# CHAPTER 11


# NOISE AND NOISE REJECTION


## INTRODUCTION

In general, noise is any unsteady component of a signal which causes the instantaneous value to differ from the *true value.* (Finite response time effects, leading to dynamic error, are part of an instrument's response characteristics and are not considered to be noise.) In electrical signals, noise often appears as a highly erratic component superimposed on the *desired* signal. If the noise signal amplitude is generally lower than the *desired* signal amplitude, then the signal may look like the signal shown in Figure 1.
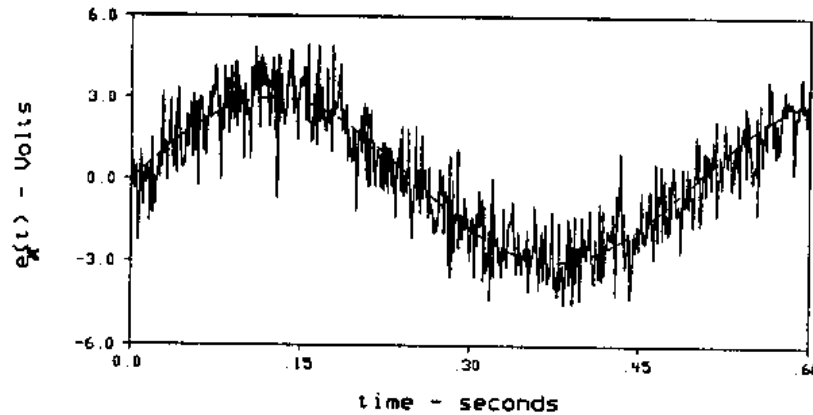


Figure 1: Sinusoidal Signal with Noise.

Noise is often random in nature and thus it is described in terms of its average behavior (see the last section of Chapter 8). In particular we describe a random signal in terms of its power spectral density, $(\phi_x(f))$, which shows how the average signal power is distributed over a range of frequencies, or in terms of its average power, or mean square value. Since we assume the average signal power to be the power dissipated when the signal voltage is connected across a 1 $\Omega$ resistor, the numerical values of signal power and signal mean square value are equal, only the units differ. To determine the signal power we can use either the time history or the power spectral density (Parseval's Theorem). Let the signal be x(t), then the average signal power or mean square voltage is:

$$\overline{x^2(t)} = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x^2(t)\, dt = \int_0^\infty \phi_x(f)\, df \tag{1}$$

Note: the bar notation, $\bar{\phantom{x}}$, denotes a time average taken over many oscillations of the signal. Consider now the case shown in Figure 1, where $x(t) = s(t) + n(t)$. $s(t)$ is the signal we wish to measure and $n(t)$ is the noise signal. $x(t)$ is the signal we actually measure. If $s(t)$ and $n(t)$ are independent of one another, the mean square voltage of $x(t)$ is the sum of the mean square of voltage of $s(t)$ and the mean square voltage of $n(t)$:

$$\overline{x^2(t)} = \underset{\substack{desired \\ signal\ power}}{\overline{s^2(t)}} + \underset{\substack{noise \\ power}}{\overline{n^2(t)}} \tag{2}$$

Here we have assumed that the noise is added on to the desired signal. This may not always be the case. However, in this chapter we will generally assume that the noise is additive.

Often noise and indeed other signals are described in terms of their *root mean square* (rms) voltage. This is the square root of the mean square value:

$$n(t)_{rms} = \sqrt{\overline{n^2(t)}} \tag{3}$$

When noise is harmonic in nature, i.e., if $n(t) = A\sin(\omega t + \delta)$, where $\delta$ is an arbitrary phase, as would be the case if we were picking up line noise in the measurement circuit, then:

$$n(t)_{rms} = \sqrt{\overline{n^2(t)}} = \frac{A}{\sqrt{2}} \tag{4}$$

(The proof is left as an exercise for the student.)

When we take measurements of random signals we can calculate the mean and variance and standard deviation. How do these relate to the mean square and root mean square voltages? Recall that the variance of a signal is:

$$\sigma_n^2 = E[(n - \mu_n)^2] = E[n^2 - 2n\mu_n + \mu_n^2] = E[n^2] - 2\mu_n\,E[n] + \mu_n^2 \tag{5}$$

where E[.] denotes average value of. Recognizing that $E[n^2] = \overline{n^2}$, and $E[n] = \mu_n$, we see that:

$$\sigma_n^2 = \overline{n^2(t)} - \mu_n^2 \tag{6}$$

Hence, if the signal has a mean value of zero ($\mu_n = 0$) then the variance is equal to the mean square value. In general:

$$\overline{n^2(t)} = \sigma_n^2 + \mu_n^2 \tag{7}$$

The rms value of the signal is:

$$n(t)_{rms} = \sqrt{\sigma_n^2 + \mu_n^2} \qquad (8)$$

When the mean value is zero, the rms value equals the standard deviation of the signal, $\sigma_n$.

### *Signal to Noise Ratio*

Both the desired signal, s(t), and the noise, n(t), appear at the same point in a system and are measured across the same impedance. Therefore we define the signal to noise *power* ratio as:

$$S/N = \frac{\overline{s^2(t)}}{\overline{n^2(t)}} = \frac{\text{signal power}}{\text{noise power}} \ . \qquad (9)$$

It is common to express the signal to noise ratio in decibels. Thus:

$$S/N\big|_{dB} = 10 \ \log_{10} \frac{\text{signal power}}{\text{noise power}} \qquad (10)$$

and

$$S/N\big|_{dB} = 20 \ \log_{10} \frac{\text{signal rms}}{\text{noise rms}} \qquad (11)$$

Note that decibels are strictly defined in terms of power (or voltage squared), hence the factor of 10. If the quantity is expressed in terms of rms voltage, then a factor of 20 is used (as we did when drawing Bode plots).

### *System Noisiness*

The *noisiness* of a system (or a system element) is called the noise figure (NF) which is defined in terms of the input and output signal to noise power ratios.

$$NF = \frac{(S/N)_{input}}{(S/N)_{output}} \qquad (12)$$

where $NF \geq 1$. Note that the S/N values must be referred to the same bandwidth for (12) to be meaningful. That is, if the output bandwidth is restricted by a filter or low bandwidth amplifier, this must be taken into account. The noise figure is often given in dB:

$$NF_{dB} = 10\log_{10} NF \qquad (13)$$

## EXAMPLE

The vibration of the side of a car engine block is measured with a piezoelectric accelerometer connected to a charge amplifier, which is connected to a low-pass filter and a data acquisition system. The main part of the signal is a periodic component whose fundamental frequency is the engine rpm. Added to this signal is some line noise at 60 Hz. that is due to a poor measurement setup, and some broadband electronic noise and quantization noise introduced by the analog to digital converter; this noise is random in nature, with a power spectral density as shown in Figure 2.
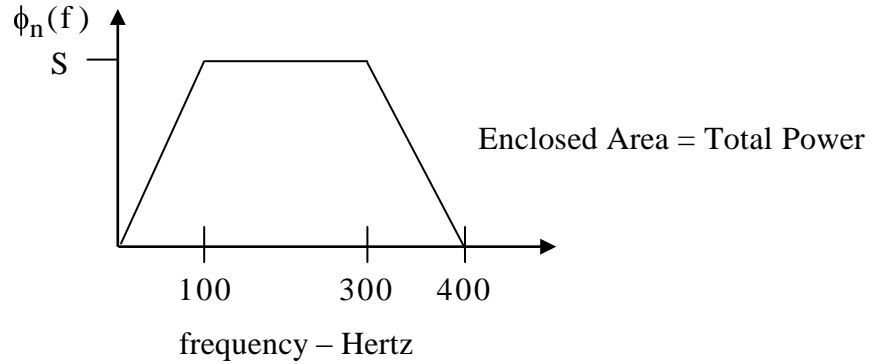


