Inspiration for ALM assignment

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Pension Company

We consider a pension company with a simplified balance consisting of

Assets

- Present value of assets at time t, $A(t)$ (market value)

Liabilities

- Present value of liabilites at time t, $L(t)$ (market value, different from article)
- Free capital at time t, $B(t)$

Financial market - P-dynamics

$$
\begin{aligned}
d\beta(t) &= r(t)\beta(t)dt \\
dr(t) &= \kappa(\theta^P - r(t))dt + \sigma_r dW_1^P(t) \\
dS(t) &= \mu S(t)dt + \sigma_s S(t) dW_3^P(t)
\end{aligned}
$$

$$
dW_3^P(t) = \rho dW_1^P(t) + \sqrt{1-\rho^2} dW_2^P(t)
$$

Financial market - Q-dynamics

$$dr(t) = \kappa(\theta - r(t))dt + \sigma_r dW_1(t)$$
$$dS(t) = r(t)S(t)dt + \sigma_s S(t)dW_3(t)$$

$\theta = \theta^P - \frac{\lambda\sigma_r}{\kappa}$

$dW_3(t) = \rho dW_1(t) + \sqrt{1-\rho^2}dW_2(t)$

Should we expect "$+$" or "$-$" sign for λ? Hint: which sign would imply positive risk premiums for ZCBs?

Financial market - ZCB dynamics

P-dynamics

$$dp(t,T) = p(t,T)(r(t) - \lambda\sigma_r B(t,T))dt - \sigma_r B(t,T)dW_1^P(t)$$

Q-dynamics

$$dp(t,T) = p(t,T)r(t)dt - \sigma_r B(t,T)dW_1(t)$$

$$B(t,T) = \frac{1}{\kappa}\left(1 - e^{-\kappa(T-t)}\right)$$

Financial market - Asset dynamics

Investment in bank, stocks and ZCBs

$$dA(t) = c_\beta(t)d\beta(t) + c_s(t)dS(t) + \sum_{j=1}^{T} c_{ij}(t)dp(t, i+j)$$

for $t \in [i, i+1)$.

Note: We need only to simulate under P in order to obtain $A(t)$ for $t \in [0, T]$. Why no need to simulate under Q?

Financial market - Liability model

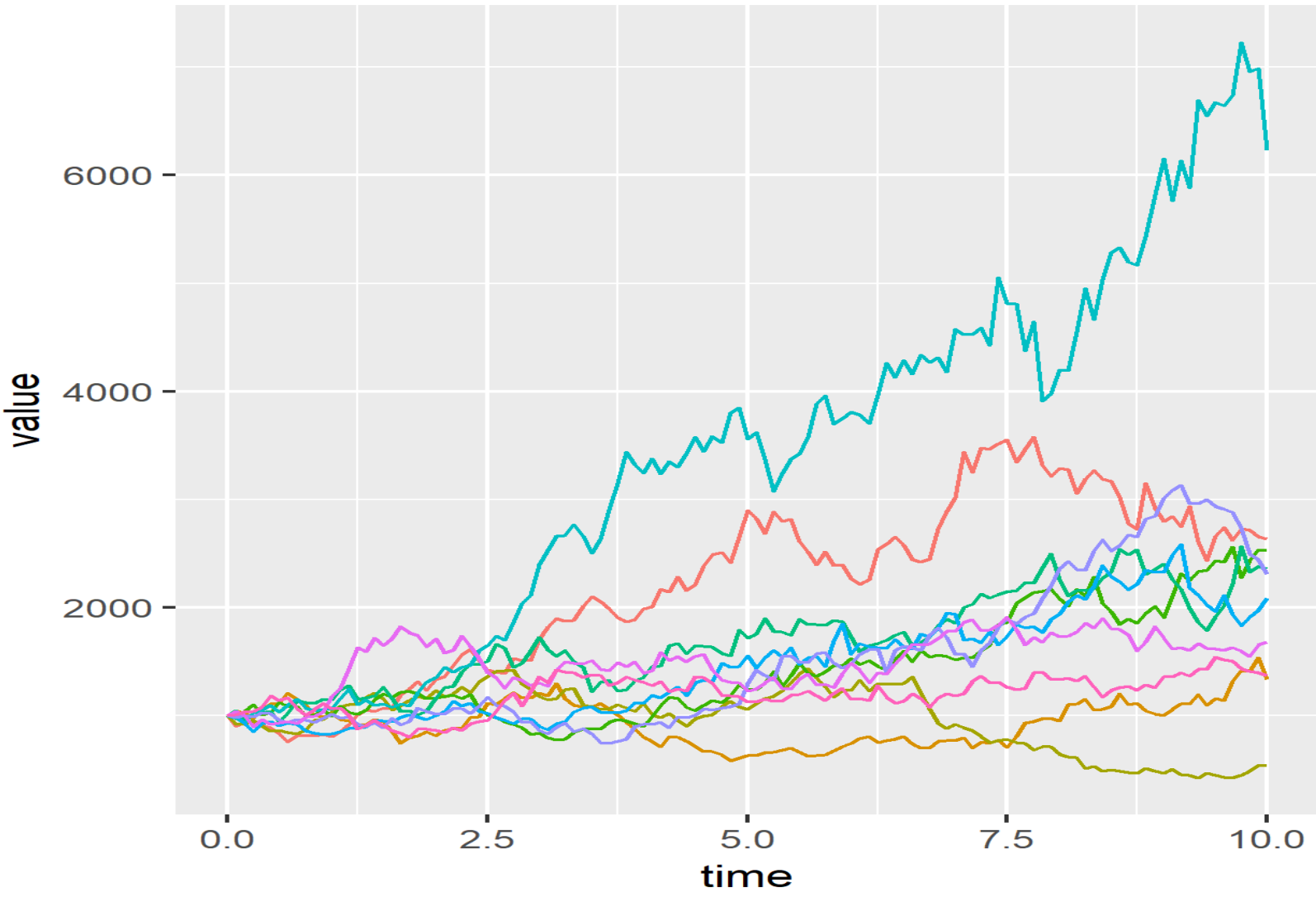
Simple savings contract.

