



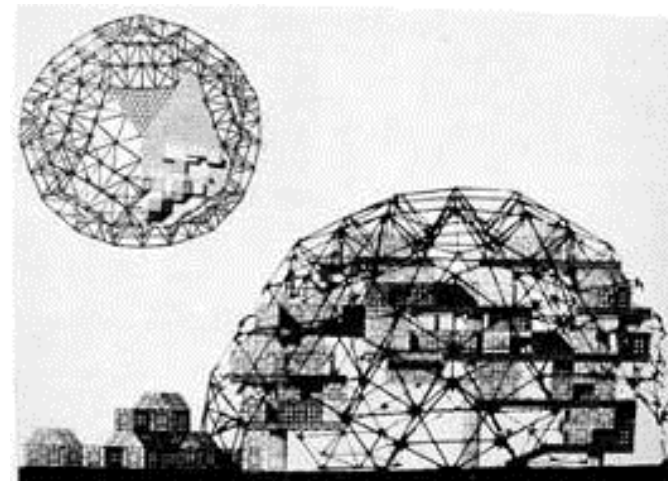
# Quantitative Risk Analysis using Monte Carlo Simulation

**Subject Coordinator:**  
**Dr Lihai Zhang**

**CONTACT:**

**Department of Infrastructure  
Engineering (Room B307)**

**[lihzhang@unimelb.edu.au](mailto:lihzhang@unimelb.edu.au)**





## **Codified Design – How safe is safe enough?**

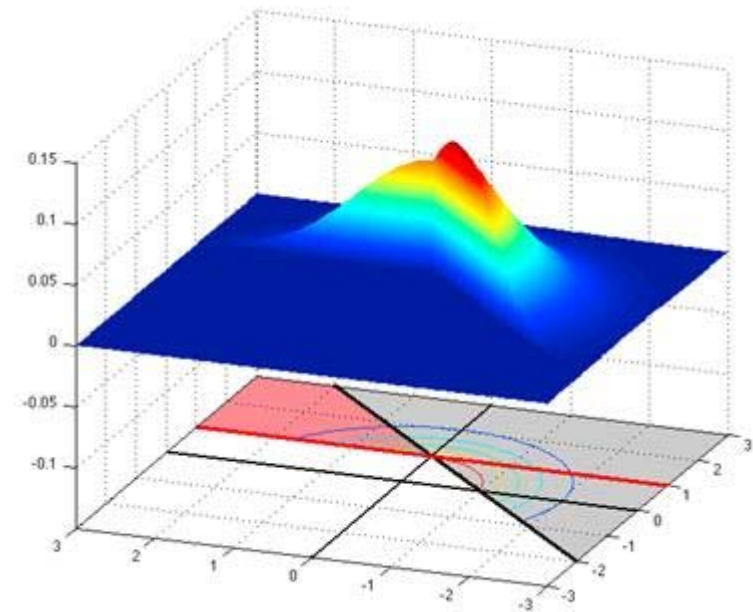
- This judgement must be made by code writers when
  - Identifying those natural and artificial forces that must be considered to ensure adequate safety and serviceability.
  - Providing criteria for achieving minimum required levels of structural resistance to these forces.

***What has this judgement been based on?***





- The deterministic approach
- Probabilistic approach

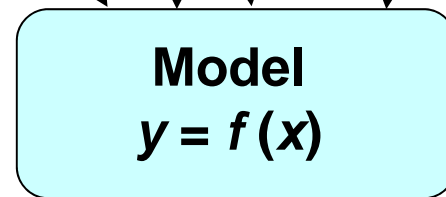




- DETERMINISTIC Approaches
  - Deterministic calculations always give same answers ( $y_1, y_2$ ) if input values ( $x_1, x_2, x_3$ ) do not change

**Input**  
(no variability)

$x_1$   $x_2$   $x_3$  . . .  $x_n$



**Output**  
(no variability)

$y_1$   $y_2$  . . .  $y_n$

Calculations might be

$$y_1 = x_1 + 3 * x_2^2 - 0.07 * \ln(x_3)$$

$$y_2 = 2 * x_1 - 3 * x_2^2$$

. . . . .



