



THE UNIVERSITY OF

MELBOURNE

Library Course Work Collections

Author/s:

Infrastructure Engineering

Title:

Engineering Risk Analysis, 2016 Semester 1, CVEN30008

Date:

2016

Persistent Link:

<http://hdl.handle.net/11343/127998>

THE UNIVERSITY OF MELBOURNE
DEPARTMENT OF INFRASTRUCTURE ENGINEERING

Examination (Semester 1 2016)

CVEN30008 ENGINEERING RISK ANALYSIS
Pass and Honours

Exam Duration: TWO (2) hours
Reading Time: 15 minutes

This paper has (13) pages

Authorised materials :

- Only electronic calculators approved by the School of Engineering can be used.

Instructions to Invigilators:

- Closed book examination.
- This examination paper is to be collected together with answer script books.

Instructions to students:

- All questions should be attempted.
- Marks allocated to each question are as indicated.
- Total marks for the examination equal 120.

Paper not to be removed from examination room

SECTION A

Answers to Section A should be written on standard exam script books

- A1. Describe the difference between a hazard and a risk. (4 marks)
- A2. Briefly describe why the business risk register is useful in risk management process. (12 marks)
- A3. What are the principles and process for managing risk described in AS/NZS/ISO 31000:2009 respectively? (20 marks)
- A4. List and briefly describe different risk treatment responses? (6 marks)
- A5. What is the purpose of risk evaluation? Briefly describe how to conduct risk evaluation? (10 marks)
- A6. What is “Project Review” in project risk management and why is it important? (8 marks)

END OF SECTION A
(Total marks for Section A = 60)

Continue to the next page for Section B

SECTION B**Answers to Section B should be written on standard exam script books**

B1 In a random pattern of eight bits used to test a micro-circuit, each bit is equally likely to be 0 or 1. Assume the values of the bits are independent.

(a) What is the probability that none of eight bits are 1?

(4 marks)

(b) What is the probability that at least five of the bits are 1?

(6 marks)

(Total marks for Question B1 =10)

B2 The failure loads (in MPa) of tensile adhesion tests on a sample of 5 alloy specimens are shown in Table 1.

(a) Use the data to find the 95 percent Confidence Interval for the population mean μ .

(6 marks)

(b) Assume that a new much larger sample of 50 was used. The new sample mean was 14.3 and new sample standard deviation was 0.7. Using this larger sample data, calculate the 95 percent Confidence Interval of the mean.

(4 marks)

| Sample values |
|---------------|
| 15.4 |
| 15.3 |
| 14.4 |
| 13.8 |
| 15.6 |

Table 1

(Total marks for Question B2 =10)

- B3 Eight independent dissolution rate measurements of a certain chemical were taken at a temperature of 0°C , the mean and standard deviation for this set of samples were found to be 2.5 and 0.5 respectively. Another seven independent measurements for the dissolution rate were taken at a temperature of 10°C with mean of 3 and standard deviation of 0.2. Can you conclude that the dissolution rate at 10°C is greater than that at 0°C at a significance level of 5%?

(15 marks)

(Total marks for Question B3 = 15)

B4 A new chemical process has been developed that may increase the yield over that of the current process. The current process is known to have a mean yield of 80 and a standard deviation of 5, where the units are the percentage of a theoretical maximum. If the mean yield of the new process is shown to be greater than 80, the new process will be put into production. It is proposed to run the new process 50 times and test the hypothesis that the mean yield of the new process is greater than 80 at a significance level of 5%. What is the power of this test? What is your suggestion based on the calculated power?

Assume that the mean yield of the new process is in fact 81 and its standard deviation is the same as that of the current process ($\sigma=5$).

(15 marks)

(Total marks for Question B4 =15)

- B5 A simply supported concrete beam is loaded with a load P which can be represented by the following relationship

$$P = 5P_1 + 3P_2 - P_3$$

where P_1 , P_2 and P_3 are statistically independent normal variables as shown in Table 2. Assuming the beam has been found to have a mean allowable load of 100 kN with a coefficient of variation (COV) of 0.1, determine the probability of failure of this beam.

