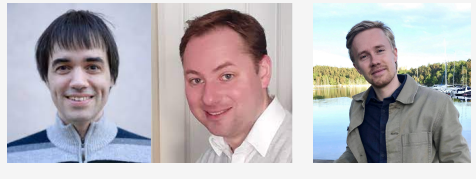
BAYESIAN INFERENCE WITH PHYSICS INFORMED PRIORS

FROM SIMULATIONS

Journée des thèses, IAP, 29.03.2024

Simon Ding in collaboration with **Ludvig Doeser**

supervised by **Guilhem Lavaux** (IAP) & **Jens Jasche** (Stockholm University)



BAYESIAN INFERENCE WITH PHYSICS INFORMED PRIORS

FROM SIMULATIONS

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BAYESIAN INFERENCE

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Given some observational data, we want to learn about:

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- cosmological parameters

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$$P(\theta|\mathrm{data}) = \frac{P(\mathrm{data}|\theta)P(\theta)}{P(\mathrm{data})} \qquad \mathrm{data} = \mathrm{Model}(\theta) + \mathrm{noise}$$

BAYESIAN INFERENCE

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- galaxy formation and evolution

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BAYESIAN INFERENCE

- cosmological parameters
- $\delta_m(x) \rightarrow \delta_g(x)$

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How to choose $P(\theta)$?

BAYESIAN INFERENCE

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1. Wide uniform prior

BAYESIAN INFERENCE

How to choose $P(\theta)$?

1. Wide uniform prior
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BAYESIAN INFERENCE

How to choose $P(\theta)$?

1. Wide uniform prior
2. Physical prior
3. Hand-tuned prior

BAYESIAN INFERENCE
WITH
PHYSICS INFORMED PRIORS FROM SIMULATIONS

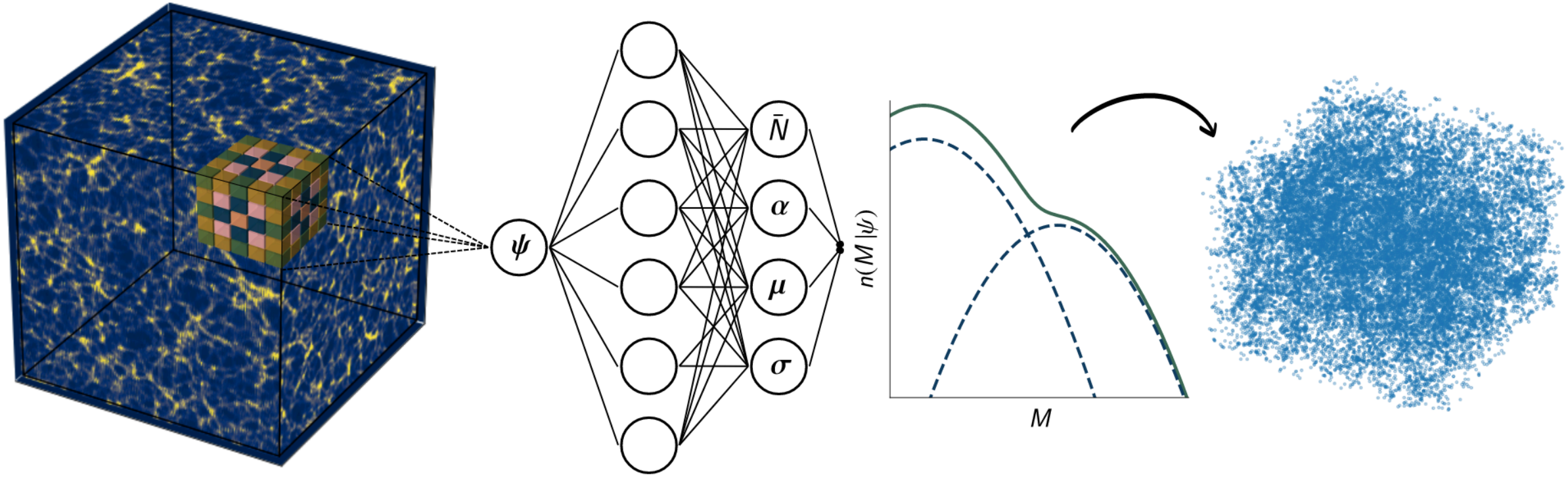
PHYSICS INFORMED PRIORS FROM SIMULATIONS

Galaxy bias model: $\delta_m(x) \rightarrow \delta_g(x)$

PHYSICS INFORMED PRIORS FROM SIMULATIONS

Galaxy bias model: $\delta_g(x) = b_1 \delta_m(x)$

PHYSICS INFORMED PRIORS FROM SIMULATIONS



overdensity field

PineTree forward model

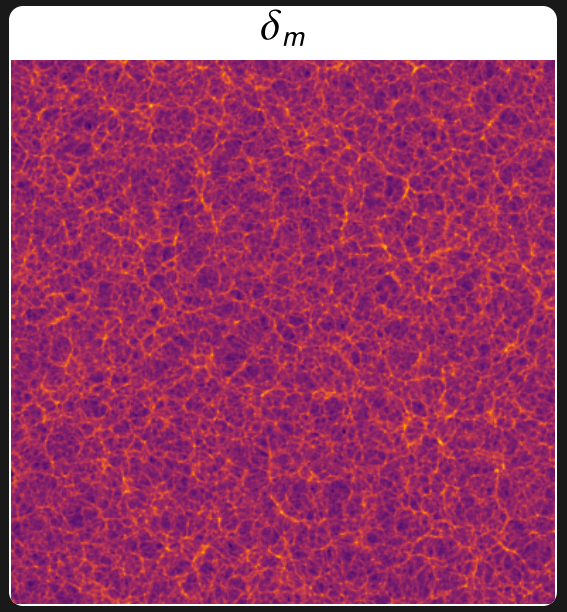
halo catalogue

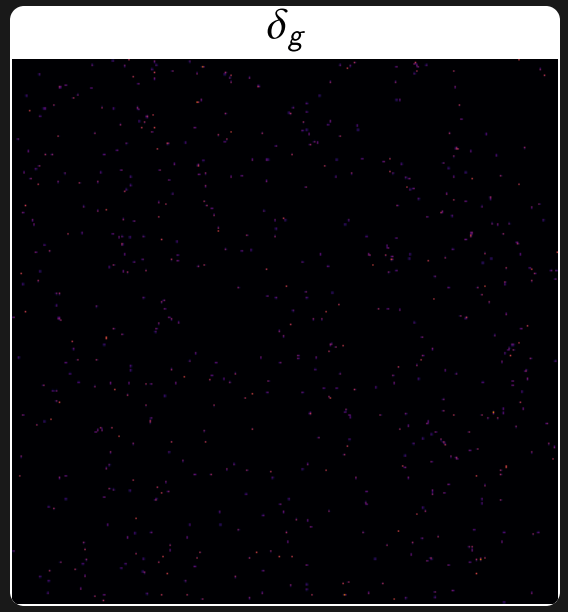
PHYSICS INFORMED PRIORS FROM SIMULATIONS

Galaxy bias model: $\delta_m(x) \rightarrow \delta_g(x)$

$\Rightarrow$ Use $N-$body simulations as guidance

PHYSICS INFORMED PRIORS FROM SIMULATIONS





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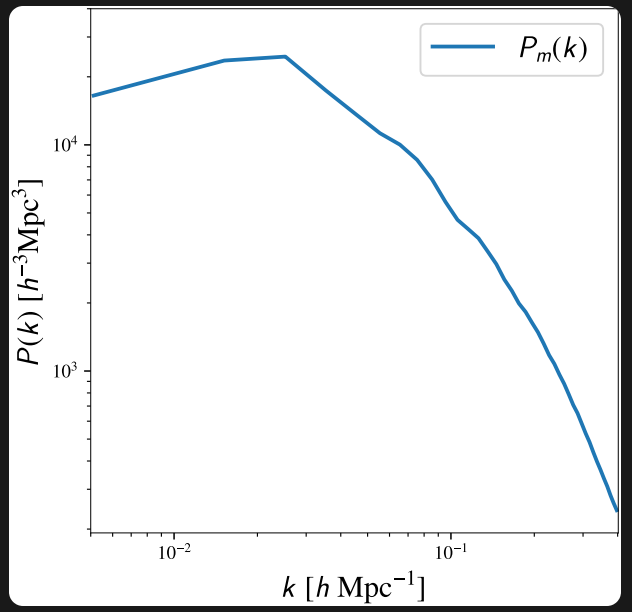

