The University of Melbourne

Department of Computing and Information Systems

COMP90051
Statistical Machine Learning
November 2016

Identical examination papers: None

Exam duration: Three hours

Reading time: Fifteen minutes

Length: This paper has 7 pages including this cover page.

Authorised materials: None

Calculators: Not permitted

Instructions to invigilators: Students may not remove any part of the examination paper from the examination room. Students should be supplied with the exam paper and a script book, and with additional script books on request.

Instructions to students: This exam is worth a total of 50 marks and counts for 50% of your final grade. Please answer all questions in the script book provided, starting each question on a new page. Please write your student ID on the front of each script book you use. When you are finished, place the exam paper inside the front cover of the script book.

Library: This paper will be held in the Baillieu Library.

Student id:

Examiner's use only:

Q1	*Q2*	*Q3*	*Q4*	*Q5*	*Q6*	*Q7*	*Q8*	*Q9*	*Q10*	*Q11*	*Q12*	*Q13*

COMP90051 Statistical Machine Learning
Final Exam

Semester 2, 2016

Total marks: 50

Students must attempt all questions

Section A: Short Answer Questions [12 marks]

Answer each of the questions in this section as briefly as possible. Expect to answer each sub-question in a couple of lines.

Question 1: General Machine Learning [12 marks]

1. What is the difference between *marginal* and *conditional probability distributions*? [1 mark]
2. What algorithm is used for efficient *marginalisation* on *probabilistic graphical models* (PGMs)? [1 mark]
3. Name two methods (considered in the class) that can be used to find *maximum likelihood estimate* of *parameters of a probabilistic model*. [1 mark]
4. In what situation do we need to use *expectation maximisation* to train a *PGM* (as opposed to directly doing maximum likelihood)? [1 mark]
5. State *Bayes' rule*, as it applies to Bayesian modelling, and identify the *posterior*, *likelihood*, *prior* and *marginal likelihood (or evidence)*. Define all mathematical symbols introduced. [1 mark]
6. Explain the view of *frequentist supervised learning* as an *optimisation problem*. [1 mark]
7. Give an example (either using a diagram or description in words) of a *dataset* that a *perceptron* cannot *classify* with perfect accuracy. [1 mark]
8. Explain the advantage of using a *sigmoid function* rather than a *step function* as the *activation function* in a *neural network*. [1 mark]
9. Define a *kernel* in the context of *SVM* learning. [1 mark]
10. What are *support vectors*? [1 mark]
11. Name two applications of *dimensionality reduction*. [1 mark]
12. Briefly describe how *active learning* differs from regular *supervised learning*. [1 mark]

Section B: Method Questions [12 marks]

In this section you are asked to demonstrate your conceptual understanding of a subset of the methods that we have studied in this subject.

Question 2: Linear Models [2 marks]

1. What is the difference between *linear regression* and *logistic regression*, in terms of how they make *predictions* (after *training*)? [1 mark]

2. Why does the algorithm for *training logistic regression* involve *gradient descent*, while *linear regression* does not? [1 mark]

Question 3: Classifier Combination [2 marks]

Please write the following in your script book, and there connect each dot on the left with one dot on the right, to create the best possible correspondence

Boosting	○	○	A *semi-supervised learner*	[0.5 marks]
Bagging	○	○	Focus *base classifiers* on hard *examples*	[0.5 marks]
Stacking	○	○	*Bootstrap aggregated ensemble*	[0.5 marks]
Self training	○	○	Layer of *learners* feed into next layer	[0.5 marks]

Question 4: Model selection [3 marks]

This question is on *model selection* for machine learning models.

1. Outline how *heldout (validation) data* is used to select between several models, under the *frequentist* paradigm. [1 mark]

2. With reference to the above, explain why the *training* data cannot be used for *frequentist model selection*. [1 mark]

3. *Bayesian model selection* does not need *heldout data*. Outline how *Bayesian model selection* works, and why *heldout data* is not needed. [1 mark]

Question 5: Gradient Descent [3 marks]

1. Write down the *gradient descent* algorithm for minimising *training error* (also known as *total loss*). You can leave the function in a generic form $L(\theta)$. Explain your notation and any parameters you introduce. [2 marks]

2. Explain the difference between *gradient descent* and *stochastic gradient descent* algorithms. [1 mark]

Question 6: Semi-Supervised Learning [2 marks]

Outline the steps involved in *self training*. [2 marks]

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Section C: Numeric Questions [16 marks]

In this section you are asked to demonstrate your understanding of a subset of the methods that we have studied in this subject, in being able to perform calculations.

