Lecture 8. Deep Learning. Convolutional ANNs. Autoencoders

COMP90051 Statistical Machine Learning

Semester 2, 2019
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THE UNIVERSITY OF MELBOURNE

This lecture

- Deep learning
  - Representation capacity
  - Deep models and representation learning
- Convolutional Neural Networks
  - Convolution operator
  - Elements of a convolution-based network
- Autoencoders
  - Learning efficient coding

Deep Learning and Representation Learning

Hidden layers viewed as
feature space transformation

Representational capacity

- ANNs with a single hidden layer are universal approximators
- For example, such ANNs can represent any Boolean function

$$OR(x_1, x_2) \qquad u = g(x_1 + x_2 - 0.5)$$

$$AND(x_1, x_2) \qquad u = g(x_1 + x_2 - 1.5)$$

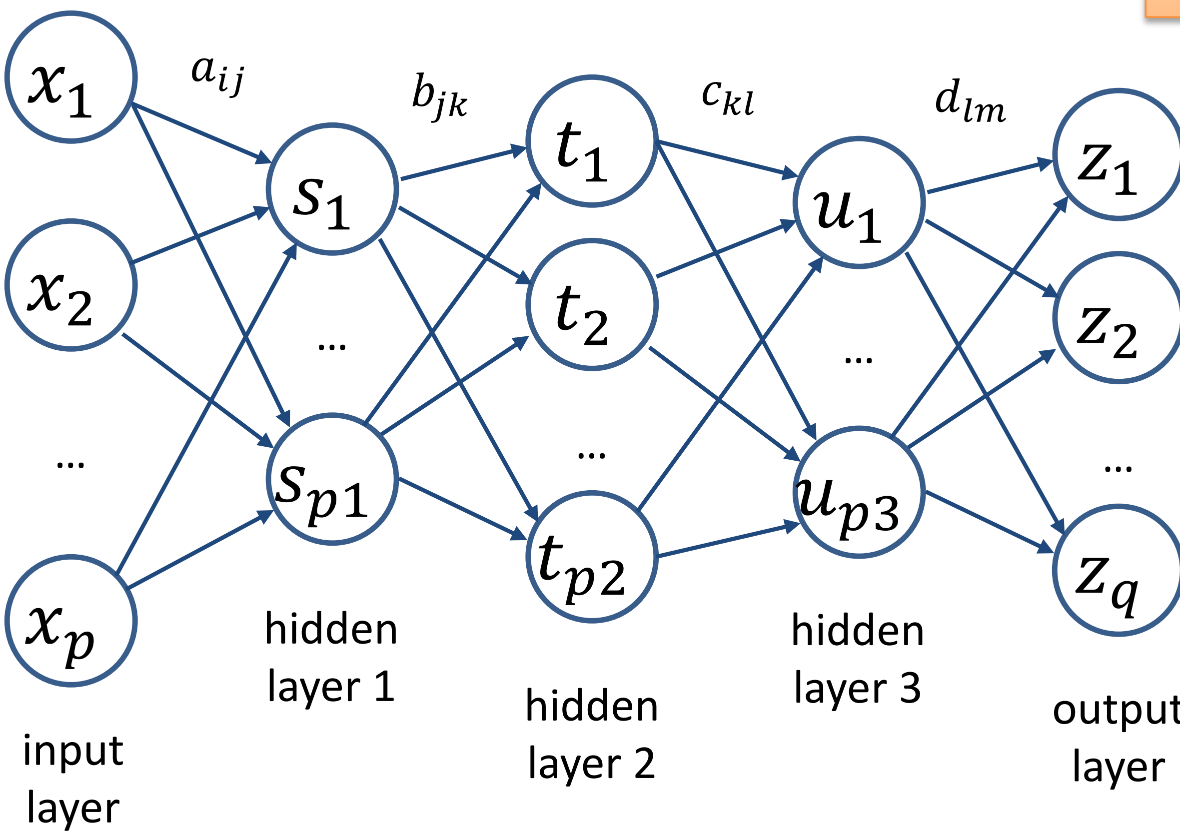
$$NOT(x_1) \qquad u = g(-x_1)$$

$g(r) = 1$ if $r \geq 0$ and $g(r) = 0$ otherwise

- Any Boolean function over m variables can be implemented using a hidden layer with up to 2^m elements
- More *efficient* to stack several hidden layers

Deep networks

"Depth" refers to number of hidden layers



$$\boldsymbol{s} = \tanh(\boldsymbol{A}'\boldsymbol{x}) \quad \boldsymbol{t} = \tanh(\boldsymbol{B}'\boldsymbol{s}) \quad \boldsymbol{u} = \tanh(\boldsymbol{C}'\boldsymbol{t}) \quad \boldsymbol{z} = \tanh(\boldsymbol{D}'\boldsymbol{u})$$

Deep ANNs as representation learning

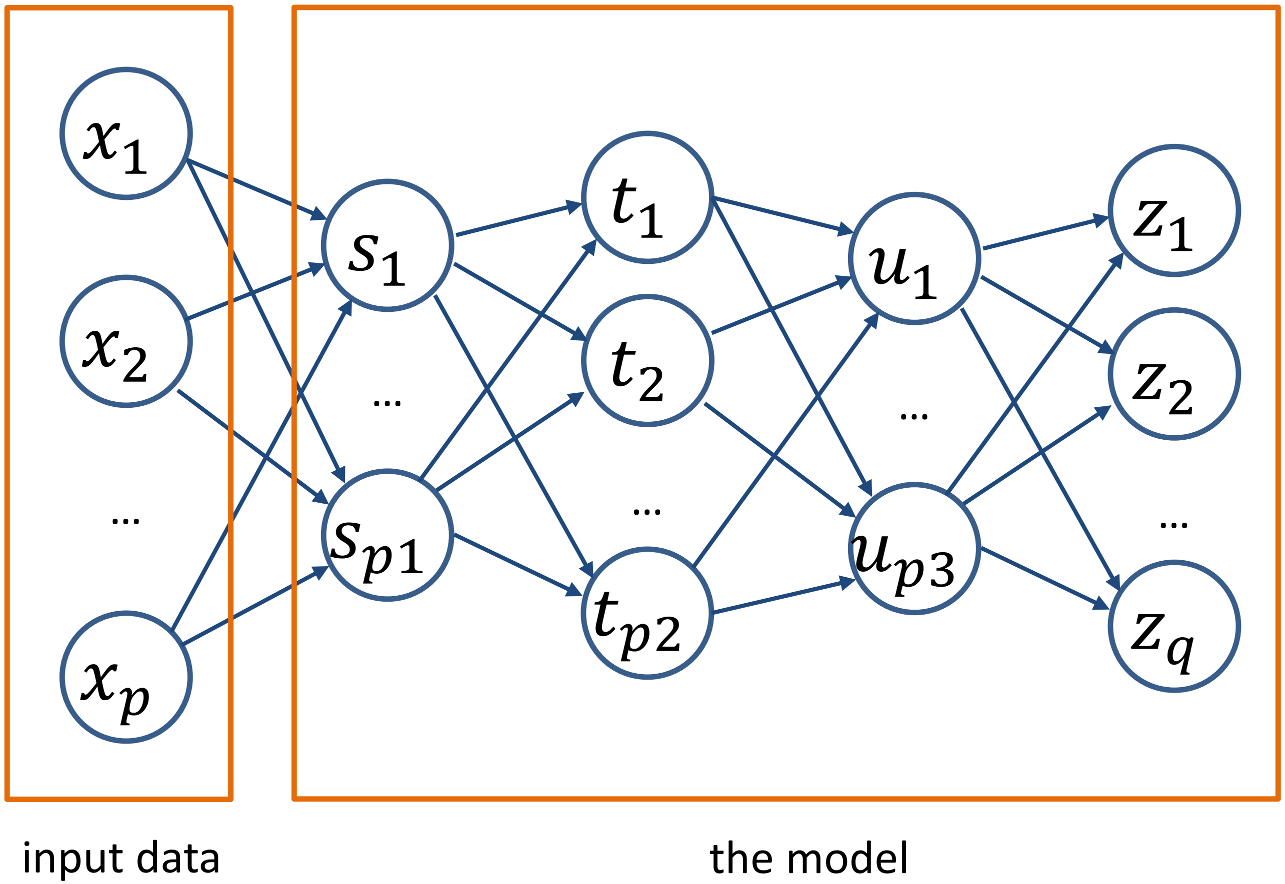
- Consecutive layers form *representations* of the input of increasing complexity
- An ANN can have a simple *linear* output layer, but using complex *non-linear* representation

$$\boldsymbol{z} = \tanh\left(\boldsymbol{D}'\left(\tanh\left(\boldsymbol{C}'\left(\tanh\left(\boldsymbol{B}'\left(\tanh\left(\boldsymbol{A}'\boldsymbol{x}\right)\right)\right)\right)\right)\right)\right)$$

