Linearity

Definition

A linear function $f(x)$ is one with the property

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

Problems

1. Prove that $f(x) = \beta x$ is linear.

$$f(x) = \beta x$$
$$f(ax_1 + bx_2) = \beta(ax_1 + bx_2)$$
$$f(ax_1 + bx_2) = a\beta x_1 + b\beta x_2$$
$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$
$$QED$$

2. Prove that $f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$ is *not* linear.

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$f'(x) = \beta_1 + \beta_2 * 2x_2$$
$$f''(x) = \beta_2 * 2$$

since $f''(x)$ is not equal to 0 therefor it is not linear. however the linearity is not determined by the predictor x but rather the coefficient β,

$$f(\beta) = \beta_0 + \beta_1 x + \beta_2 x^2$$
$$f'(\beta) = 1 + x_1 + x_2^2$$
$$f''(\beta) = 0$$
$$QED$$