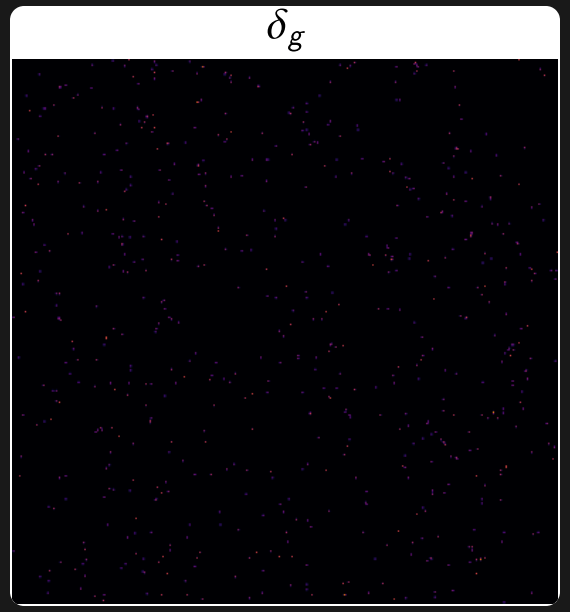

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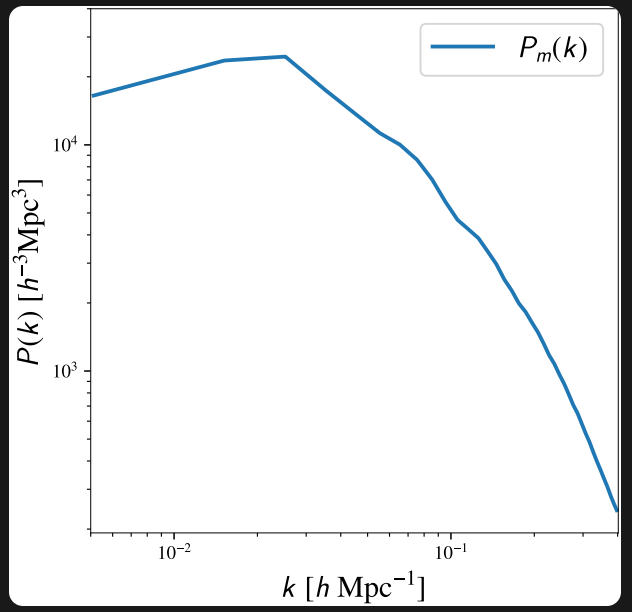

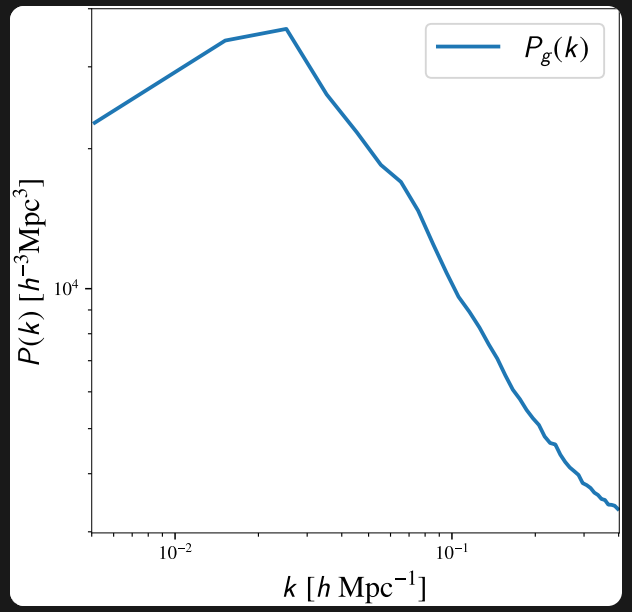

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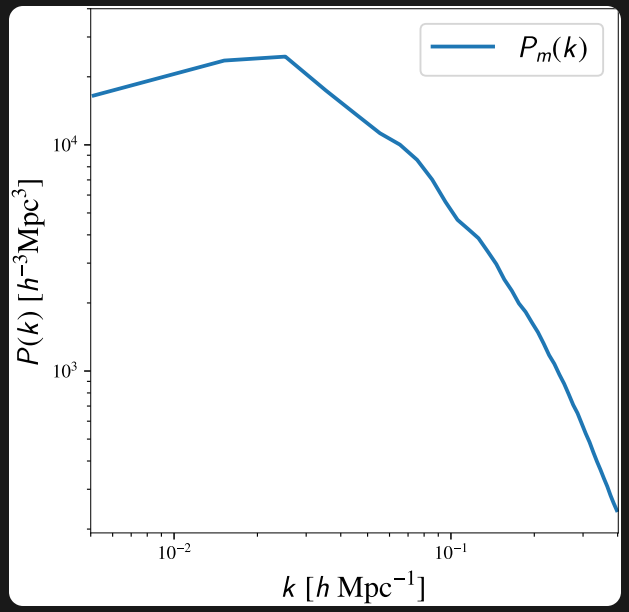

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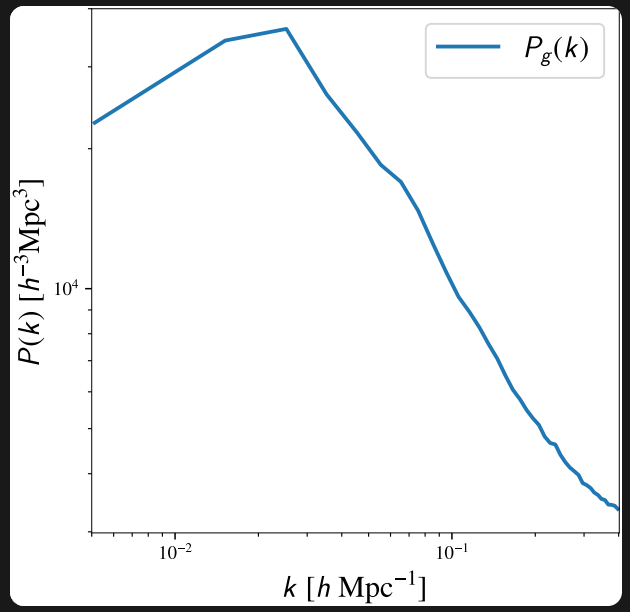

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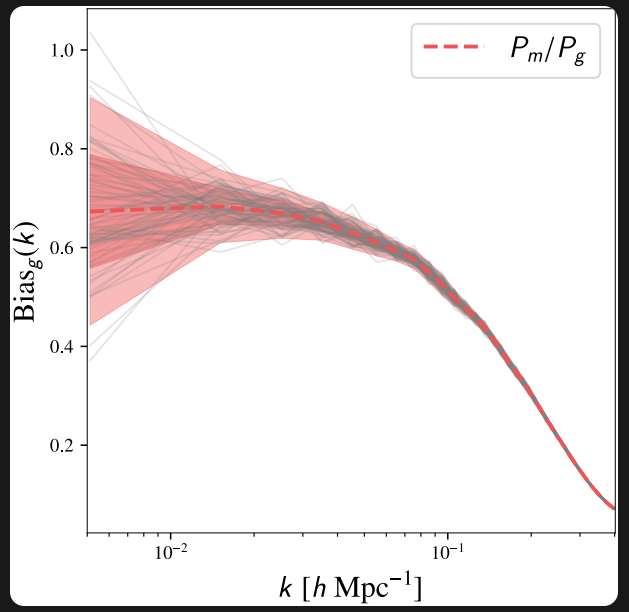

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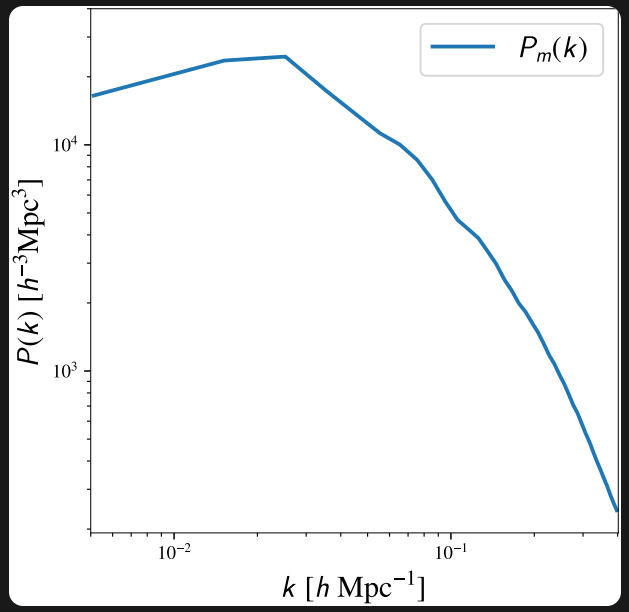

