The University of Melbourne
CVEN30008 Engineering Risk Analysis

Tutorial 5 Continuous Distribution

Quality Risk

1. **The lifetime of a battery in a certain application is normally distributed with mean μ = 16 hours and standard deviation σ = 2 hours**
   a) **What is the probability that a battery will last more than 19 hours?**
   b) **What is the probability that the lifetime of a battery is between 14.5 and 17 hours?**
   c) **Verify your results by using Matlab**

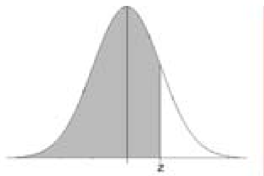
Solution:

a) $\mu = 16$, $\sigma = 2$
For $19 < X$

$$P = (19 < X < \infty) = \Phi\left(\frac{\infty - 16}{2}\right) - \Phi\left(\frac{19 - 16}{2}\right) = 1 - \Phi(1.5)$$

From Standard Normal Cumulative Probability Table

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

$$\Phi(1.5) = 0.9332$$

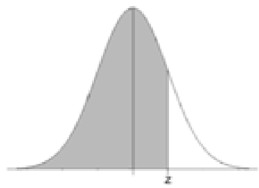
$$P == 1 - \Phi(1.5) = 1 - 0.9332 = 0.0668$$

b) For 14.5 < X < 17

$$P = (14.5 < X < 17) = \Phi\left(\frac{17-16}{2}\right) - \Phi\left(\frac{14.5-16}{2}\right)$$
$$= \Phi(0.5) - \Phi(-0.75)$$

From Standard Normal Cumulative Probability Table

Standard Normal Cumulative Probability Table

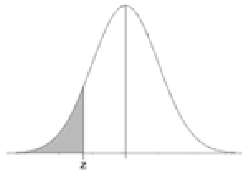


Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

$$\Phi(0.5) = 0.6915$$

Standard Normal Cumulative Probability Table



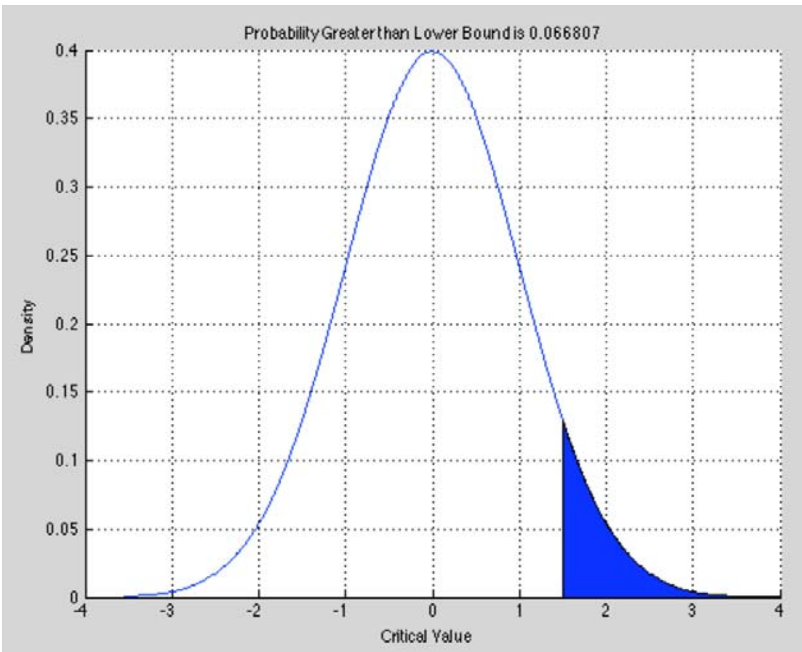
Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776