$$L(T) = premium * (1 + q)^T$$

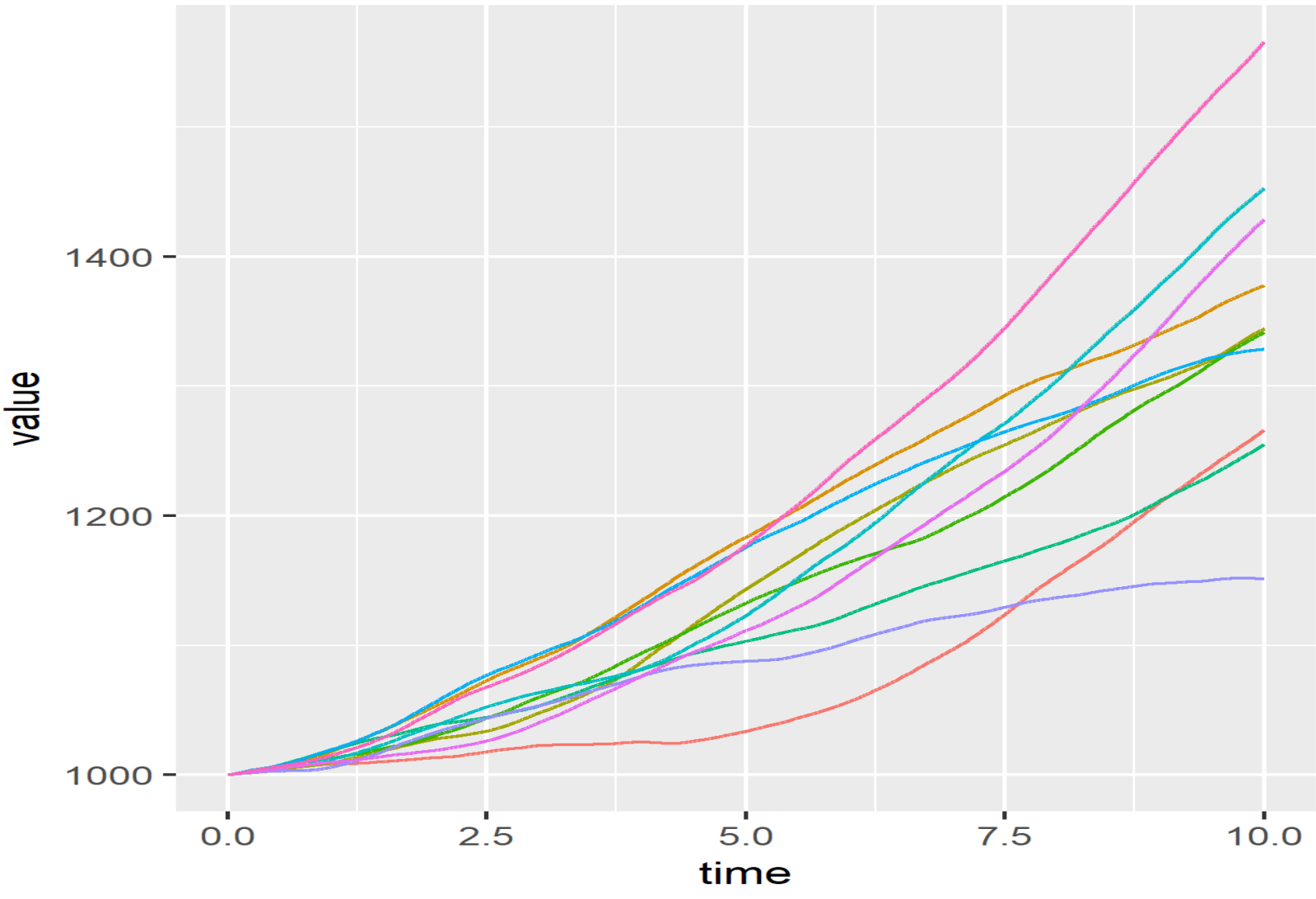
Where *premium* is initial payment and q is a constant "guaranteed" interest rate.

