

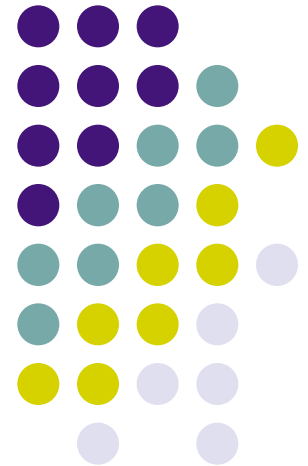
# COMP20003

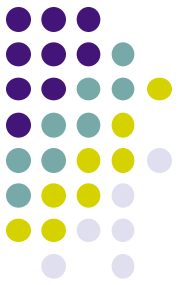
## Algorithms and Data Structures

### Traversing Trees and Graphs

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Semester 2

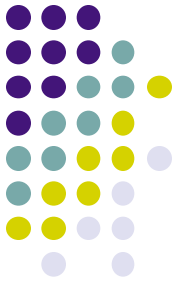




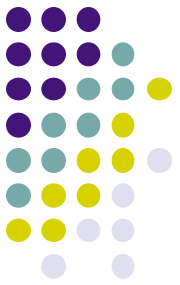
# Traversal

- Traverse: to pass or move over, along, or through.
- Tree traversal: the process of visiting (examining or updating) each node exactly once, in a systematic way.
- Graph traversal: the process of visiting all the nodes in a graph.
- Tree traversal is a special case of graph traversal.

# Traversal

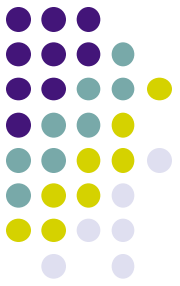


# Graph traversal vs. Tree traversal



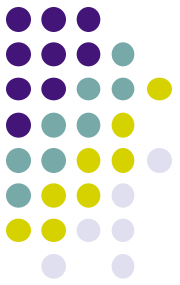
- Graph traversal: complications due to:
  - Possible cycles.
  - Not necessarily connected.

# Starting with trees: bst dfs traversal depth-first search



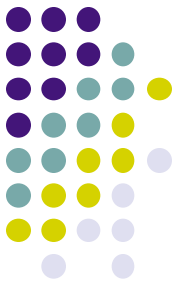
- Depth-first tree search can be done as:
  - In-order
  - Pre-order
  - Post-order

# Recursive in-order search: binary tree



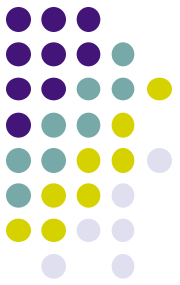
```
void
inorder (node_t    *t)
{
    if (t==NULL)  return () ;
    inorder (t->left) ;
    visit (t) ;    /* e.g. print value */
    inorder (t->right) ;
}
```

# Recursive pre-order search: binary tree



```
void
preorder (node_t    *t)
{
    if (t==NULL)  return();
    visit(t);     /* visit first */
    preorder(t->left);
    preorder(t->right);
}
```

