GRAVITATION

Class IX

Gravitational force of the earth or Gravity

A force is necessary to produce motion in a body.

A body dropped from a height falls towards the earth. It indicates that the earth exerts a force of attraction (called gravity) on the body.

In fact, the earth attracts all the objects towards its centre.

The force with which the earth pulls the objects towards it is called the gravitational force of the earth or gravity.

The gravitational force between the sun and the earth keeps the earth in uniform circular motion around the sun.

The gravitational force of the earth (or gravity) is responsible –

- for holding the atmosphere above the earth
- for the rain falling to the earth
- for flow of water in the rivers
- for keeping us firmly on the earth,
- for the revolution of moon around the earth, etc.

NEWTON'S UNIVERSAL LAW OF GRAVITATION

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

The direction of force is along the line joining the centres of the two bodies.

Let two bodies A and B of masses m_1 and m_2 are placed at a distance of 'r' from each other. Let 'F' be the force of attraction between the bodies.

Then,

(i) the force of attraction is directly proportional the product of their masses

i.e. $F \propto m_1 \times m_2$

(ii) the force of attraction is inversely proportional the square of the distance between them

i.e. $F \propto \frac{1}{r^2}$

$$\therefore \quad F \propto \frac{m_1 \times m_2}{r^2}$$

or $$\boxed{F = G\,\frac{m_1 \times m_2}{r^2}}$$

where G is a constant called as "universal gravitational constant"

Properties of gravitational force

1. Gravitational force is the weakest force in nature.
2. It is an attractive force. (Unlike electrostatic and magnetic force; they are both attractive and repulsive)
3. It is a mutual force. (First body attracts the second body and the second body attracts the first body with equal force)
4. It is a central force. (Acts along the line joining the centres of the bodies)
5. It is mass and distance dependent.
6. It obeys inverse square law.
7. It is a long range force. (It decreases with distance as per inverse square law and becomes zero only at infinite distance – like electrostatic and magnetic force)
8. It does not depend on the medium between the interacting bodies. (There is no gravitational shielding)

Gravitational force is the weakest force of all.

If two bodies of mass 1 kg each are separated by 1 m from each other, then, $F = G = 6.67 \times 10^{-11}$ N

Though the various objects on this earth attract one another constantly, they do not cause any visible motion because the gravitational force of attraction between them is very, very small.

Imagine a situation on the earth if the objects continuously move towards each other due to gravitational force between them !

When both the objects are very big, having very large masses, then the gravitational force of attraction between them becomes extremely large.

The gravitational force due to earth on an object of mass 1 kg placed on the ground is 9.8 N.

The gravitational force between the earth and the moon even at a large distance is very large and is 2×10^{20} N.

Universal Gravitational Constant (G)

$$\boxed{G = F \frac{r^2}{m_1 \times m_2}}$$

- The value of Universal Gravitational Constant G is 6.67×10^{-11} N m^2 Kg^{-2}.
- The value of G does not depend on the medium between two bodies.
- The value of G is same throughout the Universe and hence the name.
- Universal Gravitational Constant G is different from acceleration due to gravity g. Therefore correct symbol must be used accordingly.
- SI unit of G is N m^2 Kg^{-2}.
- If two bodies of mass 1 kg each are separated by 1 m from each other, then, G = F.
- Therefore, Universal gravitational constant G is numerically equal to the force of gravitation which exists between two bodies of unit masses kept at a unit distance from each other.

Acceleration due to gravity (g)

Acceleration due to gravity is defined as the uniform acceleration produced in a freely falling body due to the gravitational force of the earth.

Acceleration due to gravity (g) = $9.8\ m/s^2 = 980\ cm/s^2$.

Calculation of acceleration due to gravity (g)

Suppose a body of mass 'm' is placed on the earth of mass 'M' and radius 'R'.

According to Newton's universal law of gravitation,

Force exerted by the earth on the body is given by

$$F = G\frac{M \times m}{R^2}$$

This force exerted by the earth produces an acceleration on the body.

Therefore, $F = mg$ (g - acceleration due to gravity)

From the two equations, we have

$$mg = G\frac{M \times m}{R^2} \quad \text{or} \quad \boxed{g = \frac{G M}{R^2}}$$

FREE FALL

When a body falls from a height towards the earth only under the influence of the gravitational force (with no other forces acting on it), the body is said to have a free fall.

