Q1 (a)

$$M+7=2m$$
$$\ell = L-9$$
$$L+7 = M+1$$

(b) Writing the above 3 equations, and the given equation, as

$$M + 0\times L - 2\times m + 0\times \ell = -7$$
$$0\times M + L + 0\times m - 1\times \ell = 9$$
$$1\times M - L + 0\times m - 1\times \ell = 6$$
$$1\times M - L - 1\times m - 1\times \ell = 0$$

Reading off the coefficient matrix on the LHS, and the vector on the RHS, gives the stated augmented matrix.

(c)
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & -7 \\ 0 & 1 & 0 & -1 & 9 \\ 1 & -1 & 0 & 0 & 6 \\ 1 & -1 & -1 & 1 & 0 \end{array}\right]\begin{array}{l} \\ \\ R_3-R_1 \\ R_4-R_1\end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & -7 \\ 0 & 1 & 0 & -1 & 9 \\ 0 & -1 & 2 & 0 & 13 \\ 0 & -1 & 1 & 1 & 7 \end{array}\right]\begin{array}{l} \\ \\ R_3+R_2 \\ R_4+R_2\end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & -7 \\ 0 & 1 & 0 & -1 & 9 \\ 0 & 0 & 2 & -1 & 22 \\ 0 & 0 & 1 & 0 & 16 \end{array}\right] R_3 \leftrightarrow R_4 \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & -7 \\ 0 & 1 & 0 & -1 & 9 \\ 0 & 0 & 1 & 0 & 16 \\ 0 & 0 & 2 & -1 & 22 \end{array}\right]\begin{array}{l} \\ \\ \\ R_4-2R_3\end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & -7 \\ 0 & 1 & 0 & -1 & 9 \\ 0 & 0 & 1 & 0 & 16 \\ 0 & 0 & 0 & -1 & -10 \end{array}\right]$$

Back substitution gives

$\ell = 10$, $m = 16$

$L = 9 + 10 = 19$

$M = -7 + 2\times 16 = 25$

Q2 (a) Let $B = B_{r \times s}$. Then we have

$$A_{p\times q} B_{r \times s} A_{p \times q}$$

This requires

$$q = r, \quad p = s$$

and so B has size $q \times p$.

(b)
(i) $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Here rows/columns correspond to vertices a, b, c, d in order.

(ii) We need to compute in the top left corner of A^3. Now

$$A^2 = \begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 6 & \cdots \\ \vdots & \end{bmatrix}$$

Hence the number of walks is 6.

(c) $$Z = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 3 \\ 3 & 0 & 3 \end{bmatrix}$$ row swap

$$\sim \begin{bmatrix} 3 & 0 & 3 \\ 3 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} R_2 - R_1 \sim \begin{bmatrix} 3 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus ~~the~~ rank is 1.

3 (a) $$\left(\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&0\\5&1\end{bmatrix}\right)^{-1} = \begin{bmatrix}1&0\\5&1\end{bmatrix}^{-1}\begin{bmatrix}1&2\\0&1\end{bmatrix}^{-1}$$

$$= \begin{bmatrix}1&0\\-5&1\end{bmatrix}\begin{bmatrix}1&-2\\0&1\end{bmatrix} = \begin{bmatrix}1&-2\\-5&11\end{bmatrix}$$

(b) To decode the message, we compute

$$\begin{bmatrix}1&-2\\-5&11\end{bmatrix}\begin{bmatrix}24&51\\11&24\end{bmatrix} = \begin{bmatrix}2&3\\1&9\end{bmatrix}$$

Using the correspondence $A \leftrightarrow 1$, $B \leftrightarrow 2$ etc. we read off that the message was

BACI

(c) First HUGS $\longleftrightarrow \begin{bmatrix}8&7\\21&19\end{bmatrix}$

The sent message is then

$$\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&0\\5&1\end{bmatrix}\begin{bmatrix}8&7\\21&19\end{bmatrix}$$

$$= \begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}8&7\\61&54\end{bmatrix} = \begin{bmatrix}130&115\\61&54\end{bmatrix}$$

so the received message would be

130, 61, 115, 54.

