ALM i praksis - Dag 2

1. og 2. november 2016

Den Danske Aktuarforening

Indhold – Blok B

- **Feedback på øvelse**

- **Renteafdækning**
  - **Renteafdækningsinstrumenter**
  - **Solvens 2 diskonteringskurven og dennes risici**

- **Øvelse**
  - **Afdækning af passivernes rentefølesomhed**

- **Case study: ATP's nye livrenteprodukt**

Renteafdækning

Den mest almindelige swap er en par-swap

- **En par-swap er sammensat af to instrumenter**

  - **En ‘bullet’ obligation**
    - Fast, årlig kupon + tilbagebetaling af hovedstol ved udløb

  - **En ‘floater’ obligation**
    - Variabel, halvårlig kupon + tilbagebetaling af hovedstol ved udløb

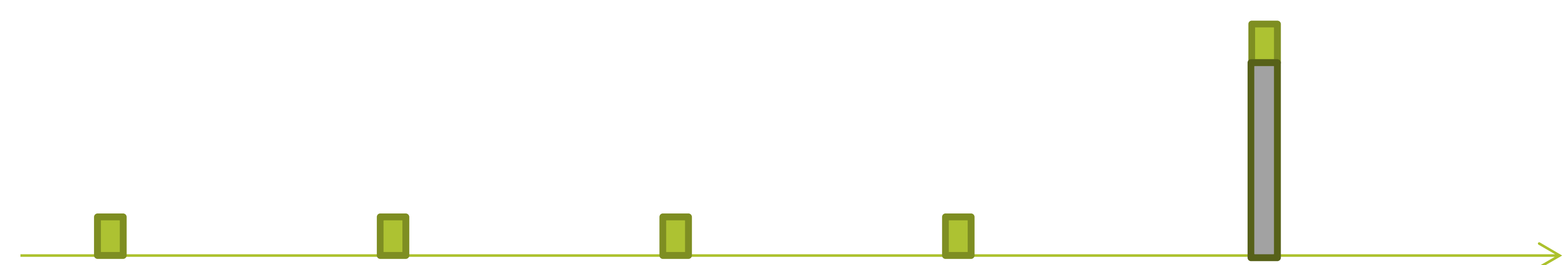
Bullet bond (stående lån)

- **En 'bullet' obligation består af**
  - **en række faste, årlig kuponbetalinger**
  - **tilbagebetaling af hovedstol ved udløb**

$$cf = \{c_{T_1}, cf_{T_2}, \ldots, cf_{T_n}\}, \text{ hvor } cf_{T_1} = \cdots = cf_{T_{n-1}} = k \text{ og } cf_{T_n} = 100 + k$$

  - **høj varighed**

$$PV(t, R_t(\cdot)) = \sum_{i=1}^{n} cf_{T_i} \cdot (1 + R_t(T_i))^{-(T_i - t)}$$



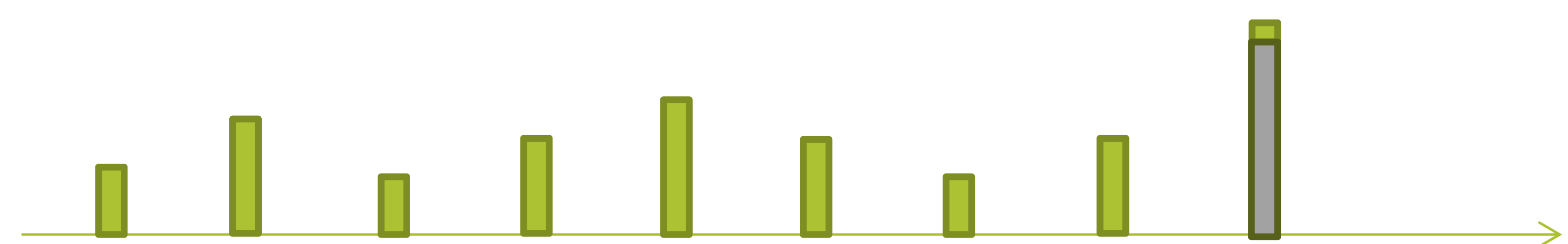
Floater

- **En 'floater' obligation består af**
  - **Variabel, halvårlig kupon**
  - **tilbagebetaling af hovedstol ved udløb**
- **Kuponen fastsættes forud for et halvt år ad gangen**
  - **så nutidsværdien bliver 100 ved hver 'fixing'**
- **Sidste gang kuponen bliver fastsat (til tid T_{n-1}) gælder der derfor**

$$PV_{T_{n-1}} = \left(100 + k_{T_{n-1}}\right) \cdot \left(1 + R_{T_{n-1}}(T_n)\right)^{-(T_n - T_{n-1})} \equiv 100$$

  - **Dette gælder rekursivt tilbage til den første kupon**

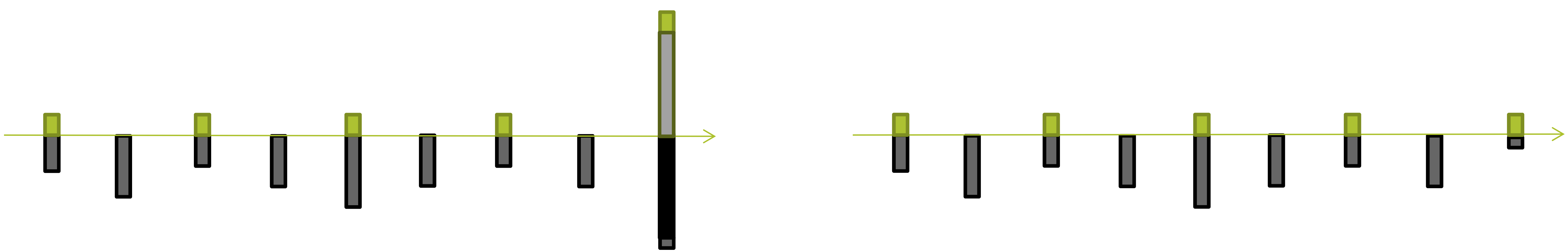
$$k_0 = \frac{100}{\left(1 + R_t(T_1)\right)^{-(T_1 - t)}} - 100$$



  - **Lav (ingen) varighed**

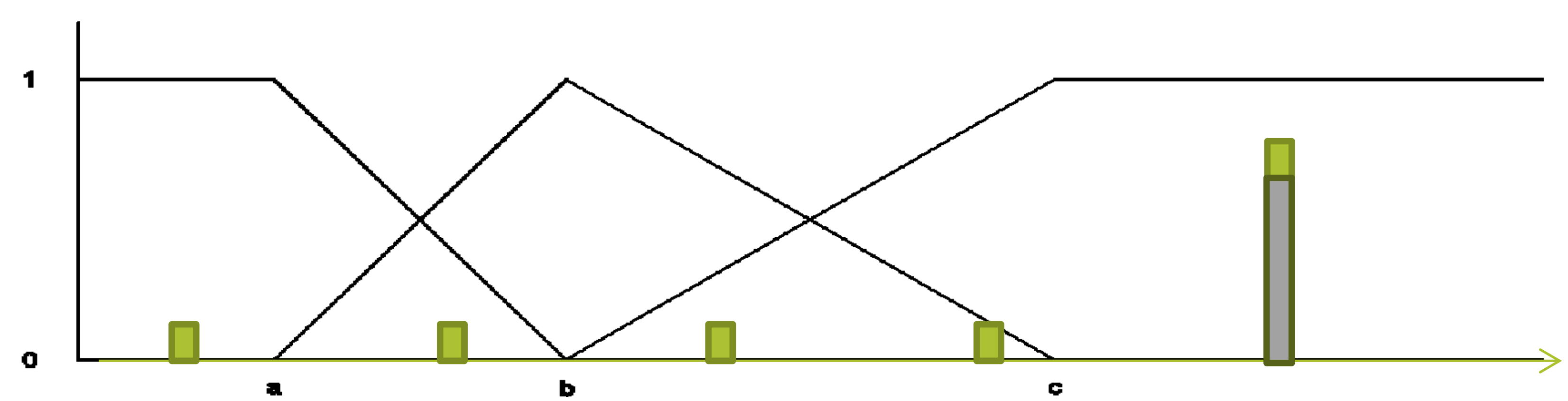
Receiver swap

- I en receiver-swap modtager man betalingerne fra bullet-obligationen og betaler den variable rente på floateren
  - Ved indgåelse er markedsværdien af hvert ben 100 ('par'), dvs. den samlede værdi af par-swappen er nul.
  - Hovedstolen på hvert ben er den samme, så hovedstolene udveksles ikke
- Man bytter med andre ord en flydende rente for en fast – heraf navnet 'swap'



Ulemper ved par swaps

- Par swaps er det mest benyttede afdækningsinstrument
- Kan være komplekse at fortolke, da de bidrager til nøglevarigheden i alle løbetidssegmenter
- Fører til højere gearing af balancen
  - kan medføre 'shorting' i de korte løbetider ved høje renteniveauer



Nulkupon swaps

- **I kurset benytter vi i stedet nulkupon swaps**
- **En nulkupon-swap består af**
  - **En ‘Nulkupon’ obligation**
    - Én fast betaling til tid T

$$cf = \{cf_T\}, \text{ hvor } cf_T = 100$$

$$PV\left(t, R_t(T)\right) = cf_T \cdot \left(1 + R_t(T)\right)^{-(T-t)}$$

  - **Den samme ‘floater’ som for par swaps**
    - Hovedstolen er ikke ‘par’ …
    - … men PV af det nulkupon-obligationen ved udstedelse …
    - … så den samlede værdi af swappen er nul ved udstedelse

Solvens 2 diskonteringskurven

- **Under Solvens 2 anvendes en modificeret diskonteringskurve**
  - **udgangspunktet er den observerede rentekurve**

- **Rentekurven modificeres efter**
  - ***last liquid point*: længste løbetid, som i praksis kan afdækkes**
  - ***ultimate forward rate*: den langsigtede (ukendte) rente i *fremtiden***
  - ***converence period*: overgangsperioden fra *LLP* til *UFR* er realiseret**
  - **hertil kommer nogle ”tillæg” som ikke omhandles her**

- **I Danmark er**
  - ***last liquid point = 20 år***
  - ***ultimate forward rate = 4,2 pct.***
  - ***convergence period = 10 år***

Forward-renter

- **Diskonteringsfaktoren* for en betaling til tid *T* er givet ved**
  - $P(T) = \left(1 + R(T)\right)^{-T}$
- **For $T_i = \{1,2,3,\ldots\}$ kan diskonteringsfaktoren skrives som**
  - $P(T_i) = \prod_{j=1}^{i}\left(1 + f_j\right)^{-1}$
- **hvor forwardrenterne f_j er givet ved**
  - $f_j = \frac{P(T_j)}{P(T_{j+1})} - 1$