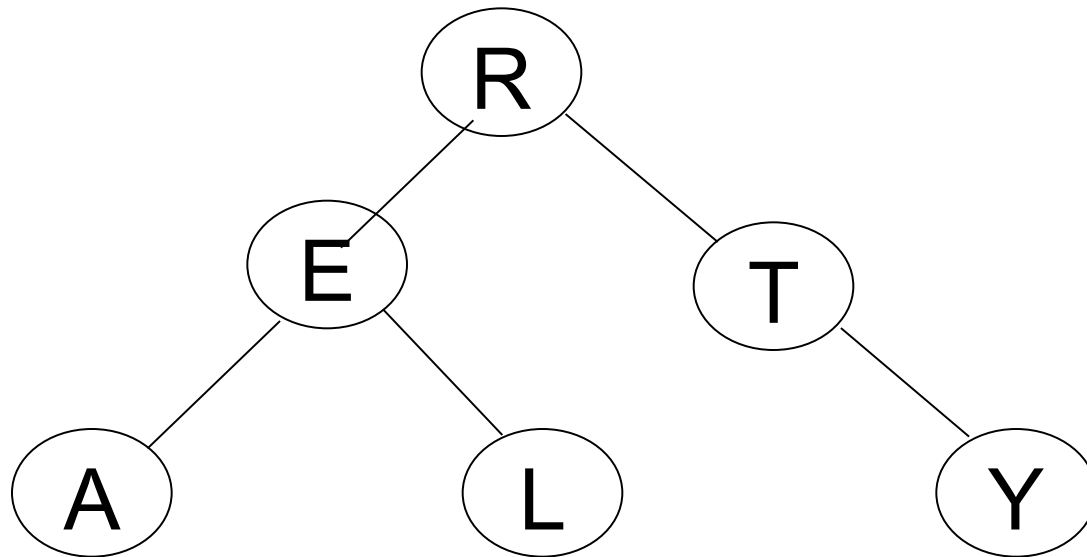
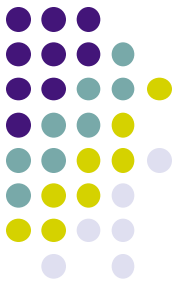
# Non-recursive pre-order search: DFS - explicit stack



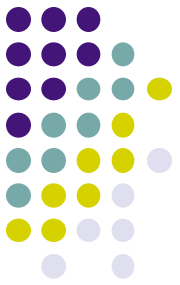
```
void
preorder(stack_t  *st,node_t  *t)
{
    push(st,t) ;
    while(!stackempty(st))
    {
        t= pop(st) ; visit(t) ;
        if(t->r != NULL) push(st,t->r) ;
        if(t->l != NULL) push(st,t->l) ;
    }
}
/* note: stack contains pointers into the tree */
```



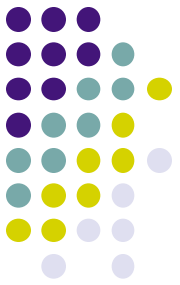
# Depth-first search vs. breadth-first search



# Breadth-first tree search: use a queue



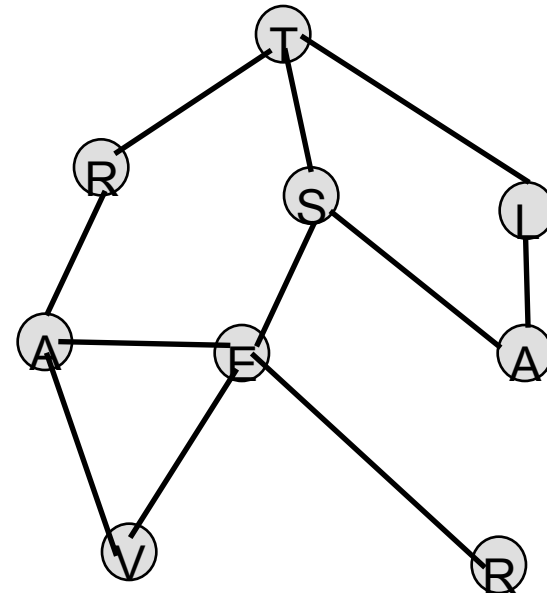
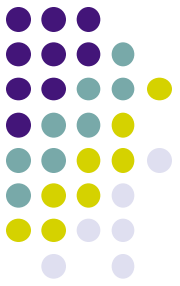
```
void
preorder(queue *Q,node_t *t)
{
    enQ(Q,t);
    while(!emptyQ(Q))
    {
        t = deQ(Q); visit(t);
        if(t->l != NULL) enQ(Q,t->l);
        if(t->r != NULL) enQ(Q,t->r);
    }
}
/* note: queue contains pointers into the tree */
```



# Tree traversal: assumptions

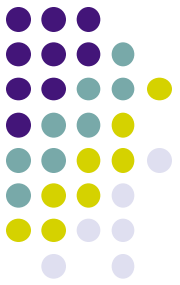
- Assumes every node is reachable from the root.
- Assumes every node has only one parent, can only be visited once.
- Graph traversal needs to make sure that:
  - Every node is reached.
  - Every node is visited only once.

