The University of Melbourne
CVEN30008 Engineering Risk Analysis

Hypothesis Testing Part 2

1. **Two methods are being considered for a paint manufacturing process, in order to increase production. In a random sample of 100 days, the mean daily production using the first method was 625 tons and the standard deviation was 40 tons. In a random sample of 64 days, the mean daily production using the second method was 640 tons and the standard deviation was 50 tons. Assume the samples are independent. Can you conclude that the second method yields a greater mean daily production? Use MATLAB to verify your results.**

H_0: μ_x- $\mu_y \geq 0$ versus H_1: μ_x- $\mu_y < 0$

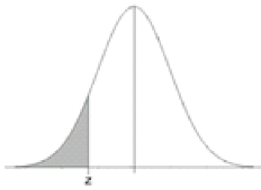
$n_x = 100, \bar{X} = 625, \sigma_x = 40$

$n_y = 64, \bar{Y} = 640, \sigma_y = 50$

$$z = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{\frac{{\sigma_x}^2}{n_x} + \frac{{\sigma_y}^2}{n_y}}} = \frac{625 - 640 - 0}{\sqrt{\frac{40^2}{100} + \frac{50^2}{64}}} = -2.02$$

P value estimated from Z table

Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183

From the Z table, $P(Z<-2.02) = 0.0217$

Since the significant level $\alpha = 0.05$ which is greater than 0.0217.

Because $P < \alpha$.

We reject H_0,

Conclusion: we conclude that the second method yields the grater mean daily production

MATLAB

Command Window

```
Left tail test

p =

    0.0216


alpha =

    0.0500

Since p <= alpha, Reject H_0
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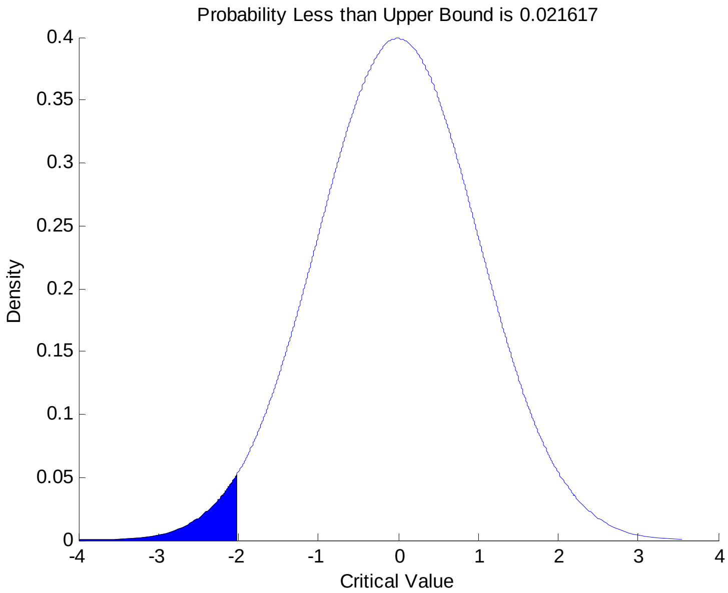

2. A crayon manufacturer is comparing the effects of two kinds of yellow dye on the brittleness of crayons. Dye B is more expensive that dye A, but it is thought that it might produce a stronger crayon. Three crayons are tested with dye A, while four crayons are tested with dye B. The results are as follows:

Dye A: 2.0 1.2 3.0

Dye B: 3.0 3.2 2.6 3.4

Can you conclude that the mean strength of crayons made with dye B is greater than that of crayons made with dye A based on a significant level of 0.05? Verify your results by using MATLAB.

Answer:

Let

Let $X_1, \ldots. X_4$ represent the strength of crayons made with dye A, and
Let $Y_1, \ldots. Y_4$ represent the strength of crayons made with dye B

$n_x = 3, \bar{X} = \frac{1}{n_x}\sum_{i=1}^{n_x} x_i = 2.07, S_x = \sqrt{\frac{1}{n_x-1}\sum_{i=1}^{n_x}(x_i - \bar{X})^2} = 0.90,$

$n_y = 4, \bar{Y} = \frac{1}{n_y}\sum_{i=1}^{n_y} y_i = 3.05, S_y = \sqrt{\frac{1}{n_y-1}\sum_{i=1}^{n_y}(y_i - \bar{Y})^2} = 0.34$

$$\frac{S_y}{S_x} = \frac{0.90}{0.34} = 2.64 > 2$$

Hence its un-pooled test.

H_0: μ_x- $\mu_y \geq 0$ versus H_1: μ_x- $\mu_y < 0$

$$t = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{\frac{{S_x}^2}{n_x} + \frac{{S_y}^2}{n_y}}} = \frac{2.07 - 3.05 - 0}{\sqrt{\frac{0.90^2}{3} + \frac{0.34^2}{4}}} = -1.795$$

Degree of freedom:

$$\nu = \frac{\left[\frac{{S_x}^2}{n_x} + \frac{{S_y}^2}{n_y}\right]^2}{\left[\left(\frac{{S_x}^2}{n_x}\right)^2 / (n_x - 1)\right] + \left[\left(\frac{{S_y}^2}{n_y}\right)^2 / (n_y - 1)\right]} = 2.4$$

Rounded down to the nearest integer

The degree of freedom = 2.

Because the sample size is smaller than 30, it is small-sample test (t test).

Take the absolute value of t, P value estimated from t table:

Degrees of Freedom	Combined Area α in Two Tails					
	0.250	0.100	0.050	0.025	0.010	0.005
1	2.4142	6.3138	12.7062	25.4517	63.6567	127.3213
2	1.6036	2.9200	4.3027	6.2053	9.9248	14.0890
3	1.4226	2.3534	3.1824	4.1765	5.8409	7.4533
4	1.3444	2.1318	2.7764	3.4954	4.6041	5.5976
5	1.3009	2.0150	2.5706	3.1634	4.0321	4.7733
6	1.2733	1.9432	2.4469	2.9687	3.7074	4.3168
7	1.2543	1.8946	2.3646	2.8412	3.4995	4.0293
8	1.2403	1.8595	2.3060	2.7515	3.3554	3.8325
9	1.2297	1.8331	2.2622	2.6850	3.2498	3.6897
10	1.2213	1.8125	2.2281	2.6338	3.1693	3.5814
11	1.2145	1.7959	2.2010	2.5931	3.1058	3.4966
12	1.2089	1.7823	2.1788	2.5600	3.0545	3.4284
13	1.2041	1.7709	2.1604	2.5326	3.0123	3.3725
14	1.2001	1.7613	2.1448	2.5096	2.9768	3.3257
15	1.1967	1.7531	2.1314	2.4899	2.9467	3.2860
16	1.1937	1.7459	2.1199	2.4729	2.9208	3.2520
17	1.1910	1.7396	2.1098	2.4581	2.8982	3.2224
18	1.1887	1.7341	2.1009	2.4450	2.8784	3.1966
19	1.1866	1.7291	2.0930	2.4334	2.8609	3.1737
20	1.1848	1.7247	2.0860	2.4231	2.8453	3.1534

From the t table, for t = 1.6036, P = α/2 = 0.125; for t = 2.9200, P = α/2 = 0.05. (Because it is two-tailed t table, while the question is one-tailed test, we need to divide the α value as shown in the second row by 2).

We know $1.6036 < t = 1.794 < 2.920$

Hence, $0.125 > P(t > 1.794) > 0.05$.

Since $P(t < -1.794) = P(t > 1.794)$,

$0.125 > P(t < -1.794) > 0.05$

Because the significant level α = 0.05 which is smaller than $P(t < -1.794)$.

$P > \alpha$

We do not reject H_0,

We cannot conclude that the mean strength of crayons made with dye B is greater than that of crayons made with dye A, based on a significant level of 0.05.

MATLAB

Command Window

