Challenge 5

a. $L_r = \{x \in \{e, n, s, w\}^* \mid$ x contains number of e = number of w and number of n = number of s$\}$.

b. Assume the language is context-free, we can get:

$$W = w^p e^p s^p n^p \text{ in } L_r, \text{ where p is the pumping length.}$$

By the Pumping Lemma, the following conditions must be satisfied:

$$uvxyz = w^p e^p s^p n^p, \text{ where } uv^i xy^i z \text{ in } L_r \text{ for all i}$$

$$|vy| > 0.$$

$$|vxy| <= p.$$

The combination vxy contains one or two types of symbol in W, otherwise |vxy| is greater than p.

In the first case, if vxy only contains one symbol in W, then v and y should have the same symbol. The number of one symbol in W can differ from another one. Let's assume the one of the symbol is w, the number of ws can be different from the number of es, which proves $uv^i xy^i z$ is not in L_r.

Secondly, if there are two types of symbol in vxy, since there is no way to satisfy both opposite directions and to be adjacent with each other in W, the number of steps of opposite directions is always different (e.g. number of ws != number of es). which proves $uv^i xy^i z$ is not in L_r again. Therefore, this are two contradictions of Pumping Lemma which proves L_r is not context-free.

c. G -> A n A G A s A | A s A G A n A | ε

Where A -> w A | e A | ε

G' -> B w B G' B e B | B e B G' B w B | ε

Where B -> n B | s B | ε