Celestial Holography

1/13/25 - 1/16/25 @ Sanya

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soft physics book: 1703.05448

celestial lectures: 2108.04801

survey up to '21: 2111.11392

short summary: 2310.04932

4 × 1.5 hour lectures

1) Motivation + IR Triangle Primer
2) Asymptotic Symmetries & Soft Theorems
3) Celestial Amplitudes & the Holographic Dictionary
4) Holographic Symmetry Algebras & Future Directions

Lecture 1: Motivation + IR Triangle Primer

Goals: Why flat holography?

What's different about flat spacetimes?
- set up Penrose diagram for $\mathbb{R}^{1,3}$
- motivate IR triangle

How have people tackled these questions?
- flat lim of AdS
- Carrollian CFTs
- Celestial CFT

$$\begin{tikzcd} & \text{Quantum Gravity} \arrow[dr, "\text{consistency in UV}"'] & \\ \text{BH thermo.} \arrow[r] & \text{Holography} & \text{String Theory} \arrow[l, "\text{AdS/CFT correspondence}"'] \end{tikzcd}$$

Holographic Principle: A theory of quantum gravity can be encoded in a lower dim theory w/out gravity at the spacetime boundary.

Celestial Holography: want to apply the holo. princ. to $\Lambda = 0$ spacetimes.

Two Approaches

Top Down : Find stringy construction.

Bottom Up : Match symmetries, identify consistency conditions.

we will follow this approach

Carrollian CFTs

dim. red. to Celestial CFT

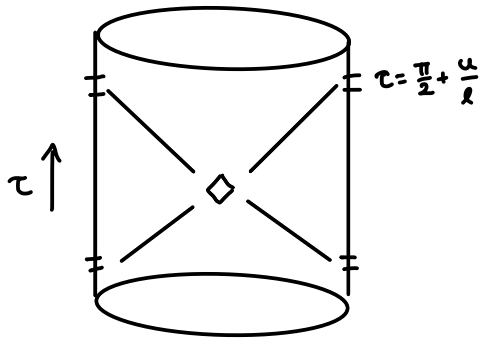
- causal structure of the boundary is different

- $\Lambda = 0$ spacetimes have an enhanced asymptotic sym. group

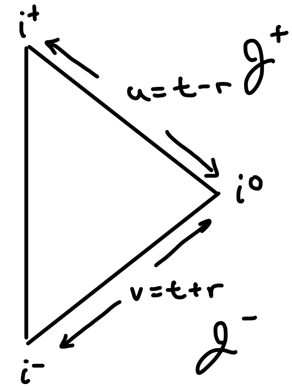
IR Triangle

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$\Lambda < 0$



$\Lambda = 0$



$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right)dt^2 + \left(1 + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + 2r^2 \gamma_{z\bar{z}}\, dz\, d\bar{z}$$

$$= -\left(1 + \frac{r^2}{\ell^2}\right) du^2 - 2du\,dr + 2r^2 \gamma_{z\bar{z}}\, dz\, d\bar{z}$$

flat limit $\longrightarrow -du^2 - 2du\,dr + 2r^2\gamma_{z\bar{z}}\, dz\, d\bar{z}$

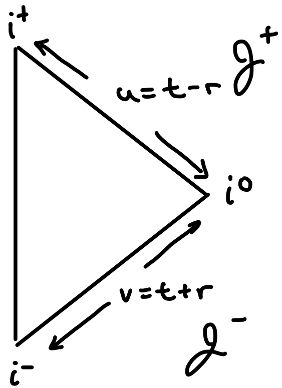
large r $\longrightarrow \sim r^2\left(\frac{-1}{\ell^2} du^2 + 2\gamma_{z\bar{z}}\, dz\, d\bar{z}\right)$

$z = e^{i\phi}\tan\frac{\theta}{2}$, $\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$

