Quantitative Risk Analysis
Estimation of Sample Size and Power

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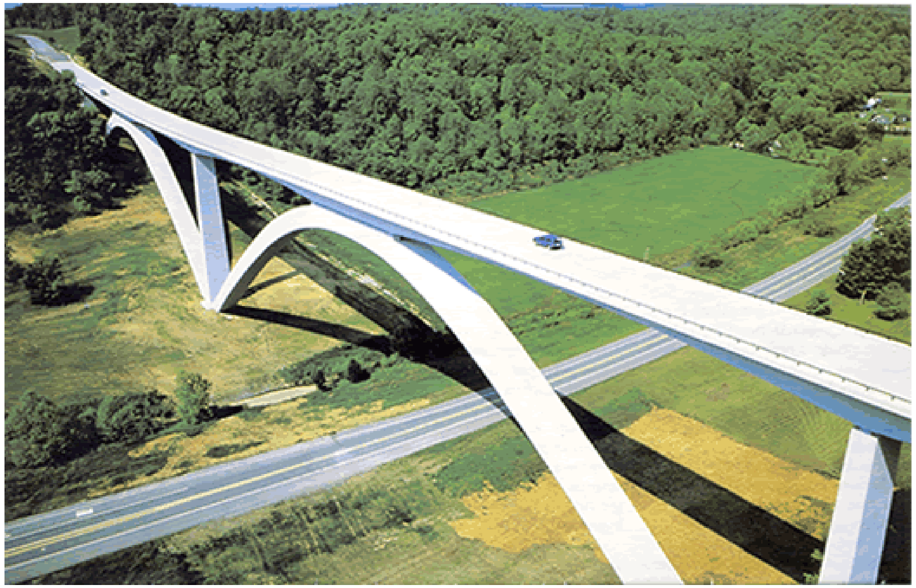


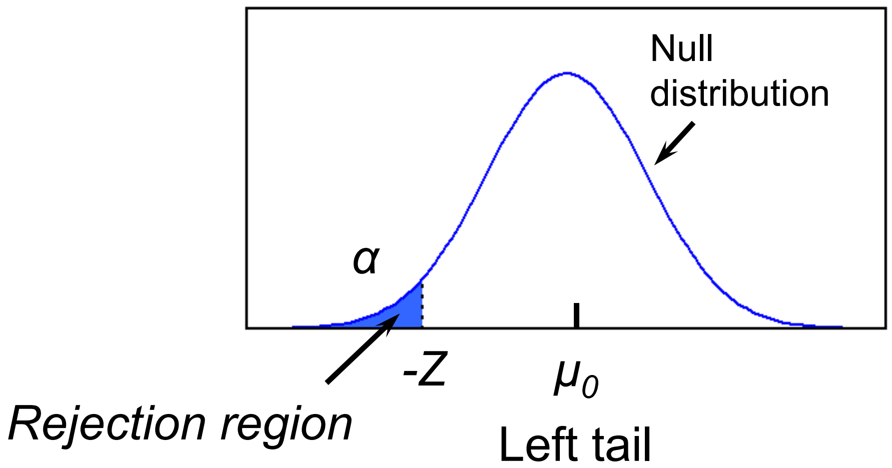
• Limitations of Hypothesis Testing

Hypothesis testing involving a significance level α, has two types of errors:

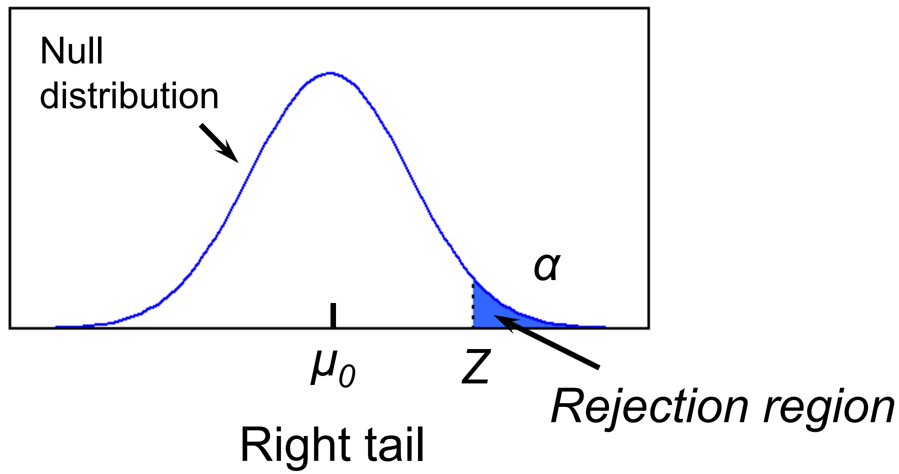
- **Type I error**: H_0 is rejected when it is True.
- **Type II error**: H_0 is not rejected when it is False.

