Quantum Logic Gate between a Solid State Quantum Bit and a Photon

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Abstract

In this report, we take a detailed look at the paper "A quantum logic gate between a solid-state quantum bit and a photon" by H. Kim, E. Waks, et al. The authors demonstrated a quantum logic gate operation between a solid-state Quantum Dot (QD) and a photonic qubit. Specifically, they experimentally implemented a Controlled-NOT (CNOT) gate on a strongly coupled QD-cavity system, where the QD system conditionally flips the polarization of a photon. Here we mathematically model the QD-cavity system and show why the polarization of the photon flips. We reproduce some of the experimental results using the mathematical models used in their experiments. Finally, we also provide a simulation of the system by implementing the overall Hamiltonian using QuTiP demonstrating the behavior of the controlled-NOT gate.

1 Introduction

Quantum Dots are very popular as solid-state qubits. An important property of QD's are that they can be coupled to cavities in the strong coupling regime. In strong-coupling regime, the QD-cavity system can be used to realize quantum logic operations on a photonic qubit. In this report, we will show how a QD strongly coupled to an optical nano-cavity can be used to implement a CNOT quantum logic operation on a photon.

1.1 The Quantum Dot

The device is composed of an indium arsenide (InAs) QD strongly coupled to a photonic cavity.The QD is essentially a three-level system consisting of ground state $|g\rangle$, and two exciton states ($|+\rangle$ and $|-\rangle$). The optical transition from ground state to the exciton states are denoted by σ_+ and σ_-. By applying a magnetic field in the sample growth direction, the σ_+ transition can be tuned on resonance with the cavity while the σ_- transition remains detuned. The states $|g\rangle, |-\rangle$ are the states of the qubit, whereas the σ_+ transition is used to couple the qubit to the photon.

1.2 The CNOT Gate

The required CNOT operation is achieved by utilizing the strong dependence of the photonic crystal cavity reflection coefficient on the qubit state of the QD. The QD acts as the control qubit and the photon is the target qubit on which the CNOT operation is performed. So, if the QD is in state $|-\rangle$, then the reflection co-efficient is r is -1. In this case, the photon experiences a bit flip (H $\rightarrow$ V and V $\rightarrow$ H). Similarly, if the QD is in $|g\rangle$, and both the photon and the σ_+ transition are resonant with the cavity in the strong coupling regime, then we get r as 1. This results in no change in state of photon or bit.

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2 Cavity Reflection Coefficient

We can show a CNOT operation in the QD-Cavity system by calculating the reflection coefficients along the horizontal and vertical polarization directions. Basically, we have three systems to account for, first is the QD qubit system, second the cavity to which it is coupled, and third the input field which provides the photon. So, let us define $\hat{a}_x, \hat{a}_y$ as the input field annihilation operators and $\hat{b}_x, \hat{b}_y$ as the output field annihilation operators, where x,y-directions are parallel and orthogonal to the cavity orthogonal axis. We also define $\hat{a}$ as the cavity annihilation operator. Then we can write the standard cavity input-output relations:

$$\hat{b}_x = \hat{a}_x - \sqrt{\kappa}\hat{a}$$
$$\hat{b}_y = \hat{a}_y$$

We can see from the expression that the reflection co-efficient(ratio between input-output field amplitudes, which is $\langle \hat{b}_y \rangle / \langle \hat{a}_y \rangle$) in the y-direction is 1. But to calculate $r_x = \langle \hat{b}_x \rangle / \langle \hat{a}_x \rangle$ we need to find steady state relation between $\hat{a}_x$ and $\hat{a}$. For that, we use the Heisenberg-Langevin equations of motion. As the σ_- transition in the QD is highly detuned, we basically have a two-level QD system that is coupled to a cavity. If we assume this system is closed, the Hamiltonian will be-

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_c}{2}\hat{w} + \hbar g(\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a})$$

where $\hat{s} = |g\rangle\langle +|$ is the QD lowering operator and $\hat{w} = |+\rangle\langle +| - |g\rangle\langle g|$ is the population difference operator. Here ω_c and ω_a are the cavity and QD resonant frequencies. But as we are dealing with an open quantum system, we need to include losses in both cavity and QD, along with accounting for the interaction between the cavity and the input field. For that, we will include an interaction Hamiltonian between the cavity and input field[1]

$$H_{int} = i\hbar\sqrt{\kappa}(\hat{a}_x \hat{a}^\dagger - \hat{a}\hat{a}_x^\dagger)$$

We will also introduce two collapse operators to account for the dissipation process in our system. These are $U_1 = \sqrt{\kappa}\hat{a}$ and $U_2 = \sqrt{\Gamma}\hat{s}$, where κ is the cavity decay-rate and Γ is the QD spontaneous emission rate. So, now our total Hamiltonian will be

$$\begin{aligned}\hat{H}_{tot} =& \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_a}{2}\hat{w} + \hbar g(\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a}) + \hbar\omega \hat{a}_x^\dagger \hat{a}_x \\ &+ i\hbar\sqrt{\kappa}(\hat{a}_x \hat{a}^\dagger - \hat{a}\hat{a}_x^\dagger)\end{aligned}$$

