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Analysis of the EH11 Mode in an Optical Fibre

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Specifications: n1=1.46, n2=1.45, λ= 820 nm, a=5.1 µm

Abstract- Propagation modes have been identified and investigated to characterise wave propagation through a step-index cylindrical fibre. The fibre considered has n_1=1.46, n_2=1.45 and a radius of a=5.1 µm. The ray of interest has λ= 820 nm. This fibre has a V-parameter of 6.666, and supports 14 propagation modes. The electric field and resultant intensity within the cladding and core were determined and plotted for the EH11 mode. The waveguide dispersion was then determined using three adjacent wavelengths, finding D_w = 13.3 ps/nmkm. From the ratio of energy in the core and cladding the effective refractive index has been calculated at n_{eff} = 1.4593, which was higher than the directly calculated value at n_{eff} = 1.4556.

V-Parameter

The V-parameter defines the relationship between radial momenta in the core and cladding. It is analogous with potential V in the time-independent Schrodinger equation, and thus can be considered as an optical potential for trapping light. In this fibre, V = 6.666 which means the fibre is multimodal as V > 2.405.

Mode Identification and Selection

Mode	β	n_{eff}
TE01	11168259.1	1.457536
TM01	11168205.8	1.457529
TM02	11126620.6	1.452102
TE02	11126545.2	1.452092
HE11	11179639.6	1.459021
EH11	11153460.3	1.455605
HE12	11148568.2	1.454966
HE21	11168222.5	1.457531
EH21	11135718.2	1.453289
HE22	11126554.2	1.452093
HE31	11153425.0	1.455600
EH31	11115505.7	1.450651
HE41	11135639.7	1.453279
HE51	11115366.5	1.450633

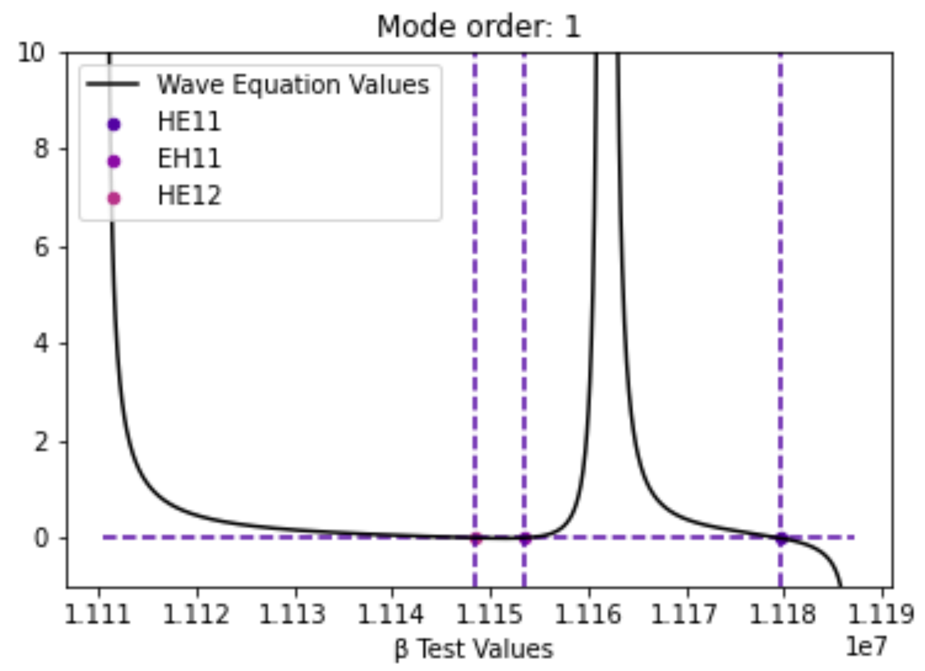


***Figure 1 (above):** Identified propagation constant values which provide solutions to the wave equation. Intersections between the left and right hand side of the equation identify these solutions.* ***Figure 2 (left):** Table of identified propagating modes for each available order with selected mode highlighted.*

Interference of propagating light generates discrete modes, that can be described by their field distributions and propagation constant values (β). Supported modes are identified through numerical solutions to the wave equation. 14 modes have been identified, aligning roughly with the predicted number for our calculated V-parameter.

