# Project 1: From the Ising model to nano-granular systems

You investigated in the class the ferromagnetic-paramagnetic transition as well as the first-order transition seen in the magnetic response of a ferromagnet where you adopted the two dimensional Ising model. The goal of this project is to carry out similar investigations using more complex models that may better correspond to realistic systems. Refer to the instructions below for the details.

1. There are many ways that one can think of in generalizing the two dimensional Ising model. Some of these ways are listed below. Consider these points systematically so that you can reach sound conclusions:

   - System size: Dealing with a realistic macroscopic system is computationally intractable. You should see the effect of the finite size you consider and how the properties are affected by the size you choose.
   - Dimension: The spins you studied before were arranged in two dimensions. Consider the arrangement in a three-dimensional cubic lattice.
   - Nearest-neighbours interactions: Include the effect of the interactions with further neighbours. You can adopt a similar formula for the energy of interaction, but you should reduce the strength of interaction as the spins get further apart. In other words, you may reduce the value of $J$ that was taken to be unity for nearest-neighbours interactions. You should justify your choice of the amount by which you reduce $J$.
   - General spin direction: Allow the spins to take an arbitrary direction in space (i.e., not only restricted to the two-valued state of 'up' and 'down'). In this case, each spin is represented by a unit vector $\mathbf{s}$. The interaction energy between the $i^{\text{th}}$ and the $j^{\text{th}}$ spins can then be written as

     $$E_{ij} = -J\ \mathbf{s}_i \cdot \mathbf{s}_j.$$ [1]

     Note that this reduces to the Ising model energy if you restrict the directions of the spins to be along a specific axis.
   - Type of interaction: You can make the spin-spin interaction more realistic by modelling each spin as a perfect magnetic dipole. Hence, their interaction will adopt the form of the dipolar interaction:

     $$E = -\frac{\mu_0}{4\pi}\ \frac{3(\mathbf{s}_i \cdot \hat{\mathbf{r}})(\mathbf{s}_j \cdot \hat{\mathbf{r}}) - \mathbf{s}_i \cdot \mathbf{s}_j}{r^3}$$

     where $r$ is the distance between the two spins and $\hat{\mathbf{r}}$ is a unit vector along their separation line. Note that we are assuming here, unlike the previous cases, that the spin vectors are not dimensionless, but they carry the unit of a magnetic moment.
   - Random systems: In your previous analysis, you assumed that all spins are located on a regular cubic or square lattices. Study other regular arrangements. Also, consider the case where the spins are randomly located in space.

[1] This model is referred to as the classical Heisenberg model.

- Ask you own question: Think of other points, that are not mentioned above, and consider their effects.
- Combine the effects: Combine two or more of the effects described above. What is the expected outcome upon the combination?

2. In all variant models and systems described above, study the effect of these variations on the following:
   - The existence of a Ferromagnetic-paramagnetic transition and the corresponding Transition temperature.
   - Magnetic response to external magnetic field and the existence of hysteresis.
   - The heat capacity of the system as a function of the temperature.
   - The magnetic susceptibility of the system as a function of the applied field.
   - Think of other properties that you wish to study and see how they change with the aforementioned variations.

Try to do your best with your investigations and discussion. You may not be able to consider all points mentioned above, but you must cover four points, at least. The rest are optional. You should understand that the research path is endless, so, a careful balance between the number of points to consider and the level of discussion should be considered.

For further reading and inspirations, you can spend time reading the attached articles while you celebrate the Eid break.

Happy Eid. Enjoy your project!