Figure 2: Power Spectral Density of the Random Component of the Signal

The measured signal can be modeled as:

$$x(t) = p(t) + B\sin(2\pi 60t) + n(t)$$

where p(t) is the periodic vibration signal and n(t) is the random component of the noise. Let's now calculate the signal to noise ratio in dB. Since p(t) is periodic it can be expressed as a Fourier Series.

$$p(t) = \frac{A_o}{2} + \sum_{k=1}^{\infty} M_k \cos(k\omega_1 t - \theta_k) \tag{14a}$$

where $\omega_1 = 2\pi/T$ rad/s, and T is the period of the periodic signal. 60/T is the engine rpm. The *power* of the kth component of this periodic engine signal is:

$$\frac{M_k^2}{T_o} \int_0^T \cos^2(k\omega_1 t - \theta_k) dt = \frac{M_k^2}{2} \ .$$

Note that we will stop italicizing *power,* understanding that we are using the term loosely in this context. The total power of the periodic signal is the sum of the power of the individual components:

$$\text{power of } p(t) = \frac{A_o^2}{4} + \sum_{k=1}^{\infty} \frac{M_k^2}{2} \ . \tag{14b}$$

We can plot these individual powers versus frequency ($2\pi k/T$ rad/s). This is called a *power spectrum,* an example of one is plotted in Figure 3. The term *discrete* is used because there is only power at discrete frequencies (multiples of $\omega_1$).
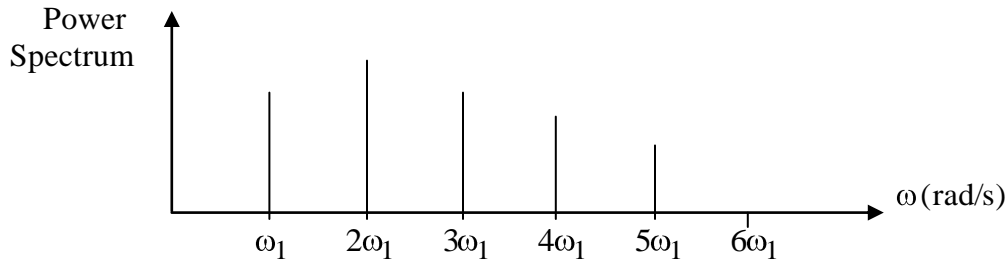


Figure 3:    Discrete Power Spectrum

The power of the 60 Hertz component (let's call it $\overline{m^2(t)}$) is $B^2/2$ because it is a pure sine wave. The power of the random component is the integral of the power spectral density function, $\phi_n(f)$, which is shown in Figure 2. Recall from the earlier chapter that the power spectral density is the distribution of the average signal power across frequency; therefore, the signal power is the integral of this function, i.e., the area under the curve.

$$\text{power of } n(t) = \overline{n^2(t)} = \int_0^{\infty} \phi_n(f) \, df = 300 \, S \qquad \text{(see Figure 2).}$$

Treating p(t) as the signal that we wish to measure, sometimes referred to as the *desired* signal, and n(t) and the power line component as *the noise,* the signal to noise ratio in dB is:

$$S/N\big|_{dB} = 10\log_{10}\left(\frac{\overline{p^2(t)}}{\overline{n^2(t)} + \overline{m^2(t)}}\right) = 10\log_{10}\left(\frac{\dfrac{A_o^2}{4} + \sum_{k=1}^{\infty} \dfrac{M_k^2}{2}}{300\,S + \dfrac{B^2}{2}}\right) \ .$$

## NOISE SOURCES

Noise can arise either from internal system elements or from external disturbances. The important identifiable sources of noise are summarized below:

*Fundamental Internal Noise Sources:*

1. **Johnson (Thermal)** noise is due to the random motion of electrons in a conductor. The *power* spectral density of the Johnson noise current signal is constant, given by,

$$\phi_{i_J}(f) = \frac{4\,k\,T}{R} \tag{15}$$

   where       T = Absolute temperature, K
   R = Resistance, ohms
   k = Boltzmann constant, $1.38 \times 10^{-23}$ Joule/K

   The corresponding *power* spectral density of the Johnson noise voltage signal is:

$$\phi_J(f) = 4\,k\,T\,R \tag{16}$$

   Thermal noise can be reduced only by reducing the temperature or the bandwidth of the measuring system.

2. ***Shot*** noise is due to the quantized nature of electrons emitted in vacuum tubes, semiconductors, etc. Again it has a constant *power* spectral density given by, if the signal measured is current,

$$\phi_{i_S}(f) = 2\,I\,e \tag{17}$$

   where   I = Current flow due to some random electron emission process in Amperes,
   and     e = Electron charge, $1.59 \times 10^{-19}$ Coulomb

   The corresponding *power* spectral density of the shot noise voltage signal measured across a resistance R is,

$$\phi_S(f) = 2\,I\,e\,R^2 \tag{18}$$

   Note that shot noise can only be reduced by reducing the system bandwidth.

3. ***Flicker*** noise occurs whenever electron conduction occurs in a conducting medium and is particularly important in semiconductors. It is not well understood physically. Its *power* spectral density has been observed to take the form:

$$\phi_{i_{NF}}(f) = \frac{C}{f^n} \tag{19}$$

   where C = Material dependent constant, f = frequency, Hz, $n \approx 1$.

Flicker noise is normally important only at low frequencies (below 1 KHz).
Moral: Avoid DC measurements when extremely small signals are to be measured.

