Formal Languages & Automata Theory

Module 1 - Intro. to formal language Theory and Regular Languages

DFA - Deterministic Finite Automata

$M = (Q, \Sigma, \delta, q_0, F)$

Q = set of all states
Σ = inputs
δ = transition function = $\boxed{Q \times \Sigma \rightarrow Q}$
q_0 = start state
F = set of final states

* Trap state necessary.

Operations on Regular Languages

Union - $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
Concatenation - $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$
Star - $A^* = \{x_1 x_2 x_3 \dots x_k \mid k \geq 0 \text{ \& each } x_i \in A\}$

* ~~Regular languages are closed under union & concatenation & star.~~

Intersection - $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Complement - Flip final states to non-final and vice versa.

Regular languages are closed under all of these operations.

NFA - Non-deterministic Finite Automata

$M = (Q, \Sigma, \delta, q_0, F)$
Q = set of states
Σ = inputs
δ = transition function = $\boxed{Q \times \Sigma \rightarrow 2^Q}$
q_0 = start state
F = set of final states.

→ Dedicate ~~an~~ separate edges for inputs ~~for~~ in the given condition.

Conversion of NFA to DFA

→ All DFAs are NFAs. But vice versa may not be true.

Steps: (Subset construction* method)

1) Draw NFA
2) Transition table of NFA
3) Transition table of DFA from NFA.
   → Start with start state, same transition.
   → If a new state formed, find the transition of that new state.
   → Repeat until all states ~~are~~ formed are reachable to every other state
4) Draw DFA.

* There should not be any ϕ in DFA.
* Every state containing final state of NFA is final for DFA.

ϵ-NFA

$M = \{Q, \Sigma, \delta, q_0, F\}$

Q = set of states
Σ = inputs
δ = transition table = $\boxed{Q \times \Sigma \cup \epsilon \rightarrow 2^Q}$
q_0 = start state
F = set of final states

* Every state on ϵ goes to itself.

ϵ-Closure: (ϵ^*) - All the states that can be reached from a particular state only by seeing the ϵ symbol.

Conversion of ϵ-NFA to NFA

1) Draw ϵ-NFA
2) Transition table of ϵ-NFA:
   → First find the ϵ^* of a particular state, then transitions with a given input, and ϵ^* from transitions obtained through given input.
   → Repeat for every state, and every input.
3) ~~Draw~~ transition table of NFA:
   → Write down distinct states from final ϵ^*.
4) Draw NFA.

* Every state ~~containing~~ reaching final state, only through ϵ of ϵ-NFA are final states of NFA.

Extended transition function ($\delta^* / \hat{\delta}$)

DFA: $Q \times \Sigma^* \rightarrow Q$
1) $\delta^*(q, \epsilon) \rightarrow q$
2) $\delta^*(q, xa) \rightarrow \delta(\delta^*(q, x), a)$

Equivalence of DFA & NFA (Proof)

If $D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D\}$ is the DFA constructed from NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ by subset construction then $L(D) = L(N)$

i.e., $\delta_D^*(\{q_0\}, w) = \delta_N^*(q_0, w)$

Base case: $|w| = 0 \Rightarrow \epsilon$

$\delta_D^*(\{q_0\}, \epsilon) = \{q_0\}$

$\delta_N^*(q_0, \epsilon) = \{q_0\}$

Induction Hypothesis

Let the same be true for $|w| \leq n$

$$\delta_D^*(\{q_0\}, x) = \delta_N^*(q_0, x)$$

Proof: Let $w = xa$ $\quad |w| = n+1$

$$\delta_D^*(\{q_0\}, x) = \delta_N^*(q_0, x) = \{P_1, P_2, \ldots, P_k\}$$

$$\delta_D^*(\{q_0\}, xa) = \delta_D(\delta_D^*(\{q_0\}, x), a)$$

$$= \delta_D(\{P_1, P_2, \ldots, P_k\}, a)$$

$$= \bigcup_{i=1}^{k} \delta_D(P_i, a)$$

$$= [r_0, r_1, \ldots r_k]$$

$$\delta_N^*(q_0, xa) = \delta_N(\delta_N^*(q_0, x), a)$$

$$= \delta_N(\{P_1, P_2, \ldots, P_k\}, a)$$

$$= \bigcup_{i=1}^{k} \delta_N(P_i, a)$$

$$= \{r_0, r_1, \ldots, r_k\}$$

$$\therefore \delta_N^*(q_0, xa) = \delta_D^*(\{q_0\}, xa)$$

Steps to identify equivalence

1) For pair of states $\{q_i, q_j\}$, $a \in \Sigma$ where $\delta(q_i, a) = q_a$ & $\delta(q_j, a) = q_b$, then $\{q_a, q_b\}$ should both be either intermediate or final state.
2) If initial is final in one automaton, same should be the case with other as well.

Grammar

$G = (V, T, S, P)$

V = set of variables (non-terminals)

T = set of terminals

S = start symbol

P = Production rules for terminals & non-terminals.

Regular Grammar

Right linear	Left linear
$A \rightarrow xB$	$A \rightarrow Bx$
$A \rightarrow x$	$A \rightarrow x$

$A, B \in V$ & $x \in T$.

Equivalence of RG & DFA

RG - (V, T, S, P)
DFA - $(Q, \Sigma, \delta, q_0, F)$

$V \rightarrow Q$
$T \rightarrow \Sigma$
$S \rightarrow q_0$
$P \rightarrow \delta$

Suppose $A_1 \longrightarrow xA_2$
$(A_1, x) = A_2$

If $A \rightarrow x$, then point A towards the final state.