# The University of Melbourne
# CVEN30008 Engineering Risk Analysis

## Tutorial 7

## Confidence Intervals

1. **Quality risk:**
   **A supplier sells synthetic fibers to a manufacturing company. A random sample of 81 fibers is selected from a shipment (first shipment). The average breaking strength of them is 29N, and the standard deviation is 9N.**

**<u>Answer:</u>**

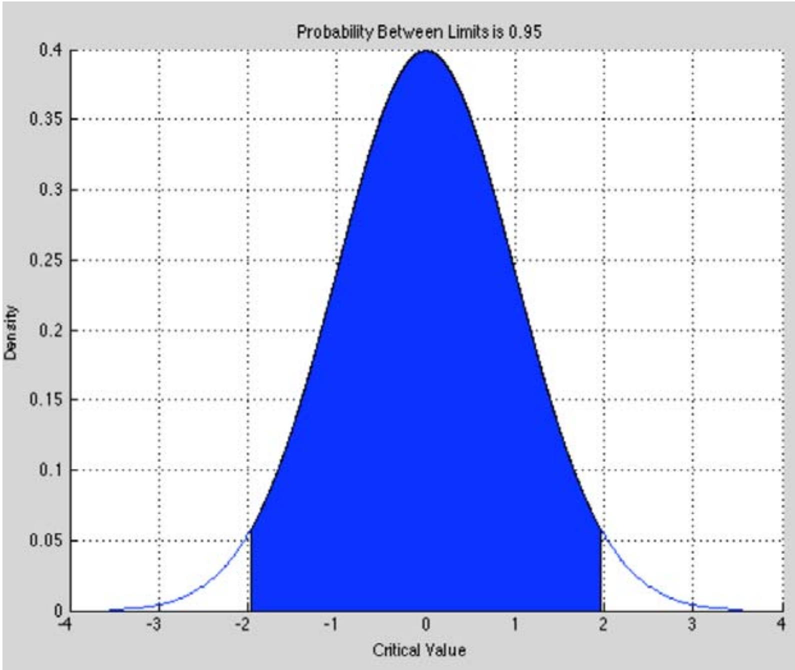
a). Find a 95% confidence interval for the mean breaking strength of all the fibers in the shipment. Verify your results by using MATLAB.

From the question, we know the sample size is 81 which is greater than 30, hence, it's large-sample. The equation used to find the confidence interval is $\bar{X} \pm Z\frac{S}{\sqrt{n}}$
For 95% confidence interval, $Z$ is 1.96. We know $S$ = 9N, and $\bar{X}$ = 29N.
95% confidence interval = $29 \pm 1.96\frac{9}{\sqrt{81}} = \ (27.04, 30.96)$

MATLAB

Command Window

```
Z_alphaon2 =

  -1.9600


CI =

  27.0400   30.9600

fx >>
```




*b). How many fibers must be sampled so that a 95% confidence interval specifies the mean to within ±1*

The 95% confidence interval for part *b*) is $29 \pm 1.96\frac{9}{\sqrt{n}}$ and sample size *n* is the value we need to determine

For the mean to within $\pm 1$

The interval is $29 \pm 1 = 29 \pm 1.96\frac{9}{\sqrt{n}} \Rightarrow 1 = 1.96\frac{9}{\sqrt{n}}$

*n* = 312 (Rounded up to nearest integer)

*c). What is the confidence level of the interval (27.5, 30.5) based on the sample size is 81?*

For part c), we need to determine the *Z* value which gives an interval of the mean to within the range (27.5, 30.5).

Hence, $\bar{X} \pm Z\frac{S}{\sqrt{n}} = (27.5, 30.5) = 29 \pm Z\frac{9}{\sqrt{81}}$

$$27.5 = 29 - Z\frac{9}{\sqrt{81}}$$

$$Z = 1.5$$

Since α(*Z*>1.5) = α(*Z*<-1.5)

For Z = -1.5, the α/2 value is 0.0663.

The confidence interval is 1 – α = 1 – 2 x 0.0668 = 86.6%

The confidence interval is 86.6% to have a mean to within the range of (27.5, 30.5).

