JANUARY 2022
28 Friday
Week 5 ■ 028-337

DECEMBER 2021
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19 20 21 22 23 24 25
26 27 28 29 30 31

Formal Languages & Automata Theory

Module 4 - More on Context-Free Languages

PDA = Finite State Machine + stack
↳ Push Down Automata

$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$
Q - set of states
Σ - input symbols
Γ - Stack symbols
δ - Transition function
q_0 - Start state
z_0 - Start stack symbol
F - final state set.

DPDA - $\delta : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$
NPDA - $\delta : Q \times \{\Sigma \cup \epsilon\} \times \Gamma \rightarrow 2^Q \times \Gamma^*$

$\delta(q, a, X)$.
q - state in Q
a - input symbol in Σ or $a = \epsilon$
X - Stack symbol in Γ

Output of $\delta = (p, \gamma)$
p - new state.
γ - string of stack symbols that replaces X

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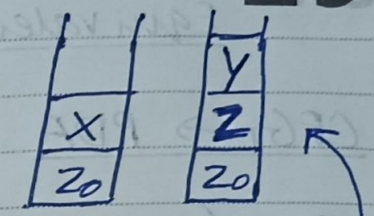
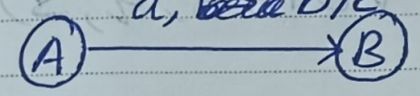
at the top of stack
$\gamma = \epsilon$ POP
$\gamma = X$ Stack unchanged
$\gamma = YZ$ Y is PUSHed
Z made as a replacement for X.





$\delta(A, a, b) = (B, c)$

a = input symbol from Σ
b = topmost element in stack that is popped ($b = \epsilon \rightarrow$ nothing popped)
c = symbol pushed onto the stack. ($c = \epsilon \rightarrow$ nothing pushed)

For PDA to accept a string (either one should be true)
→ Should reach final state $\vdash^*(q_F, \epsilon, \alpha)$
→ Stack must be empty. $\vdash^*(q, \epsilon, \epsilon)$

Instantaneous Description of PDA (ID)

(q, w, γ) $\vdash$ $\vdash^*$
$\vdash$: Turnstile notation (one move)
$\vdash^*$ ↳ many moves

Sunday 30
Week 6 ■ 030-335

If $\delta(q, a, X) = (p, \alpha)$
$(q, aw, X\beta) = (p, w, \alpha\beta)$

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Equivalence of PDAs & CFGs

CFG → PDA

$G = (V, T, P, S)$
PDA = $(\{q\}, T, V \cup T, \delta, q, S)$

1) For variable A,
$\delta(q, \epsilon, A) = \{(q, \beta) \mid A \to \beta \text{ is in } P\}$

2) For terminal a,
$\delta(q, a, a) = (q, \epsilon)$

PDA → CFG

PDA, $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$
$G = (V, \Sigma, P, S)$

V - variables
1) Start symbol S.
2) $[pxq]$ p, q - states
x - stack symbol

P - productions
1) For all states q
$S \to [q_0 Z_0 q]$

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Tuesday 01
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2) Let $\delta(q, a, X) = (r, Y_1 Y_2 \cdots Y_k)$
$[q\, x\, r_k] \to a\, [r\, Y_1 r_1][r_1 Y_2 r_2] \cdots [r_{k-1} Y_k r_k]$
For all states $r_1, r_2, \cdots r_k$

Let $\delta(q, a, X) = (r, \epsilon)$
$[q\, x\, r] \to a$

Let $\delta(q, \epsilon, x) = (r, \epsilon)$
$[q\, x\, r] \to \epsilon$

Pumping Lemma for CFL

L - CFL
p - constant for pumping
z - string in L
$|z| \geq p$
Split $z = uvwxy$
i) $|vwx| \leq p$
ii) $vx \neq \epsilon$ or $|vx| \geq 1$
iii) For all $i \geq 0$, $uv^i wx^i y \in L$

Closure Properties of CFL

1) Union $(S \to S_1 \mid S_2)$
2) Concatenation $(S \to S_1 S_2)$
3) Kleene closure $(S \to S_1 S \mid \epsilon)$

$S_1 \to aS_1b \mid \epsilon$
$S_2 \to bS_2c \mid \epsilon$