

## How to calculate $R^2$ :

**Problem:** Suppose a company's owner wants to forecast sales on the basis of advertising expenses. The owner would like to review the relationship between sales and the amount spent on advertising. Below is the information on sales and advertising expense for the last four months:

Month	Advertising expense(x) (\$ million)	Sales revenue(y) (\$ million)
July	2	7
August	1	3
September	3	8
October	4	10

**Solution:**

b) Let, Estimated Simple linear regression Model,

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \varepsilon_i$$

we know that,

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{10}{4} = 2.5 \text{ and } \bar{y} = \frac{\sum_{i=1}^n y_i}{n} = \frac{28}{4} = 7$$

Advertising expense(x)	Sales revenue(y)	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
2	7	-0.5	0	0.25	0
1	3	-1.5	-4	2.25	6
3	8	0.5	1	0.25	0.5
4	10	1.5	3	2.25	4.5
				$\sum_{i=1}^4 (x_i - \bar{x})^2 = 5$	$\sum_{i=1}^4 (x_i - \bar{x})(y_i - \bar{y}) = 11$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Now if we put the values the Estimated Simple linear regression Model,

$$\hat{Y}_i = 1.5 + 2.2X_i + \varepsilon_i$$

After that,

We may calculate

$\hat{Y}_i$	$\varepsilon = (Y_i - \hat{Y}_i)$	$(Y_i - \hat{Y}_i)^2$	$(Y_i - \bar{Y})^2$
5.9	1.1	1.21	0
3.7	.7	0.49	16
8.1	.1	0.01	1
10.3	.3	0.09	9
		$\sum(Y_i - \hat{Y}_i)^2 = 1.8$	$\sum(Y_i - \bar{Y})^2 = 26$

$$\begin{aligned}
 R^2 &= 1 - \frac{\sum(y_i - \hat{y})^2}{\sum(y_i - \bar{y})^2} \\
 &= 1 - \frac{1.8}{26} \\
 &= \mathbf{0.9308}
 \end{aligned}$$

Therefore,  $R^2=93.08\%$  which implies about 93% variation of dependent variable can be explained by the independent variables.