(10 marks)

| | Mean (kN) | Standard deviation (kN) |
|-------|-----------|-------------------------|
| P_1 | 10 | 1 |
| P_2 | 15 | 2 |
| P_3 | 5 | 0.2 |

Table 2

(Total marks for Question B5 =10)

END OF SECTION B

(Total marks for Section B = 60)

- END OF EXAM -

(Total marks of exam = 120)

Appendix

Formulae

General

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{X})^2}$$

Probabilities and Distributions:

$$Z = \frac{\ln x - \lambda_x}{\xi_x}, \quad \lambda_x = \ln \mu_x - \frac{1}{2} \xi_x^2, \quad \text{if } COV = \frac{\sigma}{\mu} \leq 0.3, \quad \xi_x = COV$$

$$P(X = x, n|p) = \binom{n}{x} p^x (1-p)^{n-x}, \quad \binom{n}{x} = \frac{n!}{x! (n-x)!}$$

$$P(x \text{ occurrences in time } t) = \frac{(vt)^x}{x!} e^{-vt}, \quad x = 0, 1, 2, \dots$$

Confidence interval:

$$\mu = \bar{X} \pm z_{\alpha/2} \frac{S}{\sqrt{n}}$$

$$\mu = \bar{X} \pm t_{n-1, \alpha/2} \frac{S}{\sqrt{n}}$$

Hypothesis Testing:

$$Z = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}}}$$

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}} \quad t = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

$$t = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{S_p \sqrt{1/n_X + 1/n_Y}}$$

$$S_p = \sqrt{\frac{(n_X - 1)S_X^2 + (n_Y - 1)S_Y^2}{n_X + n_Y - 2}}$$

$$t = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{S_X^2 / n_X + S_Y^2 / n_Y}}$$

$$\nu = \frac{\left[s_x^2 / n_X + s_y^2 / n_Y \right]^2}{\left[\left(s_x^2 / n_X \right)^2 / (n_X - 1) \right] + \left[\left(s_y^2 / n_Y \right)^2 / (n_Y - 1) \right]}$$

$$\mu_0 + Z \frac{\sigma}{\sqrt{n}} = \mu_A + Z_A \frac{\sigma}{\sqrt{n}}$$

Simple linear regression

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left[\sum_{i=1}^n (x_i - \bar{x})^2 \right] \left[\sum_{i=1}^n (y_i - \bar{y})^2 \right]}}$$

$$\beta_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

Engineering Reliability

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

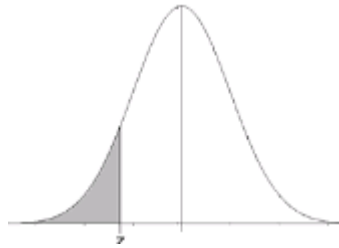
$$p_f = P(Z < 0) = 1 - \Phi(\beta)$$

$$Y = a_1 X_1 + a_2 X_2 + \dots + a_n X_n$$

$$\mu_Y = \sum_{i=1}^n a_i \mu_{X_i}$$

$$\sigma_Y^2 = \sum_{i=1}^n a_i^2 \sigma_{X_i}^2$$

$$p(t) = [1 - G(u_0 - a^*)] \lambda(t) \quad \lambda(t) = \frac{f(t)}{1 - \int_0^t f(t) dt}$$

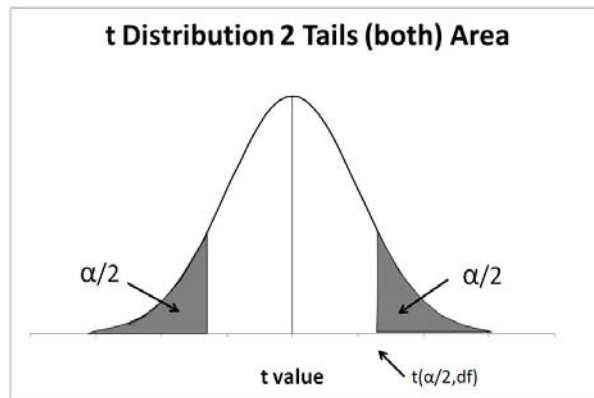
Cumulative probabilities for NEGATIVE z-values

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| -3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| -3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| -3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| -3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| -2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| -2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| -2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| -2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| -2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| -2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| -2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| -2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| -2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| -2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| -1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| -1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| -1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| -1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| -1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| -1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| -1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| -1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| -1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| -1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| -0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| -0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| -0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| -0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| -0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| -0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| -0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| -0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| -0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |

A normal distribution curve is shown. The area under the curve to the left of a point labeled z on the horizontal axis is shaded gray. The vertical axis passes through the peak of the curve.