$t = u + \ell \arctan\frac{r}{\ell}$

$\Lambda \to 0,\ \ell \to \infty$ where $\ell^2 = \frac{3}{|\Lambda|}$

$c \sim \frac{1}{\ell}$ carrollian limit



starting w/ null coordinates

$$u = t - r \quad , \quad v = t + r$$

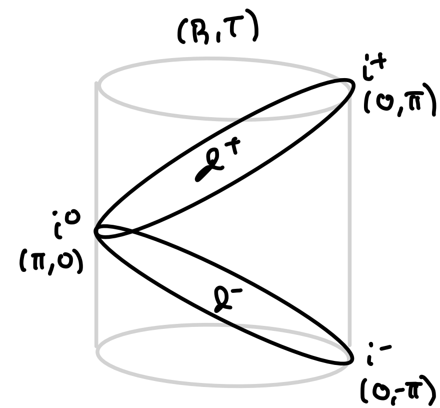
we introduce rescaled coords.

$$u = \tan U \quad , \quad v = \tan V$$

w/ $U, V \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Then the metric in terms of

$$T = U + V \quad , \quad R = V - U$$

is conformal to a patch of $S^3 \times \mathbb{R}$



- massive particles begin at i^- and end at i^+
- massless particles begin at $\mathcal{J}^-$ and end at $\mathcal{J}^+$
- spacelike geodesics end at i^0

Asymptotically flat spacetimes

- general soln's to $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

- have the same conf. boundary as $\mathbb{R}^{1,3}$ (ignoring bh's)

- have a larger asymptotic symmetry group

$$ds^2 = \underbrace{-du^2 - 2du\,dr + 2r^2\gamma_{z\bar{z}}}_{\text{flat metric}} + \frac{2m_B}{r}du^2 + \left(D^z C_{zz} + \frac{1}{r}\left[\frac{4}{3}N_z - \frac{1}{4}D_z(C_{zz}C^{zz})\right]\right)du\,dz + \text{c.c.}$$

$$+\, r C_{zz}\,dz^2 + \text{c.c.} + \ldots$$

Bondi mass

angular mom. asp.

cov. deriv. on S^2

radiative dof

subleading in r

$\partial_u m_B$, $\partial_u N_z$ constrained by eom

$$\text{Asymptotic Symmetries} = \frac{\text{Allowed Symmetries}}{\text{Trivial Symmetries}}$$

Bondi, van der Burg, Metzner, Sachs '62

look at diffeos ξ which preserve gauge s.t. $\mathcal{L}_\xi g_{\mu\nu} \sim$ same falloffs as above

the # of ξ which act non-triv. on rad. data is ∞!

$$\xi = f\partial_u - \frac{1}{r}(D^z f\partial_z + D^{\bar z} f\partial_{\bar z}) + D^z D_z f\partial_r$$
$$+ (1+\frac{u}{2r})Y^z\partial_z - \frac{u}{2r}D^{\bar z} D_z Y^z\partial_{\bar z}$$
$$- \frac{1}{2}(u+r)D_z Y^z\partial_r + \frac{u}{2}D_z Y^z\partial_u + \text{c.c.} + \ldots$$

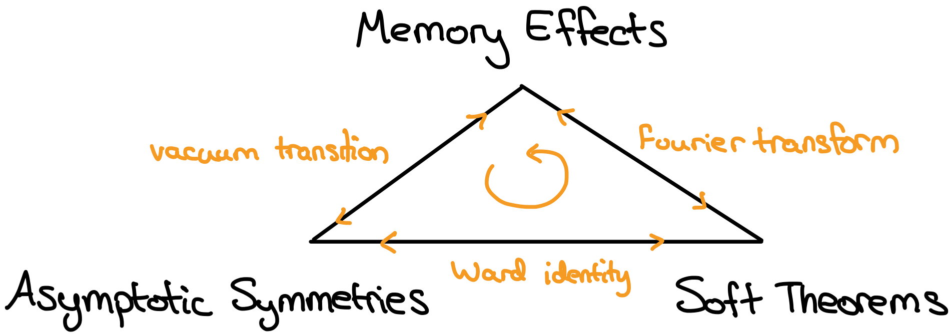
$f(z,\bar z)$ supertranslations ∞!

