# MAST10007 Linear Algebra
# MATLAB Test

**Test duration: 45 minutes**

**This paper has 6 pages (+ blank page for intermediate working)**

Please complete ***all*** the following details.

**Name:** ...............................................

**Student Number:** ...............................................

**Tutor's Name:** ...............................................

**Lab Time:** ...............................................

**Instructions to Students:**

This test is designed to evaluate your comprehension of concepts in linear algebra, and your ability to calculate efficiently with the aid of MATLAB. Some questions test your understanding of the material covered in lectures, and do not necessarily require MATLAB. No partial credit is given, so please carefully check anything typed into MATLAB, and check the output of programs used.

Any rough working must be done on this paper (last page in blank for this purpose), but only the final answer is marked.

The number of marks for each question is indicated and the total number of marks is 20.

Some MATLAB commands:

- rref($A$) gives the fully reduced row echelon form of $A$
- $A'$ is the transpose of the matrix $A$
- det($A$) gives the determinant of the matrix $A$
- eye($n$) gives the identity matrix of size $n \times n$
- inv($A$) gives the inverse of $A$
- ones($p, q$) gives the $p \times q$ matrix of all 1's
- zeros($p, q$) gives the $p \times q$ matrix of all 0's
- diag($v$) gives the diagonal matrix with diagonal corresponding to the vector $v$ written in Matlab notation
- The command v = B(:,3) selects the third column of B, for example
- dot($u, v$) gives the dot product of the vectors $u$ and $v$.

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

1. You are given the following table of data:

| x | y |
|---|---|
| 0 | 1.1 |
| 1 | 0.5 |
| 3 | 2.2 |
| 5 | 4.6 |
| 6 | 5.9 |

(a) With

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 5 \\ 1 & 6 \end{bmatrix}$$

compute $A^T A$.

(b) With $A$ as above and $\mathbf{y} = \begin{bmatrix} 1.1 \\ 0.5 \\ 2.2 \\ 4.6 \\ 5.9 \end{bmatrix}$ calculate $A^T \mathbf{y}$.

(c) Calculate the least square line of best fit to the table of data.

(d) From your answer to (c), give an estimate to the value of $y$ when $x = 4$.

[4 marks]

2. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 3 & 1 \\ -1 & 1 & -2 & 4 & 0 \\ -2 & 3 & -5 & 15 & 1 \\ -6 & 11 & 11 & 3 & 1 \\ -2 & 5 & 7 & 1 & 1 \end{bmatrix}$$

(a) Give a basis for the row space in terms of the original rows of $A$.

(b) Write the 3rd row of $A$ as a linear combination of the first two rows.

(c) Calculate a basis for the solution space of $A^T$.

(d) Consider the matrix

$$B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

where the entries 0, 1 are now considered as elements of the finite field $\mathbb{Z}_2$. Using mod 2 arithmetic, circle any vector below belonging to the solution space of $B$

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

[4 marks]

3. Consider the matrix

$$
X = \begin{bmatrix}
16 & 0 & 0 & 0 & 0 & 7 & 7 & 7 & 7 & 7 \\
0 & 16 & 0 & 0 & 0 & 7 & 6 & 7 & 7 & 7 \\
0 & 0 & 16 & 0 & 0 & 7 & 7 & 5 & 7 & 7 \\
0 & 0 & 0 & 16 & 0 & 7 & 7 & 7 & 4 & 7 \\
0 & 0 & 0 & 0 & 16 & 7 & 7 & 7 & 7 & 3 \\
16 & 16 & 16 & 16 & 16 & 0 & 0 & 0 & 0 & 0 \\
16 & 16 & 16 & 16 & 16 & 0 & -1 & 0 & 0 & 0 \\
16 & 16 & 16 & 16 & 16 & 0 & 0 & -2 & 0 & 0 \\
16 & 16 & 16 & 16 & 16 & 0 & 0 & 0 & -3 & 0 \\
16 & 16 & 16 & 16 & 16 & 0 & 0 & 0 & 0 & -4
\end{bmatrix}
$$

(a) Calculate the dot product of the 6th row and the 10th column of $A$.

(b) Let $Y = X^{-1}$. Calculate $\det(\frac{1}{4}Y^{-1})$, giving your answer as an integer.

(c) Calculate the unit vectors corresponding to the 2nd and 3rd columns.

(d) Calculate the orthonormal projection of the vector corresponding to the first column onto the subspace spanned by the second and third columns.

[4 marks]

4. A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ has a standard matrix representation

$$A_T = \begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \end{bmatrix}$$

(a) Write down the image of the three unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ under $T$.

(b) Draw on a graph the image under the linear transformation $T$ of the unit cube formed by the three unit vectors. Your graph should consist of the image of the corners, and the edges connecting the corners.

(c) Consider the basis of $\mathbb{R}^3$ given by $B = \{(1,0,0), (0,1,0), (2,2,1)\}$. Calculate $[T]_{S,B}$.

(d) Write the kernel of $T$ as a span.

[4 marks]

5. You are given that the transition matrix $P_{C,B}$ from a basis $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4\}$ to a basis $C = \{\mathbf{c}_1\, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4\}$ is

$$P_{C,B} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Calculate the co-ordinates of $2\mathbf{b}_2 - \mathbf{b}_4$ in the basis of $C$.

(b) Calculate the transition matrix $P_{B,C}$.

(c) Suppose

$$\mathbf{b}_1 = (2, 1, 0, 0), \quad \mathbf{b}_2 = (0, 2, 1, 0), \quad \mathbf{b}_3 = (0, 0, 2, 1), \quad \mathbf{b}_1 = (0, 0, 0, -1).$$

Calculate $P_{S,C}$.

(d) From your answer to (c), write down $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4$.

[4 marks]