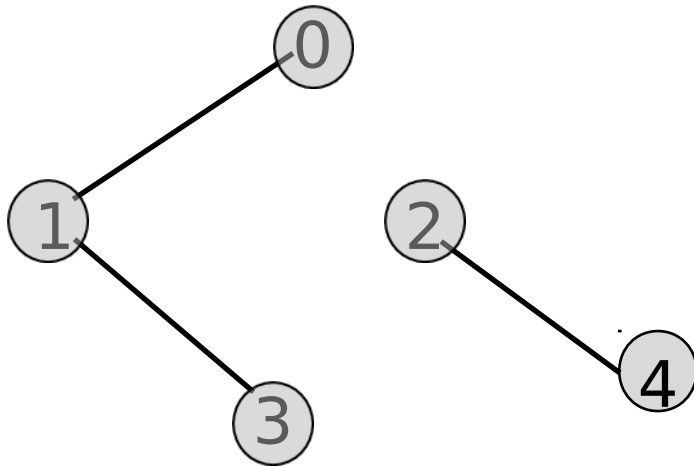
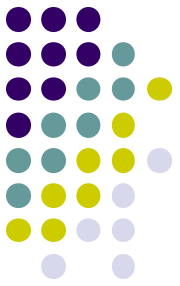


Weighted undirected graph

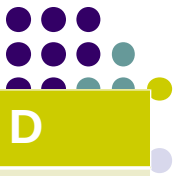


713

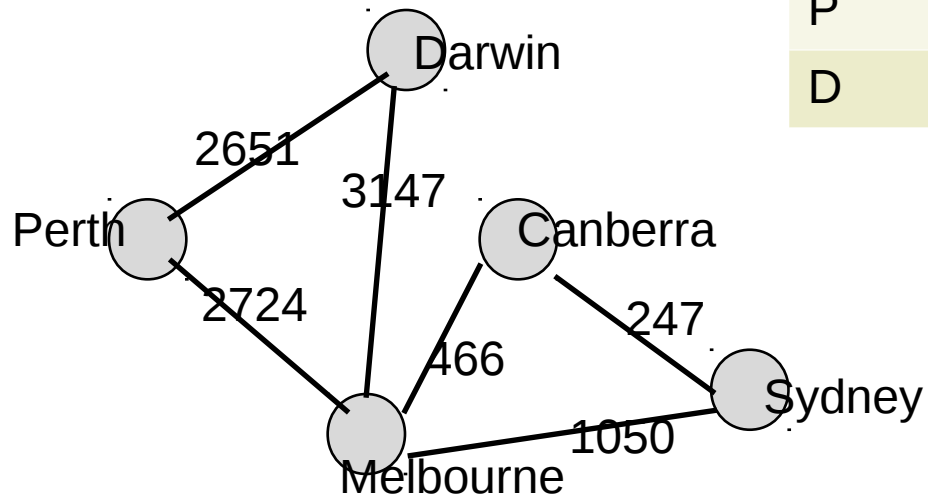
Array representation of weighted undirected graph



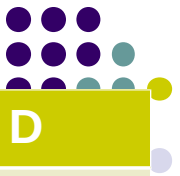
A	0	1	2	3	4
0		25			
1	25			40	
2					10
3		40			
4			10		



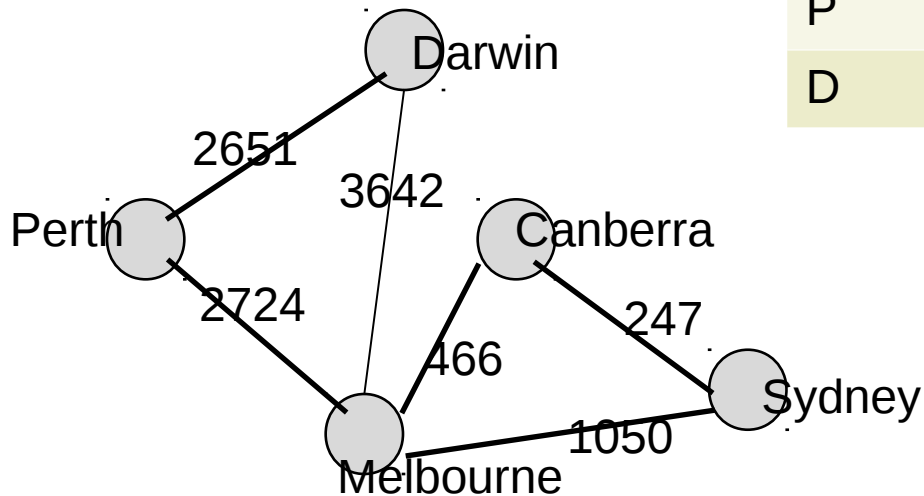
	M	C	S	P	D
M	0	466	1050	2724	3642
C	466	0	247		
S	1050	247	0		
P	2724			0	2651
D	3642			2651	0