$Y(z)$ superrotations

vs. Poincare $\quad f = c_1 + c_2\frac{z+\bar z}{1+z\bar z} + c_3\frac{i(\bar z - z)}{1+z\bar z} + c_4\frac{1-z\bar z}{1+z\bar z}$

10! $\quad Y = a + bz + cz^2$

IR Triangle



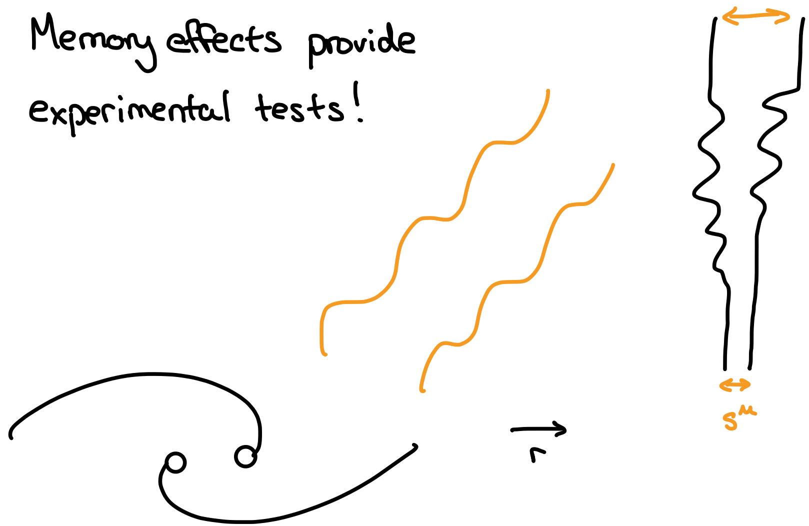
focus of next lecture

- leading E&M
- leading grav.
- subleading grav.

...

pattern → predictions

Memory effects provide experimental tests!



$$\partial_\tau^2 s^\mu = R^\mu{}_{\lambda\rho\nu} t^\lambda t^\rho s^\nu$$

geod. dev.

$$\downarrow$$

$$\partial_u^2 s^{\bar z} = \frac{\gamma^{z\bar z}}{2r}\,\partial_u^2 C_{zz} s^z$$

$$\downarrow$$

$$\Delta s^{\bar z} = \frac{\gamma^{z\bar z}}{2r}\,\Delta C_{zz} s^z$$

$t^\lambda \partial_\lambda \sim \partial_u$, $\tau \sim u$

$R_{zuzu} \sim -\frac{1}{2} r \partial_u^2 C_{zz}$

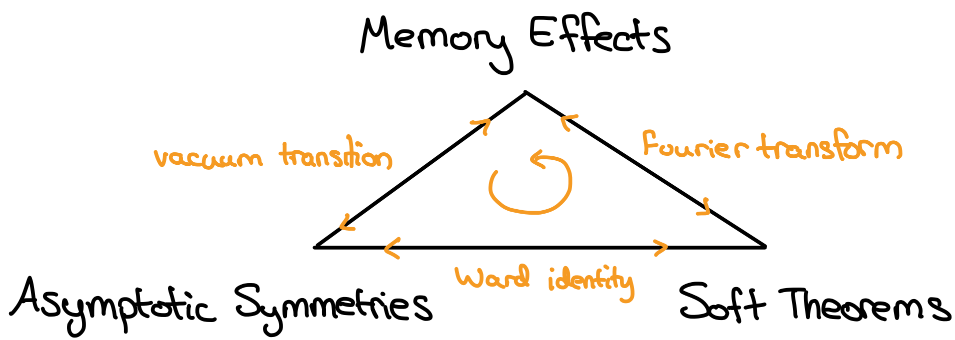
Strom. lec. ex. 13

$\exists$ non-triv. tail behavior of grav. waveform

- $\rightarrow$ meas. w/ asymp. detectors — mem. effect
- $\rightarrow$ $\Theta(u) \xrightarrow{\text{F.T.}} \frac{1}{\omega}$ — soft pole
- $\rightarrow$ $\Delta C_{zz} = -2 D_z^2 \Delta C$ — vac. trans.