In order to **minimise** the probability of **Type I error**:

- Select a small significance level, α (e.g. $\alpha \leq 0.05$)



$$H_0 : \mu \geq \mu_0$$



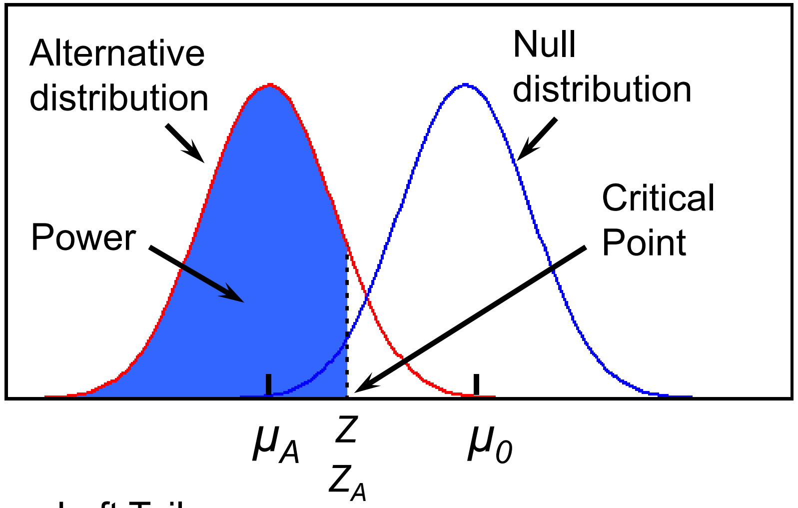
$$H_0 : \mu \leq \mu_0$$

Estimation of Power

The **Power** is the probability of ***avoiding*** **Type II error**:

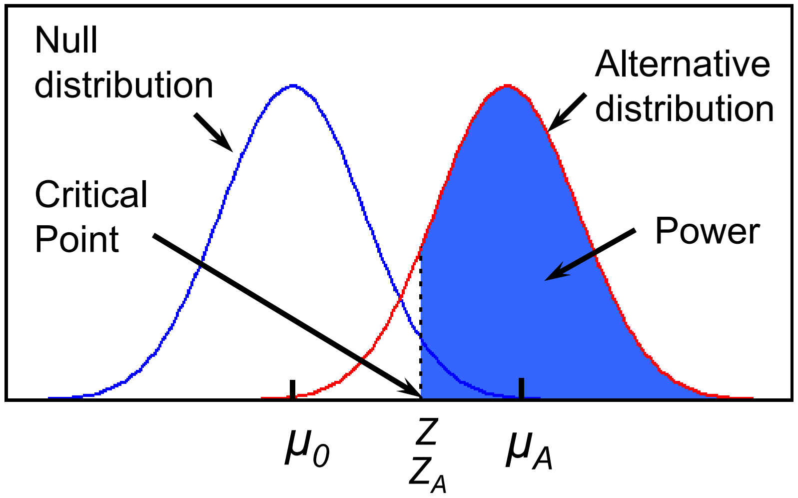
$$\text{Power} = 1 - \text{P(Type II error)}$$

Power ≥ 0.8 is generally considered to be acceptable



Left Tail

$$H_0 : \mu \geq \mu_0$$



Right Tail

$$H_0 : \mu \leq \mu_0$$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Estimation of Power

The **Power** is the probability of ***avoiding*** **Type II error**:

$$\text{Power} = 1 - \text{P}(\text{Type II error})$$

To calculate the power:

- Step 1: Determine H_0 and H_1
- Step 2: Select α and obtain Z
- Step 3: Approximate σ (through a preliminary sample or a sample of a similar population), and sample size, n

Note: While conducting the test, the sample is not yet drawn, and therefore σ needs to be assumed.

