Binary Number System

(Continued from previous slide..)

Example

$10101_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) \times (1 \times 2^0)$

$= 16 + 0 + 4 + 0 + 1$

$= \quad 21_{10}$

Oct al N um ber Syst em

(Continued from previous slide..)

- Since there are only 8 digits, 3 bits = 8) sufficient to represent any octal number in binary (2^3

Exam ple

$$2057_8 = (2 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$

$$= 1024 + 0 + 40 + 7$$

$$= 1071_{10}$$

Hexadecim al N um ber Syst em

Characterist ics

§ A positional number system

§ Has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Hence its base = 16

§ The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively

§ The maximum value of a single digit is 15 (one less than the value of the base)

(Continued on next slide)

Hexadecimal Number System

(Continued from previous slide..)

- Each position of a digit represents a specific power of the base (16)
- Since there are only 16 digits, 4 bits ($2^4 = 16$) are sufficient to represent any hexadecimal number in binary

Example

$$1AF_{16} = (1 \times 16^2) + (A \times 16^1) + (F \times 16^0)$$
$$= 1 \times 256 + 10 \times 16 + 15 \times 1$$
$$= 256 + 160 + 15$$
$$= 431_{10}$$

Converting a Number of Another Base to a Decimal Number

Method

Step 1: Determine the column (positional) value of each digit

Step 2: Multiply the obtained column values by the digits in the corresponding columns

Step 3: Calculate the sum of these products

(Continued on next slide)

Convert ing a N um ber of Anot her Base t o a Decim al N um ber

(Continued from previous slide..)

Exam ple

$4706_8 = ?_{10}$

$4706_8 = 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0$

Common values multiplied by the corresponding digits

$= 4 \times 512 + 7 \times 64 + 0 + 6 \times 1$

$= 2048 + 448 + 0 + 6$ ← **Sum of these products**

$= 2502_{10}$

Converting a Decimal Number to a Number of Another Base

(Continued from previous slide..)

Example

$952_{10} = ?_8$

Solution:

		Remainders
8	952	
	119	0
	14	7
	1	6
	0	1

Hence, $952_{10} = 1670_8$

Converting a Number of Some Base to a Number of Another Base

Method

Step 1: Convert the original number to a decimal number (base 10)

Step 2: Convert the decimal number so obtained to the new base number

(Continued on next slide)

Converting a Number of Some Base to a Number of Another Base

(Continued from previous slide..)

Example

$545_6 = ?_4$

Solution:

Step 1: Convert from base 6 to base 10

$545_6 = 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0$
$= 5 \times 36 + 4 \times 6 + 5 \times 1$
$= 180 + 24 + 5$
$= 209_{10}$

(Continued on next slide)

Converting a Number of Some Base to a Number of Another Base

(Continued from previous slide..)

Step 2: Convert 209_{10} to base 4

4	209	Remainders
	52	1
	13	0
	3	1
	0	3

Hence, $209_{10} = 3101_4$

So, $545_6 = 209_{10} = 3101_4$

Thus, $545_6 = 3101_4$

Short cut Met hod for Convert ing a Binary N um ber t o it s Equivalent Oct al N um ber

Met hod

Step 1: Divide the digits into groups of three starting from the right

Step 2: Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion

(Continued on next slide)

Short cut Met hod for Convert ing a Binary N um ber t o it s Equivalent Oct al N um ber

(Continued from previous slide..)

Exam ple

$1101010_2 = ?_8$

Step 1: Divide the binary digits into groups of 3 starting from right

<u>001</u><u>101</u><u>010</u>

Step 2: Convert each group into one octal digit

$001_2 = 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1$
$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5$
$010_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$

Hence, $1101010_2 = 152_8$

Short cut Method for Converting an Octal Number to Its Equivalent Binary Number

Method

Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)

Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

(Continued on next slide)

Short cut Method for Converting an Octal Number to Its Equivalent Binary Number

(Continued from previous slide..)

Example

$562_8 = ?_2$

Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2$, $6_8 = 110_2$, $2_8 = 010_2$

Step 2: Combine the binary groups

$562_8 = \underline{101}\ \ \underline{110}\ \ \underline{010}$

5 6 2

Hence, $562_8 = 101110010_2$

Short cut Met hod for Convert ing a Binary

Met hod

Step 1: Divide the binary digits into groups of four starting from the right

Step 2: Combine each group of four binary digits to one hexadecimal digit

(Continued on next slide)

Short cut Met hod for Convert ing a Binary N um ber t o it s Equivalent Hexadecim al N um ber

(Continued from previous slide..)

Exam ple

$111101_2 = ?_{16}$

Step 1: Divide the binary digits into groups of four starting from the right

0011 1101

Step 2: Convert each group into a hexadecimal digit

$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 3_{10} = D_{16}$

Hence, $111101_2 = 3D_{16}$

Short cut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

Method

Step 1: Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number

Step 2: Combine all the resulting binary groups (of 4 digits each) in a single binary number

(Continued on next slide)

Short cut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

(Continued from previous slide..)

Example

$2AB_{16} = ?_2$

Step 1: Convert each hexadecimal digit to a 4 digit binary number

$2_{16} = {}^{2}{}_{10} = 0010_2$

$A_{16} = 10_{10} = 1010_2$

$B_{16} = 11_{10} = 1011_2$

Short cut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

(Continued from previous slide..)

Step 2: Combine the binary groups

$2AB_{16} = \underline{0010}\quad\underline{1010}\quad\underline{1011}$

2 A B

Hence, $2AB_{16} = 001010101011_2$

Formation of Fractional Numbers in Binary Number System (Example)

Binary Point ↓

Position	4	3	2	1	0	.	-1	-2	-3	-4
Position Value	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
Quantity Represented	16	8	4	2	1		$^1/_2$	$^1/_4$	$^1/_8$	$^1/_{16}$

(Continued on next slide)

Formation of Fractional Numbers in Binary Number System (Example)

(Continued from previous slide..)

Example

$$110.101_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$
$$= 4 + 2 + 0 + 0.5 + 0 + 0.125$$
$$= 6.625_{10}$$

Formation of Fractional Numbers in Octal Number System (Example)

Octal Point

Position	3	2	1	0	.	-1	-2	-3
Position Value	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}
Quantity Represented	512	64	8	1		$1/8$	$1/64$	$1/512$

(Continued on next slide)

Formation of Fractional Numbers in Octal Number System (Example)

(Continued from previous slide..)

Example

$127.54_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 5 \times 8^{-1} + 4 \times 8^{-2}$

$= 64 + 16 + 7 + {}^5/_8 + {}^4/_{64}$

$= 87 + 0.625 + 0.0625$

$= 87.6875_{10}$