JANUARY 2024

MATHEMATICS-I

Time Allowed: 2.5 Hours *Full Marks: 60*

Answer to Question No. 1 of Group A must be written in the main answer script. In Question No. 1, out of 2 marks for each MCQ, 1 marks is allotted for right answer and 1 marks is allotted for correct explanation of the answer.

Answer any Five (05) Questions from Group-B.

Group-A

1. Choose the correct answer from the given alternatives and explain your answer (any ten) $2\times10=20$

i) If for the vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}|=1$, $|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=\sqrt{3}$, then angle between the vectors $\vec{a}$ and $\vec{b}$ is

(a) 90^0 (b) 60^0 (c) 45^0 (d) 30^0.

ii) If one root of the equation $x^2-6x+m=0$ be double the other, then the value of m is (a) 4 (b) 6 (c) 8 (d) -8.

iii) The value of $2^{\log_2 5}+9^{\log_3\sqrt{3}}$ is

(a) 9 (b) 7 (c) 8 (d) none of these.

iv) The value of the expression $\omega^2(1+i\omega)(i\omega-1)$ is

(a) 1 (b) -2 (c) -1 (d) 0.

v) The value of $\hat{k}\cdot(\hat{i}\times\hat{j})$ is

(a) 1 (b) 0 (c) -1 (d) none of these.

vi) If $z=2+i\sqrt{3}$, then $z\bar{z}$ is (a) 7 (b) 1 (c) -7 (d) 0.

vii) The coefficient of x^3 in the expansion of $(1+3x+3x^2+x^3)^{10}$ is

(a) ${}^{10}C_3$ (b) ${}^{10}C_2$ (c) ${}^{30}C_3$ (d) ${}^{30}C_2$.

viii) If the vectors $2\hat{i}-3\hat{j}+\hat{k}$ and $m\hat{i}-\hat{j}+m\hat{k}$ are perpendicular to each other, then the value of m is (a) 1 (b) -1 (c) 2 (d) -2.

ix) If $\cos\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=0$, then the value of x is

(a) 0 (b) 1 (c) $\frac{4}{5}$ (d) $\frac{1}{5}$.

x) If $\cos 3x = \sin 2x$, then $x =$

(a) 15^0 (b) 18^0 (c) 30^0 (d) $22\frac{1}{2}^0$.

xi) If $f(x-2)=2x^2+3x-5$, then $f(-1)=$

(a) 0 (b) 1 (c) -1 (d) 2.

xii) The domain of the function $\frac{1}{\sqrt{(x-2)(3-x)}}$ is

(a) $2\le x\le 3$ (b) $2<x\le 3$ (c) $2\le x<3$ (d) $2<x<3$.

xiii) $\lim_{x\to\frac{\pi}{2}}\frac{\cot x}{\frac{\pi}{2}-x}=$ (a) -1 (b) 0 (c) 1 (d) none of these.

xiv) If $f(x)=\log e^x+e^{\log x}$, then $f'(x)$ is

(a) e^x+1 (b) e^x+x (c) 2 (d) none of these.

xv) The function $(3-x)(x-1)$ is maximum for $x=$

(a) 1 (b) 2 (c) 3 (d) 4.

Group-B

Answer any Five (05) Questions

2. i) If α and β be the roots of the equation $x^2-3x+2=0$, find the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

ii) The fifth term in the expansion of $\left(x^2-\frac{1}{x}\right)^n$ is independent of x. Find n.

iii) Prove that $\sqrt{i}+\sqrt{-i}=\sqrt{2}$, where $i=\sqrt{-1}$. $(3+3+2)$

3. i) If $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=3\hat{i}-4\hat{j}+2\hat{k}$, find the projection of $\vec{a}+\vec{c}$ in the direction of $\vec{b}$.

ii) Prove that $2\log(a+b)=2\log a+\log\left(1+2\frac{b}{a}+\frac{b^2}{a^2}\right)$.

iii) If $\omega^3=1$ and $1+\omega+\omega^2=0$, find the value of $\omega^{2022}+\omega^{2023}+\omega^{2024}$.

iv) if $\tan\frac{\theta}{2}=\frac{3}{4}$, find the value of $\sin\theta$. $(2+3+1+2)$

4. i) If $\frac{1}{\log_3 x}=\frac{1}{9}$, find the value of x.

ii) Find the number of terms in the expansion of $(x+y)^7(x-y)^7$.

iii) Find the modulus of $(a-ib)^2$, where $i=\sqrt{-1}$.

iv) Prove that $\sec^2\left(\tan^{-1}\sqrt{5}\right)+\operatorname{cosec}^2\left(\cot^{-1}5\right)=32$. $(2+2+2+2)$

5. i) Find a unit vector perpendicular to both the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$.

ii) If one root of the equation $x^2+ax+8=0$ is 4 and the roots of the equation $x^2+ax+b=0$ are equal, find the value of b.

iii) If $\tan x\tan 5x=1$, prove that $\tan 3x=1$. $(3+3+2)$

6. i) The position vectors of A, B, C, D are given by the vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}$, $3\hat{i}+5\hat{j}-2\hat{k}$ and $\hat{k}-\hat{j}$. Prove that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel vectors.

ii) If $\tan(A+B)=\frac{1}{2}$ and $\tan(A-B)=\frac{1}{3}$, find the value of $\tan 2A$.

iii) Show that $\frac{\sin(x+y)}{\sin(x-y)}=\frac{\tan x+\tan y}{\tan x-\tan y}$. $(3+2+3)$

7. i) If $f(x)=\log_2 x$ and $\varphi(x)=x^2$, find $f\{\varphi(2)\}$.

ii) If $y=x^5$ and $x^2\frac{d^2y}{dx^2}=ay$, find the value of a.

iii) Find the derivative of x^6 with respect to x^2.

iv) If $y=\log_{\cot x}\tan x$, prove that $\frac{dy}{dx}=0$. $(2+2+2+2)$

8. i) Evaluate : $\lim\limits_{x\to 0}\dfrac{3^x-1}{\sqrt{9+x}-3}$.

ii) Prove that $\sin 3x \operatorname{cosec} x - \cos 3x \sec x = 2$.

iii) Prove that the function $\log\left(x+\sqrt{x^2+1}\right)$ is an odd function.

iv) Find the value of $\sin\left(\dfrac{1}{2}\cos^{-1}\dfrac{1}{2}\right)$. $(3+2+2+1)$

9. i) A parachutist falls through a distance $x=\log_e\left(6-5e^{-t}\right)$ in the t^{th} second of its motion. Find $\dfrac{dx}{dt}$ at $t=0$.

ii) If $\sin^4 x+\sin^2 x=1$, prove that $\cot^4 x+\cot^2 x=1$.

iii) If $y=e^{\sin^{-1}t}$ and $x=e^{-\cos^{-1}t}$, prove that $\dfrac{dy}{dx}$ is constant. $(3+3+2)$