# Traversing a connected graph (depth first)

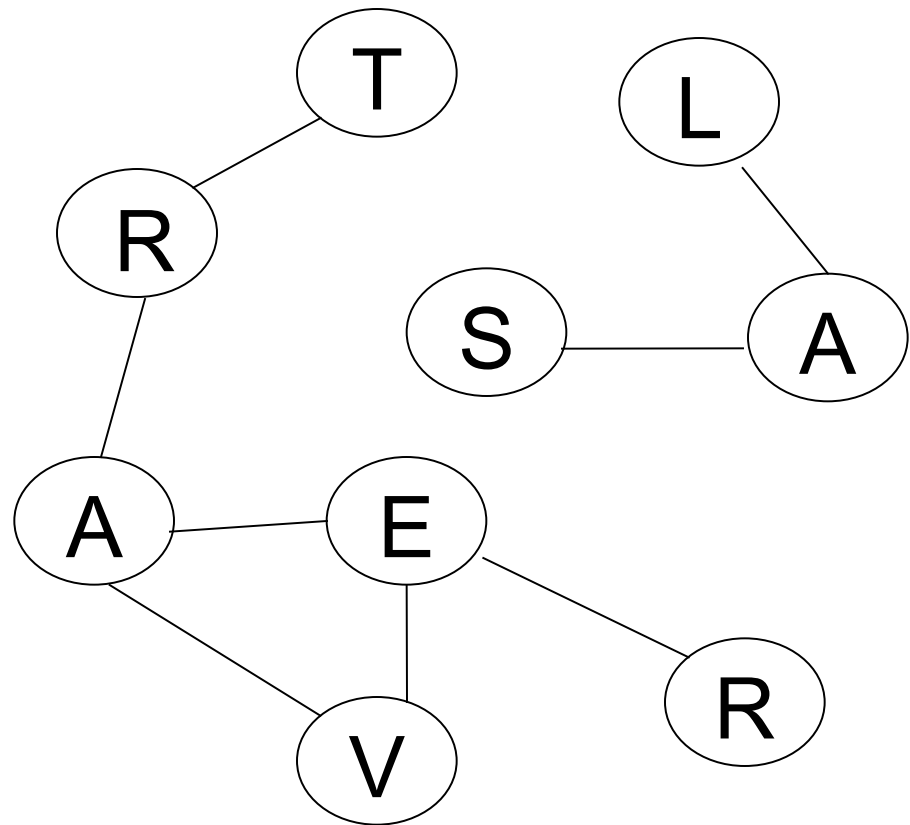


- Need to mark nodes as visited.

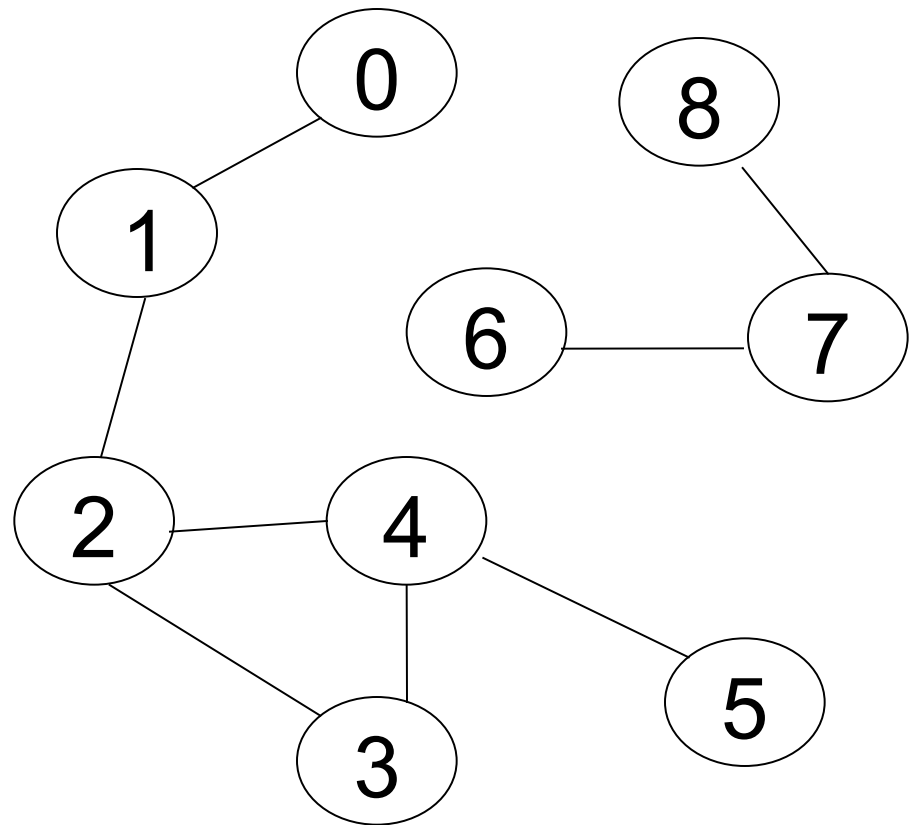
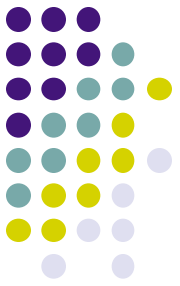
# Traversing an **un**connected graph (depth first)