4(a) (i) $\underset{\sim}{x} = t(1,0,-1) + s(1,1,0) \qquad t, s \in \mathbb{R}.$

(ii) For the Cartesian equation, we compute a normal

$$\underset{\sim}{n} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \underset{\sim}{i}\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} - \underset{\sim}{j}\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} + \underset{\sim}{k}\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$$

This tells us that the Cartesian eq^{n} is

$$\underset{\sim}{x} \cdot \underset{\sim}{n} = 0 \qquad \Rightarrow \qquad x - y + z = 0.$$

4(b)(i) area parallelogram

$$= (\text{base}) \times (\text{height})$$

$$= \|\underset{\sim}{a}\| \, \|\underset{\sim}{b}\| \sin\theta$$

(ii) We first translate the parallelogram so that one corner is at the origin.

Subtracting $(1,1,1)$ gives

$$(0,0,0), \quad (1,1,1), \quad (0,1,1), \quad (1,2,2).$$

This then is the setting of the above with $\underset{\sim}{a} = (1,1,1)$ and $\underset{\sim}{b} = (0,1,1)$. We have

$$\|\underset{\sim}{a}\| = \sqrt{3}, \quad \|\underset{\sim}{b}\| = \sqrt{2}, \quad \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1 - \left(\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\|\underset{\sim}{a}\|\,\|\underset{\sim}{b}\|}\right)^2}$$

$$= \sqrt{1 - \left(\frac{2}{\sqrt{6}}\right)^2} = \sqrt{\frac{1}{3}}$$

some student may use the cross product formula

Thus area parallelogram $= \sqrt{3}\sqrt{2}\sqrt{\frac{1}{3}} = \sqrt{2}$

Q5 (a)

- dimension zero. This corresponds to the single point $(0,0)$ in $\mathbb{R}^2$, which is the origin.
- dimension one. Such subspaces correspond to lines through the origin.
- dimension two. This case corresponds to all of $\mathbb{R}^2$.

(b) (i) A general polynomial in P_2 is of the form

$$p(x) = a + bx + cx^2$$

Reversing the order of the coefficients gives

$$q(x) = c + bx + ax^2$$

For $p(x)$ to be palindromic, we require $p(x) = q(x)$ and thus $a = c$, giving that a general palindromic polynomial is of the form

$$a + bx + ax^2.$$

(ii) The correspondence between members of P_2 and vectors $(a,b,c) \in \mathbb{R}^3$ is

$$a + bx + cx^2 \longleftrightarrow (a, b, c)$$

Thus

$$a + bx + ax^2 \longleftrightarrow (a, b, a)$$

It follows that

set of palindromic polynomials in P_2 $\longleftrightarrow \{(a,b,a) \in \mathbb{R}^3\}$

$$= \text{Span}\{(1,0,1), (0,1,0)\}.$$

(iii) Let $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$ be two general vectors in S, so that

$$\underset{\sim}{v}_1 = (a_1, b_1, a_1) \qquad a_1, b_1 \in \mathbb{R}$$
$$\underset{\sim}{v}_2 = (a_2, b_2, a_2) \qquad a_2, b_2 \in \mathbb{R}.$$

We then have

$$\begin{aligned}\underset{\sim}{v}_1 + \underset{\sim}{v}_2 &= (a_1, b_1, a_1) + (a_2, b_2, a_2) \\ &= (a_1 + a_2, b_1 + b_2, a_1 + a_2) \\ &= (a_3, b_3, a_3) \qquad a_3 = a_1 + a_2 \in \mathbb{R} \\ &\in S \qquad b_3 = b_1 + b_2 \in \mathbb{R}\end{aligned}$$

Hence S is closed under vector addition.

