HW1 STT 465
(MSU, Fall, 2018)

Due Sunday, Sept 16th (D2L)

Topics: marginal and conditional probability, Bayes theorem, exchangeability.

Background. Consider a disease controlled by a single DNA locus with two possible alleles A and B. In this setting any individual can have one of the three following genotypes: AA, AB and BB.

The following table give the probability of disease p(Di=1 | Gi) for each of the three genotypes

Genotype (Gi)	p(Di=1|Gi)
AA	0.0
AB	0.1
BB	0.9

If θ denotes the frequency of allele A, under random mating, the genotype frequencies in the population are: $p(AA)= \theta^2$, $p(BB)= (1-\theta)^2$, $p(AB)= 2\theta(1-\theta)$

Question 1

In population I $\theta=0.9$, compute and report the prevalence of disease in this population.

Question 2

An individual in population I has developed the disease, what is the probability that the genotype of that individual is AA, AB or BB?

Question 3

In population II the frequency of allele A is 0.95. An individual is healthy, what is the probability that this individual comes from population I (assume that, a priori, individuals are equally likely to come from population 1 or 2).

Question 4.

Provide frequencies for the joint distribution of the two Bernoulli random variables (i.e., probabilities in a 2x2 contingency table) that:

- satisfy IID (identically and independently distributed),
- satisfy exchangeability but not IID
- do not satisfy IID and are not exchangeable.

Explain your reasoning.

Question 5: Marginal Vs Conditional Independence

Consider a system of three Bernoulli random variables (X,Y,Z). P(Z=1)=0.6. The following tables give the conditional distributions p(X,Y|Z)

p(X,Y|Z=0)

	Y=0	Y=1
X=0	0.06	0.24
X=1	0.14	0.56

p(X,Y|Z=1)

	Y=0	Y=1
X=0	0.12	0.28
X=1	0.18	0.42

5.a) Are (X,Y) conditionally independent?

5.b) Are (X,Y) independent?

5.c) Are (X,Y) exchangeable?