Optical Communications Project - Methods

Group 8

1 Fibre Specification & V-Parameter

The refractive index of the core n_1 was 1.46 and the refractive index of the cladding n_2 was 1.45. The wavelength of light in the fibre λ was 800nm. The radius of the fibre core a was 5.1μm. The associated V-parameter was calculated using the following equation, where the wavenumber k_0=$2\pi/\lambda$:

$$V = ak_0\sqrt{(n_1^2 - n_2^2)} = 6.66.$$

2 Identifying the modes

To identify the relevant modes that can propagate for our given V-parameter required finding mode solutions from the eigenvalue equation.

$$\left[\frac{J_m'(pa)}{paJ_m(pa)} + \frac{K_m'(pa)}{paK_m(pa)}\right]\left[\frac{J_m'(pa)}{paJ_m(pa)} + \frac{n_1^2}{n_2^2}\frac{K_m'(pa)}{paK_m(pa)}\right] = m^2\left(\frac{1}{(qa)^2} + \frac{1}{(qa)^2}\right)\left(\frac{1}{(qa)^2} + \frac{n_1^2}{n_2^2}\frac{1}{(qa)^2}\right)$$

Where a is the wire radius, and p and q are respectively the radial component of the real wavevector in the fibre core and the radial component of the imaginary wavevector in the cladding. The first order differentials of the Bessel functions can be found using:

$$J_m'(x) = \mp J_{m\pm1}(x) \pm \frac{m}{x}J_m(x)$$

$$K_m'(x) = -K_{m\pm1}(x) \pm \frac{m}{x}K_m(x)$$

β parameterises the eigenvalue equation, according to the relationships:

$$p^2 = n_1^2k_0^2 - \beta^2, q^2 = \beta^2 - n_2^2k_0^2$$

Solutions to the eigenvalue equation exist for specific values of β, these values describe individual propagation modes. We defined functions for the left hand side and right hand side of Equation 1, and a subject function as the difference between them. We then plotted these difference functions for each mode within the plausible β range:

$$n_2k_0 < \beta < n_1k_0$$

Plots were used to generate initial guesses allowed β values, i.e. points at which the subject function was zero and the wave equation solution was satisfied. Root finding was used over our subject function, through scipy.optimise.root with these initial guesses. Repeated roots were removed via the np.unique function. Roots were reordered in terms of identified beta from largest values down using np.flip. This identified list of modes was then assigned to the ordered list of potential modes by mode number.

3 Plotting the Electric Field Components of the EH11 Mode

The β and n_{eff} of each mode was found in the previous task, and the mode selected for further investigation was the EH11 mode. In order to determine the electric field distribution within the fibre, the amplitude coefficients (A, B, C, D) were required. Due to the arbitrary nature of the amplitudes, amplitude A could be set to unity to deduce the remaining amplitudes (B,C,D) using the following equations,

$$A\left[\frac{i\beta m}{\omega\mu_0}\left(\frac{1}{(pa)^2} + \frac{1}{(qa)^2}\right)\right] = B\left[\frac{J_m'(pa)}{paJ_m(pa)} + \frac{K_m'(qa)}{qaK_m(qa)}\right]$$

$$\frac{C}{A} = \frac{J_m(pa)}{K_m(qa)}$$

$$\frac{D}{B} = \frac{J_m(pa)}{K_m(qa)},$$

Components of $\mathbf{E}$ in the Core (r≤a)	Components of the $\mathbf{E}$ in the cladding (r>a)
$E_z(r) = AJ_m(pr)$	$E_z(r) = CK_m(qr)$
$E_r(r) = \frac{-i\beta}{p^2}\left(ApJ'_m(pr) + i\omega\frac{m\mu_0}{\beta r}BJ_m(pr)\right)$	$E_r(r) = \frac{i\beta}{q^2}\left(CqK'_m(qr) + i\omega\frac{m\mu_0}{\beta r}DK_m(qr)\right)$
$E_\phi(r) = \frac{-i\beta}{p^2}\left(\frac{im}{r}AJ_m(pr) - \omega\frac{\mu_0}{\beta}pBJ'_m(pr)\right)$	$E_\phi(r) = \frac{i\beta}{q^2}\left(\frac{im}{r}CK_m(qr) - \omega\frac{\mu_0}{\beta}qDK'_m(qr)\right)$

where $\omega = ck_0$ is the angular frequency of the light. Functions for the three electric field components (z, r, ϕ) as a function of the distance r from the centre of the fibre were created using the equations below.
For each $\mathbf{E}$ component, a list of 100 values between $0 - 1.4a$ was created and the core values were appended to the list for $r \leq a$ and the cladding values were appended for $r > a$. A square grid between -1.4a to +1.4a was created using numpy.meshgrid, upon which a contour map of each field component was plotted using matplotlib.pyplot.contourf.

4 Spatial intensity distribution of the EH11 Mode

In order to find the spatial distribution of the total intensity of the EH11 Mode. The electric fields in radial and tangential directions deduced from Section 3 need to be taken the square modulus and summed up. In addition, only the real part of electric fields should be considered to be plotted. As a result, the total Intensity equation is:

$$Intensity = |Re(E_r)|^2 + |Re(E_\phi)|^2$$

5 Numerical Evaluation of Waveguide Dispersion

To calculate the dispersion, we used a finite difference method to approximate the second derivative of n_{eff}:

$$[\frac{\partial^2 n_{eff}}{\partial\lambda^2}]_w \approx \frac{n_{eff}(\lambda+h) - 2n_{eff}(\lambda) + n_{eff}(\lambda-h)}{h^2}$$

And then it follows that:

$$D_w = -\frac{\lambda}{c}[\frac{\partial^2 n_{eff}}{\partial\lambda^2}]_w$$

In our calculation, we set $h = 20nm$ so the three different values of n_{eff} are calculated by varying the initial guesses and iterating through the method described in Section 2. The results are substituted to the above equations and the numerical estimation for D_w is given to be $13.3 ps/nm \cdot km$.

6 Finding n_{eff}

n_{eff} can be calculated from the ratios between energy proportions in the core and cladding via,

$$n_{eff} = \sqrt{n_1^2\Gamma(V) + n_2^2(1 - \Gamma(V))}$$

Where Γ is the ratio of energy transmitted through the core and total energy within the core and cladding. Intensities within the core ($0 < r < a$), and cladding ($a < r < \infty$) were determined using the functions produced in section 5-6. Intensities were calculated across a wider radius than in these sections to give a fair representation of energies in the cladding. The upper limit of distance from core was 100x core radius as the cladding field beyond this distance can be neglected. Intensities were sampled and summed over these regions, and the ratio of the energy transmitted through the core and the total is used to find $n_{eff} = 1.4593$.

7 Acknowledgements

01705580 completed Task 3 of identifying the modes and Task 8 as seen in Sections 2 and 6. 01701107 selected the mode (Task 4), found the corresponding amplitudes and plotted the electric field components (Task 5) as seen in Section 3. 02335292 plotted the spatial distribution of the selected mode using the electric fields from Section 2 (Task 6), as seen in Section 4. 01726262 evaluated the numerical estimation for D_w (Task 7), as seen in Section 5. This group used notes from Lectures 3, 4 and 6 from the Optical Communications Course (Dunsby et al.).