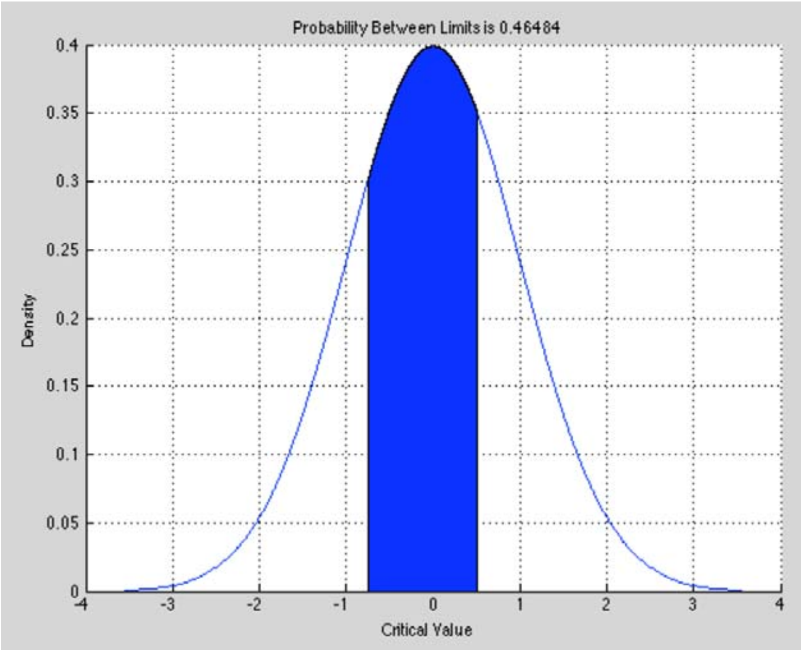
$$\Phi(-0.75) = 0.2266$$

$$\Phi(0.5) - \Phi(-0.75) = 0.6915 - 0.2266 = 0.4649$$

c) For $19 < X$



For $14.5 < X < 17$



2. Based on lognormal distribution, repeat question 1

Solution

a) $COV = {}^{\sigma}/_{\mu} = {}^{2}/_{16} = 0.125 \leq 0.3$

$\xi = COV = 0.125,$

$\lambda = \ln(\mu) - 0.5, \xi^2 = \ln(16) - 0.5 \times 0.125^2 = 2.7648$

For 19 < X

$$P = (19 < X < \infty) = \Phi\left(\frac{ln(\infty) - 2.7648}{0.125}\right) - \Phi\left(\frac{ln(19) - 2.7648}{0.125}\right)$$

$$\approx 1 - \Phi(1.44)$$

From Standard Normal Cumulative Probability Table

$$\Phi(1.44) = 0.9251$$

$$P = 1 - \Phi(1.44) = 1 - 0.9251 = 0.0749$$

b) $P = (14.5 < X < 17) = \Phi\left(\frac{ln(17)-2.76}{0.125}\right) - \Phi\left(\frac{ln(14.5)-2.76}{0.125}\right) =$

$\Phi(0.55) - \Phi(-0.73)$

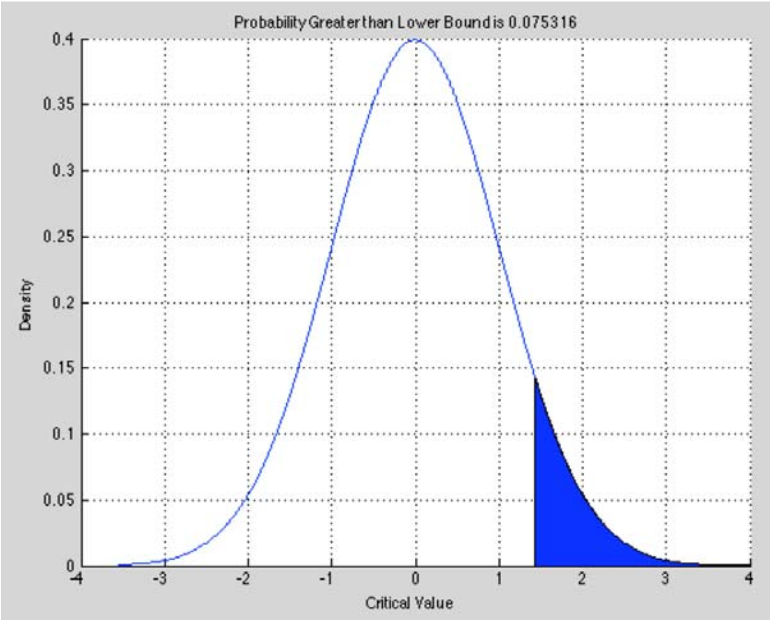
From Standard Normal Cumulative Probability Table