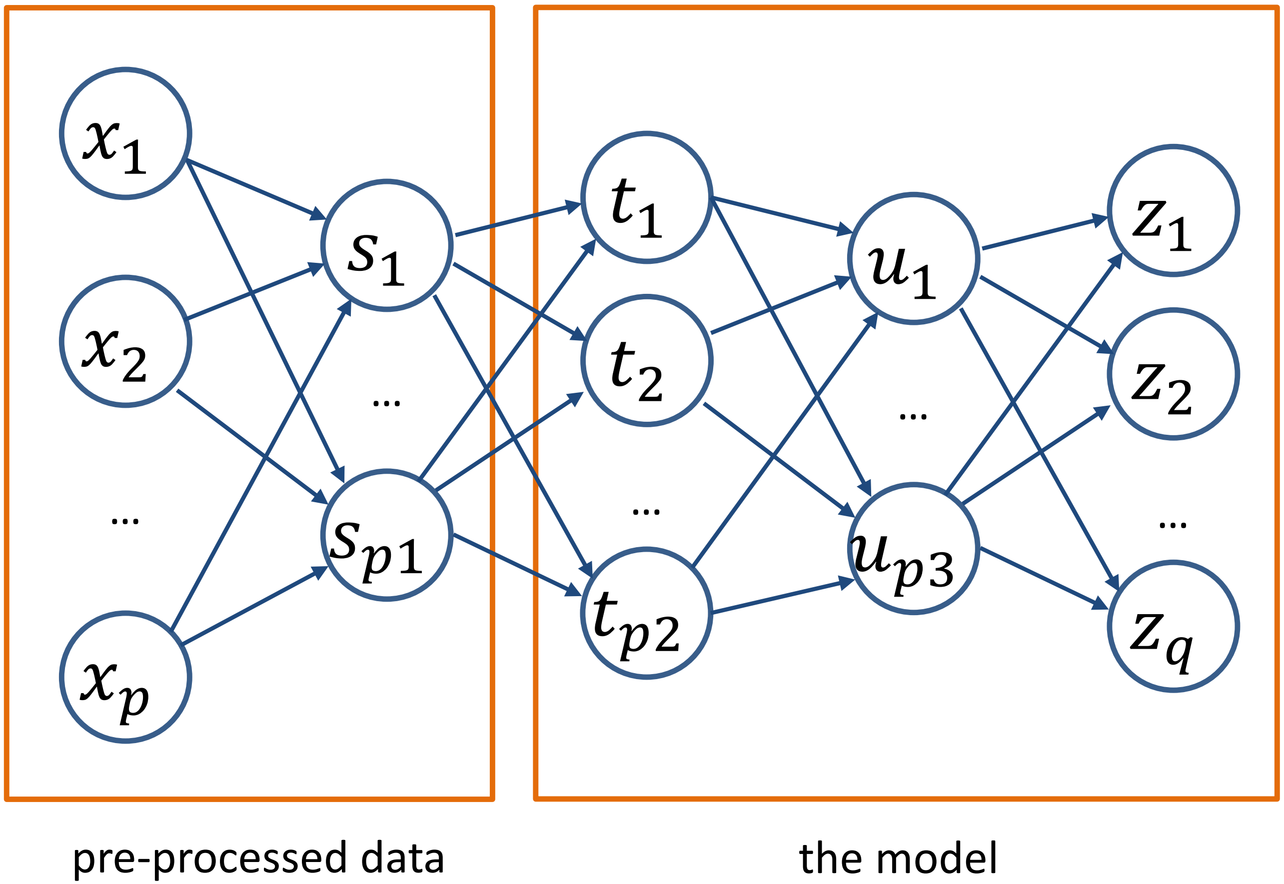
- Equivalently, a hidden layer can be thought of as the transformed feature space, e.g., $\boldsymbol{u} = \varphi(\boldsymbol{x})$
- Parameters of such a transformation are learned from data

