Coursework 1B (2015–2016)

Attempt ALL six questions

1. Find the truth value of

$$A \wedge (B \Rightarrow \neg C)$$

if A is false, and B and C are true.

What is the truth value if the brackets are removed?

I begin to do this question by writing the order of precedence from the highest to the lowest:

$\neg$, $\wedge$ and $\Rightarrow$

A = False, B = True and C = True

Now I am going to remove the brackets:

$F \wedge (T \Rightarrow \neg T)$
$F \wedge (T \Rightarrow F)$
$F \wedge F$
F

The truth value of $A \wedge (B \Rightarrow \neg C)$ is False.

Now I am going to work out the truth value of $F \wedge T \Rightarrow \neg T$.

$F \wedge T \Rightarrow \neg T$

$F \wedge T \Rightarrow F$

$F \Rightarrow F$

T

= The truth value $F \wedge T \Rightarrow \neg T$ of is also 'True'.

2. Consider the proposition

$$X \wedge (Y \Rightarrow Z) \wedge (X \Rightarrow Y) \Rightarrow Y \vee Z$$

(a) Interpreting the symbols as atomic sentences according to

X: maths is
compulsory Y: I have
to think hard Z: I get
smarter

express this compound proposition as an argument in natural English.

Connectives from the compound proposition $X \wedge (Y \Rightarrow Z) \wedge (X \Rightarrow Y) \Rightarrow Y \vee Z$:

'∧' = 'AND'
'⇒' = 'IMPLIES'
'∨' = 'OR'

Compound proposition translated as an argument in Natural English using the interpreted symbols:

Maths is compulsory. If I have to think hard then I get smarter and If maths is compulsory, then I have to think hard. Therefore, that I have to think hard or I get smarter.

(b) By means of a truth table, find out whether the argument is valid.

X	∧	(Y	⇒	Z)	∧	(X	⇒	Y)	⇒	Y	∨	Z
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	F	T	T	T	T	T	T	T	F
T	F	F	T	T	F	T	F	F	T	F	T	T
T	F	F	T	F	F	T	F	F	T	F	F	F
F	F	T	T	T	F	F	T	T	T	T	T	T
F	F	T	F	F	F	F	T	T	T	T	T	F
F	F	F	T	T	F	F	T	F	T	F	T	T
F	F	F	T	F	F	F	T	F	T	F	F	F
1	10	2	8	3	11	4	9	5	13	6	12	7

= Using the truth table I am able to find out that the argument is valid as the column 13 in the truth table is a 'Tautology 'because column 13 reveals only 'TRUE' values.

3. Consider the following collection of statements: -

I have tested my app. If I tested my app and it failed the test, I will not try to sell it to customers. If the test was faulty then my app failed the test.

(a) Choose symbols to represent the atomic sentences in the above argument, and hence express the statements in terms of propositional calculus.

A: I have tested my app
B: My app failed the test
C: I will not try to sell it to customers
D: The test was faulty

I have chosen the symbols to represent the atomic sentence above. Now I am going to express the statements in terms of propositional calculus.

$A\ (A \wedge B \Rightarrow C)$ and $D \Rightarrow B$

(b) Construct a formal proof (table of assertions and justifications) showing that the statement *If the test was faulty then I will not try to sell my app to customers follows* from these hypotheses.
Hint Consider the deduction theorem.

If the test was faulty then I will not try to sell my app to customers follows: $D \Rightarrow \neg C$

I added the new statement into my previous propositional calculus: $A\ (A \wedge B \Rightarrow C)\ D \Rightarrow B \vdash D \Rightarrow C$

I considered $A\ (A \wedge B \Rightarrow C)\ D \Rightarrow B \vdash D \Rightarrow C$ as H_1, H_2 and H_3 respectively, I then get the following hypotheses:

$H_1 = A$
$H_2 = A \wedge B \Rightarrow C$
$H_3 = D \Rightarrow B$
$H_4 = D$
$\vdash = C$

I can now construct the table of assertions and justifications:

Assertions	Justifications
1. D	H_4
2. $D \Rightarrow B$	H_3
3. B	Modus Ponens, 1, 2
4. A	H_1
5. $A \wedge B$	3, 4, Conjunction
6. $A \wedge B \Rightarrow C$	H_2
7. C	5, 6, Modus Ponens

(c) How many rows would be required by a truth table to show this? (Do <u>NOT</u> construct the truth table!)

To calculate how many rows would be required by a truth table I used the following equation:

Number of rows in truth table: 2^n

In my statement there are 4 types of sentences A, B, C and D. I used the equation above:

$2^4 = 2x2x2x2 = 16$

From this I can tell that 16 rows would be required by a truth table.

4. A computer enthusiast has a collection of 25 computers, on each of which he has installed at least one of Windows, Linux and Android operating systems. If 13 have Windows, 11 have Android, 12 have Linux, 4 have Windows and Android, 5 have Android and Linux and 3 have Windows and Linux, how many have Linux but neither of the other two operating systems?
(5 marks)

To do this I started by denoting the question as: There are 3 sets of operating systems Windows, Linux and Android and altogether there are 25 computers which have the operating systems installed on them.

