The University of Melbourne
CVEN30008 Engineering Risk Analysis

Tutorial 10
(I) Power and Sample Size

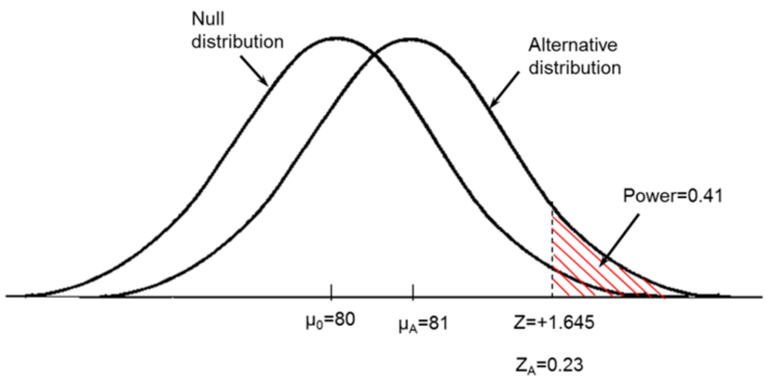
1. **A new chemical process has been developed that may increase the yield over that of the current process. The current process is known to have a mean yield of 80 and a standard deviation of 5, where the units are the percentage of a theoretical maximum. If the mean yield of the new process is shown to be greater than 80, the new process will be put into production. It is proposed to run the new process 50 times and test the hypothesis that the mean yield of the new process is greater than 80 at a significance level of 5%. What is the power of this test? What is your suggestion based on the calculated power?**
   Assume that the mean yield of the new process is in fact 81 and its standard deviation is the same as that of the current process (σ=5).

Answer:

μ_0=80, σ=5, n=50, μ_A=81

H_0: $\mu \leq 80$ vs H_1: $\mu>80$

$$\mu_0 + Z\frac{\sigma}{\sqrt{n}} = \mu_A + Z_A\frac{\sigma}{\sqrt{n}}$$



α=0.05, right tail test => z=+1.645

$$80 + 1.645\frac{5}{\sqrt{50}} = 81 + Z_A\frac{5}{\sqrt{50}}$$

Z_A=0.23

Power=Area to the right of Z_A=0.23

Power=0.41

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The test power is too low (Power<<0.8). It is suggested not to invest time and money to conduct the test since it has a high chance to fail.

(II) Simple Linear Regression

2. **A structural engineer is investigating the dynamic response of a concrete slab subject to projectile impact. The measurements indicated in the following table were obtained from 5 tests.**

Mass of Impactor (kg)	Maximum contact force (kN)	Maximum Displacement (mm)
1	6.7	0.5
5	20	0.8
10	50	4
20	120	8
50	250	12

Answers:

a). Compute the correlation between mass of impactor and maximum contact force

Let x = mass of impactor, y = maximum contact force

$$\bar{x} = \frac{1}{n_x = 5}\sum_{i=1}^{n_x} x_i = 17.2$$

$$\bar{y} = \frac{1}{n_y = 5}\sum_{i=1}^{n_y} y_i = 89.34$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{[\sum(x-\bar{x})^2][\sum(y-\bar{y})^2]}} = \frac{7823.46}{7859.65} = 0.995$$

b). Compute the correlation between mass of impactor and maximum displacement

Let x = mass of impactor, z = maximum displacement

$$\bar{x} = \frac{1}{n_x = 5}\sum_{i=1}^{n_x} x_i = 17.2$$

$$\bar{z} = \frac{1}{n_z = 5}\sum_{i=1}^{n_z} z_i = 5.06$$

$$r = \frac{\sum(x-\bar{x})(z-\bar{z})}{\sqrt{[\sum(x-\bar{x})^2][\sum(z-\bar{z})^2]}} = \frac{369.34}{387.09} = 0.954$$

c). Compute the least-squares line for predicting maximum contact force from mass of impactor

$$\beta_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{7823.46}{1546.8} = 5.0578$$

$$\beta_0 = \bar{y} - \beta_1\bar{x} = 89.34 - 5.0578 * 17.2 = 2.3452$$

The least squares line is:

$$y = 5.0578x + 2.3452$$

d). What is the maximum contact force if the impactor mass is 25 kg?

