HW1 STT 465
(MSU, Fall, 2015)

Return a pdf (typed or hand-written, but readable) of your HW to gustavoc@msu.edu no later than Wednesday, Sept 16th 10:00 am.

Question 1

Consider the genetics-disease model discussed in our 2nd lecture

Genotype	Disease Status
AA	H (healthy)
AB	H
BB	D (disease)

This model is called a recessive model because the effect of the 'disease' allele (B) is only expressed in the homozygous genotype (BB). Heterozygous (AB) have the same phenotype as that of the dominant homozygous (AA).

Mating: when two individuals mate, each produces a gamete that carries one allele (either A or B in this bi-allelic model). We assume that individuals mate at random (i.e. the likelihood of a mating is not affected by the genotype) and at the meiosis (the process that produces gametes) alleles are also sampled at random. Consequently individuals with genotypes AA, AB and BB will give A gametes with probabilities 1, 0.5 and 0 and B gametes with probabilities 0, 0.5 and 1, respectively. With this we have that the probabilities of the genotypes can be derive from the allele frequencies

p(AA)=P(A)P(A), p(AB|BA)=2p(A)p(B), p(BB)=p(B)p(B) (Hardy-Weinberg equilibrium).

Considering the above assumptions:

1.1. Derive and report the conditional probability of the health status (Si) of a progeny given the genotype of one of the parents (G0), that is p(Si|G0). Note: the second parent is assumed to be drawn at random from the population.

1.2. Derive and present the joint distribution of the health status of two half sibs (i.e., a pair of individuals that share one parent), that is p(S1,S2|G0).
Hint: assume conditional independence of the genotype of the two offspring given the parent's genotype, that is p(G1,G2|G0)=p(G1|G0)p(G2|G0).

1.3. Derive and report the marginal joint probability of the disease status of two half sibs, that is p(S1,S2).

1.4. Derive and report the conditional probability of S2 given S1, that is p(S1|S2).

Question 2

2.1. Are S1 and S2 conditionally (i.e. given the genotype of the known parent) independent?

2.2. Are S1 and S2 IID?

2.3. Are S1 and S2 exchangeable?

2.4. Bayesian learning: how does the probability of disease of S2 changes if you know that S1=D)?