We investigated the propagation behaviours of the EH11 mode. The behaviour of EH and HE modes is best described by a skew ray trajectory (Fig. 3). For EH modes we expect to find A > B, such that the E field z-component dominates over the H field z-component.

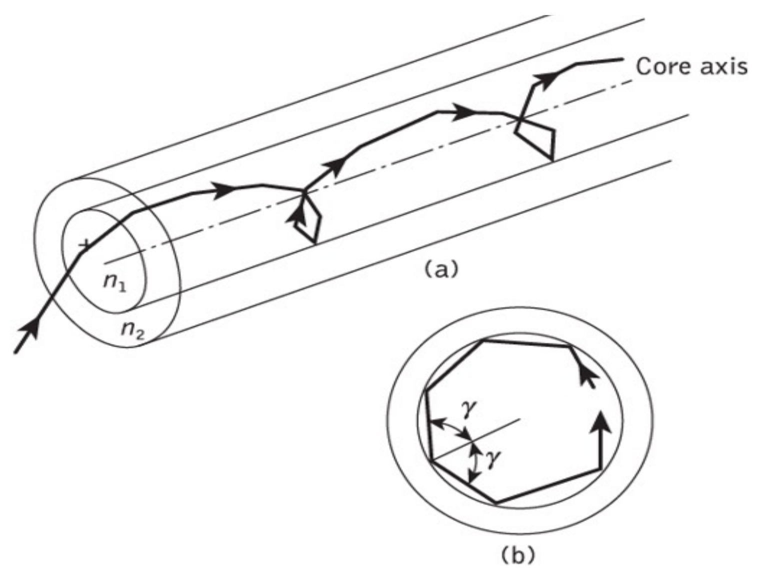


***Figure 3:** Illustration of skew ray propagation [1].*

Electric Field Distribution

- The amplitude coefficients (A,B,C,D) of the electric (E) and magnetic (H) fields in the fibre for the EH11 mode were found by setting A=1 to give B = 0.0039j, C = -51.1027, D= -0.2012j, where |A|>|B| as expected
- Amplitude maps of r, φ, z components of the EH11 E-field were plotted against the radial component *r* i.e., in the plane perpendicular to the fibre axis.
- When the plots showed zero field, such as the Re[E_r(r)] case, the imaginary component was extracted, plotted and shown to be non-zero. This was due to the inherent time-dependence of the field. This is not explicit as the field equations originate from a time-independent wave equation. The 3 non-zero plots are shown in Figures 4-6 (right).
- The z and φ components were found to be purely real and the r component purely imaginary, suggesting that the E_r field is minimum at maximum E_z and $E_φ$, and vice versa. Hence E_r may be in anti-phase with E_z and $E_φ$.