Now if we rotate this Hamiltonian to the reference frame rotating at field frequency ω, then we will get

$$\begin{aligned}\hat{H}_{tot} =& \hbar\Delta_c \hat{a}^\dagger \hat{a} + \frac{\hbar\Delta_a}{2}\hat{w} + \hbar g(\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a}) + \\ & i\hbar\sqrt{\kappa}(\hat{a}_x \hat{a}^\dagger - \hat{a}\hat{a}_x^\dagger)\end{aligned}$$

where $\Delta_c = \omega_c - \omega$ and $\Delta_a = \omega_c - \omega$. Now using the Lindbald equation in the Heisenberg picture for any observable A is:

$$\dot{A} = \frac{i}{\hbar}[H, A] + \sum_j (U_j^\dagger A U_j - \frac{1}{2}\{U_j^\dagger A U_j\})$$

Now we will substitute all the terms to find the equations of motion for the operators $\hat{a}, \hat{s}, \hat{w}$:-

$$\dot{\hat{a}} = -(i\Delta_c + \frac{\kappa}{2})\hat{a} - ig\hat{s} + \sqrt{\kappa}\hat{a}_x \tag{1}$$

$$\dot{\hat{s}} = -(i\Delta_a + \frac{\Gamma}{2})\hat{s} - ig\hat{w}\hat{a} \tag{2}$$

$$\dot{\hat{w}} = -\Gamma(\hat{w} + \hat{I}) + 2ig(\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a}) \tag{3}$$

The last equation can be directly integrated, and assuming that at time t=0, $\hat{w}(t) = \hat{w}_0$, we will get

$$\hat{w}(t) = \hat{w}_0 e^{-\Gamma t} + (e^{-\Gamma t} - 1)\hat{I} + \hat{R} \tag{4}$$

where $\hat{R} = 2ig\int_0^t (\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a})dt$. Next, to evaluate $\hat{R}$ and proceed further, we need to start with a specific state of QD, that is, either $|g\rangle$ or $|-\rangle$.

2.1 Initial State is $|g\rangle$

Taking the expectation of both sides of the equations of motion of $\hat{a}, \hat{s}$

$$\langle\dot{\hat{a}}\rangle = -(i\Delta_c + \frac{\kappa}{2})\langle\hat{a}\rangle - ig\hat{s} + \sqrt{\kappa}\hat{a}_x \tag{5}$$

$$\dot{\hat{s}} = -(i\Delta_a + \frac{\Gamma}{2})\hat{s} - ig\langle\hat{w}\hat{a}\rangle \tag{6}$$

We then substitute the expression of $\hat{w}(t)$ derived and simplify using the following two properties-

- $\langle\hat{w}_0\hat{O}\rangle = -\langle\hat{O}\rangle$ for any observable O. We can easily show this as $\langle g|\hat{w}_0\hat{O}|g\rangle = -\langle g|\hat{O}|g\rangle = \langle O\rangle$

- $\langle\hat{R}\hat{a}\rangle = 0$ under the assumption that the input field is a single photon source or in the weak-field regime. This result can be understood from the expression of $\hat{R} = 2ig\int_0^t (\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a})dt$. We can see that either we will have two-photon annihilation and one raising operation in QD or one photon annihilation and creation along and lowering in QD. So, if we start with the ground state in QD and one photon, this term will always vanish.

So we are left with the equations

$$\langle\dot{\hat{a}}\rangle = -(i\Delta_c + \frac{\kappa}{2})\langle\hat{a}\rangle - ig\hat{s} + \sqrt{\kappa}\hat{a}_x \tag{7}$$

$$\dot{\hat{s}} = -(i\Delta_a + \gamma)\hat{s} - ig\langle\hat{a}\rangle \tag{8}$$

where $\gamma = \Gamma/2 + 1/T_2$ is the QD homogeneous linewidth, T_2 being pure dephasing time. We can solve for the mean cavity field in terms of the mean input field under steady state(the derivatives are all zero) as:

$$\langle\hat{a}\rangle = \frac{\sqrt{\kappa}(i\Delta_a + \gamma)}{(i\Delta_c + \kappa/2)(i\Delta_a + \gamma) + g^2}\langle\hat{a}_x\rangle \tag{9}$$

We want to calculate the reflection coefficient, which is:

$$r(w) = \frac{\langle\hat{b}_x\rangle}{\langle\hat{a}_x\rangle} = 1 - \frac{\kappa(i\Delta_a + \gamma)}{(i\Delta_c + \kappa/2)(i\Delta_a + \gamma) + g^2} \tag{10}$$