713



	M	C	S	P	D
M	0	466	1050	2724	3642
C	466	0	247		
S	1050	247	0		
P	2724			0	2651
D	3642			2651	0

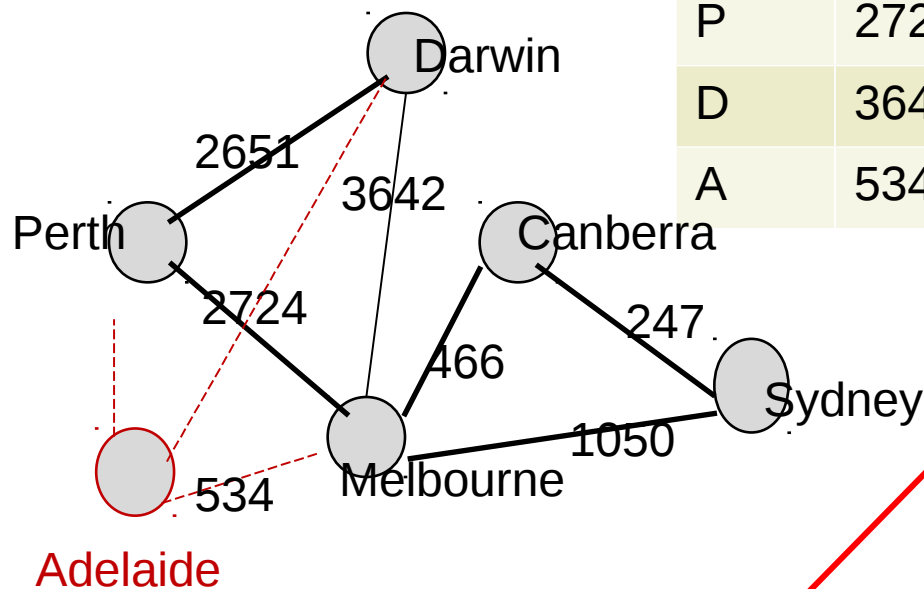


Canberra to Darwin?

$$466 + 3642 = 4108$$

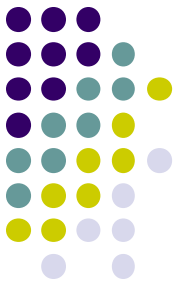


	M	C	S	P	D	A
M	0	466	1050	2724	3642	534
C	466	0	247			1160
S	1050	247	0			1163
P	2724			0	2651	2134
D	3642			2651	0	2620
A	534	1160	1163	2134	2620	0



Canberra to Darwin?
Still $466 + 3642$?
Or $466 + 534 + 2724$?

Shortest route between Melbourne and Dubbo?



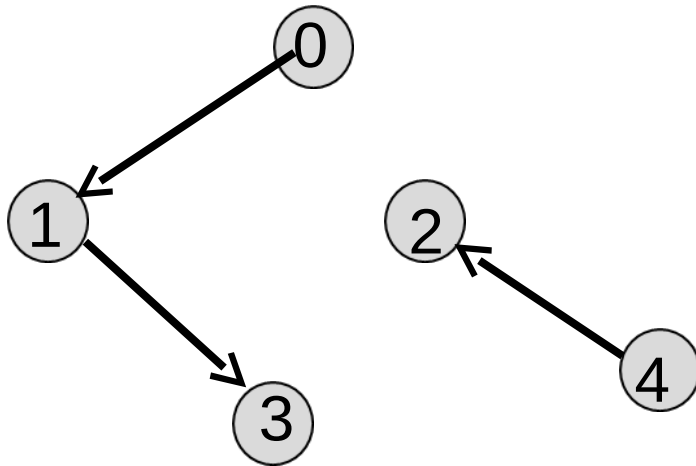
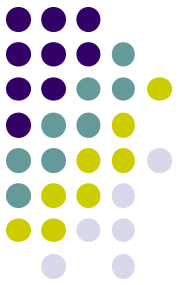
Considerations:

- Distance
- Kind of road
- Traffic points

Shortest route between Melbourne and Dubbo?

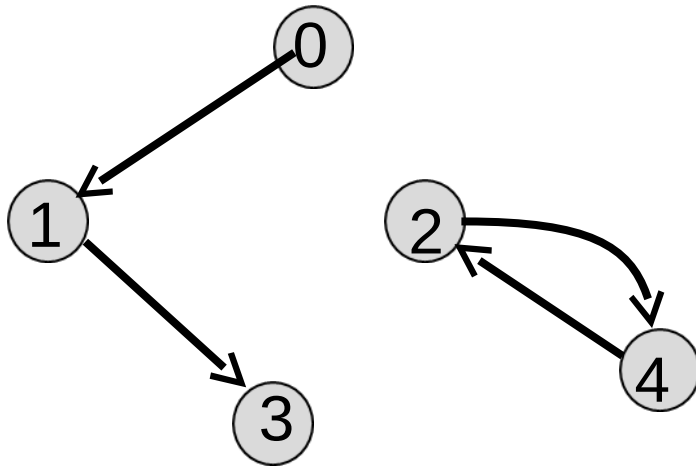
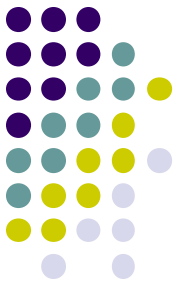


Array representation: weighted directed graph



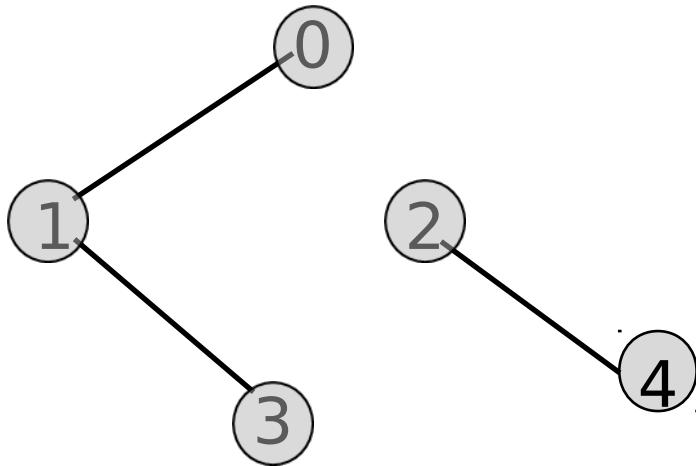
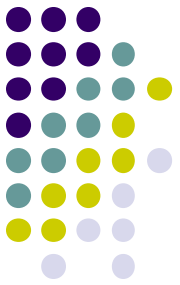
A	0	1	2	3	4
0		25			
1				40	
2					
3					
4			10		