The body having a free fall is called a 'freely falling body'.

Galileo proved that the acceleration of an object falling freely towards the earth does not depend on the mass of the object.

Imagine that vacuum is created by evacuating the air from the glass jar.

The feather and the coin fall with same acceleration and reach the ground at the same time.

Acceleration due to gravity on the earth is calculated as follows:

Mass of the earth $= 6 \times 10^{24}$ kg

Radius of the earth $= 6.4 \times 10^{6}$ m

Gravitational constant $= 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$g = \frac{GM}{R^2}$$

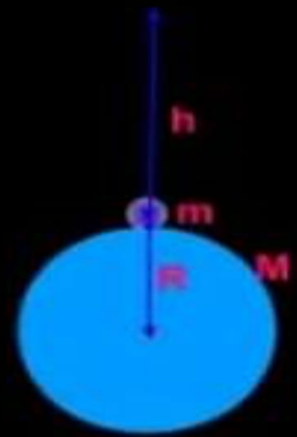
Substituting the values, we get $g = 9.8$ m/s^2

For simplified calculations we can take g as 10 m/s^2

Variation of acceleration due to gravity (g)

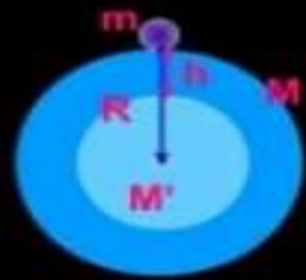
1. Acceleration due to gravity decrease with altitude.

$$g = \frac{GM}{R^2} \qquad g' = \frac{GM}{(R+h)^2}$$



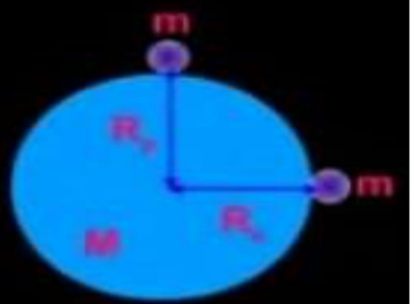
2. Acceleration due to gravity decrease with depth.