- **Akademisk anvendes ofte kontinuert rentekonvention**
  - $P^c(T_i) = e^{-R^c(T)T} = e^{-\left(f_1{}^c + f_2{}^c + \cdots + f_i{}^c\right)}$
  - $f_j{}^c = ln\left(\frac{P_{T_j}{}^c}{P_{T_{j+1}}{}^c}\right)$

**P* fordi det er prisen på en nulkupon obligation

Smith-Wilson Ekstrapolering - 1

- **Ekstrapolation i diskonteringsfaktorer**

$$P(t) = e^{-UFR \cdot t} + \sum_{j=1}^{N} \zeta_j \cdot W(t, u_j), \quad t \geq 0$$

- **hvor vægtene ζ kalibreres til markedsdata**
- **Wilson-funktionerne *W* er givet ved**

$$W(t, u_j) = e^{-UFR \cdot (t+u_j)} \cdot \left\{ \alpha \cdot \min(t, u_j) - 0.5 \cdot e^{-\alpha \cdot \max(t, u_j)} \cdot \left(e^{\alpha \cdot \min(t, u_j)} - e^{-\alpha \cdot \min(t, u_j)} \right) \right\}$$

The following notation holds:

- *N*, the number of zero coupon bonds with known price function
- m_i, *i*=1, 2, ... *N*, the market prices of the zero coupon bonds
- u_i, *i*=1, 2, ... *N*, the maturities of the zero coupon bonds with known prices
- *t*, the term to maturity in the price function
- *UFR*, the ultimate unconditional forward rate, continuously compounded
- *α*, mean reversion, a measure for the speed of convergence to the UFR
- ζ_i, *i*=1, 2, ... *N*, parameters to fit the actual yield curve

Kilde: *A Technical Note on the Smith-Wilson Method*, Finanstilsynet Norge, Juli 2010

Smith-Wilson Ekstrapolering - 2

- Input er den observerede markedskurve, $\tilde{R}_{u_i}$,
- løbetiderne u_i
- samt et 'gæt' på konvergensparameteren, α

$$m_i = P(u_i) = \exp(-u_i \cdot \tilde{R}_{u_i}) \quad \text{for continuously compounded rates}$$

- Ud fra disse kan vægtene ζ bestemmes ved at løse det lineære ligningssystem

$$m_1 = P(u_1) = e^{-UFR \cdot u_1} + \sum_{j=1}^{N} \zeta_j \cdot W(u_1, u_j)$$

$$m_2 = P(u_2) = e^{-UFR \cdot u_2} + \sum_{j=1}^{N} \zeta_j \cdot W(u_2, u_j)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$m_N = P(u_N) = e^{-UFR \cdot u_N} + \sum_{j=1}^{N} \zeta_j \cdot W(u_N, u_j)$$

Smith-Wilson Ekstrapolering - 3

- **Den danske kurve er fastlagt ved**
  - ***last liquid point***: $T_{cut-off} = 20$
    - løbetider: $u_i = \{1,2,3, \dots, 20\}$
  - ***convergence period***: $T_{conv} = 10$
  - ***Ultimate Forward Rate***: $UFR = 0{,}042$
  - **Tolerance:** $tol = 0{,}0003$
  - **Start alpha:** $\alpha_0 = 0{,}1$
  - **Alpha-skridtlænge:** $\Delta\alpha = 0{,}01$

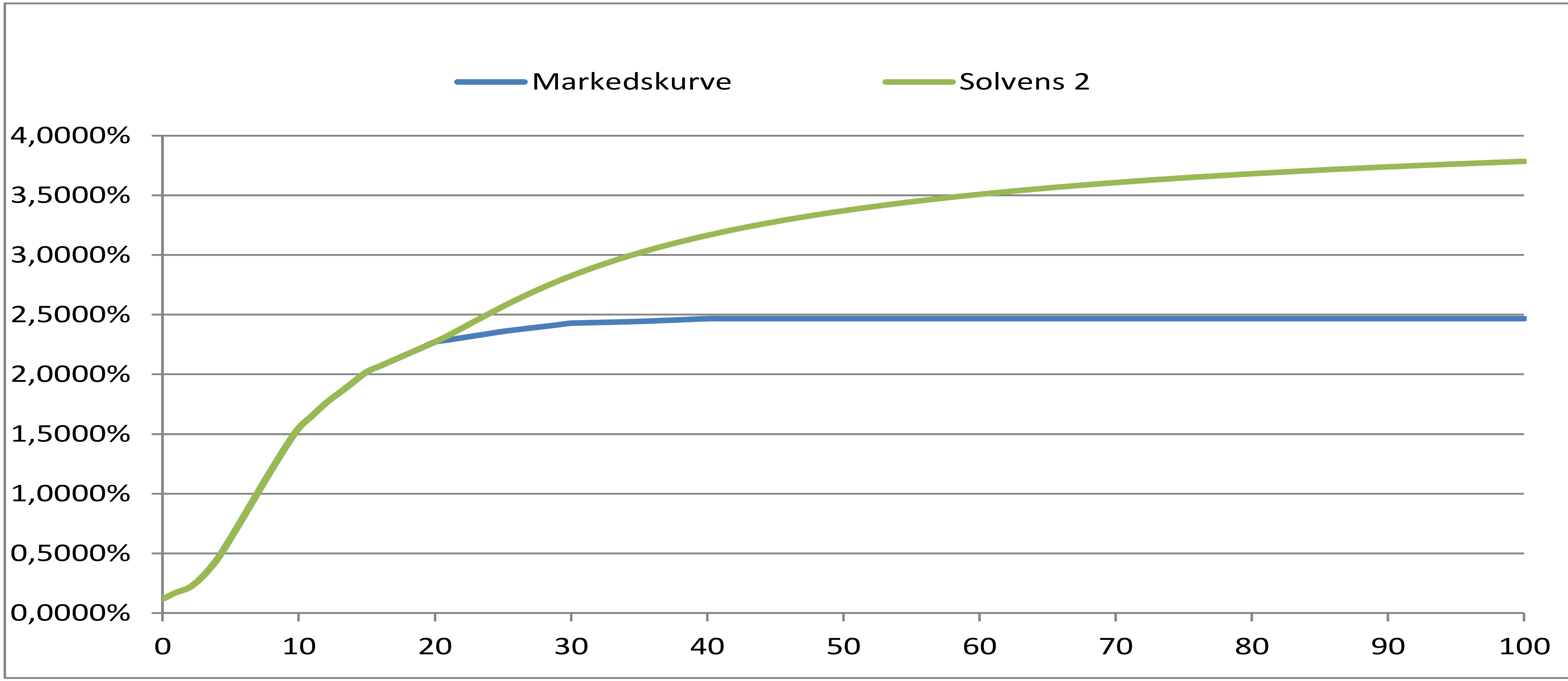
- **Det sker ved at iterere over konvergensparameteren α indtil forward-renten i 30 års punktet afviger mindre end 3 bp fra UFR**
  - **forward-rente til løbetid T:** $F_\alpha(T) = \frac{P_\alpha(T)}{P_\alpha(T+1)} - 1$

$$\alpha_{UFR} = \alpha_0 + j \cdot \Delta\alpha$$

$$j = \min_{i \in \{1,2,\dots\}} \left\{ \left| F_{\alpha_0 + i\cdot\Delta\alpha}\left(T_{cut-off} + T_{conv}\right) - UFR \right| < tol \right\}$$

I øvelserne bruger vi et fast α=0,38

Smith-Wilson Ekstrapolering - 3



Startbalance – Selskab med fuldt afdækning

```
=============================================================================
ASSETS 01-07-2016        mv    shr    dur   LIABILITIES             mv    shr    dur
-----------------------------------------------------------------------------
05YZeroSwap               0     0%   4759   Annuity             458046   100% 57768
10YZeroSwap               0     0%   5661                       ------------------
15YZeroSwap              -0    -0%  -2202   Total reserve       458046    92%
20YZeroSwap               0     0%  49460   Bonus Potential      41954     8%   109%
25YZeroSwap              -0    -0%     -0
30YZeroSwap              -0    -0%     -0
BondPF               241706    48%  14440
CashPF                   -0    -0%      0
EquityPF             258294    52%     -0   Delta1Key1            3259
TaxPF                     0     0%     -0   Delta1Key2            5987
                                            Delta1Key3           -2294
                                            Delta1Key4           51206
                                            Delta1Key5            0.00
                                            Delta1Key6            0.00
                                            IB                   18145
                                            KB                   23810
                                             RetroRsrv          476190
                                            TotAccount          476190
-----------------------------------------------------------------------------
Balance              500000         72117 (125%)                500000       458046
=============================================================================
```