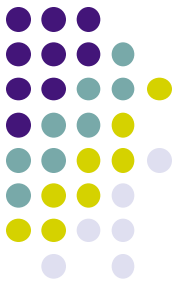
- Need to traverse each connected component.
- Still need to mark nodes as visited.



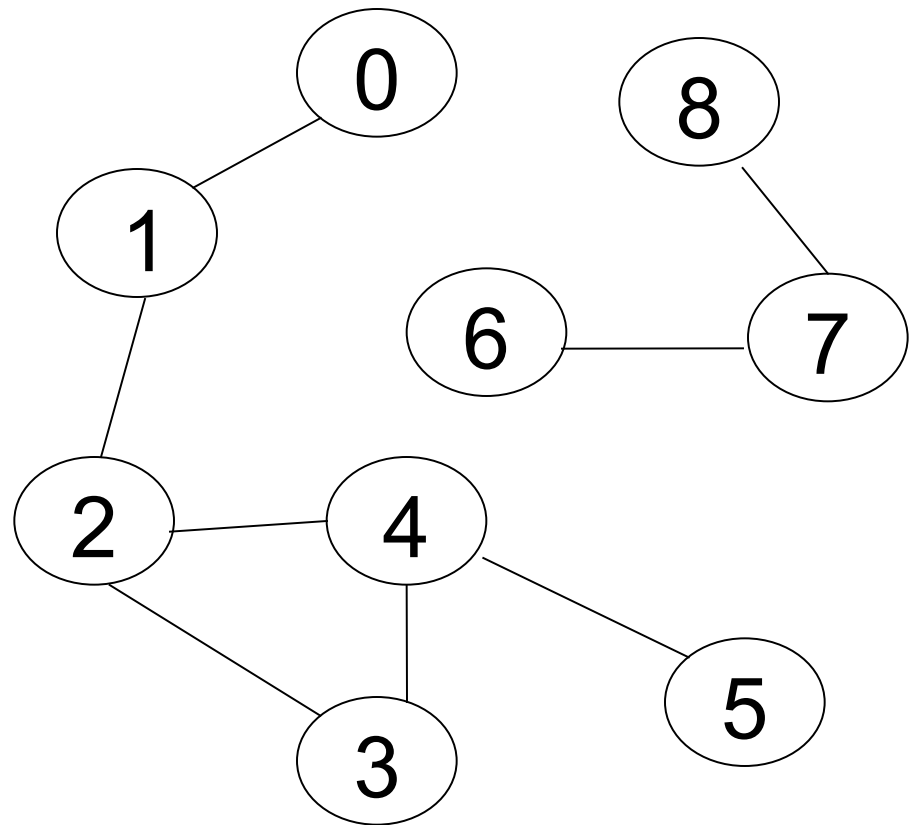
# Traversing an **un**connected graph (depth first)



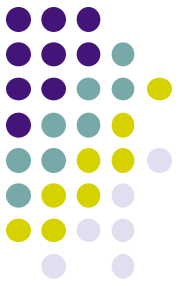
# Traversing an unconnected graph (depth first)



```
int  order=0;
```

