

## **EXERCISE SOLUTION**

1. Given N friends, each one can remain single or can be paired up with some other friend. Each friend can be paired only once. Find out the total number of ways in which friends can remain single or can be paired up.

Note: Since answer can be very large, return your answer mod  $10^9+7$ .

Example:

Input: N = 3 Output: 4 Explanation: {1}, {2}, {3} : All single {1}, {2,3} : 2 and 3 paired but 1

is single. {1,2}, {3} : 1 and 2 are paired but 3 is single. {1,3}, {2} : 1 and 3 are paired but 2

is single. Note that {1,2} and {2,1} are considered same.

# Returns count of ways

# n people can remain

# single or paired up.

def friend(n):

c = [0 for i in range(n + 1)]

# Filling dp[] in bottom-up manner using

# recursive formula explained above.

```
for i in range(n + 1):
```

```
    if(i <= 2):
```

```
        c[i] = i
```

```
    else:
```

```
        c[i] = c[i - 1] + (i - 1) * c[i - 2]
```

```
return c[n]
```

```
# Driver code
```

```
n = 3
```

```
print(friend(n))
```

```
'''
```

```
Time Complexity : O(n)
```

```
Auxiliary Space : O(n)
```

```
Another approach: (Using recursion)
```

```
'''
```

.....X.....X.....X.....

'''

2. Given a gold mine of  $n*m$  dimensions. Each field in this mine contains a positive integer which is the amount of gold in tons. Initially the miner is at first column but can be at any row. He can move only (right->,right up /,right down\)

that is from a given cell, the miner can move to the cell diagonally up towards the right or right or diagonally down towards the right. Find out maximum amount of gold he can collect.

Example:

Input : mat[][] = {{1, 3, 3}, {2, 1, 4}, {0, 6, 4}};Output : 12 {(1,0)->(2,1)-

>(1,2)}Input: mat[][] = { {1, 3, 1, 5}, {2, 2, 4, 1}, {5, 0, 2, 3}, {0, 6,

1, 2}};Output : 16(2,0) -> (1,1) -> (1,2) -> (0,3) OR(2,0) -> (3,1) -> (2,2) -> (2,3)Input : mat[][] =

{{10, 33, 13, 15}, {22, 21, 04, 1}, {5, 0, 2, 3}, {0, 6, 14, 2}};Output

: 83

'''

MAX = 100

# Returns maximum amount of

# gold that can be collected

```

# when journey started from
# first column and moves
# allowed are right, right-up
# and right-down
def getMaxGold(gold, m, n):

    # Create a table for storing
    # intermediate results
    # and initialize all cells to 0.
    # The first row of
    # goldMineTable gives the
    # maximum gold that the miner
    # can collect when starts that row
    goldTable = [[0 for i in range(n)]
                  for j in range(m)]

    for col in range(n-1, -1, -1):
        for row in range(m):

            # Gold collected on going to
            # the cell on the right(->)
            if (col == n-1):
                right = 0

```

else:

right = goldTable[row][col+1]

# Gold collected on going to

# the cell to right up (/)

if (row == 0 or col == n-1):

right\_up = 0

else:

right\_up = goldTable[row-1][col+1]

# Gold collected on going to

# the cell to right down (\)

if (row == m-1 or col == n-1):

right\_down = 0

else:

right\_down = goldTable[row+1][col+1]

# Max gold collected from taking

# either of the above 3 paths

goldTable[row][col] = gold[row][col] + max(right, right\_up, right\_down)

# The max amount of gold

# collected will be the max

```
# value in first column of all rows
```

```
res = goldTable[0][0]
```

```
for i in range(1, m):
```

```
    res = max(res, goldTable[i][0])
```

```
return res
```

```
# Driver code
```

```
gold = [[10, 33, 13, 15],
```

```
        [22, 21, 4, 1],
```

```
        [5, 0, 2, 3],
```

```
        [0, 6, 14, 2]]
```

```
m = 4
```

```
n = 4
```

```
print(getMaxGold(gold, m, n))
```

```
'''
```

```
Time Complexity : $O(m*n)$ 
```

```
Space Complexity : $O(m*n)$ 
```

'''

.....X.....X.....X.....

'''

3. Given an integer K and a queue of integers, we need to reverse the order of the first K elements of the queue, leaving the other elements in the same relative order.

Only following standard operations are allowed on queue.

- enqueue(x) : Add an item x to rear of queue
- dequeue() : Remove an item from front of queue
- size() : Returns number of elements in queue.
- front() : Finds front item.

Example:

Input:5 31 2 3 4 5Output: 3 2 1 4 5Explanation: After reversing the given input from the 3rd

position the resultant output will be 3 2 1 4 5.

'''

```
from queue import Queue
```

```
# Function to reverse the first K
```

```
# elements of the Queue

def reverseQueueFirstKElements(k, Queue):

    if (Queue.empty() == True or

        k > Queue.qsize()):

        return

    if (k <= 0):

        return

    Stack = []

    # put the first K elements

    # into a Stack

    for i in range(k):

        Stack.append(Queue.queue[0])

        Queue.get()

    # Enqueue the contents of stack

    # at the back of the queue

    while (len(Stack) != 0 ):

        Queue.put(Stack[-1])

        Stack.pop()

    # Remove the remaining elements and
```



```
# enqueue them at the end of the Queue
```

```
for i in range(Queue.qsize() - k):
```

```
    Queue.put(Queue.queue[0])
```

```
    Queue.get()
```

```
# Utility Function to print the Queue
```

```
def Print(Queue):
```

```
    while (not Queue.empty()):
```

```
        print(Queue.queue[0], end = " ")
```

```
        Queue.get()
```

```
# Driver code
```

```
if __name__ == '__main__':
```

```
    Queue = Queue()
```

```
    Queue.put(1)
```

```
    Queue.put(2)
```

```
    Queue.put(3)
```

```
    Queue.put(4)
```

```
    Queue.put(5)
```

```
    k = 3
```

```
    reverseQueueFirstKElements(k, Queue)
```

Print(Queue)

'''

Time Complexity:  $O(n+k)$ .