Q6. (a) 3. This is the number of leading entries in RE form of A.

(b) 2. This is the number of columns in A which do not have leading entries.

(c) row space A $=$ Span$\{(1,0,2,0,0), (0,1,1,0,1), (0,0,0,1,0)\}$

(d) $\underset{\sim}{b} = 7\underset{\sim}{a}_1 - 2\underset{\sim}{a}_2 - \underset{\sim}{a}_3$

(e) Let $\underset{\sim}{x} = (x_1, x_2, x_3, x_4, x_5)$. There is no leading entry for x_3 or x_5, so we set $x_3 = s$, $x_5 = t$, $s, t \in \mathbb{R}$

Back substitution then gives

$$x_4 = -1$$

$$x_2 = -x_3 - x_5 - 2 = -s - t - 2$$

$$x_1 = -2x_3 + 7 = -2s + 7$$

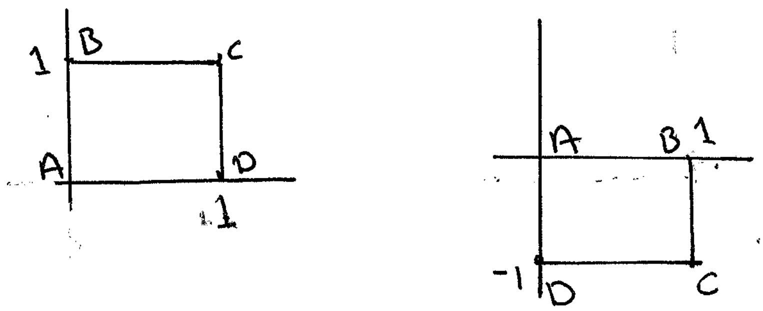
Hence the general solution is

$$\{(-2s+7, -s-t-2, s, -1, t), \quad s, t \in \mathbb{R}\}.$$

(f) The equation $A\underset{\sim}{x} = \underset{\sim}{c}$ will not have a solution. The final line of the RE form of $[A \mid \underline{c}]$ will read $[0\ 0\ 0\ 0\ 0 \mid \alpha]$ for α nonzero, or equivalently $0 = \alpha$, which is inconsistent.

Q7 (a) (i) $A_T = \left[T\begin{bmatrix}1\\0\end{bmatrix} \; T\begin{bmatrix}0\\1\end{bmatrix} \right] = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$

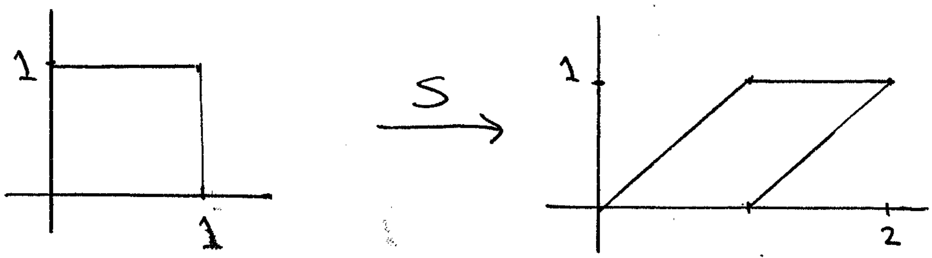
(ii)



(iii) T rotates by $-\pi/2$

Since an eigenspace is a line in $\mathbb{R}^2$ left unchanged by T, and a rotation by $-\pi/2$ leaves no line unchanged, T does not have any eigenspaces.

Q7 (b) (i)



(ii) We must have $|\det A_s| = 1$ for the area to be unchanged

(iii) This corresponds to the identity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, since a shear by -1 unit in the x-direction is the inverse of a shear by $+1$ unit in the x-direction.

8 (a) T is the orthogonal projection onto the plane spanned by $\{(1,0,1), (-1,1,1)\}$.

