Daffodil International University

Department of Computer Science and Engineering (CSE)
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Normal and Exponential Distribution Mathematical Examples Lecture Sheet

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Section: 61_L

Normal Distribution Mathematical Examples

Problem 1: If X ~ N(4, 9), find P(X > 6) using normal distribution formula.

Solution:

We know, when a variable X follows a normal distribution with mean μ and variance σ^2, this is denoted by,

$$X \sim N(\mu, \sigma^2)$$

To use normal distribution formula, let $Z = \frac{X-\mu}{\sigma} = \frac{X-4}{3}$

Now,

$$P(X > 6) = 1 - P(X < 6)$$

$$Or, P(X > 6) = 1 - \varphi\left(\frac{6-4}{3}\right)$$

$$Or, P(X > 6) = 1 - \varphi(0.67)$$

$$Or, P(X > 6) = 1 - 0.7486$$

$$Or, P(X > 6) = 0.2514$$

Answer: The probability of P(X > 6) is 0.2514 or 25.14%.

Problem 2: The working lives of a particular brand of electric light bulb are distributed with a mean of 1200 hours and a standard deviation of 200 hours. What is the probability of a bulb lasting more than 1150 hours? Use normal distribution formula.

Solution:

Let X, the working life, is distributed using normal distribution formula as,

$$X \sim N(\mu, \sigma^2)$$

$$Or, X \sim N(1200, 200^2)$$

Now,

$$P(X > 1150) = 1 - P(X < 1150)$$

$$Or, P(X > 6) = 1 - \varphi\left(\frac{1150-1200}{200}\right)$$

$$Or, P(X > 6) = 1 - \varphi(-0.25)$$

$$Or, P(X > 6) = 1 - 0.4013$$

$$Or, P(X > 6) = 0.5987$$

Answer: The probability of a bulb lasting more than 1150 hours is 0.5987 or 59.87%.

Problem 3: What will be the probability density function of normal distribution for the data; x = 3, μ = 4 and σ = 2?

Solution:

Given,

Variable, x = 3
Mean, μ = 4 and
Standard Deviation, σ = 2

We know, probability distribution function for normal distribution,

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$Or, f(3,4,2) = \frac{1}{2\sqrt{2\pi}} e^{\frac{-(3-4)^2}{2\times 2^2}}$$

$$Or, f(3,4,2) = 0.1760$$

So, the probability density function, f(x, μ, σ) = 0.1760

(Ans.)

Exponential Distribution Mathematical Examples

Problem 1: A postal clerk spends an average of 4 minutes with their customer. The time has exponential distribution. Find the value of the function at x = 5 by using the exponential function formula.

Solution:

Given,

Mean, μ = 4 and
Variable, x = 5

We know,

$$Change\ Rate, \lambda = \frac{1}{\mu}$$

$$Or, \lambda = \frac{1}{4}$$

$$Or, \lambda = 0.25$$

Again,

$$f(x,\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$Or, f(5, 0.25) = 0.25e^{-0.25\times 5}$$

$$Or, f(5, 0.25) = 0.0716$$

Answer: The value of the function at x = 5 is 0.0716

<u>Problem 2:</u> A person spends an average of 10 minutes on a counter. The time has exponential distribution. Find the value of the function at x = 7 by using the exponential function formula.

<u>Solution:</u>

Mean, μ = 10 and
Variable, x = 7

We know,

$$Change\ Rate, \lambda = \frac{1}{\mu}$$

$$Or, \lambda = \frac{1}{10}$$

$$Or, \lambda = 0.1$$

Again,

$$f(x,\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$Or, f(7, 0.1) = 0.1e^{-0.1\times 7}$$

$$Or, f(7, 0.1) = 0.04966$$

Answer: The value of the function at x = 7 is 0.04966