PHYC90045 Introduction to Quantum Computing

<u>Assignment 2</u>

<u>Due: 5pm Thursday 24th October, 2019</u>

<u>Submission: place in the assignment box "PHYC90045 Introduction to Quantum Computing" outside Tutorial Room 207 (Physics Podium, David Caro Building). Refer to the "Circuit Submission Instructions" document on LMS.</u>

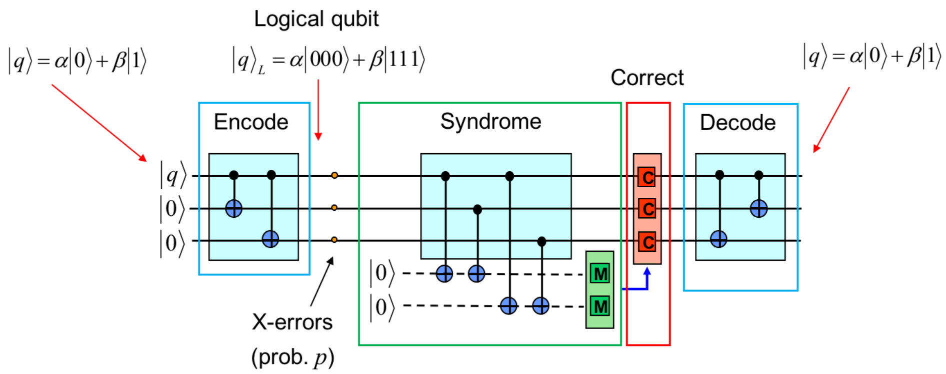
Welcome to Assignment 2 for PHYC90045 *Introduction to Quantum Computing*.

Instructions: Work on your own, attempt all questions. Hand in your completed written work (with name and student number on the front) on or before the due date as per instructions above (late penalties will apply). The QUI circuits you create for this project must be saved with the indicated filenames (including your student number as specified).

Total marks = 40

1. [2 marks]

Show how you can convert the 3 qubit bit-flip quantum error correction code (shown below) into a phase-flip (i.e. corrects a Z error) by introducing single qubit gates.



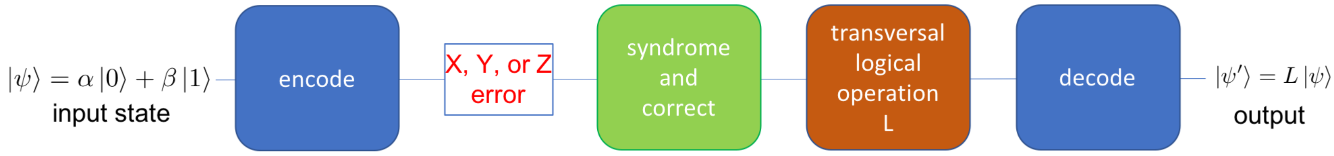
Write down the set of stabilisers. Program the phase-flip error correction circuit into the QUI, together with a Toffoli-based correction circuit. Submit your circuit as "<Student number>_Assignment-2_Q1".

2. [2 marks]

Consider the 5-bit code represented in terms of the following set of stabilisers:

$$\begin{array}{c} ZXXZI \\ IZXXZ \\ ZIZXX \\ XZIZX \end{array}$$

Code the circuit in the QUI for the following QEC schematic (below) for a given input state, including a correction circuit for a single X, Y or Z error after the encoding step, and a transversal logical operation L = Y prior to the decode step.



Submit your circuit as "Student number_Assignment-2_Q2".