**A Note on Calculating the Power of the Noise Component of the Output Signal.**

In equations (15)-(19), the power spectral densities for Johnson, shot and flicker noise are given. The system which is creating this noise will act as a filter and so the power spectral density of the noise component of the output signal $(\phi_{n-out}(f))$ is:

$$\phi_{n-out}(f) = \left|T(j2\pi f)\right|^2 \phi_{n-in}(f) \ ,$$

where $\left|T(j2\pi f)\right|$ is the magnitude of the system filter that operates on the noise, and $\phi_{n-in}(f)$ is the power spectral density of the Johnson, shot or flicker noise prior to this filtering. To calculate the power, we must integrate $\phi_{n-out}(f)$. So the *power* (or mean square voltage) of the noise component of the output signal is:

$$\overline{n^2(t)} = \int_0^\infty \phi_{n-out}(f)\,df = \int_0^\infty \left|T(j2\pi f)\right|^2 \phi_{n-in}(f)\,df$$

**EXAMPLE**

Suppose the noise was caused by Johnson noise with R=10 MΩ, T=160° C, and the system behaved as an ideal filter:

$$\left|T(j2\pi f)\right| = 20 \text{ for } 100 < f < 1000,$$
$$= 0 \ \text{ for } f < 100 \ \text{ and } \ f > 1000 \text{ Hz.}$$

Calculate the mean square voltage of the noise signal coming out of the system.
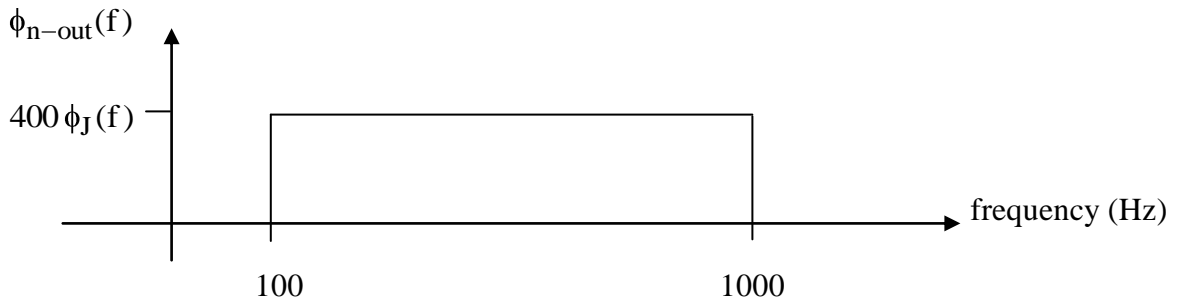
**Solution**



Figure 4: Power Spectral Density of the Noise on the System Output

The total power (or mean square value) is the area under the output power spectral density, which is plotted in Fig. 4.

$$\overline{n^2(t)} = \int_0^\infty \phi_{n-out}(f)\,df = \int_0^\infty |T(j2\pi f)|^2 \phi_{n-in}(f)\,df = \int_{100}^{1000} (20)^2 4kTR\,df$$

$$= 900 \times 400 \times 4 \times 1.38\,10^{-23}\,(273+160)\,10^{+7} = 6.617 \times 10^{-8}\quad \text{Volts}^2$$

*Environmental (External) Noise Sources:*

Many external sources of electrical noise exist which can influence the output of a measurement system.

1.  60 Hz (and higher harmonics at 120, 180, 240 Hz. etc.) picked up from lights, motors, and other devices.

2.  Radio & TV stations $\approx 10^6 - 10^8$ Hz.

3.  Lightning, arc welders, auto ignition systems and other "spark" generators produce wide band noise.

4.  Mechanical vibrations affect the output of certain transducers. This is normally at frequencies below 20 Hz.

Johnson noise and shot noise are independent of frequency, i.e., their power spectral density is constant. This is referred to as *white noise,* which is illustrated in Figure 5.
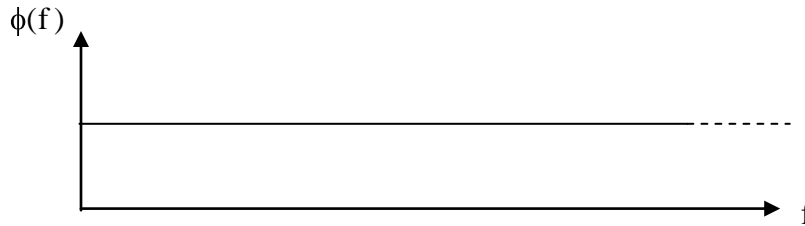


Figure 5:  Power Spectral Density of White Noise

As noted above, spark sources generate wide band noise. This is often nearly white in character. Flicker noise and the noise induced by other environmental sources have a strong frequency dependence as illustrated in Figure 6.
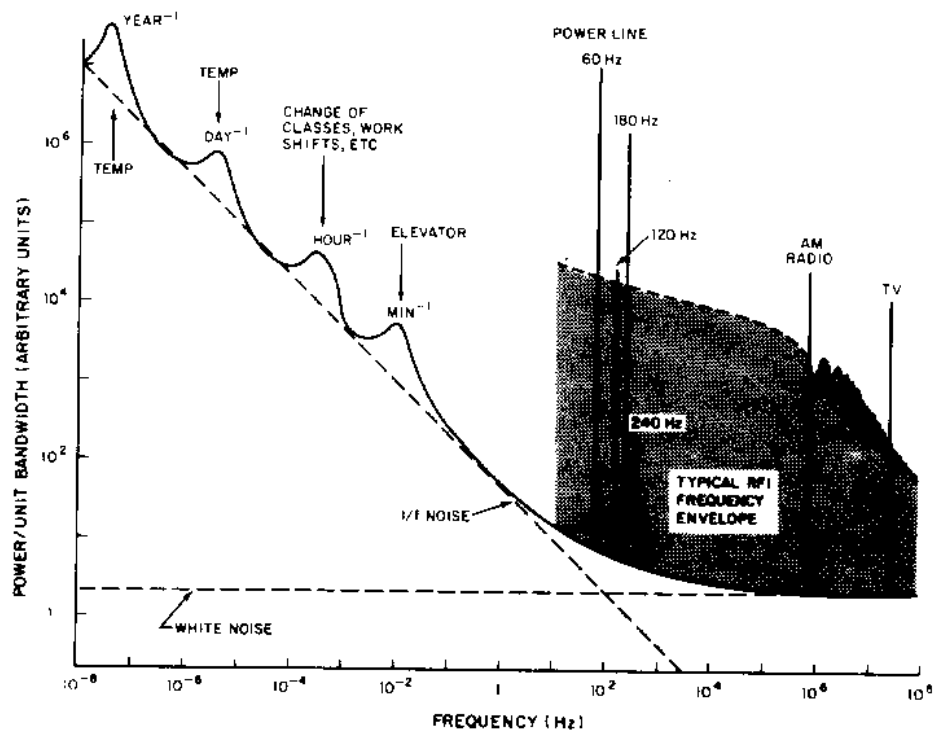
Figure 6:  The Spectrum of Common Environmental Noise
(From Vol. 1 - Handbook of Measurement Science,
P. Sydenham, ed., John Wiley and Sons, 1982.)

Noise from internal sources can be reduced by filtering. Thermal noise can, in addition, be reduced by cooling the device involved. Flicker noise can be overcome by chopping the signal at a high frequency so that AC rather than DC measurements can be made.

Environmental noise can in principle be made negligible, although this may be very difficult to do in practice. The remainder of our discussion will be concerned with environmental noise and ways of minimizing it. Particular attention will be paid to 60 Hz noise, since this is the most common problem and often the most difficult to minimize by filtering.

## COUPLING FROM EXTERNAL NOISE SOURCES

External sources of electrical power will cause measurement errors if this power is coupled into a measurement circuit. Accurate modeling of the coupling and computation of its effect in a given case is usually impossible, but it is possible to develop simple models which illustrate the general problems which can occur. This in turn provides a physical foundation for developing useful methods of noise reduction.

