PH504M: Practice Problems

Vikram Khaire

22 Januray 2025

Projectile Motion with Air Resistance

Simulate the motion of a projectile considering air resistance that is proportional to the velocity of the projectile. The drag force is modeled as:

$$F_{\text{drag}} = -k \cdot v$$

Where:

- k is the drag coefficient (kg/s),
- v is the velocity of the projectile (m/s).

The equation of motion is modified to include this drag force. The equation of motion in the x-direction (horizontal) and y-direction (vertical) are:

$$m\frac{d^2x}{dt^2} = -k \cdot \frac{dx}{dt}$$

$$m\frac{d^2y}{dt^2} = -mg - k \cdot \frac{dy}{dt}$$

Where:

- m is the mass of the projectile (kg),
- g is the gravitational acceleration (m/s^2),
- $x(t)$ and $y(t)$ are the horizontal and vertical positions of the projectile, respectively,
- $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are the horizontal and vertical components of velocity,
- $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$ are the accelerations in the x- and y-directions, respectively.

The velocity of the projectile is given by:

$$v = \sqrt{v_x^2 + v_y^2}$$

Where:

- $v_x = \frac{dx}{dt}$ is the horizontal velocity,
- $v_y = \frac{dy}{dt}$ is the vertical velocity.

Task:

1. Simulate the motion of the projectile using a numerical method.
2. Plot the trajectory of the projectile, showing the distance x (horizontal) vs. height y (vertical). Also, overplot a trajectory with no drag force (i.e. make $k = 0$.)
3. Plot the drag force as a function of time, showing how it changes during the projectile's flight.

```
# Use the following parameters to check
# Parameters
v0 = 50  # Initial velocity in m/s
angle = 45  # Launch angle in degrees
m = 1  # Mass in kg
k = 0.2  # Drag coefficient in kg/s
g = 9.81  # Gravitational acceleration in m/s^2
dt = 0.01  # Time step in seconds
t_max = 10  # Maximum simulation time in seconds
```