Quantitative Risk Analysis

Estimation of Sample Size and Power

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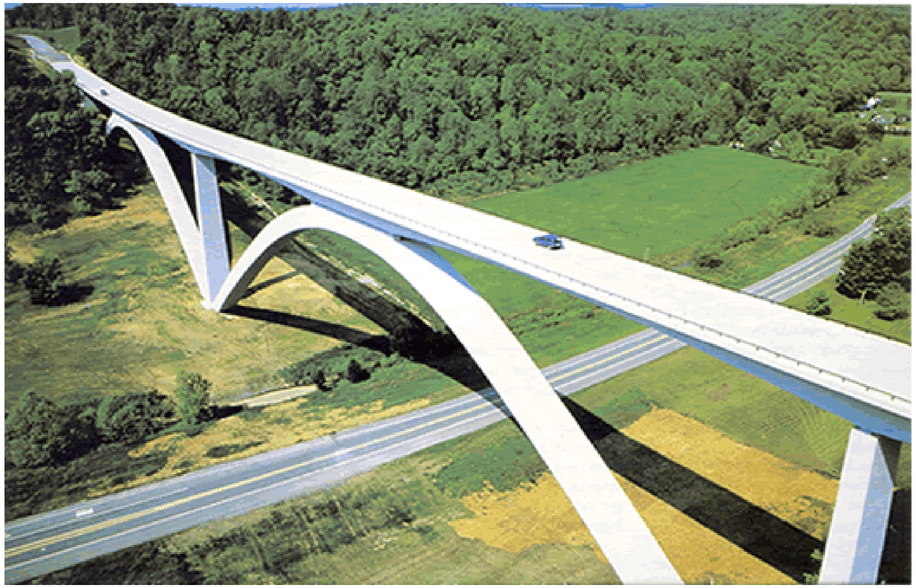


- **Limitations of Hypothesis Testing**

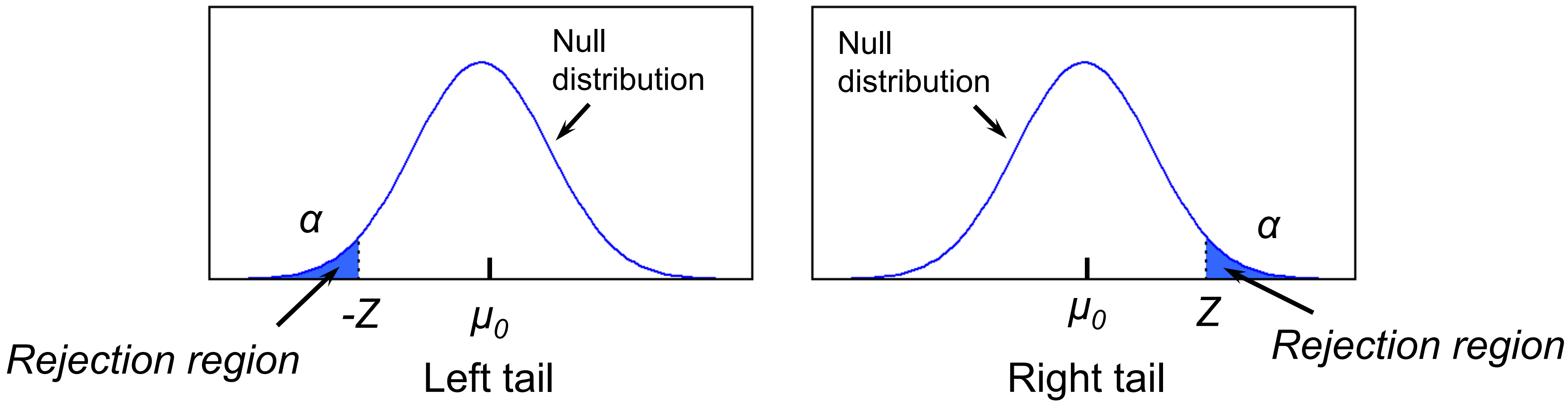
Hypothesis testing involving a significance level α, has two types of errors:

- **Type I error**: H_0 is rejected when it is True.
- **Type II error**: H_0 is not rejected when it is False.