Lecture 2: Asymptotic Symmetries & Soft Theorems

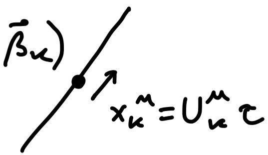
Goal: Demonstrate Asymptotic sym. $\Leftrightarrow$ soft thm. for U(1) example



focus of this lecture

Let us start by considering the electromagnetic field for a set of moving point charges with charge Q_k and 4-velocity U_k^μ

$$U_k^\mu = \gamma_k(1, \vec{\beta}_k)$$

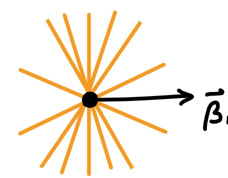


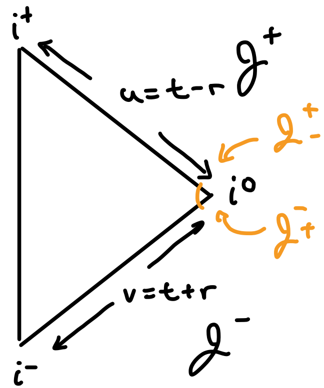
$$j_\mu^M(x) = \sum_{k=1}^{n} Q_k \int d\tau \, U_{k,\mu} \, \delta^{(4)}(x^\nu - U_k^\nu \tau)$$

The Liénard-Wiechert solution to $\nabla^\mu F_{\mu\nu} = e^2 j_\nu^M$ has

$$F_{rt}(t, \vec{x}) = \frac{e^2}{4\pi} \sum_{k=1}^{n} \frac{Q_k \gamma_k (r - t\hat{x}\cdot\vec{\beta}_k)}{|\gamma_k^2 (t - r\hat{x}\cdot\vec{\beta}_k)^2 - t^2 + r^2|^{3/2}}$$

$\vec{E}$ field lines at $t=0$





Now let's examine how things behave near the conf. bndy.

recall: $r \to \infty$, u fixed $\;\to \mathcal{J}^+$

$r \to \infty$, v fixed $\;\to \mathcal{J}^-$

also $F_{rt} = F_{ru} = F_{rv}$ due to anti-sym

$$F_{ru}\big|_{\mathcal{J}^+} = \frac{e^2}{4\pi r^2} \sum_{k=1}^{n} \frac{Q_k}{\gamma_k^2 (1 - \hat{x}\cdot\vec{\beta}_k)^2}$$

$$F_{rv}\big|_{\mathcal{J}^-} = \frac{e^2}{4\pi r^2} \sum_{k=1}^{n} \frac{Q_k}{\gamma_k^2 (1 + \hat{x}\cdot\vec{\beta}_k)^2}$$

$$\lim_{r\to\infty} r^2 F_{ru}(\hat{x})\big|_{\mathcal{J}^+_-} = \lim_{r\to\infty} r^2 F_{rv}(-\hat{x})\big|_{\mathcal{J}^-_+}$$

antipodal matching!

this will play an important role

What is the ASG for U(1) gauge theory?

$$S = \frac{-1}{4e^2}\int d^4x \sqrt{-g}\, F_{\mu\nu}F^{\mu\nu} + S_M$$

$$\xrightarrow{\delta/\delta A_\mu} \quad \nabla^\mu F_{\mu\nu} = e^2 j^M_\nu \quad \text{where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \text{ and } \nabla^\mu j^M_\mu = 0$$

Now there is also a gauge sym $\delta_\varepsilon A_\mu = \partial_\mu \varepsilon(u,r,z,\bar{z})$ we should gauge fix

$\nabla^\mu A_\mu = 0$ still allows ε s.t. $\Box \varepsilon = 0$

residual gauge dof.

