Numerical Methods: Problem Set: Fall 2023

Chapter 01	Error Analysis

Definitions: Exact and approximate numbers, significant digits, errors, absolute errors, relative errors and percentage errors, round off error.

C.W:

1. An approximate value of π is 3.1428571 and true value is 3.1415926. Find the absolute, relative and percentage errors.
2. Find the absolute, relative and percentage errors of the number 8.6 if both of its digits are correct.
3. Evaluate the sum $S = \sqrt{2} + \sqrt{3} + \sqrt{5}$ to 4 significant digits and find its absolute, relative and percentage errors.

H.W:

1. Three approximations of the number $\frac{1}{3}$ are given as 0.32, 0.33 and 0.34. Find the best approximation among the three.
2. Find the absolute, relative and percentage errors of the number 7.8 if both of its digits are correct.
3. Evaluate the sum $S = \sqrt{2} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its E_A, E_R & E_P.

Chapter 02:	The solution of algebraic & Transcendental equations

Definitions: Algebraic & Transcendental equation, roots of an equation. Bisection Method, Newton Rapshon Method &Fixed point iteration Method's derivation, Algorithm of aforementioned three methods.

C.W:

1. Find the solution of the equation $x^2 - 4x + 3 = 0$ by using Bisection Method.
2. Find the root of the equation $x^3 - x - 1 = 0$ by using Bisection method correct up to two decimal places.
3. Find the root of the equation $xe^x = 1$ by using Bisection method correct up to three decimal places on the interval (0, 1).
4. Find the root of the equation $e^x - 4\sin x = 0$ by using Bisection method correct up to four decimal places.
5. Find the real root of the equation $x^3 + x^2 - 1 = 0$ on the interval [0, 1] with an accuracy of 10^{-4} using fixed point iteration method.
6. Find the real root of the equation $\sin x - 5x + 2 = 0$ that lies on [0, 1] using fixed point iteration method.
7. Find the real root of the equation $e^x \tan x = 1$ Using fixed point iteration method.
8. Using Newton-Raphson method, find the root of the equation $x^4 - x - 10 = 0$ which is nearer to correct to $x = 2$ three decimals places.

9. Find the root of the equation $x\sin x + \cos x = 0$,using Newton-Rapshon method.

H.W:

1. Find the roots of the following equations by using bisection Method:

   (a) $x^3 + x^2 - 1 = 0$ (b) $\cos x - \ln x = 0$ (c) $xe^x - \cos x = 0$

2. Find the roots of the following equations by using Fixed point iteration Method:

   (a) $x^4 - x - 10 = 0$ (b) $\cos x - \ln x = 0$ (c) $3x - \log_{10} x - 16 = 0$

3. Find the roots of the following equations by using Fixed point iteration Method:

   (a) $e^x - 4x = 0$ (b) $x^3 + x^2 - 1 = 0$ (c) $e^x \tan x = 1$

Chapter 03:	Interpolation

Definition: Forward difference, Backward difference, interpolation, Newton's interpolation for equal and unequal intervals, divided difference & Lagrange's formula.

C.W:

1. Construct a difference table to find the polynomial of the data $(1,1),(2,8),(3,27),(4,64),$ $(5,125),(6,216),(7,343),(8,512)$, considering appropriate method. Also find r, where $(9,r)$ is given.

2. From the following table of yearly premiums for policies maturing at quinquennial ages, estimate the premiums foe policies maturing at the age of 46 years.

Age(x)	45	50	55	60	65
Premium(y)	2.871	2.404	2.083	1.862	1.712

3. The values of $\sin x$ are given below for different values of x, find the value of $\sin 38^0$.

x	15	20	25	30	35	40
$y = \sin x$	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

4. The population of a town in the last six censuses was as given below. Estimate the population for the year 1946.

Year(x)	1911	1921	1931	1941	1951	1961
Population in thousands(y)	12	15	20	27	39	52

5. Using Newton's divided difference estimate $f(8)$ & $f(15)$ from the following table.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

6. Using Lagrange's formula estimate $f(x)$ from the following table.

x	0	1	2	5
$f(x)$	2	3	12	147

7. Using Lagrange's formula estimate $f(10)$ from the following table.

x	5	6	9	11
$f(x)$	12	13	14	16

8. Using Lagrange's formula estimate $\sqrt[3]{55}$ from the following table.

x	50	52	54	56
$f(x)=\sqrt[3]{x}$	3.684	3.732	3.779	3.825

H.W:

1. Construct a difference table to find the polynomial of the data $(1,1),(2,8),(3,27),(4,64)$ $,(5,125),(6,216),(7,343)$, $(8,512)$, considering appropriate method. Also find s, where $(10,s)$ is given.

2. The population of a village in the last six censuses was recorded as follows. Estimate the population for the year 1945.

Year(x)	1941	1951	1961	1971	1981	1991
Population(y)	2500	2800	3200	3700	4350	5225

3. The values of $\sin x$ are given below for different values of x, find the value of $\sin 18^0$.

x	15	20	25	30	35	40
$y=\sin x$	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

4. Estimate the production of cotton in the year 1935 from the data given below.

Year(x)	1931	1932	1934	1936	1938
Production in millions of bales(y)	17.1	13.0	14.0	9.6	12.4

5. In a company the number of persons whose daily wage are as follows:

Daily wage in Tk.	0-20	20-40	40-60	60-80	80-100
No. of persons	120	145	200	250	150

Find the number of persons whose daily wage is between TK. 40 and TK.50.