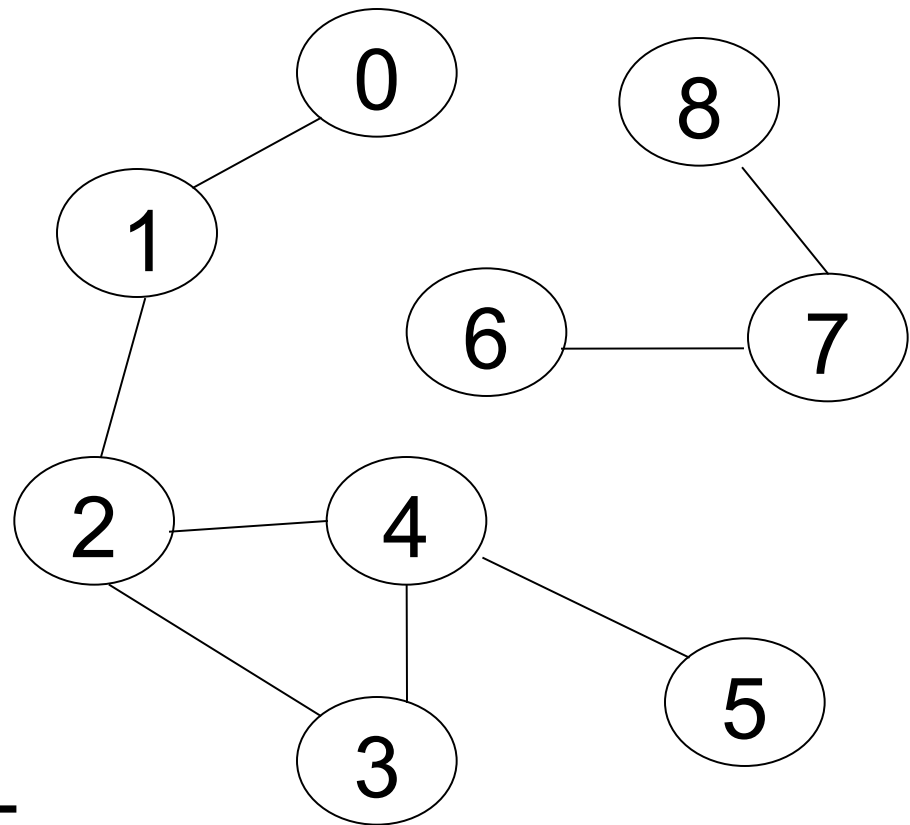


# Traversing an unconnected graph (depth first)



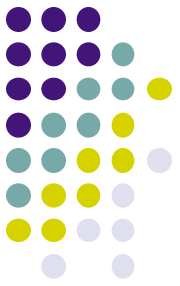
Matrix

	0	1	2	3	4	5	6	7	8
0		T							
1	T		T						
2		T		T	T				
3			T		T				
4		T	T			T			
5						T			
6							T		
7								T	
8									T