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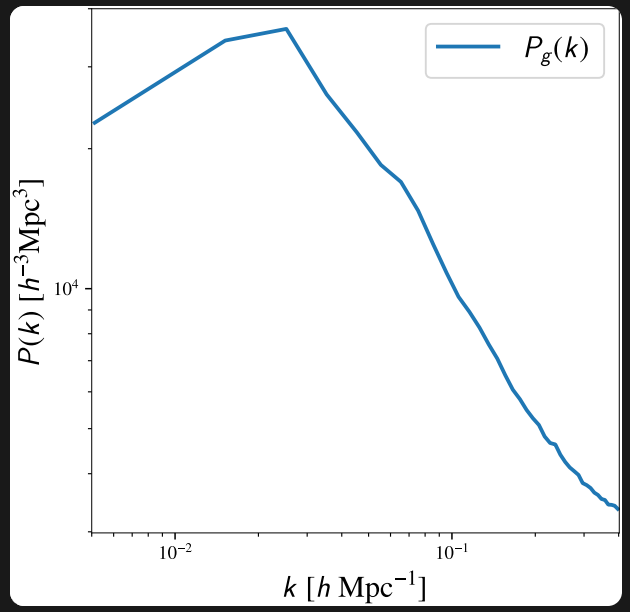

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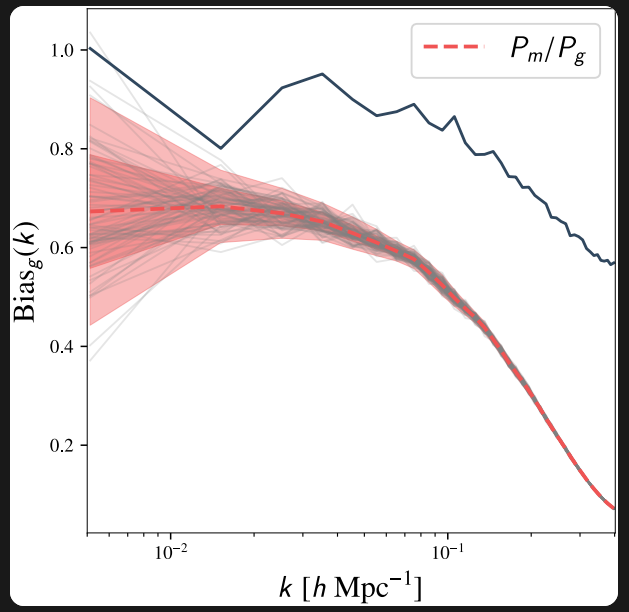

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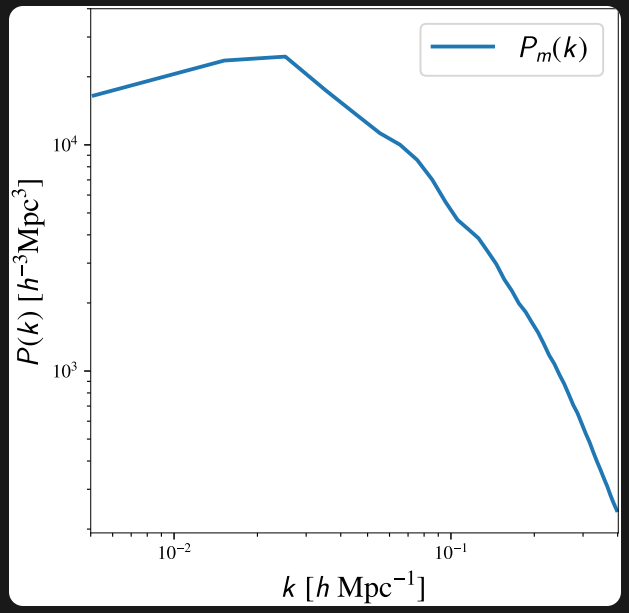

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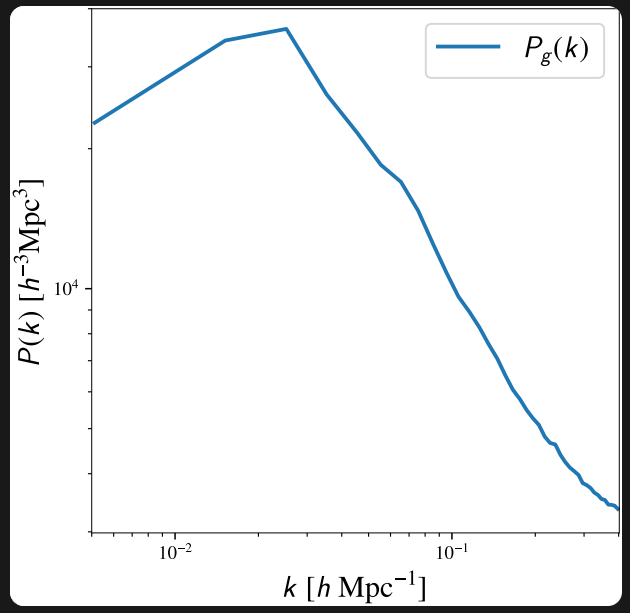

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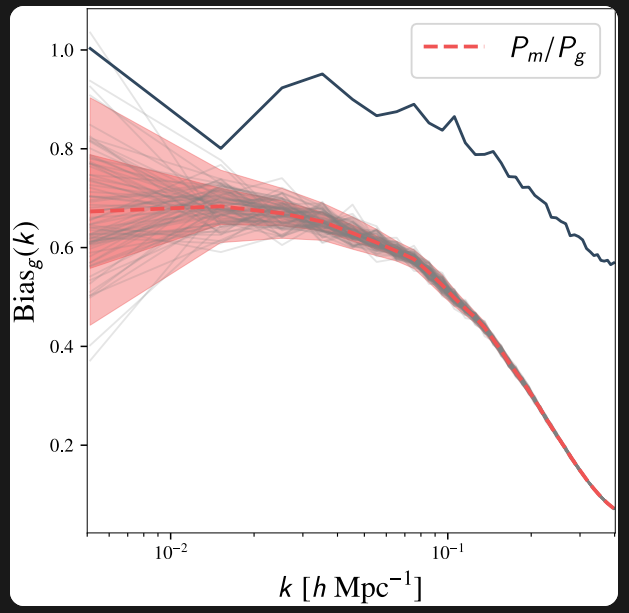

$\Rightarrow$ New constraint from simulations $r = \dfrac{P_m(k)}{P_g(k)}$

PHYSICS INFORMED PRIORS FROM SIMULATIONS

Conditional independent constraint r :

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Conditional independent constraint r : $P(\mathrm{data}|r) = P(\mathrm{data})$

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Conditional independent constraint r : $P(\mathrm{data}|r) = P(\mathrm{data})$

$$P(\theta|\mathrm{data}, r) = \frac{P(\mathrm{data}, r|\theta)P(\theta)}{P(\mathrm{data})}$$

PHYSICS INFORMED PRIORS FROM SIMULATIONS

Conditional independent constraint r : $P(\mathrm{data}|r) = P(\mathrm{data})$

$$P(\theta|\mathrm{data}, r) = \frac{P(\mathrm{data}, r|\theta)P(\theta)}{P(\mathrm{data})}$$

$$= \frac{P(\mathrm{data}|\theta)}{P(\mathrm{data})} \frac{P(r|\theta)P(\theta)}{P(r)} = \frac{P(\mathrm{data}|\theta)P(\theta|r)}{P(\mathrm{data})}$$

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$$\propto P(\mathrm{data}|\theta)P(r|\theta)P(\theta)$$

RObust **B**ayesian **IN**ference with **P**hysics-**i**nformed **P**rior

ROBIN-PiP

TOY MODEL EXAMPLE

FIELD-LEVEL INFERENCE FROM GALAXY SURVEYS WITH BORG

Jasche & Wandelt (2013), Jasche, Leclercq & Wandelt (2015),
Lavaux & Jasche (2016), Jasche & Lavaux (2019)

