

Probability dist-  $\Rightarrow$  ① PMS | CDF Probability Dist With Discrete R.V.  
✓ ② PDF/CDF Probability dist with Continuous R.V.  
③ CDF

PMS

- ① Uniform (Coin, Dice)
- ② Bernoli Dist (Coin)  $\Rightarrow$  Binary outcome
- ③ Binomial Dist  $\Rightarrow$  n try with binary outcome
- ④ multinomial Dist  $\Rightarrow$  n try with multiple outcome

5 Possion Distribution (time)

CDF  
(Cumulative Dist-  
function)

PDF  $\Rightarrow$  1 normal Distribution  
2 SND = 3 Log normal Dist-  
= 4 Exponantial Distr-  
= 5 Pareto Distr-  
= 6 Power law Distr-  
=

$\Rightarrow$  imp

$\Rightarrow$  Skewness

$\Rightarrow$  CDF

Power law

Pareto | Exponantial

⑦ t-distr., F-distr.; Chi-Square Distr.



Probability Distribution.

① Probability Density function

$$F(x) = P(a \leq x \leq b) = \int_a^b f(x) dx \geq 0$$

Range  
↓  
 $[a, b]$

② Probability Mass function

$$P(x) = P(X = x)$$

↑  
Random var. (Discrete Random var.)

③

### Cumulative Distribution function (CDF)

$$F(x) = P(X \leq x)$$

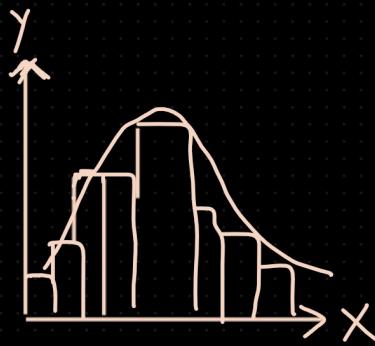
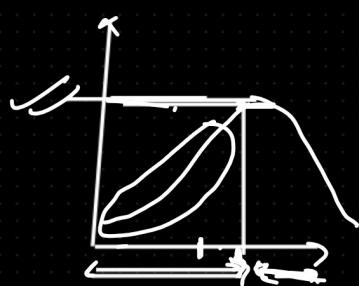
H H  $\Rightarrow$   
H T  $\Rightarrow$   
T H  $\Rightarrow$   
T T  $\Rightarrow$

$$0 \leq x \leq 2$$

$$H \leq 2$$

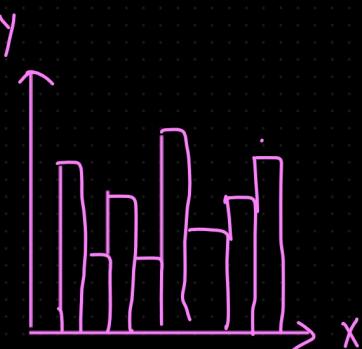
$$\boxed{\begin{array}{l} H = 0 \\ H = 1 \\ H = 2 \end{array}}$$

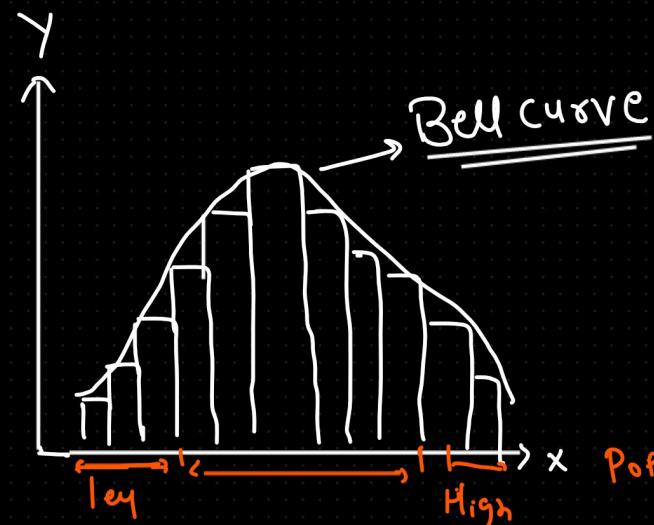
Data  $\Rightarrow$  Distributed (spread out) in different way.



left skewed Data

- ① income of the people
- ② retirement age of people





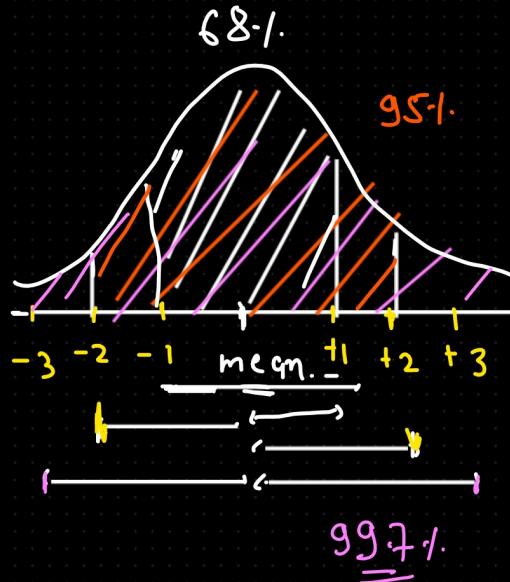
$$\boxed{\text{mean} = \text{median} = \text{mode}}$$

(Perfectly normal Dist)

Symmetry

- 1 height of people
- 2 size of things produced by machine.
- 3 errors in measurement
- 4 blood pressure
- 5 marks in the test

## Empirical rule



① S.D.