# Traversing an unconnected graph (depth first)



Adjacency List

0 → 1

1 → 2

2 → 3 → 4 → 1

3 → 2 → 4

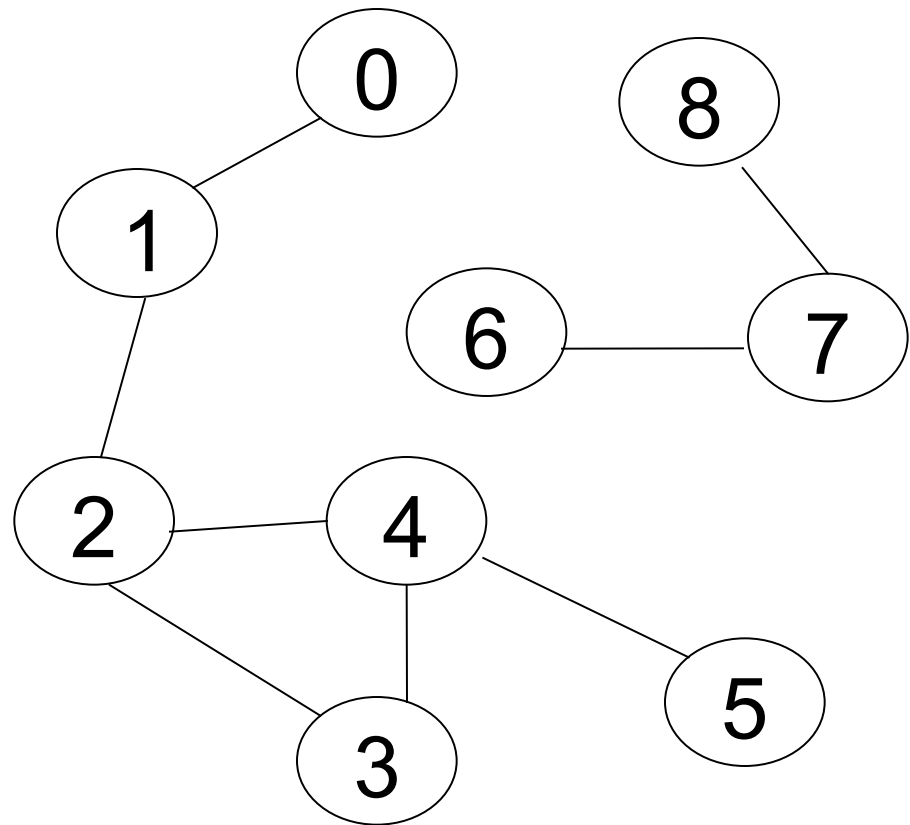
4 → 3 → 2 → 5

5 → 4

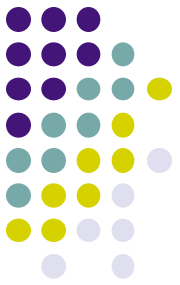
6 → 7

7 → 8 → 6

8 → 7



# visited[] array: keeping track of what's been done



```
/* invoke an array to track whether or not a  
node has already been visited */
```

```
int  visited[V];
```

```
listdfs()
```

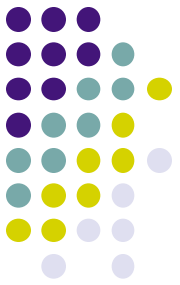
```
{
```

```
    int k;
```

```
    /* initialize - no nodes yet visited */
```

```
    for(k=0;k<V;k++)  visited[k]= 0;
```

```
}
```

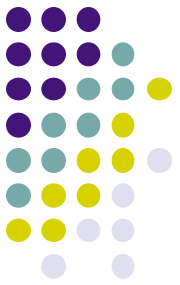


# Adjacency list node

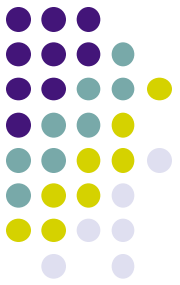
```
/* adjacency list is an array of pointers to  
nodes; node is struct with value (nodeID)  
and next ptr*/
```

```
struct node{  
    int  value;  
    struct node *next;  
};  
struct node    *adj[V];
```

# Visiting nodes; updating the `visited[]` array



```
int  visited[V];
int  order=0;  /*keeps track of the order in
                which nodes are visited */
visit(int k)
{
    visited[k] = ++order;
    for(t=adj[k]; t!=NULL; t=t->next)
        if(!visited[t->v]) visit(t->v);
}
```