Under the condition that both the QD and the cavity are resonant with the field($\Delta_a = \Delta_c = 0$), we will have:

$$r(w) = \frac{C-1}{C+1} \tag{11}$$

$$\text{where } C = \frac{2g^2}{\gamma\kappa} \tag{12}$$

2.2 Initial State is $|-\rangle$

Here we will be working with the assumption that the lifetime of the state $|-\rangle$ is long compared to the dynamics of the input field. This is fairly correct as the lifetime was measured in the order of hundreds of picoseconds, where the input field was given for 80 picoseconds. So, if the QD will remain in state $|-\rangle$, then $\langle\hat{s}\rangle = 0$. This means that the QD is decoupled from the cavity, and the system reduces to a bare cavity driven by an input field. Therefore eqns (1),(2),(3) will reduce to only one:-

$$\langle\dot{\hat{a}}\rangle = -(i\Delta_c + \kappa/2)\langle a\rangle + \sqrt{\kappa}\langle\hat{a}_x\rangle \tag{13}$$

Taking the steady state solution of this equation-

$$\langle\hat{a}\rangle = \frac{\sqrt{\kappa}}{(i\Delta_c + \kappa/2)}\langle\hat{a}_x\rangle \tag{14}$$

this gives the reflection co-efficient as-

$$r(w) = 1 - \frac{\kappa}{i\Delta_c + \kappa/2} \tag{15}$$

$$\text{when } \Delta_c = 0, \text{we will have } r(w) = -1 \tag{16}$$

2.3 Final Relations in H/V basis

Writing the output and input operators in the H/V basis, we will get relations like:

$$\hat{b}_H = \frac{\hat{b}_x + \hat{b}_y}{\sqrt{2}}; \tag{17}$$

$$\hat{b}_V = \frac{\hat{b}_y - \hat{b}_x}{\sqrt{2}} \tag{18}$$

Similar relations for $\hat{a}_x, \hat{a}_y$ will exist. Then we will attain the following relations in terms of H/V basis:

$$\langle\hat{b}_H\rangle = \frac{1+r(w)}{2}\langle\hat{a}_H\rangle + \frac{1-r(w)}{2}\langle\hat{a}_V\rangle \tag{19}$$

$$\langle\hat{b}_V\rangle = \frac{1-r(w)}{2}\langle\hat{a}_H\rangle + \frac{1+r(w)}{2}\langle\hat{a}_V\rangle \tag{20}$$

We can observe that in the case when QD is in $|g\rangle$, we had r(w)=1 in the strong coupling regime, and the eqns (23),(24) show no flipping between the horizontal and vertical polarized photons. When the QD is in $|-\rangle$, we have r(w)=-1, showing a NOT behavior of the polarization's photon.

3 Reflection Spectrum

The intensities reflected from the cavity are of specific interest to us. We will calculate the intensities along the polarization axes (H and V) for a non-monochromatic field in general and then look at the same for the narrowband probe laser.

For a non-monochromatic field, each of its frequency components will interact with the cavity independently. The mean input and output field amplitude in the x-direction can be written as follows

$$\langle\hat{a}_x(t)\rangle = \int \varepsilon_x(\omega)e^{-i\omega t}d\omega,$$

$$\langle\hat{b}_x(t)\rangle = \int r(\omega)\varepsilon_x(\omega)e^{-i\omega t}d\omega$$

$$\text{Similarly,} \qquad \langle\hat{b}_y(t)\rangle = \int \varepsilon_y(\omega)e^{-i\omega t}d\omega$$

where $\varepsilon_x(w)$ is the Fourier component of the field amplitude along the x-direction. Using equations 19 and 20, we arrive at the following expressions that will help us calculate the relevant intensities.

$$\left\langle\hat{b}_H(t)\right\rangle = \int \left[\frac{1+r(\omega)}{2}\varepsilon_H(\omega) + \frac{1-r(\omega)}{2}\varepsilon_V(\omega)\right]e^{-i\omega t}d\omega$$

$$\left\langle\hat{b}_V(t)\right\rangle = \int \left[\frac{1-r(\omega)}{2}\varepsilon_H(\omega) + \frac{1+r(\omega)}{2}\varepsilon_V(\omega)\right]e^{-i\omega t}d\omega$$

Defining the total intensity of light reflected by the cavity in H polarization

$$W_H = \left\langle\hat{b}_H^\dagger\hat{b}_H\right\rangle$$

and using Parseval's theorem

$$W_H = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1+r(\omega)}{2}\varepsilon_H(\omega) + \frac{1-r(\omega)}{2}\varepsilon_V(\omega)\right|^2 d\omega$$

$$W_V = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1-r(\omega)}{2}\varepsilon_H(\omega) + \frac{1+r(\omega)}{2}\varepsilon_V(\omega)\right|^2 d\omega$$