# Probabilistic Approaches

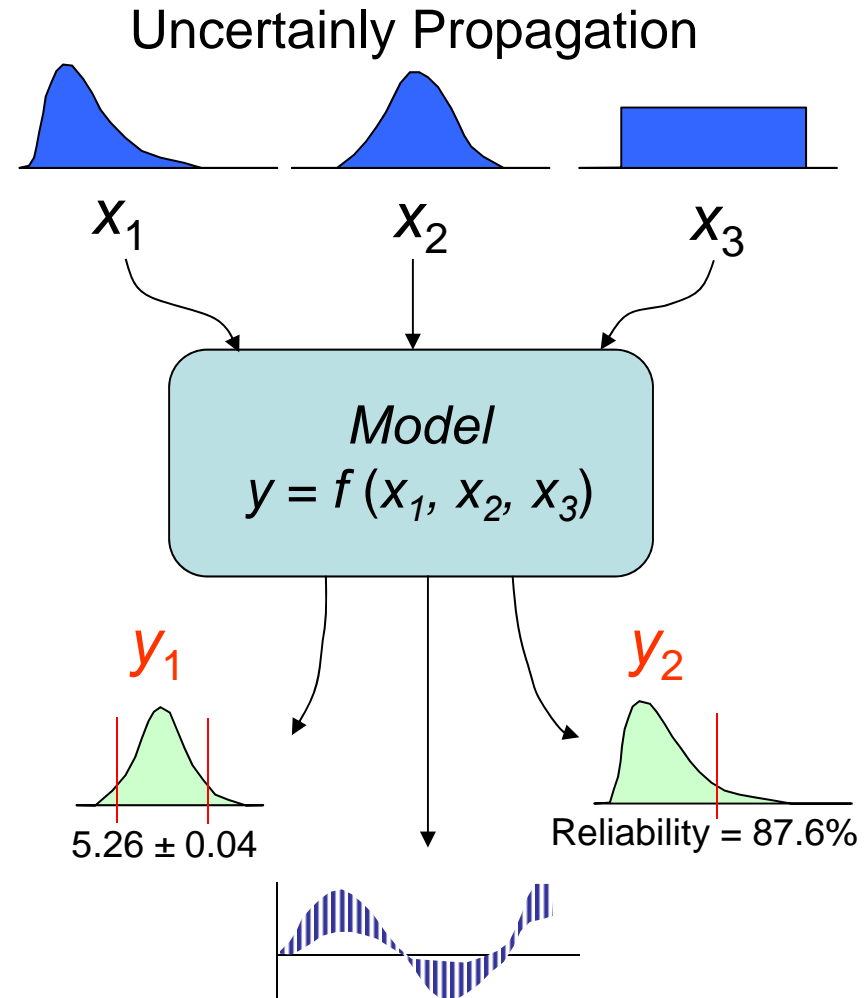
- When **inputs** ( $x_1, x_2, x_3$ ) are random variables then **outputs** ( $y_1, y_2$ ) are also random variables.

$x_1$  has a right skewed distribution,

$x_2 \dots$

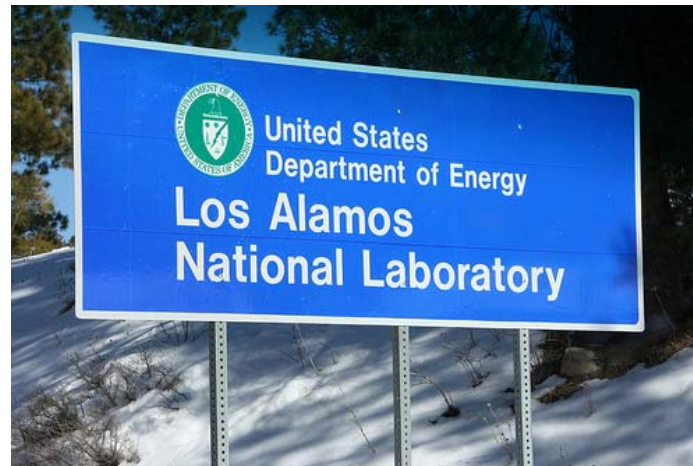
$y_1$  has a Normal distribution,

$y_2 \dots$





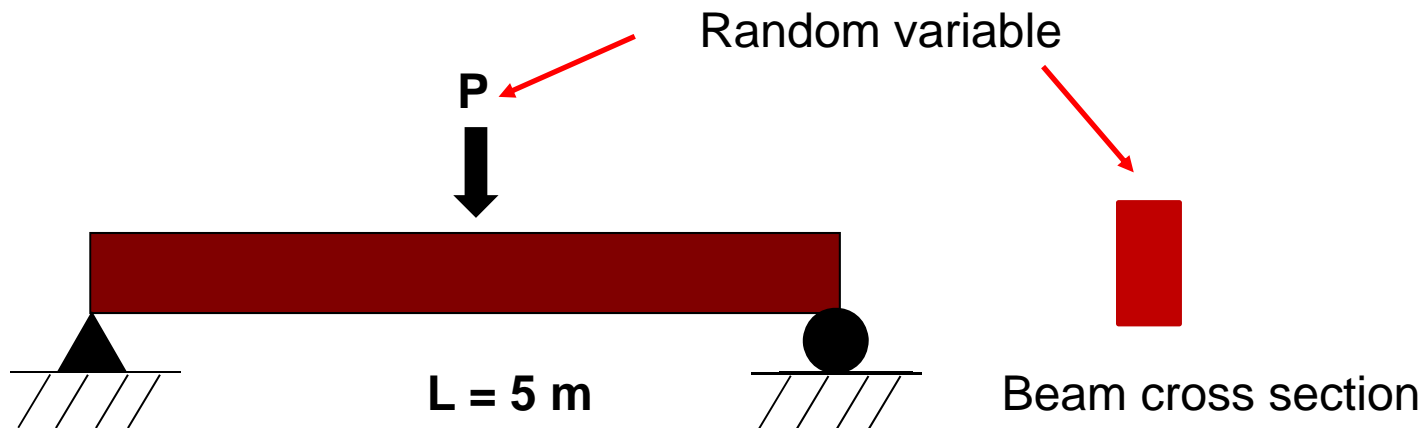
A **Monte Carlo method** is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results .