Estimation of sample size and power

In order to **minimise** the probability of **Type I error**:

- Select a small significance level, α (e.g. $\alpha \leq 0.05$)



$$H_0 : \mu \geq \mu_0$$

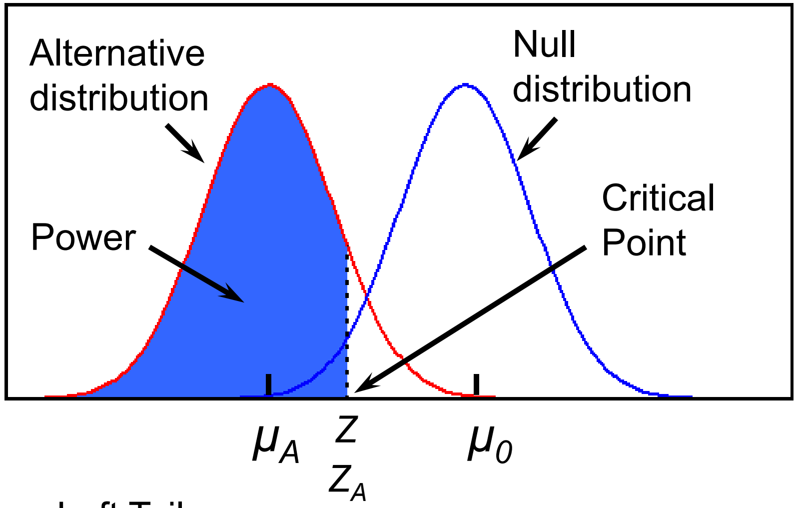
$$H_0 : \mu \leq \mu_0$$

Estimation of Power

The **Power** is the probability of ***avoiding*** **Type II error**:

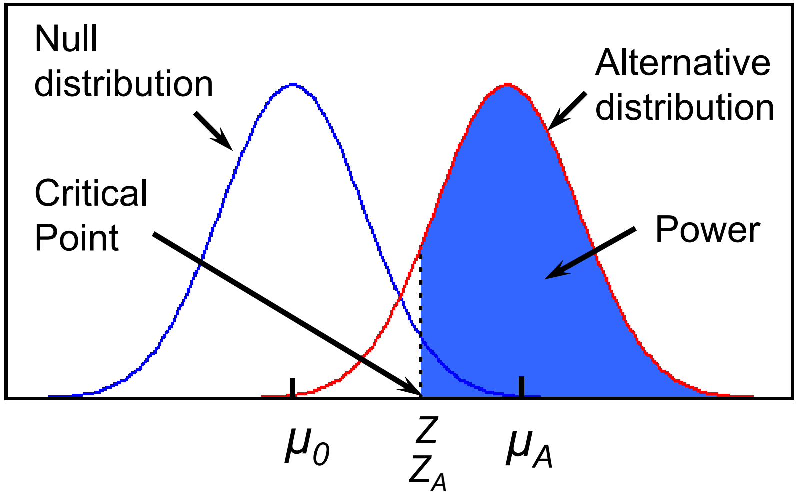
$$\text{Power} = 1 - \text{P(Type II error)}$$

Power ≥ 0.8 is generally considered to be acceptable



Left Tail

$$H_0 : \mu \geq \mu_0$$



Right Tail

$$H_0 : \mu \leq \mu_0$$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Estimation of Power

The **Power** is the probability of ***avoiding*** **Type II error**:

$$\text{Power} = 1 - \text{P}(\text{Type II error})$$

To calculate the power:

- Step 1: Determine H_0 and H_1
- Step 2: Select α and obtain Z
- Step 3: Approximate σ (through a preliminary sample or a sample of a similar population), and sample size, n

Note: While conducting the test, the sample is not yet drawn, and therefore σ needs to be assumed.

