Assignment 1

Challenge 1

Let ‘S’ be the statement of A

S: A=> (¬B ∧ ¬C)

A	B	C	S
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

There is only one possibility that can be deduced from A’s statement, which is ‘A is a knight and B and C are both knaves’. No enough information is provided to deduce what B and C are when ‘A is a knave’.

Challenge 2

Question 1

¬φ ≡ ¬ (((P => S) ∧ (Q => R) ∧ (R => P)) => S)

≡ ¬ (¬ ((¬P ∨ S) ∧ (¬Q ∨ R) ∧ (¬R ∨ P)) ∨ S)

≡ ¬¬ ((¬P ∨ S) ∧ (¬Q ∨ R) ∧ (¬R ∨ P)) ∧ ¬S)

≡ (¬P ∨ S) ∧ (¬Q ∨ R) ∧ (¬R ∨ P) ∧ ¬S

Question 2

Let P, S, Q and R be 0, the negation formula is now (1 ∨ 0) ∧ (1 ∨ 0) ∧ (1 ∨ 0) ∧ 1, which equals true. Therefore, the negation formula is satisfiable and the original formula is non-valid.

Question 3

Negation ≡ ¬ ((((P ∨ Q) => S) ∧ (¬P => (R => Q) ∧ (R ∨ S)) => S)

≡ ¬ (¬ ((¬ (P ∨ Q) ∨ S) ∧ (¬¬P ∨ (¬R ∨ Q)) ∧ (R ∨ S)) ∨ S)

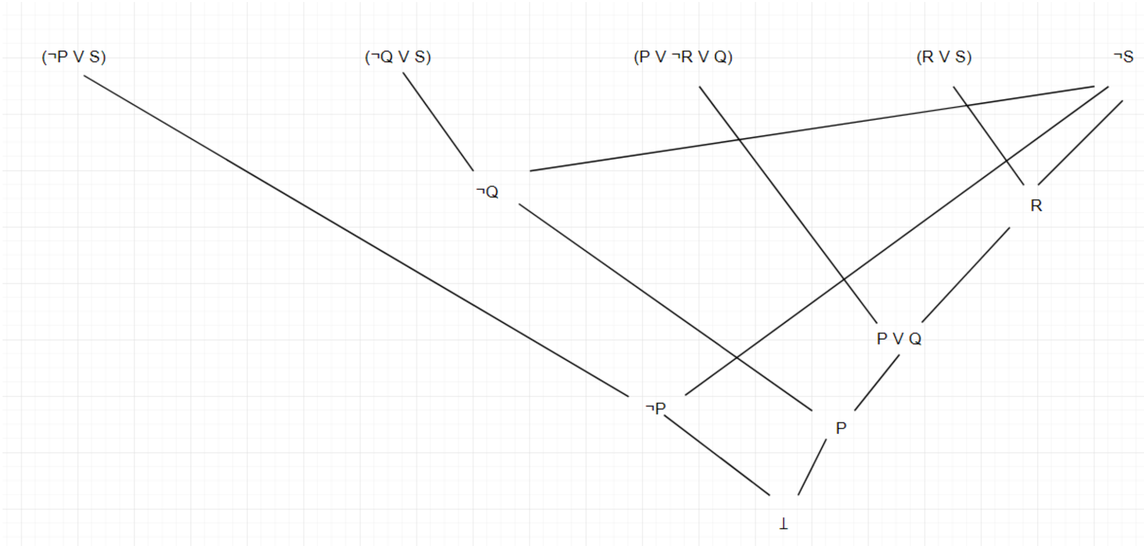
≡¬¬ ((¬ (P ∨ Q) ∨ S) ∧ (¬¬P ∨ (¬R ∨ Q)) ∧ (R ∨ S)) ∧ ¬S

≡ (((¬P ∧ ¬Q) ∨ S) ∧ (P ∨ (¬R ∨ Q)) ∧ (R ∨ S)) ∧ ¬S

≡ ((¬P ∧ ¬Q) ∨ S) ∧ (P ∨ ¬R ∨ Q) ∧ (R ∨ S) ∧ ¬S

≡ (¬P V S) Λ (¬Q V S) Λ (P V ¬R V Q) Λ (R V S) Λ ¬S

Question 4



⊥ can be deriving as the graph shown above, therefore the negation formula is unsatisfiable and the original formula is valid.

Challenge 3

[∀x ∀y (P(x, y) => P(h(x), h(h(y))))] => ∀x (P(x, h(x)) Λ P(h(h(x)), x))

Satisfiable

Let h(x) = x * x, P(x, y) means x = y, D = {0}

Therefore, we can get

[P(0, 0) => P(0, 0)] => (P(0, 0) Λ P(0, 0))

Which can be transform to true and false form

(true => true) => (true Λ true)

true => true

This is always true if the formula follows this interpretation, hence the formula is satisfiable because there is an interpretation showing true.

