PHYC90045 Introduction to Quantum Computing

Week 9

Lecture 17
Optimization problems, Encoding problems as energies, Quadratic Binary Optimization (QUBO), Problem embedding

Lecture 18
Adiabatic Quantum Computing

Lab 9
Using DWave machines

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Adiabatic Quantum Computation

Physics 90045
Lecture 21

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Overview

In this lecture we will cover/review

- Quantum Adiabatic Processes
- The problem Hamiltonian
- Avoided crossings and energy gap
- The adiabatic theorem

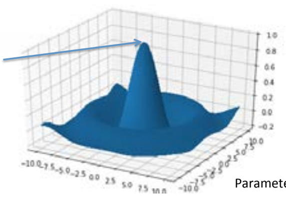
D-Wave systems

Image: D-Wave Systems

- 2000 qubit quantum "annealers"
- Sold quantum computers to Lockheed Martin, Google, NASA, Los Alamos National Laboratory ($15 million each)
- We will be using Dwave machines (remotely) during the labs

Review: Optimization Problems

Given some cost-function or "objective function" we would like to maximize/minimize. Often the inputs/parameters are constrained.



We have seen in previous lecture how to map these onto QUBO problem Hamiltonians.

Adiabatic Processes

- Start In the known ground state of a simple Hamiltonian
- Slowly change the Hamiltonian to the problem Hamiltonian
- Provided the change has been "slowly enough" the system will remain in the ground state, and we will have found the ground state of the problem Hamiltonian.

$$H(s) = (1-s)H_x + sH_p \qquad 0 \le s \le 1$$

$$s = t/T$$

Bad analogy: Moving a glass full of water slowly enough means that you keep the water in the glass. If you move it too fast all the water slops out.

Note: Quantum adiabatic process is not the same as thermodynamic adiabatic process (Q=0)

Hamiltonian of transverse field

$$H_x \propto B_x \sum_i X_i$$

Transverse field Hamiltonian

Eg. Three electrons in a transverse field:

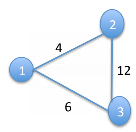
$$H_x = g\mu_B B_x (X_1 + X_2 + X_3)$$

We call this the transverse Hamiltonian, $\mathbf{H_x}$.

Problem Hamiltonian

We have seen many examples of problem Hamiltonians in our discussions of QUBO Problems. Two-body because that's what nature gives us.

Eg. Number partitioning for the set **{1, 2, 3}**:



$$H = 4Z_1Z_2 + 6Z_1Z_3 + 12Z_2Z_3 + 14I$$

Finding minimum energy state will solve the problem!

We call this the problem Hamiltonian, $\mathbf{H_p}$.

Quantum "Annealing"

Definition according to Google:

Annealing: heat (metal or glass) and allow it to cool slowly, in order to remove internal stresses and toughen it. "Copper tubes must be annealed after bending or they will be brittle"

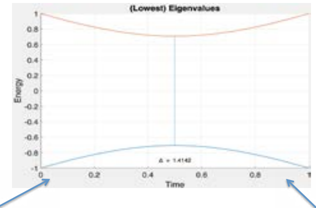
$$H(s) = (1-s)H_x + sH_p$$

Transverse field plays the role of temperature. Causes excitations, strength is slowly being lowered.

Problem Hamiltonian defines the energy landscape of the problem we want to solve.

Example: Just one Qubit

$$H(s) = (1-s)H_x + sH_p$$
$$s = t/T$$



$$H_x = X$$
$$H_p = Z$$

"Landau-Zener" avoided crossing

System is originally in the state: $|-\rangle$

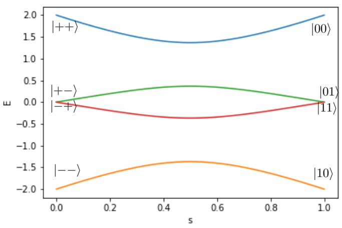
Time

System ends up in the state: $|1\rangle$

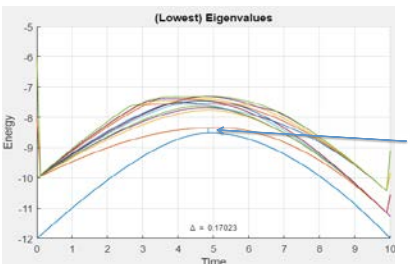
Two qubit example

$$H_p = Z_1 Z_2 + Z_1$$
$$H_x = X_1 + X_2$$

$$H(s) = (1-s)H_x + sH_p \qquad s = t/T$$



More complicated problem Hamiltonians



Avoided crossing Energy gap

Adiabatic Theorem

How slowly is slowly enough? Adiabatic critereon.