# Example dfs graph traversal

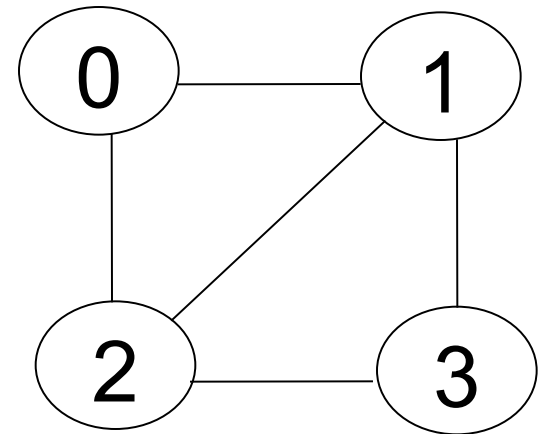
Adjacency List

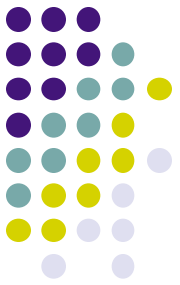
$0 \rightarrow 1 \rightarrow 2$

$1 \rightarrow 0 \rightarrow 2 \rightarrow 3$

$2 \rightarrow 0 \rightarrow 1 \rightarrow 3$

$3 \rightarrow 1 \rightarrow 2$





# Example dfs graph traversal

Adjacency List

0 → 6

1 → 4 → 7

2 → 8

3 → 5 → 8

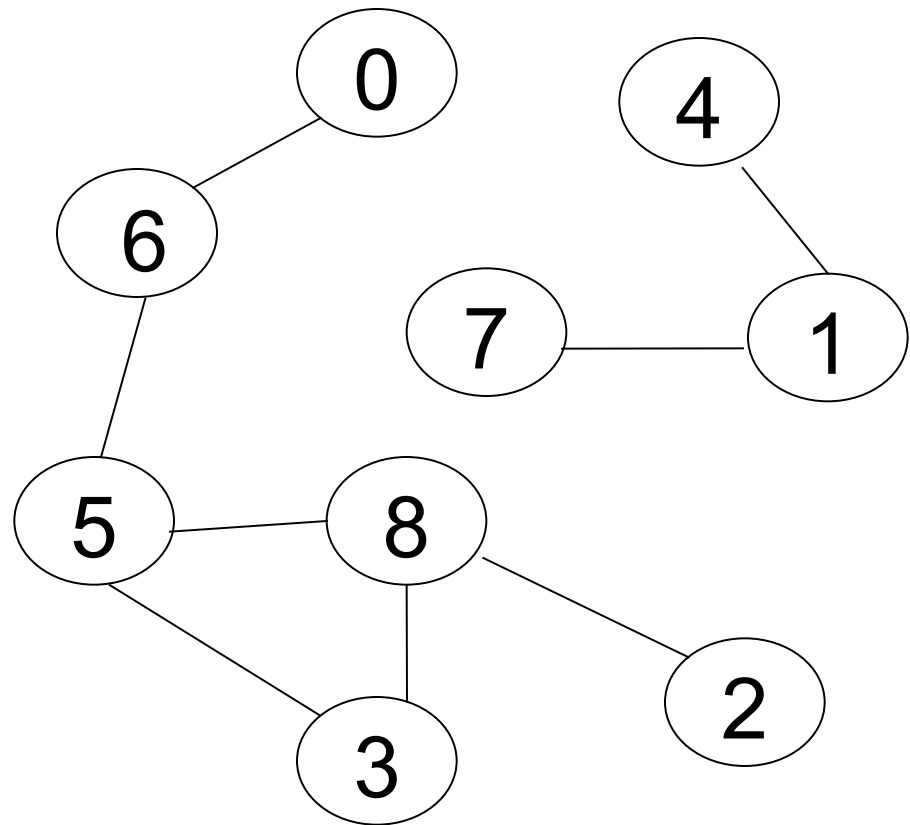
4 → 1

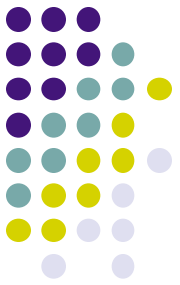
5 → 3 → 6 → 8

6 → 0 → 5

7 → 1

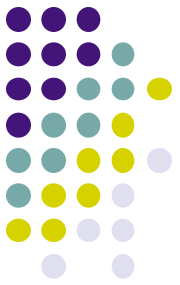
8 → 2 → 3 → 5





# Graph dfs: Analysis

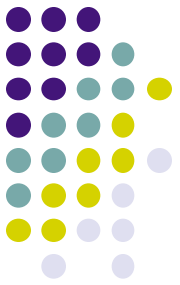
- Fill in the `visited[]` array:
- Examine (at most) each edge twice:



# Graph dfs: Analysis

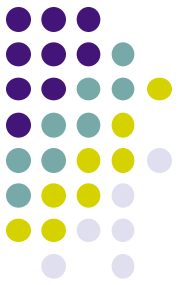
- Fill in the `visited[]` array:
  - $|V|$
- Examine (at most) each edge twice:
  - $|E|$
- Overall:  $|V| + |E|$





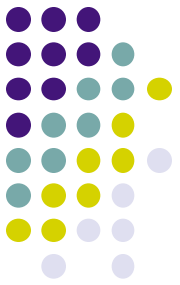
# Graph *breadth*-first search

- Again, modify the tree bfs, to make sure that:
  - 
  -



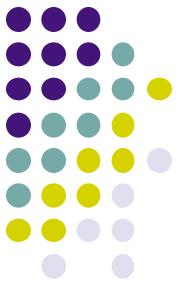
# Graph *breadth*-first search

- Again, modify the analogous tree search, to make sure that:
  - Every node is visited, even if the graph is not connected, and
  - Every node is visited only once



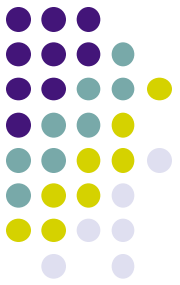
# bfs visit()

```
int visited[V]; int order=0;
visit(int k){
    struct node *t;
    enQ(Q,k);
    while(!Qempty(Q)){
        k = deQ(Q);
        if(!visited[k]){
            visited[k]=++order;
            for(t=adj[k];t!=NULL; t=t->next)
                if(!visited[t->num]) enQ(Q,t->num);
        }
    }
}
```



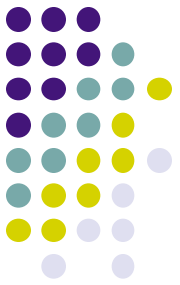
# Breadth-first graph search

```
int  visited[V];  
listbfs()  
{  
    int k;  
    for(k=0;k<V;k++)  visited[k]= 0;  
    for(k=0;k<V;k++)  
        if(!visited[k])  visit(k);  
}
```



# Weighted graphs

- So far, we have used somewhat arbitrary ordering of the nodes (determined by position in adjacency list or matrix).
- For weighted graphs, it might be nice to get the nodes out in order of distance.



# Example weighted graph bfs

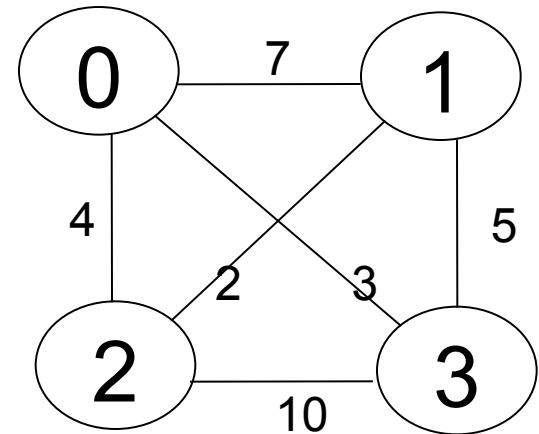
Adjacency List

$0 \rightarrow 1 \rightarrow 2$

$1 \rightarrow 0 \rightarrow 2 \rightarrow 3$

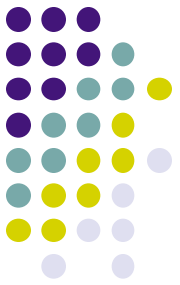
$2 \rightarrow 0 \rightarrow 1 \rightarrow 3$

$3 \rightarrow 1 \rightarrow 2$



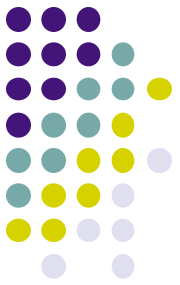
Previous visit order from node 0:

But if these are restaurants and nightclubs,  
and we want to go to a nearby nightclub  
from restaurant 0...



# Priority Queues

- We can still use a queue, but we make that a priority queue (PQ).



- Chapter 5, Skiena book