MAST10007 (Summer) Assignment 2 Due Mon. Jan. 18th 1.30 p.m.

Please leave your assignment in your tutor's box located near the north entrance to the Richard Berry building. Make sure that you include your name, your student number and your tutor's name.

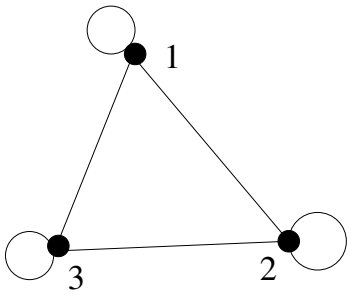
1. (a) Let A be a square matrix. State the condition on the determinant of A that guarantees A is invertible.

   (b) Show that the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1-\alpha^2 & 1-\beta^2 & 1-\gamma^2 \end{bmatrix}$$

   is singular for all values of α, β, γ.

2. Consider the following graph, with nodes labelled 1,2 and 3.



   (a) Write down the corresponding adjacency matrix, A say.

   (b) Use matrix algebra to compute the number of walks from node 1 back to itself using exactly 3 edges.

   (c) List the walks corresponding to the walks in (b). As a hint on a convenient notation, the walk going around the loop at 1, then going along the edge from 1 to 3, then going back along the edge from 3 to 1 could be written (1)(13)(31).

3. A message consisting of three words, with spaces removed, is first written in terms of numbers by the correspondence $A \leftrightarrow 1$, $B \leftrightarrow 2$ etc. The message is then made into a 4×4 matrix by placing it down columns in order, and then coded by multiplication on the left by the matrix product

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 3 & 1 & 0 \\ 2 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If the received message was

$$22, 74, 70, 73, 71, 281, 340, 314, 72, 269, 300, 293, 24, 113, 178, 106$$

what was the original message? You are required to compute the required matrix inverses using the algorithm from lectures, but you may use Matlab to carry out the required matrix multiplications.

4. A hiker begins at the origin, and walks in a straight line in the direction of the vector $(1, 1)$ for $2\sqrt{2}$ kilometers, then in the direction of the vector $(1, -1)$ for $\frac{3}{2}\sqrt{2}$ kilometers, and then in a straight line back to the origin.

   (a) Plot the path of the hiker in the xy-plane.

   (b) Calculate the total distance travelled by the hiker.

   (c) Calculate the area enclosed by the path of the hiker.