Noise from external sources is coupled into instrumentation circuits by various means:

> *Inductive coupling* from power lines. This can be reduced by physical separation of the instrument circuit from the power line and by using twisted wire connecting cables in the circuit.

> *Capacitance coupling* from power lines. This can be reduced by physical separation and by using a grounded conductive shield.

Both types of coupling can induce *series mode* as well as *common mode* noise in a circuit. Another important cause of such induced noise is multiple grounds in a circuit which can lead to induced noise through *ground loops.* We will explore the significance of these concepts here.

*Inductive (Electromagnetic) Coupling*

Noise voltage in a measurement system can be generated by a mutual inductance between the measurement circuit and a nearby power circuit (AC). The noise voltage will depend on the overlapping length of the two circuits and will occur *even if the measurement circuit is isolated from earth ground.* This source of noise is especially common in factories where many high current power lines are needed to supply power to motors and other heavy duty electrical equipment.

In Figure 7 is shown a power line in close proximity to one lead of a measurement circuit, represented by its Thevenin equivalent voltage source $E_{Th}$ and impedance $Z_{Th}$. There will be some mutual inductance M between the lead and power line, leading to a *series-mode* voltage $V_{SM} = M \, di/dt$ where i is the power line current. This will be in series with, and thus add to, the voltage produced by the transducer. Here we have ignored any induced voltage on the Low lead. If present, this could partially or completely offset the error introduced on the High lead as will be noted below.
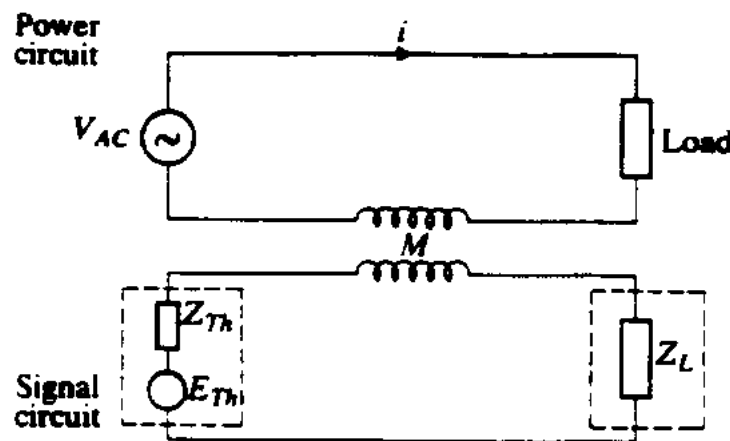


Figure 7: Inductive Coupling

Inductive coupling can, of course, be reduced by physical separation since mutual inductance is inversely proportional to separation distance. Further reduction can be accomplished by using *twisted pairs* and *short cable lengths.*
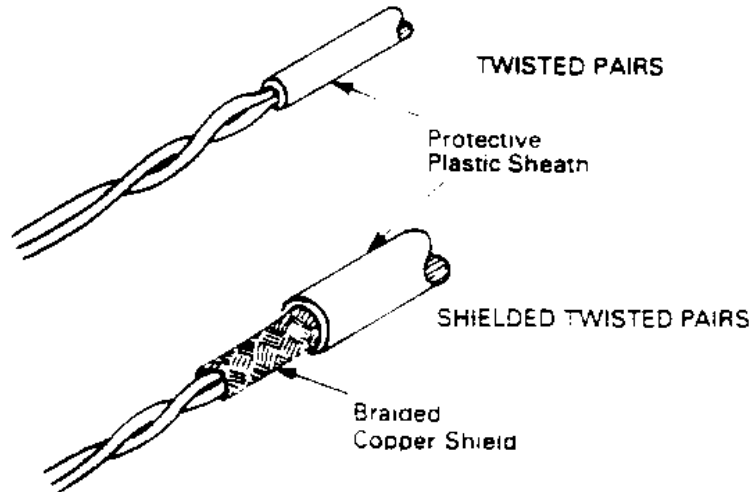


Figure 8:  Twisted Pair Cable

Twisted pair cables reduce the interference effect by insuring that both the High and Low electrical leads have similar induced voltages so that the differential voltage due to the transducer output is minimally affected.

(b)  *Capacitive (Electrostatic) Coupling*

Voltages can also be introduced in a circuit through capacitive coupling. In this case a power line (or other AC source) and the circuit's electrical leads act as the elements of capacitors. This is illustrated in Figure 9. Note that the four capacitors shown in the figure represent those effectively formed by the various electrical leads present.
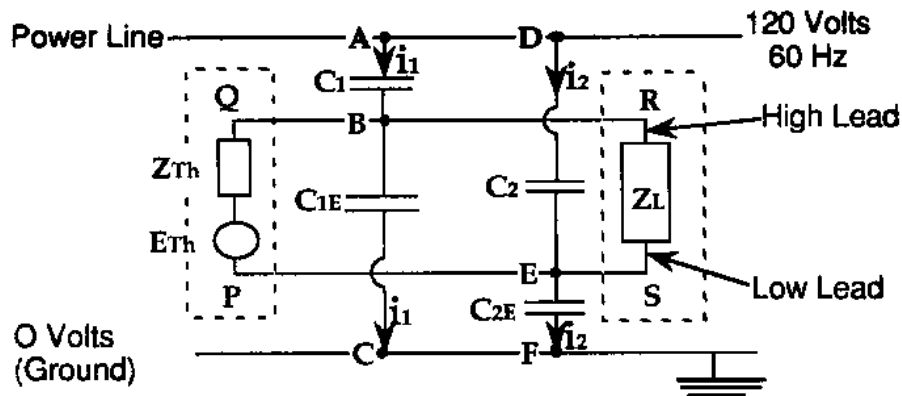


Figure 9:  Capacitive Coupling

Assume first that $V_B = 0$ (induced voltage at B = 0). Then if $Z_L \gg Z_{Th}$, the voltage drop across $Z_{Th}$ is approximately 0 and

$$V_Q = V_E + E_{Th} \tag{20}$$

while $V_S = V_E$. Thus the induced *common mode* voltage appearing at both Q and S is

$$V_{CM} = V_E \tag{21}$$

Thus the total voltage at Q is

$$V_Q = E_{Th} + V_B + V_E \tag{22}$$

Now assume $E_{Th} = 0$. The voltage difference across $Z_L$ is $V_B - V_E$ which is the induced *series mode* voltage

$$V_{SM} = V_B - V_E \tag{23}$$

$V_{SM}$ will add to $E_{Th}$. When the latter is non-zero, both common mode and series mode noise voltages have been induced in the measurement circuit by capacitive coupling. Here the readout instrument is detecting the differential voltage, so common mode noise should not, *theoretically,* affect the measured result.

