School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 11

9–13 October 2017

The plan

Reminder: A great introduction to context-free languages is available under "Readings Online".

The exercises

75. Give context-free grammars for the following languages. Assume the alphabet is $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

    (a) $\{w \mid w$ starts and ends with the same symbol$\}$

    (b) $\{w \mid$ the length of w is odd$\}$

    (c) $\{w \mid$ the length of w is odd and its middle symbol is $\mathtt{0}\}$

    (d) $\{w \mid w$ is a palindrome$\}$

76. Construct a context-free grammar for $M = \{\mathtt{a}^i\mathtt{b}^j \mid i > 0, j > 0, i \neq j\}$.

77. Show that the class of context-free languages is closed under the regular operations: union, concatenation, and Kleene star. Hint: Show how context-free grammars for A and B can be manipulated to produce context-free grammars for $A \cup B$, AB, and A^*. Be careful: The variables used in A and in B could overlap.

78. If we consider English words the "symbols" (or primitives) of English, we can use a context-free grammar to try to capture certain classes of sentences and phrases. For example, we can consider articles (A), nouns (N), adjectives (Q), intransitive verbs (IV), transitive verbs (TV), noun phrases (NP), verb phrases (VP), and sentences (S). List 5–10 sentences generated by this grammar:

S	$\to$	$NP\ VP$	N	$\to$	cat	IV	$\to$	hides
A	$\to$	a	N	$\to$	dog	IV	$\to$	runs
A	$\to$	the	Q	$\to$	lazy	IV	$\to$	sleeps
NP	$\to$	$A\ N$	Q	$\to$	quick	TV	$\to$	chases
NP	$\to$	$A\ Q\ N$	VP	$\to$	IV	TV	$\to$	hides
N	$\to$	bone	VP	$\to$	$TV\ NP$	TV	$\to$	likes

    Are they all meaningful? Discuss "well-formed" versus "meaningful".

79. How would you change the grammar from the previous question so that "adverbial modifiers" such as "angrily" or "happily" can be used? For example, we would like to be able to generate sentences like "the dog barks constantly" and "the black cat sleeps quietly".

80. Give a context-free grammar for $\{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i = j \vee j = k$ where $i, j, k \geq 0\}$. Is your grammar ambiguous? Why or why not?

81. Consider the context-free grammar $G = (\{S, A, B\}, \{\mathtt{a}, \mathtt{b}\}, R, S)$ with rules R:

$$\begin{array}{lcl} S & \to & A\ B\ A \\ A & \to & \mathtt{a}\ A \mid \epsilon \\ B & \to & \mathtt{b}\ B \mid \epsilon \end{array}$$

(a) Show that G is ambiguous.

(b) The language generated by G is regular; give a regular expression for $L(G)$.

(c) Give an unambiguous context-free grammar, equivalent to G. Hint: As an intermediate step, you may want to build a DFA for $L(G)$.

82. Construct a push-down automaton which recognises the language $M = \{\mathtt{a}^i\mathtt{b}\mathtt{a}^j \mid i > j \geq 0\}$.

83. The Grok Worksheet for Week 11 asks you to complete a Post machine emulator written in Haskell, and then to apply it. This is of interest because of Assignment 2, which asks you to construct certain state machines and submit them, in the form of Haskell representations. If you get the simulator to work, this will allow you to test your Post machines thoroughly before submission.