Consider the asymptotic expansion

$$\mathcal{O}(u,r,z,\bar{z}) = \sum_n r^{-n}\,\mathcal{O}^{(n)}(u,z,\bar{z})$$

Then solving $\Box\mathcal{E}=0$ order-by-order gives

$$(\Box\mathcal{E})^{(n)} = 2(n-2)\partial_u \mathcal{E}^{(n-1)} + \left[D^2 + (n-2)(n-3)\right]\mathcal{E}^{(n-2)}$$

$\mathcal{E}^{(1)}(u,z,\bar{z})$ free data

Can use this to set $A_u^{(1)}=0$. Then

$$A_u \sim \mathcal{O}\left(\tfrac{1}{r^2}\right) \qquad A_r \sim \mathcal{O}\left(\tfrac{1}{r^2}\right) \qquad A_A \sim \mathcal{O}(1)$$

This residual gauge fixing still allows for a non-zero $\varepsilon^{(0)}(z,\bar{z})$.

$$Q_\varepsilon = \frac{1}{e^2}\int_{i^0} \varepsilon(z,\bar{z}) \star F$$

generates the non-trivial $\delta_\varepsilon A_A = \partial_A \varepsilon$ respecting our b.c.'s

$$\Rightarrow \quad ASG \ni \text{large } U(1) \text{ gauge trans.}$$

Meanwhile bc of antipodal matching of $F_{ru}|_{\mathcal{I}^+_-}$ and $F_{ru}|_{\mathcal{I}^-_+}$:

$$\underbrace{Q^+_\varepsilon = \frac{1}{e^2}\int_{\mathcal{I}^+_-} \varepsilon \star F = \frac{1}{e^2}\int_{\mathcal{I}^-_+} \varepsilon \star F = Q^-_\varepsilon}_{\text{our Ward id.}} \qquad \text{if} \quad \varepsilon(z,\bar{z})|_{\mathcal{I}^+_-} = \varepsilon(z,\bar{z})|_{\mathcal{I}^-_+}$$

antipodal pts.

Can we see $\langle \text{out} | Q^+_\varepsilon S - S Q^+_\varepsilon | \text{in} \rangle = 0$ in S-matrix elements?

leading r-behavior of u-component of eom:

$$\partial_u F^{(2)}_{ru} + D^z F^{(0)}_{uz} + D^{\bar{z}} F^{(0)}_{u\bar{z}} + e^2 j^{(2)}_u = 0 \qquad \text{(assume } m=0 \text{ charges)}$$

$$\downarrow$$

$$Q^+_\varepsilon = -\frac{1}{e^2} \int_{\mathcal{I}^+} \underbrace{du\, d^2z \left(\partial_z \varepsilon F^{(0)}_{u\bar{z}} + \partial_{\bar{z}} \varepsilon F^{(0)}_{uz} \right)}_{Q_S \text{ (photon field)}} + \int_{\mathcal{I}^+} \underbrace{du\, d^2z\, \varepsilon \gamma_{z\bar{z}} j^{(2)}_u}_{Q_H \text{ (meas. charge)}}$$

ditto for Q^-_ε.

Now $\langle \text{out} | Q_S^+$ will change the state to one w/ additional photon

$\langle \text{out} | Q_H^+$ no change in particle # soft thm!

$$A_\mu(x) = e\sum_{\alpha=\pm}\int\frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}\left[\varepsilon_\mu^{\alpha *}(\vec{q})\,a_\alpha(\vec{q})\,e^{iq\cdot x} + \varepsilon_\mu^{\alpha}(\vec{q})\,a_\alpha(\vec{q})^\dagger e^{-iq\cdot x}\right]$$

for our gauge $F_{uz}^{(0)} = \partial_u A_z^{(0)} - \cancel{\partial_z A_u^{(0)}}$ where $A_z^{(0)} = \lim_{r\to\infty}\partial_z x^\mu A_\mu(x)$

now at large r fixed u: $e^{iq\cdot x} = e^{-i\omega_q u - i\omega_q r(1-\cos\theta)} \to e^{-i\omega_q u}\times\frac{1}{\omega_q r}\frac{\delta(\theta)}{\sin\theta}$