Capacitive coupling can be reduced by separating the source of the interference from the instrument circuit. When necessary it can be treated by surrounding the instrument circuit with a grounded screen.
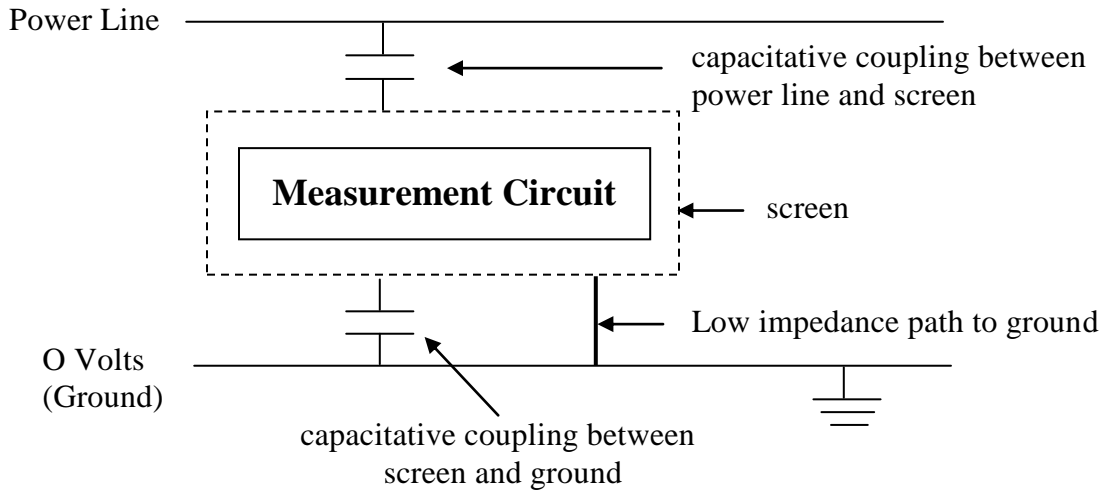


Figure 10:  Shielding by Means of a Conducting Screen

The screen provides a low impedance path to ground for the induced currents and minimizes interference in the measurement circuit. In actual circuits such a screen is provided by using shielded cable and having all circuit elements in an aluminum or other electrically conducting housing. The cable and housings then act as a continuous conductor screen.

(c) *Ground Loops*

Finally we wish to consider the problems which can arise in grounded circuits. Consider Figure 11.
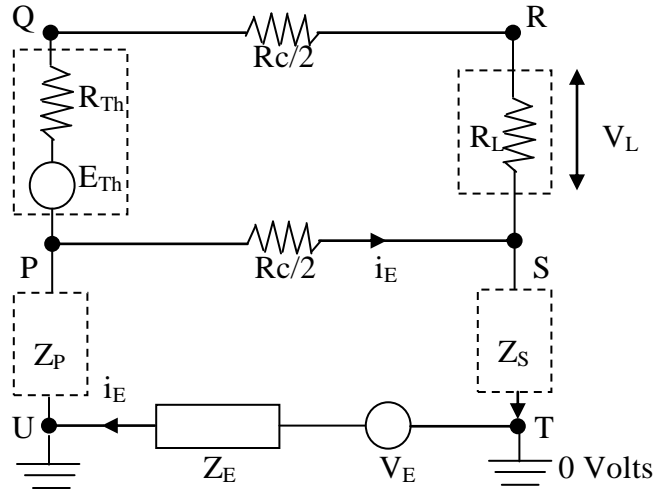


Figure 11: Ground Loop due to Multiple Grounds

Here Q and P represent the High and Low outputs of a transducer. R and S are the corresponding High and Low input voltages to the readout instrument. These voltages are floating unless the circuit is deliberately grounded at P or S. Suppose that it is in fact grounded at both points with leads or other connectors having impedances $Z_P$ and $Z_S$. $Z_S$ is connected to point T which is a true ground at 0 V, but $Z_P$ is connected to point U which is isolated from true ground by the impedance $Z_E$ and voltage source $V_E$. The voltage $V_E$ can in fact be the induced voltage in a conducting wire. (Rather large voltages can be magnetically induced even in highly conducting wires.) Since this wire is connected to both T and U, a *ground loop* is formed. $V_E$ causes a current $i_E$ in the ground loop and corresponding voltage drops across the various impedances. (Note that current flow in the instrument circuit is assumed negligible, since $R_L$ is typically large compared to the other impedances.) The voltage drop $V_S - 0 = i_E Z_S = V_{CM}$ is the common mode voltage, which appears at S and will also add to the Thevenin voltage for the transducer and appear at Q. The series mode voltage drop $V_P - V_S = i_E R_c / 2 = V_{SM}$ will also add to the transducer output and appear at Q. These noise components can be eliminated by connecting the circuit to ground at only one point, namely point T.

Ground loops are often a problem in measurement circuits which contain several active elements with each plugged into a separate power socket with a grounded plug. Power line grounds are often unreliable and special precautions may be needed to avoid ground loops in such cases.

## NOISE REDUCTION AND REJECTION

We have already discussed the importance of proper shielding and grounding. Other methods of noise reduction include:

  (a)  Filtering to limit noise outside the signal bandwidth.

  (b)  Modulation to move the signal into a spectral region where noise can be more easily filtered out.

  (c)  Differential amplification to minimize common mode noise using an amplifier with a high common mode rejection ratio.

  (d)  Averaging of periodic signals by adding samples of the signal. The random noise averages to zero if enough samples are taken. For N samples the signal to noise ratio in dB improves by $10\log_{10} N$.

Each of these methods will be considered in this section.

(a) *Filtering*

Filtering is an obvious way to reduce noise. Consider the case of white noise (as exemplified by Johnson noise, for example) and the system behaving like an ideal band-pass filter with a gain of 1 and a bandwidth of $\Delta f$ Hz. Here the mean square noise voltage is directly proportional to system bandwidth

$$\overline{n^2(t)} = c\Delta f \tag{24}$$

where c is a constant. Reducing the bandwidth by means of a suitable filter will reduce the noise voltage correspondingly. The same principle will apply to noise with a more complex power spectral density. However, only noise outside the frequency region of the signal spectrum can be reduced in this way.