$$\begin{aligned} L(t) &= E^Q \left[e^{\int_t^T r(s)ds} L(T) | \mathcal{F}_t \right] \\ &= E^Q \left[e^{\int_t^T r(s)ds} | \mathcal{F}_t \right] L(T) \\ &= p(t, T) L(T) \end{aligned}$$

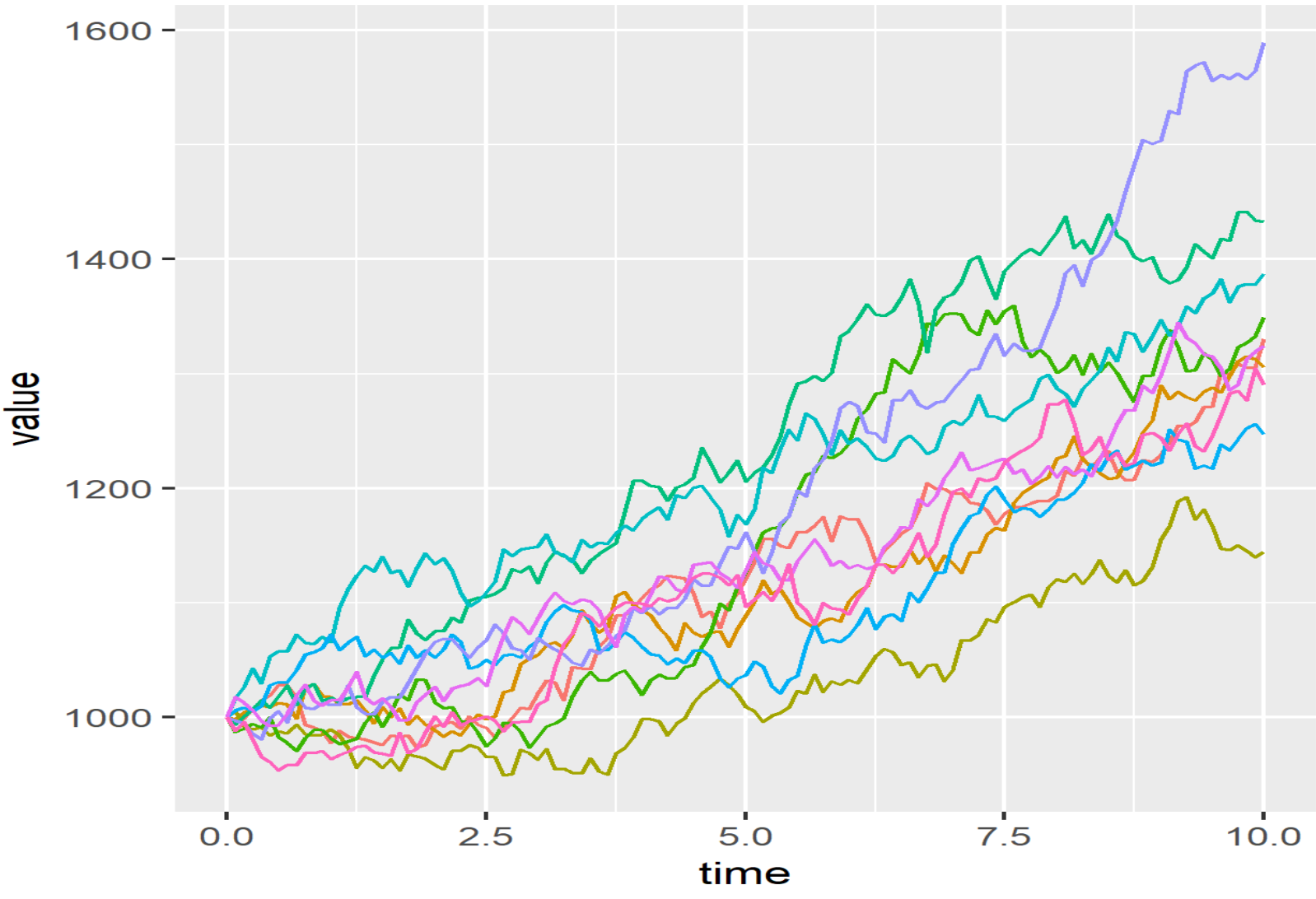
Simulated asset paths - 100% Stock, dt = 1/12