For a vertically polarized input field, $\varepsilon_H(\omega) = 0$. Thus the above equations are reduced to

$$W_{V\to H} = \int_{-\infty}^{\infty}\left|\frac{1-r(\omega)}{2}\right|^2 S_{in}^V(\omega)d\omega \tag{21}$$

$$W_{V\to V} = \int_{-\infty}^{\infty}\left|\frac{1+r(\omega)}{2}\right|^2 S_{\text{in}}^V(\omega)d\omega \tag{22}$$

where $S_{in}^V(\omega) = |\varepsilon_V(\omega)|^2/2\pi$ is the input power spectrum. Similarly, for a horizontally polarized input field, $\varepsilon_H(\omega) = 0$, and we have

$$W_{H\to H} = \int_{-\infty}^{\infty}\left|\frac{1+r(\omega)}{2}\right|^2 S_{in}^H(\omega)d\omega \tag{23}$$

$$W_{H\to V} = \int_{-\infty}^{\infty}\left|\frac{1-r(\omega)}{2}\right|^2 S_{in}^H(\omega)d\omega \tag{24}$$

where $S_{in}^H(\omega) = |\varepsilon_H(\omega)|^2/2\pi$ These relations are generally true for a broadband laser (i.e. non-monochromatic field). However, the reflection coefficient and intensities will be modified for real QDs which exhibit spectral diffusion. This can be modeled by taking $\omega_a' = \omega_0 + \beta$ where ω_0 is the average transition frequency of the σ_+ transition and β is a zero-mean random variable that describes fluctuations in the QD resonant frequency due to spectral diffusion[2]. The reflection coefficient is modified to

$$r(\omega,\beta) = 1 - \frac{\kappa\left[i\left(\Delta_a^0+\beta\right)+\gamma\right]}{\left[i\Delta_c+\kappa/2\right]\left[i\left(\Delta_a^0+\beta\right)+\gamma\right]+g^2}$$

where $\Delta_a^0 = \omega - \omega_0$ is the mean detuning between the QD and the cavity. The intensities (21-24) now dependent on β must be averaged over all possible β, thus giving

$$W_{V\to H} = \int_{-\infty}^{\infty} P(\beta) \int_{-\infty}^{\infty} \left|\frac{1-r(\omega,\beta)}{2}\right|^2 S_{in}^V(\omega)d\omega d\beta$$
$$W_{V\to V} = \int_{-\infty}^{\infty} P(\beta) \int_{-\infty}^{\infty} \left|\frac{1+r(\omega,\beta)}{2}\right|^2 S_{in}^V(\omega)d\omega d\beta$$
$$W_{H\to H} = \int_{-\infty}^{\infty} P(\beta) \int_{-\infty}^{\infty} \left|\frac{1+r(\omega,\beta)}{2}\right|^2 S_{in}^H(\omega)d\omega d\beta$$
$$W_{H\to V} = \int_{-\infty}^{\infty} P(\beta) \int_{-\infty}^{\infty} \left|\frac{1-r(\omega,\beta)}{2}\right|^2 S_{in}^H(\omega)d\omega d\beta$$

where $P(\beta) = \frac{1}{\sqrt{2\pi\gamma_I^2}}\exp\left(-\frac{\beta^2}{2\gamma_I^2}\right)$ is a Gaussian probability distribution function where the standard deviation (γ_I) acts as the inhomogeneous linewidth of the QD. The plots in the appendix 2 show the numerically fitted and reproduced curve.

4 Calculating the Probabilities from the Plots

We wish to calculate the probabilities $P_{i\to j}$ where $i,j \in [H,V]$ from the plots of the reflected spectrums. Here $P_{i\to j}$ is the probability that a photon with initial polarization state "**i**" is detected in the polarization direction "*j*."

In the figure(1.a), we have the intensity plots for when the input polarization is H and measured polarization is V. Cavity reflected spectra is measured with a π pump pulse for 80ps(blue circles) and 4ns(red squares) pump-probe delay. The solid curves represent the numerical fits. The red curve corresponds to $W^g_{H\to V}$, the blue one is for $W^-_{H\to V}$, the black curve plots $W^\pi_{H\to V}$, and the green plots $W^\infty_{H\to V}$. $W^g_{H\to V}$ represents a numerical fit to the data of 4ns delay. Then we define $W^-_{H\to V}$ as the ideal intensity distribution when the QD is in $|-\rangle$, that is when the QD is decoupled from the cavity-field system. We can obtain this from $W^g_{H\to V}$ by setting **g=0**. Then the numerical fit for the 80ps data is given by the plot of $W^\pi_{H\to V}$. And the distribution $W^\infty_{H\to V} = \lim_{g\to\infty} W^g_{H\to V}$, which corresponds to spectrum under very strong coupling. It os not identically zero because of some background level reflection.
Note the different extremums of the figure(a), namely