Question 7: Artificial Neural Networks [2 marks]

Consider a 2-class classification problem and *perceptron* training algorithm. Assume you have a 3-feature prediction function

$$f(x) = w_1x_1 + w_2x_2 + w_3x_3$$

and current weights $\mathbf{w} = [2, 3, 4]'$. For simplicity assume that there is no bias term. Given an *example* $\mathbf{x} = [-2, 3, 1]'$ and *label* $y = -1$, what are the updated *weights*? Introduce additional *parameter(s)* if required, and specify which values were used for these parameter(s).

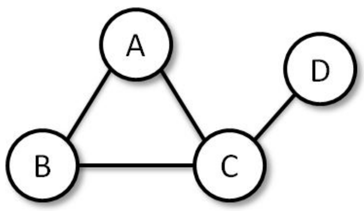
Question 8: Properties of Kernels [3 marks]

1. What is the *kernel trick* and what is the benefit of using it? [1.5 marks]
2. Suppose $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^m$, $k : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is a kernel, $c \in \mathbb{R}$, and $c > 0$. Prove that $ck(\boldsymbol{u}, \boldsymbol{v})$ is a kernel. [1.5 marks]

Question 9: Probabilistic Inference [3 marks]

Consider the following *undirected PGM*

A	B	C	f(A,B,C)
T	T	T	5
T	T	F	3
T	F	T	3
T	F	F	1
F	T	T	3
F	T	F	1
F	F	T	1
F	F	F	0



C	D	g(C,D)
T	T	0
T	F	4
F	T	4
F	F	0

where the truth tables associated with cliques are the model's *potentials* $f(A, B, C)$ and $g(C, D)$ respectively, and as such are not normalised conditional probability tables (as you get for *directed PGMs*).

1. For arbitrary truth values a, b, c, d, write an expression for $\Pr(A = a, B = b, C = c, D = d)$ in terms of $f(a, b, c), g(c, d)$, and normalising constant $Z = \sum_{A,B,C,D} f(A, B, C)g(C, D)$ only (just using these three expressions, without using the truth tables). [1 mark]
2. Now using the tables, calculate the *normalising constant* $Z = \sum_{A,B,C,D} f(A, B, C)g(C, D)$. [1 mark]
3. Calculate $\Pr(A = F, C = F)$. You may leave your answer as a fraction. (If you were unable to compute the normalising constant in the previous part, leave it as Z in your workings here.) [1 mark]

Question 10: Bayesian Posterior Updating [4 marks]

Consider a random variable over $X \in 0, 1, 2, \ldots$ governed by a *Poisson*(λ) distribution. That is, X has probability mass function $p(x|\lambda) = \frac{\lambda^x \exp(-\lambda)}{x!}$. We set as the prior distribution the *Gamma*(α, β) distribution with probability density function $\frac{\lambda^{\alpha-1}\exp(-\lambda\beta)}{Z(\alpha,\beta)}$ where $Z(\alpha, \beta)$ is a normalising constant and $\alpha, \beta > 0$ are constants. The *mode* of the *Gamma*(α, β) is $\frac{\alpha-1}{\beta}$ for $\alpha \geq 1$, and 0 otherwise.

1. Prove that this is a conjugate prior and likelihood, i.e., the posterior $p(\lambda|x, \alpha, \beta)$ has a *Gamma* distribution. [2 marks]

2. After a single observation $X = x$, what is the resulting posterior $p(\lambda|x, \alpha, \beta)$? I.e., what are the new parameters, α', β', of the posterior $Gamma(\alpha', \beta')$ in terms of α, β, x? [1 mark]

3. Explain whether the posterior distribution is more or less informative than the *posterior mode*; consider in your answer the effect of the number of observed data points. [1 mark]

Question 11: Principal Component Analysis [4 marks]

In this question, you will be performing *Principal Component Analysis (PCA)* over the following 2D dataset: $\mathbf{x}_1 = [-1, 0]'$, $\mathbf{x}_2 = [2, 1]'$, $\mathbf{x}_3 = [1, 2]'$, $\mathbf{x}_4 = [0, 3]'$, $\mathbf{x}_5 = [3, 4]'$. Show your working when answering parts 2 to 4.