Renteafdæk-
ning i swaps

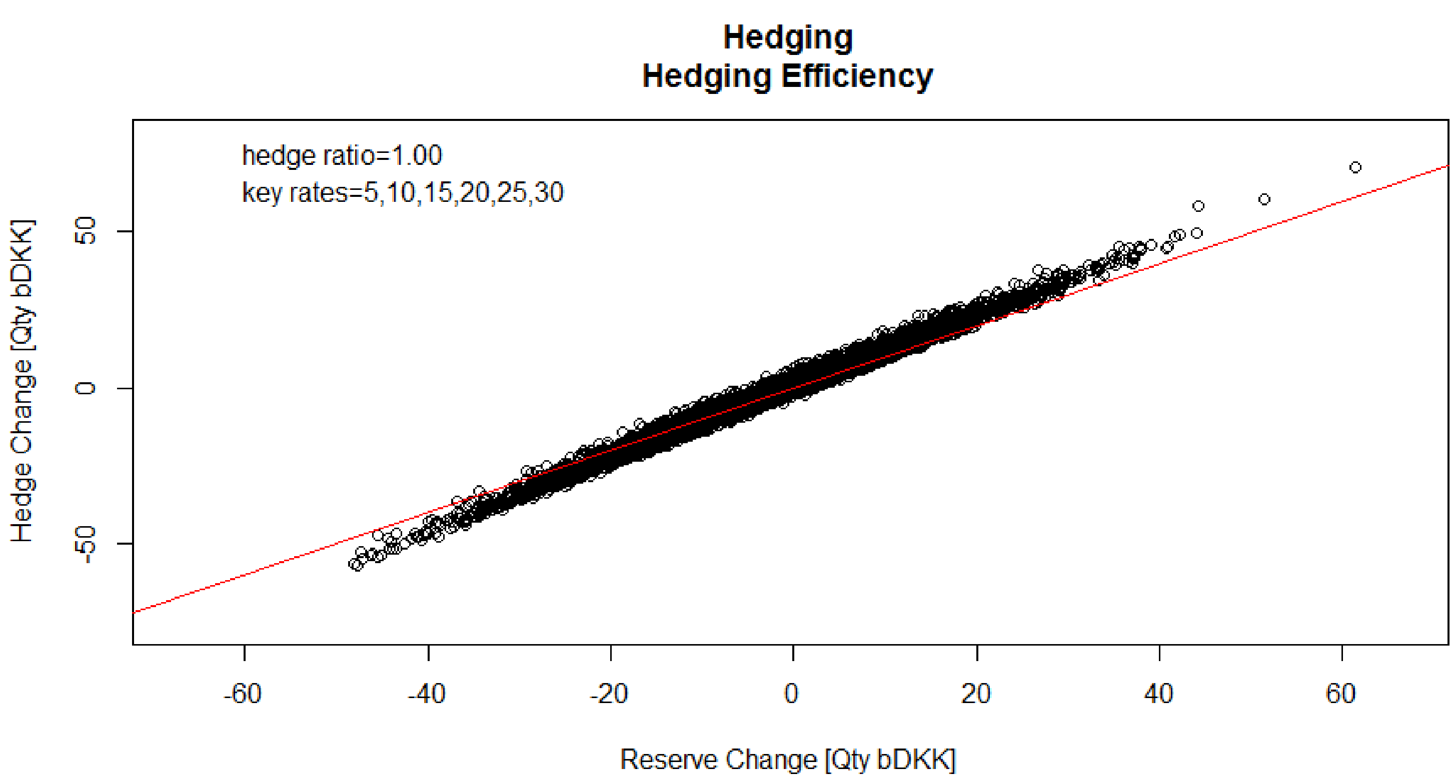
Negativ rentefølsomhed i
15-års punktet!

Rentefølsomheden er nul
for løbetider over 20 år

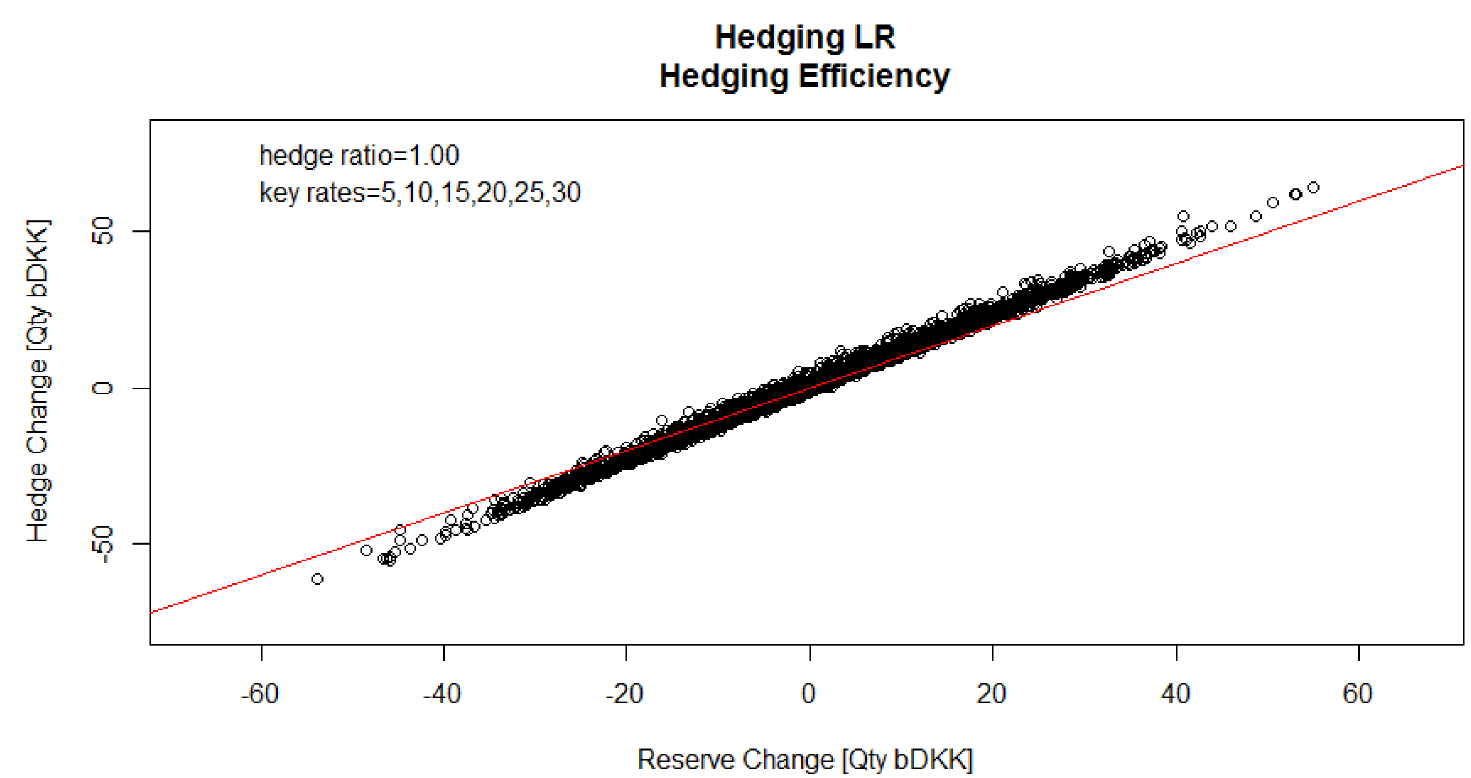
Afdækningen fungerer under alle renteantagelser

- **For begge renteantagelser gælder**
  - **Tæt korrelation mellem afkast på afdækning og ændring i hensættelse**
  - **Hældningen passer ikke helt (hvorfor?)**