Bias terms are omitted for simplicity

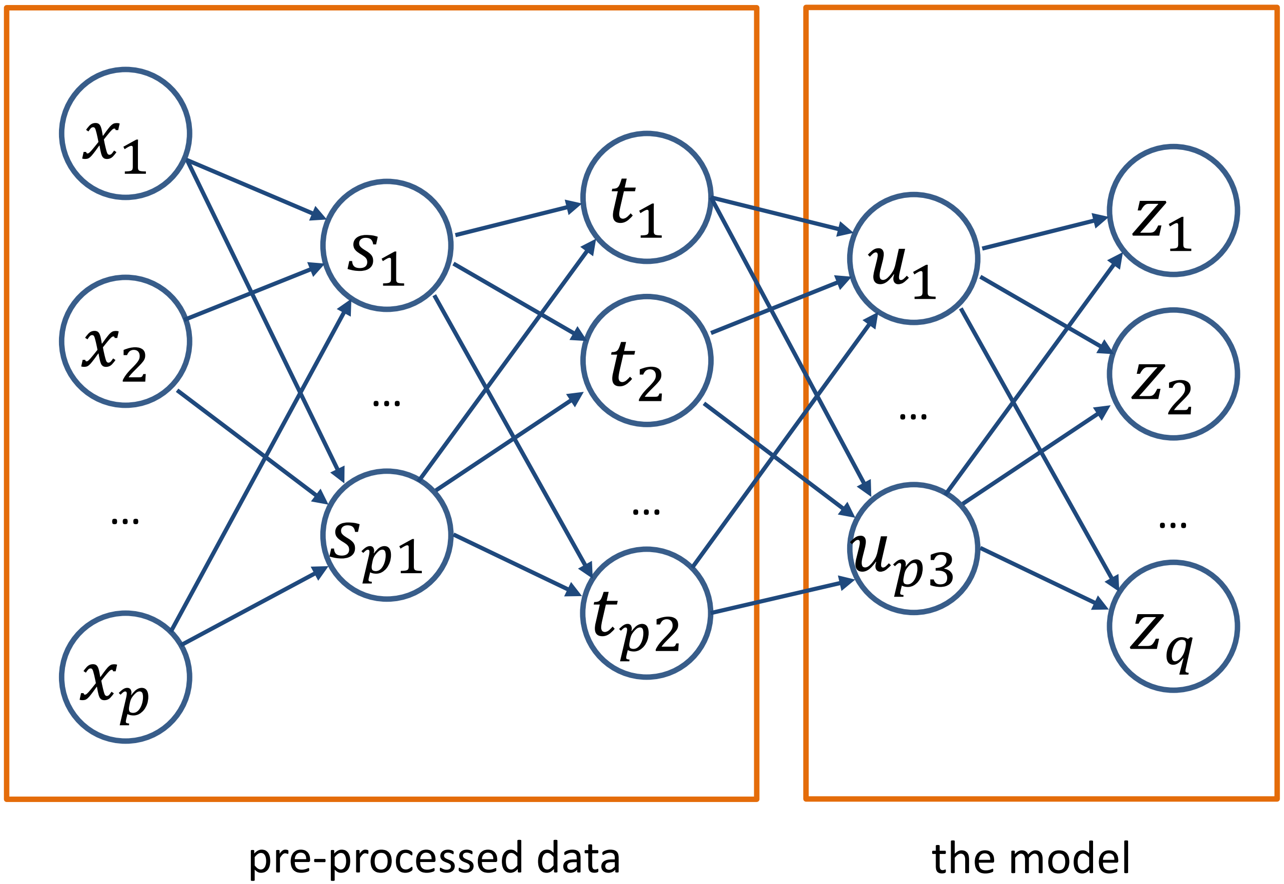
ANN layers as data transformation



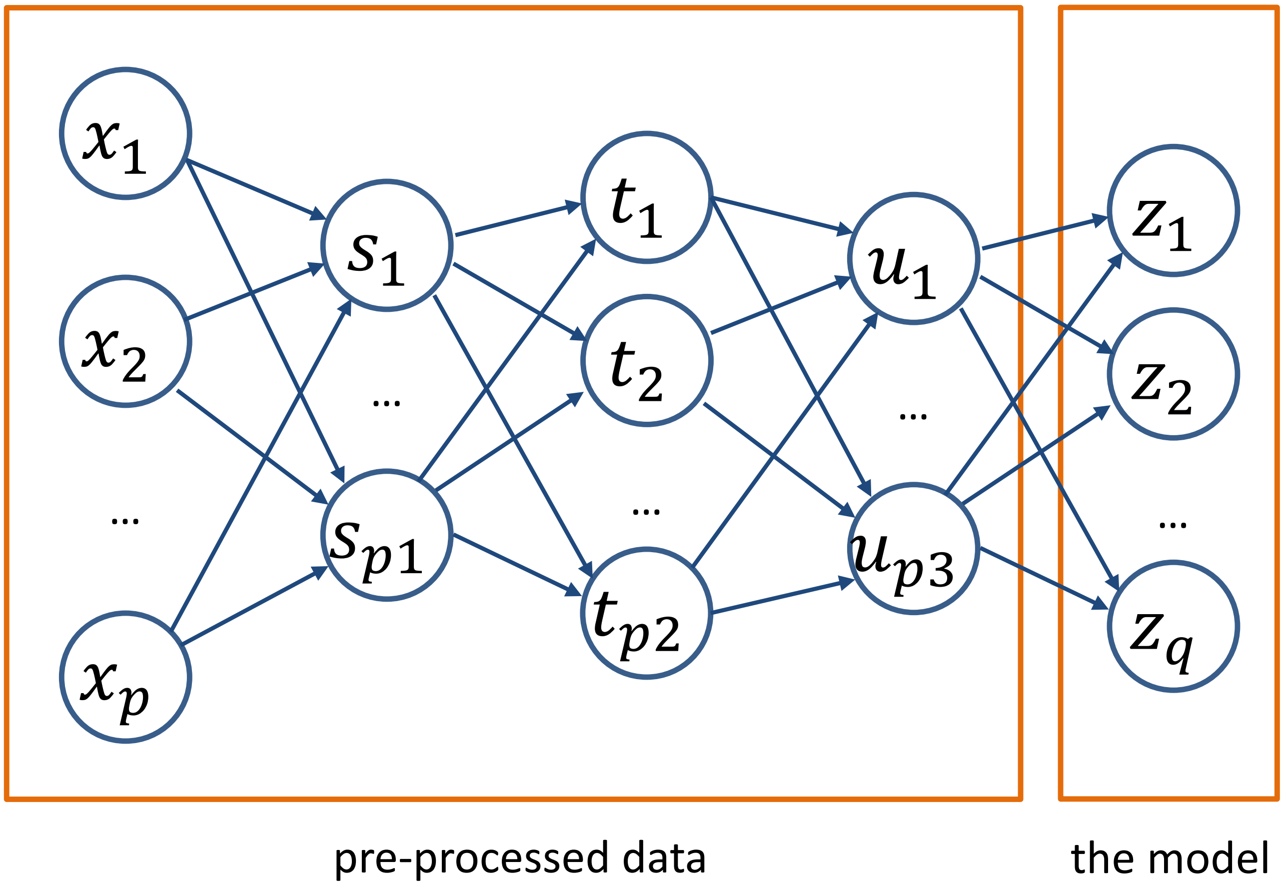
ANN layers as data transformation



ANN layers as data transformation



ANN layers as data transformation

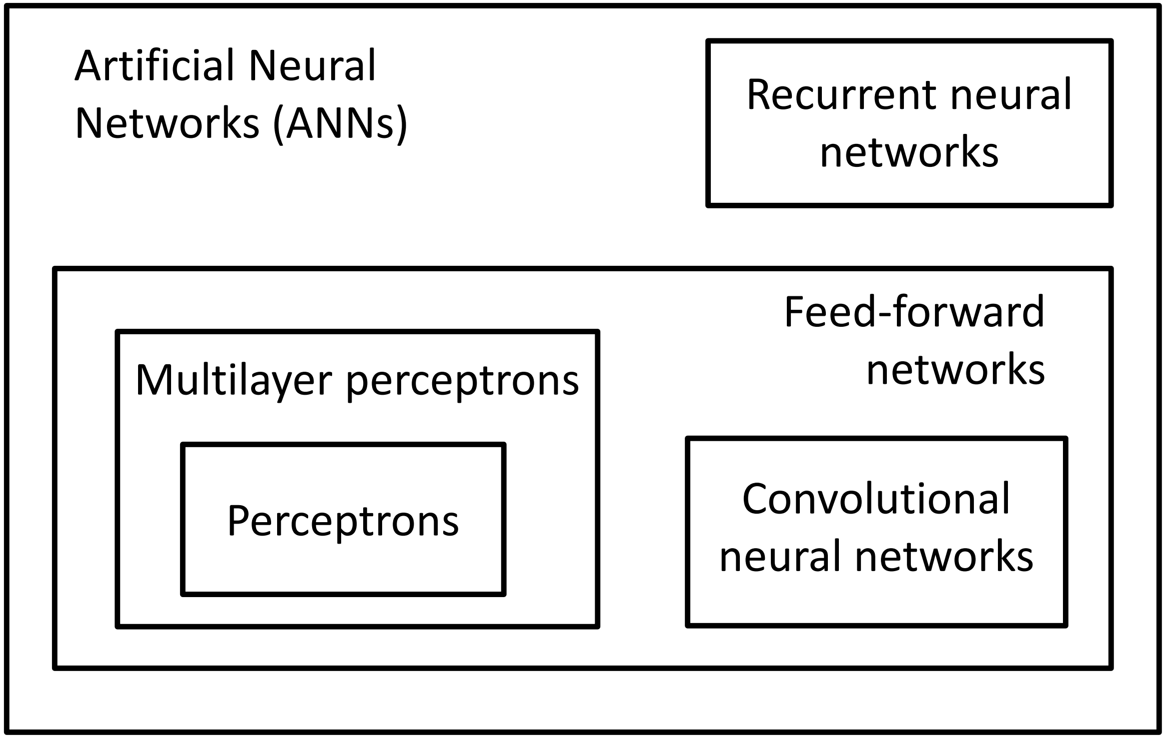


Depth vs width

- A single infinitely wide layer in theory gives a universal approximator
- However (empirically) depth yields more accurate models

  Biological inspiration from the eye:
  - first detect small edges and color patches;
  - compose these into smaller shapes;
  - building to more complex detectors, of e.g. textures, faces, etc.
- Seek to mimic layered complexity in a network
- However *vanishing gradient problem* affects learning with very deep models