$$g = \frac{G M}{R^2} \qquad g' = \frac{G M'}{(R-h)^2}$$



3. Acceleration due to gravity is greater at the poles and less at the equator.

$$g_p = \frac{G M}{R_p^2} \qquad g_e = \frac{G M}{R_e^2}$$



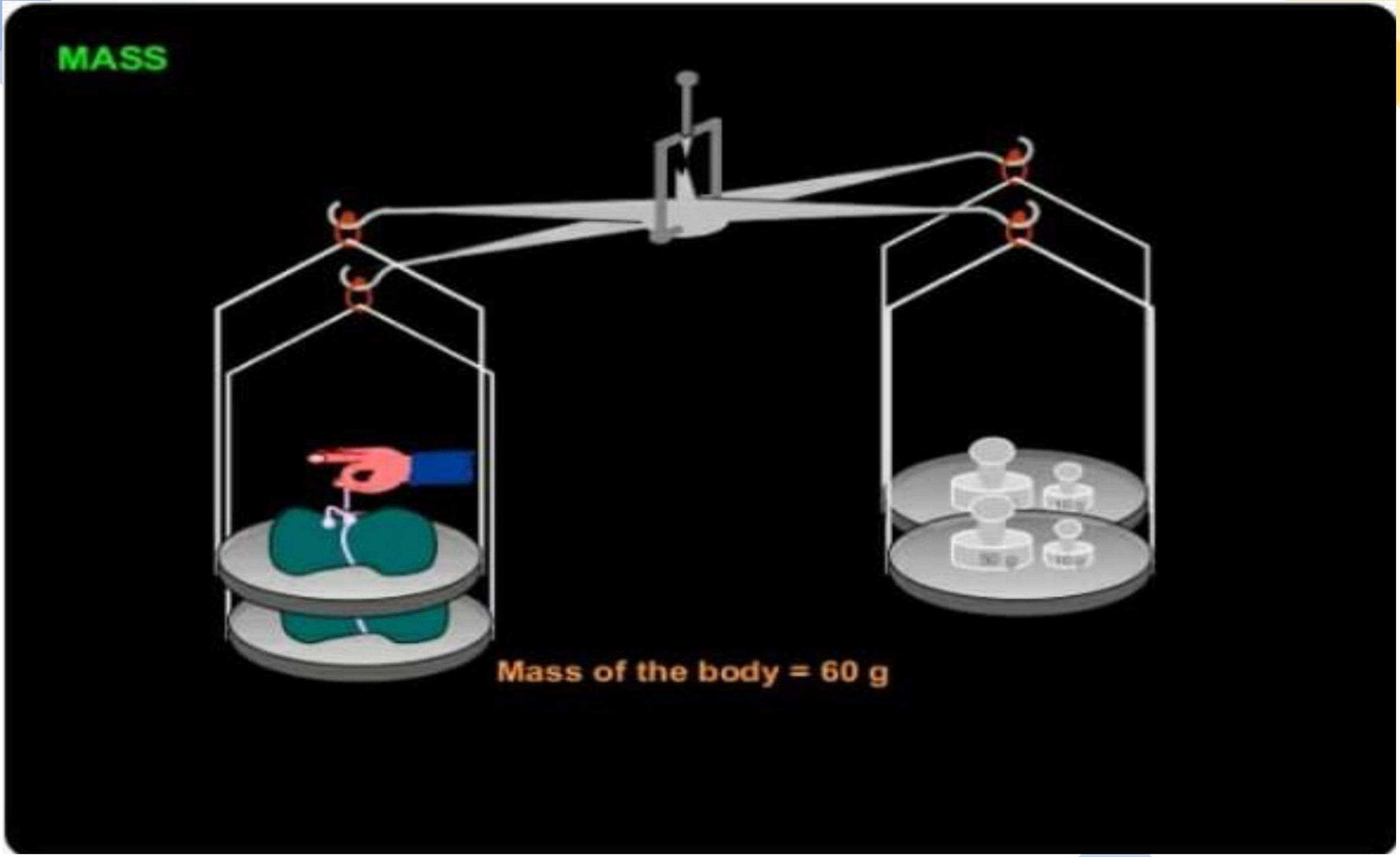
Earth is slightly flattened at the poles and bulging at the equator. The radius of the earth at the poles is 21 km less than that at the equator.

i.e. $R_p < R_e$

Therefore, from the above equations, G and M being same and g is inversely proportional to the square of the radius,

$g_p > g_e$

$g_p = 9.823\ m/s^2$, $g_e = 9.789\ m/s^2$ and average value of $g = 9.8\ m/s^2$


MASS
Mass of the body = 60 g

The mass of a body is the quantity of matter contained in it.

Mass of a body is a measure of inertia of the body and hence called inertial mass.

Mass is a scalar quantity.