The term **Monte Carlo** was coined in the 1940s by physicists working on nuclear weapon projects in the [Los Alamos National Laboratory](#).



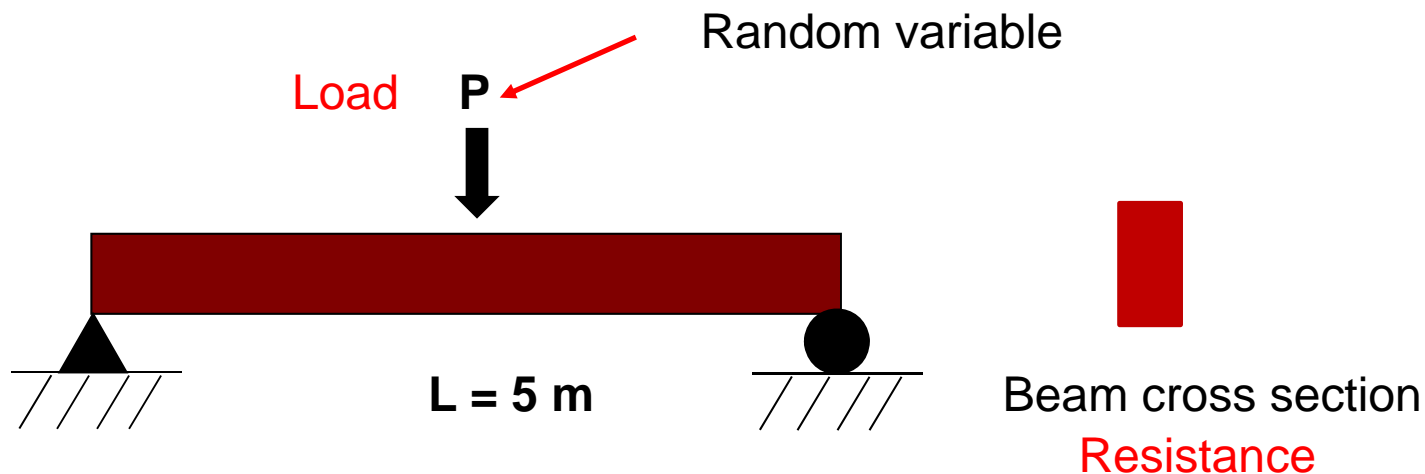
- Monte Carlo Simulation
  1. Defining all the random variables (continued)





- Monte Carlo Simulation
  2. Quantifying the probabilistic characteristics of the random variables.

A simply supported timber beam of length  $5\text{ m}$  is loaded with a central load  $P$  with  $\mu_S = 3\text{ kN}$  and  $\sigma_S = 1\text{ kN}$ . The bending strength of similar beams has been found to have a allowable load of  $5\text{ kN}$ . Determine the probability of failure.