Where 'n' is the total number of elements in the queue and 'k' is the number of elements to be reversed.

This is because firstly the whole queue is emptied into the stack and after that first

'k' elements are emptied and enqueued in the same way.

Auxiliary Space :Use of stack to store values for the purpose of reversing-:  $O(n)$

'''

.....X.....X.....X.....

4. Given a string S. The task is to print all permutations of a given string.

Examples:

Input: ABSG

Output:ABGS ABSG AGBS AGSB ASBG ASGB BAGS BASG BGAS BGSA BSAG BSGA

GABS GASB GBAS GBSA GSAB GSBA SABG SAGB SBAG SBGA

SGAB SGBA

Explanation:Given string ABSG has 24 permutations

'''

# Function to find permutations of a given string

from itertools import permutations

```
def allPermutations(str):
```

```
    permList = permutations(str)
```

```
    # print all permutations
```

```
    for perm in list(permList):
```

```
        print (''.join(perm))
```

```
# Driver program
```

```
if __name__ == "__main__":
```

```
    str = 'ABSG'
```

```
    allPermutations(str)
```

```
'''
```

Expected Time Complexity:  $O(n! * n)$

Expected Space Complexity:  $O(n)$

```
'''
```

```
.....X.....X.....X.....
```

'''

5. Given the root of a binary tree. Check whether it is a BST or not.

Note: We are considering that BSTs cannot contain duplicate Nodes.

A BST is defined as follows:

- The left subtree of a node contains only nodes with keys less than the node's key.
- The right subtree of a node contains only nodes with keys greater than the node's key.
- Both the left and right subtrees must also be binary search trees.

Example:

Input: 2 / \ 1 3

Output: 1 Explanation: The left subtree of root node contains node with key lesser than the root node's key and the right subtree of root node contains node with key greater than the root node's key. Hence, the tree is a BST.

'''

""" Program to check if a given Binary

Tree is balanced like a Red-Black Tree """

# Helper function that allocates a new

# node with the given data and None

# left and right poers.

```
class newNode:
```

```
    # Construct to create a new node
```

```
    def __init__(self, key):
```

```
        self.data = key
```

```
        self.left = None
```

```
        self.right = None
```

```
# Returns true if given tree is BST.
```

```
def isBST(root, l = None, r = None):
```

```
    # Base condition
```

```
    if (root == None) :
```

```
        return True
```

```
    # if left node exist then check it has
```

```
    # correct data or not i.e. left node's data
```

```
    # should be less than root's data
```

```
    if (l != None and root.data <= l.data) :
```

```
        return False
```

```
    # if right node exist then check it has
```

```
    # correct data or not i.e. right node's data
```

```

# should be greater than root's data
if (r != None and root.data >= r.data) :
    return False

# check recursively for every node.
return isBST(root.left, l, root) and \
    isBST(root.right, root, r)

```

# Driver Code

```

if __name__ == '__main__':
    root = newNode(2)
    root.left = newNode(1)
    root.right = newNode(3)
    #root.right.left = newNode(1)
    #root.right.right = newNode(4)
    #root.right.left.left = newNode(40)
    if (isBST(root, None, None)):
        print("Is BST")
    else:
        print("Not a BST")

```

'''

Time Complexity:  $O(n)$

Auxiliary Space:  $O(1)$  if Function Call Stack size is not considered, otherwise  $O(n)$

.....X.....X.....X.....

6. Given a sorted array `arr` containing  $n$  elements with possibly duplicate elements, the task is to find indexes of first and last occurrences of an element  $x$  in the given array.

Example:

Input:  $n=9$ ,  $x=5$ , `arr[] = { 1, 3, 5, 5, 5, 5, 67, 123, 125 }`

Output: 2 5  
Explanation: First occurrence of 5 is at index 2 and last occurrence of 5 is at index 5.

'''

# last occurrences of a number in

# a given sorted array

# if  $x$  is present in `arr[]` then

# returns the index of FIRST

# occurrence of  $x$  in `arr[0..n-1]`,

# otherwise returns -1

def first(arr, low, high, x, n) :

```

if(high >= low) :
    mid = low + (high - low) // 2
    if( ( mid == 0 or x > arr[mid - 1]) and arr[mid] == x) :
        return mid
    elif(x > arr[mid]) :
        return first(arr, (mid + 1), high, x, n)
    else :
        return first(arr, low, (mid - 1), x, n)

return -1

```

# if x is present in arr[] then

# returns the index of LAST occurrence

# of x in arr[0..n-1], otherwise

# returns -1

```
def last(arr, low, high, x, n) :
```

```

    if (high >= low) :
        mid = low + (high - low) // 2
        if (( mid == n - 1 or x < arr[mid + 1]) and arr[mid] == x) :
            return mid
        elif (x < arr[mid]) :
            return last(arr, low, (mid - 1), x, n)

```



else :

return last(arr, (mid + 1), high, x, n)

return -1

# Driver program

arr = [1, 3, 5, 5, 5, 5, 67, 123, 125]

n = len(arr)

x = 5

print("First Occurrence = ",

first(arr, 0, n - 1, x, n))

print("Last Occurrence = ",

last(arr, 0, n - 1, x, n))

'''

Time Complexity :  $O(\log n)$

Auxiliary Space :  $O(\log n)$

'''

.....X.....X.....X.....

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