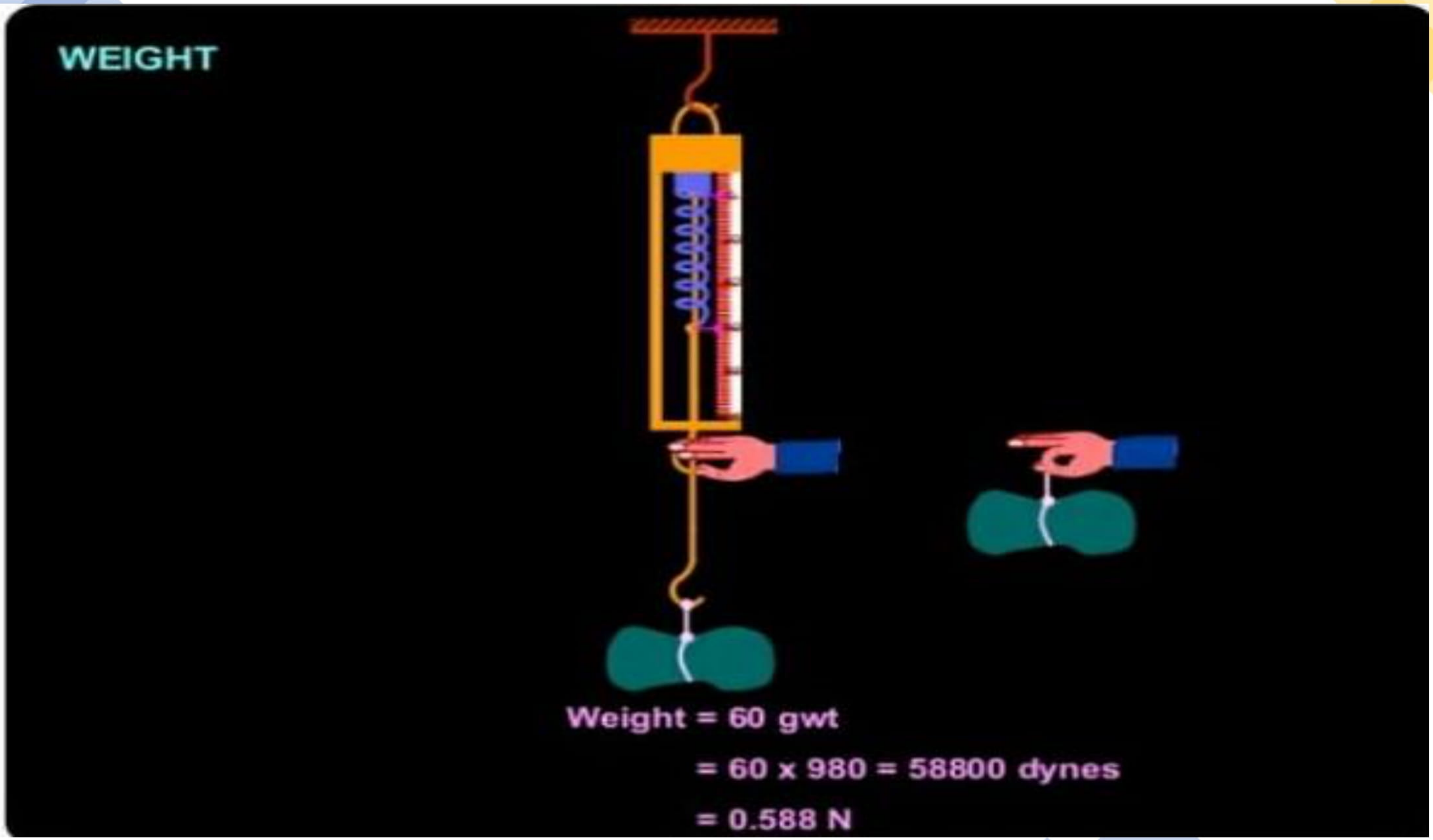
SI unit of mass is kilogramme or kg.

CGS unit of mass is gramme or g.

Mass of a body is constant and does not change from place to place.

Mass is measured by a beam or common balance.

Mass of a body can not be zero.


WEIGHT
Weight = 60 gwt
= 60 x 980 = 58800 dynes
= 0.588 N

The weight of a body on the earth is the force with which it is attracted towards the centre of the earth.

Weight = mass x acceleration due to gravity

$$W = m \times g$$

SI unit of weight is 'newton' or N

Weight of 1 kg mass is 9.8 newton.

Weight is a vector quantity since it has both magnitude and direction.

Weight changes from place to place.

Weight is measured by a spring balance.

Weight of a body can be zero. When a body is taken to the centre of the earth, acceleration due to gravity at the centre is zero and hence weight is zero.

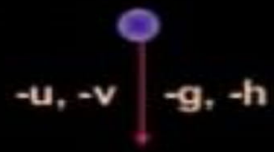
On the moon, acceleration due to gravity is nearly 1/6th of that on the earth and hence the weight of a body on the moon is 1/6th of its weight on the earth.

EQUATIONS OF VERTICAL MOTION (Under the influence of g)

Let a body be thrown vertically downward with initial velocity 'u'. Let the final velocity of the body after time 't' be 'v'. Let 'h' be the height (vertical distance) covered by the body and 'g' be the acceleration due to gravity.