3. [2+3+3 = 8 marks]

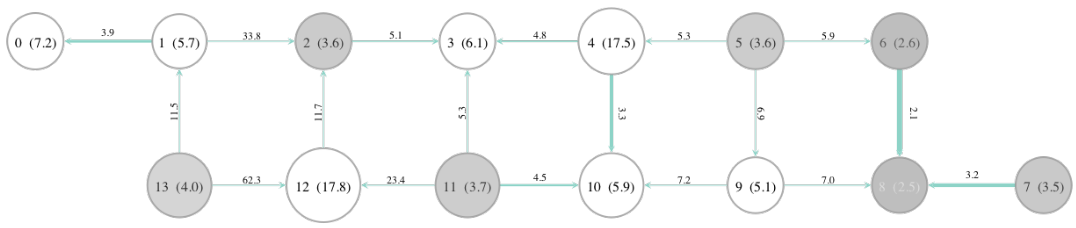
We have seen that many quantum algorithms rely on the Toffoli gate. In practice, most quantum computer architectures do not implement a 3-qubit gate and so the Toffoli gate must be constructed from (compiled into) a decomposition involving 1 and 2-qubit gates.

a) Using an IBM 5-qubit QC implement the best Toffoli gate you can (averaging over 1000 instances). Provide full details of the program (text code and circuit image), and the results (probability histogram). Your circuit must work on an arbitrary input state, and you will be judged on the final fidelity obtained for the 110 case.

b) Assume the main source of error in a) is an effective Y-rotation (uniform small angles) after all gates. Using the QUI, find the rotation angle (% of pi) that produces a distribution of probabilities most closely matching your results in a). Submit your circuit as "<Student number>_Assignment-2_Q3b".

c) The IBM 14-qubit QC *Melbourne* has a different connection architecture (see below). Using any qubits, program a Toffoli gate using QISKIT and compile and run on the hardware (averaging over 1000 instances, start in 110 state). Your circuit must work on an arbitrary state, and you will be judged on the final fidelity obtained for the 110 case. Compare the fidelity with that obtained in a) and explain the difference. Provide full details of the program (text code and compiled circuit image), and the results (probability histogram).

"IBMQ16-Melbourne"



4. [6+3+2 = 11 marks]

Consider the three qubit state given by

$$|K\rangle = \frac{1}{\sqrt{3}}|001\rangle - \frac{1}{\sqrt{7}}|010\rangle + \sqrt{\frac{11}{21}}|100\rangle$$

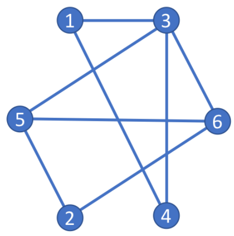
a) Using QUI, implement a circuit to create this state using only single qubit gates and CNOTs. Submit your circuit as "<Student_number>_Assignment-2_Q4a" and briefly explain the reasoning you used to create this circuit.

b) On an IBM-Q device of your choice, make a choice of qubits and implement this circuit, attempting to achieve the highest fidelity state possible. Describe which device, and which qubits you chose, and any additional optimization which you made.

c) Measure each of the qubits in the Z-basis, and record the results. Plot the measurement results. Comment on the results you obtained, including any errors.

5. [1+1+1+2+3+1 = 9 marks]

A particular problem reduces to graph partitioning on the following 6-vertex graph:

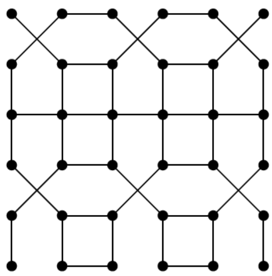


a) Draw the possible graph partition solutions (i.e. partitions where the vertices can be placed into two equal subsets with the minimum number of edges between the subsets), and write out the quantum versions of the solutions.