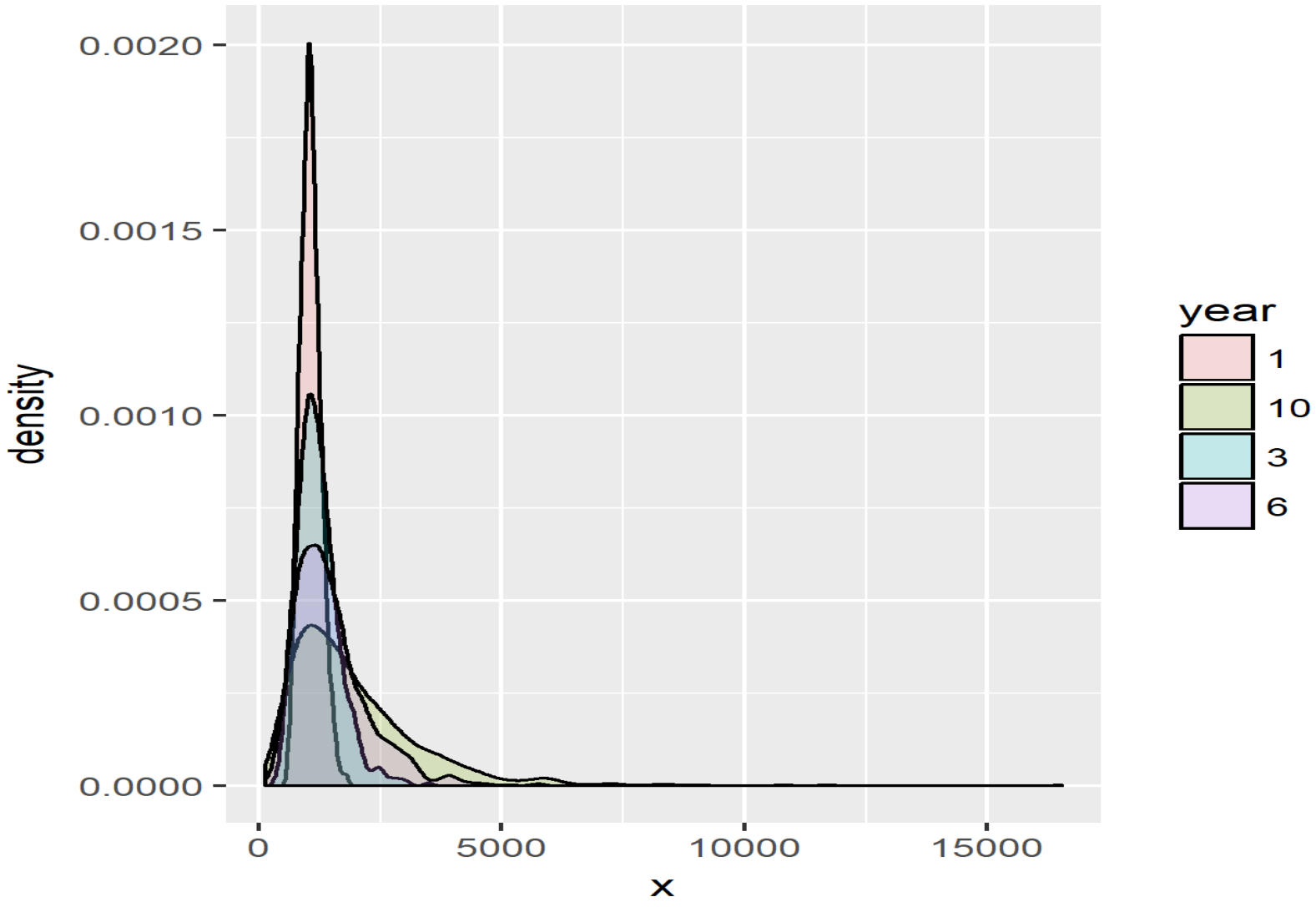
Simulated asset paths - 100% Bank, dt = 1/12