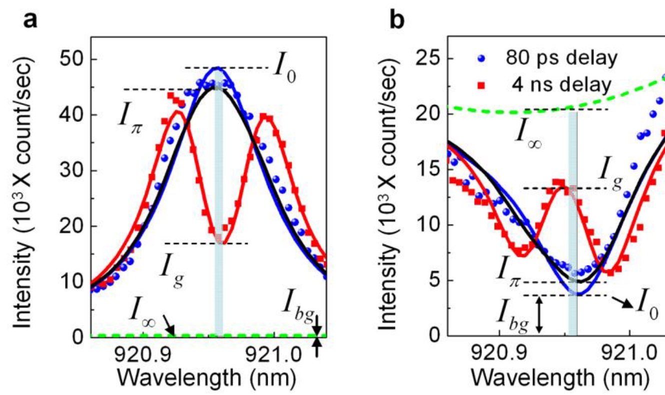

Figure 1: *Cavity spectra are measured with a π pump pulse for 80 ps (blue circles) and 4 ns (red squares) pump-probe delay for (a) input polarization of H and measured polarizations of V and (b) V and V.*

$I_o, I_\pi, I_g, I_\infty$. The probability values $P_{i\to j}$ when input-output polarizations are orthonormal are given by:

$$P^0_{i\to j} = \frac{(I_g - I_\infty)}{(I_o - I_\infty)} \text{ given QD is in } |g\rangle \tag{25}$$

$$P^\pi_{i\to j} = \frac{(I_\pi - I_\infty)}{(I_o - I_\infty)} \text{ given QD is in } |-\rangle \tag{26}$$

This can be understood by thinking $(I_o - I_\infty)$ as the total number of photons that we had given to the system, where $(I_g - I_\infty)$ and $(I_\pi - I_\infty)$ represent the photons that were reflected in the respective cases. Theoretically, we should have $P^0_{i\to j} = 0$ and $P^\pi_{i\to j} = 1$.

In the figure(1.b), we have the intensity plots for when the input polarization is V and measured polarization is V. Similarly, we can obtain the plots $W^g_{V\to V}$, $W^-_{V\to V}$, $W^\pi_{V\to V}$, and $W^\infty_{V\to V}$ shown in figure(b). The plots are conjugate of the ones shown in figure(a), because for some input polarization, the output of one polarization should be the conjugate of the other. Here we note that the background level intensity $_bg$ is quite high because direct surface reflection is observed significantly in the port that measures the same input and output polarization as surface reflection doesn't rotate the polarization.
Then for same input-output polarization, we have:

$$P^0_{i\to j} = \frac{(I_g - I_o)}{(I_\infty - I_o)} \text{ given QD is in } |g\rangle \tag{27}$$

$$P^\pi_{i\to j} = \frac{(I_\pi - I_o)}{(I_\infty - I_o)} \text{ given QD is in } |-\rangle \tag{28}$$

A Appendix

A.1 Deriving the Heisenberg-Langevin equations of motion from the Lindblad master equation

We have the total rotated Hamiltonian as:

$$\begin{aligned}\hat{H}_{tot} =& \hbar\Delta_c \hat{a}^\dagger \hat{a} + \frac{\hbar\Delta_a}{2}\hat{w} + \hbar g(\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a}) + \\ & i\hbar\sqrt{\kappa}(\hat{a}_x \hat{a}^\dagger - \hat{a}\hat{a}_x^\dagger)\end{aligned}$$

We use the Lindbald equation in Heisenberg picture , to get the equation of motion for operator $\hat{a}$ as:-

$$\begin{aligned}\dot{\hat{a}} = -i[\hat{a}, \Delta_c \hat{a}^\dagger \hat{a} + \frac{\Delta_a}{2}\hat{w} + g(\hat{a}^\dagger \hat{s} + \hat{s}^\dagger \hat{a}) + i\sqrt{\kappa}(\hat{a}_x \hat{a}^\dagger - \hat{a}\hat{a}_x^\dagger] \\ + \kappa(\hat{a}^\dagger \hat{a}^2 - \frac{1}{2}\hat{a}^\dagger \hat{a}^2 - \frac{1}{2}\hat{a}\hat{a}^\dagger \hat{a})\end{aligned}$$

The other terms will be zero as $[\hat{a}, \hat{s}] = 0$ and $[\hat{a}, \hat{w}] = 0$. After simplifying this we will arrive at Eqn(1).