Estimation of Power

- Step 4: Identify the critical point:

$$\text{Critical Point} = \mu_0 + Z\frac{\sigma}{\sqrt{n}}$$

- Step 5: Assume an alternative mean, μ_A (usually close to μ_0) for the alternative distribution

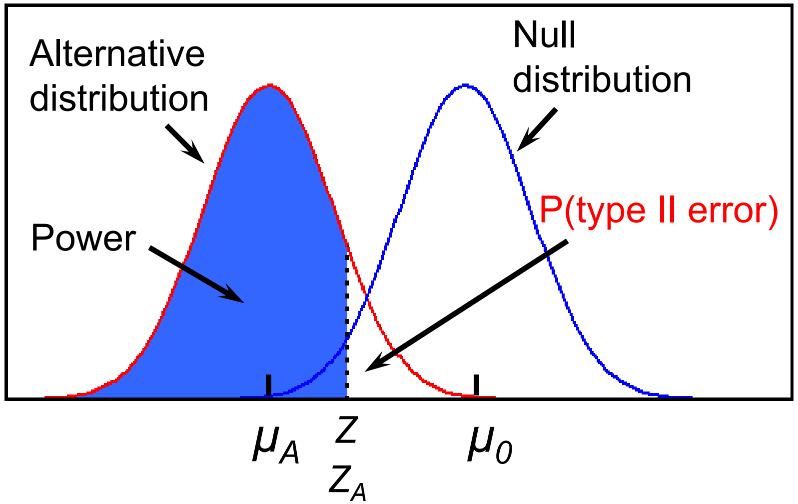
Estimation of Power

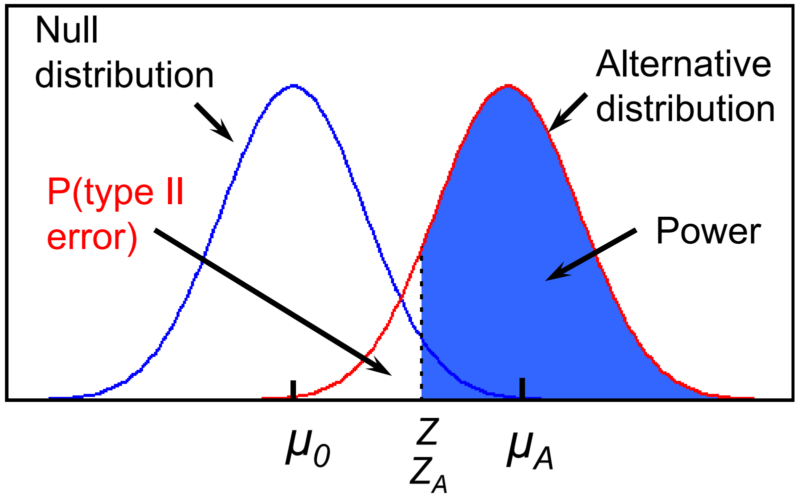
- Step 6: Using the critical point identified in Step 4, define Z_A for the alternative distribution

$$Z_A = \frac{(\text{Critical Point} - \mu_A)\sqrt{n}}{\sigma}$$

Estimation of Power

- Step 7: Calculate P(Type II error) using Z_A :





Left Tail $H_0 : \mu \geq \mu_0$

Right Tail $H_0 : \mu \leq \mu_0$

Power=Area to the left of Z_A

Power=Area to the right of Z_A

Note: Power ≥ 0.8 is generally considered to be acceptable

Example 1: Calculation of Power (Risk of Concrete Failure)

A decision needs to be made concerning the production of high strength concrete. Find the power: with a significance level, α of 5%, and the hypothesis testing consists of H_0: $\mu \leq$ 80 MPa and H_1: $\mu >$ 80 MPa; the alternative mean, μ_A, is 82 MPa, and assuming that the sample size, n = 50 and the standard deviation, σ = 5 MPa. The production will commence if Power ≥ 0.8 in order to reduce Type II error.

Solution

Solution

- Step 1: H_0 and H_1 are given
- Step 2: $\alpha = 0.05$ and therefore $Z = 1.645$
- Step 3: σ is given as 5
- Step 4: Therefore the critical point is