Array representation: weighted directed graph



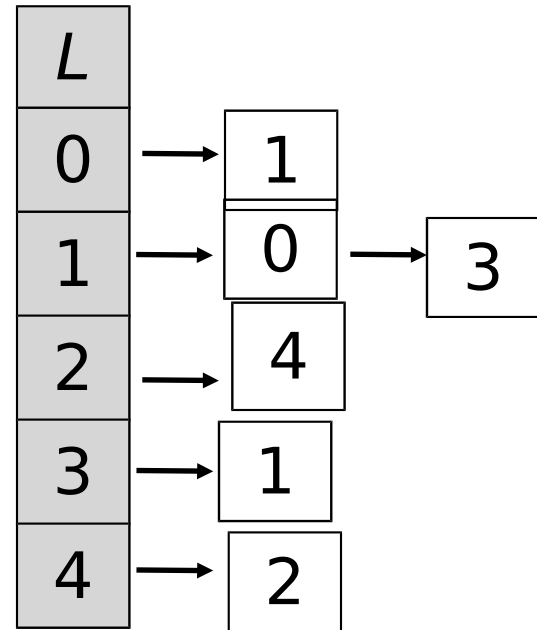
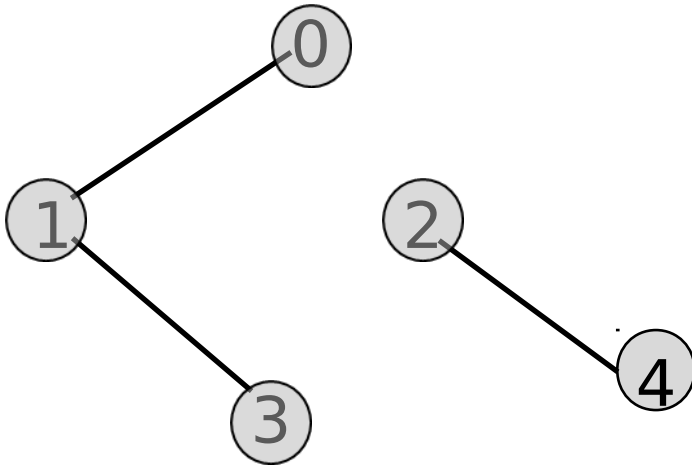
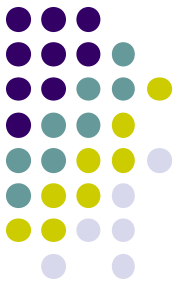
A	0	1	2	3	4
0		25			
1				40	
2					15
3					
4			10		

Array representation: initialization

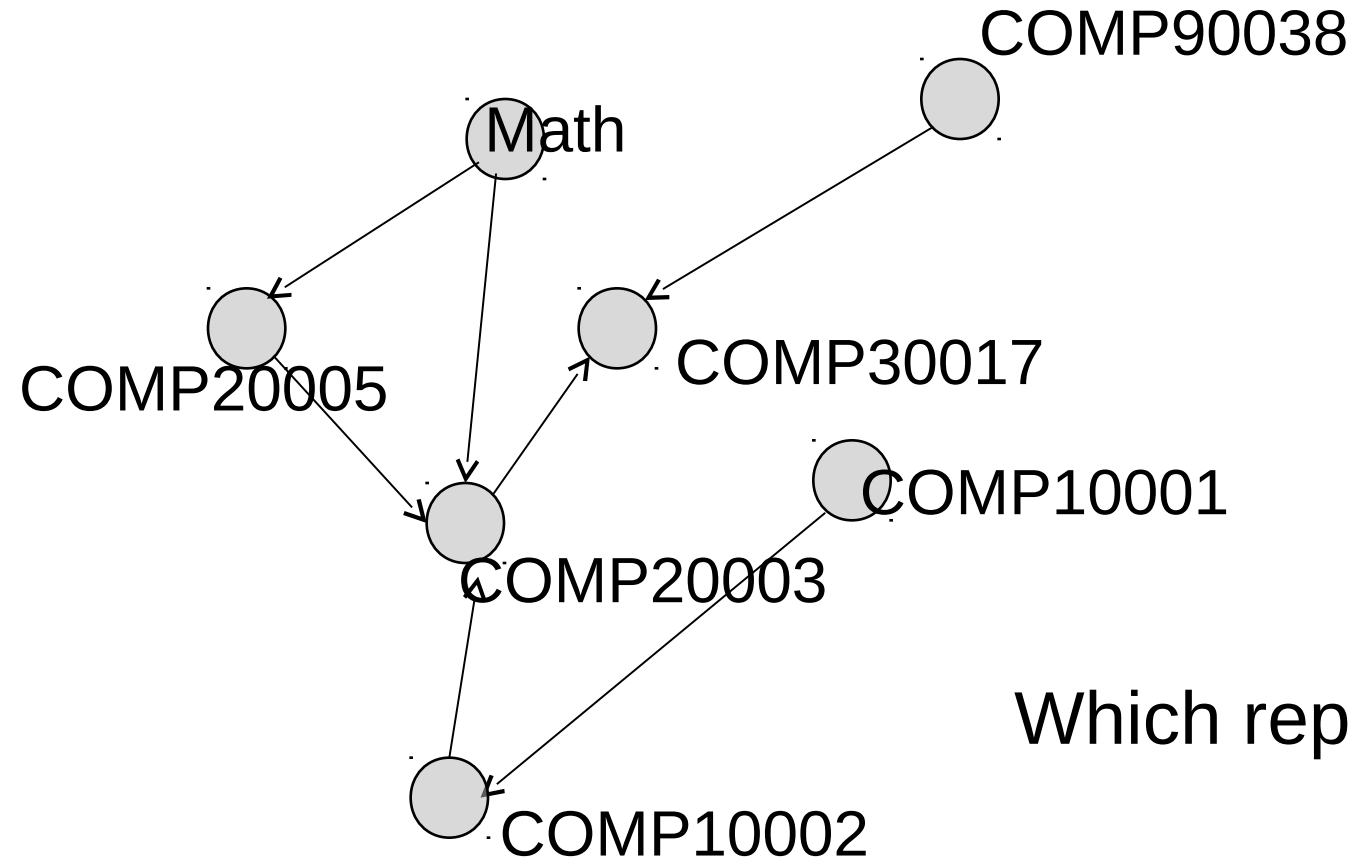
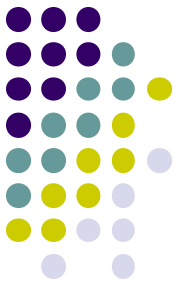


