020-156 Summer Semester 2009 - Solutions

1 (a) (i) Not possible ①

(ii) $A\underset{\sim}{v} = \begin{bmatrix} -3 & 1 & 3 \\ 4 & -2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$ ①

(iii) $\underset{\sim}{v}\,\underset{\sim}{v}^T = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ ①

(iv) $\underset{\sim}{v}^T \underset{\sim}{v} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 3$ ①

(b) (i)
$$\begin{aligned} -3x + y + 3z &= 0 \\ 4x - 2y &= 0 \end{aligned}$$ ①

(ii)
$$\begin{aligned} 6x + 2y &= -1 \\ 2x - y &= 2 \end{aligned}$$ ①

2 (a) $\det H = \det \begin{bmatrix} 1 & 1 & 1 \\ \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ \cos^2\alpha & \cos^2\beta & \cos^2\gamma \end{bmatrix}$

($\frac{1}{2}$) $= \det \begin{bmatrix} 1 & 1 & 1 \\ \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ 1 & 1 & 1 \end{bmatrix}$ adding row ② to row ③ and noting $\sin^2\theta + \cos^2\theta = 1$

$= \det \begin{bmatrix} 1 & 1 & 1 \\ \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ 0 & 0 & 0 \end{bmatrix}$ subtracting row ① from row ③

$= 0$

($\frac{1}{2}$) We know that a square matrix is invertible if and only if its determinant is non-zero.
But the determinant of H is zero, so it is not invertible.

(b) (i) $\det AB = \det A \det B = 2 \times \frac{3}{2} = 3$ ①

(ii) $\det A^T A = \det A^T \det A = (\det A)^2 = 4$ ①

(iii) $\det(2B) = 2^5 \det B = 2^5 \times \frac{3}{2} = 48$ ①

3(a) (i) Because $\det G = 1 \neq 0$ ①

(ii) $$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} ① - ④ \\ ② - ④ \\ ③ - ④ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} ① - ③ \\ ② - ③ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} ① - ② \\ \\ \\ \end{array}$$

Working ②

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Hence $$G^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer ①

(b) As a matrix equation the linear system reads

$$G\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \qquad 1$$

Applying G^{-1} to both sides gives

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = G^{-1}\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} \qquad 1$$

4 (a) (i) Let $\underset{\sim}{a}$ denote such a vector. Then

$$\underset{\sim}{a} = \det\begin{bmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \underset{\sim}{i}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - \underset{\sim}{j}\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + \underset{\sim}{k}\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \qquad 1\tfrac{1}{2}$$

$$= -\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$$

(ii) Since the plane passes through the origin, it is specified by

$$\underset{\sim}{a} \cdot (x, y, z) = 0$$

$$\Rightarrow (-1, 2, -1) \cdot (x, y, z) = 0 \qquad 1\tfrac{1}{2}$$

$$\Rightarrow x - 2y + z = 0$$

(b) This is the same as the area of the triangle formed by $(0,0,0)$, $(-3,2,-1)$, $(0,3,1)$, which in turn is half the area of the parallelogram formed by $(-3,2,-1)$ and $(0,3,1)$:

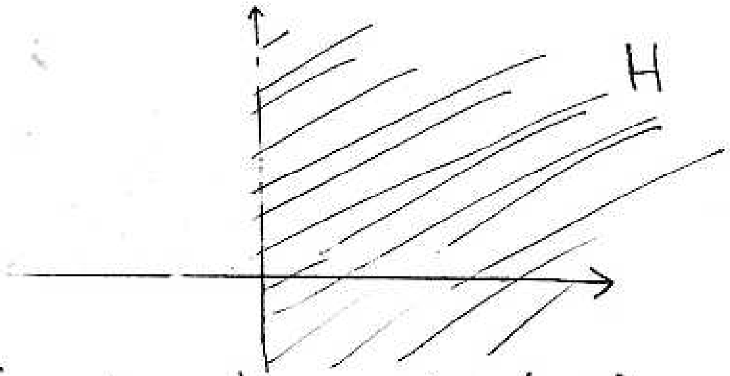
$$\frac{1}{2}\,\|(-3,2,-1)\times(0,3,1)\| = \frac{1}{2}\sqrt{\|(-3,2,-1)\|^2\|(0,3,1)\|^2-((-3,2,-1)\cdot(0,3,1))^2}$$

$$= \frac{1}{2}\sqrt{14\times 10-25}$$

3

$$= \frac{1}{2}\sqrt{115}$$

5 (a)



1

Consider the point $(1,0)$ in H. Scalar multiplication by (-1) gives the point $(-1,0)\notin H$. The set is not closed under scalar multiplication, and so is not a subspace.