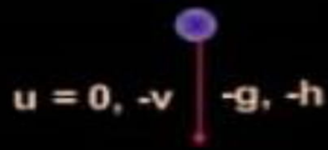
Then the equations of motion are:

$$v = u + gt$$
$$h = ut + \tfrac{1}{2} gt^2$$
$$v^2 = u^2 + 2gh$$



When a body is dropped freely, the equations of motion are:

$$v = gt$$
$$h = \tfrac{1}{2} gt^2$$
$$v^2 = 2gh$$



When a body is thrown vertically upwards, the equations of motion are:

$$v = u - gt$$
$$h = ut - \tfrac{1}{2} gt^2$$
$$v^2 = u^2 - 2gh$$

Cartesian Sign Conventions for Vertical Motion

1. The physical quantities having vertically upward motion are assigned positive values.
2. On the other hand, the physical quantities having vertically downward motion are assigned negative values.
3. Acceleration due to gravity (g) always acts in the downward direction and hence taken negative.

TIPS TO SOLVE THE NUMERICAL PROBLEMS

1. g is always taken negative.
2. When a body is dropped freely its initial velocity u = 0.
3. When a body is thrown vertically upwards, its final velocity becomes zero at the maximum height.
4. The time taken by a body to rise to the highest point is equal to the time it takes to fall from the same height.

GRAVITATION

1. How does the force of gravitation between two objects change when the distance between them is reduced to half?

Solution:

Consider the Universal law of gravitation,

According to that law, the force of attraction between two bodies is

$$F = \frac{(Gm_1m_2)}{r^2}$$

Where,

m_1 and m_2 are the masses of the two bodies.

G is the gravitational constant.

r is the distance between the two bodies.

Given that the distance is reduced to half then,

r = 1/2 r

Therefore,

$$F = \frac{(Gm_1m_2)}{r^2}$$

$$F = \frac{(Gm_1m_2)}{({}^{r}/_{2})^2}$$

$$F = \frac{(4Gm_1m_2)}{(r)^2}$$

F = 4F

Therefore once the space between the objects is reduced to half, then the force of gravitation will increase by fourfold the first force.

2. Gravitational force acts on all objects in proportion to their masses. Why then does a heavy object not fall faster than a light object?

Solution:

All objects fall from the top with a constant acceleration called acceleration due to gravity (g). This is constant on earth and therefore the value of 'g' doesn't depend on the mass of an object. Hence, heavier objects don't fall quicker than light-weight objects provided there's no air resistance.

3. What is the magnitude of the gravitational force between the earth and a 1 kg object on its surface? (Mass of the earth is 6×10^{24} kg and radius of the earth is 6.4×10^6m.)

Solution:

From Newton's law of gravitation, we know that the force of attraction between the bodies is given by

$$F = \frac{(Gm_1m_2)}{r^2}$$

Here

m_1 = mass of Earth = 6.0×10^{24}kg

m_2 = mass of the body = 1kg

r = distance between the two bodies

Radius of Earth = 6.4×10^6m

G = Universal gravitational constant = 6.67×10^{-11} Nm^2kg^{-2}

By substituting all the values in the equation

$$F = \frac{(Gm_1m_2)}{r^2}$$

$$F = \frac{6.67 \times 10^{11}(6.0 \times 10^{24} \times 1)}{(6.4 \times 10^6)^2}$$

F = 9.8 N

This shows that Earth exerts a force of 9.8 N on a body of mass 1 kg. The body

will exert an equal force of attraction of 9.8 N on the Earth.

4. The earth and the moon are attracted to each other by gravitational force. Does the earth attract the moon with a force that is greater or smaller or the same as the force with which the moon attracts the earth? Why?

Solution:

The earth attracts the moon with a force same as the force with which the moon attracts the earth. However, these forces are in opposite directions. By universal law of gravitation, the force between moon and also the sun can be

$$F = \frac{(Gm_1m_2)}{d^2}$$

Where,

d = distance between the earth and moon.

m_1 and m_2 = masses of earth and moon respectively.

5. If the moon attracts the earth, why does the earth not move towards the moon?

Solution:

According to the universal law of gravitation and Newton's third law, we all know that the force of attraction between two objects is the same, however in the opposite directions. So the earth attracts the moon with a force same as the moon attracts the earth but in opposite directions. Since earth is larger in mass compared to that of the moon, it accelerates at a rate lesser than the acceleration rate of the moon towards the Earth. Therefore, for this reason the earth does not move towards the moon.

6. What happens to the force between two objects, if

(i) The mass of one object is doubled?