Image credit: D.K Ramanah

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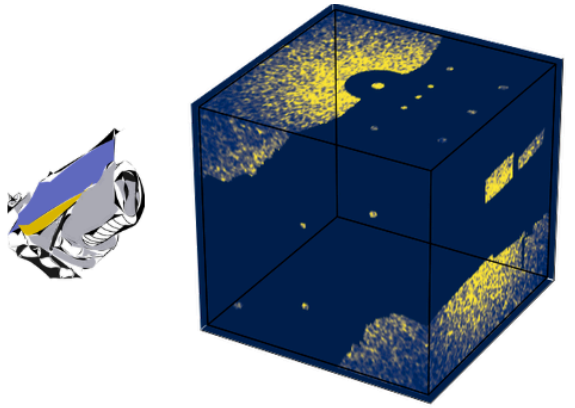

Observations

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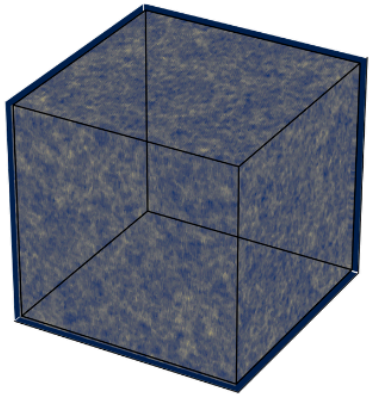
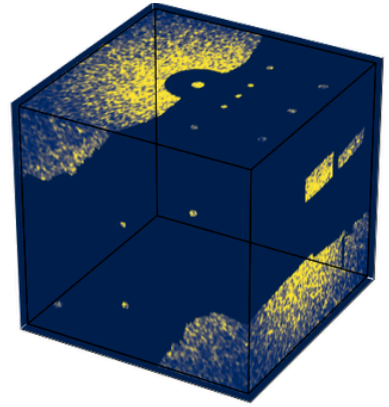


Latent parameter space (Gaussian prior)



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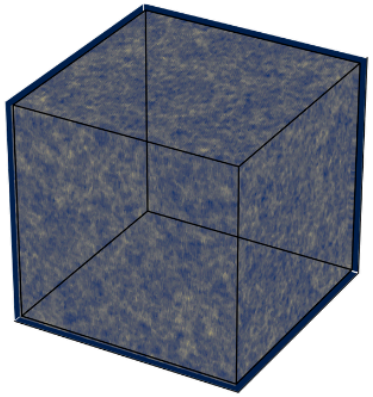

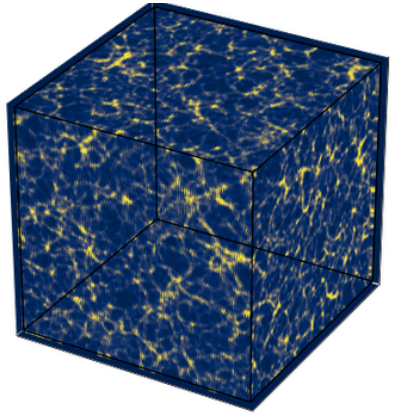

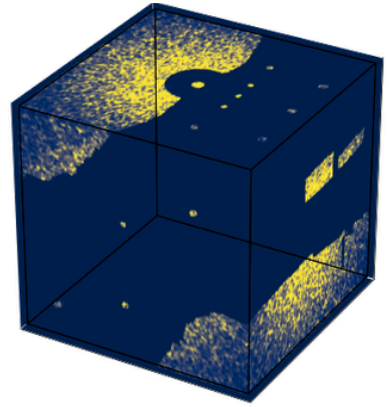

Latent parameter space (Gaussian prior)

Hamiltonian Equation

Observations

Jasche & Wandelt (2013), Jasche, Leclercq & Wandelt (2015), Lavaux & Jasche (2016), Jasche & Lavaux (2019)

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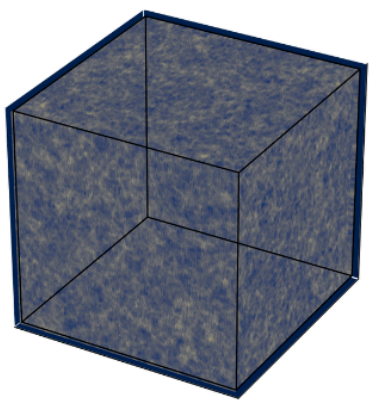

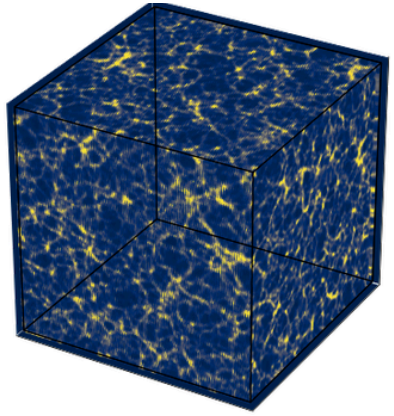

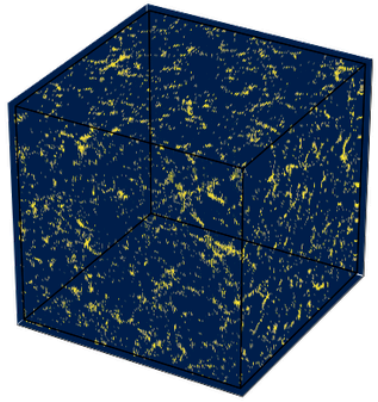

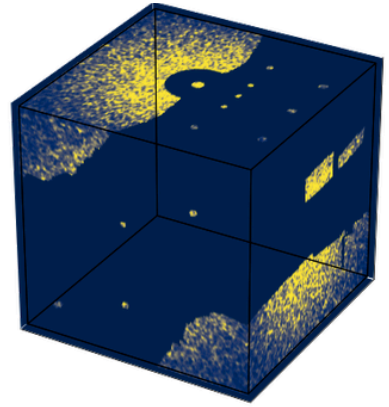

Latent parameter space (Gaussian prior)

Hamiltonian Equation

Small scale baryonic physics

Observations

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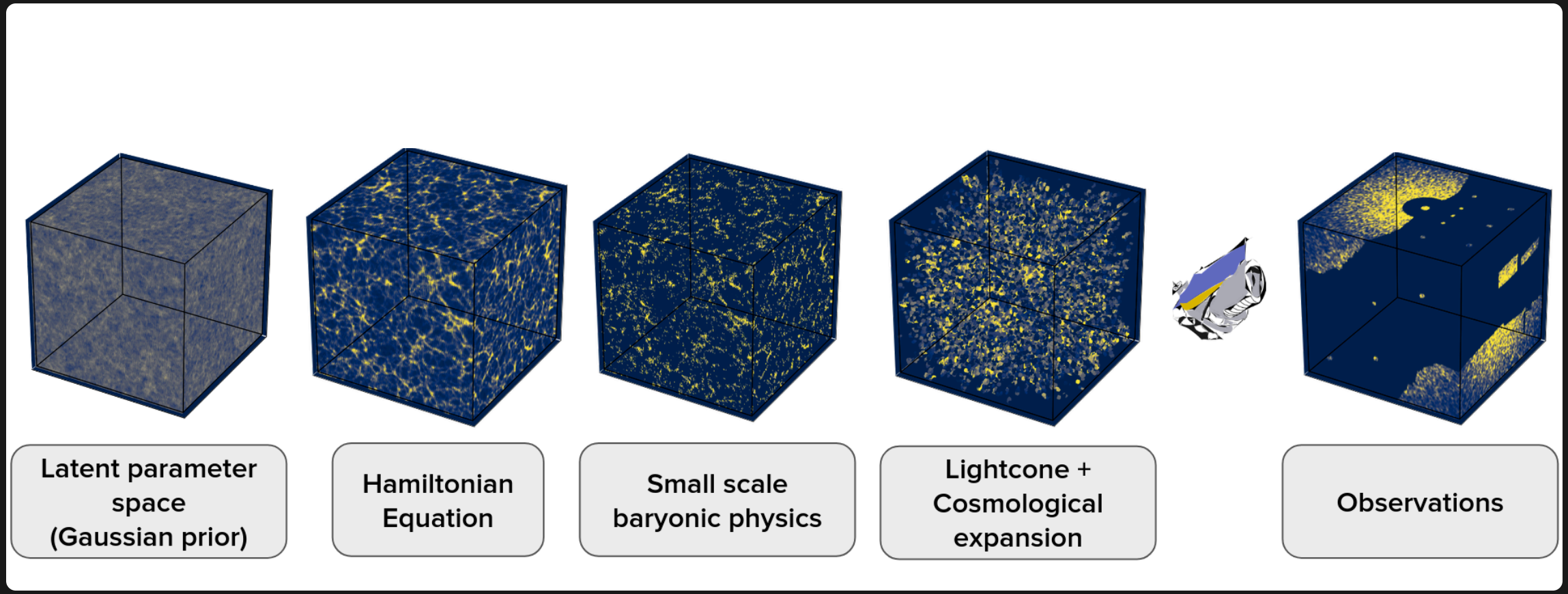