Animals in the zoo



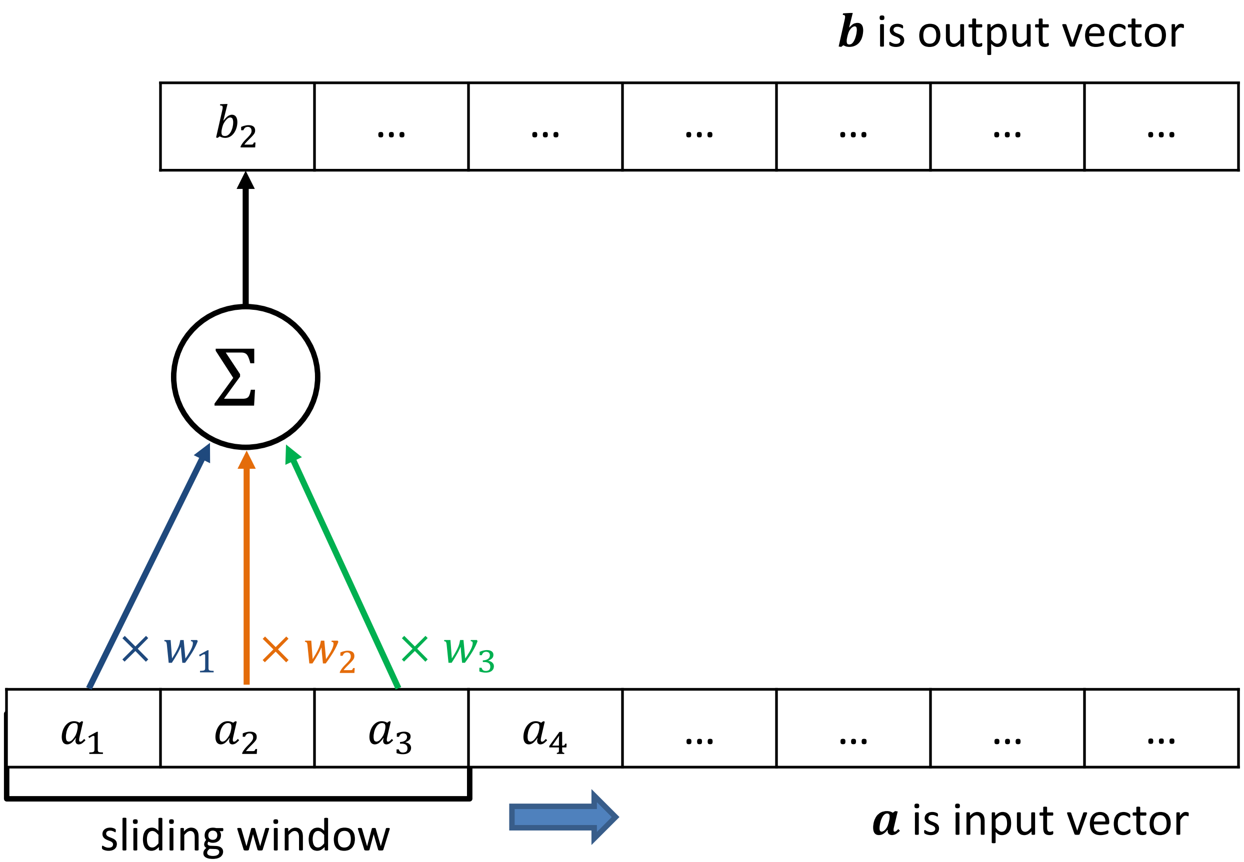
art: OpenClipartVectors at pixabay.com (CC0)

- Recurrent neural networks are not covered in this subject
- An autoencoder is an ANN trained in a specific way.
  - E.g., a multilayer perceptron can be trained as an autoencoder, or a recurrent neural network can be trained as an autoencoder.

Convolutional Neural Networks (CNN)

Based on repeated application of small filters to patches of a 2D image or range of a 1D input