Stigende renteniveau

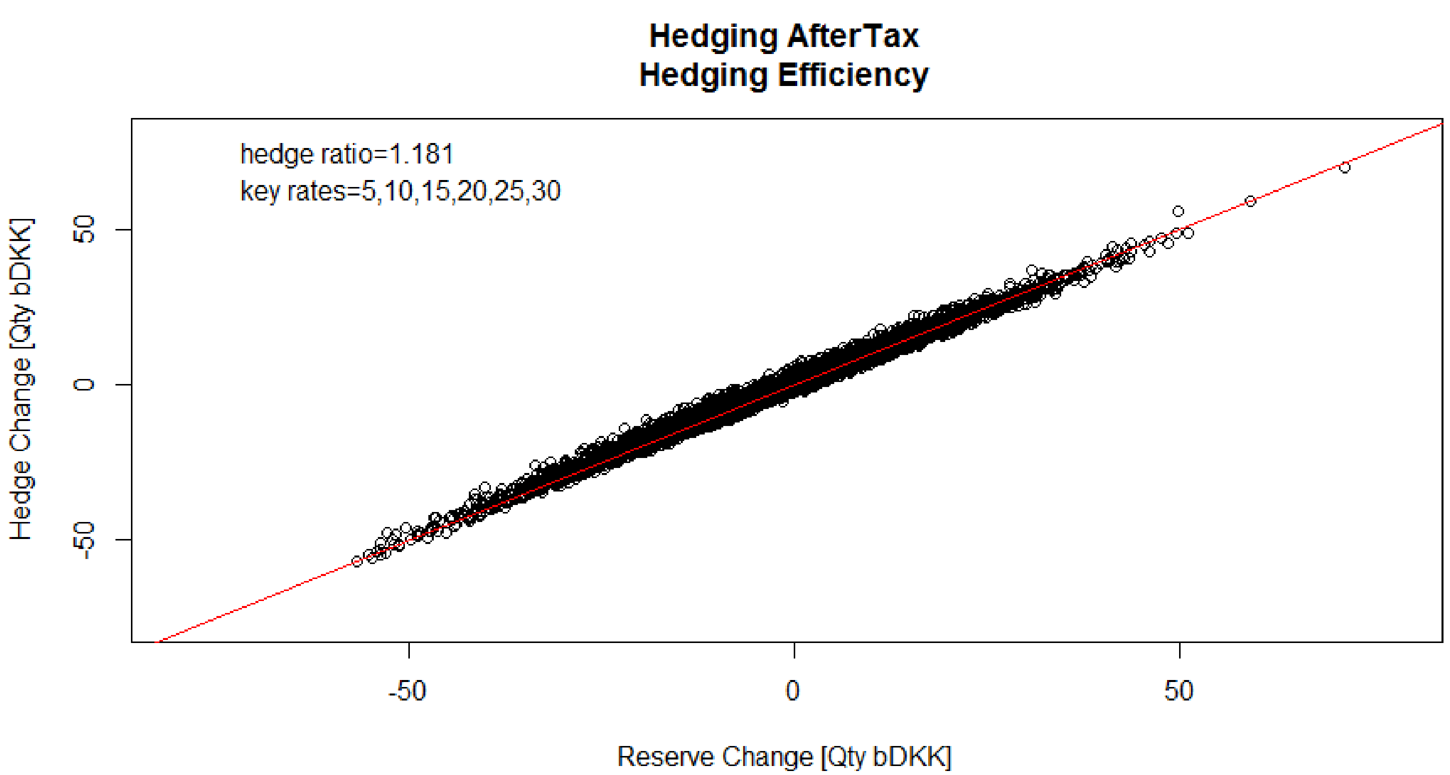


Uændret renteniveau

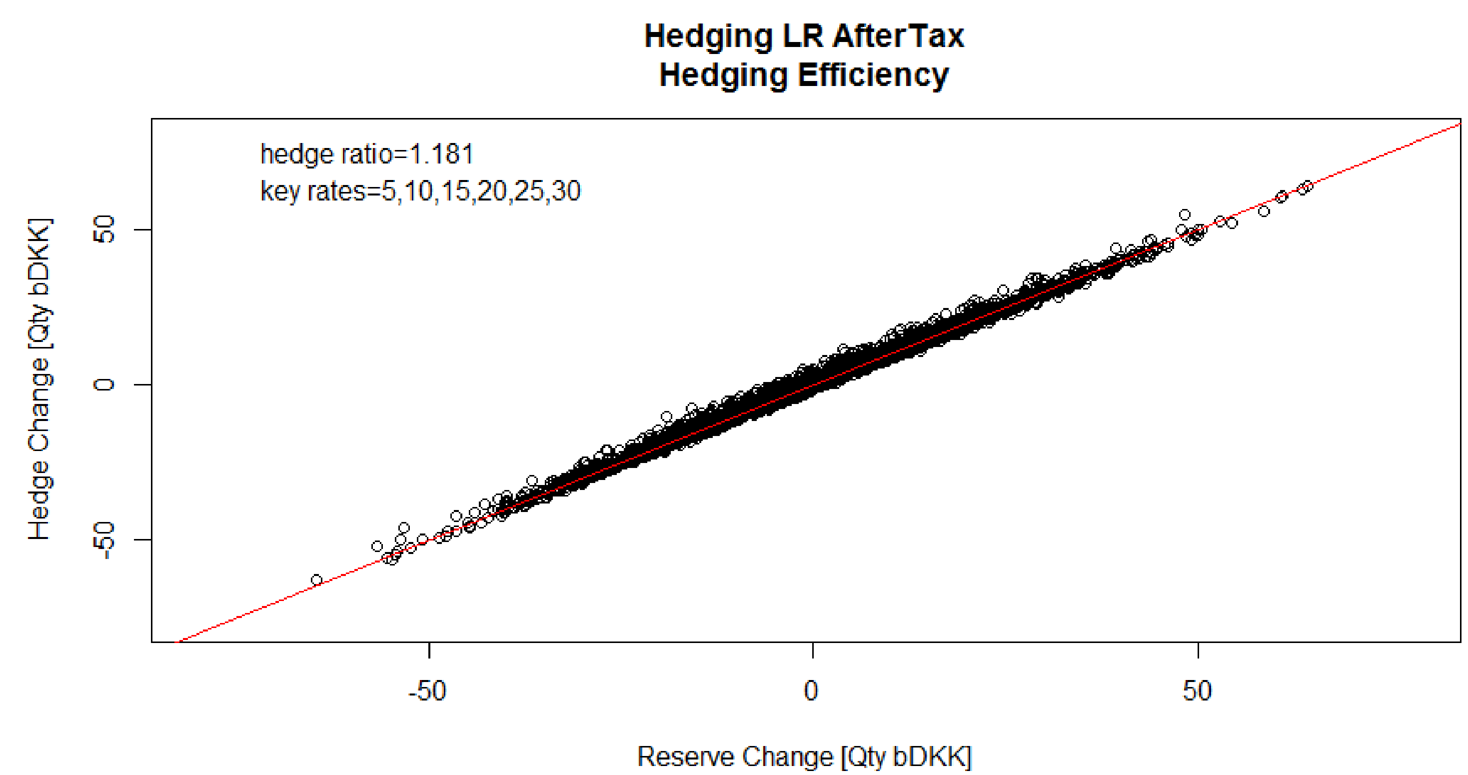


Der skal afdækkes *efter* skat …

Stigende renteniveau



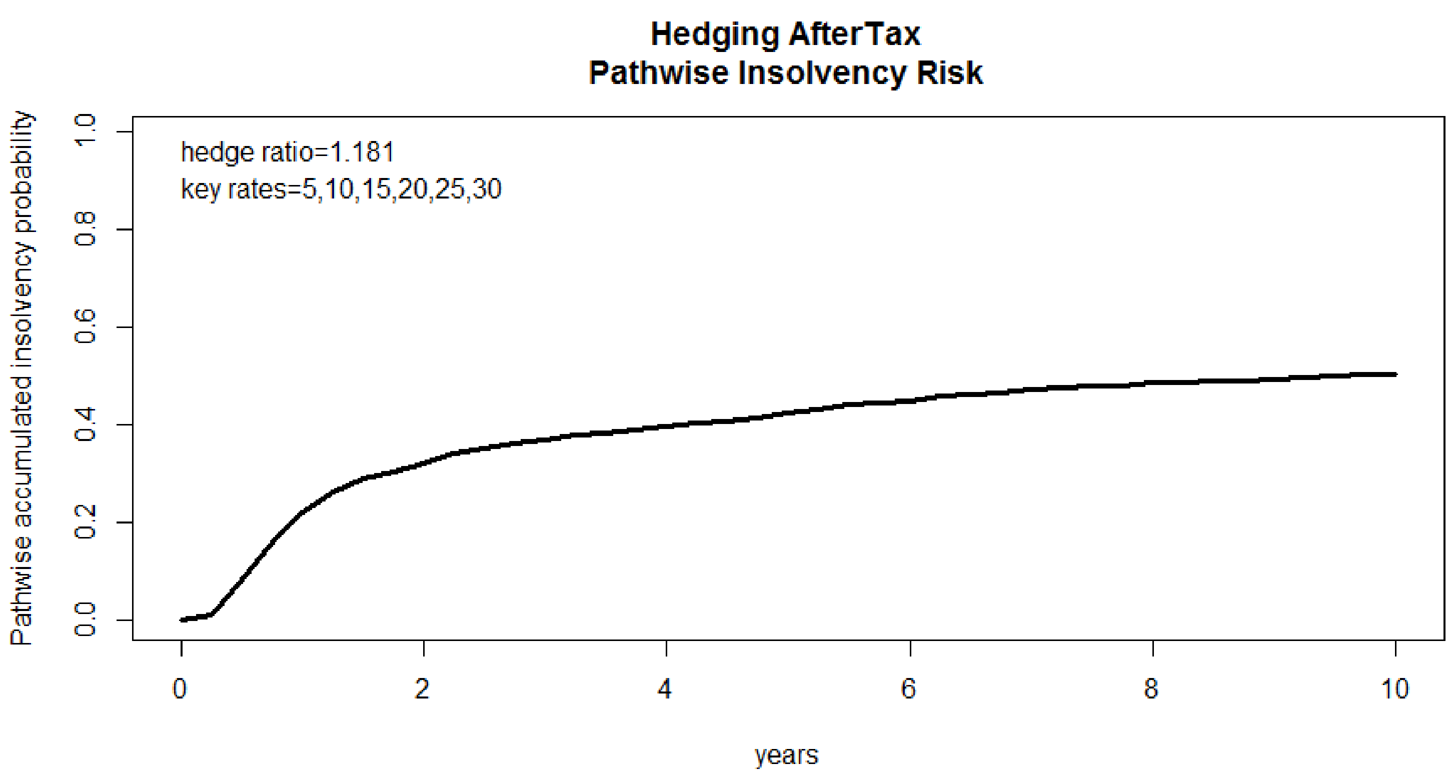
Uændret renteniveau



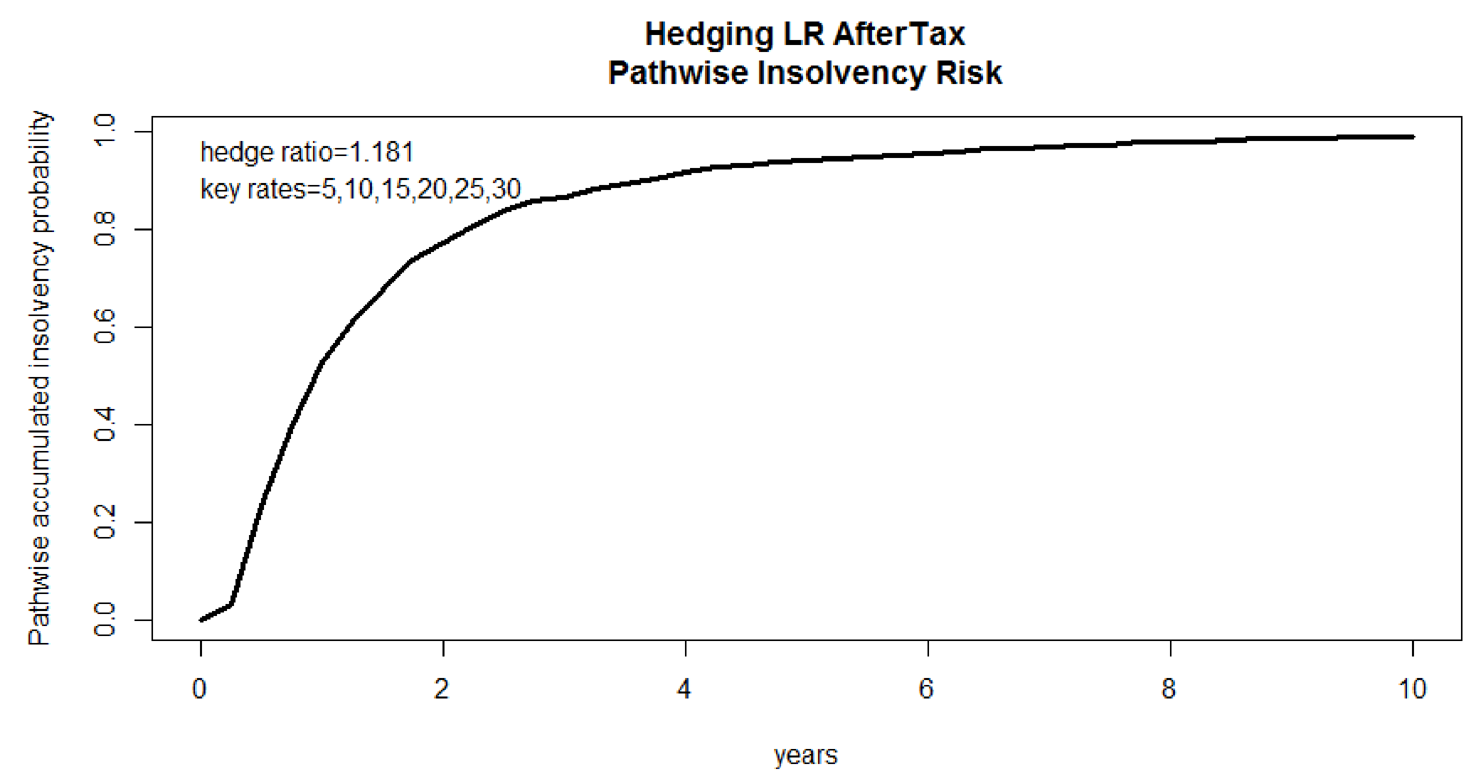
… men der er ugler i mosen!