Time derivative of Hamiltonian

m, n[th] element

$$\sum_{m\neq n} \frac{\hbar |\langle m|\,\dot{H}\,|n\rangle|}{|E_n - E_m|^2} = \sum_{m\neq n} \left|\frac{\hbar\langle m\,|\dot{n}\rangle}{E_n - E_m}\right| \ll 1$$

Energy of eigenstates:
n is the ground state,
m is every other state

Roughly speaking…

$$\sum_{m\neq n} \frac{\hbar |\langle m|\,\dot{H}\,|n\rangle|}{|E_n - E_m|^2} = \sum_{m\neq n} \left|\frac{\hbar\langle m\,|\dot{n}\rangle}{E_n - E_m}\right| \ll 1$$

For ground state, largest contribution from the smallest two eigenvalues. E_n-E_m

$$|E_n - E_m| > \Delta$$

Energy gap between ground state and first excited Eigenstate

For our linear schedule:

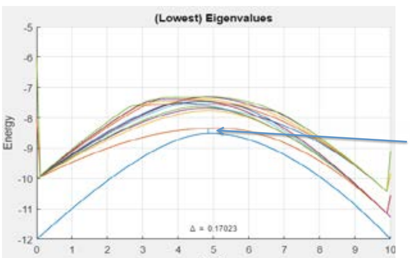
$$\dot{H}_{mn} \approx \frac{H_{mn}}{T}$$

Time required scales inversely proportional to the gap:

$$T \propto \frac{1}{\Delta}$$

For a given problem: How big is the gap? Difficult to work out in general!

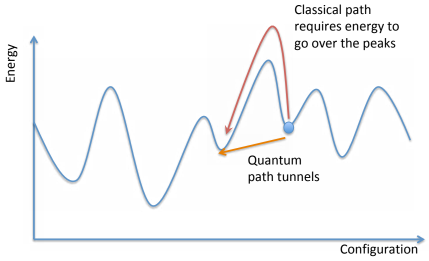
Energy gap



Energy gap

Quantum Tunneling

When might AQC give an advantage?



Quantum annealing has been shown to outperform classical annealing in the case where the barriers are high, but thin.

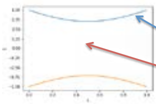
Equivalent of Grover's algorithm

Roland and Cerf demonstrated that AQC could be used to implement an unordered search:

$$H_x = I - 2\left|\phi\right\rangle\left\langle\phi\right| \qquad \left|\phi\right\rangle = \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}\left|i\right\rangle$$

$$H_p = I - 2\left|m\right\rangle\left\langle m\right|$$

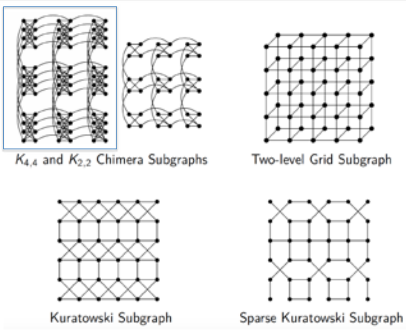
Marked state, m



Optimization, since the energy spectrum is known:
Change Hamiltonian faster when the gap is larger, faster when it is smaller.

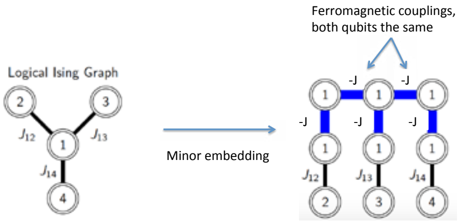
This achieves the same $O(\sqrt{N})$ speedup as Grover's algorithm.

Quantum Computer Layouts



From:
S. Tonetto
MSc thesis

Embedding computational problems



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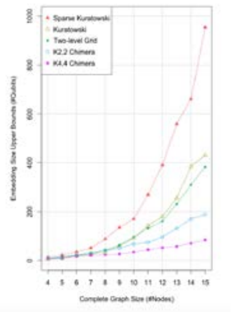
Embedding complete graph



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D-Wave systems

Image:
D-Wave Systems

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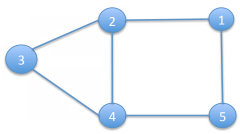
- Opportunity to use them in this week's lab!

Adiabatic from Start to Finish

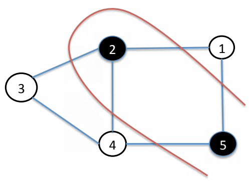
(1) Map computational problem to QUBO/Hamiltonian
(2) Embed problem on physical architecture
(3) Execute the adiabatic algorithm
(4) Read the ground state configuration for the answer to the problem

MAX-Cut Problem



Partition the nodes into two disjoint subsets (not necessarily with equal numbers of nodes in each!) so that there is the maximum number of edges between the two subsets.