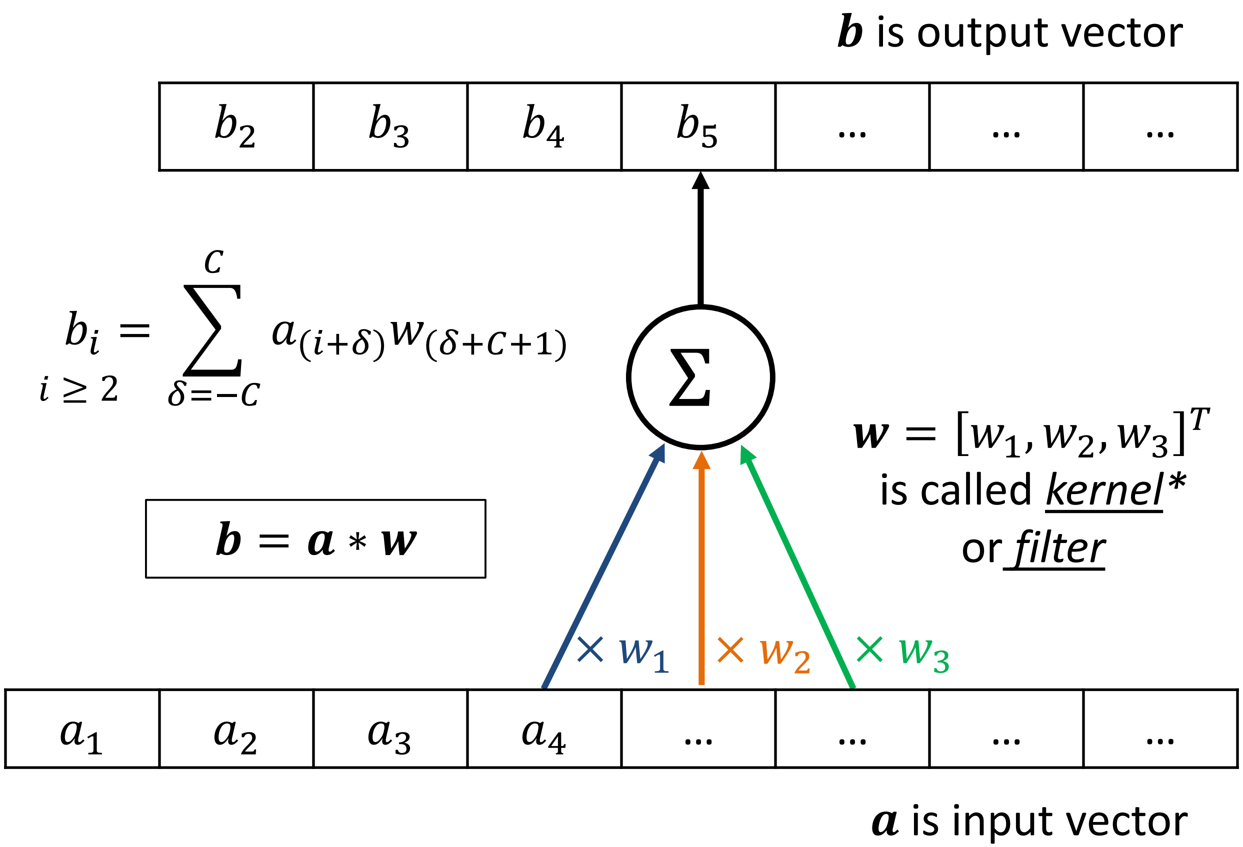
Convolution



Convolution

$$b_i = \sum_{\delta=-C}^{C} a_{(i+\delta)} w_{(\delta+C+1)}$$
$i \geq 2$

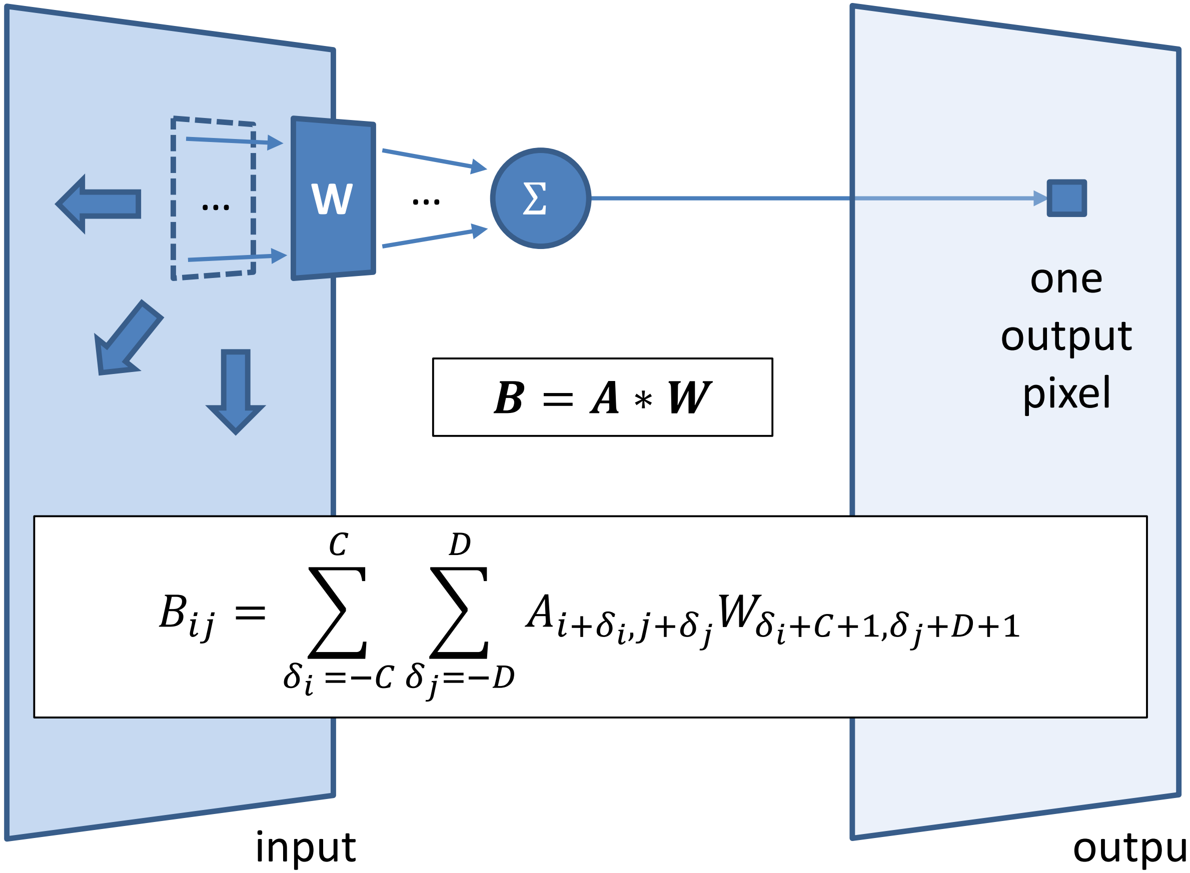
$$\boxed{\boldsymbol{b} = \boldsymbol{a} * \boldsymbol{w}}$$



$\boldsymbol{w} = [w_1, w_2, w_3]^T$ is called *kernel** or *filter*

*Later in the subject, we will also use an unrelated definition of kernel as a function representing a dot product

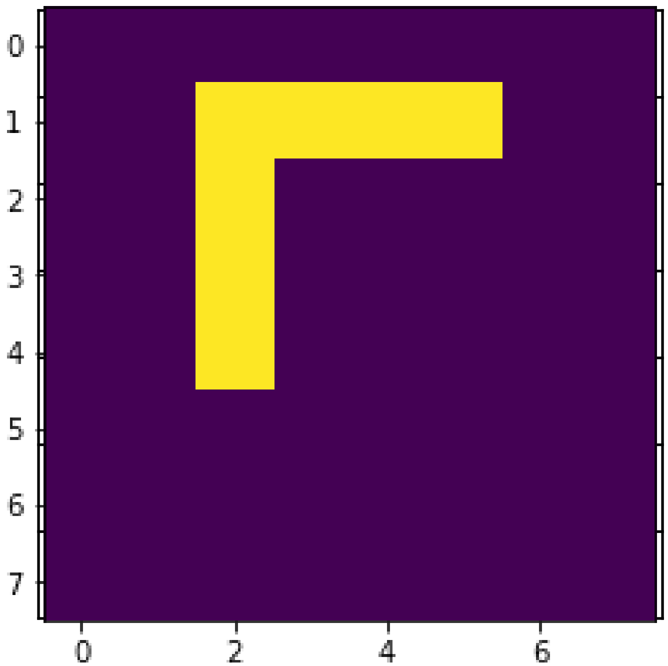
Convolution on 2D images



$$\boldsymbol{B} = \boldsymbol{A} * \boldsymbol{W}$$

$$B_{ij} = \sum_{\delta_i = -C}^{C} \sum_{\delta_j = -D}^{D} A_{i+\delta_i, j+\delta_j} W_{\delta_i + C + 1, \delta_j + D + 1}$$

