PH504M Lab 8: System of linear equations & Integration

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Problem 1: Rotating Spacecraft

In rotational dynamics, the *moment of inertia tensor* I relates the *angular momentum vector* L to the *angular velocity vector* ω using $L = I\omega$. To determine ω, we must compute the inverse of I and solve $\omega = I^{-1}L$.

Consider a rotating spacecraft with an asymmetric moment of inertia tensor:

$$I = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 5 & 1 \\ 1 & -3 & 4 \end{bmatrix}$$

Given the angular momentum vector:

$$L = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

1. Compute the inverse of I using Gauss-Jordan elimination (use the function from class notes).
2. Find angular velocity components by solving $\omega = I^{-1}L$.

Problem 2: Power Method for Dominant Eigenvalue

The power method is an iterative numerical technique used to find the eigenvalue with the largest absolute value of a square matrix, as discussed in the class.

1. Write a Python function to implement the power method for a given $n \times n$ matrix. Ensure that in each iteration, the eigenvector is normalized to prevent it from diverging. The standard normalization is done by dividing the vector by its largest absolute component.
2. The given matrix is:

$$A = \begin{bmatrix} 4 & 1 & 2 & -1 \\ 1 & 3 & 0 & 1 \\ 2 & 0 & 5 & 2 \\ -1 & 1 & 2 & 4 \end{bmatrix}$$

Use your function to find the largest absolute eigenvalue and compare your results with numpy function `numpy.linalg.eig()`. Use an initial guess of $x_0 = [1, 1, 1, 1]^T$. Run the method for 5, 10, and 15 iterations and check convergence.

Problem 3: Numerical Integration of a Gaussian Function

The Gaussian function is widely used in physics, particularly in statistical mechanics and quantum mechanics. Consider the unnormalized Gaussian function:

$$f(x) = \exp\left(\frac{-x^2}{2}\right)$$

where the mean is $\mu = 0$ and the standard deviation is $\sigma = 1$.

1. Write a Python function to compute the integral of $f(x)$ using the trapezoidal rule.
2. Numerically evaluate the integral $I = \int f(x)dx$ over the following intervals:
   - $[-1, 1]$
   - $[-2, 2]$
   - $[-3, 3]$
   - $[-\infty, \infty]$ (use a sufficiently large range to approximate infinity)
3. The exact value of the given Gaussian integral is:

$$\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$$

   Using this, confirm that for a Gaussian distribution, the area under the curve contained within $\pm 1\sigma$ is approximately 68.27%, $\pm 2\sigma$ is approximately 95.45% and $\pm 3\sigma$ is approximately 99.73%.
4. Plot the function and show the area with verticle lines encompassed by $\pm$ 1,2 and 3σ intervals.