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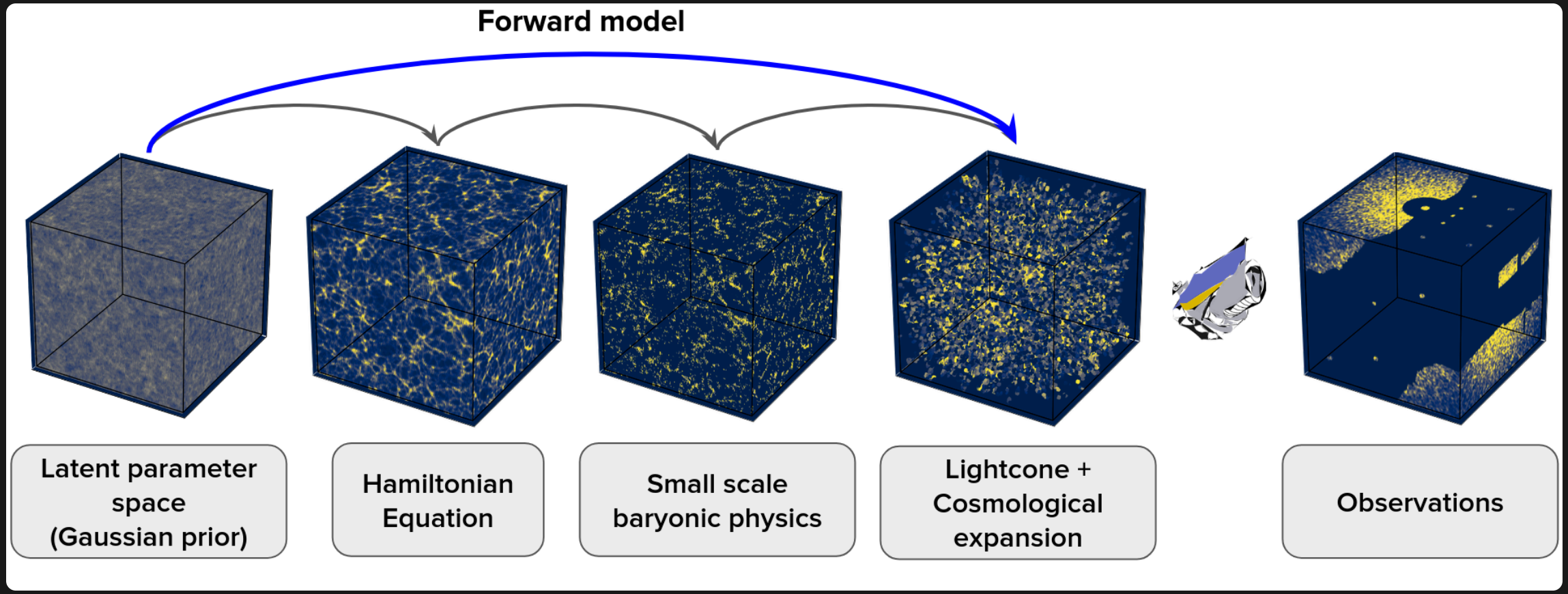

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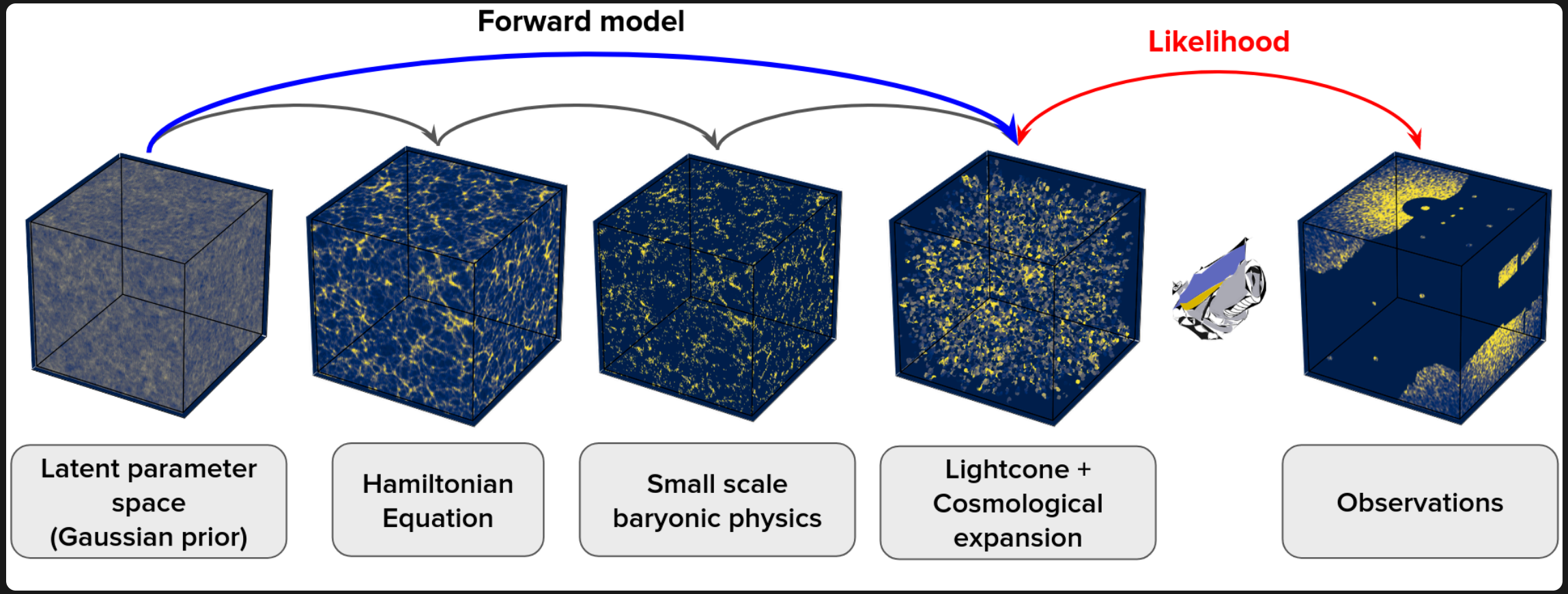

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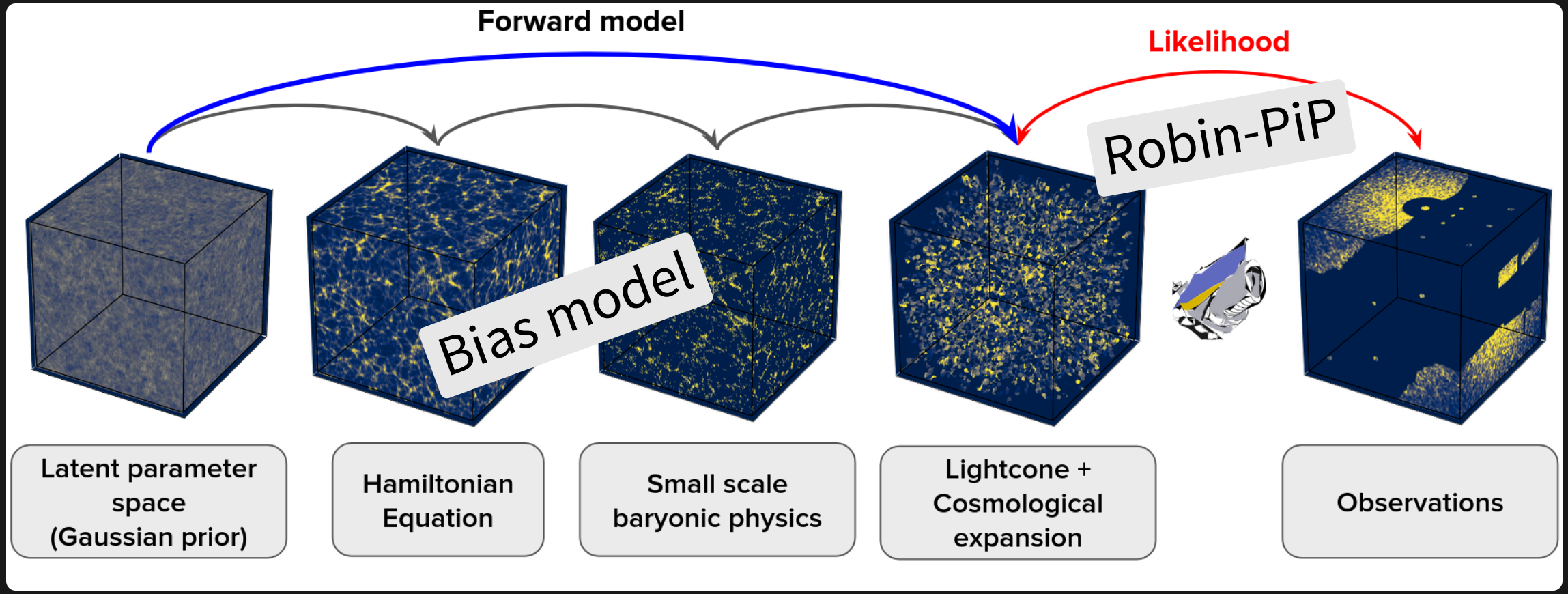