Non-valid

Let h(x) = x * x, P(x, y) means x < y, D = {2}

We can get from the formula

[P(2, 2) => P(4, 16)] => (P(2, 4) Λ P(16, 2))

Then the true and false form is

$(false \Rightarrow true) \Rightarrow (true \land false)$

$true \Rightarrow false$

This is always false if the formula follows this interpretation, hence the formula is not valid because there is an interpretation showing false.

Challenge 4

Question 1

$\forall x\ \forall y\ ((S(x) \land \neg P(y, x)) \Rightarrow H(x))$

Question 2

$\forall x\ (S(x) \Rightarrow ((\forall y\ (P(y, x) \Rightarrow R(y))) \Rightarrow H(x)))$

Question 3

$\forall x\ (S(x) \Rightarrow ((\forall y\ (P(y, x) \Rightarrow R(y))) \Rightarrow H(x)))$

Eliminate =>

$\forall x\ (\neg S(x) \lor (\neg\ (\forall y\ (\neg P(y, x) \lor R(y))) \lor H(x)))$

Eliminate negation

$\forall x\ (\neg S(x) \lor (\exists y\ (P(y, x) \land \neg R(y))) \lor H(x)))$

Eliminate existential quantifiers

$\forall x\ (\neg S(x) \lor ((P(f(x), x) \land \neg R(f(x)))) \lor H(x)))$

Drop Universal Quantifiers

$(\neg S(x) \lor ((P(f(x), x) \land \neg R(f(x)))) \lor H(x)))$

Turn to CNF

$(\neg S(x) \lor ((P(f(x), x) \lor H(x)) \land (\neg R(f(x)) \lor H(x)))$

$(\neg S(x) \lor P(f(x), x) \lor H(x)) \land (\neg S(x) \lor \neg R(f(x)) \lor H(x))$

Clausal form

$\{\{\neg S(x), P(f(x), x), H(x))\}, \{\neg S(x), \neg R(f(x)), H(x)\}\}$

Question 4

$\neg(\forall x\ \forall y\ ((S(x) \land \neg P(y, x)) \Rightarrow H(x)))$

Eliminate =>

$\neg(\forall x\ \forall y\ (\neg\ (S(x) \land \neg P(y, x)) \lor H(x)))$

Eliminate negation

$\exists x\ \exists y\ \neg\ (\neg\ (S(x) \wedge \neg P(y, x)) \vee H(x)))$

$\exists x\ \exists y\ (\neg\neg\ (S(x) \wedge \neg P(y, x)) \wedge \neg H(x)))$

$\exists x\ \exists y\ ((S(x) \wedge \neg P(y, x)) \wedge \neg H(x)))$

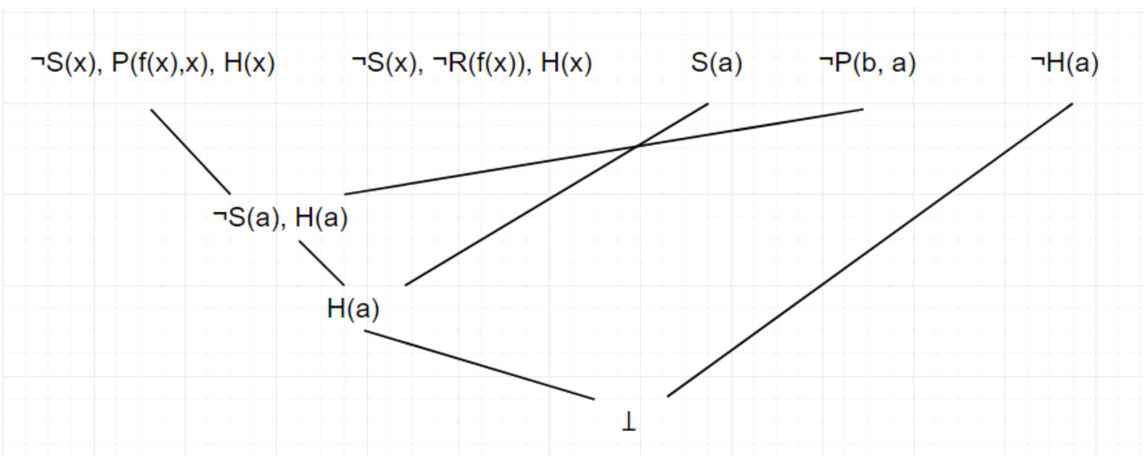
Eliminate existential quantifiers

$(S(a) \wedge \neg P(b, a) \wedge \neg H(a)))$

Clausal form

$\{\{S(a)\}, \{\neg P(b, a)\}, \{\neg H(a)\}\}$

Question 5



$\perp$ is deriving from S2 $\wedge$ ¬S1. Therefore, we can say that S1 follows from S2.