PH504M Lab 7 (Part A): χ^2 Minimization

Vikram Khaire

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Part A

1. Fitting Gravitational Acceleration Using χ^2 Minimization

A freely falling object follows the equation of motion:

$$y(t) = \frac{1}{2}gt^2$$

where:

- y is the measured position (m),
- t is the time (s),
- g is the acceleration due to gravity (m/s^2).

You are given the following experimental data with uncertainties in y:

Time t (s)	Measured y (m)	Uncertainty σ_y (m)
0.1	0.051	0.006
0.2	0.185	0.007
0.3	0.460	0.005
0.4	0.810	0.008
0.5	1.210	0.006
0.6	1.820	0.007
0.7	2.440	0.005
0.8	3.150	0.009
0.9	4.070	0.006
1.0	5.100	0.007

1. Manually vary g over a range in small steps.
2. Compute the chi-square value for each g.
3. Find the best-fit g by locating the minimum χ^2.
4. Estimate the uncertainty in g by finding where $\chi^2 = \chi^2_{\text{min}} + 1$.
5. Plot:
   - The measured data with error bars along with the best-fit curve $y = \frac{1}{2}g_{\text{best}}t^2$.
   - A χ^2 vs. g plot showing the minimum.

2. Fit a line through the data

In an experiment, the position y of a moving object is recorded at different times x. The data follows a linear relationship:

$$y = mx + c$$

where m is the velocity and c is an initial offset.

You are given the following 10 measurements:

x (s)	y (m)	Uncertainty σ_y (m)
1.0	5.8	0.15
2.0	9.5	0.30
3.0	12.4	0.23
4.0	15.9	0.38
5.0	19.5	0.30
6.0	23.8	0.45
7.0	26.5	0.38
8.0	29.9	0.53
9.0	34.0	0.45
10.0	37.2	0.60

Table 1: Experimental Data

1. Write a Python program to estimate the best-fit values of m and c by minimizing the chi-square function. Then scan the grid of m and c values to find the minimum.

2. Compute the uncertainties in m and c by finding the region where χ^2 increases by 1 from its minimum value.

3. Plot the experimental data (with error bars) along with the best-fit line.

4. Generate a contour plot of χ^2 as a function of m and c, indicating the best-fit point.

Hint:

For both slope and intercept create a grid around value 3. Use `plt.contourf()` function for the contour plot.