1

(b) $a+d=0$

With the correspondence $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \leftrightarrow (a,b,c,d)\in \mathbb{R}$

we recognise $a+d=0$ as a homogeneous linear eq^{n}. We know that the solution space of homogeneous linear equations forms a subspace of $\mathbb{R}^4$, so we suspect S is a subspace of $M_{2,2}$.

2

(c) Consider $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in S$ so that $a+d=0$.

Then, with α a scalar

$$\alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix}$$

and

$$\alpha a + \alpha d = \alpha(a+d) = 0 \quad \text{since } a+d=0$$

Hence $\alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in S$ and so S is closed under scalar multiplication. (2)

6 (a) rank $A = 3$ (1)

(b) $\{(1,0,0,1,-20), (0,1,0,-1,58), (0,0,1,0,67)\}$, (1)
using the fact that the non-zero rows in RE form are a basis for the row space

(c) $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -10 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 0 \\ 1 \end{bmatrix} \right\}$ (1)

(d) No. The rank of the matrix formed by these vectors is 3. (1)

(e) Three (1)

(f) $(3,13,2,0) = (1,3,1,2) + (-1)(-2,-10,-1,2)$ (1)

(g) Let the variables be denoted $x_1, x_2, \ldots, x_5$.
There is no leading entry for x_4 & x_5 so we set $x_5 = t$, $x_4 = s$. Back substitution gives

$$x_3 = -67x_5 = -67t$$
$$x_2 = x_4 - 58x_5 = s - 58t$$
$$x_1 = -x_4 + 20x_5 = -s + 20t$$

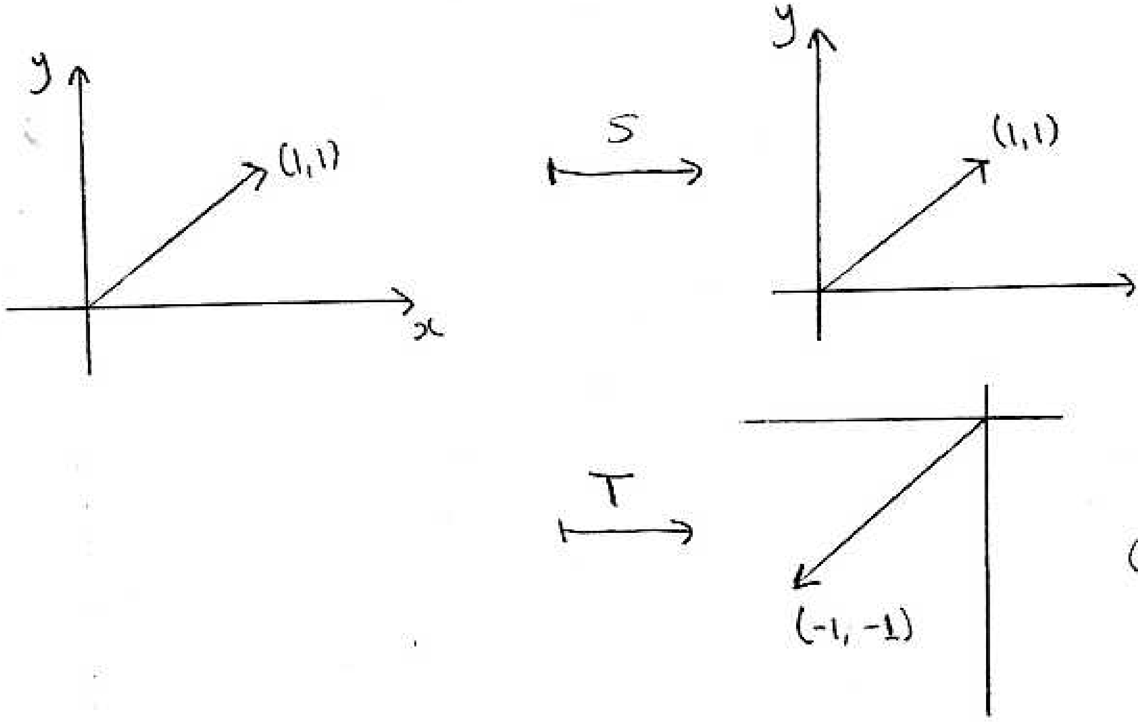
Hence

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 20 \\ -58 \\ -67 \\ 0 \\ 1 \end{bmatrix}$$ 2

so a basis for the sol$^{\underline{n}}$ space is

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 20 \\ -58 \\ -67 \\ 0 \\ 1 \end{bmatrix} \right\}$$

