Gaussian Boson Sampling

Asish Kumar Mandoi
akmandoi@iitk.ac.in
Department of Electrical Engineering
Indian Institute of Technology Kanpur

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Abstract

The development of quantum computers that can successfully carry out tasks beyond the capabilities of classical computers is considered a major technological advance. Recently, there have been significant developments in building of photonic quantum computers, in particular those with an Gaussian Boson Sampling architecture. The aim of this project is to understand the working of the GBS architecture and how GBS machines have become leading candidates for achieving a quantum advantage.

INTRODUCTION

Boson Sampling was initially proposed by Aaronson and Arkhipov[1] in 2013. It demonstrates the power of quantum over classical computation and provides evidence against the Extended Church-Turing theorem. In other words we don't need a universal quantum computer to realize quantum advantage. Since, boson sampling also favours current photonic experimental platforms, researchers have come up with methods to sample photons from a general Gaussian state[2]. In fact, the recent launch of Xanadu's Borealis device was considered an important milestone for quantum computing, as it demonstrated a quantum computational advantage experiment using quantum photonics which was featured in Nature. Among other quantum advantage achievements prior to this is the Google Quantum team's experiment as can be seen in their paper Quantum supremacy using a programmable superconducting processor[3]. While Google's experiment performed the task of random circuit sampling using a superconducting processor, Xanadu leveraged the quantum properties of light to tackle Gaussian Boson Sampling (GBS).

BACKGROUND

A photon is said to be in a squeezed state if its electric field strength ϵ for some phases ν has a quantum uncertainty smaller than that of a coherent state corresponding to the photon. To obey Heisenberg's uncertainty relation, a squeezed state must also have phases at which the electric field uncertainty is *anti-squeezed*, i.e. larger than that of a coherent state.

In quantum optics, the fundamental physical systems of interest are optical modes of the quantized electromagnetic field, often referred to as qumodes. As opposed to qubits that are described by two-dimensional Hilbert spaces, qumodes are mathematically represented by a Hilbert space of infinite dimension. A general qubit state can be written as $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$, with $|c_0|^2 + |c_1|^2 = 1$. A general qumode state can be expressed as $|\psi\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$, with $\sum_{n=0}^{\infty} |c_n|^2 = 1$. The basis states $|0\rangle, |1\rangle, |2\rangle, \ldots$ are known as Fock states, and the Fock state $|n\rangle$ has the physical interpretation of a qumode with n photons.

The state of a system of m qumodes can also be uniquely specified by its Wigner function $W(\boldsymbol{q}, \boldsymbol{p})$, where $\boldsymbol{q} \in \mathbb{R}^m$ and $\boldsymbol{p} \in \mathbb{R}^m$ are known respectively as the position and momentum quadrature vectors. Gaussian states are characterized by having a Wigner function which is a Gaussian distribution.

IMPLMENTATION OF GBS ON PHOTONIC HARDWARE

GBS is a special-purpose model of photonic quantum computation where a multi-mode Gaussian state is prepared and then measured in the Fock basis. A general pure Gaussian state can be prepared from the vacuum by a sequence of single-mode squeezing, multimode linear interferometry, and single-mode displacements. For a Gaussian state with zero mean – which can be prepared using only squeezing followed by linear interferometry – the probability $\Pr(S)$ of observing an output $S = (s_1, s_2, \ldots, s_m)$, where s_i denotes the number of photons detected in the i-th mode, is given by

$$\Pr(S) = \frac{1}{\sqrt{\det(\boldsymbol{Q})}} \frac{\mathrm{Haf}(\mathcal{A})}{s_1! s_2! \ldots s_m!}$$

where,

$$\begin{aligned} \boldsymbol{Q} &:= \boldsymbol{\Sigma} + I/2 \\ \mathcal{A} &:= \boldsymbol{X}(1 - \boldsymbol{Q}^{-1}) \\ \boldsymbol{X} &:= \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \end{aligned}$$