6. Using Newton's divided difference formula estimate $f(6)$ from the following table.

x	5	7	11	13	21
$f(x)$	150	392	1452	2366	9702

7. Using Lagrange's formula estimate $\sin 39^0$ from the following table.

x	0	10	20	30	40
$f(x) = \sin x$	0	1.1736	0.3420	0.5000	0.6428

8. Using Lagrange's formula estimate $\tan(0.15)$ from the following table.

x	0.10	0.13	0.20	0.22	0.30
$f(x) = \tan x$	0.1003	0.1307	0.2027	0.2236	0.3093

9. Using Lagrange's formula estimate $\sqrt{151}$ from the following table.

x	150	152	154	156
$f(x) = \sqrt{x}$	12.247	12.329	12.410	12.490

Chapter 04:	Curve Fitting

Definition: Lest square methods, Linear curve fitting, Nonlinear curve fitting

C.W:

1. Use the method of least squares to fit a straight line to the following data:

x	0	5	10	15	20
y	7	11	16	20	26

Estimate the value of y when $x = 25$.

2. Find the least square line $y = ax + b$ for the data points $(-1,10),(0,9),(1,7),(2,5),(-1,10),(3,4),(4,3),$ $(5,0)$ and $(6,-1)$.

3. Fit a second-degree parabola to the following data:

x	0	1	2	3	4
y	1	5	10	22	38

H.W:

1. Find the least square line $y = ax + b$ for the data :

x	-2	-1	0	1	2
y	1	2	3	3	4

Estimate the value of y when $x = 5$.

2. The table below gives the temperature T (in 0^0C) and length l (in mm) of a heated rod. If $l = a_0 + a_1 T$ find the values of a_0 and a_1 using linear least squares

T	40	50	60	70	80
l	600.5	600.6	600.8	600.9	601

3. Fit a curve of the form $y = ax^2 + bx + c$ to the data:

x	87.5	84	77.8	63.7	46.7
y	292	283	270	235	197

Chapter 05: Numerical Differentiation

Definition: Numerical derivative for Newton's Forward difference and Backward difference formulas, Range-Katta method of order four.

C.W:

1. From the following table of values of x and y, Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

2.Find the maximum and minimum values of y from the following table

x	0	1	2	3	4	5
y	0	0.25	0	2.25	16	56.25

3. Find $y(0.1)$ by Runge-Kutta method of fourth order for the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$.

H.W:

1. From the following table of values of x and y, Obtain first and second order derivatives at $x = 2.2$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

2. From the following table Find x correct to two decimal places for which y is maximum and find this value of y.

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

3. If $\frac{dy}{dx} = xy + y^2$ with $y(0) = 1$, then find $y(0.2)$ & $y(0.3)$ by using Runge-Kutta fourth order method.

Chapter 06:	Numerical Integration

Definition: Numerical Integration, General formula for numerical integration, Derivation of Trapezoidal rule, Simpson's one- third rule, Simpson's three- eight rule from general formula.

C.W:

1. Compute the definite integral $\int_{0.2}^{1.4} \left(\sin x - \ln x + e^x\right) dx$ by using various rules using 6 equidistant subintervals correct up to three decimal places.

2. Determine $\int_{4}^{5.2} \ln x \, dx$ by Simpson's 3/8 rule and Weddle's rule considering the number of intervals six. Find true value and then compare and comment on it.

3. Evaluate $\int_{0}^{6} f(x)\, dx$ by using trapezoidal rule where the values of $f(x)$ are given by the following table:

x	0	1	2	3	4	5	6
$y = f(x)$	0.146	0.161	0.176	0.190	0.204	0.217	0.230

H.W:

1. Calculate the value of the integral $I = \int_{0}^{1} \frac{x\, dx}{1 + x^2}$ by taking seven equidistant ordinates, using the Simpson's 1/3 rule and trapezoidal rule. Find the exact value of I and then compare and comment on it.

2. Compute $\int_{1}^{2} x^2 dx$ by Simpson's one third rule and compare with exact value.

3. Using Simpson's three-eight rule evaluate the integrals $\int_{0.2}^{1.4} \left(\sin x + e^{2x}\right) dx$ and hence find the errors.

4. Using Simpson's 3/8 th rule find the value of $I = \int_{0}^{1} \frac{\ln(1 + x^2)\, dx}{1 + x^2}$. Also, find the percentage errors and compare to the exact solution.

Chapter 03:	Solution to system of linear equations

Definition: System of linear equations, n-tuple, solution, strictly diagonally dominant, iteration, Gauss seidel and Gauss Jordan Methods

C.W:

1. Check the system whether it is strictly diagonally dominant.

$$20x_1 + 2x_2 + 6x_3 = 28$$
$$x_1 + 20x_2 + 9x_3 = -23$$
$$2x_1 - 7x_2 - 20x_3 = -57$$

2. Solve the following SLE with the help of **Gauss-Jordan/ Gauss-Seidel iteration method**

$$3x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 3x_2 + 2x_3 = 9$$
$$7x_1 + 5x_2 - x_3 = 11$$

3. Solve the equations with the help of **Gauss-Jordan/ Gauss-Seidel iteration method**

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

H.W:

1. Check the system whether it is strictly diagonally dominant.

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

2. Solve the following equations with the help of **Gauss-Jordan/ Gauss-Seidel iteration method**

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Note: Presentation topics: Write down the algorithm and C code for the Bisection, Newton Rapshon &Iteration Methods and also for Gauss Jordan and Gauss seidel.