7. (a)



1

1

(b) $S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ Hence $A_S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ 1

$R\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ Hence $A_R = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ 1

8. (a) $A_T = \begin{bmatrix} 3 & -1 & -6 \\ -2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix}$ (1)

(b) $\begin{bmatrix} 3 & -1 & -6 \\ -2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix} \begin{array}{l} \\ ② + \frac{2}{3}① \\ ③ - ① \end{array} \sim \begin{bmatrix} 3 & -1 & -6 \\ 0 & 1/3 & 1 \\ 0 & 4 & 12 \end{bmatrix} \begin{array}{l} \\ \\ ③ - 12 \times ② \end{array}$

$\sim \begin{bmatrix} 3 & -1 & -6 \\ 0 & 1/3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

We know that Ker(T) is the same as the solution space of A_T. For this, x_3 has no leading entry, so set

$$x_3 = t.$$

Back substitution gives $\frac{1}{3}x_2 = -x_3 = -t \Rightarrow x_2 = -3t$,

$$3x_1 = x_2 + 6x_3 \Rightarrow 3x_1 = -3t + 6t = 3t$$
$$\Rightarrow x_1 = t$$

Hence the sol^n space is (3)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ -3t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$$

so a basis is $\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \right\}$ This is a line through the origin in $\mathbb{R}^3$.

(c) T is not invertible since Rank T = 2 < 3 (dimension of $\mathbb{R}^3$). (1)

(d) We know that $\mathrm{Im}(T)$ is the same as the column space of A_T. Since the leading entries are in columns ① & ②, a basis is

$1\frac{1}{2}$

$$\left\{ \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \right\}$$

This corresponds to a plane through the origin in $\mathbb{R}^3$.

(e) We observe that

$$(4,-3,0) = (3,-2,3) + (-1)(-1,1,3)$$

and so is in the span of the basis vectors for $\mathrm{Im}(T)$, and is thus an element of $\mathrm{Im}(T)$. We read off from this that

$$\left[(4,-3,0)\right]_{\mathcal{C}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$1\frac{1}{2}$

9. (a) We know that $\mathcal{B}$ is a basis if and only if

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

has rank 3. Now

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{matrix} \\ ② - ① \\ ③ - ① \end{matrix} \sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

which indeed has rank 3.

1

(b) We can write down the transition matrix $P_{S,\mathcal{B}}$ from $\mathcal{B}$ to the standard basis:

$$P_{S,\mathcal{B}} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

1

We know that

$$P_{B,S} = P_{S,B}^{-1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ 2 - 1 \\ 3 - 1 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ (-1)2 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} 1 + 3 \\ 2 + 2\times 3 \\ \\ \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} 1 - 2 \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

Hence $P_{S,B}^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} = P_{B,S}$ 2

(c) $[\underset{\sim}{v}]_B = \begin{bmatrix} 1 & 1 & -1 \\ -1 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ 2

10. (a) $\det(A-\lambda I) = \begin{vmatrix} 7-\lambda & -2 \\ 15 & -4-\lambda \end{vmatrix}$

$= (7-\lambda)(-1)(4+\lambda) - (-2)\times 15$

$= (\lambda-7)(\lambda+4)+30 = \lambda^2-3\lambda+2 = (\lambda-2)(\lambda-1)$ 2

(b) Eigenvalues $\lambda=2$, $\lambda=1$.