(b)
$$T\underset{\sim}{i} = \frac{1}{2}\left((1,0,0)\cdot(1,0,1)\right)(1,0,1) + \frac{1}{3}\left((1,0,0)\cdot(-1,1,1)\right)(-1,1,1)$$
$$= \frac{1}{2}(1,0,1) - \frac{1}{3}(-1,1,1) = \left(\frac{5}{6}, -\frac{1}{3}, \frac{1}{6}\right)$$

$$T\underset{\sim}{j} = \frac{1}{2}\left((0,1,0)\cdot(1,0,1)\right)(1,0,1) + \frac{1}{3}\left((0,1,0)\cdot(-1,1,1)\right)(-1,1,1)$$
$$= \frac{1}{3}(-1,1,1)$$

$$T\underset{\sim}{k} = \frac{1}{2}\left((0,0,1)\cdot(1,0,1)\right)(1,0,1) + \frac{1}{3}\left((0,0,1)\cdot(-1,1,1)\right)(-1,1,1)$$
$$= \frac{1}{2}(1,0,1) + \frac{1}{3}(-1,1,1) = \left(\frac{1}{6}, \frac{1}{3}, \frac{5}{6}\right)$$

Thus $A_T = [T\underset{\sim}{i} \quad T\underset{\sim}{j} \quad T\underset{\sim}{k}]$

$$= \begin{bmatrix} \frac{5}{6} & -1/3 & 1/6 \\ -1/3 & 1/3 & 1/3 \\ 1/6 & 1/3 & 5/6 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & -2 & 1 \\ -2 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

(c) From the interpretation as an orthogonal projection

$$\operatorname{Im} T = \operatorname{Span}\{(1,0,1),\ (-1,1,1)\}$$

Hence a basis is

$$\{(1,0,1),\ (-1,1,1)\}$$

students may also compute the column space of A_T.

(d) Since $(1,0,1)\times(-1,1,1)$ is orthogonal to both $(1,0,1)$ and $(-1,1,1)$ we have $T(1,0,1)\times(-1,1,1) = \underset{\sim}{0}$ and thus $(1,0,1)\times(-1,1,1) \in \operatorname{Ker} T$.

9 (a) We have $[\underset{\sim}{x}]_B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. But

$$[\underset{\sim}{x}]_C = P_{C,B}[\underset{\sim}{x}]_B = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Thus $\underset{\sim}{x} = \frac{1}{2}\underset{\sim}{c}_3$

(b) $P_{B,C} = P_{C,B}^{-1}$. To compute the inverse

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ R_2 + R_3 \\ \\ \end{matrix} \sim \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} R_1 + R_2 \\ \\ \\ \end{matrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Hence $P_{B,C} = 2\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $P_{S,C} = [\underset{\sim}{c}_1 \ \underset{\sim}{c}_2 \ \underset{\sim}{c}_3] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

(d) $[\underset{\sim}{y}]_S = P_{S,C} P_{C,B} [\underset{\sim}{y}]_B$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

Hence $\underset{\sim}{y} = \frac{1}{2}(1, 0, 3)$

10 (a) We have $u_3 = -u_1 - u_2$
$v_3 = -v_1 - v_2$

Thus

$$\langle (u_1,u_2),(v_1,v_2)\rangle = u_1v_1 + u_2v_2 + (-u_1-u_2)(-v_1-v_2)$$
$$= u_1v_1 + u_2v_2 + (u_1v_1 + u_1v_2 + u_2v_1 + u_2v_2)$$
$$= 2u_1v_1 + u_1v_2 + u_2v_1 + 2u_2v_2$$

(b) We read off that

$$\langle (u_1,u_2),(v_1,v_2)\rangle = [u_1 \ u_2]\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

(c) Let the 2x2 matrix be denoted A.

We require that

- A be symmetric
- the eigenvalues of A be positive.