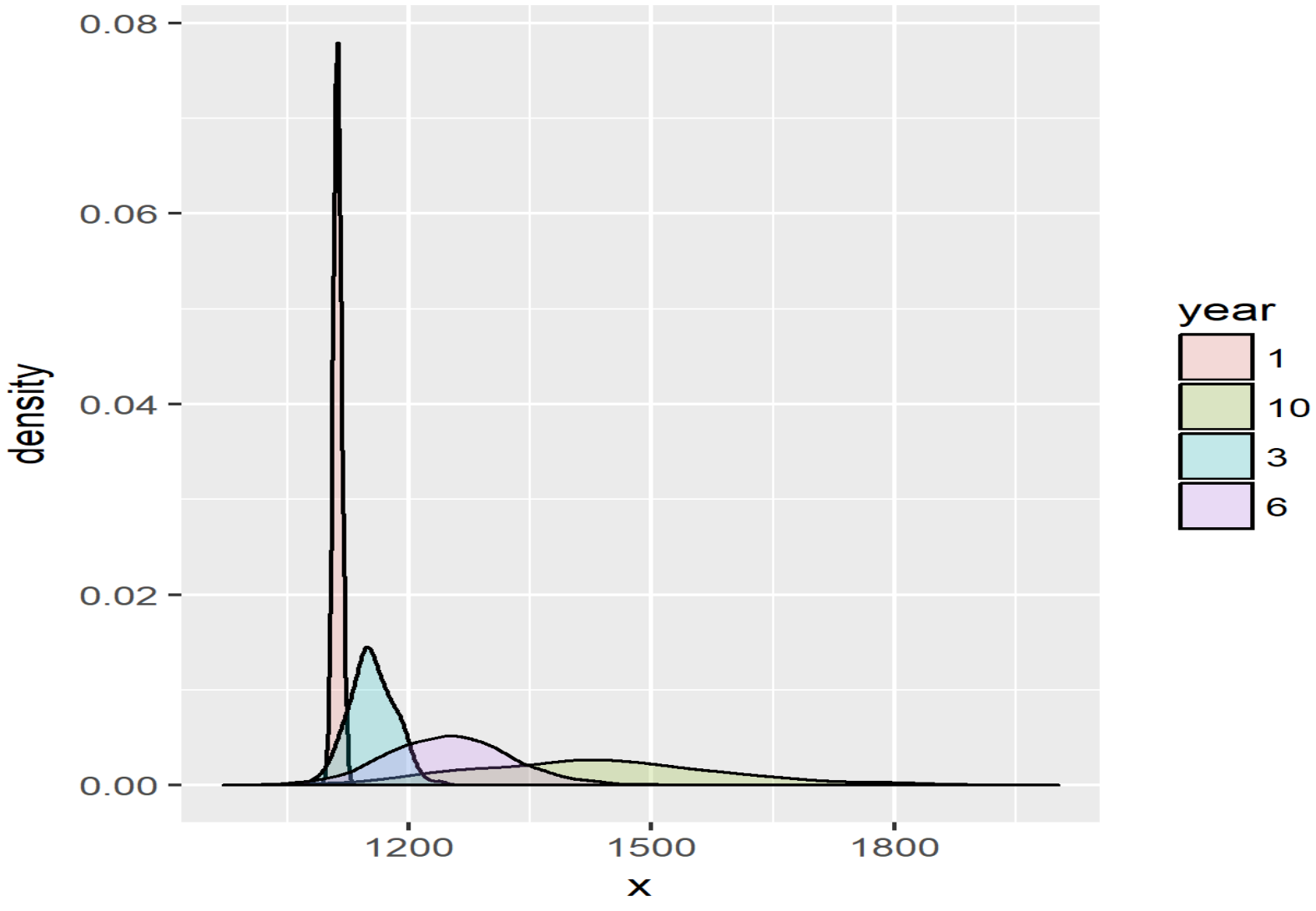
Simulated asset paths - 100% Bonds, dt = 1/12