A	0	1	2	3	4
0	∞	25	∞	∞	∞
1	25	∞	∞	40	∞
2	∞	∞	∞	∞	10
3	∞	40	∞	∞	∞
4	∞	∞	10	∞	∞

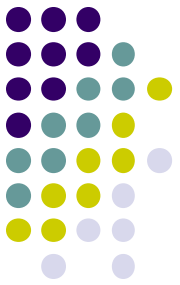
Adjacency list representation



Directed graph: Subject prerequisites

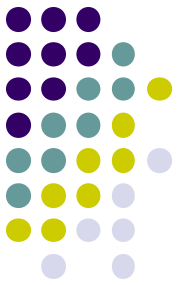


Which representation?



Size of matrix and list

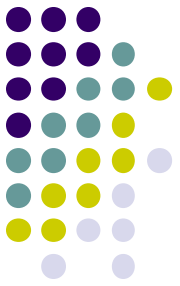
- Size of representation in terms of
 - $|V|$ number of vertices
 - $|E|$ number of edges
- Matrix
 - $O(??)$
- Adjacency list
 - $O(??)$



Size of matrix and list

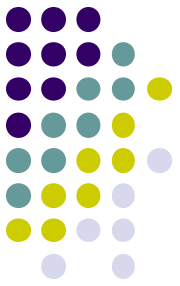
- Matrix
 - $O(|V|^2)$
- Adjacency list
 - $O(|V|+|E|)$
- Dense graph, lots of edges, use matrix representation.
- Sparse graph, use list.

Some interesting graph path problems



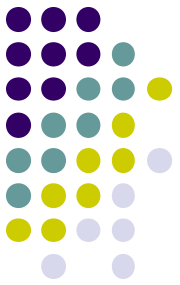
- Reachability.
- Single shortest path.
- Single source shortest path.
- All pairs shortest paths.
- Travelling Salesman Problem.