Filters as feature detectors

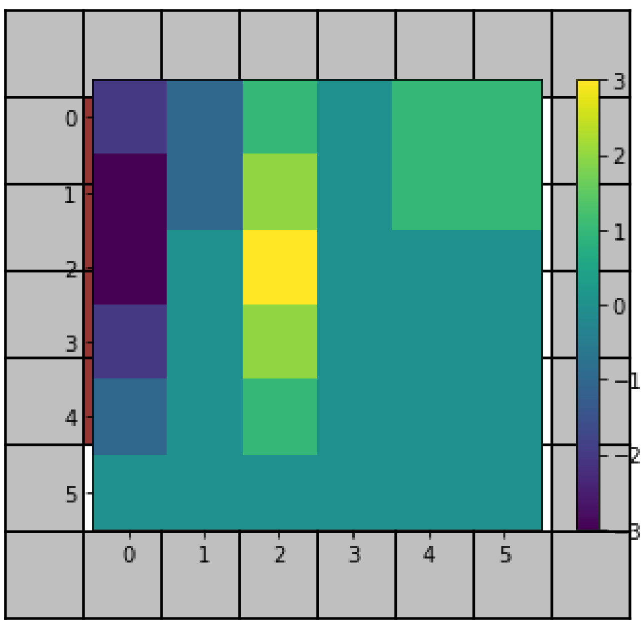


A is input image

convolve with a vertical edge filter

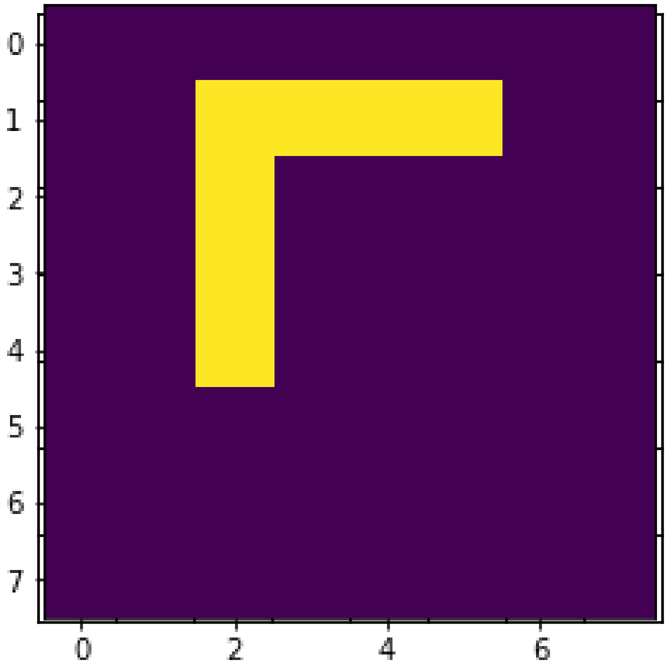
-1	0	1
-1	0	1
-1	0	1

activation function



filtered image

Filters as feature detectors

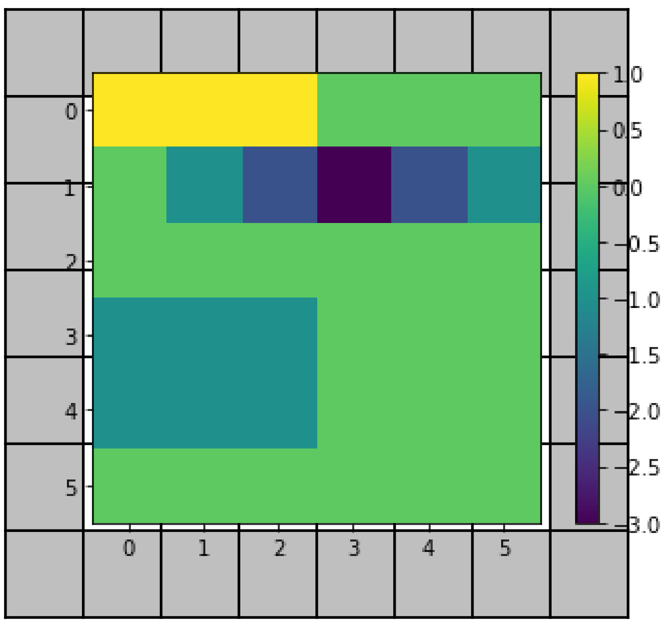


$\boldsymbol{A}$ is input image

convolve with a horizontal edge filter

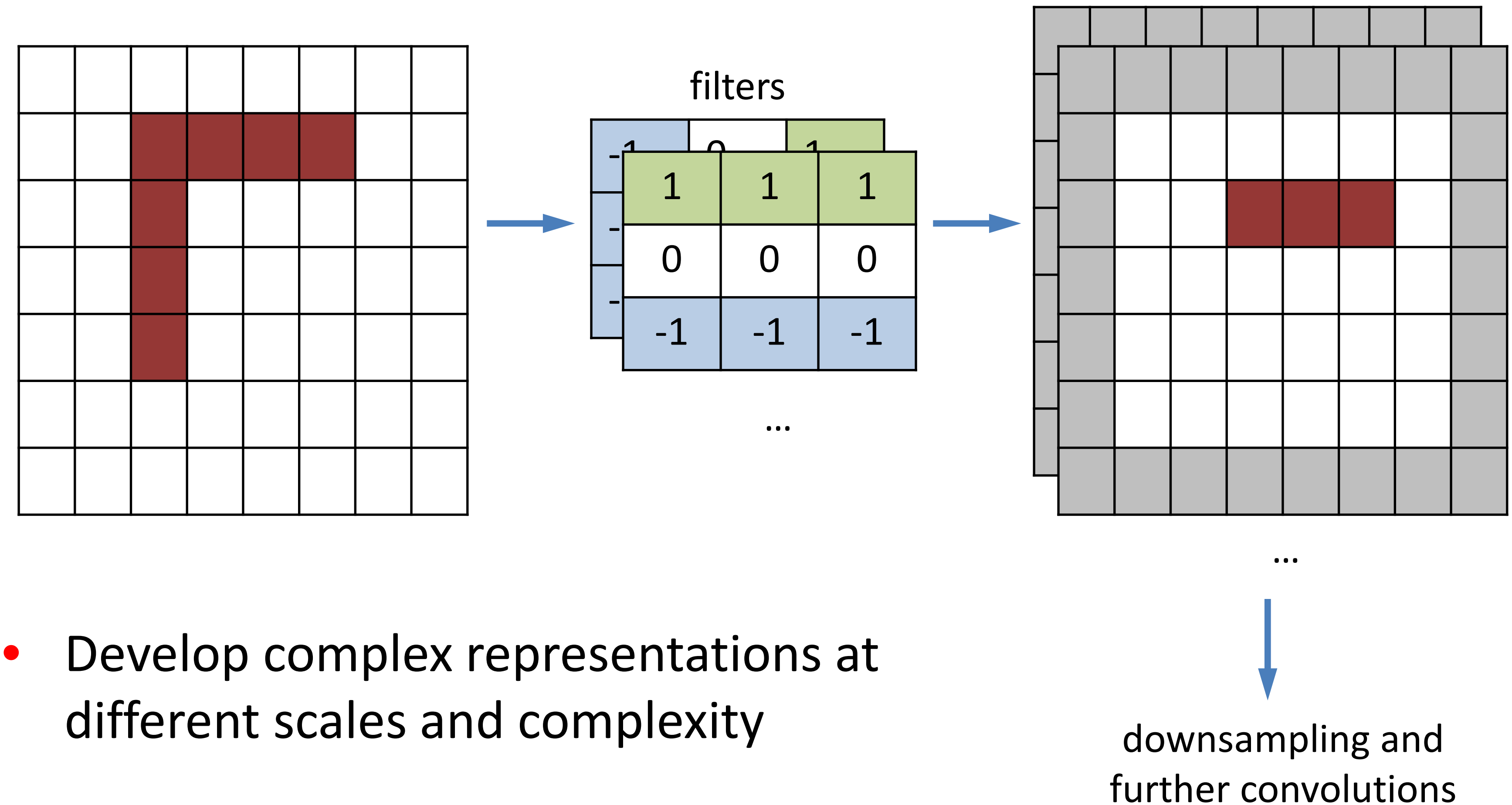
1	1	1
0	0	0
-1	-1	-1

activation function



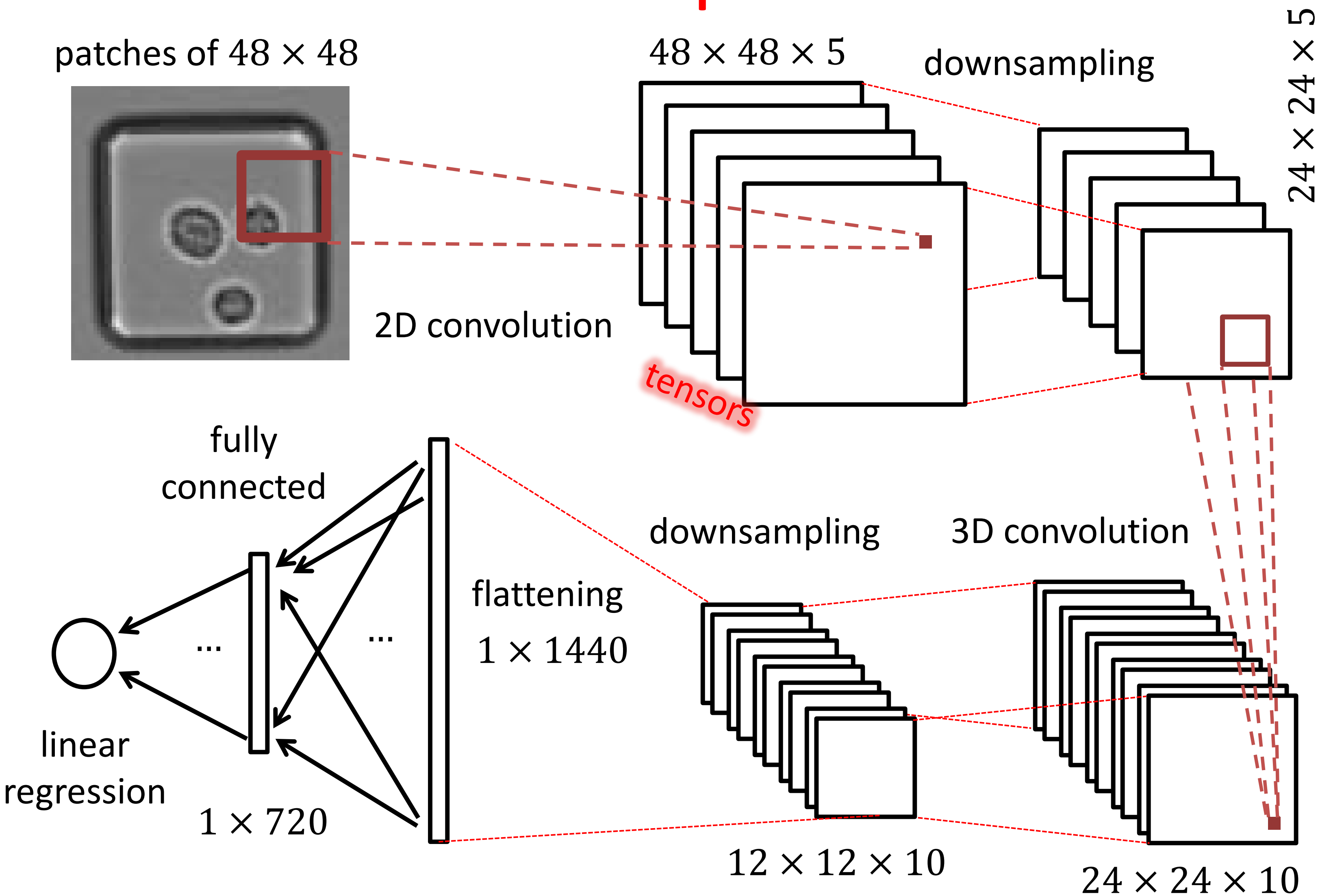
filtered image

Stacking convolutions



- Develop complex representations at different scales and complexity
- Filters are learned from training data!