Estimation of Power

- Step 4: Identify the critical point:

$$\text{Critical Point} = \mu_0 + Z\frac{\sigma}{\sqrt{n}}$$

- Step 5: Assume an alternative mean, μ_A (usually close to μ_0) for the alternative distribution

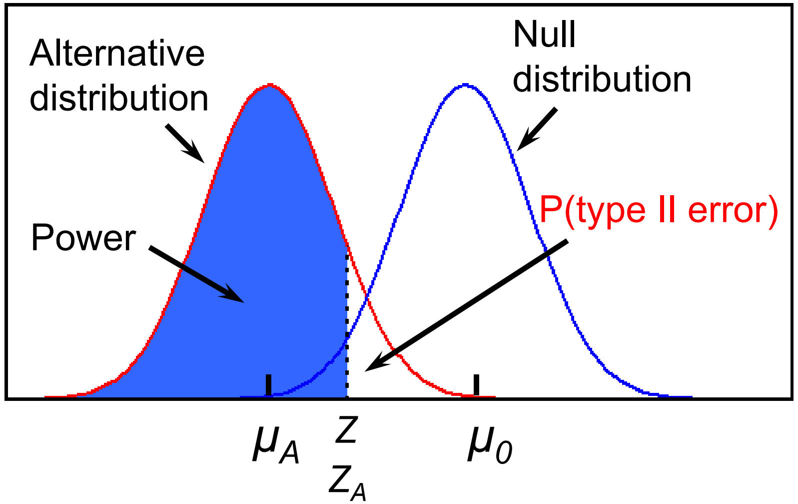
Estimation of Power

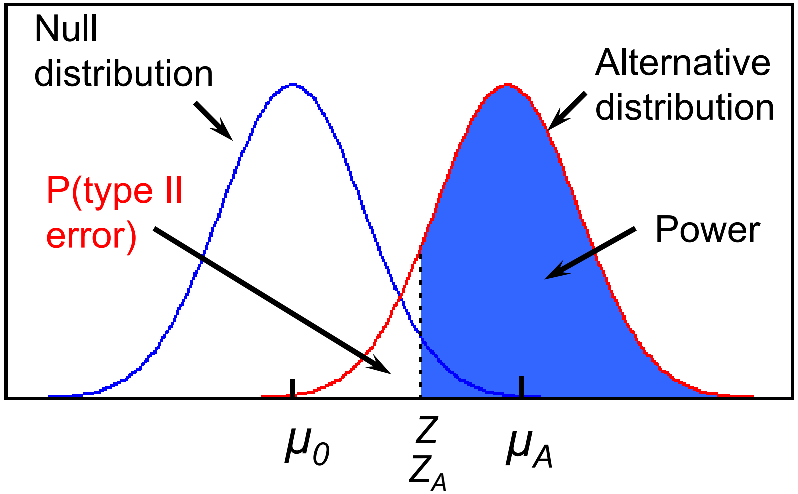
- Step 6: Using the critical point identified in Step 4, define Z_A for the alternative distribution

$$Z_A = \frac{(\text{Critical Point} - \mu_A)\sqrt{n}}{\sigma}$$

Estimation of Power

- Step 7: Calculate P(Type II error) using Z_A :