Other interesting graph problems

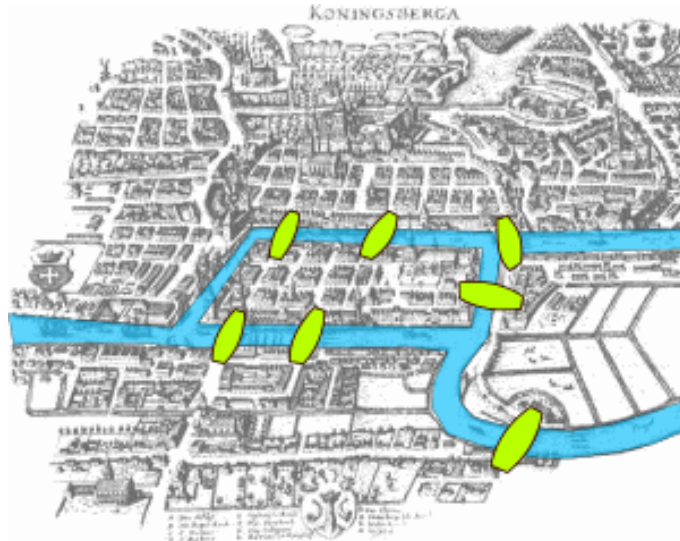


- Minimum Spanning Tree
- Topological sort
- Map coloring
- Matching

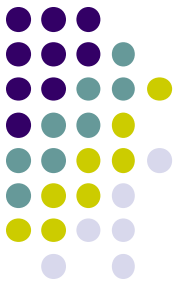
History



- Graph theory started with Euler (1736) who was asked to find a nice path across the seven Königsberg bridges

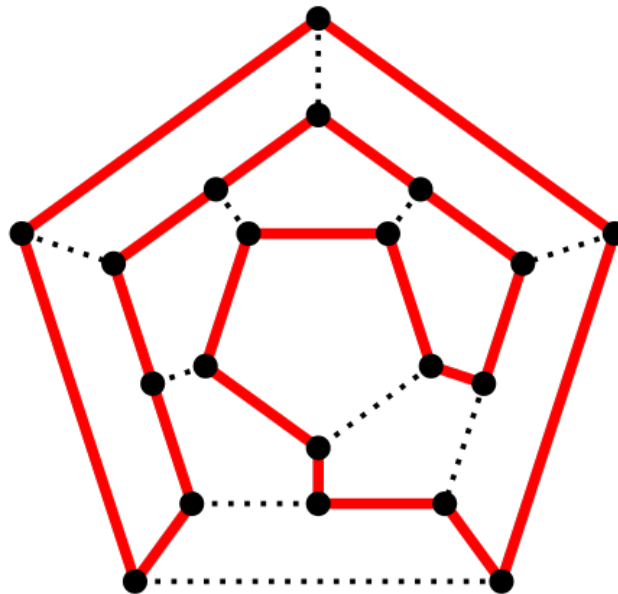


- The (Eulerian) path should cross over each of the seven bridges exactly once

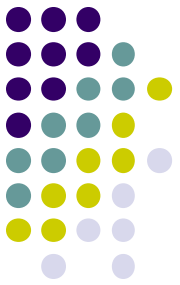


History

- Another early precursor was Sir William Rowan Hamilton (1805-1865)

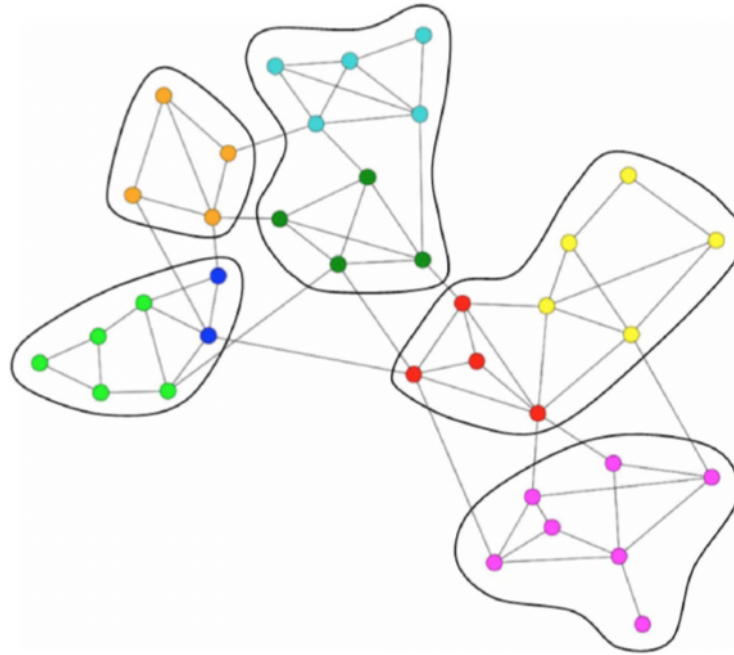


- In 1859 he developed a toy based on finding a path visiting all cities in a graph exactly once and sold it to a toy maker in Dublin. It never was a big success.