```
Left tail test

p =

    0.1073


alpha =

    0.0500

Since p > alpha, we do not reject H_0
```

3. **Two formulations of a certain coating, designed to inhibit corrosion, are being tested. For each of eight pipes, half the pipe is coated with formulation A and the other half is coated with formulation B. Each pipe is exposed to a salt environment for 500 hours. Afterward, the corrosion loss (in μm) is measured for each formulation on each pipe.**

   Formulation A: 197 161 144 162 185 154 136 130

   Formulation B: 204 182 140 178 183 163 156 143

   Can you conclude that the mean amount of corrosion differs between the two formulations at 5% level of significance? Verify your results by using MATLAB.

<u>**Answer:**</u>

Let Difference = Formulation A – Formulation B, hence:

Formulation A	197	161	144	162	185	154	136	130
Formulation B	204	182	140	178	183	163	156	143
Difference (D)	-7	-21	4	-16	2	-9	-20	-13

$n_D = 8$, $\overline{D} = \frac{1}{n_D}\sum_{i=1}^{n_D} d_i = -10$, $S_D = \sqrt{\frac{1}{n_D-1}\sum_{i=1}^{n_D}(d_i - \overline{D})^2} = 9.38$, degree of freedom = n_D -1 = 7

H_0: $\mu_D = 0$ versus H_1: $\mu_D \neq 0$

Because the sample size is greater than 30, it is small-sample test (*t* test).

$$t = \frac{\overline{D} - \mu_D}{S_D/\sqrt{n_D}} = \frac{-10 - 0}{9.38/\sqrt{8}} = -3.015$$

Take the absolute value of *t*, P value estimated from *t* table:

Degrees of Freedom	Combined Area α in Two Tails					
	0.250	0.100	0.050	0.025	0.010	0.005
1	2.4142	6.3138	12.7062	25.4517	63.6567	127.3213
2	1.6036	2.9200	4.3027	6.2053	9.9248	14.0890
3	1.4226	2.3534	3.1824	4.1765	5.8409	7.4533
4	1.3444	2.1318	2.7764	3.4954	4.6041	5.5976
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18	1.1887	1.7341	2.1009	2.4450	2.8784	3.1966
19	1.1866	1.7291	2.0930	2.4334	2.8609	3.1737
20	1.1848	1.7247	2.0860	2.4231	2.8453	3.1534

From the t table, for $t = 2.8412$, P = α = 0.025; for $t = 3.4995$, P = α = 0.01. (Because it is two-tailed t table, while the question is two-tailed test, we DO NOT need to divide the α value as shown in the second row by 2).

We know $2.8412 < t = 3.015 < 3.4995$

Hence, $0.025 > P > 0.01$.

Because the significant level α = 0.05 which is greater than P.

$P < \alpha$

We reject H_0,

We can conclude that the mean amount of corrosion differs between the two formulations.

MATLAB

Command Window

```
Two tailed test

p =

    0.0195


alpha =

    0.0500

Since p <= alpha, we reject H_0
```