- Insolvens synes alligevel uundgåelig
  - fordi afdækningen ikke amortiserer som hensættelsen
  - der er simpelthen ikke penge nok til at dække den finansielle forpligtelser

Stigende renteniveau

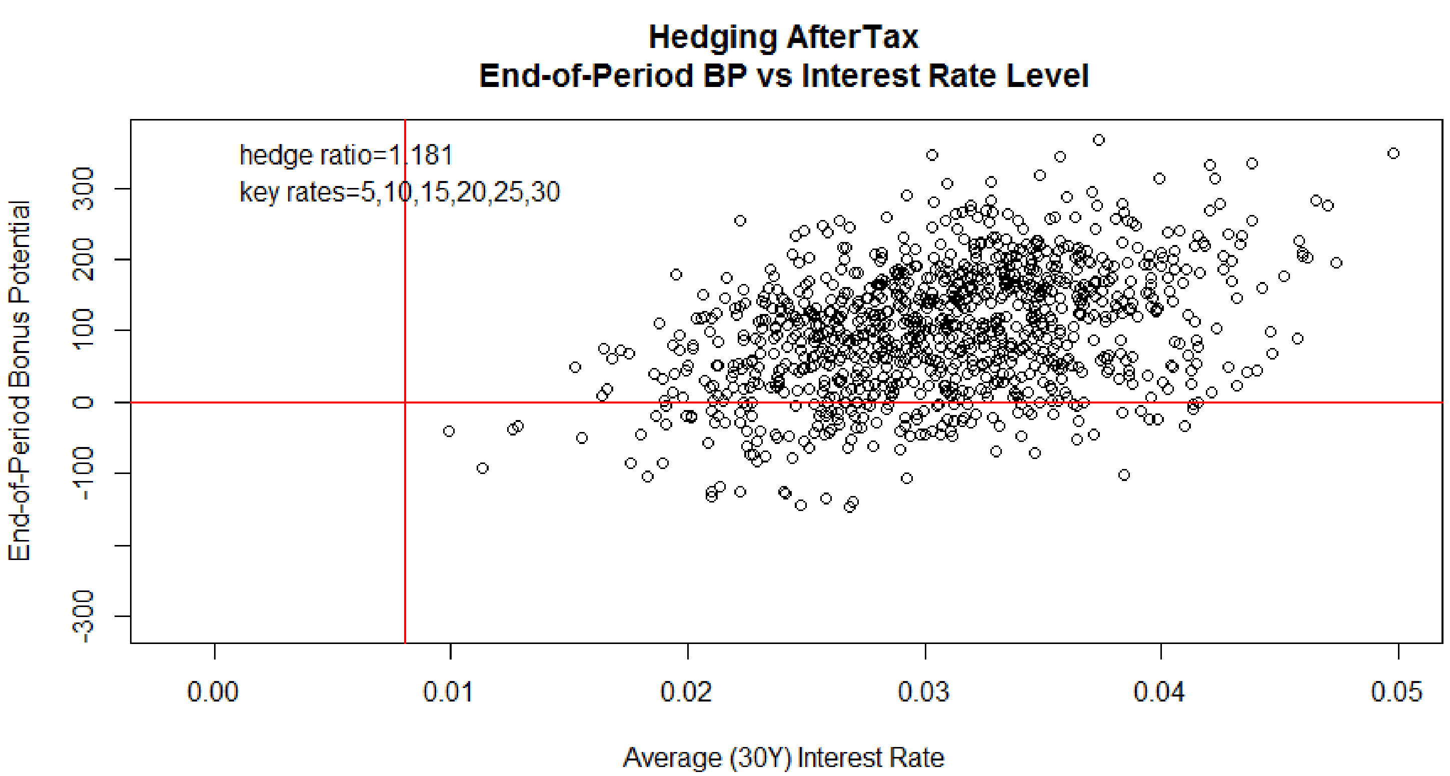


Uændret renteniveau

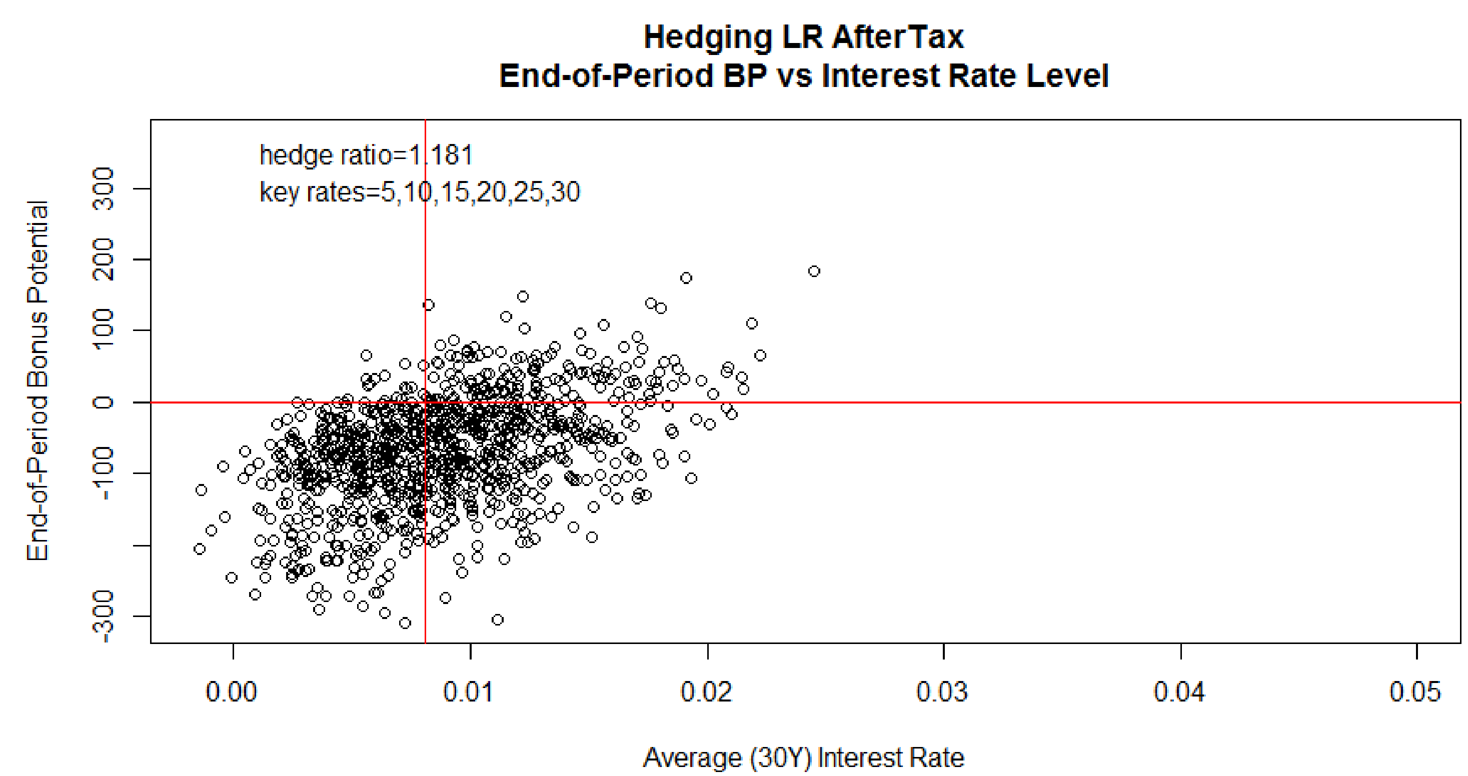


Bonuspotentiale efter 10 vs gennemsnitlig 30-årig rente

Stigende renteniveau



Uændret renteniveau



Øvelse: Renteafdækning *revisited*

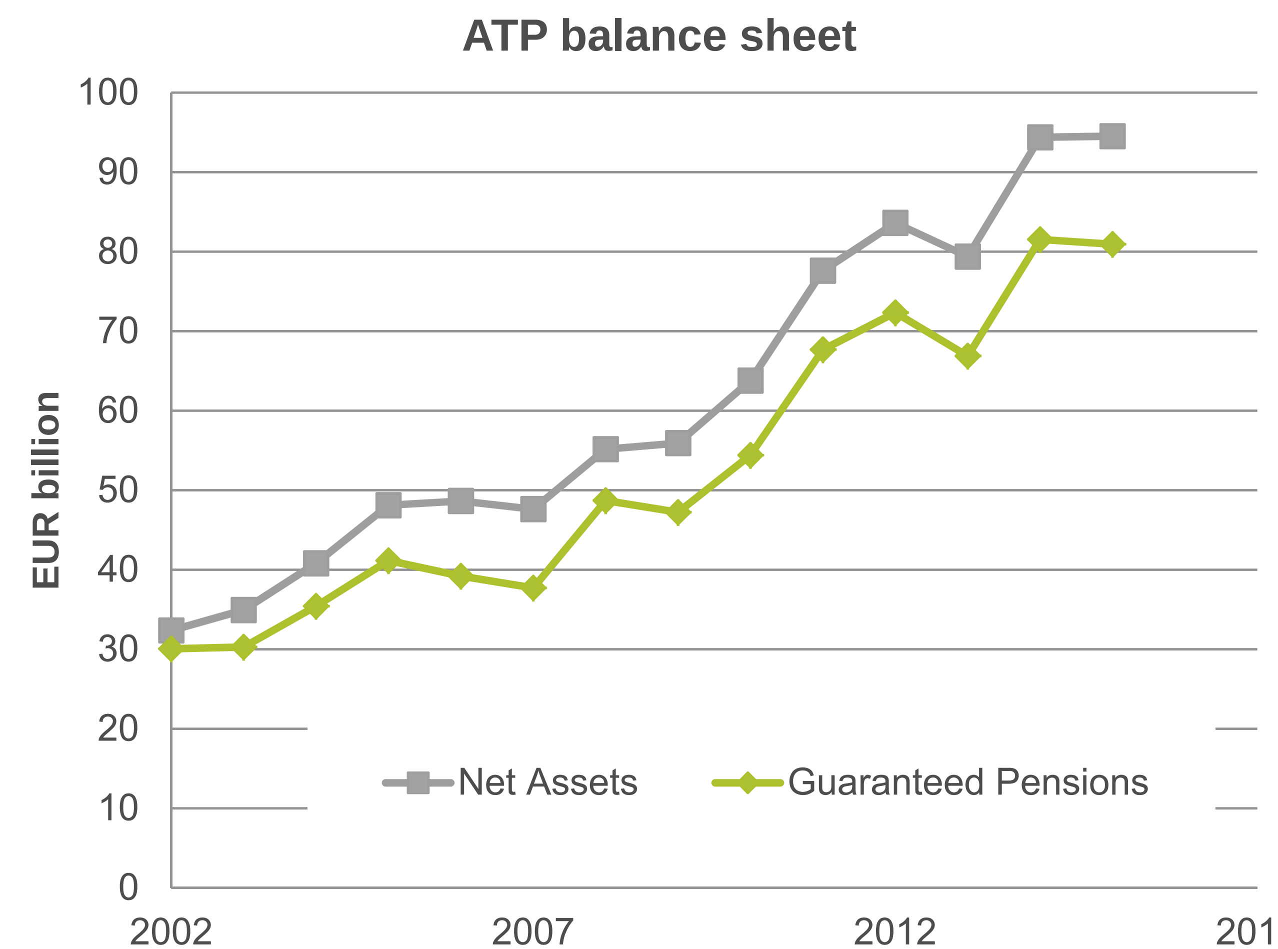
- **Opgave**
  - **Diskutér hvordan den del af forpligtelserne, som *ikke* kan afdækkes bør påvirke**
    - Selskabets afdækningsstrategi
    - Selskabets investeringsstrategi. I eksemplerne investerede selskabet 50 pct. i aktier og 50 pct. i obligationer.
  - **Vælg hvilke løbetider, jeres selskab afdækker**
    - vælg 2-5 løbetider
    - begrund jeres valg
  - **Vælg hvilken afdækningsgrad* jeres selskab skal anvende**
    - begrund jeres valg