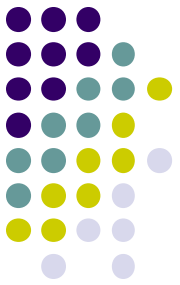
Applications

- But now graph theory is used for finding communities in networks

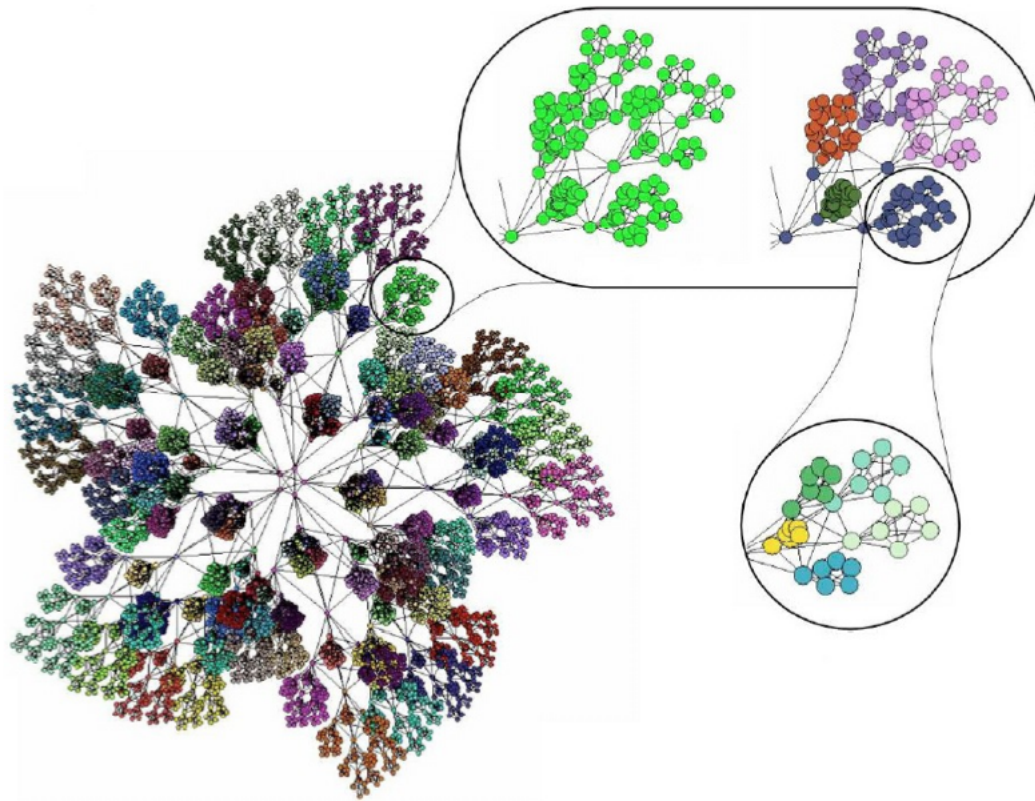


- where we want to detect hierarchies of substructures

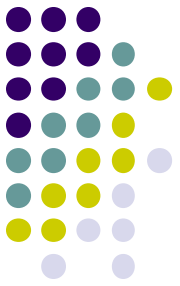
Applications



- and their sizes can become quite big ...



Applications



- It is also used for ranking (ordering) hyperlinks

The screenshot shows a Google search for "university of melbourne". The search bar at the top contains the text "university of melbourne" and a search button. Below the search bar, the results are displayed. The first result is "The University of Melbourne, Australia - Australia's best ...", which includes the website "www.unimelb.edu.au/". Below this, there is a location pin for "Parkville VIC 3010 (03) 9035 5511". To the right of the search results is a map showing the location of the University of Melbourne. Below the map is a detailed card for the University of Melbourne, which includes the university's name, a star rating, and a "Directions" button. The card also provides a brief description of the university, its address, motto, enrollment, and key personnel.

