Double Angle Trigonometric Identities: Derivation and Context

1. Sine Double Angle

Identity:

$$\sin(2x) = 2\sin(x)\cos(x)$$

Derivation:

Start from the compound angle formula for sine:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

Let $A = x$ and $B = x$:

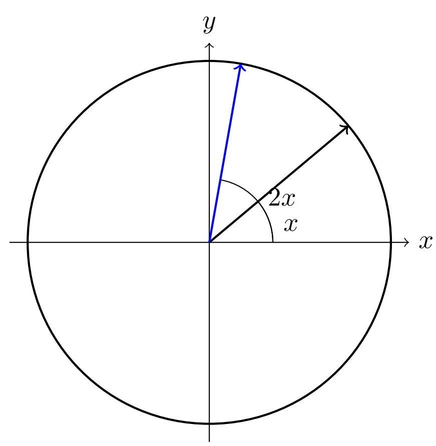
$$\sin(x + x) = \sin x \cos x + \cos x \sin x = 2\sin x \cos x$$

Thus,

$$\sin(2x) = 2\sin x \cos x$$

Geometric Meaning:

If you imagine a right triangle with angle x, $\sin x$ and $\cos x$ are the ratios of sides. Doubling the angle effectively "mixes" these ratios.



2. Cosine Double Angle

Identity:

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\cos(2x) = 1 - 2\sin^2(x)$$

Derivation:

Start from the compound angle formula for cosine:

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

Let $A = x$, $B = x$:

$$\cos(x+x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$$

Alternate forms use the Pythagorean identity ($\sin^2 x + \cos^2 x = 1$):

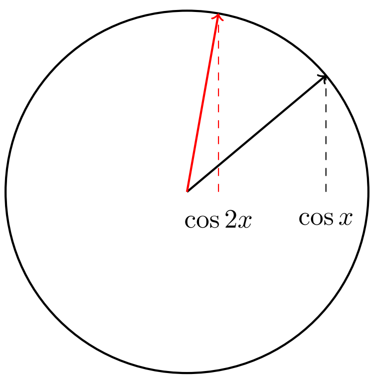
$$\cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$$

or

$$\cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

Geometric Meaning:

Cosine relates to the horizontal projection of a point on a circle. Doubling the angle changes the projection, and these formulas describe that change.



3. Tangent Double Angle

Identity:

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

Derivation:

Start from the compound angle formula for tangent:

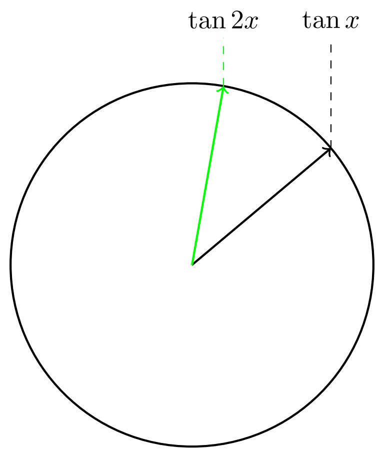
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Let $A = x$, $B = x$:

$$\tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x \tan x} = \frac{2\tan x}{1 - \tan^2 x}$$

Geometric Meaning:

Tangent is the ratio of sine to cosine, which can also be thought of as the slope of the radius at angle x. Doubling the angle changes this slope, and the formula describes how.



Summary

All double angle identities are derived from the compound angle formulas. Geometrically, they describe how the basic trigonometric ratios change when the angle is doubled.