A.2 Reproducing the Cavity Spectrum

Using the model assumed for spectral diffusion in the paper, the cavity spectrum can be successfully regenerated from the fitted parameters $g = 2\pi 12.9$ GHz, $\kappa = 2\pi 31.9$ GHz, and $\gamma_I = 2\pi 5.2$ GHz. The code below generates a plot (Figure 2) similar to the cavity spectrum given in the paper.

```python
import numpy as np
from scipy.integrate import quad
import matplotlib.pyplot as plt

c = 3e8
w0 = c/920.97          # Experimental resonant frequency
wa = w0                # QD transition frequency
wc = w0                # Cavity transition frequency
g = 2*np.pi*12.9       # Cavity-QD coupling strength
kappa = 2*np.pi*31.9   # Cavity energy decay rate
gamma = 2*0.943        # Exciton decay rate for the sigma+ transition
gamma_I = 2*np.pi*5.2  # Inhomogenous linewidth

def P(x):
  return 1/np.sqrt(2*np.pi*gamma_I**2) * np.exp(-x**2/(2*gamma_I**2))

def I_VtoH(x, w):
  return P(x) * abs((kappa * complex(gamma, w - wa + x))/(complex(kappa/2, w - wc) * complex(gamma, w - wa + x) + g**2))**2/4

f = np.vectorize(I_VtoH)

w = np.linspace(int(w0-500), int(w0+500), 1000)
int_f = np.vectorize(lambda w: quad(f, -np.inf, np.inf, args=(w), epsrel=1e-2)[0])
res = int_f(w)
plt.plot(c/w, res)
plt.xlim(920, 922)
plt.xlabel('Wavelength (nm)')
plt.ylabel('Intensity (counts/s)')
plt.show()
```

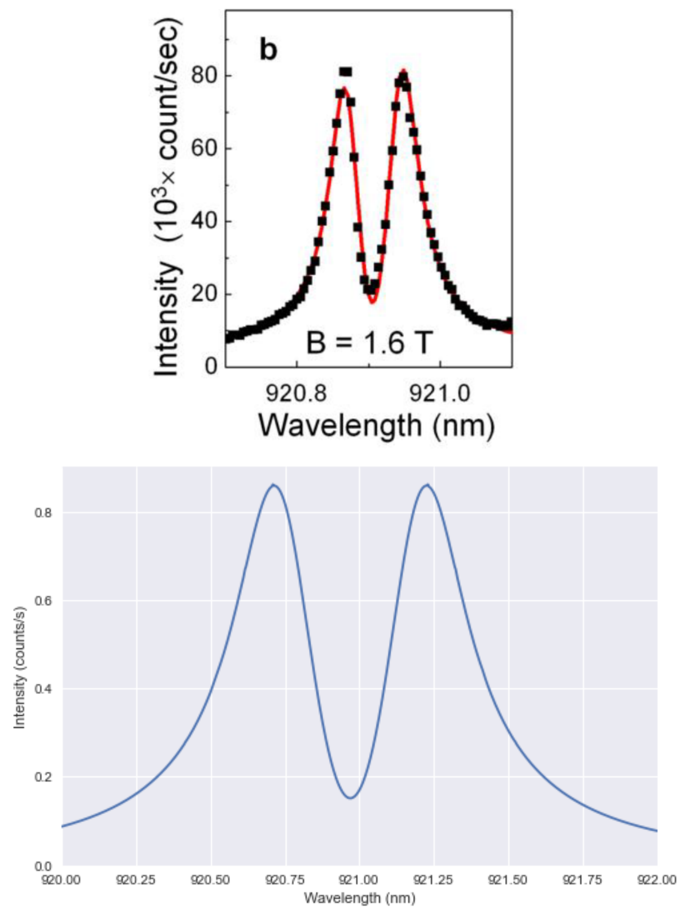

Figure 2: *Intensity vs wavelength (detuning of the pump laser) plots where the QD is in ground state and is highly coupled with the cavity. The first plot is taken from [2]*

Figure 3 corresponds to the QD $|-\rangle$ state i.e. when there is zero coupling between QD and cavity. It can be obtained by running the same code above only setting g to 0 this time. This plot shows very high similarity with all of the zero coupling intensity plots in the paper.

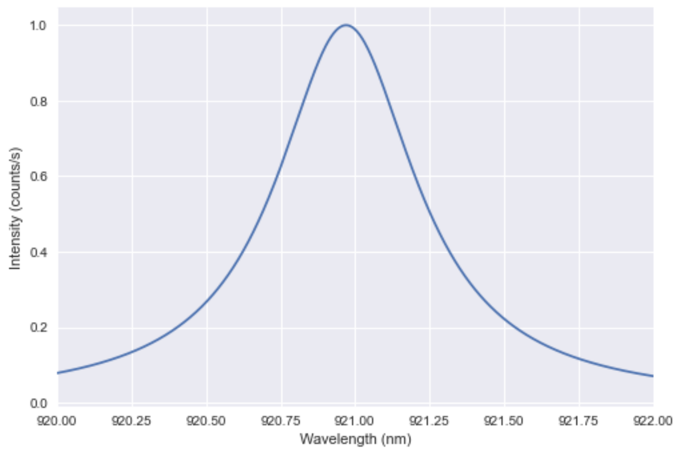

Figure 3: *Intensity vs wavelength (detuning of the pump laser) plot with no coupling between the QD and the cavity.*