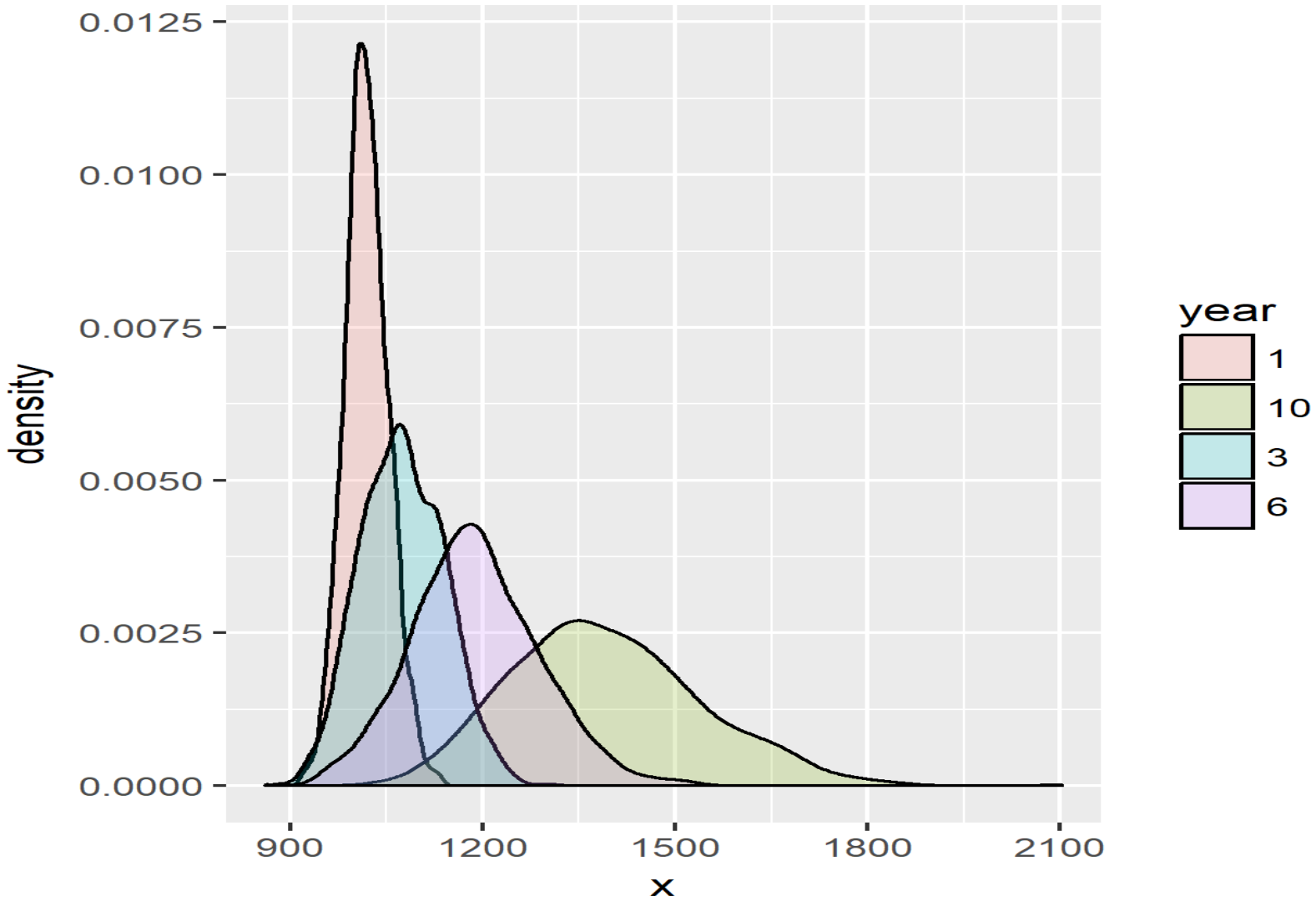
Asset distribution - 100% Stock, dt = 1/12



Asset distribution - 100% Bank, dt = 1/12



Asset distribution - 100% Bonds, dt = 1/12



Remarks on previous plots

Combination of bank, stocks and bonds influence

- Skewness of asset distribution at different times (median vs mean)
- $P(A(T) < L(T))$

Results are very sensitive to changes in model parameters

- given current parameters: approx 10% stocks and 90% to minimize $P(A(T) < L(T))$
- increasing κ: approx 100% bank to minimize $P(A(T) < L(T))$

However, usually we also wish to create som profit!

