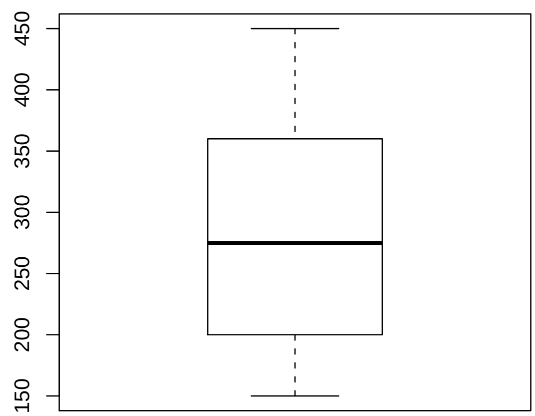
Solutions for 5.6 Exercises

1. (a)
```
> textbook <- c(200, 175, 450, 300, 350, 250, 150, 200, 320, 370,
+     404, 250)
```
```
> boxplot(textbook)
```



Assumptions

- Presumably the observations are a random sample (since we are told that), although we can't be sure since we don't know exactly how the data were collected. For instance, if all the students were in the same class then they may tend to buy the same textbook. Or if the survey was conducted in the bookshop then we may conjecture that these students tend to spend more than the average student, some of whom may not buy any textbooks.
- The data are reasonably symmetric and are consistent enough with a normal distribution. With only 12 observations, there needs to be substantial departure from normality to abandon the assumption.
- The data do not refer to the whole population.

One undesirable aspect of the data is that different values have apparently been rounded to different levels of accuracy – some to the nearest 50, some to the nearest 10, etc. It would have been better if the students had been given clear instructions on this, rather than being left to their own judgment.

(b)
```
> n <- length(textbook)
> mean <- mean(textbook)
```
Standard error of the mean:
```
> sem <- sd(textbook)/sqrt(n)
```
t distribution quantile:
```
> tquantile <- qt(c(0.025,0.975),df=n-1)
```
95% confidence interval:
```
> mean + sem*tquantile
```
```
[1] 223.8806 345.9527
```

(c) If we were to conduct the survey a very large number of times, approximately 95 out of 100 such intervals would contain the true population mean.

Less technically: we are 95% confident that the mean amount spent on textbooks by students at the university was between $224 and $346.

2. This is somewhat like the *Swain vs. Alabama* example in Chapter 4. We start with a model: $X \sim Bi(100, p)$, where X is the number of heads, and p is the unknown probability of spinning a head. We conduct a hypothesis test stated as $H_0 : p = 0.5$ versus $H_1 : p \neq 0.5$. Once we get the experimental result of $\hat{p} = 0.68$ we have to determine the P-value. Since 0.68 is much larger than 0.5, we expect a very small P-value.

The true P-value is

```
> 2 * (1 - pbinom(68, size = 100, prob = 0.5))
[1] 0.0001831432
```

3. (a) The hypothesis is $H_0 : \mu = 5.0$ vs. $H_1 : \mu < 5.0$. The test is conducted as follows:

```
> dis.oxygen <- c(5.1, 4.9, 5.6, 4.2, 4.8, 4.5, 5.3, 5.2)
> t.test(dis.oxygen, mu = 5, alternative = "less")

        One Sample t-test

data:  dis.oxygen
t = -0.314, df = 7, p-value = 0.3813
alternative hypothesis: true mean is less than 5
95 percent confidence interval:
     -Inf 5.251691
sample estimates:
mean of x
     4.95
```

The P-value is 0.38, so we do not reject the null hypothesis, which means there is insufficient evidence to claim the dissolved oxygen is less than 5.0. With a sample mean of 4.95, this is not surprising. Notice that the lower confidence limit is infinite—a consequence of a one-sided test.

(b) The hypothesis is $H_0 : \mu = 0.5$ vs. $H_1 : \mu > 5.0$. The test is conducted as follows:

```
> t.test(dis.oxygen, mu = 5, alternative = "greater")

        One Sample t-test

data:  dis.oxygen
t = -0.314, df = 7, p-value = 0.6187
alternative hypothesis: true mean is greater than 5
95 percent confidence interval:
 4.648309      Inf
sample estimates:
mean of x
     4.95
```

With a P-value of 0.62, we will also fail to reject the null hypothesis here. This means there is insufficient evidence to claim the dissolved oxygen is bigger than 5.0.

(c) Comparing the two tests, we do not have enough evidence to show the oxygen level is either greater than or less than 5.0. Note that each group would consider themselves justified in failing to reject their alternative hypotheses. This does not mean that the fish are definitely safe, or definitely in danger.

4. (a) H_0: the bonding quality for the new cement is equal to the bonding quality for the current cement mix. H_1: the new cement mix has better bonding quality.
   (b) H_0: unemployment rate is 10%. H_1: unemployment rate differs from 10%.
   (c) H_0: users of vitamin C suffer flu at same rate as the rest of the population. H_1: users of vitamin C suffer flu at a lower rate.
   (d) H_0: defect rate is at the nominal rate. H_1: defect rate is not at the nominal rate.
   (e) H_0: the new strain has the same resistance as the existing variety. H_1: the new strain is more resistant than existing variety.