Google university of melbourne Nir

Web Maps Images News Videos More Search tools

About 161,000,000 results (0.58 seconds)

The University of Melbourne, Australia - Australia's best ...
www.unimelb.edu.au/
Oldest Victorian university (established 1855) offering a vast range of coursework and research programs. Melbourne has campuses in both the metropolitan ...
4.7 ★★★★★ 72 Google reviews · Write a review

Parkville VIC 3010
(03) 9035 5511

Results from unimelb.edu.au

Course Search
Undergraduate courses - Graduate course - Bachelor of Science - ...

Graduate course
Master of Engineering - Master of Science - Master of Finance - ...

Staff
Staff page in the Staff website. ... Find people (staff directory) ...

Undergraduate courses
Bachelor of Science - Bachelor of Arts - Bachelor of Commerce - ...

Students
Manage my admin - Ask Unimelb - Explore Unimelb - Plan my future

Web Search
Support the Campaign ... The University of Melbourne ...

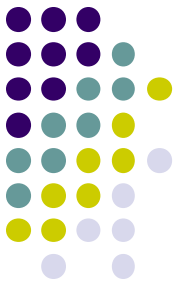
University of Melbourne ★
Public university in Melbourne, Victoria

The University of Melbourne is an Australian public research university located in Melbourne, Victoria. Founded in 1853, it is Australia's second oldest university and the oldest in Victoria.
[Wikipedia](#)

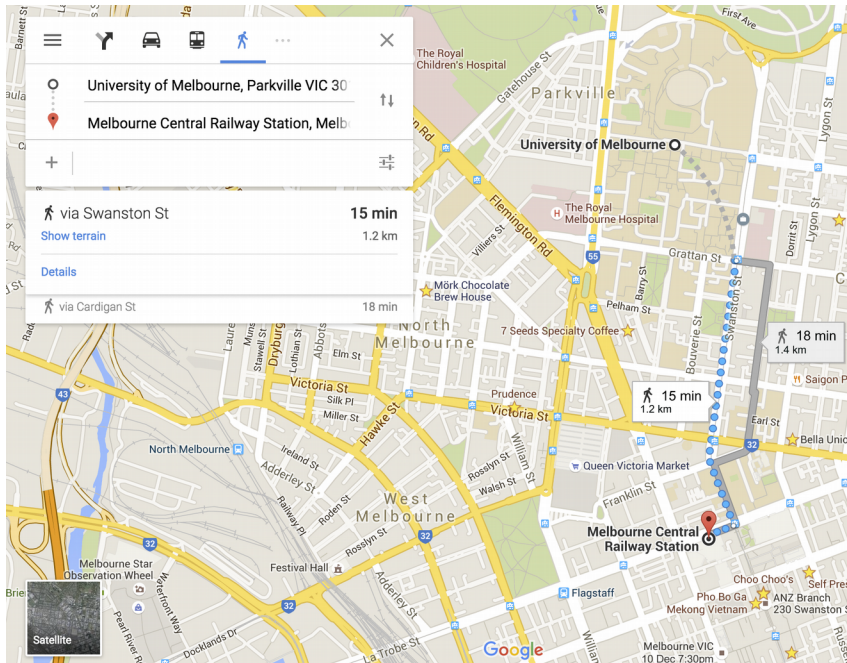
Address: Parkville VIC 3010
Motto: Postera Crescam Laude
Enrollment: 42,653 (2014)
Chancellor: Elizabeth Alexander
Phone: (03) 9035 5511
Founder: Hugh Childers
Vice-chancellor: Glyn Davis

University of Melbourne - Wikipedia, the free encyclopedia
https://en.wikipedia.org/wiki/University_of_Melbourne
The University of Melbourne (informally Melbourne University or simply Melbourne) is an Australian public research university located in Melbourne, Victoria.

Applications



- or by your GPS to find the shortest path home ...



- ... and the list should go on for hours