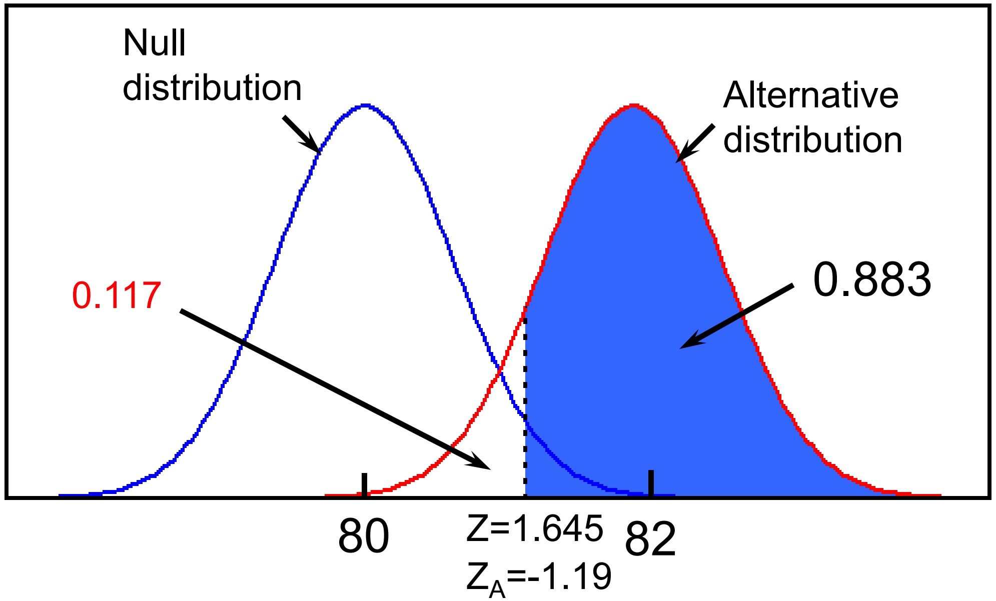
$$80 + 1.645\frac{5}{\sqrt{50}} = 81.16$$

- Step 5: $\mu_A = 82$
- Step 6: $Z_A = \frac{(81.16-82)}{\frac{5}{\sqrt{50}}} = -1.19$

Solution

- Step 7: Since Z_A = -1.19, P(Type II error) = 0.117
- Step 8: $\text{Power} = 1 - 0.117 = 0.883$

The production can commence



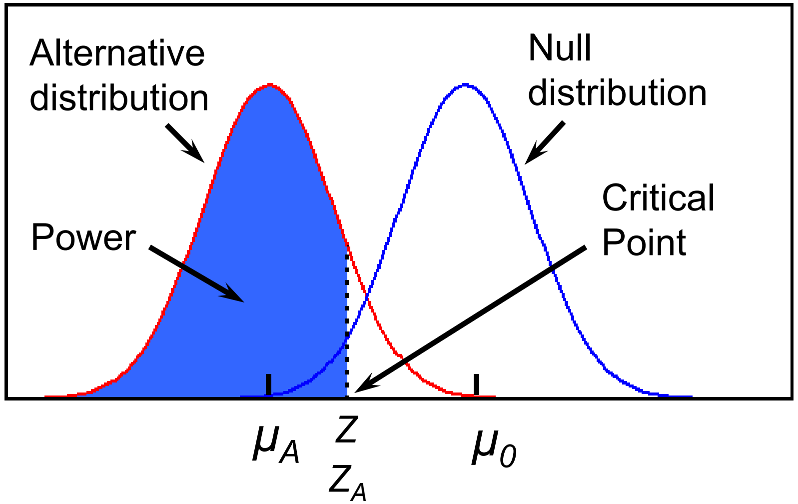
Estimation of Power

MATLAB function to calculate Power

sampsizepwr ('test type (z or t)', [μ, σ], μ_A, [], n, 'tail', 'both/right/left', 'alpha', α)

sampsizepwr('z',[80,5],82,[],50,'tail','right','alpha',0.05)

Estimation of Sample Size

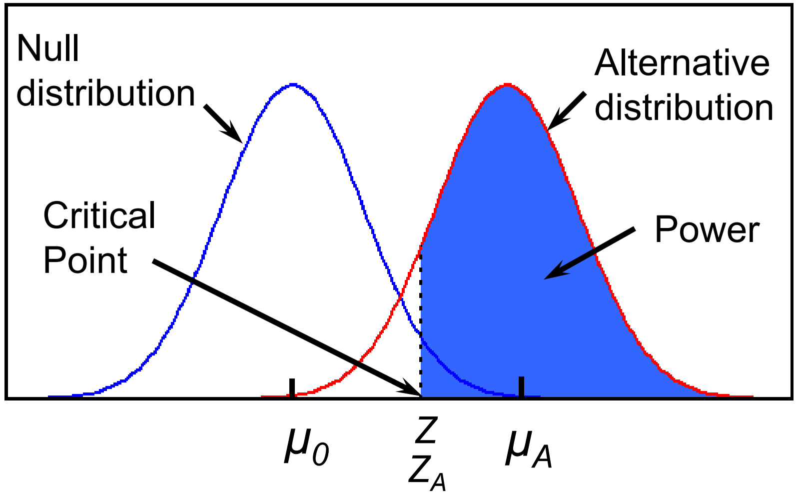


Left Tail $H_0 : \mu \geq \mu_0$

$$\text{Critical Point} = \mu_0 + Z\frac{\sigma}{\sqrt{n}} \qquad \text{Critical Point} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Estimation of Sample Size