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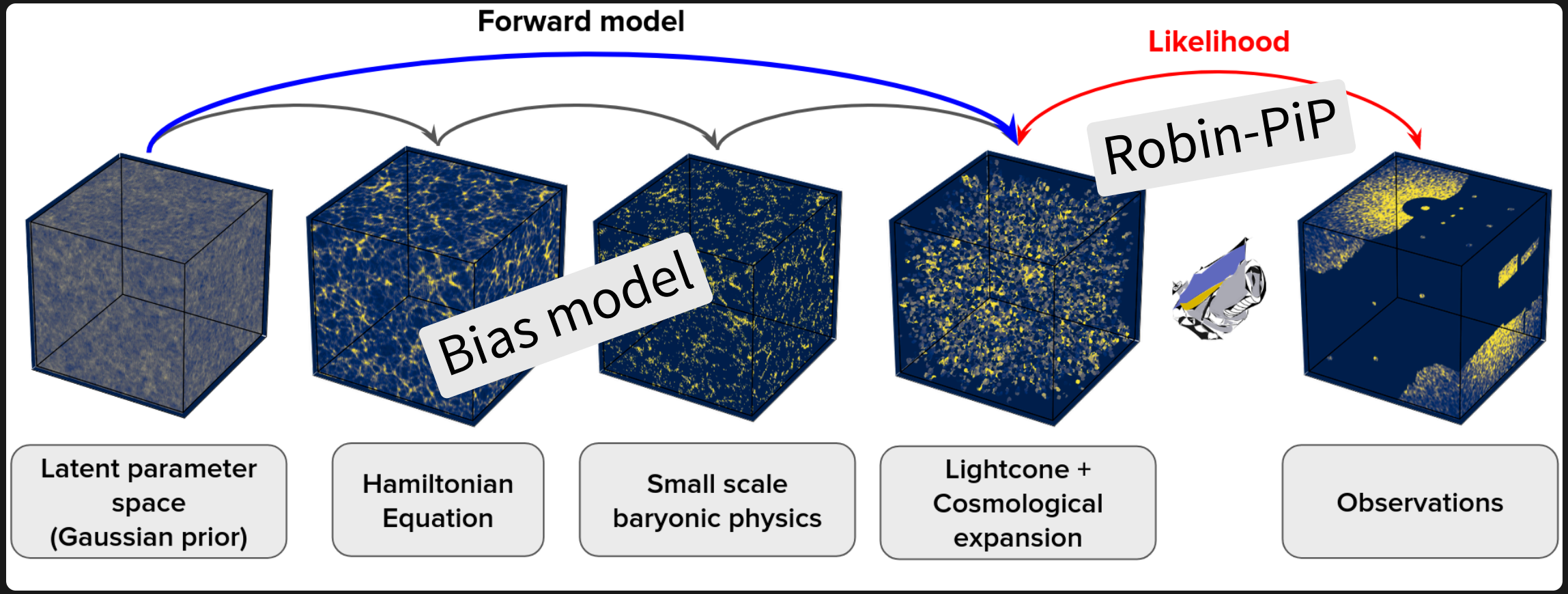

$\approx 2.1 \times 10^6$ parameters

Jasche & Wandelt (2013), Jasche, Leclercq & Wandelt (2015), Lavaux & Jasche (2016), Jasche & Lavaux (2019)

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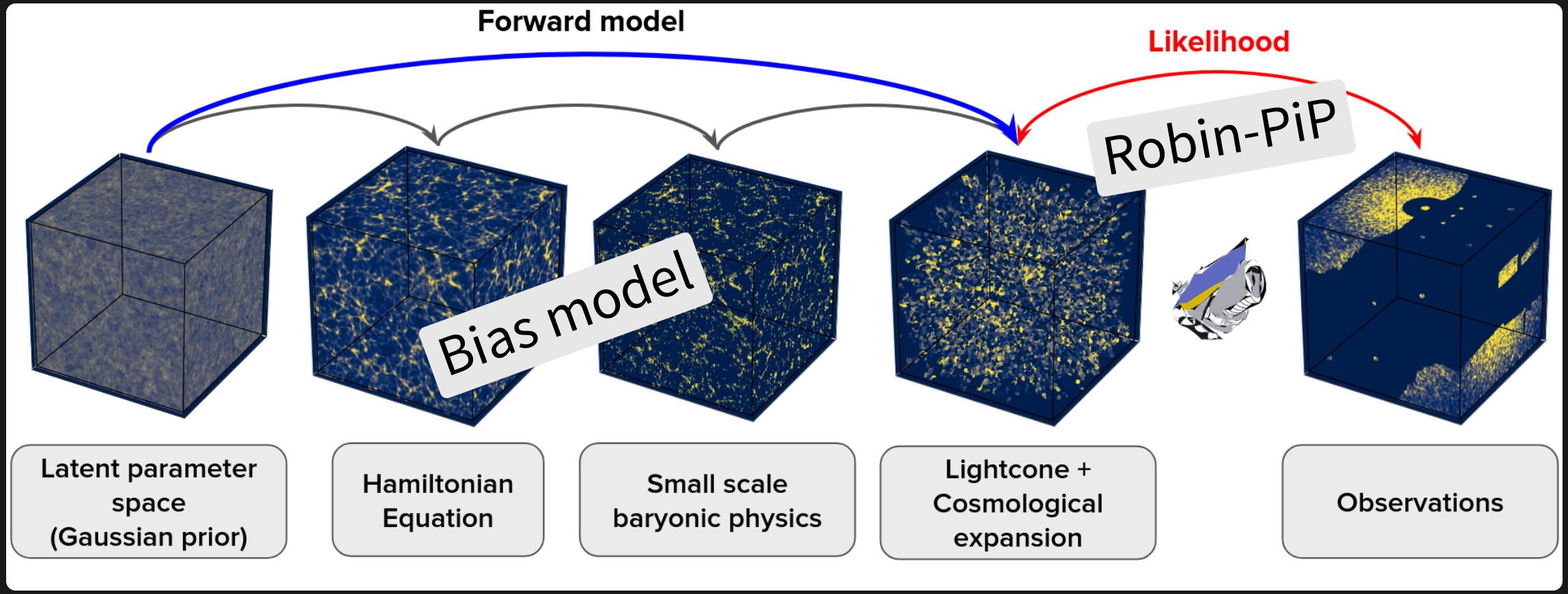

$\approx 2.1 \times 10^6$ parameters

Use self-consistent simulations

Jasche & Wandelt (2013), Jasche, Leclercq & Wandelt (2015), Lavaux & Jasche (2016), Jasche & Lavaux (2019)