CNN for computer vision



Implemented by Jizhizi Li
based on LeNet5: http://deeplearning.net/tutorial/lenet.html

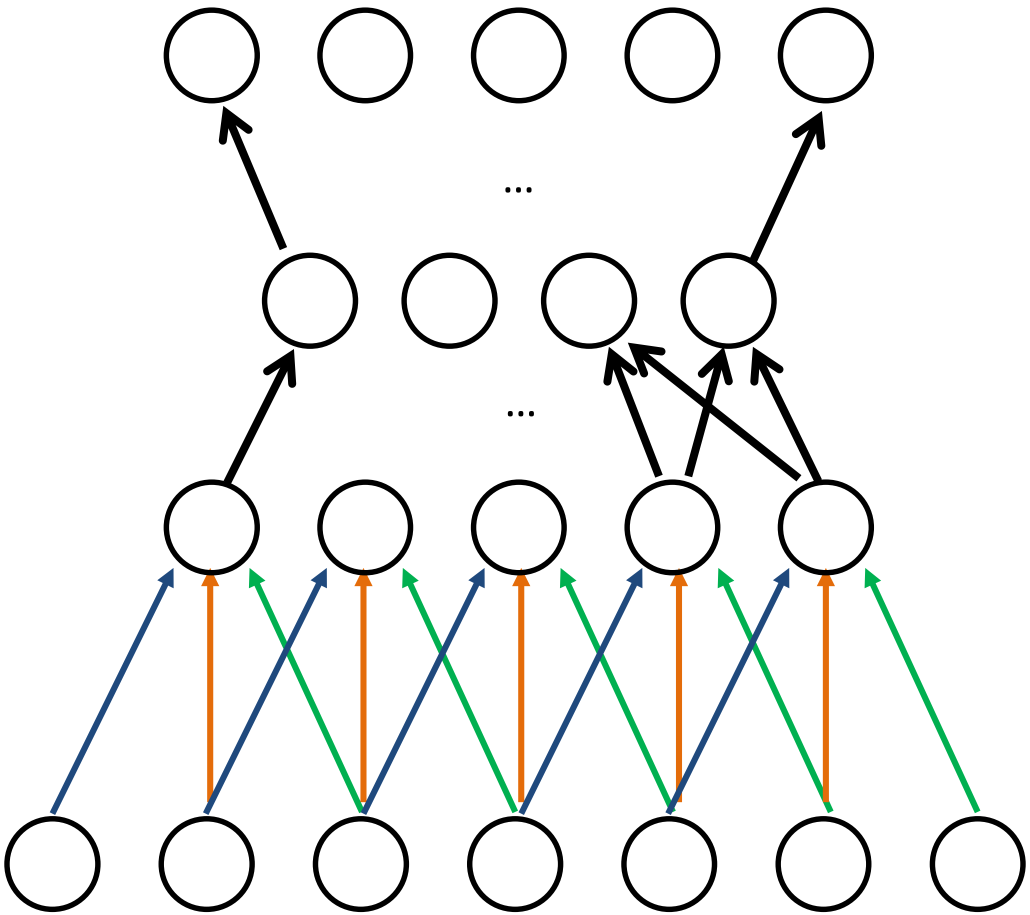
Components of a CNN

- Convolutional layers
  - Complex input representations based on convolution operation
  - Filter weights are learned from training data
- Downsampling, usually via Max Pooling
  - Re-scales to smaller resolution, limits parameter explosion
- Fully connected parts and output layer
  - Merges representations together

Downsampling via max pooling

- Special type of processing layer. For an $m \times m$ patch
$$v = \max(u_{11}, u_{12}, \dots, u_{mm})$$
- Strictly speaking, not everywhere differentiable. Instead, gradient is defined according to "sub-gradient"
  - Tiny changes in values of u_{ij} that is not max do not change v
  - If u_{ij} is max value, tiny changes in that value change v linearly
  - Use $\frac{\partial v}{\partial u_{ij}} = 1$ if $u_{ij} = v$, and $\frac{\partial v}{\partial u_{ij}} = 0$ otherwise
- Forward pass records maximising element, which is then used in the backward pass during back-propagation

Convolution as a regulariser



Fully connected, unrestricted

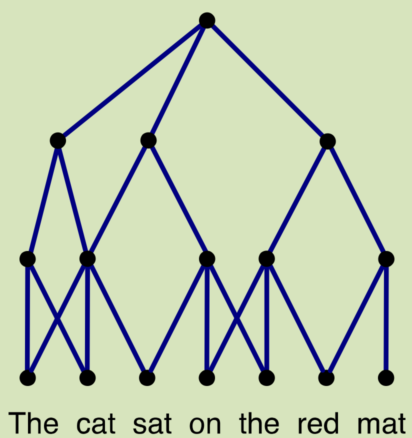
Fully connected, unrestricted

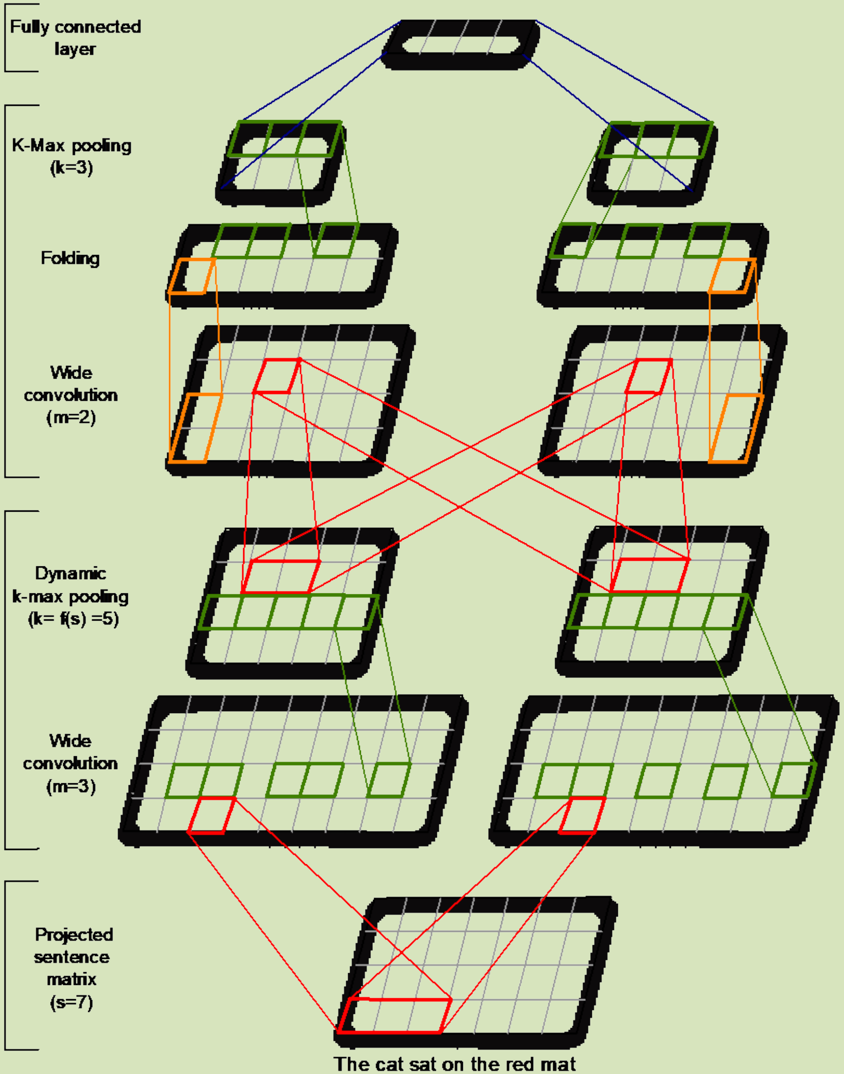
Restriction: same color – same weight

Document classification (Kalchbrenner et al, 2014)