I denoted:
W as Windows
L as Linux
A as Android

I then write the general statement:

$| W | = 13$
$| A | = 11$
$| L | = 12$

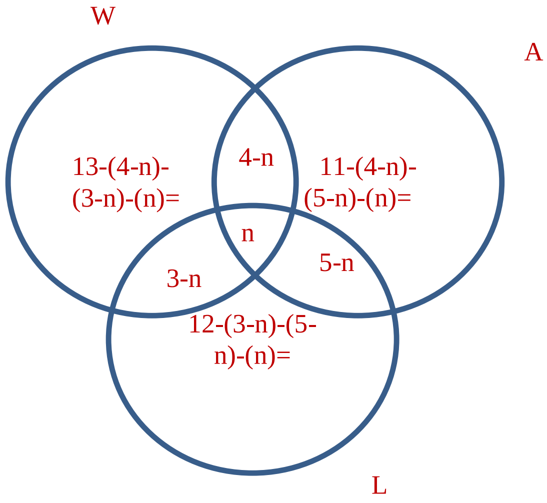
This gives me the total of 25 computers which the operating systems are installed on and now I can write the conclusion as:

$| W \cup A \cup L | = 25$

I know that there are 4 computers which have Windows and A installed, 5 computers which have Android and Linux installed and 3 which have Windows and Linux installed on them. Now I can write the following statement:

$| W \wedge A | = 4$
$| A \wedge L | = 5$
$| W \wedge L | = 3$

Now I can construct a Venn Diagram and fill in the information I have.

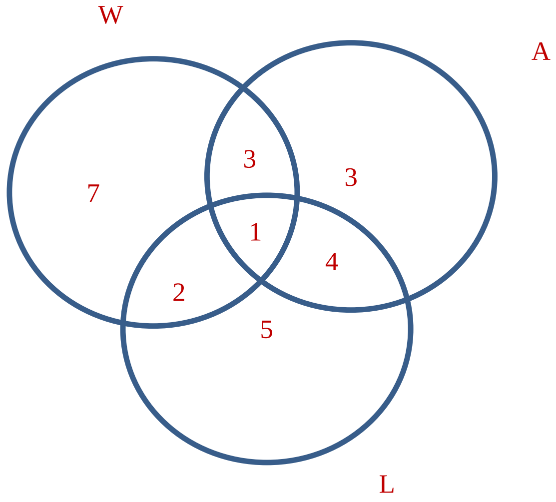


Now I can use the following formula to substitute in the values I know from the question which will help me find the n value in the Venn Diagram.

$$| W \cup A \cup L | = | W | + | A | + | L | - | W \wedge A | - | A \wedge L | - | W \wedge L | + | W \cup A \cup L |$$

$$| 25 | = | 13 | + | 11 | + | 12 | - | 4 | – | 5 | – | 3 | + | W \cup A \cup L |$$
$$| 25 | = | 24 | + | W \cup A \cup L |$$
$$| 25 | - | 24 | = | W \cup A \cup L |$$
$$| 1 | = | W \cup A \cup L |$$

I can now fill the Venn diagram with the n values along with the values I already knew from the questions to find the answer.



From the Venn diagram I can see that there are 5 computers which have Linux operating systems installed but neither of the other two operating systems therefore the answer is 5.

5. (a) Use a hybrid truth/membership table to show that if $C \subseteq A$ and $C \subseteq B$ then $C \subseteq (A \cup B)$.

C	⊆	A	∧	C	⊆	B	⇒	C	⊆	(A	∨	B)
0	T	0	T	0	T	0	T	0	T	0	0	0
0	T	0	T	0	T	1	T	0	T	0	1	1
0	T	1	T	0	T	0	T	0	T	1	1	0
0	T	1	T	0	T	1	T	0	T	1	1	1
1	F	0	F	1	F	0	T	1	F	0	0	0
1	F	0	F	1	T	1	T	1	T	0	1	1
1	T	1	F	1	F	0	T	1	T	1	1	0
1	T	1	T	1	T	1	T	1	T	1	1	1
1	8	2	11	3	9	7	13	5	12	6	10	4

Column 1,2 and 7 are reference values.
Column 3,5,6 and 4 are copied referenced values.
Column 8 = 1 ⊆ 2
Column 9 = 3 ⊆ 7
Column 10 = 6 U 4
Column 11 = 8 ∧ 9
Column 12 = 5 ⊆10
Column 13 = 11 ⇒ 12

The column 13 vales are always 'TRUE', therefore the proposition is a 'Tautology' and shown $C \subseteq A$ and $C \subseteq B$ then $C \subseteq (A \cup B)$.

(b) Give an example of a situation where the relationship $C \subseteq (A \cup B)$ holds, even though $C \subseteq A$ and $C \subseteq B$ do not, where A, B and C are subsets of the universal set $\{1, 2, 3, 4\}$.

∪ = [1,2,3,4]

A = [1,2]
B = [3]
C = [1,2,3]

A ∪ B = (1,2,3)

6. a $[1 \ldots n]$ is an array of integers, and N is the set $\{1, 2, \ldots, n - 1\}$.

   (a) Express the statement the elements of the array are in increasing order in the language of predicate calculus.

   $$\forall i \in \{1, \ldots, N - 1\} | a[i] \leq a[i + 1]$$

   (b) Give the negation of this statement in predicate calculus.

   $$\exists ! \, E \, \{1, \ldots, N - 1\} | a[\, i \,] > a[i + 1]$$

(2 marks)