$$*\text{Afdækningsgrad} = \frac{Varighed\ af\ swaps}{Varighed\ af\ GY}$$

Case Study: ATP’s nye pensionsprodukt

Market value accounting and its implications for ATP

- **Market value accounting since 2003**
  - **"Pure" market rate discounting**
    - Long-dated liabilities discounted at 30Y rate
    - Allows delta-hedging (in normal markets)
  - **Huge interest rate sensitivity**
  - **Fully hedged in swaps and bonds**
  - **Decrease in interest rates increased value of liabilities dramatically.**



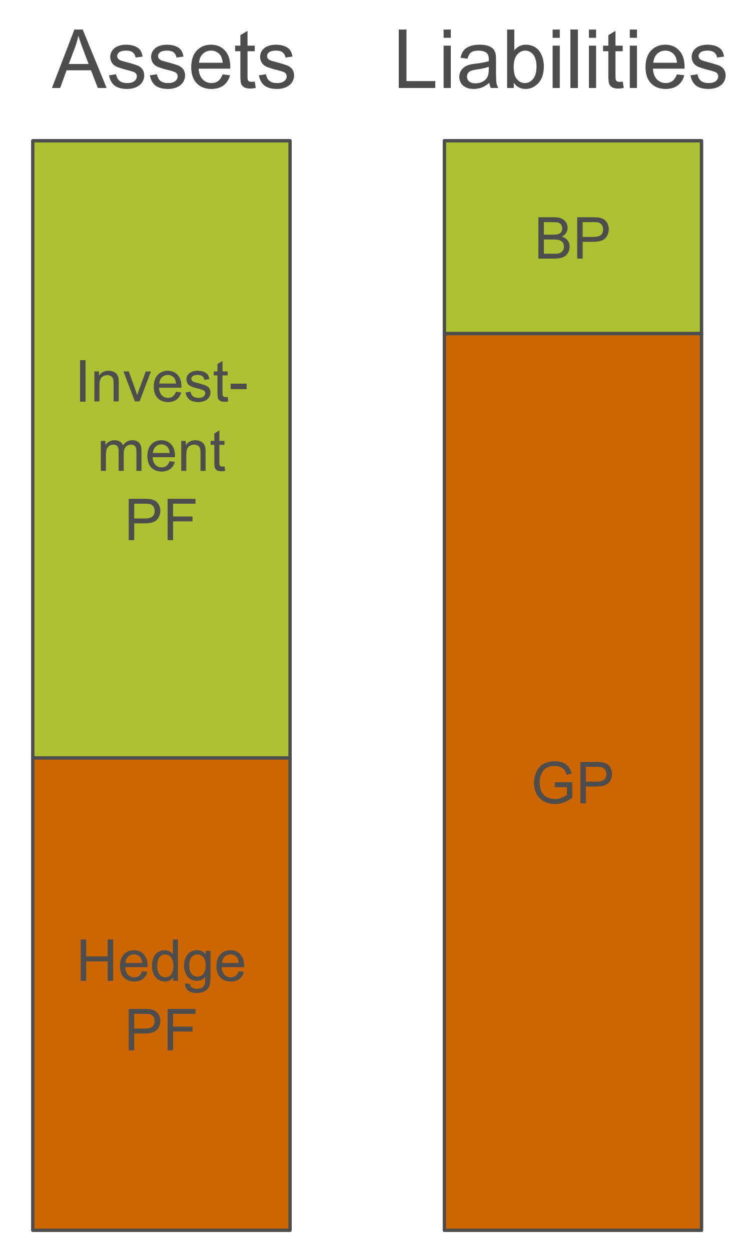
- **Discounting curve under Solvency 2**
  - **Long-dated liabilities valued at UFR**
  - ***Long-dated liabilities cannot be hedged***
    - Discounted value ≠ value of (delta) hedge

The problem

- **The old annuity product at ATP**
  - **80 pct. of contribution converted to nominal life-long annuity at the time of payment**
  - **Annuity level (tariff) updated annually to reflect current market rates and life expectancy.**
- **Large hedge demand at long-dated maturities**
  - **Increasingly difficult – and costly – to maintain the necessary hedge**
  - **Long-dated liabilities non-hedgeable (due to "semi" market rate discounting).**
- **The Board of ATP wants guarantees!**
  - **Not an option to move to unit-link type products**
  - **"Could you please design a hedgeable life-long guarantee".**

… and one more thing

- **"Please make sure to preserve the business model"**
  - **The liability side of the balance sheet is very simple**
  - **… allowing a very sophisticated asset side**
  - **Accommodation of all guarantees in one (simple) business model.**
- **Implication 1: Type of guarantee**
  - **All pension rights in the form of "guaranteed annual pension"**
  - **No individual unit-link accounts.**
- **Implication 2: Same status of new and old guarantees**
  - **Collective risk sharing of financial and biometric risks**
  - **New and old guarantees should entail same, or at least very similar, risks and have the same "claim" on free reserves (BP).**



Traditional annuity vs rolling annuity

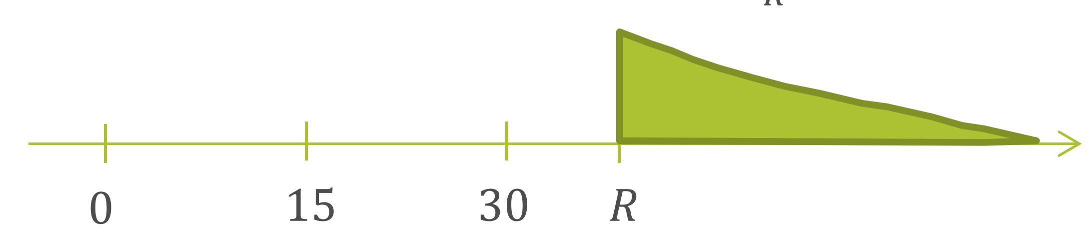
- **Consider a person paying a contribution of 100 at time 0 and retiring at R**
  - **Denote by $p_t(T)$ the price at time t of a zero-coupon bond (ZCB) maturing at time T**

- **Traditional (deferred) life-long annuity**
  - **Ignore tax, safety loadings, technical basis etc.**
  - **Guaranteed annuity level = $100/\int_R^\infty p_0(w)S(w|0)dw$.**

Expected, unit cash-flow = prob. of survival

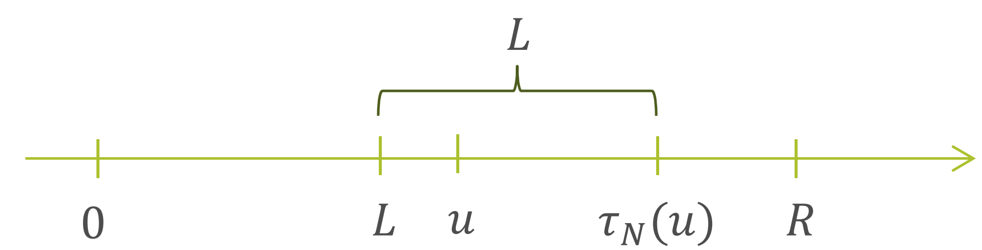
- **The rolling annuity replaces the long interest rate guarantee with shorter ones**
  - **Assume interest rate guarantee of 15 years**
  - **Initial guarantee: $z(0) = \frac{100}{\int_R^\infty S(w|0)dw}\frac{1}{p_0(15)}$**
  - **Guarantee after 15 years: $z(15) = z(0)\frac{1}{p_{15}(30)}$**
  - **Final guarantee: $z(30) = \frac{z(15)\int_R^\infty S(w|30)dw}{\int_R^\infty p_{30}(w)S(w|30)dw}$.**