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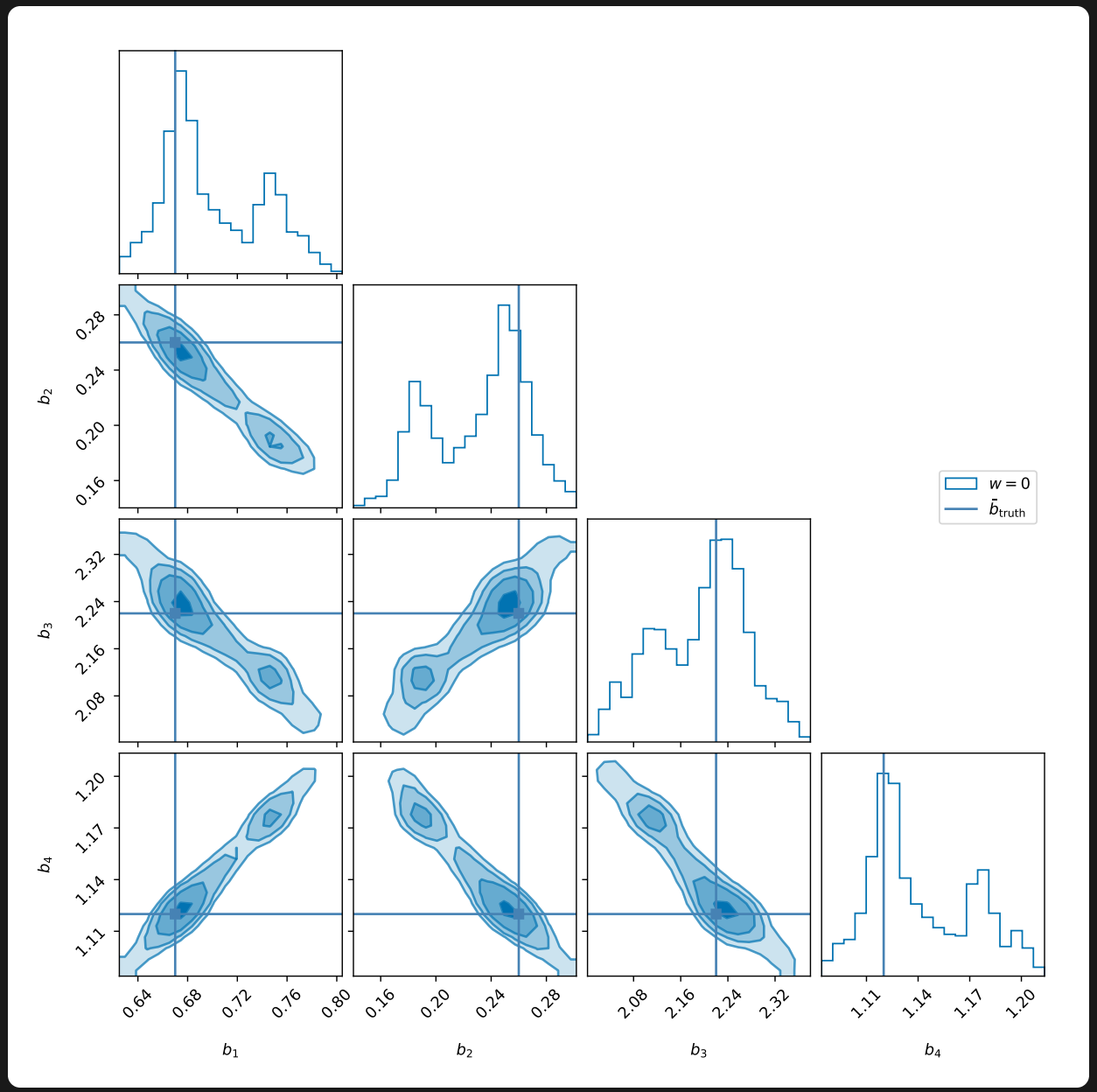
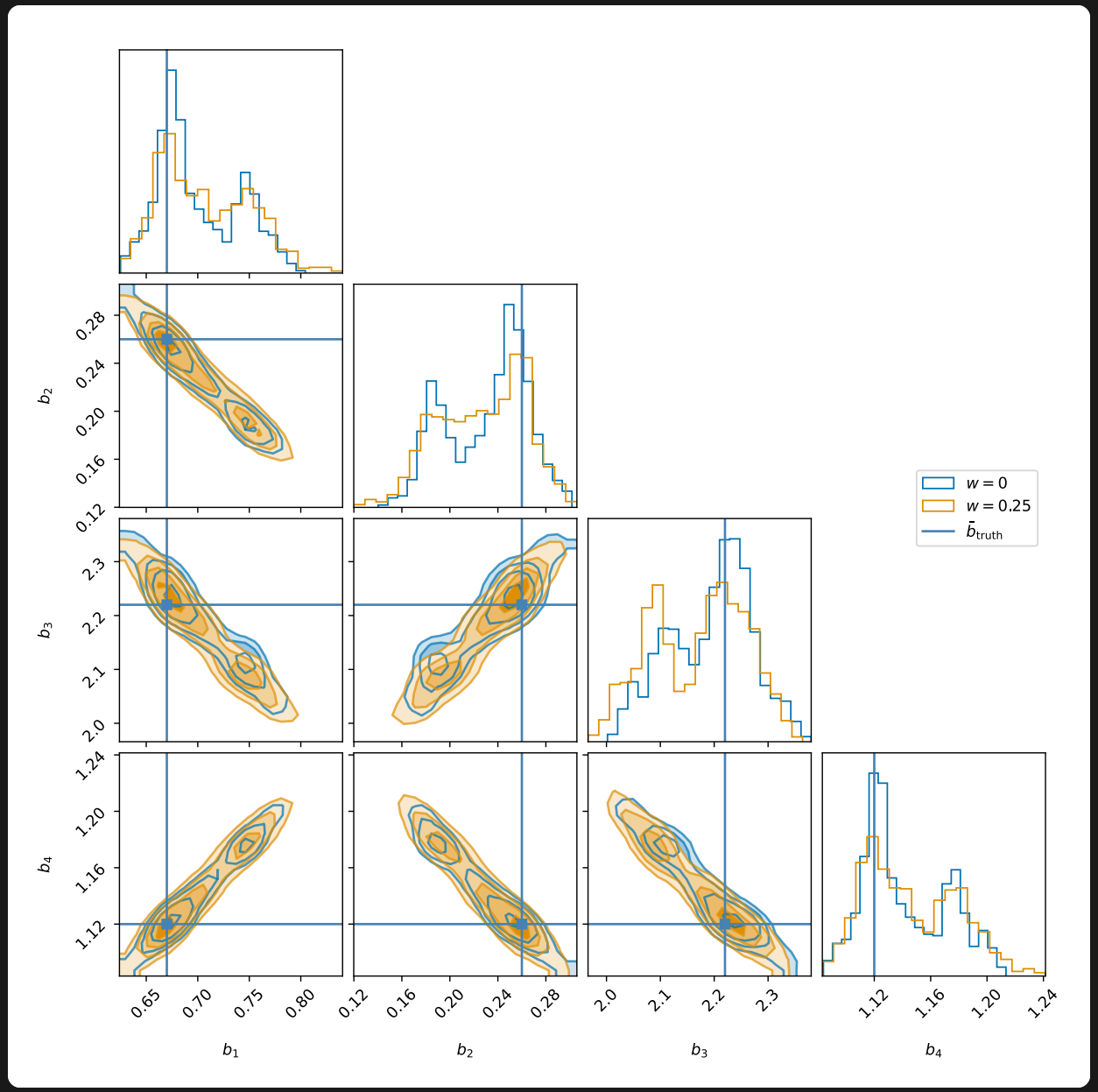
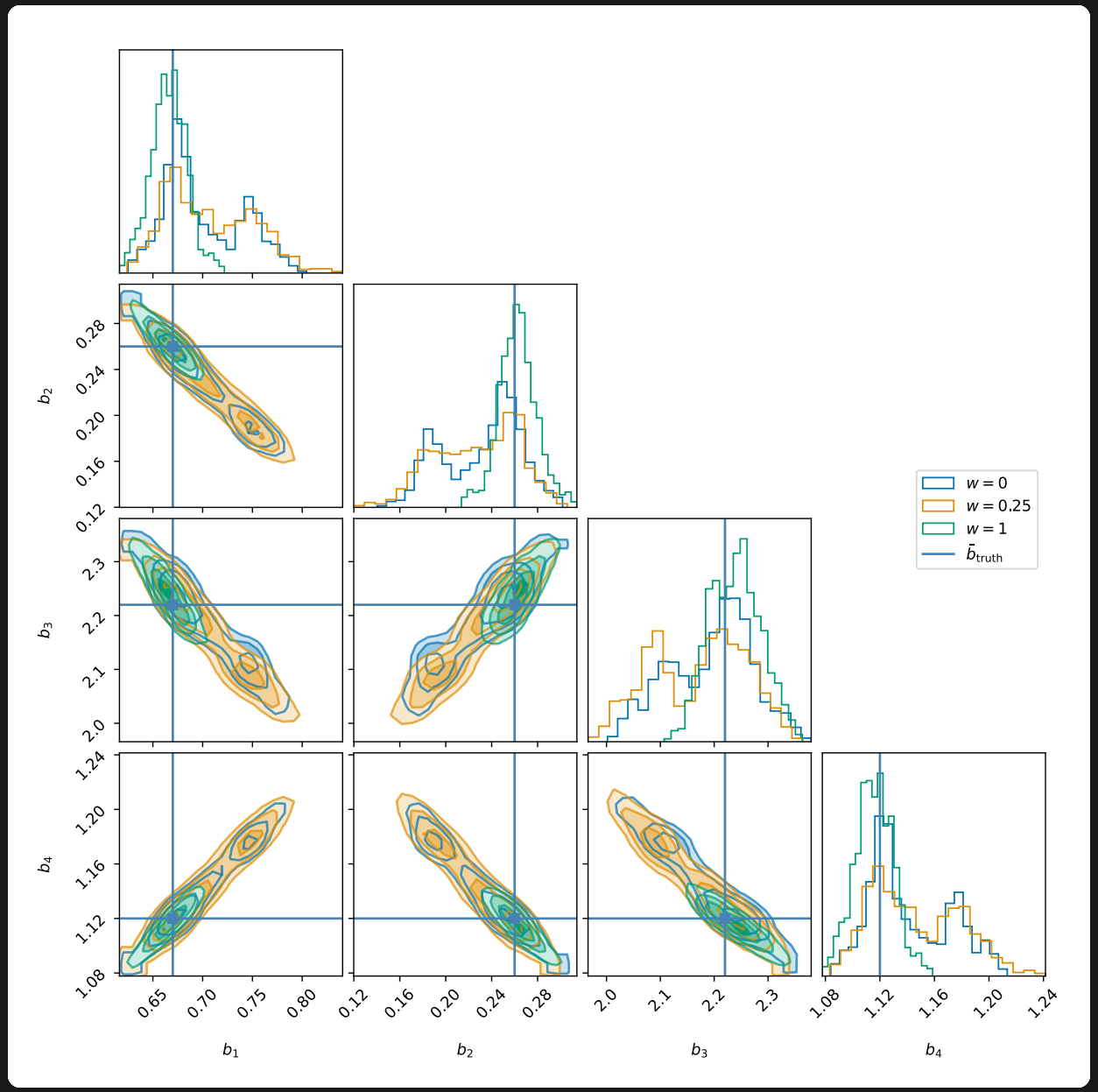
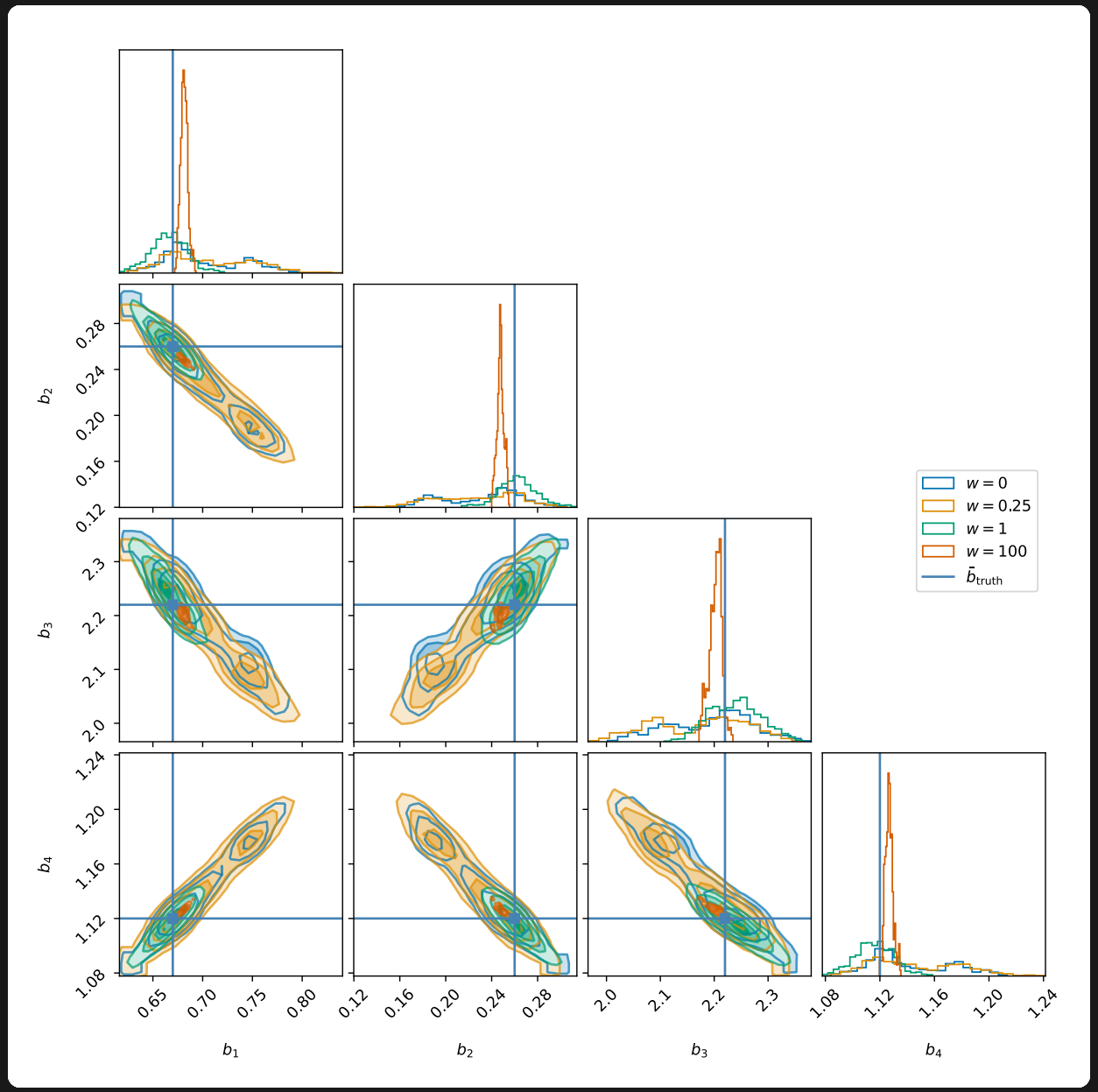