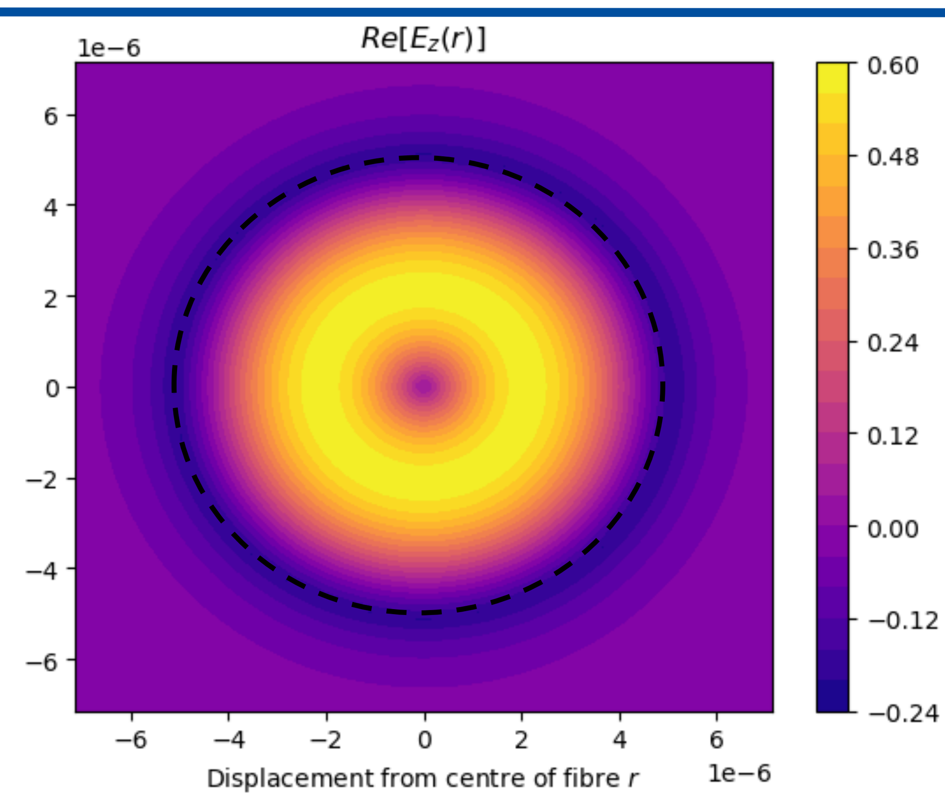


***Figure 4**: Amplitude of the purely real E field z-component. The black dashed circle indicates the core boundary. The maximum amplitude is reached within the core and then reduces to zero within 2.0 µm of cladding. This non-zero electric field along the direction of propagation (z) mode is in line with the skew ray model for EH modes (Fig.3).*

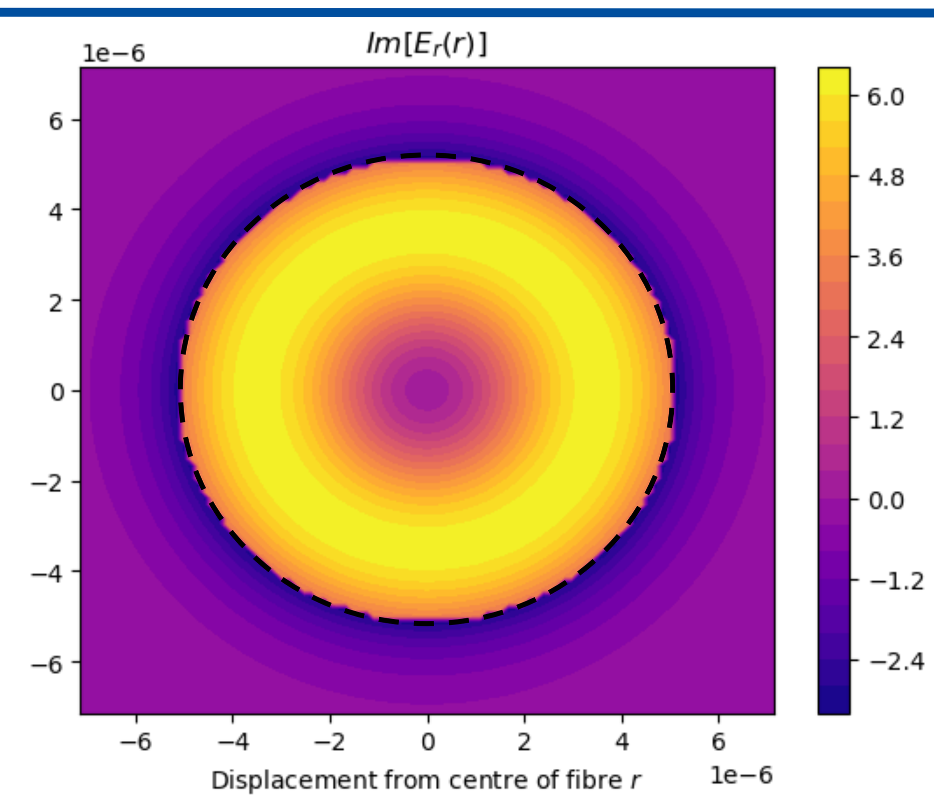


***Figure 5:** Map of the r-component of the E field, which was found to be purely imaginary . In the core, the component varies according to the associated Bessel solutions. Once in the cladding the radial E field is evanescent, as a consequence of total internal reflection at the boundary.*

