Canonical and Standard Form

- **Canonical Form –**

In Boolean algebra,Boolean function can be expressed as Canonical Disjunctive Normal Form known as **minterm** and some are expressed as Canonical Conjunctive Normal Form known as **maxterm** . In Minterm, we look for the functions where the output results in "1" while in Maxterm we look for function where the output results in "0". We perform **Sum of minterm** also known as Sum of products (SOP) . We perform **Product of Maxterm** also known as Product of sum (POS). Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form.

- **Standard Form –**

A Boolean variable can be expressed in either true form or complemented form. In standard form Boolean function will contain all the variables in either true form or complemented form while in canonical number of variables depends on the output of SOP or POS.

Truth table representing minterm and maxterm –

X	Y	Z		Minterms Product Terms		Maxterms Sum Terms
0	0	0		$m_0 = \overline{X} \cdot \overline{Y} \cdot \overline{Z} = \min\left(\overline{X},\overline{Y},\overline{Z}\right)$		$M_0 = X + Y + Z = \max\left(X,Y,Z\right)$
0	0	1		$m_1 = \overline{X} \cdot \overline{Y} \cdot Z = \min\left(\overline{X},\overline{Y},Z\right)$		$M_1 = X + Y + \overline{Z} = \max\left(X,Y,\overline{Z}\right)$
0	1	0		$m_2 = \overline{X} \cdot Y \cdot \overline{Z} = \min\left(\overline{X},Y,\overline{Z}\right)$		$M_2 = X + \overline{Y} + Z = \max\left(X,\overline{Y},Z\right)$
0	1	1		$m_3 = \overline{X} \cdot Y \cdot Z = \min\left(\overline{X},Y,Z\right)$		$M_3 = X + \overline{Y} + \overline{Z} = \max\left(X,\overline{Y},\overline{Z}\right)$
1	0	0		$m_4 = X \cdot \overline{Y} \cdot \overline{Z} = \min\left(X,\overline{Y},\overline{Z}\right)$		$M_4 = \overline{X} + Y + Z = \max\left(\overline{X},Y,Z\right)$
1	0	1		$m_5 = X \cdot \overline{Y} \cdot Z = \min\left(X,\overline{Y},Z\right)$		$M_5 = \overline{X} + Y + \overline{Z} = \max\left(\overline{X},Y,\overline{Z}\right)$
1	1	0		$m_6 = X \cdot Y \cdot \overline{Z} = \min\left(X,Y,\overline{Z}\right)$		$M_6 = \overline{X} + \overline{Y} + Z = \max\left(\overline{X},\overline{Y},Z\right)$
1	1	1		$m_7 = X \cdot Y \cdot Z = \min\left(X,Y,Z\right)$		$M_7 = \overline{X} + \overline{Y} + \overline{Z} = \max\left(\overline{X},\overline{Y},\overline{Z}\right)$

Example – Express the Boolean function F = A + B'C as standard sum of minterms.

- **Solution –**
  A = A(B + B') = AB + AB'
  This function is still missing one variable, so
  A = AB(C + C') + AB'(C + C') = ABC + ABC'+ AB'C + AB'C'
  The second term B'C is missing one variable; hence,
  B'C = B'C(A + A') = AB'C + A'B'C
  Combining all terms, we have
  F = A + B'C = ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C
  But AB'C appears twice, and
  according to theorem 1 (x + x = x), it is possible to remove one of those occurrences. Rearranging the minterms in ascending order, we finally obtain
  F = A'B'C + AB'C' + AB'C + ABC' + ABC
  = m1 + m4 + m5 + m6 + m7
  SOP is represented as Sigma(1, 4, 5, 6, 7)

Sum of minterms –

- From the above table it is clear that minterm is expressed in product format and maxterm is expressed in sum format.

- The minterms whose sum defines the Boolean function are those which give the 1's of the function in a truth table. Since the function can be either 1 or 0 for each minterm, and since there are 2^n minterms, one can calculate all the functions that can be formed with n variables to be $(2^{(2^n)})$. It is sometimes convenient to express a Boolean function in its sum of minterm form.

Example – Express the Boolean function F = xy + x'z as a product of maxterms

- **Solution –**
  F = xy + x'z
  = (xy + x')(xy + z)
  = (x + x')(y + x')(x + z)(y + z)
  = (x' + y)(x + z)(y + z)
  x' + y = x' + y + zz'
  = (x'+ y + z)(x' + y + z') x + z
  = x + z + yy'
  = (x + y + z)(x + y' + z) y + z
  = y + z + xx'
  = (x + y + z)(x' + y + z)
  F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z')
  = M0*M2*M4*M5
  POS is represented as Pi(0, 2, 4, 5)

Thank You