$w = 0$
$\vec{b}_{\text{truth}}$
b_1
b_2
b_3
b_4
0.64
0.68
0.72
0.76
0.80
0.16
0.20
0.24
0.28
2.08
2.16
2.24
2.32
1.11
1.14
1.17
1.20


$w = 0$
$w = 0.25$
$\tilde{b}_{\text{truth}}$
b_1
b_2
b_3
b_4
0.65
0.70
0.75
0.80
0.12
0.16
0.20
0.24
0.28
2.0
2.1
2.2
2.3
1.12
1.16
1.20
1.24


$w = 0$
$w = 0.25$
$w = 1$
$\bar{b}_{\text{truth}}$
b_1
b_2
b_3
b_4
0.65
0.70
0.75
0.80
0.12
0.16
0.20
0.24
0.28
2.0
2.1
2.2
2.3
1.08
1.12
1.16
1.20
1.24


$w = 0$
$w = 0.25$
$w = 1$
$w = 100$
$\bar{b}_{\text{truth}}$
b_1
b_2
b_3
b_4
0.65
0.70
0.75
0.80
0.12
0.16
0.20
0.24
0.28
2.0
2.1
2.2
2.3
1.08
1.12
1.16
1.20
1.24

BAYESIAN INFERENCE WITH PHYSICS INFORMED PRIORS

using ROBIN-PiP

BAYESIAN INFERENCE WITH PHYSICS INFORMED PRIORS

using ROBIN-PiP

- Principled way of incorporating simulations into inference

BAYESIAN INFERENCE WITH PHYSICS INFORMED PRIORS

using ROBIN-PiP

- Principled way of incorporating simulations into inference
- Model agnostic

BAYESIAN INFERENCE WITH PHYSICS INFORMED PRIORS

using ROBIN-PiP

- Principled way of incorporating simulations into inference
- Model agnostic
- Can enable direct inference of more sophisticated model
  e.g. neural networks

Back-up slides

ROBIN-PiP × BORG