Left Tail $H_0 : \mu \geq \mu_0$

Right Tail $H_0 : \mu \leq \mu_0$

Power=Area to the left of Z_A

Power=Area to the right of Z_A

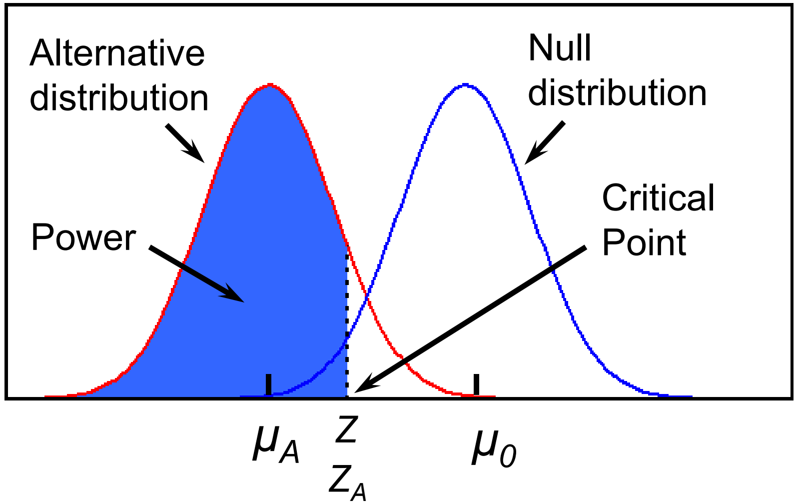
Note: Power ≥ 0.8 is generally considered to be acceptable

Example 1: Calculation of Power (Risk of Concrete Failure)

A decision needs to be made concerning the production of high strength concrete. Find the power: with a significance level, α of 5%, and the hypothesis testing consists of H_0: $\mu \leq 80$ MPa and H_1: $\mu > 80$ MPa; the alternative mean, μ_A, is 82 MPa, and assuming that the sample size, $n = 50$ and the standard deviation, $\sigma = 5$ MPa. The production will commence if Power ≥ 0.8 in order to reduce Type II error.

Solution

Estimation of Sample Size



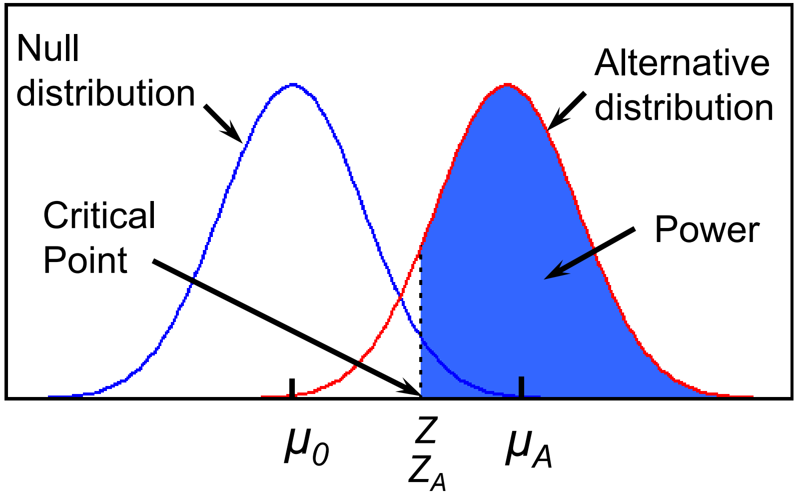
Left Tail $H_0 : \mu \geq \mu_0$

$$\text{Critical Point} = \mu_0 + Z\frac{\sigma}{\sqrt{n}}$$

$$\text{Critical Point} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Estimation of Sample Size



Right Tail $H_0 : \mu \leq \mu_0$

$$\text{Critical Point} = \mu_0 + Z\frac{\sigma}{\sqrt{n}} \qquad \text{Critical Point} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Estimation of sample size and power

In order to determine the required sample size, *n*:

- Step 1: Determine H_0 and H_1
- Step 2: Select α and obtain Z
- Step 3: Approximate σ
- Step 4: Obtain the expression of the critical point
- Step 5: Define μ_A
- Step 6: Select an acceptable Power, $\mathrm{P(Type\ II\ error)}$= 1-Power
- Step 7: Determine Z_A by using $\mathrm{P(Type\ II\ error)}$
- Step 8: Use the following equation and solve it for sample size, *n*

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Example 2: Calculation of Sample Size (Risk of Concrete Failure)

Find the sample size of the production of high strength concrete, with a significance level, α, of 5%, and the hypothesis testing consists of H_0: $\mu \leq 80$ MPa and H_1: $\mu > 80$ MPa; the alternative mean, μ_A, is 81 MPa, and assuming that the standard deviation, σ, is 7 MPa and the Power is 0.9.

Solution