(ii) The distance between the objects is doubled and tripled?

(iii) The masses of both objects are doubled?

Solution:

(i)

According to universal law of gravitation, the force between 2 objects (m_1 and m_2) is proportional to their plenty and reciprocally proportional to the sq. of the distance(R) between them.

$$F = \frac{(G2m_1m_2)}{R^2}$$

If the mass is doubled for one object.

F = 2F, so the force is also doubled.

(ii)

If the distance between the objects is doubled and tripled

If it's doubled

Hence,

$F = (Gm_1m_2)/(2R)^2$

$F = 1/4\ (Gm_1m_2)/R^2$

F = F/4

Force thus becomes one-fourth of its initial force.

Now, if it's tripled

Hence,

$F = (Gm_1m_2)/(3R)^2$

$F = 1/9\ (Gm_1m_2)/R^2$

F = F/9

Force thus becomes one-ninth of its initial force.

(iii)

If masses of both the objects are doubled, then

$$F = \frac{(G2m_1 2m_2)}{R^2}$$

F = 4F, Force will therefore be four times greater than its actual value.

7. What is the importance of universal law of gravitation?

Solution:

The universal law of gravitation explains many phenomena that were believed to be unconnected:

(i) The motion of the moon round the earth

(ii) The force that binds North American nation to the world

(iii) The tides because of the moon and therefore the Sun

(iv) The motion of planets round the Sun

8. What is the acceleration of free fall?

Solution:

When a body is in free fall, the only force acting on the body is that of the earth's gravitational force. By Newton's second law of motion, all the forces produce acceleration and thus all the objects accelerate toward the surface of the earth due gravitational attraction of the earth.

This acceleration is known as the acceleration due to gravity on the earth's surface. It's denoted by 'g' and its value is $9.8m/s^2$ and it's constant for all objects close to earth's surface (irrespective of their masses).

9. What do we call the gravitational force between the earth and an object?

Solution:

The gravitational force between the earth and an object is known as the object's weight.

10. Amit buys few grams of gold at the poles as per the instruction of one of his friends. He hands over the same when he meets him at the equator. Will the friend agree with the weight of gold bought? If not, why? [Hint: The value of g is greater at the poles than at the equator.]

Solution:

The weight of a body on the earth's surface;

W = mg (where m = mass of the body and g = acceleration due to gravity)

The value of g is larger at poles when compared to the equator. So gold can weigh less at the equator as compared to the poles.

Therefore, Amit's friend won't believe the load of the gold bought.

11. Why will a sheet of paper fall slower than one that is crumpled into a ball?

Solution:

A sheet of paper has a larger surface area when compared to a crumpled paper ball. A sheet of paper will face a lot of air resistance. Thus, a sheet of paper falls slower than the crumpled ball.

12. Gravitational force on the surface of the moon is only 1/6 as strong as gravitational force on the earth. What is the weight in newton's of a 10 kg object on the moon and on the earth?

Solution:

Given data:

Acceleration due to earth's gravity = g_e or g = 9.8 m/s^2

Object's mass, m = 10 kg

Acceleration due to moon gravity = g_m

Weight on the earth= W_e

Weight on the moon = W_m

Weight = mass x gravity

g_m = (1/6) g_e (given)

So W_m = m g_m = m x (1/6) g_e

W_m = 10 x (1/6) x 9.8 = 16.34 N

W_e = m x g_e = 10 x 9.8

W_e = 98N

13. A ball is thrown vertically upwards with a velocity of 49 m/s.

Calculate

(i) The maximum height to which it rises,

(ii) The total time it takes to return to the surface of the earth.

Solution:

Given data:

Initial velocity u = 49 m/s

Final speed v at maximum height = 0

Acceleration due to earth gravity $g = -9.8\ m/s^2$ (thus negative as ball is thrown up).

By third equation of motion,

$v^2 = u^2 - 2gs$

Substitute all the values in the above equation

$$0 = (49)^2 - 2 \times 9.8 \times s$$

$$S = \frac{(49)^2}{2 \times 9.8}$$

s = 122.5m

Total time T = Time to ascend (T_a) + Time to descend (T_d)

v = u – gt

0 = 49 – 9.8 x T_a

Ta = (49/9.8) = 5 s

Also, T_d = 5 s

Therefore T = T_a + T_d

T = 5 + 5

T = 10 s

14. A stone is released from the top of a tower of height 19.6 m. Calculate its final velocity just before touching the ground.