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

t DISTRIBUTION



Area α shown in 2 Tails. $\alpha / 2$ in each tail

Two Tail t Values for α and Degrees of Freedom

| Degrees of Freedom | Combined Area α in Two Tails | | | | | |
|--------------------|-------------------------------------|--------|---------|---------|---------|----------|
| | 0.250 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
| 1 | 2.4142 | 6.3138 | 12.7062 | 25.4517 | 63.6567 | 127.3213 |
| 2 | 1.6036 | 2.9200 | 4.3027 | 6.2053 | 9.9248 | 14.0890 |
| 3 | 1.4226 | 2.3534 | 3.1824 | 4.1765 | 5.8409 | 7.4533 |
| 4 | 1.3444 | 2.1318 | 2.7764 | 3.4954 | 4.6041 | 5.5976 |
| 5 | 1.3009 | 2.0150 | 2.5706 | 3.1634 | 4.0321 | 4.7733 |
| 6 | 1.2733 | 1.9432 | 2.4469 | 2.9687 | 3.7074 | 4.3168 |
| 7 | 1.2543 | 1.8946 | 2.3646 | 2.8412 | 3.4995 | 4.0293 |
| 8 | 1.2403 | 1.8595 | 2.3060 | 2.7515 | 3.3554 | 3.8325 |
| 9 | 1.2297 | 1.8331 | 2.2622 | 2.6850 | 3.2498 | 3.6897 |
| 10 | 1.2213 | 1.8125 | 2.2281 | 2.6338 | 3.1693 | 3.5814 |
| 11 | 1.2145 | 1.7959 | 2.2010 | 2.5931 | 3.1058 | 3.4966 |
| 12 | 1.2089 | 1.7823 | 2.1788 | 2.5600 | 3.0545 | 3.4284 |
| 13 | 1.2041 | 1.7709 | 2.1604 | 2.5326 | 3.0123 | 3.3725 |
| 14 | 1.2001 | 1.7613 | 2.1448 | 2.5096 | 2.9768 | 3.3257 |
| 15 | 1.1967 | 1.7531 | 2.1314 | 2.4899 | 2.9467 | 3.2860 |
| 16 | 1.1937 | 1.7459 | 2.1199 | 2.4729 | 2.9208 | 3.2520 |
| 17 | 1.1910 | 1.7396 | 2.1098 | 2.4581 | 2.8982 | 3.2224 |
| 18 | 1.1887 | 1.7341 | 2.1009 | 2.4450 | 2.8784 | 3.1966 |
| 19 | 1.1866 | 1.7291 | 2.0930 | 2.4334 | 2.8609 | 3.1737 |
| 20 | 1.1848 | 1.7247 | 2.0860 | 2.4231 | 2.8453 | 3.1534 |
| 21 | 1.1831 | 1.7207 | 2.0796 | 2.4138 | 2.8314 | 3.1352 |
| 22 | 1.1815 | 1.7171 | 2.0739 | 2.4055 | 2.8188 | 3.1188 |
| 23 | 1.1802 | 1.7139 | 2.0687 | 2.3979 | 2.8073 | 3.1040 |
| 24 | 1.1789 | 1.7109 | 2.0639 | 2.3909 | 2.7969 | 3.0905 |
| 25 | 1.1777 | 1.7081 | 2.0595 | 2.3846 | 2.7874 | 3.0782 |
| 26 | 1.1766 | 1.7056 | 2.0555 | 2.3788 | 2.7787 | 3.0669 |
| 27 | 1.1756 | 1.7033 | 2.0518 | 2.3734 | 2.7707 | 3.0565 |
| 28 | 1.1747 | 1.7011 | 2.0484 | 2.3685 | 2.7633 | 3.0469 |
| 29 | 1.1739 | 1.6991 | 2.0452 | 2.3638 | 2.7564 | 3.0380 |
| 30 | 1.1731 | 1.6973 | 2.0423 | 2.3596 | 2.7500 | 3.0298 |
| 40 | 1.1673 | 1.6839 | 2.0211 | 2.3289 | 2.7045 | 2.9712 |
| 50 | 1.1639 | 1.6759 | 2.0086 | 2.3109 | 2.6778 | 2.9370 |
| 60 | 1.1616 | 1.6706 | 2.0003 | 2.2990 | 2.6603 | 2.9146 |
| 70 | 1.1600 | 1.6669 | 1.9944 | 2.2906 | 2.6479 | 2.8987 |
| 80 | 1.1588 | 1.6641 | 1.9901 | 2.2844 | 2.6387 | 2.8870 |
| 90 | 1.1578 | 1.6620 | 1.9867 | 2.2795 | 2.6316 | 2.8779 |
| 100 | 1.1571 | 1.6602 | 1.9840 | 2.2757 | 2.6259 | 2.8707 |
| 200 | 1.1537 | 1.6525 | 1.9719 | 2.2584 | 2.6006 | 2.8385 |
| 500 | 1.1517 | 1.6479 | 1.9647 | 2.2482 | 2.5857 | 2.8195 |
| 10000 | 1.1504 | 1.6450 | 1.9602 | 2.2417 | 2.5763 | 2.8077 |