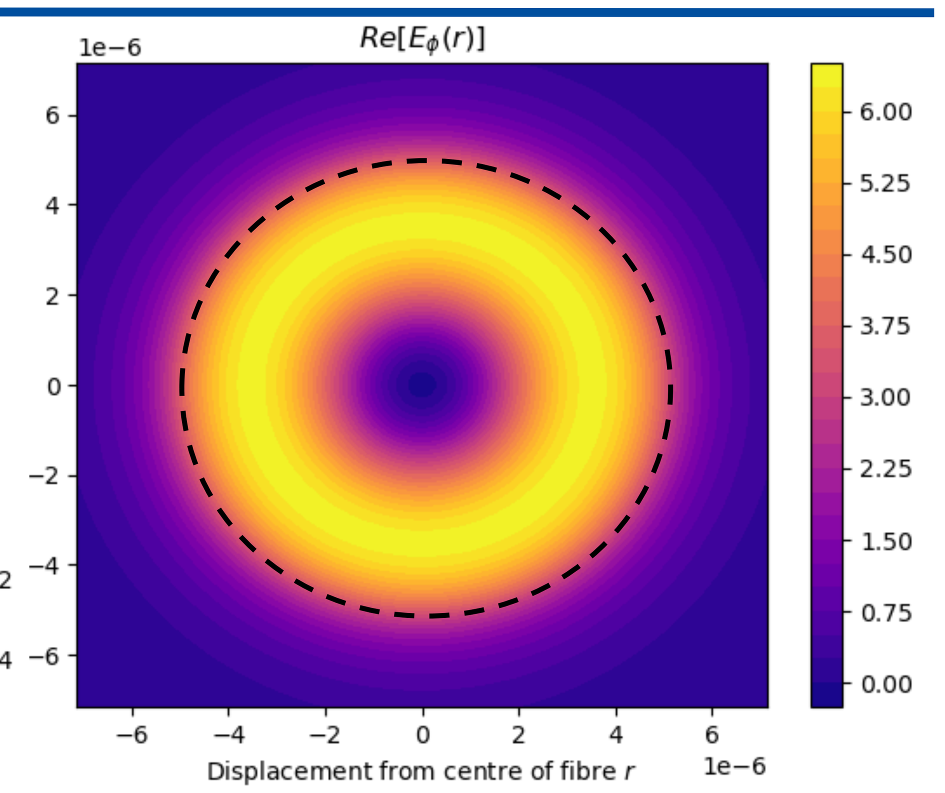
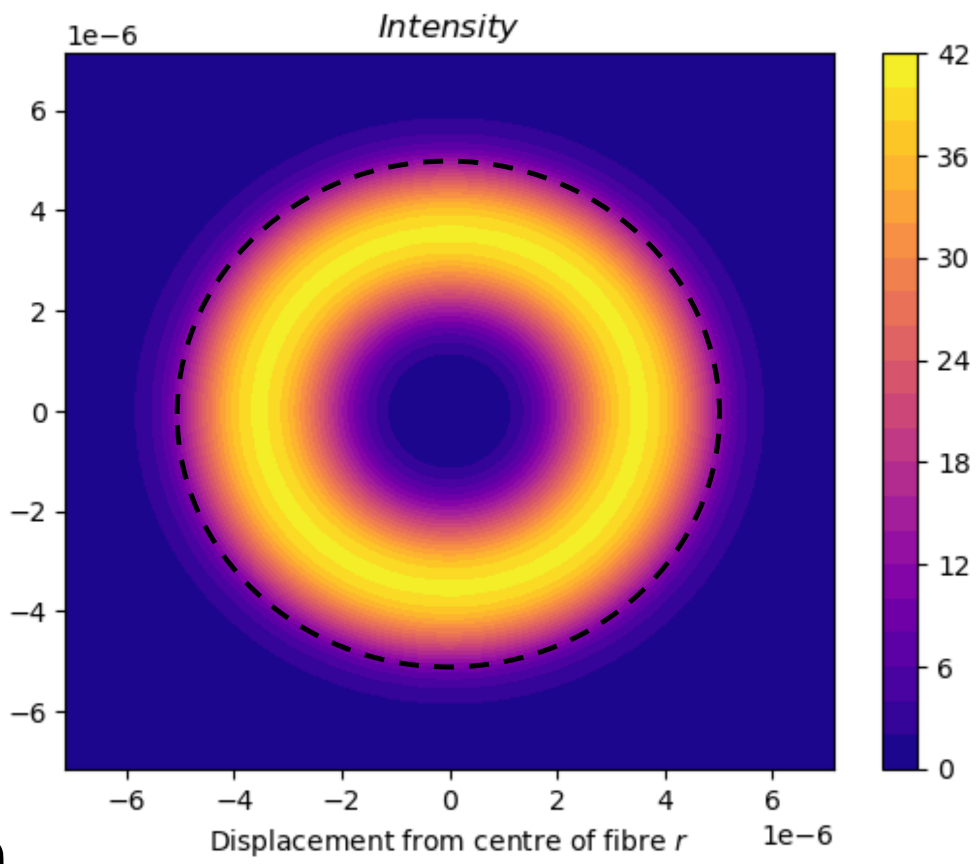