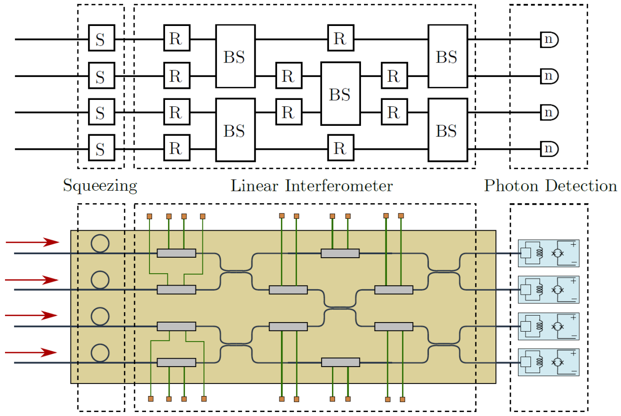

FIG. 1. A schematic illustration of a GBS circuit creating and measuring a Gaussian state with zero displacement. The top figure shows the sequence of gates that are used to create the Gaussian state: squeezing gates, followed by a linear interferometer that can be decomposed in terms of rotation (R) and beamsplitter (BS) gates. The state is then measured in the Fock basis, revealing the number of photons detected in each mode. The bottom figure is a schematic of a photonic chip realizing the same circuit. [4]

$\boldsymbol{\Sigma}$ is the covariance matrix of the Gaussian state. The matrix function $\mathrm{Haf}(\cdot)$ is the hafnian defined as

$$\mathrm{Haf}(\mathcal{A}) = \sum_{\pi \in \mathrm{PMP}} \prod_{(i,j)\in\pi} \mathcal{A}_{ij}$$

where $\mathcal{A}_{ij}$ are the entries of $\boldsymbol{\mathcal{A}}$ and PMP denotes the set of perfect matching permutations. It should be noted that computing the hafnian is a #P-hard problem [1, 2], a fact that has been leveraged to argue that, unless the polynomial hierarchy collapses to third level, it is not possible to efficiently simulate GBS using classical computers.

The diagram above describes a schematic of a GBS architecture. In this particular GBS architecture, a bright pump laser enters the chip, which is used to generate squeezing in a neighbouring mode via nonlinear effects in a microring resonator. Rotation gates are implemented using tunable phase-shifters (grey boxes with electrical contacts). Beamsplitters are implemented through evanescent coupling of waveguides that are brought close to each other. They can be made tunable by extending them to a Mach-Zehnder interferometer, which is not shown in this figure. Finally, output light is measured using photon-number-resolving detectors, which can be implemented for example using superconducting transition edge sensors. [4]

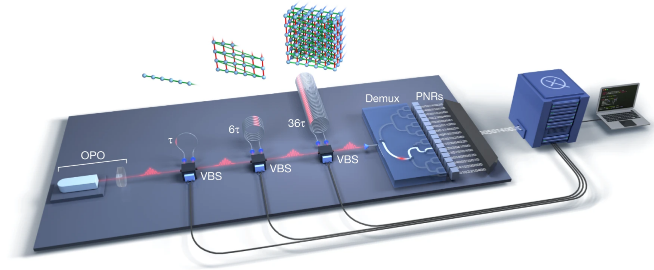

FIG. 2. High-dimensional GBS from a fully programmable photonic processor [5]

QUANTUM COMPUTATIONAL ADVANTAGE WITH A PROGRAMMABLE PHOTONIC PROCESSOR

In June 2022, Nature published an article [5] reporting quantum computational advantage using Borealis, a photonic processor offering dynamic programmability on all gates implemented, that was built by Xanadu. They carried out Gaussian boson sampling on 216 squeezed modes entangled with three-dimensional connectivity, using a time-multiplexed and photon-number-resolving architecture. They have claimed to have solved certain technological hurdles associated with time-domain multiplexing, fast electro-optical switching, high-speed photon-number-resolving detection technology and non-classical light generation, to build a scalable and programmable Gaussian boson sampler.

As shown in fig. 2, Borealis implements linear-optical transformations on a train of input squeezed-light pulses, using a sequence of three variable beamsplitters (VBSs) and phase-stabilized fibre loops that act as effective buffer memory for light, allowing interference between modes that are either temporally adjacent, or separated by six or 36 time bins. [5]

CONCLUSION

It was quickly realized that GBS offers significant versatility in the scope of problems that can be encoded into the device. GBS has applications in graph optimization, molecular docking, graph similarity, point processes, and quantum chemistry.

While Boson Sampling devices are computationally hard to simulate, they are not capable of universal quantum computing on their own. However, in combination with other components, photonic Boson Sampling, in particular, GBS has not only established the advantage of near-term quantum computers over classical computer, but it is also a key building block for a universal device.

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