Risk management - Solvency 2 and VaR

Solvency 2 SCR = free capital today must insure that risk of default during next year is less than 99.5%

In ohter words, Solvency 2 SCR = 99.5% quantile of 1-year loss distribution

Solvency 2 risk measure: *Value at Risk* (VaR)

$$Loss(t) = B(t) - e^{-\int_t^{t+1} r(s)ds} B(t+1)$$

$$CR(t) = \frac{B(t)}{SCR(t)}$$

FSA supervision of company if CR falls below regulatory level

Risk management - Example

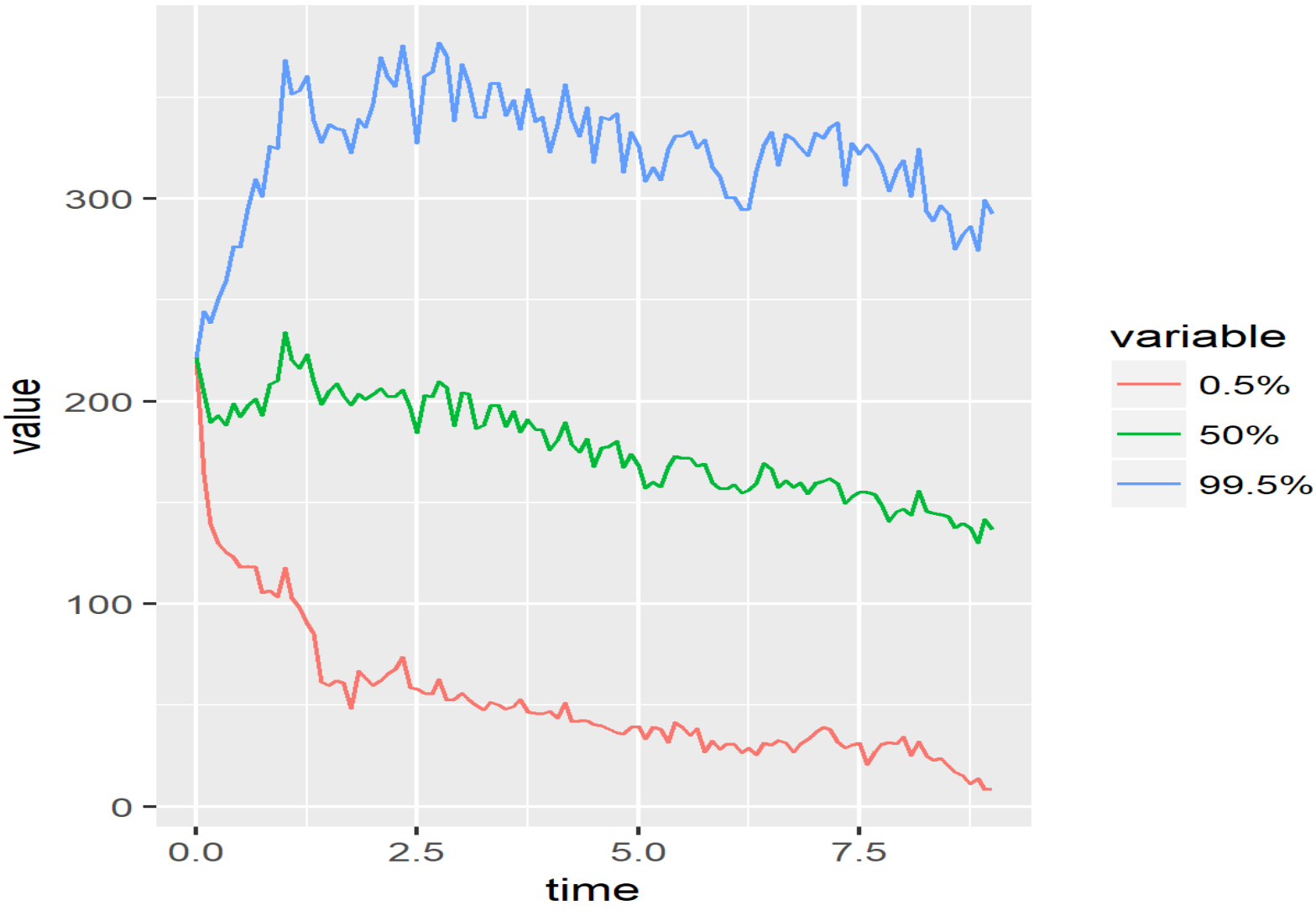
Initial premium $= 1000$

"Guaranteed" interest rate $= 2.5\%$

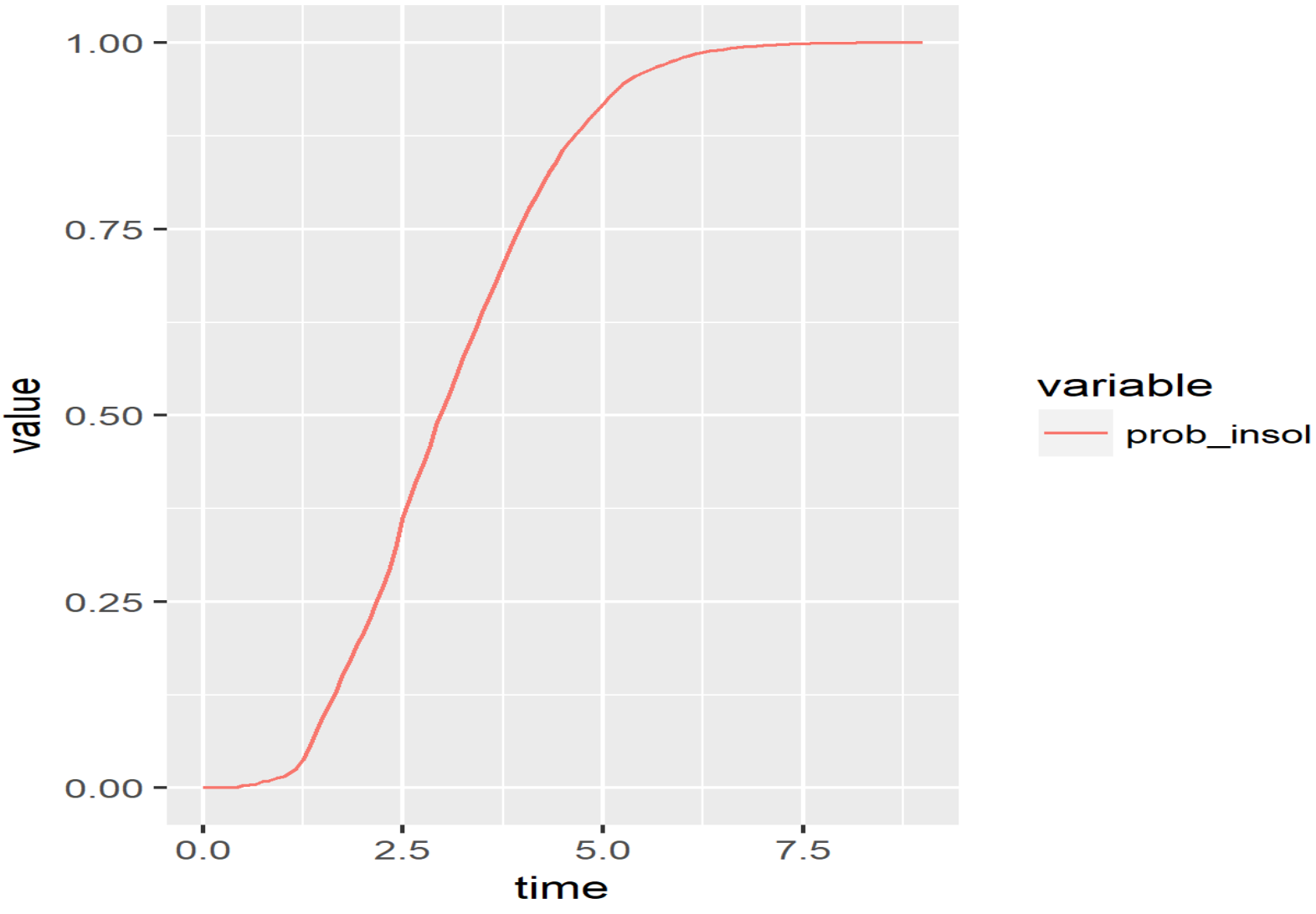
$B(0) = 100$

10% stocks 90% bonds

Risk management - example: CR paths, dt = 1/12



Risk management - example: insolvency prob, dt = 1/12



The End

Thank you!