***Figure 6:** Map of the amplitude of the purely real φ-component of the E-field, where the black dashed circle indicates the core boundary. The evanescent field in the cladding is evident – again the field reduces to zero within 2.0 µm.*

Intensity Distribution

The optical field within a waveguide can be approximated by a Gaussian intensity distribution. As shown in Fig. 7, the majority of the intensity of the EH11 mode distributes away from the centre of the core region and forms a ring-like shape in the radial and tangential plane, similar to Fig. 6. This is because in the radial direction there are only trivial solutions for the real part of the electric field.

***Figure 7 (right):** Spatial distribution of the total intensity of the EH 11 mode in the direction perpendicular to the optical fibre axis. Dashed line represents the core boundary.*



Waveguide Dispersion

Waveguide (or chromatic) dispersion D_w is the spreading out of light in the fibre is due to different wavelengths of light travelling at different speeds [2]. Using the finite difference method,

$$\left[\frac{\partial^2 n_{eff}}{\partial \lambda^2}\right]_w \approx \frac{n_{eff}(\lambda-h)-2n_{eff}(\lambda)+n_{eff}(\lambda+h)}{h^2},$$

and taking $\lambda = 820nm,\ h = 20nm,$

$$D_w = -\frac{\lambda}{c}\left[\frac{\partial^2 n_{eff}}{\partial \lambda^2}\right]_w \approx 13.3\ ps/nm \cdot km$$

Determining n_{eff}

The proportion of energy in the core and cladding were determined through summing their respective electric field intensities. The ratio between the two represents the effective index of a mode. We found n_{eff} =1.4593 which was significantly above our calculated value of 1.4556, implying a greater amount of energy in the core than anticipated. This may result from the lack of an infinite limit in our intensity sum calculations, generating a lower sum of total energy transmitted, and a high ratio of energy in the core.

Conclusion

We investigated the EH11 mode of light in an optical fibre. The {r,φ,z} E-fields were plotted by solving Bessel functions and consequently the intensity distribution of the mode was visualised. The waveguide dispersion of this fibre was found to be 13.3 ps/nmkm, which agreed with typical values [3]. However, multimode fibres typically have large radii (~25 µm) and support thousands of optical modes, and thus our fibre of 5.1 µm radius may not be ideal for practical use.

REFERENCES: [1] Dunsby C, et al. (2022) *Optical Communications Course Notes (Lecture 3)*. Imperial College London Department of Physics. [2] Méndez, A. & Morse, T.F. (2007) *Specialty optical fibers handbook*. 1st edition. Oxford, Academic Press. [3] Fosco Connect. *Dispersion in Single-Mode FIbres* (2022) Available at: https://www.fiberoptics4sale.com/blogs/wave-optics/dispersion-in-single-mode-fibers (Accessed: 10/12/2022).