MAX-Cut Problem



Partition the nodes into two disjoint subsets (not necessarily with equal numbers of nodes in each!) so that there is the maximum number of edges between the two subsets.

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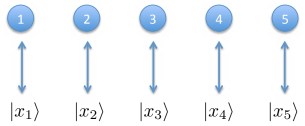
Graph Partitioning to QUBO



Qubits are $|0\rangle$ if they're in subset 0, $|1\rangle$ if they're in subset 1

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Map Problem to QUBO/Hamiltonian

Hamiltonian which counts the edges between subsets:

$$\boxed{H = \sum_{i,j \in E} \frac{Z_i Z_j - I}{2}}$$

Score -1 if edges are in different subsets
Score 0 if the edges are in the same subset

Most edges between subsets will have the minimum energy.
Ground state gives the answer to the MAX-CUT problem.

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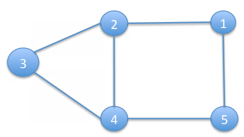
Map QUBO to Hamiltonian

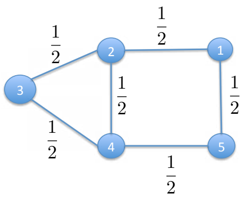


$$H = \sum_{i,j \in E} \frac{Z_i Z_j - I}{2}$$

In our case:

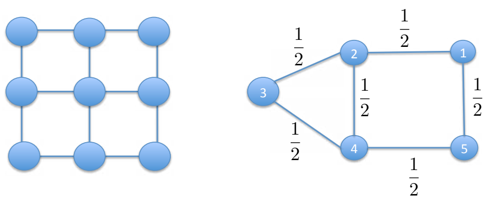
$$H = \frac{Z_1 Z_2 - I}{2} + \frac{Z_2 Z_3 - I}{2} + \frac{Z_3 Z_4 - I}{2} + \frac{Z_4 Z_5 - I}{2} + \frac{Z_5 Z_1 - I}{2} + \frac{Z_2 Z_4 - I}{2}$$

Map QUBO to Hamiltonian



Embed on Physical Architecture

Let's imagine that our quantum computing architecture was made up of a square pattern:



Actual architecture

Problem we want to embed

Embed on Physical Architecture

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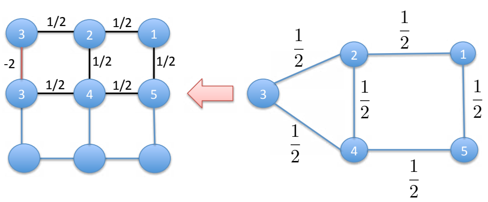

Actual architecture

Problem we want to embed

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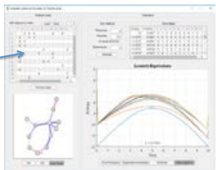
Execute algorithm

At this point you would program the couplings into your physical quantum computer, and physically perform the annealing schedule:

$$H(s) = (1-s)H_x + sH_p \qquad 0 \leq s \leq 1$$

$$s = t/T$$

In our case, we will enter the couplings in our MATLAB environment



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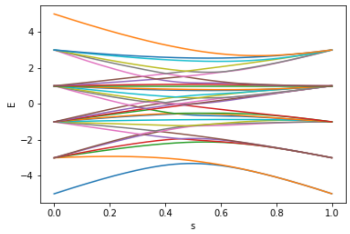
Energy for our MAX-CUT example



Prepare in $|-\rangle^{\otimes 5}$ state

In ground state, solving computational problem

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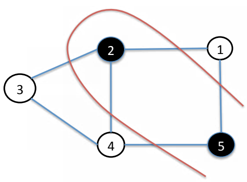
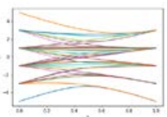
Read Ground State configuration





After the adiabatic evolution we read the state of the quantum computer. Provided we have changed our Hamiltonian slowly enough, we will be in the ground state:

$|01001\rangle$ or $|10110\rangle$

Adiabatic from Start to Finish

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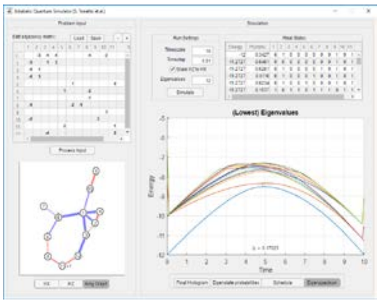
MATLAB environment in the lab



D-Wave systems

Image:
D-Wave Systems

- Opportunity to use them in this week's lab!
- Note: Friday lab times changed for this week only.

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