Right Tail $$H_0 : \mu \le \mu_0$$

$$\text{Critical Point} = \mu_0 + Z\frac{\sigma}{\sqrt{n}} \qquad \text{Critical Point} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Estimation of sample size and power

In order to determine the required sample size, n:

- Step 1: Determine H_0 and H_1
- Step 2: Select α and obtain Z
- Step 3: Approximate σ
- Step 4: Obtain the expression of the critical point
- Step 5: Define μ_A
- Step 6: Select an acceptable Power, $\mathrm{P}(\text{Type II error})$= 1-Power
- Step 7: Determine Z_A by using $\mathrm{P}(\text{Type II error})$
- Step 8: Use the following equation and solve it for sample size, n

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

Example 2: Calculation of Sample Size (Risk of Concrete Failure)

Find the sample size of the production of high strength concrete, with a significance level, α, of 5%, and the hypothesis testing consists of H_0: $\mu \leq 80$ MPa and H_1: $\mu > 80$ MPa; the alternative mean, μ_A, is 81 MPa, and assuming that the standard deviation, σ, is 7 MPa and the Power is 0.9.

Solution

Estimation of Sample Size

Solution

- Step 1: H_0: $\mu \leq 80$ MPa and H_1: $\mu > 80$ MPa
- Step 2: α = 0.05 and therefore Z = 1.645
- Step 3: σ is given as 7
- Step 4: Therefore the critical point is

$$80 + 1.645\frac{7}{\sqrt{n}}$$

- Step 5: μ_A = 81
- Step 6: Power = 0.9, $\mathrm{P(Type\ II\ error)}$= 1-0.9=0.1

Estimation of Sample Size

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

P(Type II error)= 1-0.9=0.1

Step 7: Z_A is approximately -1.28

Estimation of Sample Size

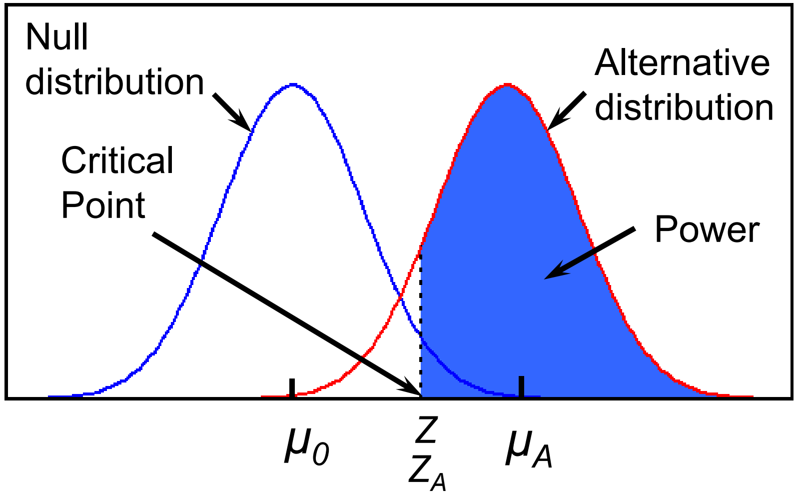
Solution

- Step 8

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$

$$80 + 1.645\frac{7}{\sqrt{n}} = 81 - 1.28\frac{7}{\sqrt{n}}$$

$$n \approx 420$$



Right Tail $\qquad H_0 : \mu \leq \mu_0$

MATLAB function to calculate Sample Size

sampsizepwr ('test type (z or t)', [μ, σ], μ_A, Power, [], 'tail', 'both/right/left', 'alpha', α)

sampsizepwr('z',[80,7],81,0.90,[],'tail','right','alpha',0.05)