$\lambda=2$ eigenvector : $\left[\begin{array}{cc|c} 5 & -2 & 0 \\ 15 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 5 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Set $y=t$. Back substitution gives $5x = 2y = 2t$

$\Rightarrow x = \frac{2}{5}t$

Taking for example $t=5$ gives the eigenvector $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

1

$\lambda=1$ eigenvector: $\left[\begin{array}{cc|c} 6 & -2 & 0 \\ 15 & -5 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 6 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Set $y=t$. Back substitution gives $x = \frac{1}{3}t$

Taking for example $t=3$ gives the eigenvector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

1

(c) A is diagonalizable because the eigenvectors are linearly independent (this is always the case when the eigenvalues are distinct)

1

$A = PDP^{-1}$ with $P = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

1

11 (a) $5x_1y_1 - x_2y_2 = [x_1 \;\; x_2]\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ (1)

(b) We require that

(i) $b = c$ (i.e. the matrix is symmetric)

(ii) $[u \;\; v]\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} \geq 0$ (2)

with equality if and only if $u = v = 0$.

Here

$$[u \;\; v]\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = [5u \;\; -v]\begin{bmatrix} u \\ v \end{bmatrix}$$

$$= 5u^2 - v^2$$ (1)

But with $u = 0$, $v = 1$ (for example) this is negative, which violates (ii).

(c) (i)

$$\underset{\sim}{x} = (\underset{\sim}{u}_1 \cdot \underset{\sim}{x})\underset{\sim}{u}_1 + (\underset{\sim}{u}_2 \cdot \underset{\sim}{x})\underset{\sim}{u}_2 + (\underset{\sim}{u}_3 \cdot \underset{\sim}{x})\underset{\sim}{u}_3$$

$$= \tfrac{1}{3}(3)\underset{\sim}{u}_1 + \tfrac{1}{3}(6)\underset{\sim}{u}_2 + \tfrac{1}{3}(9)\underset{\sim}{u}_3$$ (1)

$$= \underset{\sim}{u}_1 + 2\underset{\sim}{u}_2 + 3\underset{\sim}{u}_3$$

(ii)

$$P_{U,S} = \begin{bmatrix} \underset{\sim}{u}_1^T \\ \underset{\sim}{u}_2^T \\ \underset{\sim}{u}_3^T \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$ (1)

12. (a) $y = a + bx$ where, with

$$A = \begin{bmatrix} 1 & -3 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad \underset{\sim}{y} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 3 \end{bmatrix}$$

$$A^T A \begin{bmatrix} a \\ b \end{bmatrix} = A^T \underset{\sim}{y}$$

Now $A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 14 \end{bmatrix}$$

$\Rightarrow (A^T A)^{-1} = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/14 \end{bmatrix}$ 4

and so $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/14 \end{bmatrix} A^T \underset{\sim}{y}$

But

$A^T \underset{\sim}{y} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/14 \end{bmatrix} \begin{bmatrix} 2 \\ 12 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 6/7 \end{bmatrix}$

Hence $y = \frac{1}{2} + \frac{6}{7}x$ is the line of best fit

(b) $y = \frac{1}{2} + \frac{6}{7}x$

With $x = -2$ this gives $y = \frac{1}{2} - \frac{12}{7} \simeq -1^\circ C$.

2

13. (a) Eigenvalues: $\det(A - \lambda I) = 0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(-3-\lambda) - 16 = 0$$

$$\Rightarrow (\lambda-3)(\lambda+3) - 16 = 0$$

2

$$\Rightarrow \lambda^2 - 9 - 16 = 0 \quad \Rightarrow \lambda = \pm 5.$$

Eigenvectors: $\lambda = 5$

$$\left[\begin{array}{cc|c} -2 & 4 & 0 \\ 4 & -8 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -2 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Set $y = t$
$\Rightarrow x = 2t$

$\Rightarrow$ eigenvector $t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\Rightarrow$ normalized eigenvector $\frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\lambda = -5$

$$\left[\begin{array}{cc|c} 8 & 4 & 0 \\ 4 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 8 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Set $y = t$
$\Rightarrow x = -\frac{t}{2}$

$\Rightarrow$ eigenvector $t\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$ $\Rightarrow$ normalized eigenvector $\frac{1}{\sqrt{5}}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

~~We see that~~ $\cancel{\left(\frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) \cdot \left(\frac{1}{\sqrt{5}}\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \frac{1}{5}(-2+2) = 0}$

~~thus demonstrating that the vectors are orthogonal.~~

!

(b) The transformation scales by a factor of -5 in the direction of

2 $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, and scales by a factor of 5 in the direction of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $A^{10}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = 5^{10}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ since $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an

2 eigenvector.