A.3 Plotting reflection coefficient as a function of time

The following code plots the reflection coefficient (r) as a function of time. Figure 4 shows the convergence of r to 1 and -1 for the cases $g = 2\pi * 12.9$ GHz and $g = 0$, respectively.

```
import numpy as np
import matplotlib.pyplot as plt
from itertools import product
from qutip import *

N_res = 2                         # Number of reservoir states

# Define the parameters
delta_wa = 0              # QD transition frequency
delta_wc = 0              # Cavity transition frequency
g = 2*np.pi*12.9e9        # Cavity-QD coupling strength
kappa = 2*np.pi*31.9e9    # Cavity energy decay rate
k = np.sqrt(kappa)        # Cavity-Reservoir coupling strength
Gamma_spon = 1.887e9      # Exciton decay rate for the sigma+ transition
T_d = 7.6e-9              # Pure dephasing time

plus_state = (basis(N_res, N_res-1) + basis(N_res, N_res-2)).unit()  # state |+⟩
minus_state = (basis(N_res, N_res-1) - basis(N_res, N_res-2)).unit()  # state |-⟩

# Define the qubit states of the QD
g_state = basis(2, 0)  # ground state |g⟩
e_state = basis(2, 1)  # excited state |e⟩
qd = [g_state, e_state]

# Define the qubit states of the photon in the cavity
cav = [basis(2, 0), basis(2, 1)]

# Define the qubit states of the photon outside the cavity
res = [plus_state, minus_state]

# Define the operators
s = tensor(g_state * e_state.dag(), qeye(2), qeye(N_res))  # |g⟩⟨e|  I  I
w = tensor(e_state * e_state.dag() - g_state * g_state.dag(), qeye(2), qeye(N_res))      # |e⟩⟨e| - |g⟩⟨g|  I  I
_w_ = tensor(e_state * e_state.dag() + g_state * g_state.dag(), qeye(2), qeye(N_res))    # |e⟩⟨e| + |g⟩⟨g|  I  I

a = tensor(qeye(2), destroy(2), qeye(N_res))  # I  a  I
b = tensor(qeye(2), qeye(2), destroy(N_res))  # I  I  b
b_out = b - k*a

# Define the Hamiltonian (under RWA)
def H(g):
        return delta_wc * a.dag() * a + delta_wa * w / 2 + g * (a.dag() * s + a * s.dag()) + 1j * k * (a.dag() * b - a * b.dag())

# Define the collapse operators
c_ops = [np.sqrt(kappa) * a, np.sqrt(Gamma_spon/2) * s, np.sqrt(1/T_d) * (_w_)]

tlist = np.linspace(0, 5e-9, 1001)

# Define the initial state
psi0 = tensor(qd[0], cav[0], res[0])

# Solve master equation for finite coupling
result1 = mesolve(H(g=2*np.pi*12.9e9), psi0, tlist, c_ops, [s.dag() * s, a.dag() * a, b, b_out])

# Solve master equation for zero coupling
```

```python
result2 = mesolve(H(g=0), psi0, tlist, c_ops, [s.dag() * s, a.dag() * a, b, b_out])

plt.plot(tlist, (result1.expect[3]/result1.expect[2]).real, label='g = 2 × 12.9 GHz', color='C0')
plt.plot(tlist, (result2.expect[3]/result2.expect[2]).real, label='g = 0', color='C2')
plt.ylabel('Reflection coefficient')
plt.xlabel('Time (s)')
plt.legend()
plt.show()
```

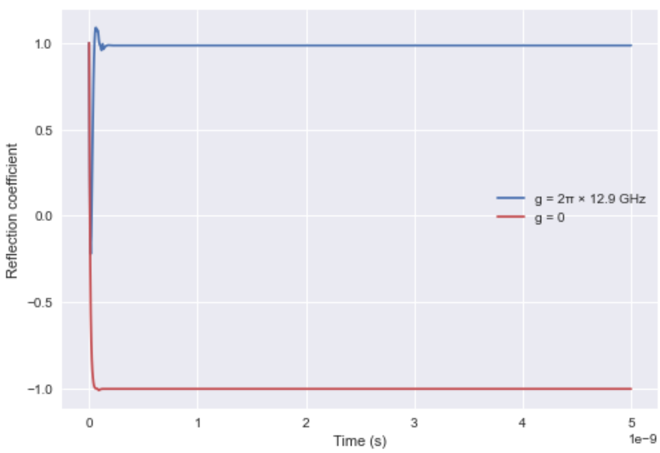

Figure 4: *Reflection coefficient vs time evolution plot with both cases of coupling values. The cavity mode is in* $|0\rangle$ *and the reservoir in* $|+\rangle$ *(or H state) initially.*