$\Phi(0.55) = 0.7088$

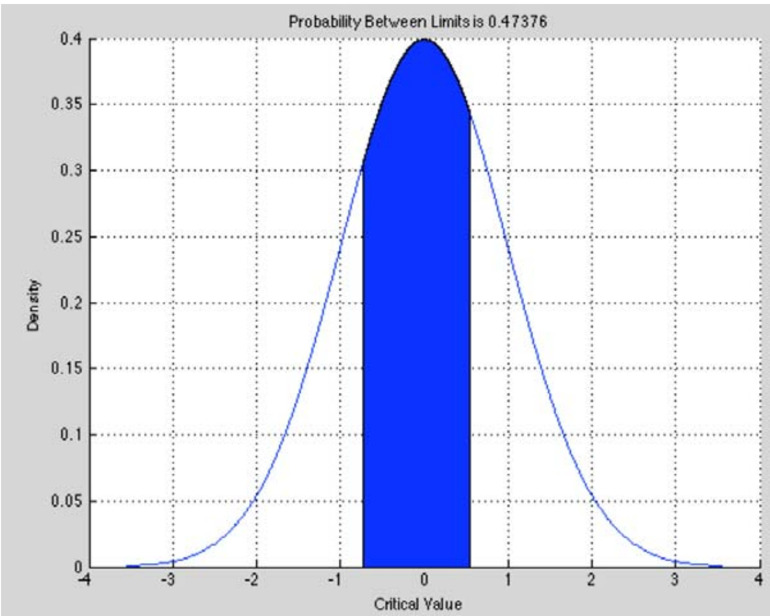
$\Phi(-0.73) = 0.2327$

$$P = \Phi(0.55) - \Phi(-0.73) = 0.7088 - 0.2327 = 0.4761$$

c) $P = (19 < X < \infty)$



$P = (14.5 < X < 17)$



3. The strength of an aluminium alloy is normally distributed with mean 10 GPa and standard deviation 1.4 GPa.

(a) What is the probability that a specimen of this alloy will have strength greater than 12 GPa?

(b) What is the probability that a specimen of this alloy will have strength smaller than 9 GPa?

(c) Verify your results by using Matlab

Solution

(a) $\mu = 10$, $\sigma = 1.4$

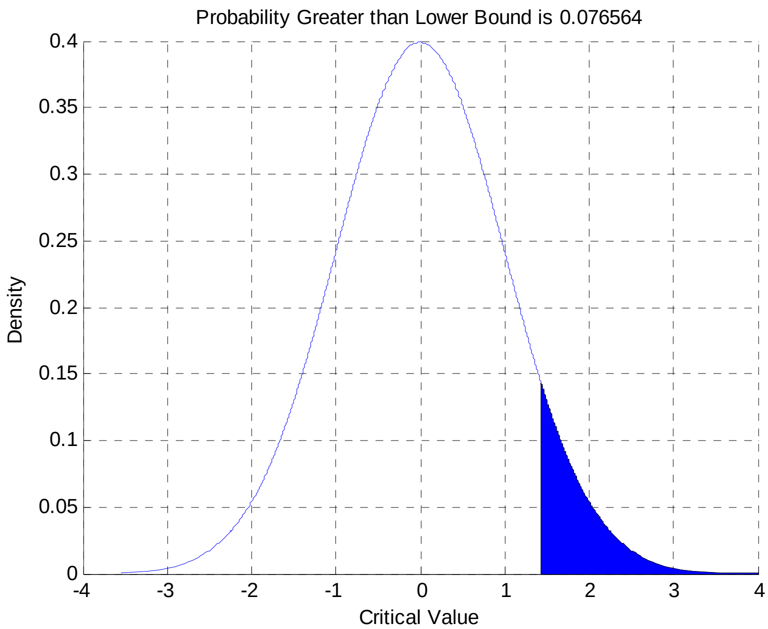
For $12 < X$

$$P = (12 < X < \infty) = 1 - \Phi\left(\frac{12-10}{1.4}\right) = 7.64\%$$

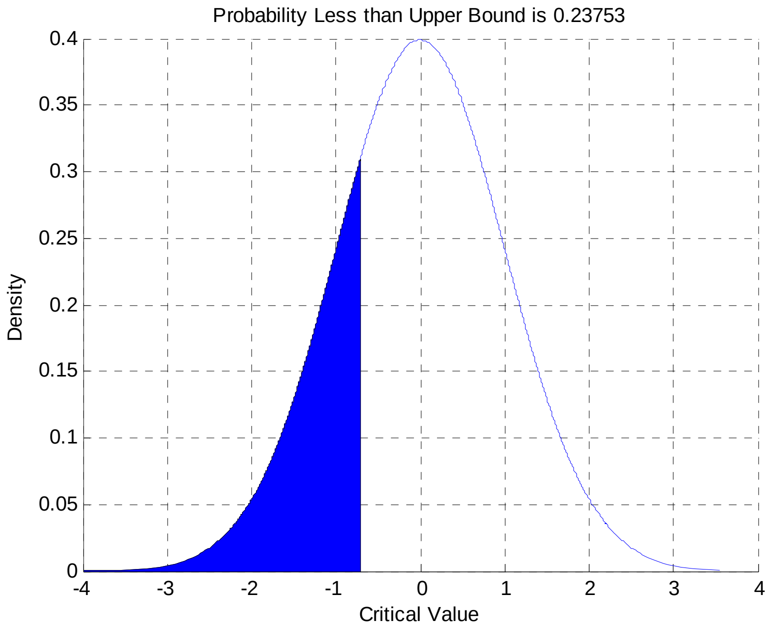
(b) For $X < 9$

$$P = (-\infty < X < 9) = \Phi\left(\frac{9-10}{1.4}\right) = 23.89$$

(c) $P = (12 < X < \infty)$



$P = (-\infty < X < 9)$


Probability Less than Upper Bound is 0.23753
Density
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-4
-3
-2
-1
0
1
2
3
4
Critical Value