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(Machine Learning)

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Chain Rule Assignment

(1) Given, $f(z) = \log_e(1+z)$ where $z = x^T x$, $x \in \mathbb{R}^d$

Sol^n:

If $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$, then, $x^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_d \end{bmatrix}$

$\therefore x^T x = \begin{bmatrix} x_1^2 & x_2^2 & \cdots & x_d^2 \end{bmatrix}$

Now, Applying the chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(\log_e(1+z)\right) \cdot \frac{d}{dx}\left(x^T x\right)$$

$$= \frac{1}{1+z} \cdot \frac{d}{dz}(1+z) \cdot \frac{d}{dx}\left(x_1^2 + x_2^2 + \cdots + x_d^2\right)$$

$$= \frac{1}{1+z} \cdot 1 \cdot \left(2x_1 + 2x_2 + \cdots + 2x_d\right)$$

$$= \frac{1}{1+z} \cdot 2\left(x_1 + x_2 + \cdots + x_d\right)$$

$$= \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

(Ans)

(2) $f(z) = e^{-\frac{z}{2}}$; where $z = g(y)$, $g(y) = y^T S^{-1} y$, $y = h(x)$,

$h(x) = x - \mu$

Soln:

Using the chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx} \quad \text{—— (1)}$$

here, $\frac{df}{dz} = \frac{d}{dz}\left(e^{-z/2}\right) = -\frac{e^{-z/2}}{2}$

Now, $\frac{dz}{dy} = \frac{d}{dy}\left(y^T S^{-1} y\right)$

$$= \lim_{h\to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h\to 0} \frac{(y^T + h) S^{-1} (y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h\to 0} \frac{(y^T S^{-1} + h S^{-1})(y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{y^T S^{-1} y} + y^T S^{-1} h + y h S^{-1} + h^2 S^{-1} - \cancel{y^T S^{-1} y}}{h}$$

$$= \lim_{h\to 0} \frac{h(y^T S^{-1} + y S^{-1} + h S^{-1})}{h} = \lim_{h\to 0} \left(y^T S^{-1} + y S^{-1} + h S^{-1}\right)$$

$$= y^T S^{-1} + y S^{-1}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x-\mu) = 1$$

$\therefore$ from eq (1),

$$\frac{df}{d\cancel{z}x} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{e^{-z/2}}{2} \cdot \left(y^T s^{-1} + y s^{-1}\right) \cdot (1)$$

$$= -\frac{e^{-z/2}}{2s} \cdot \left(y^T + y\right) \quad \text{(Ans)}$$