A.4 Bloch sphere representation of the evolving reservoir state

The code below plots the bloch sphere representation under the condition that there is no decay in the cavity mode, which is ideal but required to visualize the flip operation distinctively.

```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mpl
from itertools import product
from qutip import *

mpl.style.use('seaborn')
%config InlineBackend.figure_format = 'png'

N_res = 2                               # Number of reservoir states

# Define the parameters
delta_wa = 0 # 2*np.pi*20e9          # QD transition frequency
delta_wc = 0 # 2*np.pi*30e9          # Cavity transition frequency
g = 2*np.pi*12.9e9            # Cavity-QD coupling strength
kappa = 0            # Cavity energy decay rate
k = 2e9            # Cavity-Reservoir coupling strength
Gamma_spon = 1.887e9            # Exciton decay rate for the sigma+ transition
T_d = 7.6e-9            # Pure dephasing time

plus_state = (basis(N_res, N_res-1) + basis(N_res, N_res-2)).unit()  # state |+⟩
minus_state = (basis(N_res, N_res-1) - basis(N_res, N_res-2)).unit()  # state |-⟩

# Define the qubit states of the QD
g_state = basis(2, 0)  # ground state |g⟩
e_state = basis(2, 1)  # excited state |e⟩
qd = [g_state, e_state]

# Define the qubit states of the photon in the cavity
cav = [basis(2, 0), basis(2, 1)]

# Define the qubit states of the photon outside the cavity
res = [plus_state, minus_state]
meas_res = [tensor(qeye(2), qeye(2), res[0] * res[0].dag()), tensor(qeye(2), qeye(2), res[1] * res[1].dag())]

# Define the operators
s = tensor(g_state * e_state.dag(), qeye(2), qeye(N_res))  # |g⟩⟨e|  I  I
w = tensor(e_state * e_state.dag() - g_state * g_state.dag(), qeye(2), qeye(N_res))      # |e⟩⟨e| - |g⟩⟨g|  I  I
_w_ = tensor(e_state * e_state.dag() + g_state * g_state.dag(), qeye(2), qeye(N_res))    # |e⟩⟨e| + |g⟩⟨g|  I  I

a = tensor(qeye(2), destroy(2), qeye(N_res))  # I  a  I
b = tensor(qeye(2), qeye(2), destroy(N_res))  # I  I  b
b_out = b - np.sqrt(k)*a

meas_plus = tensor(qeye(2), qeye(2), plus_state * plus_state.dag())

# Define the Hamiltonian (under RWA)
H = delta_wc * a.dag() * a + delta_wa * w / 2 + g * (a.dag() * s + a * s.dag()) + 2j * np.pi * k * (a.dag() * b - a * b.dag())

```

```python
# Define the collapse operators
c_ops = [np.sqrt(kappa) * a, np.sqrt(Gamma_spon/2) * s, np.sqrt(1/T_d) * (_w_)]

tlist = np.linspace(0, 4.75e-9, 101)

# Define the initial state
psi0 = tensor(qd[0], cav[0], res[0])

result = mesolve(H, psi0, tlist, c_ops)
states = [result.states[t].ptrace(2) for t in range(len(result.states))]
b = Bloch()
b.add_states(result.states[0].ptrace(0))
b.add_states(states, kind='point')
b.render()
```

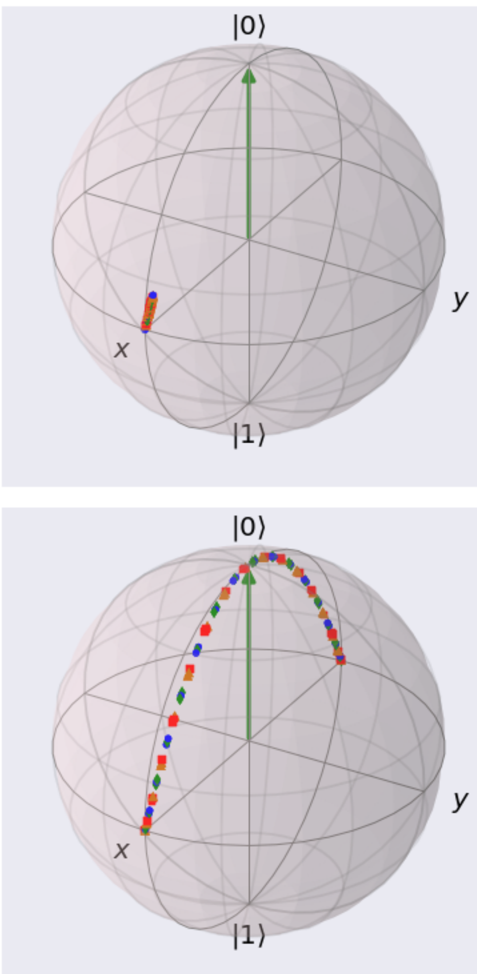

Figure 5: *Bloch sphere representation of the reservoir mode with both cases of coupling values for the whole time range of $\tilde{5}$ ns. The cavity mode is in $|0\rangle$ and the reservoir in $|+\rangle$ (or H state) initially.*

B Acknowledgement

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References

[1] G. Milburn D. Walls. *Quantum Optics*. Springer, 1994.

[2] R. Bose E. Waks H, Kim. A quantum logic gate between a solid-state quantum bit and a photon, 2013.