Expected no. of years in retirement = $\int_R^\infty S(w|0)dw$



Market value reserve

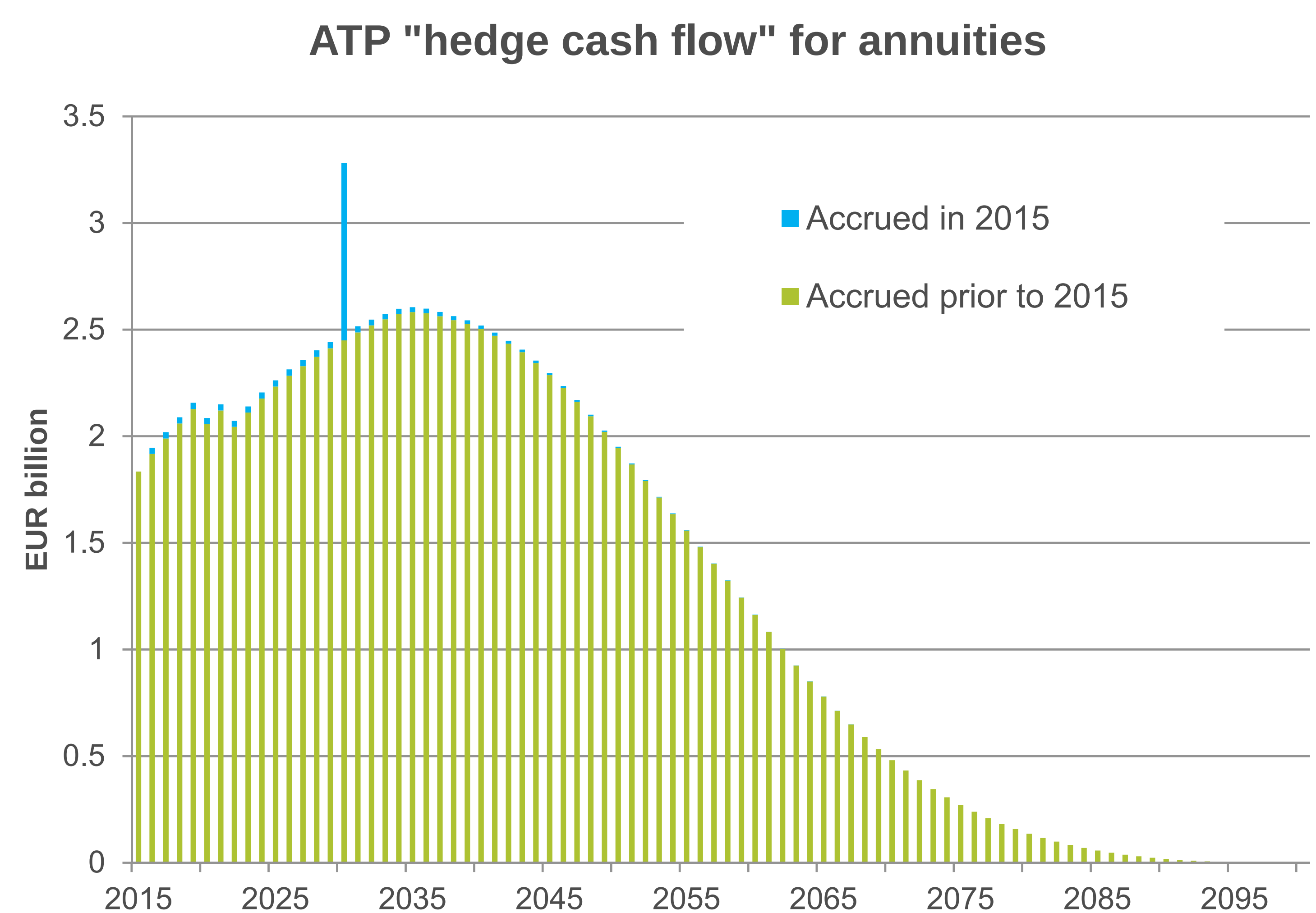
- **Consider the reserve associated with a contribution paid at time 0**
  - **Let $z(u)$ denote the guarantee at time $u \geq 0$**
  - **Prior to the final guaranteed increase, the reserve is**
    - $V(u) = z(u)e(R|u)p_u(\tau_N(u))$,
    - where $e(R|u)$ is the expected no. of years in retirement given survival to time u, and $\tau_N(u)$ is the time of the next increase.



  - **At or past the final guaranteed increase, the reserve is**
    - $V(u) = z(u) \int_{\max\{u,R\}}^{\infty} p_u(w)S(w|u)dw$, i.e. the reserve for an ordinary, life-long annuity.
- ***Before the final increase, the reserve for a cohort equals the price of a ZCB maturing at $\tau_N(u)$ with principal $\bar{z}(u) \times total\ no.of\ years\ in\ retirement$***
  - **The liability can be semi-statically hedged, i.e. hedge needs to be adjusted only every L years**
  - **For L up to 20 years, say, the hedge can be implemented in liquid markets**
  - **In practice, the reserve is based on updated mortality assumptions**
  - **Longevity risk is borne collectively, i.e. guarantees are unaffected.**

Implementation at ATP

- **Rolling annuities were implemented at ATP with effect from 1 January 2015**
  - **Guarantee period of $L = 15$ years**
  - **The effect from contributions received in 2015 can be seen as an increased "payment" in 2030**
  - **The remaining cash flow stems from ordinary annuities; both old guarantees and guarantees written in 2015 for members within 15 years of retirement.**
- **Hedgeable at large scale**
  - **The bulk of the (rolling annuity) cash flow is at maturities where market liquidity is high**
  - **Ordinary life-long annuities are issued only close to retirement.**



Example: Longevity risk

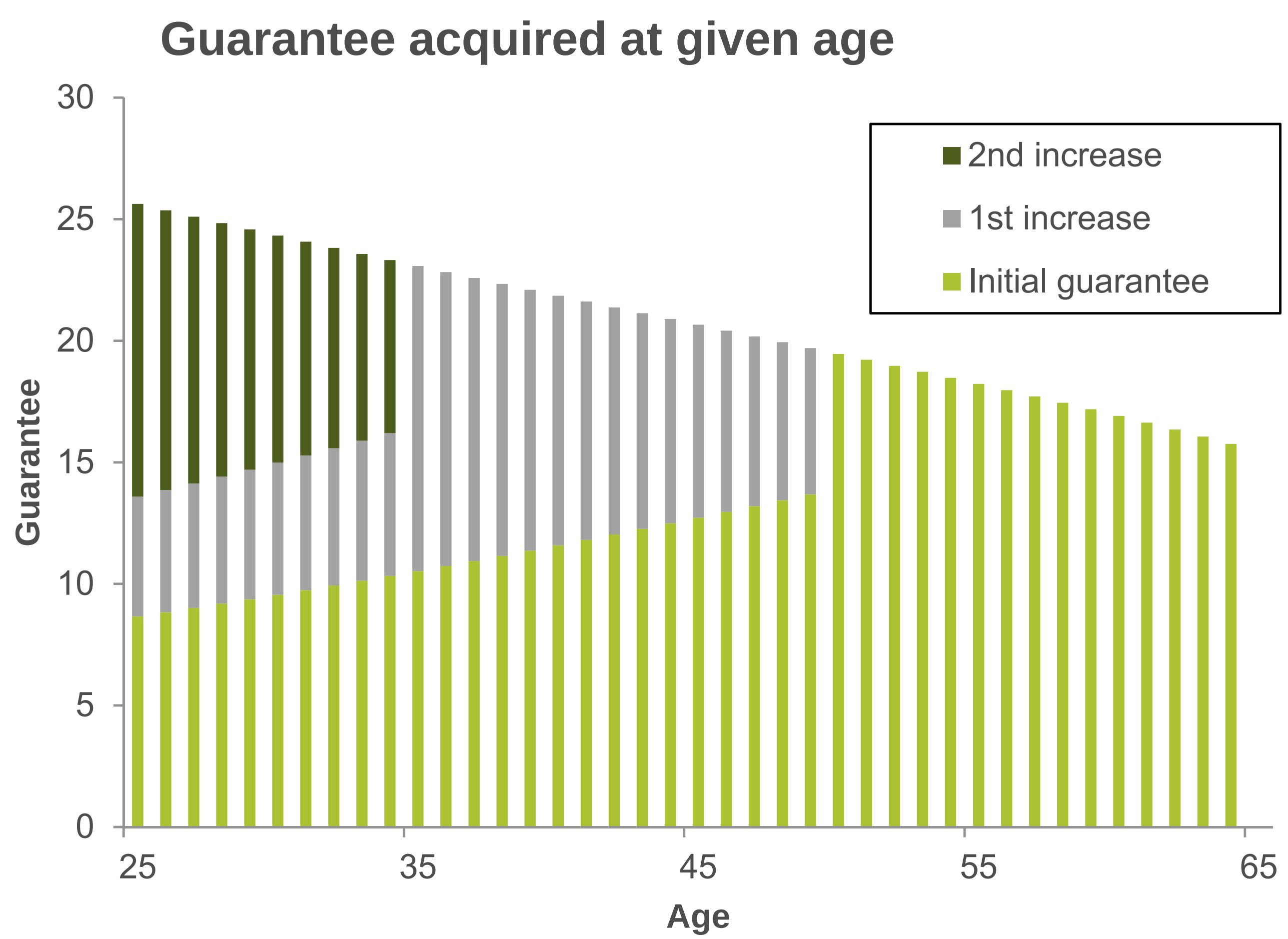
- **Table shows the relative reserve increase when applying a 20% mortality stress**
  - **GM mortality law*** : $\mu(x) = 1.5 \cdot 10^{-5} \exp(0.1 \cdot x) + 2 \cdot 10^{-4}$
  - **Stressed mortality law:** $\tilde{\mu}(x) = 0.8\,\mu(x)$
  - **Flat yield curve** : $p_t(T) = \exp(-(T-t)r)$, **for some fixed** r
  - **Single premium at age** x, **age of retirement** $R = 65$ **yrs, and guarantee period of** $L = 15$ **yrs.**

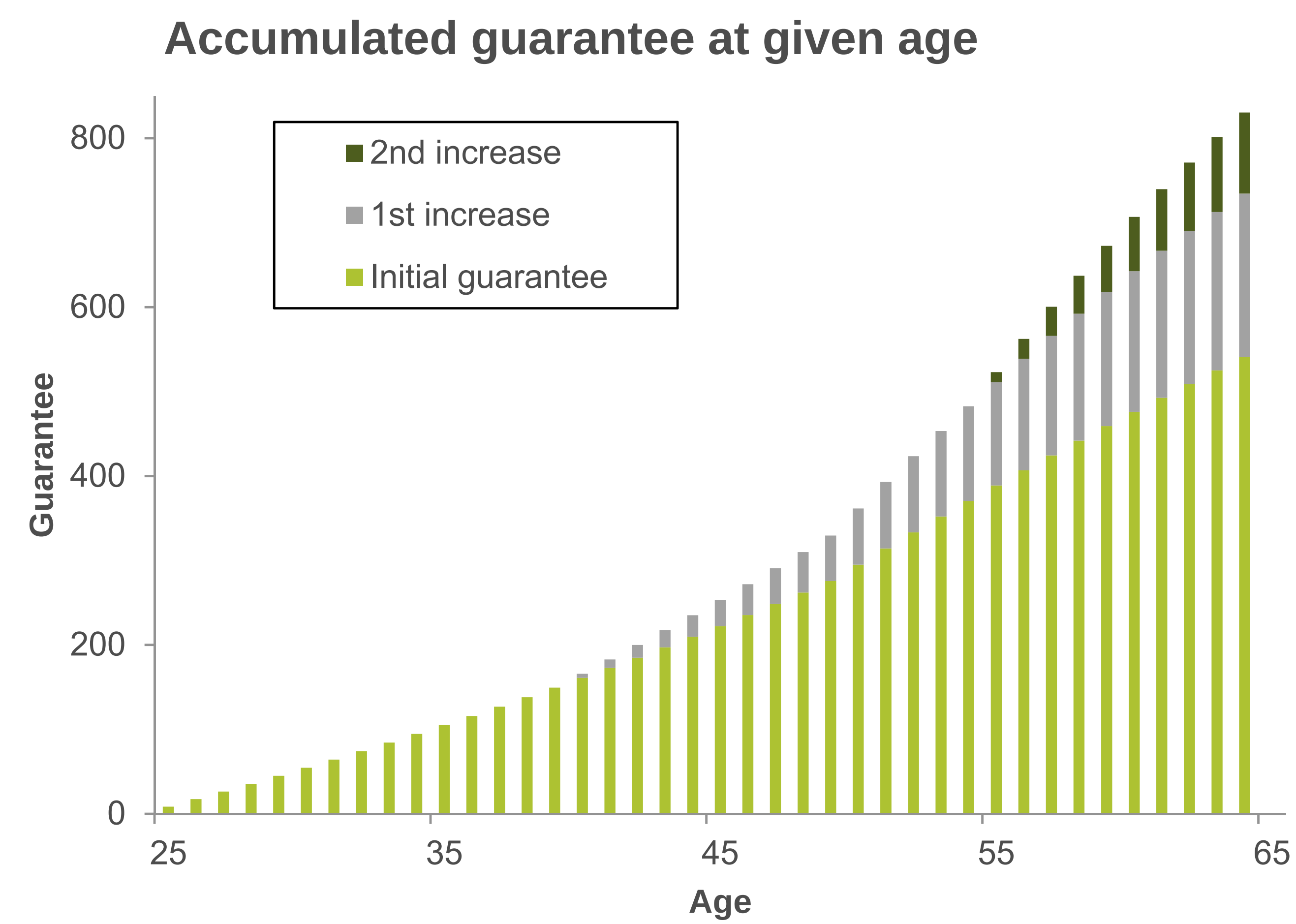
$\Delta V/V$	Age (x)						
Rate (r)	25	45	55	65	75	85	100
0%	11.4%	11.0%	10.5%	9.0%	11.6%	14.9%	19.9%
2%	11.4%	11.0%	8.7%	7.3%	10.0%	13.5%	19.1%
4%	11.4%	11.0%	7.3%	5.9%	8.7%	12.3%	18.3%