5. We need to do a paired t-test. First, compute the difference

```
> affected <- c(488, 478, 480, 426, 440, 410, 458, 460)
> not.affected <- c(484, 478, 492, 444, 436, 398, 464, 476)
> diff <- affected - not.affected
```

The test is conducted as

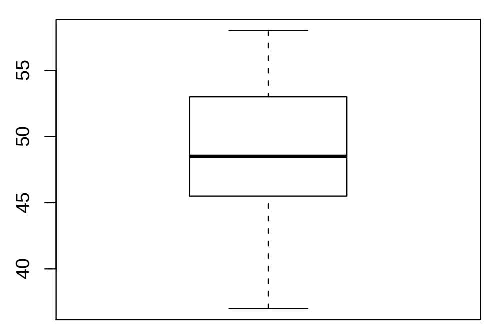
```
> t.test(diff)

        One Sample t-test

data:  diff
t = -1.053, df = 7, p-value = 0.3273
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -12.982016   4.982016
sample estimates:
mean of x
       -4
```

The P-value shows there is insufficient evidence to conclude that there is a difference.

6. (a)
```
> speed <- c(46, 43, 46, 53, 50, 57, 45, 57, 58, 53,
+             46, 46, 48, 49, 53, 49, 53, 37, 42, 38)
```

```
> quantile(speed)
   0%   25%   50%   75%  100%
37.00 45.75 48.50 53.00 58.00
```

```
> boxplot(speed)
```



(b)
```
> n <- length(speed)
> mean(speed) + sd(speed)/sqrt(n) * qt(c(0.025, 0.975), df = n -
+     1)
```
```
[1] 45.66883 51.23117
```

(c)
```
> t.test(speed, mu = 50)
```
```
        One Sample t-test

data:  speed
t = -1.1665, df = 19, p-value = 0.2579
alternative hypothesis: true mean is not equal to 50
95 percent confidence interval:
 45.66883 51.23117
sample estimates:
mean of x
    48.45
```

The test statistic is -1.17. The P-value shows that we should not reject the null hypothesis that the average speed is 50 km/h. The test is of some use, but from a safety point of view, the maximum speed may be of more interest than the average speed.

(d)
```
> direction <- c(rep("east", 10), rep("west", 10))
> t.test(speed ~ direction)
```
```
        Welch Two Sample t-test

data:  speed by direction
t = 1.8834, df = 18, p-value = 0.07591
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.5429614  9.9429614
sample estimates:
mean in group east mean in group west
              50.8               46.1
```
```
> t.test(speed ~ direction, var.equal = TRUE)
```
```
        Two Sample t-test

data:  speed by direction
t = 1.8834, df = 18, p-value = 0.07591
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.5429594  9.9429594
sample estimates:
mean in group east mean in group west
              50.8               46.1
```

For both tests, the difference between means is not significant.

Note that the confidence intervals are very similar regardless of whether equal variances is assumed. This suggests that the assumption is reasonable.

```
> speed.east <- speed[direction == "east"]
> var(speed.east)
[1] 31.06667
> speed.west <- speed[direction == "west"]
> var(speed.west)
[1] 31.21111
```

The variances are not very different—this confirms that an assumption of equal variances would be reasonable.

(e) i. 7/19 observations give speed greater than 50 km/h, and for 12/19 the speed is less than 50 km/h. In one case, we cannot decide whether the actual speed is less than or greater than 50 (assuming that speed is continuous, and measured to the nearest integer). $\hat{p} = 7/20 = 0.35$.

ii. For a confidence interval, the Agresti-Coull method will be adequate, as the estimated proportion is not close to 0 or 1. Jeffreys prior would also be fine. $n = 20$ is too small to use the Wald interval.

```
> n.tilde <- 20 + 4
> p.tilde <- (7 + 2)/n.tilde
> p.tilde + qnorm(c(0.025, 0.975)) * sqrt(p.tilde * (1 - p.tilde)/n.tilde)
[1] 0.1813141 0.5686859
```

7. (a) Let the means for the sleep and non-sleep group be μ_1 and μ_2, respectively. The null hypothesis can be stated as $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$.

(b) The pooled standard deviation is computed by

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

where $n_1 = 25, n_2 = 15$ and $s_1 = 1.32, s_2 = 1.84$.

(c) The t-statistic is computed by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $x_1 = 35.6, x_2 = 37.2$ The value is

```
> (35.6 - 37.2)/(1.532 * sqrt(1/25 + 1/15))
[1] -3.197767
```

The two-sided P-value is:

```
> 2 * pt(-3.1978, 25 + 15 - 2)
[1] 0.002790226
```

The P-value is very small, so H_0 is rejected. It can be concluded that sleep improves the recall of information on the multiple choice test, increasing the number of correctly answered questions by an average of 1.6.