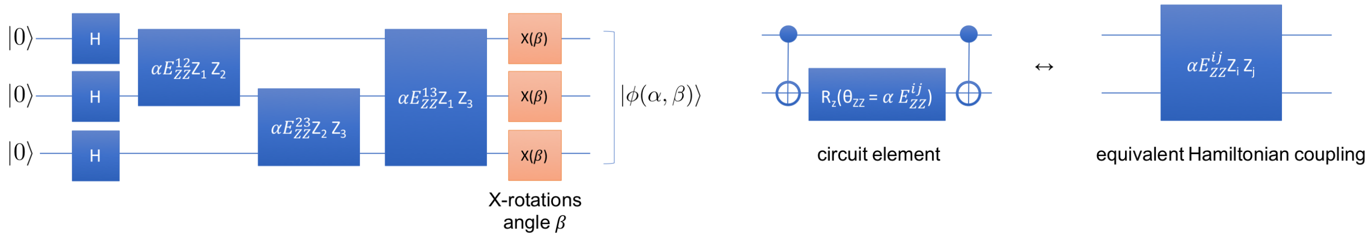
b) For the problem at hand write out the vertex Hamiltonian (H_A) and the edge Hamiltonian (H_B), setting the coefficients A = B = 1 (see lecture notes).

c) By enumerating all basis state possibilities (in an excel table) verify that the basis states corresponding to the solution graphs are the lowest energy states of the full Hamiltonian. (attach the excel table with the solutions highlighted).

d) Consider solving this problem using an adiabatic quantum computer (AQC). Find a minor embedding for the sparse-Kuratowski graph architecture (below), assuming couplings run from -1 to +1, and label all qubits and couplings. Explain how an AQC would find the solution.



e) Consider solving this problem on a universal (gate-model) quantum computer. Generalising the example below, encode the QAOA trial state corresponding to the full Hamiltonian in the QUI. Submit your circuit as "Student number_Assignment-2_Q5e".



f) Find a value of α and β that gives a significant probability of measuring the correct solution states in the output.

6. [4+2+2 = 8 marks]

Consider a problem for which the Hamiltonian can be written as:

$$\mathrm{H} = \mathrm{c_0 I} + \mathrm{c_1 Z_1} + \mathrm{c_2 Z_2} + \mathrm{c_3 Z_1 Z_2} + \mathrm{c_4 Y_1 Y_2} + \mathrm{c_5 X_1 X_2} + \mathrm{c_6 X_1 Z_2}$$

where the c's are real constants to be provided. The trial wavefunction, from which we will start our search for the lowest energy configuration, will be of the form

$$|\varphi(\theta)\rangle = (\cos(\theta)\, I - i \sin(\theta)\, X_1 Y_2)|01\rangle$$

a) The circuit to construct $|\varphi(\theta)\rangle$ differs from the circuit to get ZZ-couplings in the QAOA applications we have considered so far. Based on the circuit for a ZZ coupling, and the fact that $X = HZH$ and $Y = (\sqrt{X})^{\dagger} Z \sqrt{X}$ (where H are Hadamard operations), construct a circuit in the QUI that produces $|\varphi(\theta)\rangle$. Submit your circuit as "<Student number>_Assignment-2_Q6a".

b) In order to minimize the energy of the Hamiltonian, θ must be varied to minimize

$$\langle H \rangle = c_0 + c_1\langle Z_1\rangle + c_2\langle Z_2\rangle + c_3\langle Z_1 Z_2\rangle + c_4\langle Y_1 Y_2\rangle + c_5\langle X_1 X_2\rangle + c_6\langle X_1 Z_2\rangle$$

where for each observable $\mathcal{O}$ we must measure the quantity $\langle \mathcal{O}\rangle = \langle \varphi(\theta)|\mathcal{O}|\,\varphi(\theta)\rangle$. Note that measuring in the X or Y bases is equivalent to measuring in Z with a suitable basis-change. Modify your circuit to measure the expectation values required to determine $\langle H\rangle$ for a given value of θ. Rather than submit all the circuits, describe how you are programming the measurement of each term in $\langle H\rangle$.

c) Consider the case where the c's are given by:

c_0 = -0.470
c_1 = 0.342
c_2 = -0.446
c_3 = 0.573
c_4 = c_5 = 0.091
c_6 = 0.100

By trying out different values of θ, find the lowest energy for the Hamiltonian obtainable using your QUI circuit.