Independent of interest rate

* *Gompertz-Makeham law fitted to Danish unisex population mortality for 2011*

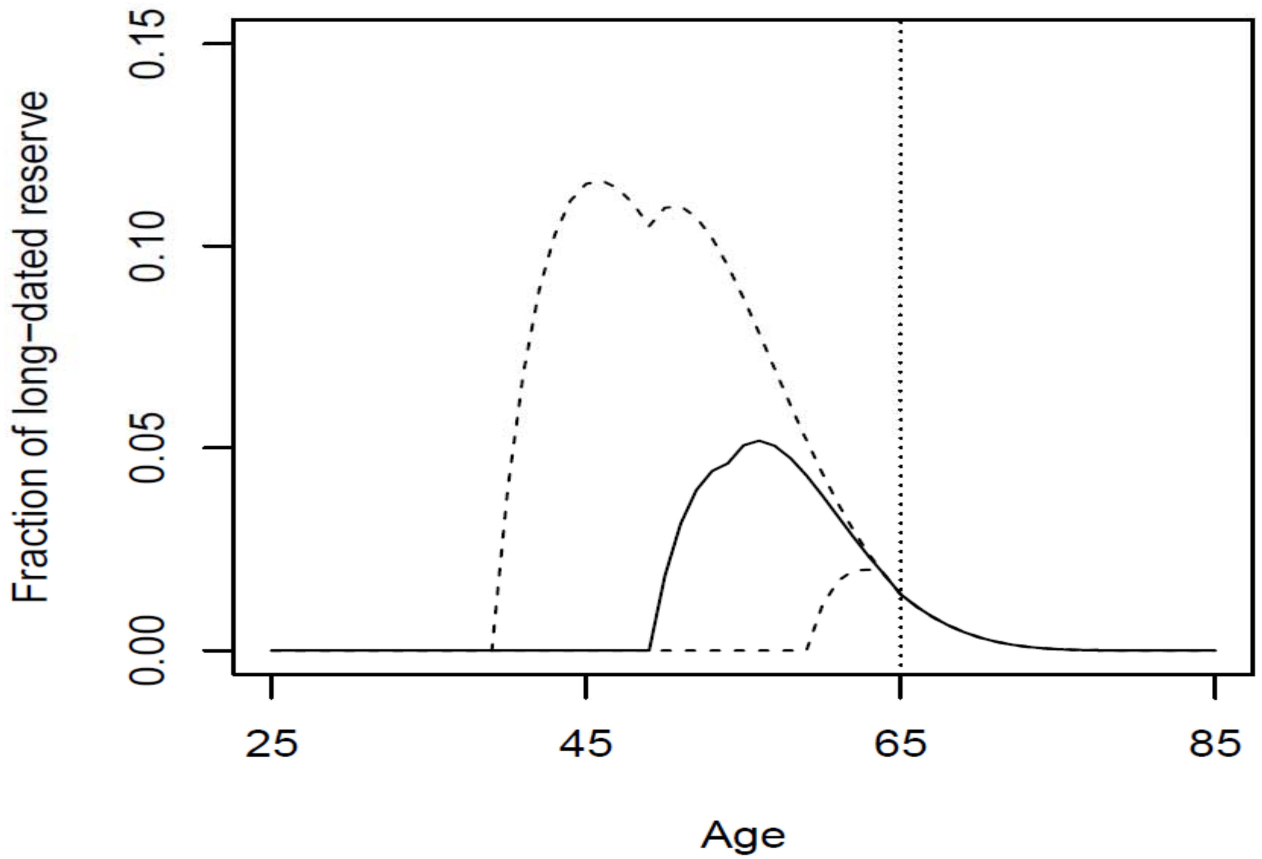
Example: Building up of guarantee

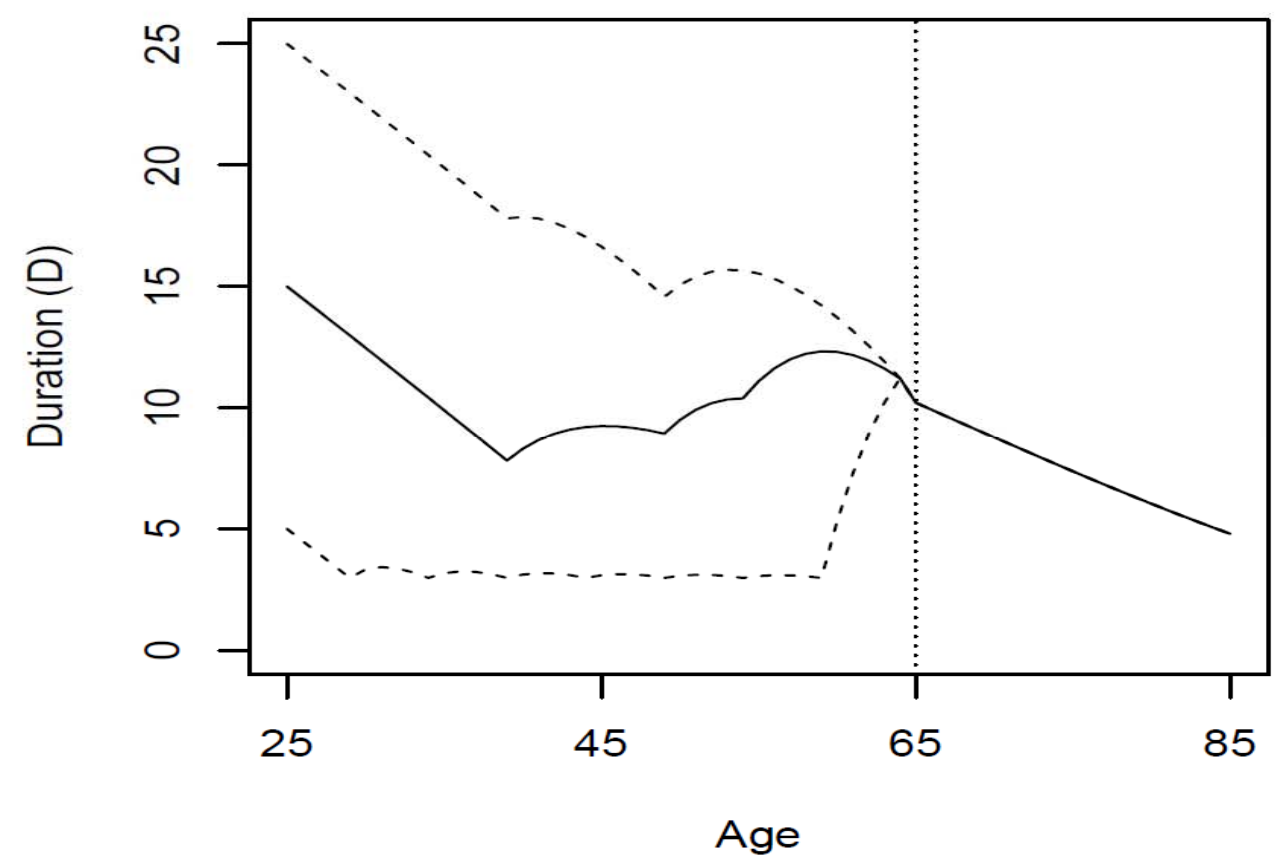




Annual contribution of 100 indexed by inflation of 2% from age 25 to age 64.
Interest rate of 3%, and guarantee period of $L = 15$ years.

Example: Duration





Left plot : Reserve for maturities over 30 years as fraction of the total reserve.

Right plot : Duration of total reserve measured in years.

In both plots the solid line represents a guarantee period of $L = 15$ *years, while the dashed lines represent guarantee periods of 5 and 25 years, respectively.*

The vertical dotted line at age $R = 65$ *years marks the age of retirement.*

Summing up

- **Initial minimum guarantee and subsequent guaranteed increases prior to retirement**
  - **Prior to the final increase, the reserve equals a zero-coupon bond maturing at the next increase**
  - **Rolling annuities can be hedged at large scale for guarantee periods of up to, say, 20 years**
  - **Keeping the duration below 20 years imply very similar financial and regulatory (S2) value**
  - **This simplifies risk management considerably**
  - **Rolling annuities have been implemented at ATP with a guarantee period of 15 years.**

- **Longevity risk can be reduced by weakening the "life expectancy guarantee"**
  - **However, the rolling annuities at ATP have full longevity risk (similar to existing annuities).**

- **Rolling annuity guarantees are intended as part of a with-profits contract**
  - **A complementing return-seeking portfolio is essential to obtain broad market exposure**
  - **The guarantees entail both longevity risk and hedging risk and thus can apply to only part of contributions**
  - **At ATP, rolling annuities are acquired for 80 pct. of contributions.**

[A] Jarner and Preisel. Long guarantees with short duration: The rolling annuity. *Scandinavian Actuarial Journal*, 2016.