Consider, for example, the situation illustrated in Figure 12 below. Here the power spectrum of the signal, $\phi_S$, contains only low frequencies. Thus a low pass filter with a sharp cutoff such as a 3rd Order Butterworth can reject most of the low level white noise and the 60 Hz noise. When viewed in the time domain, the filtered signal will be much cleaner, although some noise fluctuation will remain due to the white noise within the filter's passband. (Note that we have assumed the flicker noise to be negligible in this example.)
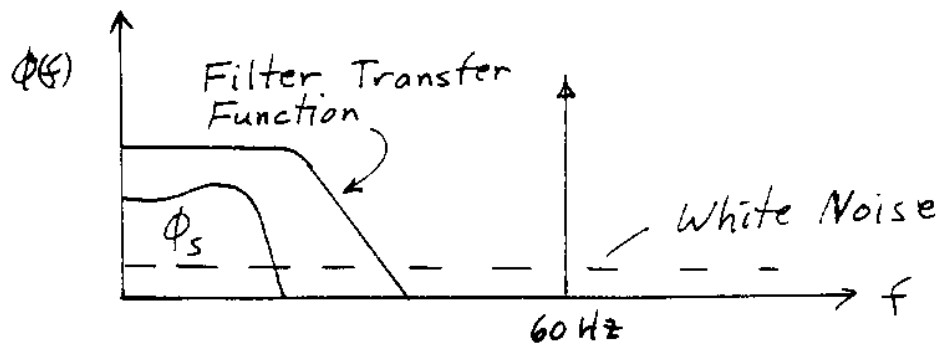
Figure 12:  Noise Rejection with a Filter
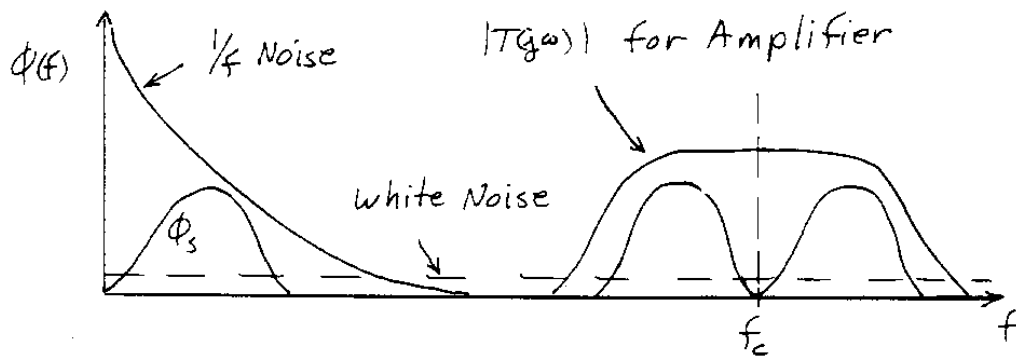
(b) *Modulation*



Figure 13:  Shifting the Signal Spectrum by Means of Modulation

A near-DC signal that is weak will be strongly affected by flicker noise and any white noise. Filtering is ineffective in this case, since the frequencies of the measured signal and the noise overlap. Modulation is effective in this case, since we can use a carrier frequency to transpose the DC signal to a frequency region unaffected by 1/f or other low frequency noise. A narrow bandpass filter can then be used to minimize the influence of white noise. In essence, this procedure allows us to employ AC amplification at the carrier frequency, as shown in Figure 13.

Any 60 Hz interference that arises *after* modulation is rejected by the AC amplifier; if the interference occurs *before* modulation, it will not be rejected since it will also be shifted to the carrier frequency. Note, however, that transposition to a higher frequency will not help the white-noise situation. This can be handled by averaging methods which are discussed in a subsequent section.

(c) *Differential Amplification*

*Common-mode noise is* often the major interference resulting from capacitance coupling and ground loops. In a measurement circuit such as that of Figure 11 with $R_L \gg R_{Th}$, we then have the simplified effective circuit of Figure 14 (assuming negligible current flow in both "loops" and $V_{SM} \approx 0$).
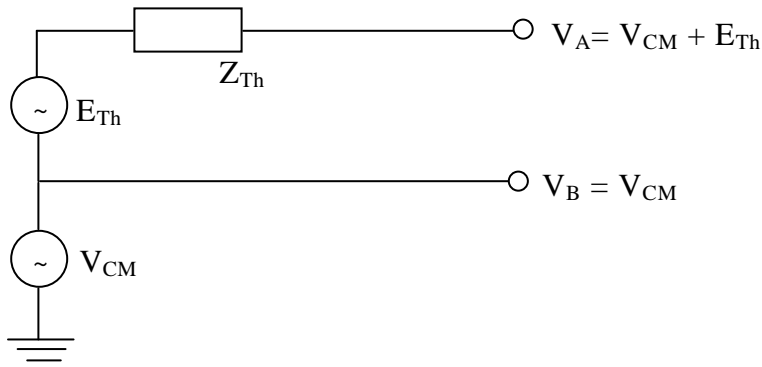


Figure 14:     Common Mode Noise at the Connections of a Transducer

Thus, if we were to make the usual assumption that $V_B = 0$ (true ground) and simply amplify $V_A$ via an inverting op-amp, we would of course also amplify the common mode noise voltage $V_{CM}$.     Note, however, that we can employ *differential amplification* as shown in Figure 15.
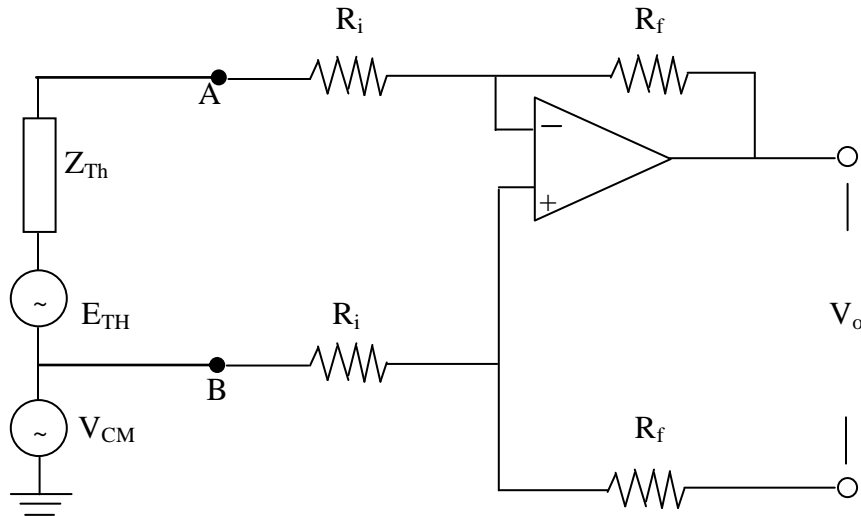


Figure 15:   Differential Amplification

Here

$$V_o = \frac{R_f}{R_i}\left(V_B - V_A\right) = -\frac{R_f}{R_i}E_{Th} \tag{25}$$

assuming $R_i \gg |Z_{Th}|$, i.e., negligible current flow in the circuit. Thus we have theoretically rejected the common mode noise.

Unfortunately, there are problems. When used in the non-inverting mode, an op-amp tends to have high common mode noise, i.e., rejection of $V_{CM}$ is imperfect. The ability of an amplifier (or an instrument) to reject common-mode noise is quantified in the common mode rejection ratio (CMRR).

$$CMRR = \frac{\text{differential gain}}{\text{common mode gain}} \tag{26}$$

For a practical operational amplifier in its open loop configuration:

$$V_o = A_{OL}(V^+ - V^-) + A_{CM}V_{CM} \tag{27}$$

where $A_{OL}$ is the open-loop gain and $A_{CM}$ is the common mode gain. Since

$$CMRR = A_{OL}/A_{CM} \tag{28}$$

we then have

$$V_o = A_{OL}\left[(V^+ - V^-) + \frac{V_{CM}}{CMRR}\right] \tag{29}$$
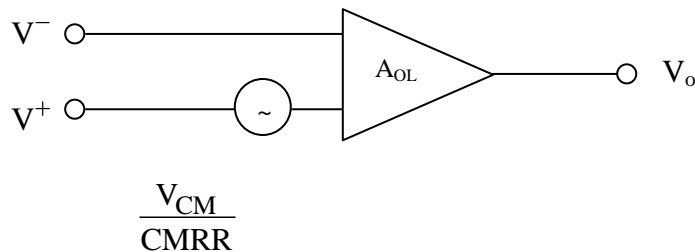
Thus an equivalent circuit is as follows:



Figure 16:  Equivalent Circuit

Non-inverting op-amp operation is plagued by common mode noise problems, thus $V_{CM}/CMRR$ is represented by a voltage source on the non-inverting terminal.