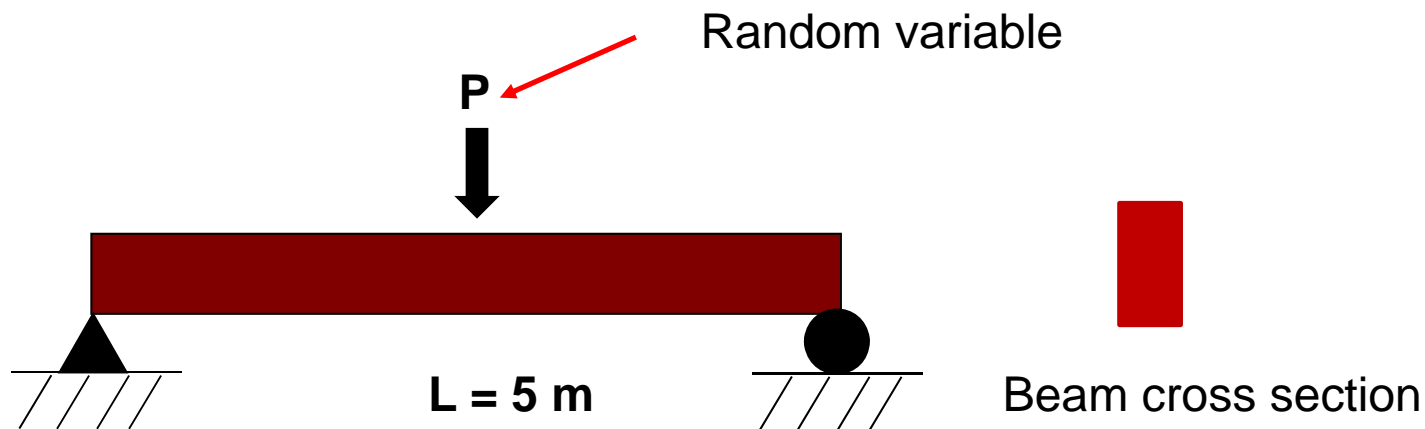






- Monte Carlo Simulation
  1. Defining random variables.
  2. Generating random numbers.
  3. Generating values of these random variables.

Load **P** with  $\mu_P = 3 \text{ kN}$  and  $\sigma_P = 1 \text{ kN}$





- Monte Carlo Simulation
  3. Generating 1000 values of these random variables.

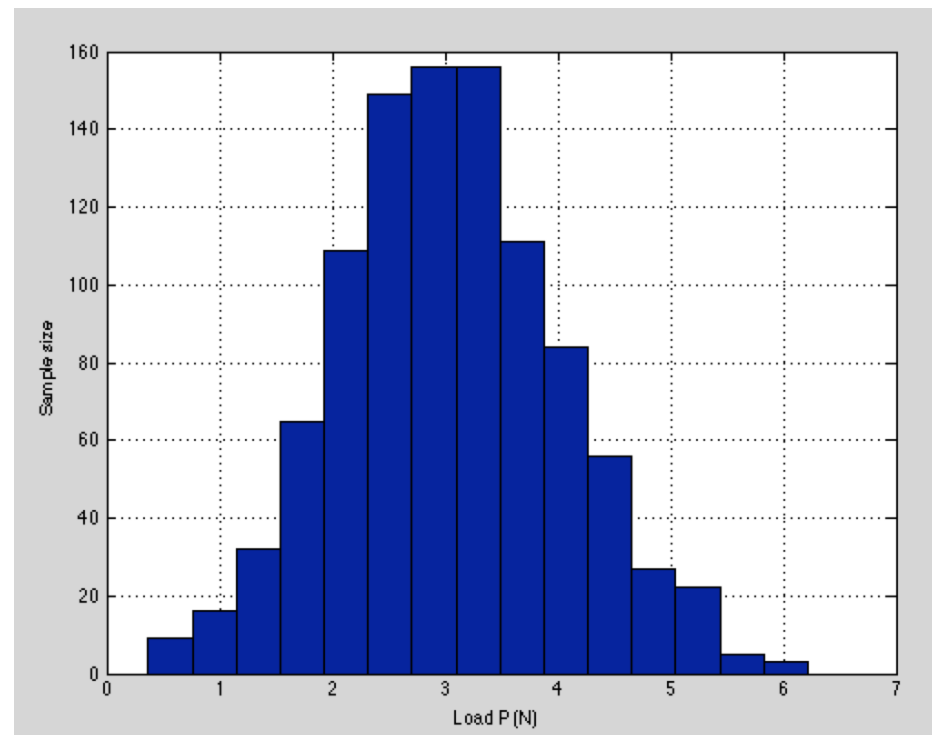
Load **P** with  $\mu_S = 3$  kN and  $\sigma_S = 1$  kN

```
1      %Monte Carlo simulation with histogram
2
3 -    clear all;
4 -    close all;
5 -    clc;
6
7      % generate sample
8 -    rng(123456);      %specifiy random seed
9 -    mu = 3;           %mean
10 -    sigma = 1;       %standard deviation
11 -    size = 1000;      %sample size
12
13 -    r = mu + sigma.*randn(size,1);    %generate sample mu and sigma
14
```



- Monte Carlo Simulation
  3. Generate histogram of 1000 samples with 15 bins

```
15 % generate histogram
16 - hist(r,15);
17 - xlabel('Load P (N)');
18 - ylabel('Sample size');
19 - grid on;
20
```





- Monte Carlo Simulation

4. Determine the probability of failure if the allowable load = 5 kN

```
21 %calculate probability of failure
22
23 - n = length(r);
24 - allowable_load = 5;
25 - counter = 0;
26
27 - for i = 1:n
28 -     if r(i) > allowable_load
29 -         counter = counter+1;
30 -     end
31 - end
32
33 - probability_failure = counter/size*100;
34 - display(probability_failure);
```

## Command Window

```
probability_failure =
    3.1000
```

**Probability of failure: 3.1%**