2. **Quality risk:**
**The same supplier sells second shipment to the manufacturing company. A sample of 10 measurements is taken. The breaking strengths of the 10 measurements, in N, are:**

**25, 26, 28, 30, 32, 29, 29, 30, 33, 26**

**Find a 95% confidence interval for the breaking strength based on the 10 samples, and verify your results by using MATLAB**

**Answer:**

n = 10
degrees of freedom = n -1 = 9
$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(25 + 26 \ldots + 26)$ =28.8N

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\bar{X} - x_i)^2} = \sqrt{\frac{1}{10-1}[(25 - 28.8)^2 - (26 - 28.8)^2 \ldots + (25 - 28.8)^2]} = 2.62$$

Since the sample size "n" is smaller than 30, hence, it's small sample.
Equation used to find the confidence interval is $\bar{X} \pm t\frac{S}{\sqrt{n}}$
For 95% confidence interval, and degrees of freedom of 9, $t$ = 2.26.
95% confidence interval = $28.8 \pm 2.26\frac{2.62}{\sqrt{10}} = (26.9,\ 30.7)$

MATLAB

```
Command Window

t_alphaon2 =

   -2.2622


CI =

   26.9285   30.6715

fx >>
```

3. **Quality risk:**
**There are third and fourth shipments delivered to the manufacturing company by the supplier at the same time. 50 samples were taken from each shipment. The average breaking strength of the fibers are 27, 31 for the third and fourth shipments respectively. The standard deivation are 9 and 10 for the third and fourth shipments respectively. Find the 95% confidence interval for the difference between the breaking strength of these two shipments. Verify your results by using MATLAB**

**Answer:**
Let $X_1, \ldots X_{50}$ represent the breaking strength of the 50 measurements taken from the four shipment, and
Let $Y_1, \ldots Y_{50}$ represent the breaking strength of the 50 measurements taken from the third shipment
We have $\bar{X} = 31$, $\bar{Y} = 27$, $s_x = 10$, $s_y = 9$, $n_x = n_y = 50$
To find the confidence interval for the difference between two means, we need to use the equation

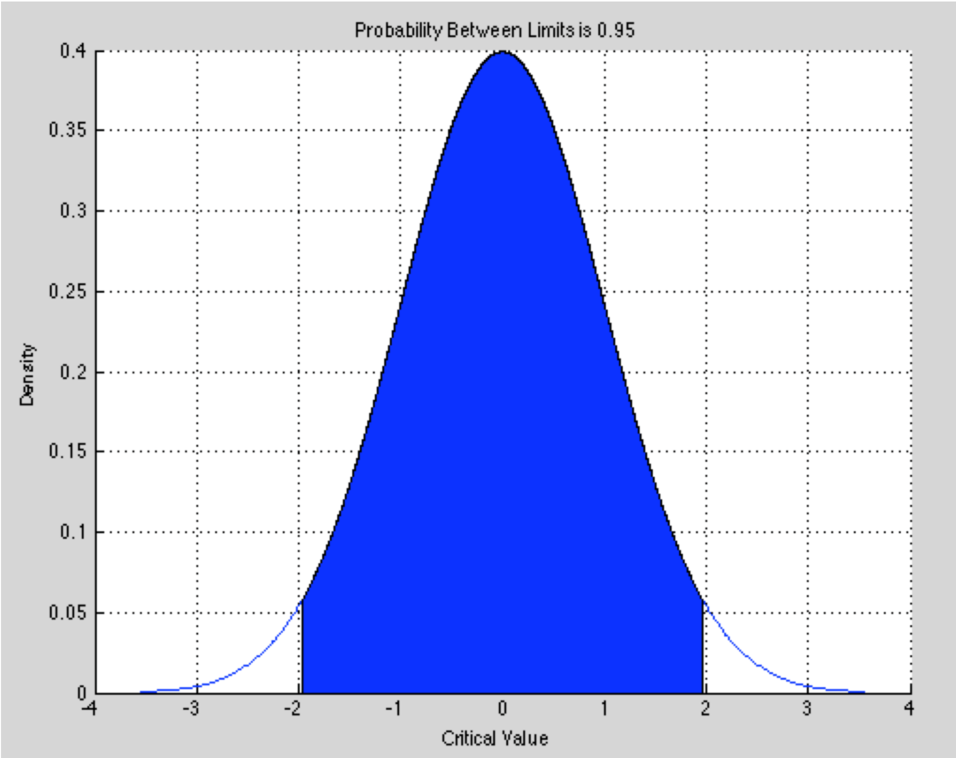
$$\bar{X} - \bar{Y} \pm Z\sqrt{\frac{{s_x}^2}{n_x} + \frac{{s_y}^2}{n_y}}$$

The 95% confidence interval for the difference between the breaking strength of these two shipments:

$$31 - 27 \pm 1.96\sqrt{\frac{10^2}{50} + \frac{9^2}{50}} = (0.27, 7.73)$$

MATLAB

```
Command Window

Z_alphaon2 =

   -1.9600


CI =

    0.2709    7.7291

fx >>
```


Probability Between Limits is 0.95
Density
Critical Value
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-4
-3
-2
-1
0
1
2
3
4