Structure of text important for classifying documents

Capture patterns of nearby words using 1d convolutions





Autoencoder

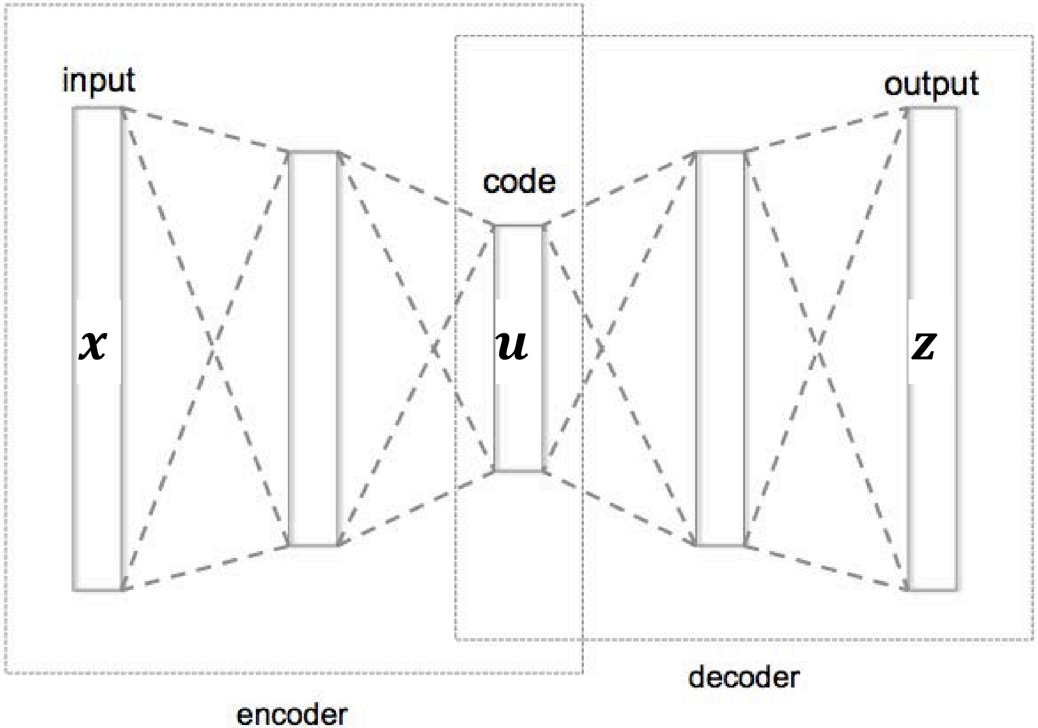
An ANN training setup that can be used for unsupervised learning, initialisation, or just efficient coding

Autoencoding idea

- Supervised learning:
  - Univariate regression: predict y from $\boldsymbol{x}$
  - Multivariate regression: predict $\boldsymbol{y}$ from $\boldsymbol{x}$
- Unsupervised learning: explore data $\boldsymbol{x}_1, \dots, \boldsymbol{x}_n$
  - No response variable
- For each $\boldsymbol{x}_i$ set $\boldsymbol{y}_i \equiv \boldsymbol{x}_i$
- Train an ANN to predict $\boldsymbol{y}_i$ from $\boldsymbol{x}_i$
- Pointless?

Autoencoder topology

- Given data without labels $\boldsymbol{x}_1, \dots, \boldsymbol{x}_n$, set $\boldsymbol{y}_i \equiv \boldsymbol{x}_i$ and train an ANN to predict $\boldsymbol{z}(\boldsymbol{x}_i) \approx \boldsymbol{x}_i$
- Set bottleneck layer $\boldsymbol{u}$ in middle "thinner" than input



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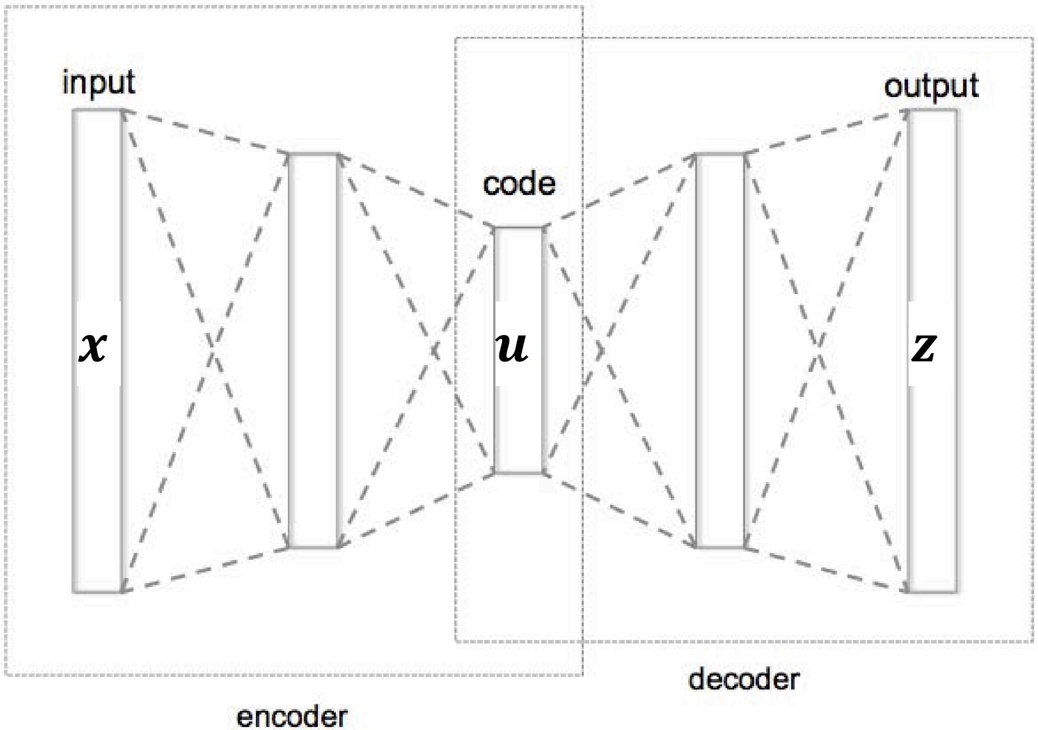
Introducing the bottleneck

- Suppose you managed to train a network that gives a good restoration of the original signal $\boldsymbol{z}(\boldsymbol{x}_i) \approx \boldsymbol{x}_i$
- This means that the data structure can be effectively described (encoded) by a lower dimensional representation $\boldsymbol{u}$



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Dimensionality reduction

- Autoencoders can be used for compression and dimensionality reduction via a non-linear transformation

- If you use linear activation functions and only one hidden layer, then the setup becomes almost that of Principal Component Analysis (stay tuned!)
  - ANN might find a different solution, doesn't use eigenvalues (directly)

Tools

- Tensorflow, Theano, Torch
  - python / lua toolkits for deep learning
  - symbolic or automatic differentiation
  - GPU support for fast compilation
  - Theano tutorials at http://deeplearning.net/tutorial/
- Various others
  - Caffe
  - CNTK
  - deeplearning4j …
- Keras: high-level Python API. Can run on top of TensorFlow, CNTK, or Theano

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  * Convolution operator
  * Elements of a convolution-based network
- Autoencoders
  * Learning efficient coding

- Workshops Week #5: Neural net topics
- Next lectures: Kernel methods