For x = 25kg,

$y = 5.0578 * 25 + 2.3452 = 128.8kN$

e). Compute the least-squares line for predicting maximum displacement from mass of impactor

$$\beta_1 = \frac{\sum(x-\bar{x})(z-\bar{z})}{\sum(x-\bar{x})^2} = \frac{369.34}{1546.8} = 0.239$$

$$\beta_0 = \bar{z} - \beta_1\bar{x} = 5.06 - 0.2388 * 17.2 = 0.953$$

The least squares line is:

$$z = 0.239x + 0.953$$

f). In order to induce a maximum displacement value of 10 mm, what is the mass of impactor?

For z = 10mm

$$x = \frac{10 - 0.953}{0.239} = 37.9kg$$

g). Verify your results by using MATLAB.

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(a) Correlation coefficient between mass of impactor and maximum contact force is:

r =

    0.9954

(b) Correlation coefficient between mass of impactor and maximum displacement is:

r =

    0.9541

Force  = p1*Mass + P2, where

p1 =

    5.0580


p2 =

    2.3450


force_predicted =

  128.7950

Displacement  = p1*Mass + P2, where

p1 =

    0.2388


p2 =

    0.9530


mass_predicted =

   37.8853
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3. **The structural engineer later on study the effect of impacting velocity on the maximum contact force. The measurements indicated in the following table were obtained from 8 tests.**

Impactoring velocity (m/s)	Maximum contact force (kN)
2	20
4	33
6	65
8	69
10	90
12	110
14	130
16	158

Is there evidence of a linear relationship between impacting velocity and maximum contact force at the 0.05 level of significance? Verify your results by using MATLAB.

<u>Answers:</u>

Let x = impacting velocity, y = maximum contact force

$$\bar{x} = \frac{1}{n_x = 8}\sum_{i=1}^{n_x} x_i = 9$$

$$\bar{y} = \frac{1}{n_y = 8}\sum_{i=1}^{n_y} y_i = 84.375$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{[\sum(x-\bar{x})^2][\sum(y-\bar{y})^2]}} = \frac{1607}{1618} = 0.993$$

Hypothesis:

H_0: $\rho = 0$ (no correlation) vs H_1: $\rho \neq 0$ (correlation exists)

Degree of freedom = $n - 2 = 8 - 2 = 6$

t test statistic:

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} = \frac{0.993}{0.048} = 20.75$$

Degrees of Freedom	Combined Area α in Two Tails					
	0.250	0.100	0.050	0.025	0.010	0.005
1	2.4142	6.3138	12.7062	25.4517	63.6567	127.3213
2	1.6036	2.9200	4.3027	6.2053	9.9248	14.0890
3	1.4226	2.3534	3.1824	4.1765	5.8409	7.4533
4	1.3444	2.1318	2.7764	3.4954	4.6041	5.5976
5	1.3009	2.0150	2.5706	3.1634	4.0321	4.7733
6	1.2733	1.9432	2.4469	2.9687	3.7074	4.3168
7	1.2543	1.8946	2.3646	2.8412	3.4995	4.0293
8	1.2403	1.8595	2.3060	2.7515	3.3554	3.8325
9	1.2297	1.8331	2.2622	2.6850	3.2498	3.6897
10	1.2213	1.8125	2.2281	2.6338	3.1693	3.5814
11	1.2145	1.7959	2.2010	2.5931	3.1058	3.4966
12	1.2089	1.7823	2.1788	2.5600	3.0545	3.4284
13	1.2041	1.7709	2.1604	2.5326	3.0123	3.3725
14	1.2001	1.7613	2.1448	2.5096	2.9768	3.3257
15	1.1967	1.7531	2.1314	2.4899	2.9467	3.2860
16	1.1937	1.7459	2.1199	2.4729	2.9208	3.2520
17	1.1910	1.7396	2.1098	2.4581	2.8982	3.2224
18	1.1887	1.7341	2.1009	2.4450	2.8784	3.1966
19	1.1866	1.7291	2.0930	2.4334	2.8609	3.1737
20	1.1848	1.7247	2.0860	2.4231	2.8453	3.1534

From the *t* table, for $t = 4.3168$, $P = \alpha = 0.005$.

Since we have $t = 20.75$ which is much greater than 4.3168,

Hence $P \ll 0.005 < 0.05$ (level of significance)

Conclusion: there is strong evidence of a linear relationship at the 0.05 level of significance.

MATLAB:

Command Window

```
t =

   20.7526

Two tailed test

p =

   8.1487e-07


alpha =

    0.0500

Since p < alpha, we reject H_0.
There is evidence of a linear relationship at the 5% level of significance.
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