⇒ 100% if my Data is normally  
Distr.

68%

$$= \boxed{\text{Mean} \pm 1\sigma}$$

Spread of the  
Data on this X-axis

② S.D. ⇒ 95%

$$\boxed{\text{mean} \pm 2\sigma}$$

Spread of the data  
on Y-axis

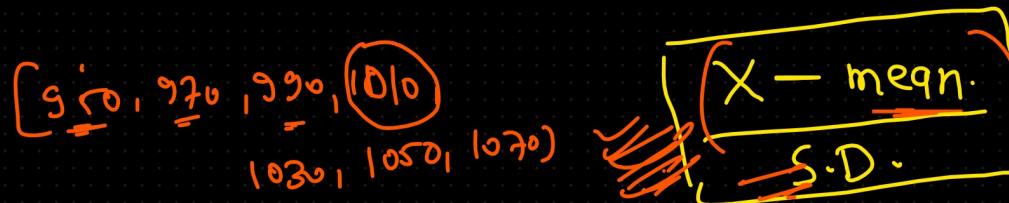
③ S.D. ⇒ 99.7%

$$\boxed{\text{mean} \pm 3\sigma}$$

↓  
Spread of the on X-axis

Standard Score :- the number of S.D. from the mean is called Standard Score

Or Z-score.



[mean = 0, std dev = 1]



Standardization  
⇒



Normal Dist

Standard normal Dist

max num  $\Rightarrow 60$ , min-threshold  $\Rightarrow 30$

Class  $\Rightarrow \{ 20, 15, 26, 32, 18, 28, 35, 14, 26, 22, 17 \}$

= fail?

Range of the is high.

$$\sum_{i=1}^N \frac{(x_i - \bar{x})^2}{N}$$

S.D.

$$\left( \frac{\bar{x} - M}{\sigma} \right)$$

$$\text{mean} = 23$$

$$\text{std.dev} = 6.6$$

$$-0.45, -1.21, 6.45, 1.36, -0.76, 0.76, 1.82, -1.36, 0.45, -0.15, -0.91$$



$\Rightarrow 68\%$

(1) reduce the range

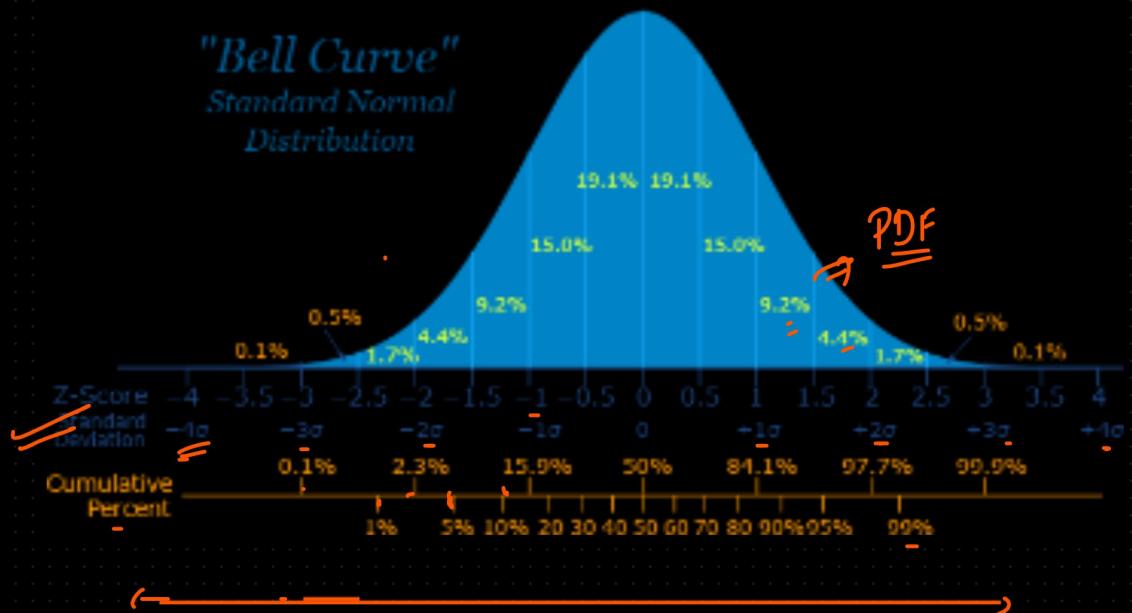
(2) to interpretation of the

Data wrt S.D.

