Tutorial 2

Let A and B be matrices of size $m \times n$ and $p \times q$, respectively. Then

- $A + B$ is defined if and only if A and B are the same size, namely $m = p$ and $n = q$
- AB is defined if and only if $n = p$, and AB is of size $m \times q$.

Q1. Let

$$A = \begin{bmatrix} 1 & 0 & -3 \\ -2 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0 & 1 \\ 2 & 3 & -6 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & 5 \end{bmatrix}, \quad D = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}.$$

Calculate, where possible:

(i) $A + B$	(ii) $C + D$	(iii) $4 + D$	(iv) $3D$
(v) AD	(vi) DA	(vii) BC	(viii) CB.

Using your knowledge of MATLAB from last week, to which of these (if any) would MATLAB give a different answer?

To find the *inverse*, A^{-1}, of a square matrix A:

- if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - bc \neq 0$, then $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- if A is larger than 2×2, then reduce the augmented matrix $0[A|I]$ to reduced row echelon form, $[R|B]$, and if $R = I$ then $B = A^{-1}$.
- if $ad - bc = 0$ or the row reduction does not give $R = I$, then A^{-1} does not exist, and A is *singular*.

Q2. Find the inverses of the following matrices, where they exist.

(i) $\begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} -3 & 5 \\ 6 & -10 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

Q3. Let B be any matrix such that $B + B^T$ is defined. Show that B is square.

Q4. If A is any square matrix satisfying $A^3 = I$, then verify that

$$(A + I)^{-1} = \frac{1}{2}(A^2 - A + I).$$

A linear system of equations can be written in the form $A\mathbf{x} = \mathbf{b}$, where A is the matrix of coefficients, $\mathbf{x}$ is the column matrix of variables, and $\mathbf{b}$ is the column matrix of the right hand side. If A is invertible, then the unique solution is given by $\mathbf{x} = A^{-1}\mathbf{b}$.

Q5. A system of linear equations is defined by

$$\begin{array}{rcrcl} 2x & - & 3y & = & 17 \\ 9x & - & 4y & = & 10 \end{array}$$

(i). Write the system as a matrix equation.

(ii). *Hence* solve the system.

Q6. The three Victorian courts [Magistrates (M), County (C) and Supreme (S)] each deal with different crimes to differing extents. The division of labor for various cases (including proportion of cases, appeals and time spent) is estimated to be:

$$\begin{array}{llllcl} \text{Theft:} & 0.80\ M & +\ 0.1\ C & +\ 0.10\ S & = & \text{number of theft arrests} \\ \text{Murder:} & 0.01\ M & & +\ 0.99\ S & = & \text{number of murder arrests} \\ \text{Assault:} & 0.01\ M & +\ 0.9\ C & +\ 0.09\ S & = & \text{number of assault arrests} \end{array}$$

(i). Use the fact that

$$\begin{bmatrix} 0.80 & 0.1 & 0.10 \\ 0.01 & 0 & 0.99 \\ 0.01 & 0.9 & 0.09 \end{bmatrix}^{-1} \approx \begin{bmatrix} 1.25 & -0.1 & -0.14 \\ -0.013 & -0.1 & 1.11 \\ -0.01 & 1.0 & 0 \end{bmatrix}$$

to solve this system if there are 400 theft arrests, 10 murder arrests, and 120 assault arrests in a given time period. Convert your answer to percentage workloads.

(ii). If the murder arrests double, which court will have the largest change in workload?