COMP20003
Algorithms and Data Structures
Distribution Counting

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Semester 2

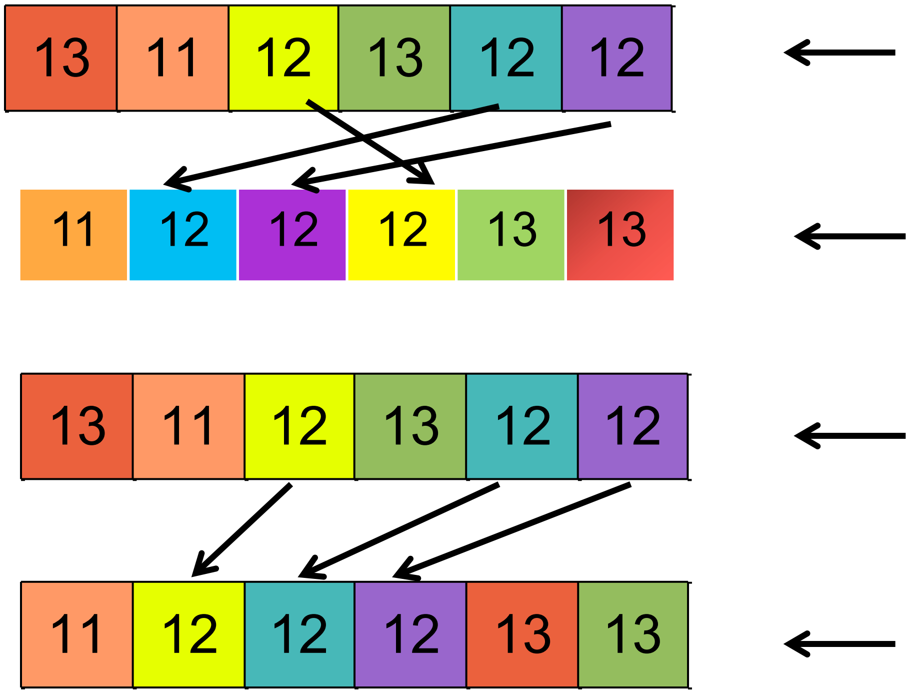
Sorting by Counting

- Distribution counting: an unusual approach to sorting.
- Later we will look at more common approaches.
- Distribution counting requires:
  - Key values to be within a certain range, *lower* to *upper*.

Sorting by Counting: Approach

- Steps in distribution counting:
  - Start with array of:
    - Records, or
    - Keys + pointers to records
  - Count number of records associated with each key value (*lower* to *upper*)
  - Redistribute array elements
- Net result:
  - Sorted array
  - Stable sort

Segue: What is a stable sort?



