MAST10007 Assignment 1 - Solutions

Q1 (a) We first rewrite the four equations so that the unknowns are on the LHS, all in the same order, and the lone constants are on the RHS. Thus we have

$$\begin{aligned} A + B \qquad\qquad &= 91 \\ A \qquad\qquad -3b &= 0 \\ -2B + a \qquad &= 0 \\ A - B - a + b &= 0 \end{aligned}$$

and from this the augmented matrix form is read off to be

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 91 \\ 1 & 0 & 0 & -3 & 0 \\ 0 & -2 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 \end{array}\right]$$

(b) We apply appropriate row operations:

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 91 \\ 1 & 0 & 0 & -3 & 0 \\ 0 & -2 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ \\ R_4 - R_1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 91 \\ 0 & -1 & 0 & -3 & -91 \\ 0 & -2 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & -91 \end{array}\right] \begin{array}{l} \\ \\ R_3 - 2R_2 \\ R_4 - 2R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 91 \\ 0 & -1 & 0 & -3 & -91 \\ 0 & 0 & 1 & 6 & 182 \\ 0 & 0 & -1 & 7 & 91 \end{array}\right] \begin{array}{l} \\ \\ \\ R_4 + R_3 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 91 \\ 0 & -1 & 0 & -3 & -91 \\ 0 & 0 & 1 & 6 & 182 \\ 0 & 0 & 0 & 13 & 273 \end{array}\right]$$

(c) Now applying back substitution

$13b = 273 \Rightarrow b = 21$

$a + 6b = 182 \Rightarrow a = 182 - 126 = 56$

$-B - 3b = -91 \Rightarrow B = 91 - 63 = 28$

$A + B = 91 \Rightarrow A = 91 - 28 = 63.$

Q2 (a) Denoting the given matrix by A, we must form $[A \mid I]$, and calculate its fully reduced row echelon form. We have

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 & 1 & 0 \\ 4 & 4 & 4 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 & -3 & 0 & 1 & 0 \\ 0 & 4 & 4 & 4 & -4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 - \frac{3}{2}R_2 \\ R_4 - 2R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 4 & 4 & 0 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ R_4 - \frac{4}{3}R_3 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & -\frac{4}{3} & 1 \end{array}\right] \begin{array}{l} \\ R_2/2 \\ R_3/3 \\ R_4/4 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/2 & 1/3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1/3 & 1/4 \end{array}\right]$$

Hence

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1/2 & 0 & 0 \\ 0 & -1/2 & 1/3 & 0 \\ 0 & 0 & -1/3 & 1/4 \end{bmatrix}$$

b) Let the matrix in (a) be denoted A.

Then we observe that the matrix in b) is

$$2A^T$$

Now, from properties of matrix algebra

$$(2A^T)^{-1} = \frac{1}{2}(A^{-1})^T$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1/2 & 0 & 0 \\ 0 & -1/2 & 1/3 & 0 \\ 0 & 0 & -1/3 & 1/4 \end{bmatrix}^T \qquad \text{using a)}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$$