Solution:

Given data:

Initial velocity

u = 0

Tower height = total distance = 19.6m

$g = 9.8\ m/s^2$

Consider third equation of motion

$v^2 = u^2 + 2gs$

$v^2 = 0 + 2 \times 9.8 \times 19.6$

$v^2 = 384.16$

$v = \sqrt{(384.16)}$

v = 19.6m/s

15. A stone is thrown vertically upward with an initial velocity of 40 m/s. Taking g = 10 m/s^2, find the maximum height reached by the stone. What is the net displacement and the total distance covered by the stone?

Solution:

Given data:

Initial velocity u = 40m/s

g = 10 m/s^2

Max height final velocity = 0

Consider third equation of motion

$v^2 = u^2 - 2gs$ [negative as the object goes up]

$0 = (40)^2 - 2 \times 10 \times s$

s = (40 x 40) / 20

Maximum height s = 80m

Total Distance = s + s = 80 + 80

Total Distance = 160m

Total displacement = 0 (The first point is the same as the last point)

16. Calculate the force of gravitation between the earth and the Sun, given that the mass of the earth = 6 × 10^{24} kg and of the Sun = 2 × 10^{30} kg. The average distance between the two is 1.5 × 10^{11} m.

Solution:

Given data:

Mass of the sun $m_s = 2 \times 10^{30}$ kg

Mass of the earth $m_e = 6 \times 10^{24}$ kg

Gravitation constant $G = 6.67 \times 10^{-11}$ N m^2/kg^2

Average distance $r = 1.5 \times 10^{11}$ m

Consider Universal law of Gravitation

$$F = \frac{(Gm_1m_2)}{d^2}$$

$$F = \frac{(6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^{30})}{(1.5 \times 10^{11})^2}$$

$F = 3.56 \times 10^{22}$ N

17. A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 m/s. Calculate when and where the two stones will meet.

Solution:

Given data:

(i) When the stone from the top of the tower is thrown,

Initial velocity u = 0

Distance travelled = x

Time taken = t

Therefore,

$$s = ut + \frac{1}{2}gt^2$$
$x = 0 + (1/2)\, gt^2$
$x = 5t^2$ ----------(a)

(ii) When the stone is thrown upwards,

Initial velocity u = 25 m/s

Distance travelled = (100 – x)

Time taken = t

$$s = ut + \frac{1}{2}gt^2$$
$(100 - x) = 25t + (1/2) \times 10 \times t^2$
$x = 100 - 25t + 5t^2$ ------- (b)

From equations (a) and (b)

$5t^2 = 100 - 25t + 5t^2$

t = (100/25) = 4sec.

After 4sec, two stones will meet

From (a)

$x = 5t^2 = 5 \times 4 \times 4 = 80$m.

Putting the value of x in (100-x)

= (100-80) = 20m.

This means that after 4sec, 2 stones meet a distance of 20 m from the ground.

18. A ball thrown up vertically returns to the thrower after 6 s. Find

(a) The velocity with which it was thrown up,

(b) The maximum height it reaches, and

(c) Its position after 4s.

Solution:

Given data:

$g = 10m/s^2$

Total time T = 6sec

$T_a = T_d = 3sec$

(a) Final velocity at maximum height v = 0

From first equation of motion:-

$v = u - gt_a$

$u = v + gt_a$

= 0 + 10 x 3

= 30m/s

The velocity with which stone was thrown up is 30m/s.

(b) From second equation of motion

$$s = ut_a - \frac{1}{2}g(t_a)^2$$

$= 30 \times 3 - (1/2) \times 10 \times (3)^2$

= 90-45 = 45m

The maximum height stone reaches is 45m.

(c) In 3sec, it reaches the maximum height.

Distance travelled in another 1sec = s'

$$s = ut_a - \frac{1}{2}g(t_a)^2$$

s = 0 + 10 x 1 x 1

s = 5m.

The distance travelled in another 1sec = 5m.

Therefore in 4sec, the position of point p (45 – 5)

= 40m from the ground.