The CMRR is usually expressed in dB so that

$$(CMRR)_{dB} = 20\log_{10}(CMRR) \tag{30}$$

Note that CMRR is a voltage ratio, hence the 20 when converting to dB.

Consider again a standard op-amp differential amplifier, but now include the effects of incomplete common-mode rejection.
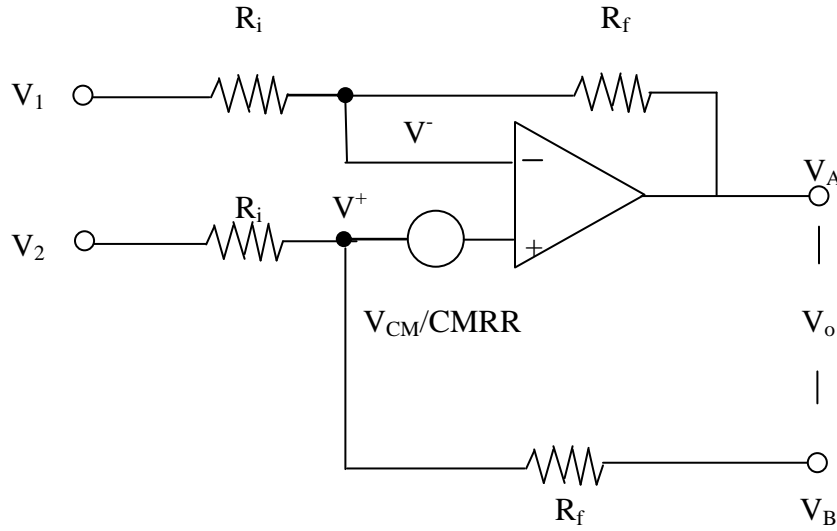


Figure 17: A Practical Differential Amplifier

Now

$$V^- = V^+ + V_{CM}/CMRR \tag{31}$$

$$V^+ = V_B + \left(\frac{R_f}{R_i + R_f}\right)(V_2 - V_B) \tag{32}$$

also

$$V^- = V_A + \left(\frac{R_f}{R_i + R_f}\right)(V_1 - V_A) \tag{33}$$

From these equations we obtain

$$(V_A - V_B)(R_i + R_f) = (R_i + R_f)\left(\frac{V_{CM}}{CMRR}\right) = R_f(V_2 - V_1) + R_f(V_A - V_B) \qquad (34)$$

and hence:

$$V_o = V_A - V_B = \frac{R_f}{R_i}(V_2 - V_1) + \left(1 + \frac{R_f}{R_i}\right)\frac{V_{CM}}{CMRR} \qquad (35)$$

Consider $V_2 - V_1 = 1\,\text{mV}$, $R_f/R_i = 10^4$, $V_{CM} = 1\,\text{V}$ and $CMRR = 10^4$ (80 dB). For these typical values it is easily shown that

$$V_o = 10 + 1.0001 \text{ Volts}$$

Thus the signal to noise voltage ratio is close to 10, corresponding to a S/N power ratio of 20 dB. While acceptable in some cases, this S/N power ratio is not adequate for precision measurements. For these applications *instrumentation amplifiers* are available which have much better common mode noise rejection characteristics and other features which improve their performance.

(d) *Signal Averaging*

Signal averaging can be employed to enhance deterministic signals buried in *random noise* or time varying signals distorted by periodic noise even in cases where the original S/N is less than one, i.e., very poor. Two averaging methods which can be used are discussed here.

*Ensemble Averaging*

The first involves *ensemble* averaging, where we have many examples of the signal. In each example the deterministic part is identical to that in the other examples; the noise part, however, is random and varies from example to example. You may have accumulated these examples by carefully repeating a measurement, or you may have a periodic signal (whose period you know) and each period is effectively an example of the signal. To improve the signal to noise ratio you simply average the examples. The success of the method is strongly dependent on being able to exactly reproduce the deterministic part of the signal in each example. If the signal is periodic, this means that the period must be known exactly.

A very useful instrument that can be used with periodic signals to do this form of averaging is the *boxcar averager*. The boxcar averager is simply an op-amp integration circuit with a switch that allows sampling of the signal on a repetitive basis.
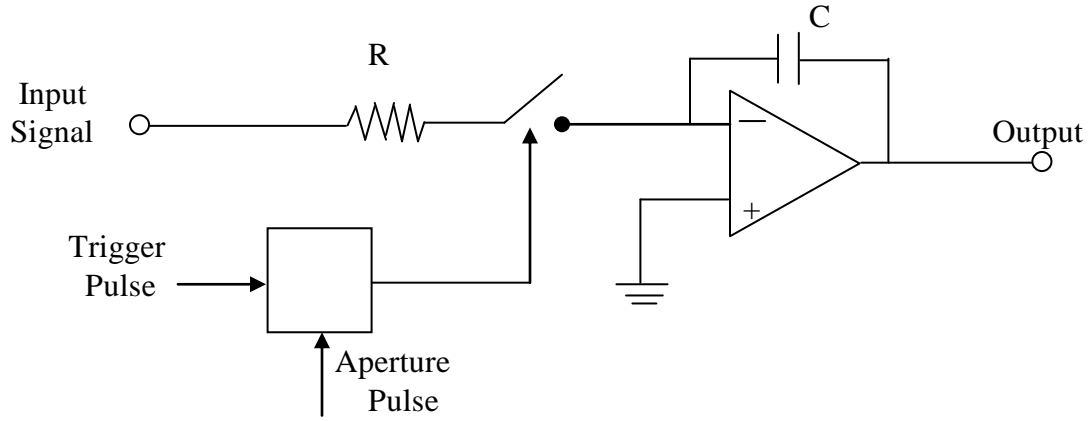
Figure 18:   Boxcar Averaging Circuit

The circuit can be set up to sample the signal for a short time, at the same position in each period. These samples are then averaged by the circuit shown in Figure 18. In Figure 19 is illustrated a typical sampling scheme which would sample the peak value of a periodic signal. If the entire signal was to be determined, the narrow sampling aperture could be scanned to map out the complete signal behavior.
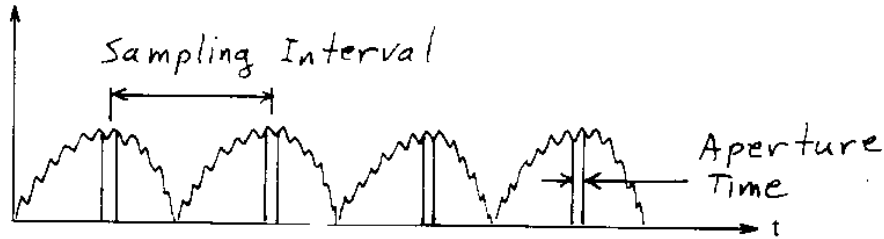