1. Plot the dataset and draw the two PCA axes. [1 mark]

2. Compute the *variance* along the first PCA dimension. [1 mark]

3. Compute the *variance* along the second PCA dimension. [1 mark]

4. Compute the *covariance* between the two dimensions in the transformed space. [1 mark]

Hints: Recall that sample variance/covariance are normalised by $\frac{1}{n-1}$ where n is the number of points. You should be able to answer this question without using the notion of eigenvalues/eigenvectors. However, if you wish to use eigenvectors note that before PCA eigenvalues of the covariance matrix of the centered data are 1 and 4.

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Section D: Design and Application Questions [10 marks]

In this section you are asked to demonstrate that you have gained a high-level understanding of the methods and algorithms covered in this subject, and can apply that understanding. Expect your answer to each question to be from one third of a page to one full page in length. These questions may require significantly more thought than those in Sections A–C and should be attempted only after having completed the earlier sections.

Question 12: Probabilistic Graphical Models [5 marks]

Your task is to design a probabilistic graphical model (PGM) to capture a real-world scenario. Consider the problem of a student graduating from University and looking for a graduate job, by writing a CV and having a lecturer write them a reference letter. The CV is based on the student's marks for the N subjects they have taken, and each mark is a reflection of the student's general skill. The reference letter is written by a lazy lecturer, basing their assessment purely on the mark obtained by the student in their subject.

1. Design a PGM to capture the above scenario, using as random variables: M_i: mark on subject i, with $i \in [1, N]$; C: quality of their CV; S: skill level; R: recommendation of reference letter. [1 mark]
2. Assuming each variable is binary (i.e., each value is *high* or *low*; or *true* or *false*), state the number of conditional probability tables and the number of free parameters in each table needed to specify the model. [1 mark]
3. CVs are not always written honestly, in that the marks may not always be reported correctly. Propose a change to the model and/or its parameterisation to best capture this situation. [1 mark]
4. Consider the role of a recruiter who wants to employ honest and skilful workers. Their task is to find the skill of the student and whether or not they are honest. What method might they use to determine these values, and what inputs would be required? [1 mark]
5. Finally, how might the conditional probability tables in the model be learned from data? What kind of data might be required, and what technique might be used to fit the parameters? [1 mark]

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Question 13: Multi-target tracking [5 marks]

Consider a *surveillance camera* overlooking a central area in a busy shopping center, and producing a video stream in real time. Hundreds of people are crossing this area throughout the day. Automated *detection and tracking software* is capable of identifying locations of pedestrians in each video frame, as well as of following customers' *trajectories*. Each person is being followed by the software from the moment this customer appears in the *field of view* of the *camera* until disappearance. In each *video frame* t, location for each identified person is represented as a 2D Euclidean point $(x_i(t), y_i(t))$, where i is the unique index for each person. For each person i, a *trajectory* is a sequence $\{(x_i(t_{0,i}), y_i(t_{0,i})), \ldots, (x_i(t_{n,i}), y_i(t_{n,i}))\}$, where $t_{0,i}$ and $t_{n,i}$ denote, respectively, frames of appearance and disappearance for this person. Customers that re-enter the field of view later are treated as new individuals.
One of the major questions that can be addressed with these data is identification of common routes. For example, it may turn out that the majority of customers are passing from south-east to north-west section of the shopping center, while a relatively small group heads towards west instead of north-west. Therefore, your task is to identify groups of similar trajectories.

Answer the following questions and justify your answers. Some of these questions will not necessarily have a single correct answer.

1. What is the main challenge for using *Gaussian Mixture models* to *cluster* the *trajectories*? [1 mark]
2. Can *Euclidean distance* be used for comparing *trajectories*? [1 mark]
3. Design a *function* that measures *similarity* between *trajectories*. [1 mark]
4. How can this *function* be used for *clustering* the *trajectories*? [1 mark]
5. What other information apart from locations $(x_i(t), y_i(t))$ could be used to provide a deeper understanding of movement patterns? [1 mark]

— *End of Exam* —