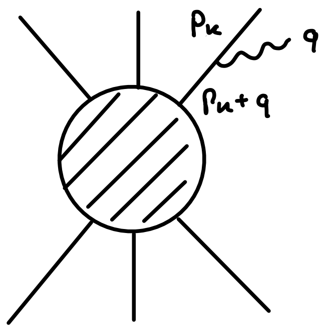
$$\Rightarrow A_z^{(0)} = \frac{-i}{8\pi^2}\frac{\sqrt{2}e}{1+z\bar{z}}\int_0^\infty d\omega_q\left[a_+(\omega_q\hat{x})e^{-i\omega_q u} - a_-(\omega_q\hat{x})^\dagger e^{i\omega_q u}\right]$$

$$\Rightarrow \int du\,F_{uz}^{(0)} = \frac{-1}{8\pi}\frac{\sqrt{2}e}{1+z\bar{z}}\lim_{\omega\to 0^+}\left[\omega a_+(\omega\hat{x}) + \omega a_-(\omega\hat{x})^\dagger\right]$$

so $\langle \text{out}|Q_S^+ S - SQ_S^-|\text{in}\rangle = -\langle\text{out}|Q_H^+ S - SQ_H^-|\text{in}\rangle$ from our Ward id

but we know this from soft thms!

Weinberg tells us these insertions have a universal form!



$$\langle out | a_+(\vec{q}) S | in \rangle = e \sum_{out-in} \frac{Q_k\, p_k \cdot \varepsilon^+}{p_k \cdot q} \langle out | S | in \rangle + \mathcal{O}(\omega_q^0)$$

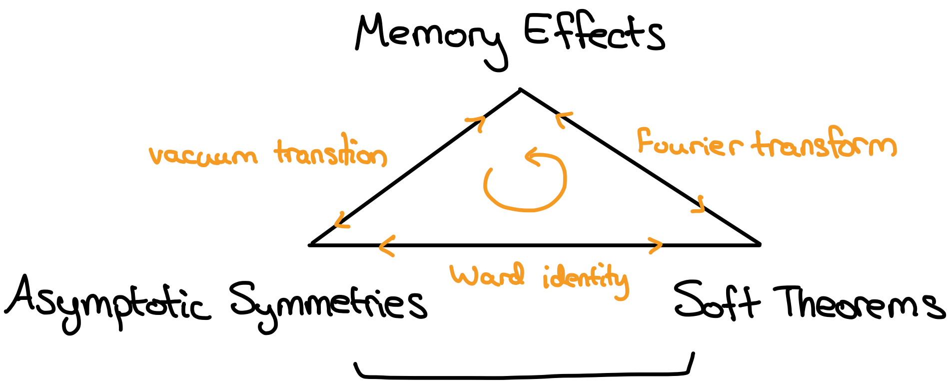
Plugging in the soft theorem

$$\langle out | \int du\, F_{uz}^{(0)} S | in \rangle = \frac{-e^2}{4\pi} \sum_{out-in} \frac{Q_k}{z - z_k} \langle out | S | in \rangle$$

we indeed have $\langle out | Q_\varepsilon^+ S - S Q_\varepsilon^- | in \rangle = 0$

Ward id $\Longleftrightarrow$ soft thm!

Looking back at the IR triangle



- leading E&M large U(1)
- leading grav. Supertranslations
- NEW! ⇒ subleading grav. Superrotations
- ...

pattern → predictions

But look! $j^+ := Q_S^+\left(\varepsilon = \frac{1}{z-\omega}\right) = -4\pi \int du\, F_{uz}$ obeys

$$\langle j(z)\, \mathcal{O}_1(z_1,\bar{z}_1)\ldots\mathcal{O}_n(z_n,\bar{z}_n)\rangle = \sum_k \frac{Q_k}{z-z_k}\langle \mathcal{O}_1(z_1,\bar{z}_1)\ldots\mathcal{O}_n(z_n,\bar{z}_n)\rangle$$

ASG $\Rightarrow$ 2D Kac-Moody sym. of S-matrix?

Next time: reorganize scattering to make these symmetries manifest!

CCFT!