(3) zero centric

$t_1, t_2$

*"Bell Curve"  
Standard Normal  
Distribution*

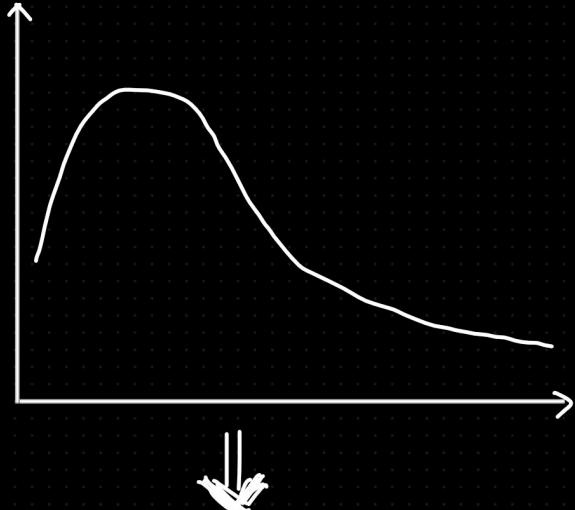


## log normal Distribution

Random variable

$\log(R.V.)$

Normal Dist-



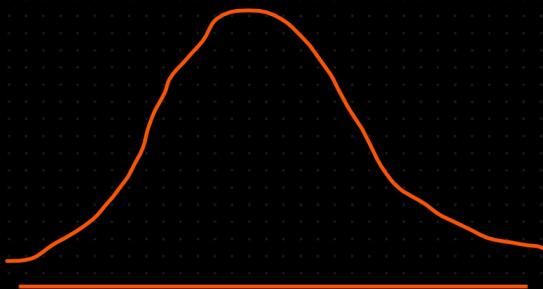
← right or pos Skewed  
Skewed data

Random vgr ⇒ Probability  
curve

$\log(x)$

↓  
normal Distribution

Natural log ⇒  $\log(x) \Rightarrow e(\text{exponential})$



e<sup>x</sup>

log → normal → exponential → log

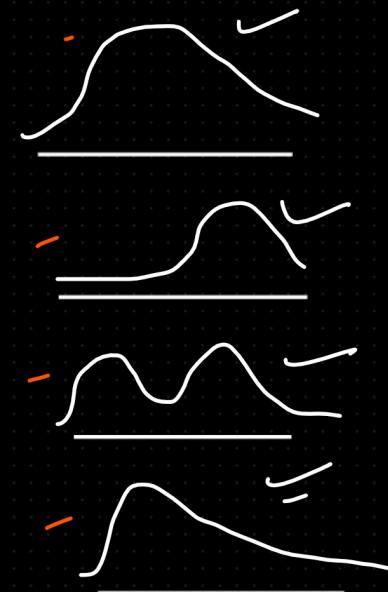
$$\Rightarrow X \sim \text{lognormal}(\mu, \sigma) \Rightarrow \text{Normal Dist. } (\mu, \sigma)$$

$n = \text{natural log} (\ln e)$

$$Y = \underline{\ln(x)}$$

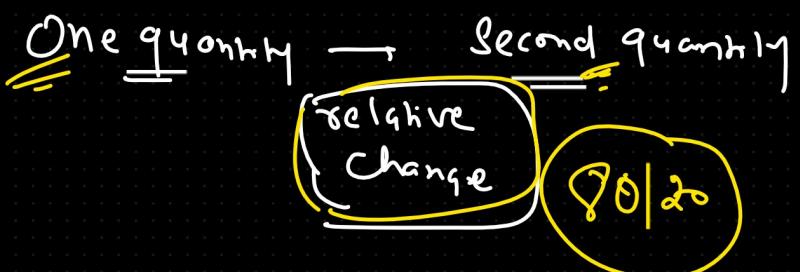
In probability theory, a **log-normal** (or **lognormal**) distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. Thus, if the random variable  $X$  is log-normally distributed, then  $Y = \ln(X)$  has a normal distribution.<sup>[1][2]</sup> Equivalently, if  $Y$  has a normal distribution, then the exponential function of  $Y$ ,  $X = \exp(Y)$ , has a log-normal distribution. A random variable which is log-normally distributed takes only positive real values. It is a convenient and useful model for measurements in exact and engineering sciences, as well as medicine, economics and other topics (e.g., energies, concentrations, lengths, prices of financial instruments, and other metrics).

$$\begin{array}{c} \textcircled{1} \text{ Power law} \\ \textcircled{2} \text{ Pareto} \end{array} \Rightarrow \boxed{\frac{80\%}{20\%}} \text{ Rule}$$



Powerlaw Distr  $\Rightarrow$  Continuous Random variable

(2-9 Quantiles)



Ex:- ① 20% of the team is responsible for winning 80% of the matches.

② 20% Product-driven 80% Sale in the Amazon.

③ 80% wealth is distributed among 20% of people

④ 80% of project completed by 20% of team.

⑤ 80% of tax paid by 20% of population.

Ex:- of Powerlaw Dist | type of the Power law  $\Rightarrow$  Pareto