13 11 12 13 12 12 ← Unsorted

11 12 12 12 13 13 ← Sorted
BUT

13 11 12 13 12 12 ← Unsorted

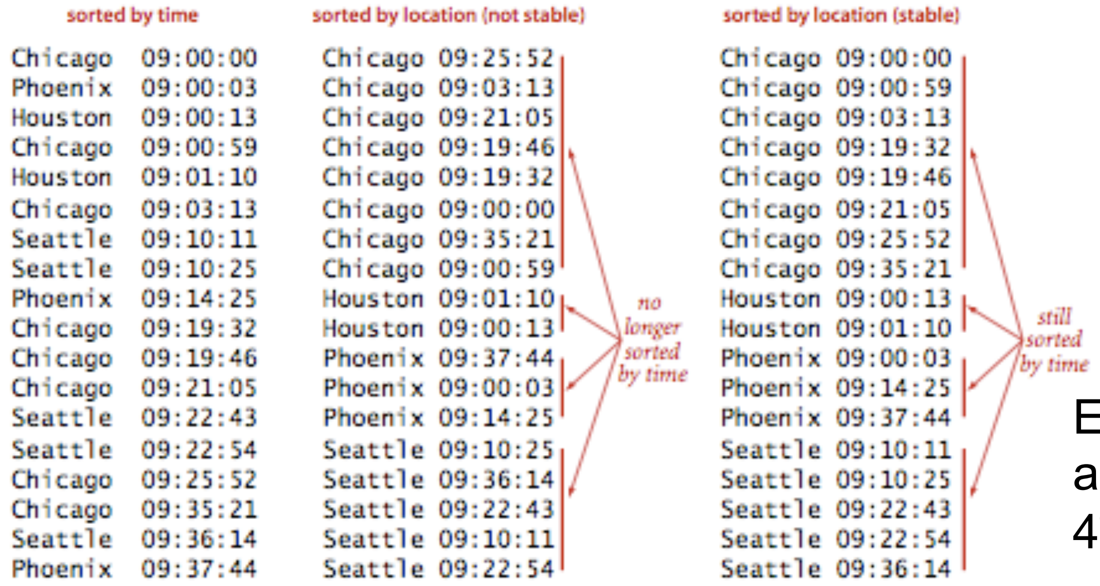
11 12 12 12 13 13 ← Stably sorted

Stable sorting: definition

- Stable sorting algorithms maintain relative order of records with equal key values.

Stable sorting: Applications

- Want file sorted on one key, and within each group, sorted on another key:



sorted by time	sorted by location (not stable)	sorted by location (stable)
Chicago 09:00:00	Chicago 09:25:52	Chicago 09:00:00
Phoenix 09:00:03	Chicago 09:03:13	Chicago 09:00:59
Houston 09:00:13	Chicago 09:21:05	Chicago 09:03:13
Chicago 09:00:59	Chicago 09:19:46	Chicago 09:19:32
Houston 09:01:10	Chicago 09:19:32	Chicago 09:19:46
Chicago 09:03:13	Chicago 09:00:00	Chicago 09:21:05
Seattle 09:10:11	Chicago 09:35:21	Chicago 09:25:52
Seattle 09:10:25	Chicago 09:00:59	Chicago 09:35:21
Phoenix 09:14:25	Houston 09:01:10	Houston 09:00:13
Chicago 09:19:32	Houston 09:00:13	Houston 09:01:10
Chicago 09:19:46	Phoenix 09:37:44	Phoenix 09:00:03
Chicago 09:21:05	Phoenix 09:00:03	Phoenix 09:14:25
Seattle 09:22:43	Phoenix 09:14:25	Phoenix 09:37:44
Seattle 09:22:54	Seattle 09:10:25	Seattle 09:10:11
Chicago 09:25:52	Seattle 09:36:14	Seattle 09:10:25
Chicago 09:35:21	Seattle 09:22:43	Seattle 09:22:43
Seattle 09:36:14	Seattle 09:10:11	Seattle 09:22:54
Phoenix 09:37:44	Seattle 09:22:54	Seattle 09:36:14

Stability when sorting on a second key

Example from Sedgewick and Wayne, Algorithms, 4th Edition, 2011

Back to Distribution Counting: Approach

- Steps in distribution counting:
  - Input: array of:
    - records, or
    - keys + pointers to records
  - Count number of records associated with each key value (*lower* to *upper*).
  - Redistribute array elements.
  - Output: stably sorted array.

Back to Distribution Counting: Example:

- Sort [4,4,2,2,0,2,1,3,2,4,3,1,4,3,1,4]
- Count records for each key [1,3,4,3,5]
  - CumulativeCount = [0,1,4,8,11]
- Redistribute
  - Create auxiliary array
  - traverse original array copying each item to position:
    - aux_array[cumulativeCount[item.key]] = item
    - Increase cumulativeCount[itemkey] + 1

Distribution Counting: Analysis

- Time:
  - Worst-case:
  - Average-case:
- Space:

Does the key range influence the complexity?

- *O*(*n*) if range *r* of keys is in *O*(*n*).
  - `count[]` array size is *r*.
  - Initialization and shuffling are *O*(*r*).
  - So if *r* > *n*…

But what about theory?

- we said weeks ago:
  - Comparison-based sorting is $\Omega(n \log n)$.
- Does distribution counting contradict that statement?

Sorting without comparing

- Other non-comparison-based sorting algorithms include:
  - LSD Radix sort
  - MSD Radix sort
  - Several others
- Drawbacks:
  - Take extra space.
  - Generally less flexible than comparison-based.
  - Can be fiddly if keys are not the same length, *e.g.* variable length strings in MSD radix.