Figure 19: Typical Sampling Scheme

If the signal period is T seconds, the ith sample can be expressed as:

$$s(t + iT) + n(t + iT) = s(t) + n_i \tag{36}$$

$s(t) = s(t + iT)$ because $s(t)$ is periodic. Each of the $n(t + iT)$ are different and denoted here by $n_i$. The average over N pulses is:

$$s(t) + \frac{1}{N} \sum_{i=0}^{N-1} n_i = s(t) + \overline{n} \tag{37}$$

where ¯ denotes average. If the noise is zero mean then its average, over a large number of samples, will approach zero, while the average signal level remains constant. As in the earlier

statistics chapter, the standard deviation of the mean is $(1/\sqrt{N})$ times the standard deviation of the data. Hence:

$$\sigma_{\bar{n}} = \frac{\sigma_n}{\sqrt{N}} \tag{38}$$

Since the mean of the noise is zero, then the mean of the averaged noise is also zero. Hence its mean square value of $\bar{n}$ is equal to

$$\sigma_{\bar{n}}^2 = \frac{\sigma_n^2}{N} \quad .$$

The signal to noise power ratio of the output of the averager is given by:

$$S/N_{out} = \frac{\overline{s^2(t)}}{\dfrac{\sigma_n^2}{N}} = N\,\frac{\overline{s^2(t)}}{\sigma_n^2} = N\ S/N_{in} \tag{39}$$

Thus the output S/N power ratio increases as the number of samples averaged increases. This is a very powerful method of reducing random noise so that low level signals can be extracted from the background noise.

If you repeat your measurements of a signal several times and store the results on a computer, you can do the averaging in software. The improvement in signal to noise power ratio is the same as for the boxcar averager, i.e., the signal to noise power ratio is directly proportional to N, the number of measurements averaged.

### Time Averaging

The second method of averaging is a time average of the signal and is really only useful when trying to remove high frequency noise from low frequency signals. The signal, x, at time, t, is replaced by:

$$x_{average}(t) = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(t)\,dt \tag{40}$$

This can be done with a Dual Slope Integrator as described below. This will remove both deterministic and random high frequency components from your signal. Hence if your desired signal is $\ll 60$ Hz., as shown in Figure 20, you can use this method to remove power line noise. Note that if your desired signal has high frequency components, this averaging will remove those components too!
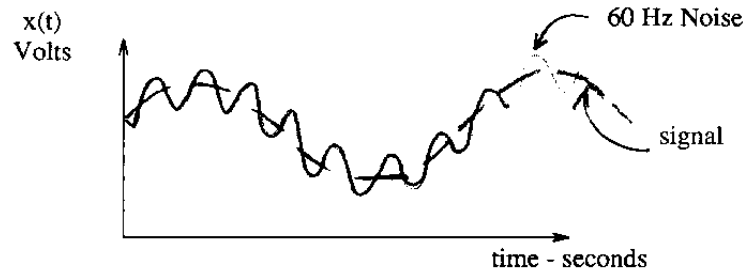
Figure 20: Signal with Significant Periodic Noise

Figures 21 and 22 illustrate the basic concept of dual slope integration. The slope of the fixed time integration is proportional to the incoming signal voltage. Hence the maximum output is also proportional to the incoming signal voltage. The decay portion slope is proportional to the constant reference voltage and hence the time taken to decay to zero is proportional to the maximum value, which is in turn proportional to the incoming signal voltage. The time taken to decay from the maximum value to zero is used as a measure of the incoming signal voltage.
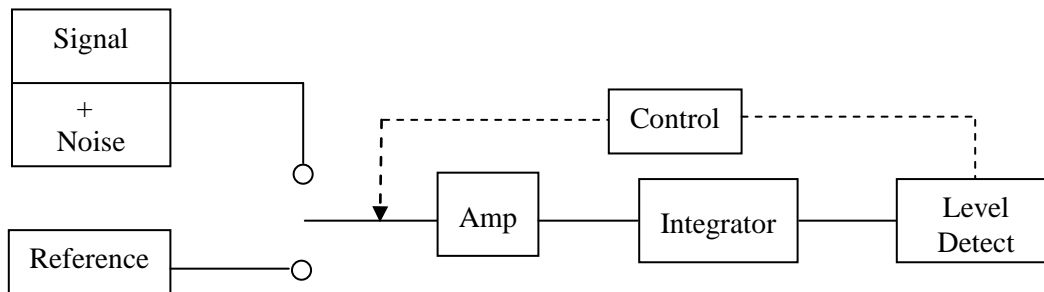


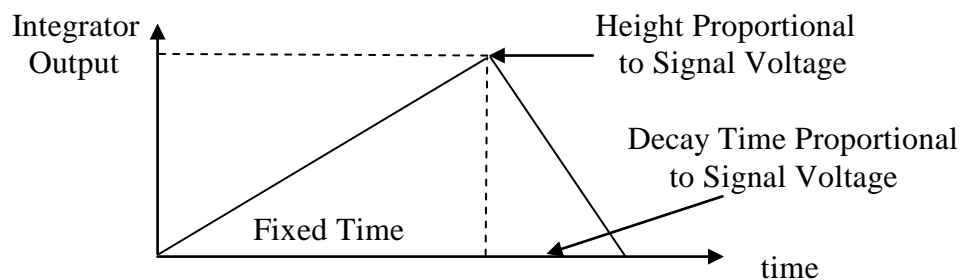Figure 21: Schematic for Dual Slope Integrator



Figure 22: Dual Slope Integration

## SUMMARY

Noise in measurements is a complex subject with many different aspects. The noise produced by various sources can be coupled into measurement circuits in a variety of ways. This leads to series mode noise from both internal and external sources which will add to the transducer's output or to common mode noise from external sources which appears at both the high and low transducer outputs. Ground loops due to multiple ground points in the measurement circuit often cause common mode noise.

A number of noise reduction techniques are available which can improve the signal to noise ratio when it is inadequate. These include:

1.  The use of twisted pair cables to reduce inductive noise.

2.  Shielding to minimize inductive and capacitive coupling.

3.  Filtering to limit noise outside the signal bandwidth.

4.  Modulation to move the signal to a frequency region where noise can be more easily reduced by filtering.

5.  Differential amplification to minimize common-mode noise.

6.  Sampling and averaging of periodic signals to reduce random noise.

7.  Integration of slowly varying signals to reduce high frequency random and periodic noise.

The power spectral densities of noise produced by internal sources are given by well known equations and can be readily computed in most cases. However, the noise from external sources can rarely be accurately quantified analytically in practice. It depends on circuit elements acting as parts of an ill-defined inductor or capacitor which cause induced voltages of unknown value. Nevertheless, the basic principles outlined here provide a framework for understanding these effects, identifying them experimentally, and minimizing them when they are present.