(d) We have

$$P_{(1,1)}((2,1)) = \frac{\langle (1,1),(1,2)\rangle}{\|(1,1)\|^2}(1,1)$$

Here

$$\|(1,1)\| = \sqrt{\langle (1,1),(1,1)\rangle} = \sqrt{2+1+1+2} = \sqrt{6}$$

$$\langle (1,1),(1,2)\rangle = 2\times 1\times 1 + 1\times 2 + 1\times 1 + 2\times 1\times 2$$
$$= 9$$

Hence

$$P_{(1,1)}((2,1)) = \frac{9}{6}(1,1) = \frac{3}{2}(1,1).$$

11 (a) We want to solve

$$A\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} \qquad \text{where} \qquad A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$$

using least squares. This means we must solve

$$A^T A\begin{bmatrix} a \\ b \end{bmatrix} = A^T\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$$

Now $$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A^T\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

Thus we have

$$\begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

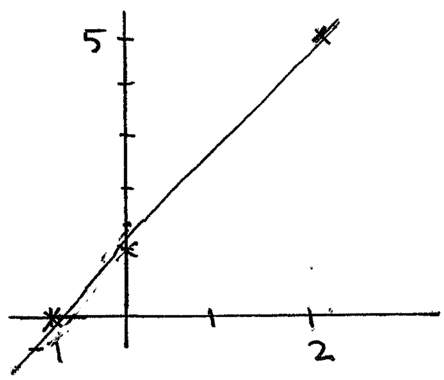
$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 10 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} 20 \\ 24 \end{bmatrix} = \begin{bmatrix} 10/7 \\ 12/7 \end{bmatrix}$$

Hence

$$R = \frac{10}{7} + \frac{12}{7}T$$

(b)



(c) When $T = 7$

$$R = \frac{10}{7} + \frac{12}{7} \times 7 \simeq 13$$

chirps/minute.

12(a) We have for the eigenvalues λ

$$\begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(-2-\lambda) + 4 = 0 \quad \Rightarrow \lambda^2 - \lambda - 2 = 0 \quad \Rightarrow (\lambda-2)(\lambda+1) = 0$$

$$\Rightarrow \lambda = 2 \text{ or } -1.$$

(b) For the eigenspaces

$\underline{\lambda = 2}$

Seek $\underset{\sim}{v}$ such that $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}\underset{\sim}{v} = \underset{\sim}{0}$.

Have $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \underset{R_2 - 2R_1}{\sim} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

With $\underset{\sim}{v} = (v_1, v_2)$ there is no leading entry for v_2, so we set $v_2 = t$, $t \in \mathbb{R}$. Back substitution implies $v_1 = 2t$, and so the eigenspace is $\{t(2,1),\ t \in \mathbb{R}\}$

$\underline{\lambda = -1}$

Seek $\underset{\sim}{v}$ such that $\begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix} \underset{\sim}{v} = \underset{\sim}{0}$.

Have $\begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix}_{R_2 - \frac{1}{2}R_1} \sim \begin{bmatrix} 4 & -2 \\ 0 & 0 \end{bmatrix}$

With $\underset{\sim}{v} = (v_1, v_2)$ there is no leading entry for v_2, so we set $v_2 = t$, $t \in \mathbb{R}$. Back substitution then gives $v_1 = \frac{t}{2}$, implying the eigenspace $\{ t(\frac{1}{2}, 1),\ t \in \mathbb{R} \}$.

(c) A is diagonalizable since the eigenvectors implied by the eigenspaces, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$, are linearly independent.

(d) We have $\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1}}_{P^{-1}}$

Thus $\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}^5 = PD^5P^{-1}$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2^5 & 0 \\ 0 & (-1)^5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 64 & -1 \\ 32 & -2 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 129 & -66 \\ 66 & -36 \end{bmatrix} = \begin{bmatrix} 43 & -22 \\ 22 & -12 \end{bmatrix}.$$